Production of two $c\bar{c}$ pairs and two mesons with charm in double-parton scattering: inclusive and correlation observables

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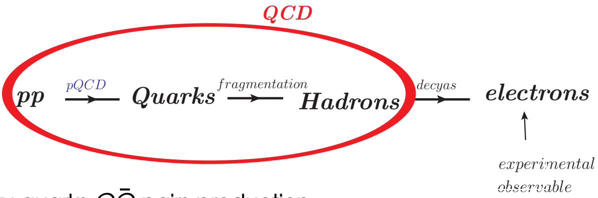
Contents

- General framework of $c\bar{c}$ production
- D meson production at LHC
- Double parton production of $c\bar{c}c\bar{c}$
- Single parton production of $c\bar{c}c\bar{c}$
  (high-energy approximation)
- Same flavour DD production in DPS
  (relevance to the recent LHCb results)
- New exact LO SPS calculations of $c\bar{c}c\bar{c}$ and DD production
- Conclusions

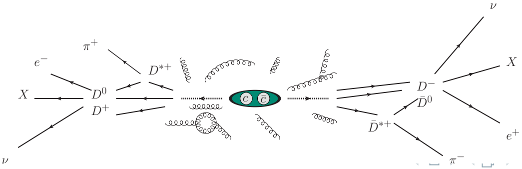
Low-x physics because c quark mass rather small

3-step process



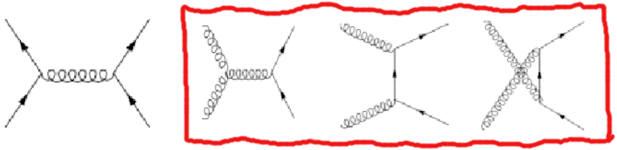
1. Heavy quarks $Q\bar{Q}$ pairs production
   - $m_c = 1.5$ GeV, $m_b = 4.75$ GeV $\longrightarrow$ perturbative QCD
2. Heavy quarks hadronization (fragmentation)
3. Semileptonic decays of D and B mesons

$$\frac{d\sigma^e}{dyd^2p} = \frac{d\sigma^Q}{dyd^2p} \otimes D_{Q\to H} \otimes f_{H\to e}$$

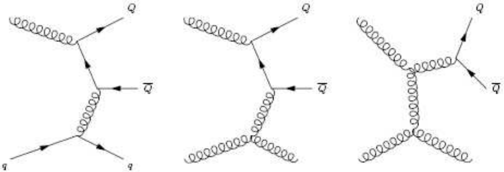


Dominant mechanisms of $Q\bar{Q}$ production

- Leading order processes contributing to $Q\bar{Q}$ production:

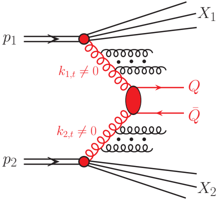


- **gluon-gluon fusion** dominant at high energies
- $q\bar{q}$ anihilation important only near the threshold
- some of next-to-leading order diagrams:



NLO contributions $\longrightarrow$ K-factor

k_t-factorization (semihard) approach



- charm and bottom quarks production at high energies $\longrightarrow$ gluon-gluon fusion
- QCD collinear approach $\rightarrow$ only inclusive one particle distributions, total cross sections

LO k_t-factorization approach $\longrightarrow \kappa_{1,t}, \kappa_{2,t} \neq 0$

$\Rightarrow Q\bar{Q}$ correlations

- multi-differential cross section

$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1,t} d^2p_{2,t}} = \sum_{i,j} \int \frac{d^2\kappa_{1,t}}{\pi} \frac{d^2\kappa_{2,t}}{\pi} \frac{1}{16\pi^2 (x_1 x_2 s)^2} \overline{|\mathcal{M}_{ij\to Q\bar{Q}}|^2}$$
$$\times \; \delta^2 \left(\vec{\kappa}_{1,t} + \vec{\kappa}_{2,t} - \vec{p}_{1,t} - \vec{p}_{2,t}\right) \mathcal{F}_i(x_1, \kappa_{1,t}^2) \, \mathcal{F}_j(x_2, \kappa_{2,t}^2)$$

- off-shell $\overline{|\mathcal{M}_{gg\to Q\bar{Q}}|^2} \longrightarrow$ Catani, Ciafaloni, Hautmann (rather long formula)
- major part of NLO corrections automatically included
- $\mathcal{F}_i(x_1, \kappa_{1,t}^2)$, $\mathcal{F}_j(x_2, \kappa_{2,t}^2)$ - unintegrated parton distributions
- $x_1 = \frac{m_{1,t}}{\sqrt{s}} \exp(y_1) + \frac{m_{2,t}}{\sqrt{s}} \exp(y_2)$,

  $x_2 = \frac{m_{1,t}}{\sqrt{s}} \exp(-y_1) + \frac{m_{2,t}}{\sqrt{s}} \exp(-y_2)$, where $m_{i,t} = \sqrt{p_{i,t}^2 + m_Q^2}$.

