Extracting sigma_effective from the LHCb double-charm measurement

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with

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[M. H. Seymour, and AS, arXiv:1308.6749]

[M. Bahr, M. Myska, M.H. Seymour, AS, JHEP 1303 (2013)]

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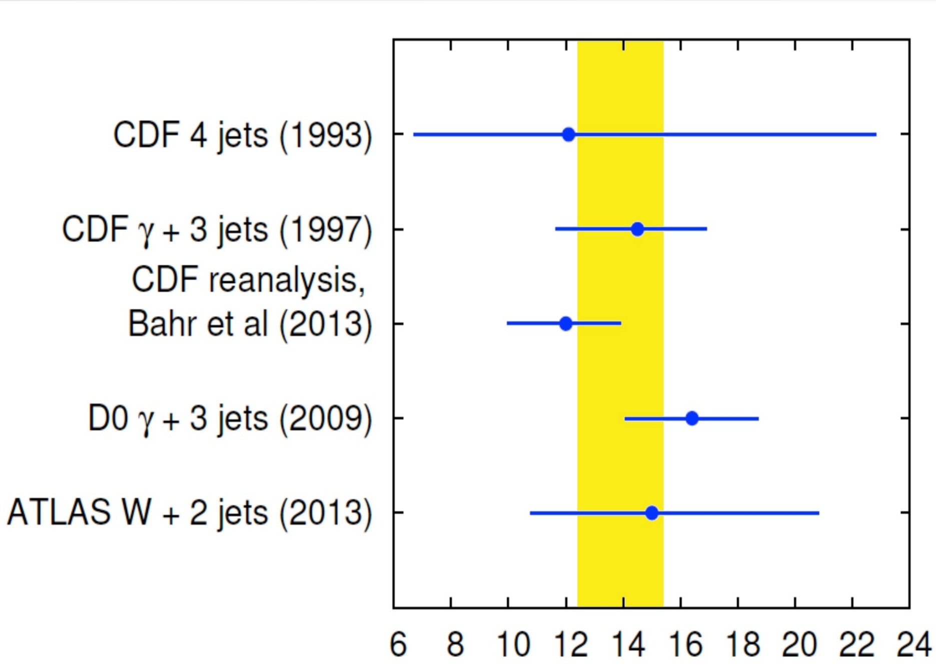
Outline

1. Motivation
2. Single- and double-inclusive cross sections
3. Extracting sigma_eff from CDF measurement
4. Extracting sigma_eff from LHCb measurement
5. Outlook and conclusions

