Double Parton Correlations and Constituent Quark Models

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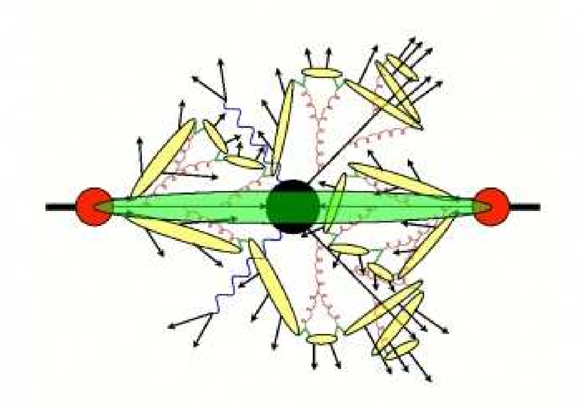
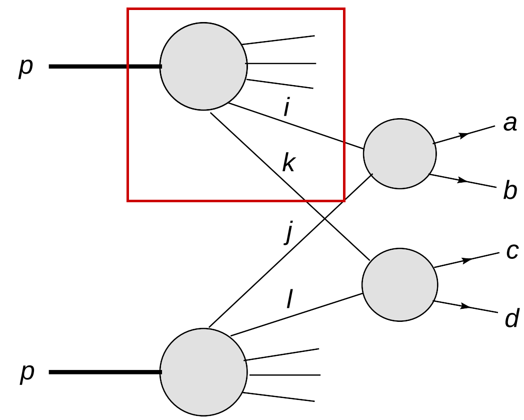
Outline

- **Are double parton correlations (DPCs) relevant in double parton scattering (DPS) @LHC?**
  Check within quark models
- **Model calculations of parton distributions** *in the valence region*: **virtues and caveats**
- **Double parton correlations in constituent quark models**
  (M. Rinaldi, S.S. and V.Vento, PRD 87, 114021 (2013))
- **Work in progress: taking Relativity into account within a Light-Front approach**
- **Conclusions**

DPS and Double Parton Distributions (dPDFs)

- In an LHC collision, MPI can occur:





- The DPS cross section is written **(N. Paver, D. Treleani, Nuovo Cimento 70A, 215 (1982))**

$$\mathrm{d}\sigma = \frac{1}{S}\sum_{i,j,k,l}\int \mathrm{d}^2\vec{z}_\perp\, F_{ik}(x_1,x_2,\vec{z}_\perp,\mu)F_{jl}(x_3,x_4,\vec{z}_\perp,\mu) \times\hat{\sigma}_{ij}(x_1x_3\sqrt{s},\mu)\hat{\sigma}_{kl}(x_2x_4\sqrt{s},\mu)$$

x_i = momentum fraction carried by the parton inside the hadron;
μ = momentum scale; $z_\perp$ = transverse distance between the two partons

- DPS: a background to be taken into account for fundamental studies;
from our point of view: the dPDF $F_{ik}(x_1,x_2,\vec{z}_\perp,\mu)$ in one of the protons is very interesting!

dPDFs and correlations

F_{ij} is usually factorized as follows ($(x_1, x_2) - z_\perp$ factorization):

$$F_{ij}(x_1, x_2, \vec{z}_\perp, \mu) = F_{ij}(x_1, x_2, \mu) T(\vec{z}_\perp, \mu)$$

AND (x_1, x_2 factorization):

$$F_{ij}(x_1, x_2, \mu) = \underbrace{\overbrace{q_i(x_1, \mu)}^{\text{PDF}} \, q_j(x_2, \mu)}_{\text{NO CORRELATION ANSATZ}} \, \theta(1 - x_1 - x_2)(1 - x_1 - x_2)^n$$

NO CORRELATION ANSATZ

This means that correlations between the quarks in the proton are neglected.

- Is this a safe approximation? Important for LHC fundamental studies
- Can one understand better the proton structure through MPI observation?