Unintegrated parton distribution functions

- k_t-factorization → replacement: $p_k(x, \mu_F^2) \longrightarrow \mathcal{F}_k(x, \kappa_t^2, \mu_F^2)$
- PDFs ⟶ UPDFs

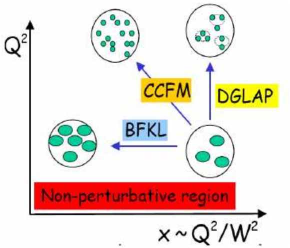
$$xp_k(x, \mu_F^2) = \int_0^\infty d\kappa_t^2 \mathcal{F}(x, \kappa_t^2, \mu_F^2)$$

- UPDFs - needed in less inclusive measurements which are sensitive to the transverse momentum of the parton

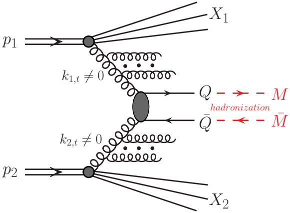
gg-fusion dominance ⇒ great test of existing unintegrated gluon densities!
especially at LHC (small-x)

several models:

- Jung, Kwiecinski (CCFM, wide x-range)
- Kimber-Martin-Ryskin (higher x-values)
- Kutak-Stasto (small-x, saturation effects)
- Ivanov-Nikolaev, GBW, Karzeev-Levin, etc.



Fragmentation functions technique



- fragmentation functions extracted from e^+e^- data
- often used: Braaten et al., Kartvelishvili et al., Peterson et al.
- rescalling transverse momentum
  at a constant rapidity (angle)

- from heavy quarks to heavy mesons:

$$\frac{d\sigma(y, p_t^M)}{dy d^2 p_t^M} \approx \int \frac{D_{Q\to M}(z)}{z^2} \cdot \frac{d\sigma(y, p_t^Q)}{dy d^2 p_t^Q} dz$$

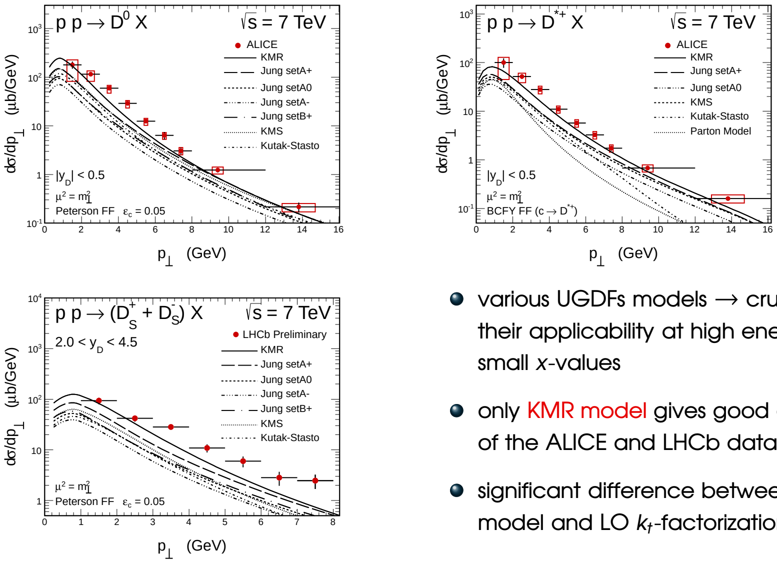
where: $p_t^Q = \frac{p_t^M}{z}$ and $z \in (0, 1)$

- **approximation:**
  rapidity unchanged in the fragmentation process $\rightarrow y_Q \approx y_M$

Production of D mesons in this framework:

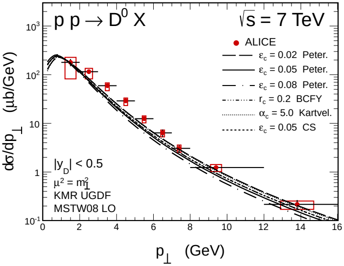
Maciula, Szczurek, Phys. Rev. **D87** (2013) 094022.