Motivation

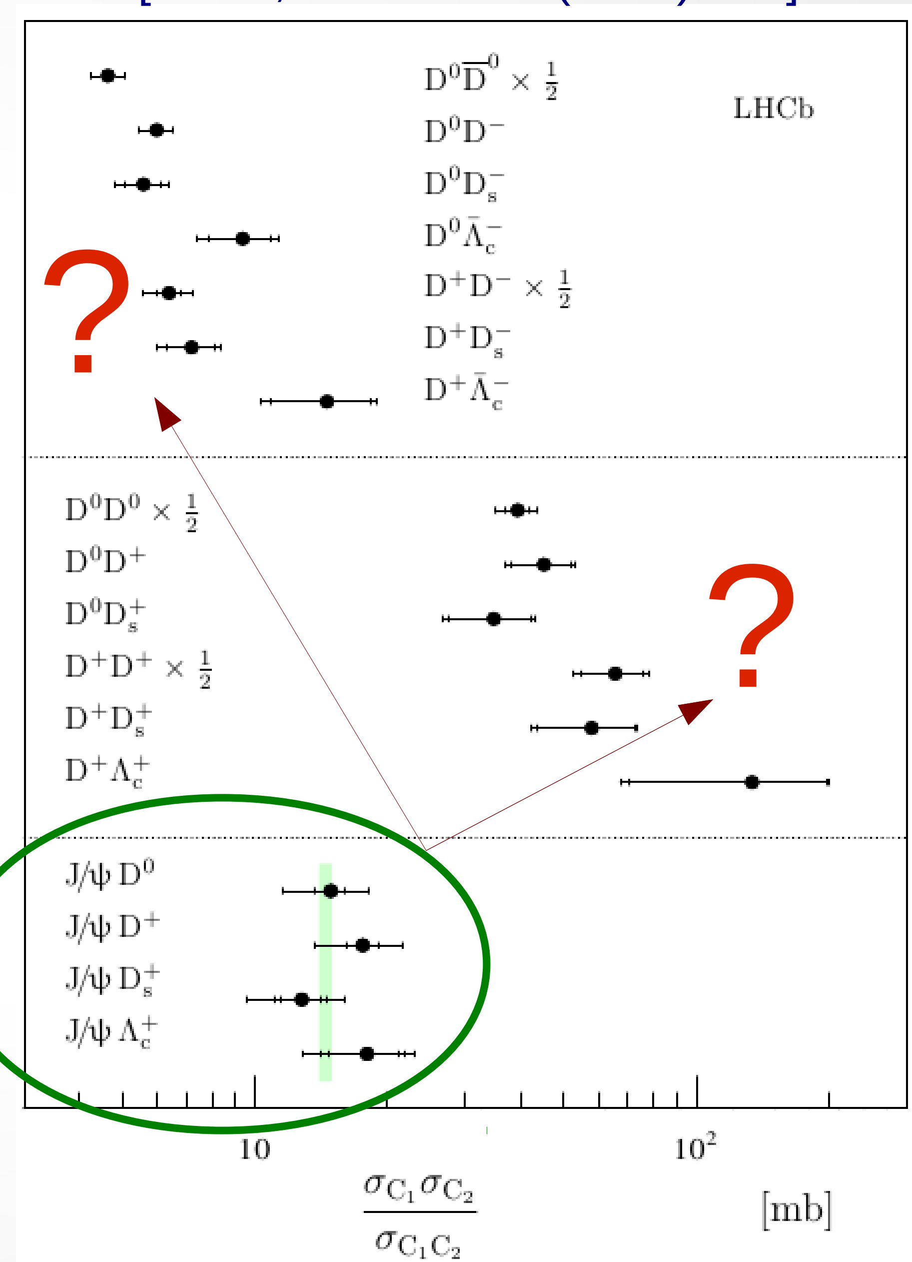


$$\sigma_{ii} = \frac{\sigma_i^2}{2\sigma_{eff}}, \qquad \sigma_{ij} = \frac{\sigma_i \sigma_j}{\sigma_{eff}}$$

Agreement with the other measurements.

Sigma eff if defined in the inclusive way should be independent of the process!

[LHCb, JHEP 1206 (2012) 141]



Observation of double charm production involving open charm

Inclusive Cross Sections

Reminder from Mike Seymour's overview talk:

Defining cross section inclusively

Single-particle inclusive cross sections:

$$\sigma_{incl\,\mathrm{i}} \equiv \frac{N_\mathrm{i}}{\mathcal{L}} \iff \sigma_{incl\,\mathrm{i}} = \sum_n n\,\sigma_{n\mathrm{i}}$$

Scatters Poissonian at fixed impact parameter:

$$\sigma_{n\mathrm{i}} = \int \mathrm{d}^2 b\, \frac{(\sigma_\mathrm{i} A(b))^n}{n!}\, \mathrm{e}^{-\sigma_\mathrm{i} A(b)}$$

Reproduces partonic cross section:

$$\begin{aligned}\sigma_{incl\,\mathrm{i}} &= \sum_n n \int \mathrm{d}^2 b\, \frac{(\sigma_\mathrm{i} A(b))^n}{n!}\, \mathrm{e}^{-\sigma_\mathrm{i} A(b)} = \int \mathrm{d}^2 b \left[\sum_n n \frac{(\sigma_\mathrm{i} A(b))^n}{n!}\right] \mathrm{e}^{-\sigma_\mathrm{i} A(b)} \\ &= \int \mathrm{d}^2 b\, \sigma_\mathrm{i} A(b) = \sigma_\mathrm{i}\end{aligned}$$