Double Parton Correlations (DPCs)

- In principle, correlations are there
- We are not alone in addressing this issue
  (see Markus' and almost all the other talks. Many published papers: Korotkikh and Snigirev (2004), Gaunt and Stirling (2010), Diehl and Schäfer (2011), Snigirev (2011), Blok et al. (2012), Schweitzer, Strikman and Weiss (2013)...)
- DPCs cannot be studied from first principles:
  dPDFs are non-perturbative quantities
- Our contribution: a quark model analysis as a possible useful tool
- A first important investigation, which motivated our work, has been presented in a modified version of the MIT bag model
  (H.-M. Chang, A.V. Manohar, and W.J. Waalewijn, PRD 87, 034009 (2013))
  (In its simplest version, the MIT bag model is an independent particle model and no correlations are found)

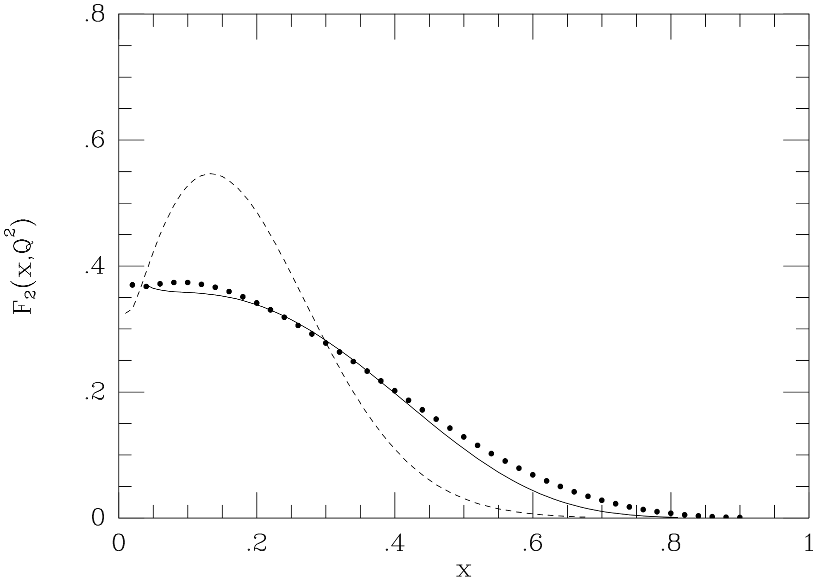
DPCs in Constituent Quark Models (CQM)

- In a potential model, effective particles are strongly bound and correlated.
  No modifications of the model properties are necessary to describe correlations

- In this sense, CQM are a proper framework to describe DPCs **BUT** their predictions are reliable in the valence region, while LHC data, for the moment, are available only for much lower values of Bjorken x
- At very low x, due to the large population of partons, the role of correlations may be less relevant **BUT** there is no quantitative theoretical estimate available

Model calculations of PDFs in the valence region - I

- CQM calculations have been proven to be able to reproduce the gross-features of experimental PDFs in the valence region:



(S.S., V. Vento and M. Traini, PLB 421 (1998) 64)

Similar expectations motivate the present investigation of dPDFs

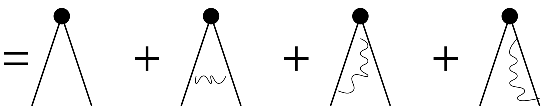
- Calculations are straightforward and results can be quite general.
  In DIS Physics, CQM calculations are a useful tool for the interpretation of data and for the planning of measurements of unknown quantities (e.g., TMDs in SiDIS, GPDs in DVCS...)

Model calculations of PDFs in the valence region - II

- In order to consistently compare data of twist-2 PDFs with the predictions of a CQM, one has to follow a 2-steps procedure:
  (firstly suggested by R.L. Jaffe and G.G. Ross, PLB 398 (1980) 313)
  1. evaluate in the model the twist-2 part of the corresponding observable, which has to be related to a low momentum scale, μ_o^2
  2. perform a perturbative QCD evolution to the DIS experimental scale, Q^2

$$f(x,\mu_o^2) \xrightarrow{R.G.E.,\ p.\ QCD} f(x,Q^2),\ DIS$$

Twist-2

$L.O. =$ 

$+\ N.L.O.\ (2\ loops)$

- *Caveat*: in the simplest CQM picture, all the gluons and sea quarks are perturbatively generated