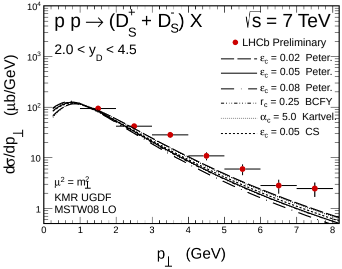
D mesons, different UGDFs

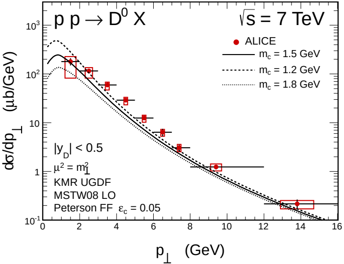


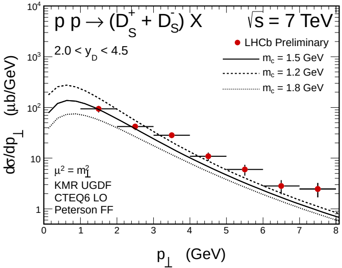
- various UGDFs models → crucial test of their applicability at high energies and small x-values
- only KMR model gives good description of the ALICE and LHCb data
- significant difference between LO parton model and LO k_t-factorization

Effects of hadronization and quark mass uncertainty for KMR UGDF



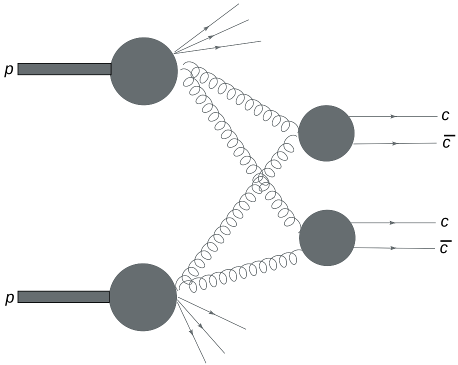






Production of two $c\bar{c}$ pairs in double-parton scattering

Consider two hard (parton) scatterings



Luszczak, Maciula, Szczurek, arXiv:1111.3255, Phys.Rev. **D85** (2012) 094034,

Maciula, Szczurek, arXiv:1301.4469, Phys. Rev. **D87** (2013) 074037.

Formalism

Consider reaction: $pp \to c\bar{c}c\bar{c}X$

Modeling double-parton scattering

Factorized form:

$$\sigma^{DPS}(pp \to c\bar{c}c\bar{c}X) = \frac{1}{2\sigma_{eff}}\sigma^{SPS}(pp \to c\bar{c}X_1) \cdot \sigma^{SPS}(pp \to c\bar{c}X_2).$$

The simple formula can be generalized to include differential distributions

$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1t} dy_3 dy_4 d^2p_{2t}} = \frac{1}{2\sigma_{eff}} \cdot \frac{d\sigma}{dy_1 dy_2 d^2p_{1t}} \cdot \frac{d\sigma}{dy_3 dy_4 d^2p_{2t}} .$$

σ_{eff} is a model parameter (15 mb).

Formalism

$$d\sigma^{DPS} = \frac{1}{2\sigma_{eff}} F_{gg}(x_1, x_2, \mu_1^2, \mu_2^2) F_{gg}(x_1' x_2', \mu_1^2, \mu_2^2)$$
$$d\sigma_{gg\to c\bar{c}}(x_1, x_1', \mu_1^2) d\sigma_{gg\to c\bar{c}}(x_2, x_2', \mu_2^2)\ dx_1 dx_2 dx_1' dx_2' \,.$$

$F_{gg}(x_1, x_2, \mu_1^2, \mu_2^2)$, $F_{gg}(x_1' x_2', \mu_1^2, \mu_2^2)$
are called double parton distributions

dPDF are subjected to special evolution equations
single scale evolution: Snigirev
double scale evolution: Ceccopieri, Gaunt-Stirling

What the σ_{eff} is?

It is much easier to understand the DPS in the impact parameter space.
Then one considers even more generalized objects:

$$\Gamma_{i,j}(x_1, x_2; \mathbf{b_1}, \mathbf{b_2}; \mu_1^2, \mu_2^2) = F_{i,j}(x_1, x_2; \mu_1^2, \mu_2^2) f(\mathbf{b_1}) f(\mathbf{b_2}) \quad .$$