Inclusive Cross Sections

Double-particle inclusive cross section is given by:

$$\sigma_{incl\mathrm{ii}} = \sum_n \tfrac{1}{2} n(n-1) \sigma_{n\mathrm{i}}$$

In the eikonal model, the double-particle inclusive cross section:

$$\sigma_{incl\mathrm{ii}} = \sum_n \tfrac{1}{2} n(n-1) \int \mathrm{d}^2 b \, \frac{(\sigma_\mathrm{i} A(b))^n}{n!} \mathrm{e}^{-\sigma_\mathrm{i} A(b)} = \frac{1}{2} \sigma_\mathrm{i}^2 \int \mathrm{d}^2 b \, A(b)^2$$

In similar way cross sections for two different particle types i and j:

$$\sigma_{incl\mathrm{ij}} = \sum_{n,m} n \, m \, \sigma_{n\mathrm{i},m\mathrm{j}} = \sigma_\mathrm{i} \, \sigma_\mathrm{j} \int \mathrm{d}^2 b \, A(b)^2$$

The effective cross section:

Process independent

$$\sigma_{eff} = \frac{\sigma_{incl\mathrm{i}}^2}{2\sigma_{incl\mathrm{ii}}} = \frac{\sigma_{incl\mathrm{i}} \, \sigma_{incl\mathrm{j}}}{\sigma_{incl\mathrm{ij}}} = \frac{1}{\int \mathrm{d}^2 b \, A(b)^2}$$

Alternative Inclusive Cross Sections

Single-particle: cross sections for events that contain one or more i

$$\sigma_{\geq 1\mathrm{i}} \equiv \sum_{n=1} \sigma_{n\mathrm{i}} = \sum_{n=1} \int \mathrm{d}^2 b \, \frac{(\sigma_\mathrm{i} A(b))^n}{n!} \, \mathrm{e}^{-\sigma_\mathrm{i} A(b)}$$

$$= \int \mathrm{d}^2 b \left[\sum_{n=1} \frac{(\sigma_\mathrm{i} A(b))^n}{n!} \right] \mathrm{e}^{-\sigma_\mathrm{i} A(b)} = \int \mathrm{d}^2 b \left(1 - \mathrm{e}^{-\sigma_\mathrm{i} A(b)} \right)$$

Double-particle: cross section events containing two or more i's:

$$\sigma_{\geq 2\mathrm{i}} \equiv \sum_{n=2} \sigma_{n\mathrm{i}} = \int \mathrm{d}^2 b \left(1 - \mathrm{e}^{-\sigma_\mathrm{i} A(b)} - \sigma_\mathrm{i} A(b) \mathrm{e}^{-\sigma_\mathrm{i} A(b)} \right)$$

The effective cross section:

$$\sigma_{eff} = \frac{\sigma_{\geq 1\mathrm{i}}^2}{2\sigma_{\geq 2\mathrm{i}}} = \frac{\left[\int \mathrm{d}^2 b \left(1 - \mathrm{e}^{-\sigma_\mathrm{i} A(b)} \right) \right]^2}{2 \int \mathrm{d}^2 b \left(1 - \mathrm{e}^{-\sigma_\mathrm{i} A(b)} - \sigma_\mathrm{i} A(b) \mathrm{e}^{-\sigma_\mathrm{i} A(b)} \right)}$$

$$\sigma_{eff} = \frac{\sigma_{\geq 1\mathrm{i}} \sigma_{\geq 1\mathrm{j}}}{\sigma_{\geq 1\mathrm{i}, \geq 1\mathrm{j}}} = \frac{\left[\int \mathrm{d}^2 b \left(1 - \mathrm{e}^{-\sigma_\mathrm{i} A(b)} \right) \right] \left[\int \mathrm{d}^2 b \left(1 - \mathrm{e}^{-\sigma_\mathrm{j} A(b)} \right) \right]}{\int \mathrm{d}^2 b \left(1 - \mathrm{e}^{-\sigma_\mathrm{i} A(b)} \right) \left(1 - \mathrm{e}^{-\sigma_\mathrm{j} A(b)} \right)}$$

Process dependent

Exclusive definition

The exclusive single-i cross section:

$$\sigma_{1\mathrm{i}} = \int \mathrm{d}^2 b\, \sigma_{\mathrm{i}} A(b)\, \mathrm{e}^{-\sigma_{\mathrm{i}} A(b)}$$