Model calculations of quark-quark dPDFs

- quark-quark dPDFs are defined through Light-Cone quantized states and fields (see, e.g., **M. Diehl, D. Ostermeier, A. Schäfer JHEP 03 (2012) 089**)
- A Non Relativistic (NR) reduction allows one to calculate them, in momentum space, in terms of intrinsic wave functions (WFs):

$$F_{12}(x_1, x_2, \vec{k}_\perp) = 3 \int d\vec{k}_1 d\vec{k}_2$$
$$\times \; \Psi^* \left(\vec{k}_1 + \frac{\vec{k}_\perp}{2}, \vec{k}_2 - \frac{\vec{k}_\perp}{2} \right) \hat{P}_{q_1}(1) \hat{P}_{q_2}(2) \, \Psi \left(\vec{k}_1 - \frac{\vec{k}_\perp}{2}, \vec{k}_2 + \frac{\vec{k}_\perp}{2} \right)$$
$$\times \; \delta \left(x_1 - \frac{k_1^+}{P^+} \right) \delta \left(x_2 - \frac{k_2^+}{P^+} \right)$$

$\hat{P}_{u(d)}(i) = \frac{1 \pm \tau_3(i)}{2}$ is a flavor projector, $a^\pm = a_0 \pm a_3$
(expression seen also in **A.V. Manhoar and W.J. Waalewijn, PRD 85 114009 (2012)**)

- We need now to choose the WF corresponding to a suitable CQM and to perform the calculation

The model of Isgur and Karl (IK)

IK is a well known model based on a One Gluon Exchange (OGE) correction to the H.O., generating a hyperfine interaction which breaks SU(6). Nucleon state (up to the 2^{nd} energy shell):

$$|N\rangle = a|^2S_{1/2}\rangle_S + b|^2S'_{1/2}\rangle_S + c|^2S_{1/2}\rangle_M + d|^4D_{1/2}\rangle_M$$

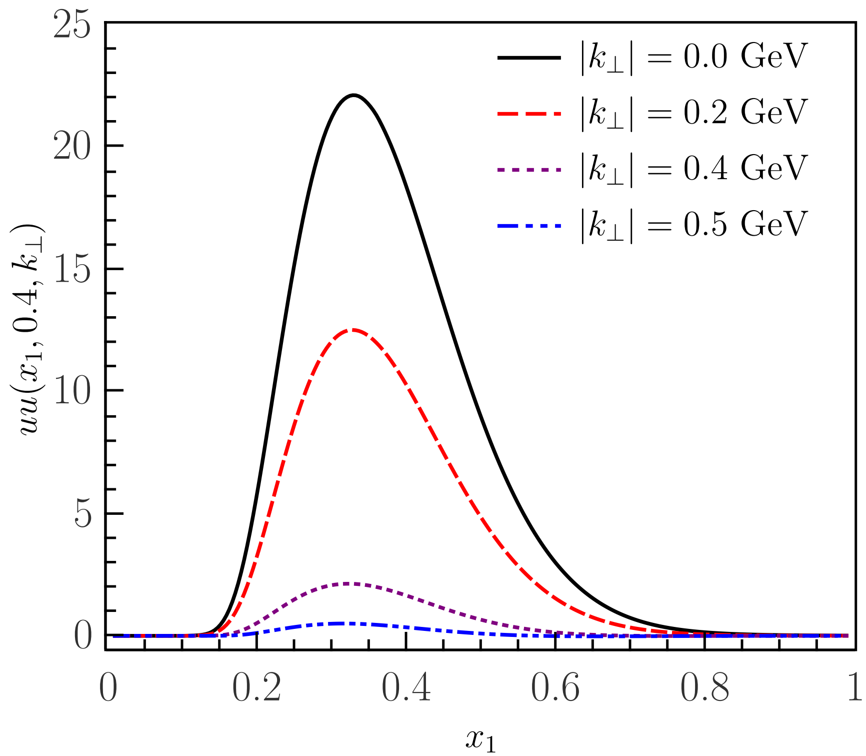
Notation: $|^{2S+1}X_J\rangle_t$; $t = A, M, S$ = symmetry type
From spectroscopy: $a = 0.931$, $b = -0.274$, $c = -0.233$, $d = -0.067$
If $a = 1$, $b = c = d = 0$, the pure H.O. is recovered.