$f(\mathbf{b_i})$ universal functions for all kinds of partons with:

$$\int f(\mathbf{b_1}) f(\mathbf{b_1} - \mathbf{b}) d^2 b_1 d^2 b = \int T(\mathbf{b}) d^2 b = 1 \ ,$$

where

$$T(\mathbf{b}) = \int f(\mathbf{b_1}) f(\mathbf{b_1} - \mathbf{b}) d^2 b_1$$

is the overlap function.
Then:

$$\sigma_{eff} = \left(\int d^2 b \, (T(b))^2 \right)^{-1} \quad .$$

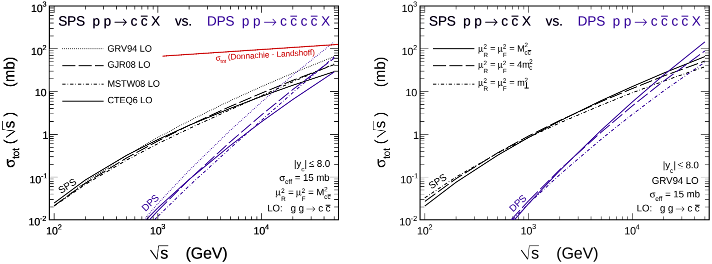
Universal function

Simple estimate of σ_{eff}

Naive estimate:

- gluon distribution in the proton:
  $\rho \propto \exp\left(-r^2/a^2\right)$
- overlap function as a function of impact parameter:
  $F(b) \propto \int d^2\, r\rho(\mathbf{r})\rho(\mathbf{r}-\mathbf{b}) \propto \exp(-b^2/2a^2)$
- Then $\sigma_{eff} = 4\pi a^2$
- If a i reflecting the proton radius
  $\sigma_{exp} > 4\pi a^2$

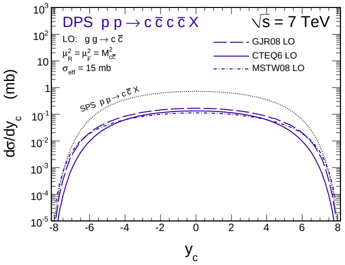
DPS results, collinear approximation

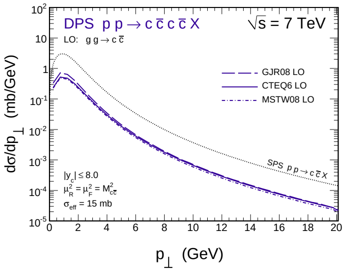


Inclusive cross section more difficult to calculate

$$\sigma_{SS}, 2\sigma_{DS} < \sigma_c^{inclusive} < \sigma_{SS} + 2\sigma_{DS}$$

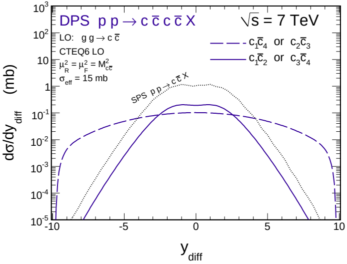
DPS results, collinear approximation

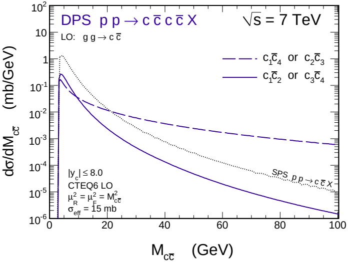




In the factorized model inclusive double-scattering distributions in y and p_t are identical as for single-$c\bar{c}$ production.

DPS results, collinear approximation

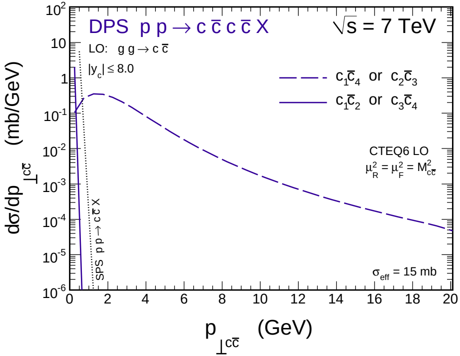




DPS: large rapidity differences, large invariant masses

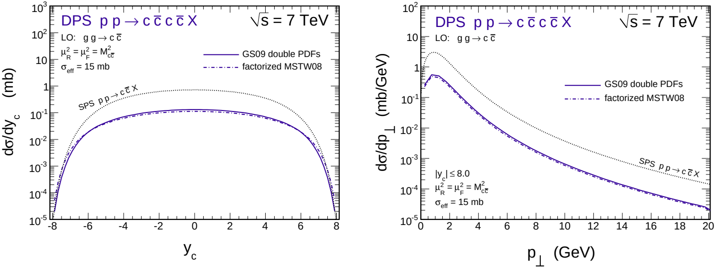
- Not possible for quarks (antiquarks)
- mesons ?
- nonphotonic electrons (muons) ?