Exclusive Double-particle cross-sections:

$$\sigma_{2\mathrm{i}} = \int \mathrm{d}^2 b\, \tfrac{1}{2} (\sigma_{\mathrm{i}} A(b))^2\, \mathrm{e}^{-\sigma_{\mathrm{i}} A(b)} \qquad \sigma_{1\mathrm{i},1\mathrm{j}} = \int \mathrm{d}^2 b\, \sigma_{\mathrm{i}} \sigma_{\mathrm{j}}\, A(b)^2\, \mathrm{e}^{-(\sigma_{\mathrm{i}}+\sigma_{\mathrm{j}}) A(b)}$$

The effective cross section:

$$\sigma_{\mathit{eff}} = \frac{\sigma_{1\mathrm{i}}^2}{2\sigma_{2\mathrm{i}}} = \frac{\left[\int \mathrm{d}^2 b\, A(b)\, \mathrm{e}^{-\sigma_{\mathrm{i}} A(b)}\right]^2}{\int \mathrm{d}^2 b\, A(b)^2\, \mathrm{e}^{-\sigma_{\mathrm{i}} A(b)}}$$

$$\sigma_{\mathit{eff}} = \frac{\sigma_{1\mathrm{i}} \sigma_{1\mathrm{j}}}{\sigma_{1\mathrm{i},1\mathrm{j}}} = \frac{\left[\int \mathrm{d}^2 b\, A(b)\, \mathrm{e}^{-\sigma_{\mathrm{i}} A(b)}\right] \left[\int \mathrm{d}^2 b\, A(b)\, \mathrm{e}^{-\sigma_{\mathrm{j}} A(b)}\right]}{\int \mathrm{d}^2 b\, A(b)^2\, \mathrm{e}^{-(\sigma_{\mathrm{i}}+\sigma_{\mathrm{j}}) A(b)}}$$

} Process dependent

Extracting sigma_effective from the CDF gamma+3jets measurement

- CDF definition of σ_{eff} required exactly two scatters, the standard definition is in the inclusive manner

$$\sigma^{ex}_{ab;2} = \frac{\sigma_a \sigma_b}{\sigma_{eff(CDF)}} \Rightarrow \sigma^{in}_{ab;2} = \frac{\sigma_a \sigma_b}{\sigma_{eff}}$$

$$\sigma_{eff(CDF)} = (14.5 \pm 1.7^{+1.7}_{-2.3})\ \text{mb}$$

← process dependent

- [Treleani, Phys. Rev. D 76 (2007) 076006] $\sigma_{eff} \approx 10.3\ \text{mb}$
- [M. Bähr, M. Myska, M. H. Seymour, and AS, JHEP 1303 (2013)]

$$\sigma_{eff} = \left(12.0 \pm 1.3^{+1.3}_{-1.5}\right)\ \text{mb}$$

← process independent

Useful for constraining models

Systematic uncertainty was significantly reduced!

(Uncertainty caused by removal of triple parton scattering was avoided)

One of the most precise results of the sigma effective.

[Talk by M. Myska at MPI@LHC 2012]

Extracting sigma_effective from the LHCb measurement

[LHCb, JHEP 1206 (2012) 141]

Let me stress that we do not question the LHCb measurements of the cross sections themselves, only the way they combine them to extract sigma effective.

Extracting sigma_effective from the LHCb measurement

Open charm cross sections:

- the previous discussion can be applied directly to the partonic cross sections to produce charm quark pairs.
- however, experiments observe the hadrons they fragment to.