IK is a suitable framework for a first CQM calculation of DPCs:

- IK is the prototype of any other CQM; low energy properties of the nucleon, such as the spectrum and the electromagnetic form factors at small momentum transfer are reproduced;
- Gross features of the standard PDFs are reproduced.

The model results correspond to a low momentum scale (hadronic scale, μ_o^2). There are only valence quarks: the scale has to be very low ($\mu_o \simeq$0.300 GeV according to NLO pQCD). Data are taken at a high momentum scale t. QCD evolution needed!

RESULTS: k_T dependence

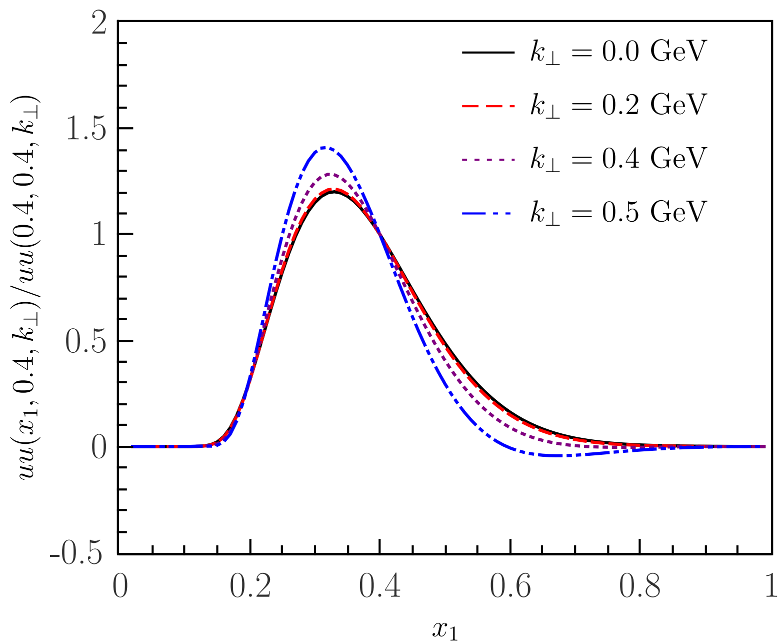


The dPDF $uu(x_1, x_2, k_\perp)$ is shown at $x_2 = 0.4$ for four values of $k_\perp$

The decrease of the dPDFs with increasing $k_\perp$ is an obvious model independent feature, found also in the MIT bag model calculation

We note on passing that our dPDF fulfills the sum rule $\int dx_1 dx_2 uu(x_1, x_2, 0) = 2$ (**J.R. Gaunt and W.J. Stirling, JHEP 03 (2010) 005**)

RESULTS: $(x_1, x_2) - k_T$ factorization



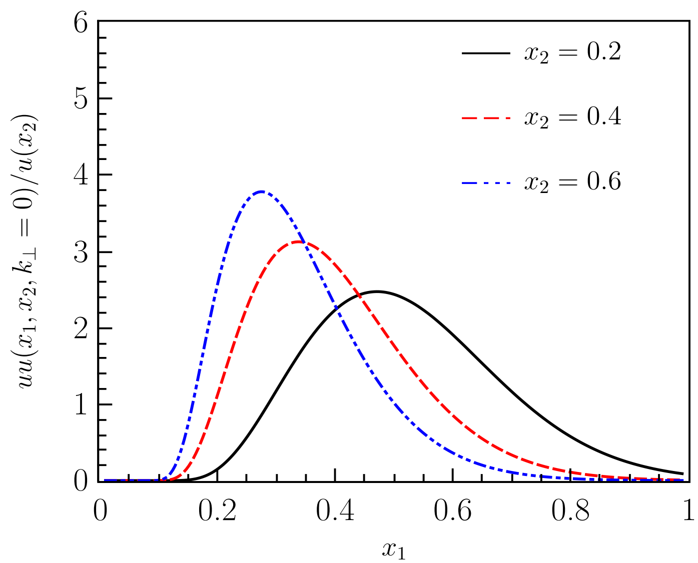
The ratio $uu(x_1, x_2 = 0.4, k_\perp)/uu(x_1 = 0.4, x_2 = 0.4, k_\perp)$,