We assume that the probability that a given charm quark produces a charmed hadron of a given species within the fiducial region of an experiment is a fixed number $p_{\mathrm{D}}^{\mathrm{c}}$ where $\mathrm{D} = \{\mathrm{D}^0, \mathrm{D}^+, \mathrm{D}_\mathrm{s}^+, \Lambda_\mathrm{c}\}$

We assume that $\mathrm{p}_{\mathrm{D}}^{\bar{\mathrm{c}}} = 0$

Within these assumptions, and using the single- and double-inclusive charm quark cross sections, it is straightforward to calculate the single- and double-inclusive charmed hadron cross sections.

$$\sigma_{\mathrm{D}} = p_{\mathrm{D}}^{\mathrm{c}} \sigma_{\mathrm{c}} \, , \sigma_{\mathrm{DD}} = (p_{\mathrm{D}}^{\mathrm{c}})^2 \sigma_{\mathrm{cc}}$$

$$\sigma_{\mathrm{D}_1\mathrm{D}_2} = 2 p_{\mathrm{D}_1}^{\mathrm{c}} p_{\mathrm{D}_2}^{\mathrm{c}} \sigma_{\mathrm{cc}}$$

Not true for the alternative inclusive or exclusive cross section definitions!

Extracting sigma_effective from the LHCb measurement

Therefore , we can use measurements of single- and double-charmed hadron production to extract sigma effective

$$\frac{\sigma_{\mathrm{D}}^2}{2\sigma_{\mathrm{DD}}} = \frac{(p_{\mathrm{D}}^{\mathrm{c}})^2\sigma_{\mathrm{c}}^2}{2(p_{\mathrm{D}}^{\mathrm{c}})^2\sigma_{\mathrm{cc}}} = \sigma_{eff}$$

$$\frac{\sigma_{\mathrm{D}_1}\sigma_{\mathrm{D}_2}}{\sigma_{\mathrm{D}_1\mathrm{D}_2}} = \frac{p_{\mathrm{D}_1}^{\mathrm{c}}p_{\mathrm{D}_2}^{\mathrm{c}}\sigma_{\mathrm{c}}^2}{2p_{\mathrm{D}_1}^{\mathrm{c}}p_{\mathrm{D}_2}^{\mathrm{c}}\sigma_{\mathrm{cc}}} = \sigma_{eff}$$

Extracting sigma_effective from the LHCb measurement

Since QCD is charge-conjugation-symmetric we might expect

$$p_{\mathrm{D}}^{\mathrm{c}} = p_{\overline{\mathrm{D}}}^{\bar{\mathrm{c}}}$$

Total number of D hadrons is expected to be the same as the number of Dbar hadrons. The numbers within a given fiducial volume are not necessarily the same. LHCb did not notice any difference between charge conjugate modes.

$$\begin{aligned} \sigma_{\mathrm{D}} &= \sigma_{\overline{\mathrm{D}}}\,, \\ \sigma_{\mathrm{DD}} &= \sigma_{\overline{\mathrm{D}}\,\overline{\mathrm{D}}}\,, \\ \sigma_{\mathrm{D}_1\mathrm{D}_2} &= \sigma_{\overline{\mathrm{D}}_1\overline{\mathrm{D}}_2}\,. \end{aligned}$$

LHCb used these results to effectively double their data set and defined

$$\begin{aligned} \sigma_{\mathrm{D,LHCb}} &\equiv \sigma_{\mathrm{D}} + \sigma_{\overline{\mathrm{D}}}\,, \\ \sigma_{\mathrm{DD,LHCb}} &\equiv \sigma_{\mathrm{DD}} + \sigma_{\overline{\mathrm{D}}\,\overline{\mathrm{D}}}\,, \\ \sigma_{\mathrm{D}_1\mathrm{D}_2,\mathrm{LHCb}} &\equiv \sigma_{\mathrm{D}_1\mathrm{D}_2} + \sigma_{\overline{\mathrm{D}}_1\overline{\mathrm{D}}_2}\,. \end{aligned}$$

Extracting sigma_effective from the LHCb measurement

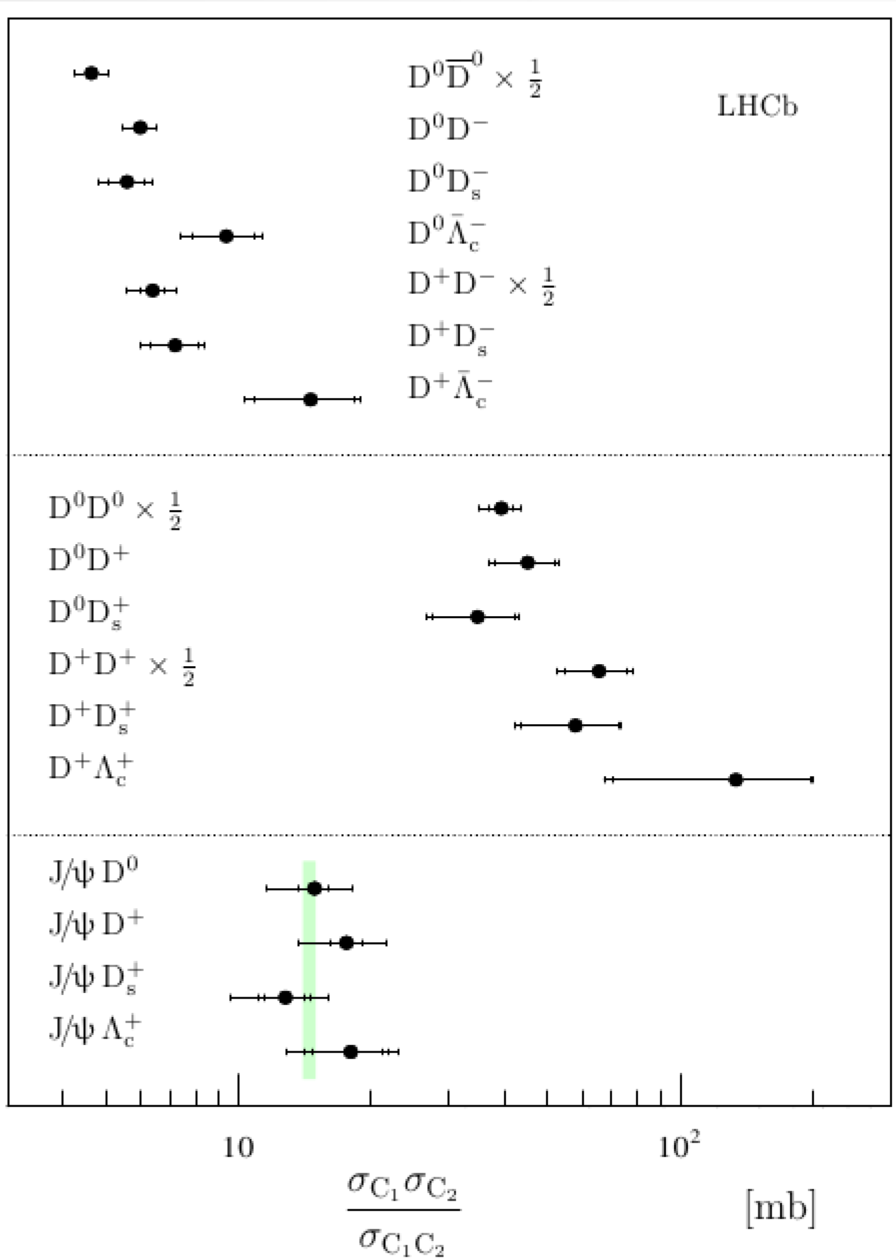
It is still possible to use these charge-conjugation-summed cross sections to extract seff but one must be careful to include an additional factor of 2:

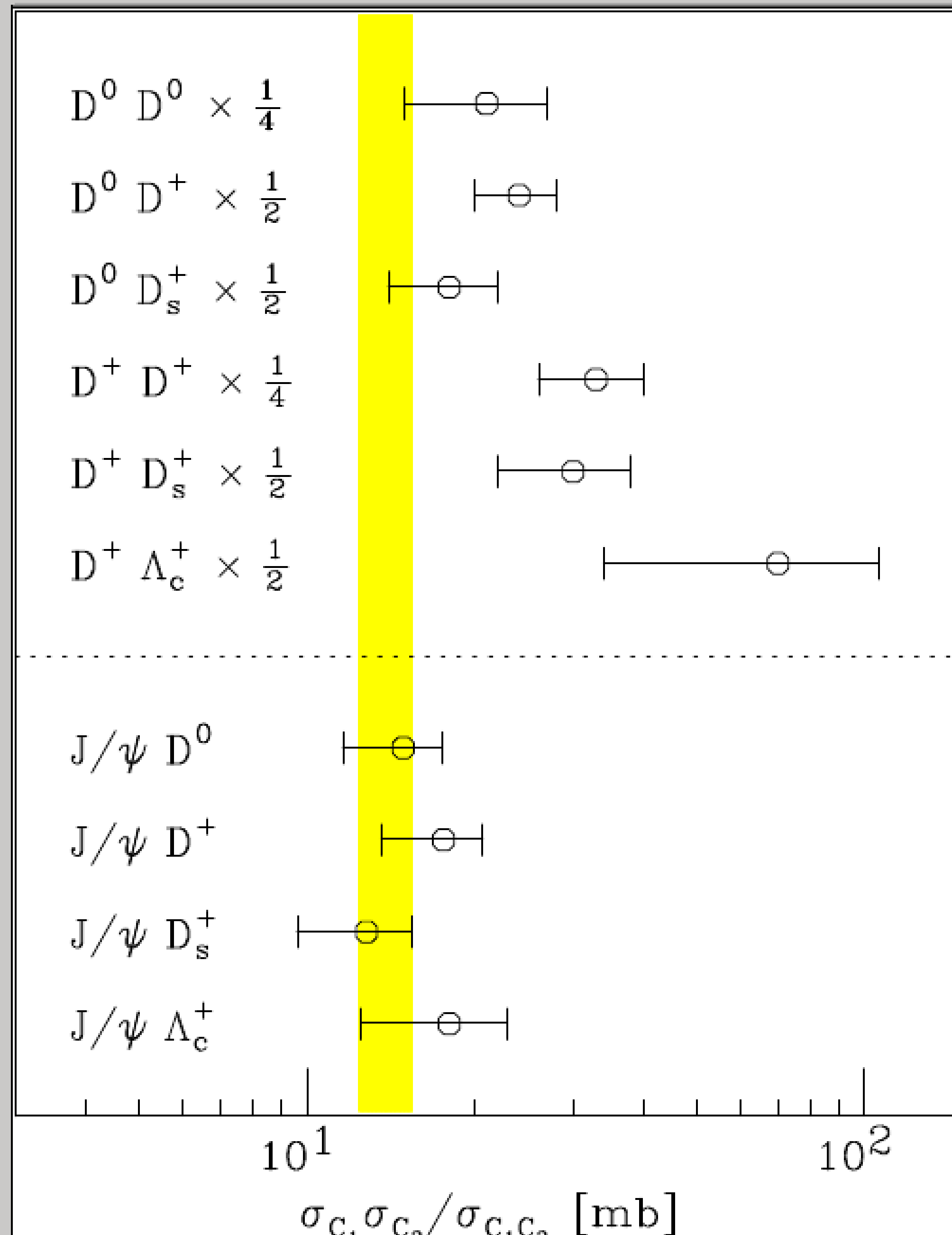
$$\frac{\sigma^2_{\mathrm{D,LHCb}}}{\mathbf{2}\times 2\sigma_{\mathrm{DD,LHCb}}} = \frac{(\sigma_{\mathrm{D}} + \sigma_{\overline{\mathrm{D}}})^2}{4(\sigma_{\mathrm{DD}} + \sigma_{\overline{\mathrm{D}}\,\overline{\mathrm{D}}})} = \frac{4\sigma^2_{\mathrm{D}}}{8\sigma_{\mathrm{DD}}} = \sigma_{eff}\,,$$

$$\frac{\sigma_{\mathrm{D}_1,\mathrm{LHCb}}\sigma_{\mathrm{D}_2,\mathrm{LHCb}}}{\mathbf{2}\times \sigma_{\mathrm{D}_1\mathrm{D}_2,\mathrm{LHCb}}} = \frac{(\sigma_{\mathrm{D}_1} + \sigma_{\overline{\mathrm{D}}_1})(\sigma_{\mathrm{D}_2} + \sigma_{\overline{\mathrm{D}}_2})}{2(\sigma_{\mathrm{D}_1\mathrm{D}_2} + \sigma_{\overline{\mathrm{D}}_1\overline{\mathrm{D}}_2})} = \frac{4\sigma_{\mathrm{D}_1}\sigma_{\mathrm{D}_2}}{4\sigma_{\mathrm{D}_1\mathrm{D}_2}} = \sigma_{eff}\,.$$

- It appears to us that LHCb have not included this factor of two and hence that their extracted values of σ_eff are too large by a factor of two.
- Charmonium channels are not subject to this factor of two. Since charmonium is self-conjugate, there is no summation to be done.
- The single charmonium, single-open charm channel contains a factor of two in the numerator but also in the denominator so it cancels.