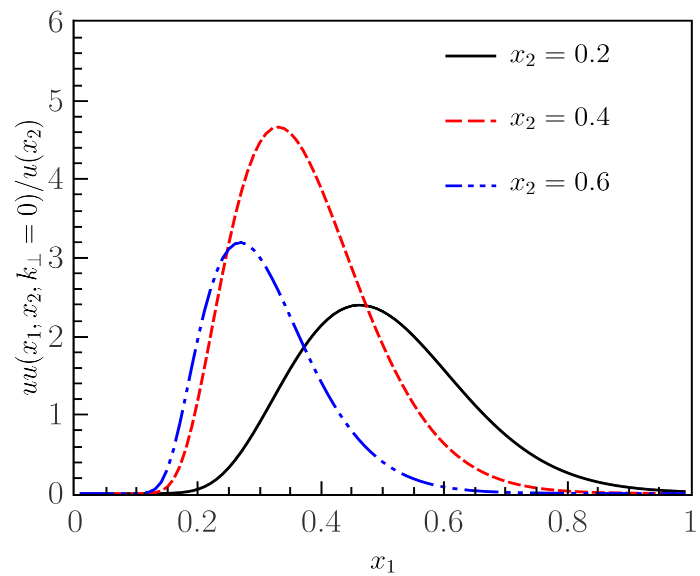
for different $k_\perp$ values: there should be no $k_\perp$ dependence if factorization worked

- mildly violated in the IK model (and in the Bag);
- valid in the H.O. model (quarks in pure relative S waves);

Model independent lesson: stronger violations would be found using model WFs with relevant higher quark orbital angular momentum components

RESULTS: $x_1 - x_2$-factorization





The ratio $uu(x_1, x_2, k_\perp = 0)/u(x_2)$, for three different values of x_2, for the H.O. (left panel) and for the IK (right panel) model.

There should be no x_2 dependence in the ratio if the $x_1 - x_2$-factorization were realized:

- The $x_1 - x_2$-factorization is badly violated
- Already found in the bag model calculation

Model independent feature

RESULTS: more on $x_1 - x_2$-factorization

Let us show the ratio:

$$r_\beta(x_1, x_2) = \frac{2uu_\beta(x_1, x_2, k_\perp = 0)}{u_\beta(x_1) u_\beta(x_2)}$$ where:

1) the dPDF depends on a parameter β:

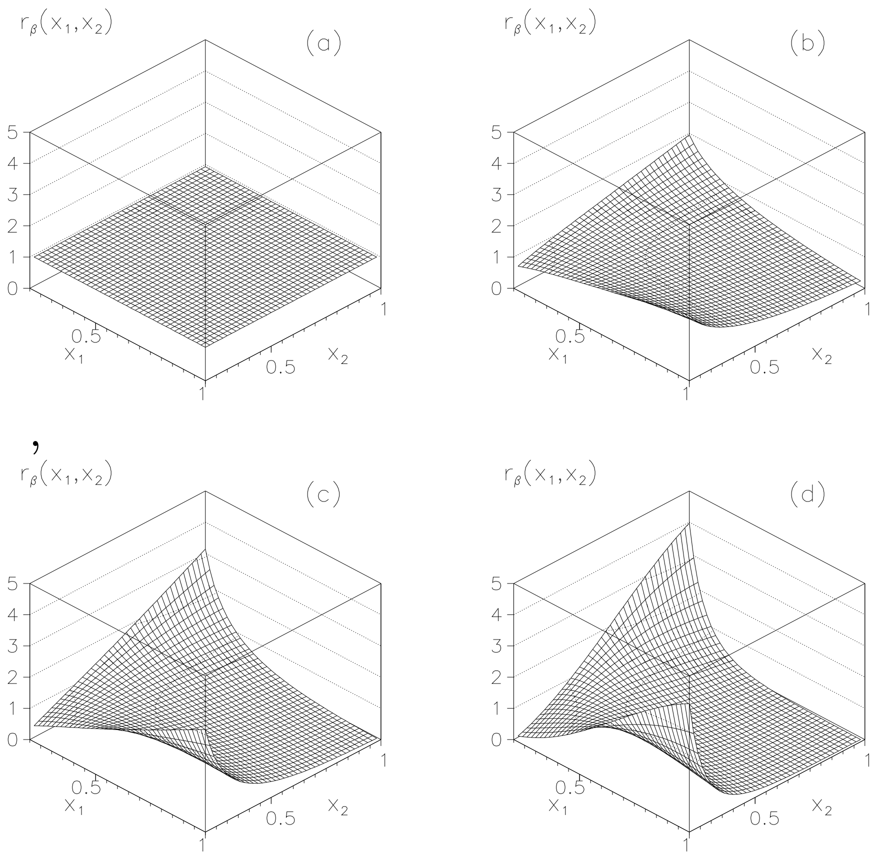
$$uu_\beta(x_1, x_2, k_\perp = 0) = 2\frac{(4-\beta)^{3/2}}{\pi^3 \alpha^6} \int d\vec{k}_1 d\vec{k}_2$$