Extracting sigma_effective from the LHCb measurement





Conclusions

- We have emphasized the importance of properly-defined inclusive cross sections – example of CDF measurement.
- We showed that LHCb extraction of σ_eff from their double-inclusive open-charm data is too large by a factor of 2.
- The additional factor of 2 has brought the results closer together.
- But results for double open charm are still significantly higher then for the other processes.
- Several issues that could be worthy of further study: triple-charm production; differences between charmed and anticharmed hadron distributions; and correlations between charm-anticharm pairs.

Opposite-sign charmed hadron pairs

Even within the assumption:

$$p_{\mathrm{D}}^{\bar{\mathrm{c}}} = p_{\overline{\mathrm{D}}}^{\mathrm{c}} = 0 \qquad \mathrm{D}\overline{D} \text{ and } \mathrm{D}_1\overline{\mathrm{D}}_2$$

pairs can come from a single ccbar pair, which have equal and opposite transverse momenta and correlated rapidities.

Therefore we cannot consider the probabilities of charm quarks to produce charmed hadrons within the fiducial region as uncorrelated.

Correlation coefficient C, such that the probabilities that a ccbar pair from a single partonic scattering produces a Ddbar pair within the fiducial region is $\mathrm{C}(\mathrm{p}_{\mathrm{D}}^{\mathrm{c}})^2$

We can show:

$$\begin{aligned} \sigma_{\mathrm{D}\overline{\mathrm{D}}} &= (p_{\mathrm{D}}^{\mathrm{c}})^2 \left(\mathcal{C}\sigma_{\mathrm{c}} + 2\sigma_{\mathrm{cc}}\right), \\ \sigma_{\mathrm{D}_1\overline{\mathrm{D}}_2} &= p_{\mathrm{D}_1}^{\mathrm{c}} p_{\mathrm{D}_2}^{\mathrm{c}} \left(\mathcal{C}\sigma_{\mathrm{c}} + 2\sigma_{\mathrm{cc}}\right). \end{aligned}$$

Opposite-sign charmed hadron pairs

$$\frac{\sigma^2_{\mathrm{D,LHCb}}}{2\sigma_{\mathrm{D\overline{D},LHCb}}} = \frac{4\sigma^2_{\mathrm{D}}}{2\sigma_{\mathrm{D\overline{D}}}} = \frac{2\sigma_{\mathit{eff}}}{\mathcal{C}\sigma_{\mathit{eff}}/\sigma_{\mathrm{c}} + 1},$$

$$\frac{\sigma_{\mathrm{D_1,LHCb}}\sigma_{\mathrm{D_2,LHCb}}}{\sigma_{\mathrm{D_1\overline{D}_2,LHCb}}} = \frac{4\sigma_{\mathrm{D_1}}\sigma_{\mathrm{D_2}}}{2\sigma_{\mathrm{D_1\overline{D}_2,LHCb}}} = \frac{2\sigma_{\mathit{eff}}}{\mathcal{C}\sigma_{\mathit{eff}}/\sigma_{\mathrm{c}} + 1}.$$

$\mathcal{C}\ and\ \sigma_{\mathit{eff}}/\sigma_{\mathrm{c}}$ are expected to be larger than 1.

Therefore, these results are expected to be smaller than \seff but without further studies to extract the values of these constants, we cannot quantify the expected size.

Backup

$$\sigma_{\geq 1\mathrm{i},\geq 1\mathrm{j}} \equiv \sum_{n,m=1} \sigma_{n\mathrm{i},m\mathrm{j}} = \int \mathrm{d}^2 b \left(1 - \mathrm{e}^{-\sigma_\mathrm{i} A(b)}\right) \left(1 - \mathrm{e}^{-\sigma_\mathrm{j} A(b)}\right)$$