$$\times e^{-2(k_1^2 + k_2^2 + \beta \vec{k}_1 \cdot \vec{k}_2)/\alpha^2} \delta\left(x_1 - \frac{k_1^+}{P^+}\right) \delta\left(x_2 - \frac{k_2^+}{P^+}\right) ,$$

2) the corresponding PDF is:

$$u_\beta(x_i) = 2\frac{(4-\beta)^{3/2}}{\pi^3 \alpha^6} \int d\vec{k}_1 d\vec{k}_2$$

$$\times e^{-2(k_1^2 + k_2^2 + \beta \vec{k}_1 \cdot \vec{k}_2)/\alpha^2} \delta\left(x_i - \frac{k_i^+}{P^+}\right) ,$$



a) $\beta = 0$: uncorrelated scenario; b) $\beta = 0.25$; c) $\beta = 0.5$;
d) $\beta = 1$: the correlated HO framework

Huge effect! May be the real situation is somewhere in between (a) and (d)...
We have to improve the model...

How to improve the approach?

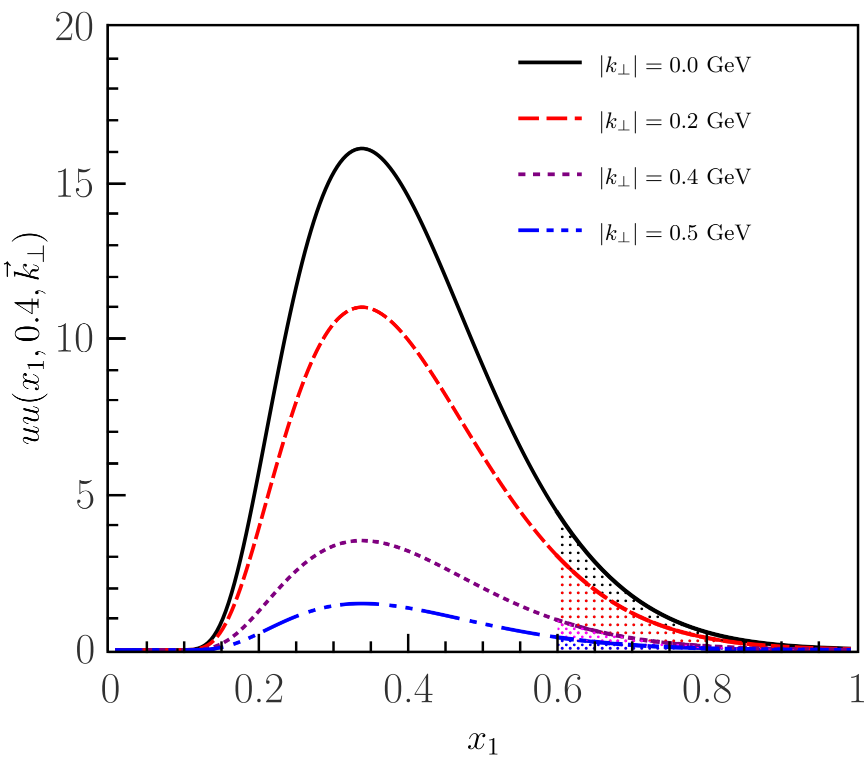
- QCD evolution (theoretically known, it can be estimated)
- Implement Relativity into the scheme
- Overcome the so called “bad support problem”: the dPDF should be zero for $x_1 + x_2 > 1$. In the CQM calculations described so far, this is not realized:

In our approach:

$$\delta\left(x_i - \frac{k_i^+}{P^+}\right) \Longrightarrow \delta\left(x_i - \frac{k_i^+}{M}\right)$$

so that one can get: $\sum_i x_i > 1$

We cannot describe properly
the off-shell “$i-$” parton in the hadron.



PERSPECTIVES: Towards a Light-Front approach

Relativity can be implemented by using a Light-Front (LF) approach, yielding the correct support. In the Relativistic Hamiltonian Dynamics (RHD) of an interacting system, introduced by Dirac (1949), one has:

- full Poincarè covariance
- fixed number of on-mass-shell constituents

Among the 3 possible forms of RHD, the we adopt the LF
(initial hypersurface: $x^+ = x_0 + x_3 = 0$; in standard, “Instant Form” QM: $x_0 = 0$).
In the LF there are several advantages. The most relevant here:

- 7 Kinematical generators (maximum number): i) three LF boosts (at variance with the dynamical nature of the Instant-form boosts), ii) $P^+, \mathbf{P}_\perp$, iii) Rotation around z.
- The LF boosts have a subgroup structure, then one gets a trivial separation of the intrinsic motion from the global one (as in the non relativistic (NR) case).
- in a peculiar construction of the Poincaré generators (Bakamjian-Thomas) it is possible to obtain a Mass equation, Schrödinger-like. A clear connection to NR.
- The IMF description of DIS is easily included.

Parton distributions in a LF approach

- In LF, boosting a state is trivial:
  In general, if the center of mass 4-momentum of one state, P, transforms under a Lorentz Transformation Λ according to $P' = \Lambda P$, the state transforms as

$$|\Psi'; P'SS_z\rangle = \mathcal{N} \sum_{S'_z} \mathcal{D}_{S_z,S'_z}(R_W(\Lambda, P/M))|\Psi'; PSS'_z\rangle \; ;$$

  If Λ is a LF boost, the Wigner rotation $\mathcal{D}(\mathcal{R}_{\mathcal{W}})$ reduces to the identity.
  Very simple and suitable to describe any hard process.

- In this context the "bad support" problem does not arise (on-shell particles):

$$\delta\left(x_i - \frac{k_i^+}{P^+}\right) \Longrightarrow \delta\left(x_i - \frac{k_i^+}{M_0}\right) \quad \text{where now:} \quad M_0 = \sum_i \sqrt{m^2 + k_i^2}$$

  with this formalism one finds always: $\sum_i x_i < 1$.

- LF has been extensively used in hadronic Physics studies (electromagnetic form factors, PDFs, GPDs, TMDs...)

Our LF calculation of dPDFs

- Now to complete this analysis we need to choose a proton wave function which is solution of the LF Mass equation
- In collaboration with M. Traini, we are presently extending to dPDFs calculations an approach used to evaluate PDFs, GPDs, TMDs...
  (see, e.g., **Pasquini, Boffi and Traini NPB 649 (2003) 243**)
- It is a hyperspherical harmonics calculation where the intrinsic proton state is

$$|N, j, j_n = 1/2\rangle = \frac{1}{\xi^5}\psi_{00}(\xi)\mathcal{Y}^{0,0}_{0,0,0}(\Omega) \times SU(6)_{sf}$$

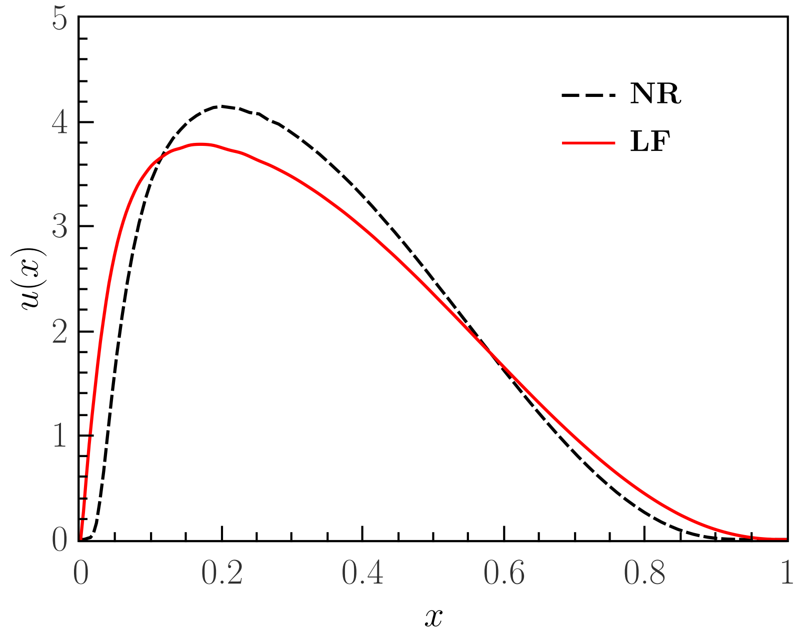
where $\psi_{00}(\xi)$ is solution of the mass equation:

$$\left(\sum_{i=1}^{3}\sqrt{\mathbf{k_i}^2 + m_i^2} - \frac{\tau}{\xi} + \kappa_l \xi\right)\psi_{00}(\xi) = M\psi_{00}(\xi)$$

and ξ is the radius of the hypersphere in 6 dimensions
($\xi = \sqrt{\rho^2 + \lambda^2}$, $\rho = (\mathbf{r_1} - \mathbf{r_2})/\sqrt{2}$, $\lambda = \sqrt{3/2}(\mathbf{r_1} + \mathbf{r_2})$)

Status of the LF calculation

- PDF limit (results in **Pasquini, Boffi and Traini NPB 649 (2003) 243**) recovered, with correct support.



- dPDFs and DPCs formalism already obtained; numerics in progress
- LF suitable to study spin correlations: LF allows a relativistic treatment of the spin

$$(0, \vec{j}_{LF}) = [\mathcal{R}_{Melosh}] \, (0, \vec{j}_{IF})$$

- LF suitable at low x: non perturbative sea can be included complicating the nucleon Fock space... Not easy but one can try

Conclusions

- **A CQM calculation of DPDs and DPCs has been presented:**
  (**M. Rinaldi, S.S. and V.Vento, PRD 87, 114021 (2013)**)
  * correlations are present from the very beginning;
  * mild violation of $(x_1, x_2) - k_\perp$ factorization, depending on orbital angular momentum in the wf;
  * strong violation of x_1, x_2 factorization;
  * dynamical origin of correlations clarified.

- **Recent developments using LF RHD (work in progress):**
  * A covariant approach with on-shell constituents
  * Correct support (important for QCD evolution)
  * Proper framework for spin correlations and low-x model calculations

Backup: dPDF, formally

$$
\begin{aligned}
F_{q_1 q_2}(x_1, x_2, \mathbf{z}_\perp) &= -8\pi M^2 \int \frac{\mathrm{d}z_1^+}{4\pi}\frac{\mathrm{d}z_2^+}{4\pi}\frac{\mathrm{d}z_3^+}{4\pi} e^{-ix_1 M z_1^+/2}\, e^{-ix_2 M z_2^+/2}\, e^{ix_1 M z_3^+/2} \\
&\times \langle P, \mathbf{p}=\mathbf{0}| \left[\bar{q}_1\left(z_1^+\frac{\bar{n}}{2}+z_\perp\right)\frac{\not{n}}{2}\right]_c \\
&\times \left[\bar{q}_2\left(z_2^+\frac{\bar{n}}{2}\right)\frac{\not{n}}{2}\right]_d q_{1,c}\left(z_3^+\frac{\bar{n}}{2}+z_\perp\right) q_{2,d}(0)|P,\mathbf{p}=\mathbf{0}\rangle\,. \qquad (1)
\end{aligned}
$$