DPS results, collinear approximation



Large transverse momenta of the cc or $\bar{c}\bar{c}$ pairs

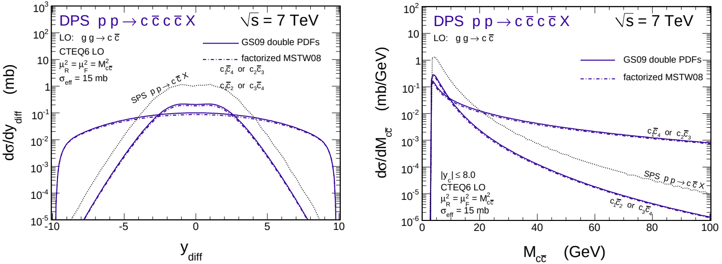
Evolution of dPDFs



Gaunt-Stirling dPDFs with evolution

very small effect of the evolution

Evolution of dPDFs



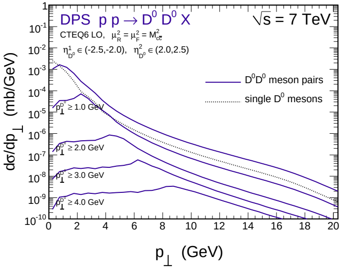
Gaunt-Stirling dPDFs with evolution

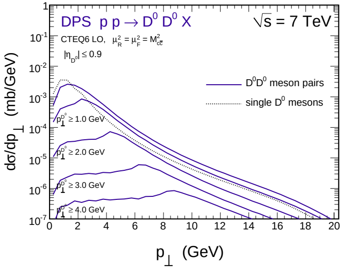
very small effect of the evolution

From quarks/antiquarks to D mesons

large transverse momentum of cc or $\bar{c}\bar{c}$

transverse $D^0 D^0$ momentum distribution





ATLAS: $-2.5 < \eta_1 < -2.0$ and $2.0 < \eta_2 < 2.5$

ALICE: $-0.9 < \eta_1, \eta_2 < 0.9$

DPS in k_t-factorization

Generalize the LO collinear approach to k_t-factorization approach.

More complicated (more kinematical variables) as momenta of outgoing partons are less correlated

We need information about each quark and antiquark

$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1,t} d^2p_{2,t} dy_3 dy_4 d^2p_{3,t} d^2p_{4,t}} = \frac{1}{2\sigma_{eff}} \cdot \frac{d\sigma}{dy_1 dy_2 d^2p_{1,t} d^2p_{2,t}} \cdot \frac{d\sigma}{dy_3 dy_3 d^2p_{3,t} d^2p_{4,t}} \quad (1)$$

12 dimensions (!)

DPS in k_t-factorization

Each individual scattering in the k_t-factorization approach

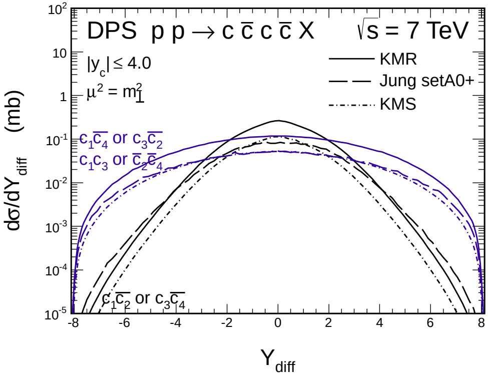
$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1,t} d^2p_{2,t}} = \frac{1}{16\pi^2 \hat{s}^2} \int \overline{|\mathcal{M}_{off}|^2} \delta\left(\vec{k}_{1t} + \vec{k}_{2t} - \vec{p}_{1t} - \vec{p}_{2t}\right) \mathcal{F}(x_1, k_{1t}^2, \mu^2) \mathcal{F}(x_2, k_{2t}^2, \mu^2) \frac{d^2k_{1t}}{\pi} \frac{d^2k_{2t}}{\pi}$$

$$\frac{d\sigma}{dy_3 dy_4 d^2p_{3,t} d^2p_{4,t}} = \frac{1}{16\pi^2 \hat{s}^2} \int \overline{|\mathcal{M}_{off}|^2} \delta\left(\vec{k}_{3t} + \vec{k}_{4t} - \vec{p}_{3t} - \vec{p}_{4t}\right) \mathcal{F}(x_3, k_{3t}^2, \mu^2) \mathcal{F}(x_4, k_{4t}^2, \mu^2) \frac{d^2k_{3t}}{\pi} \frac{d^2k_{4t}}{\pi}$$

Effectively 16 dimensions, Monte Carlo method

Maciula-Szczurek, hep-ph-1301.4469, Phys. Rev. **D87** (2013) 074039.

DPS k_t-factorization calculation



The same situation as in collinear approach

DPS k_t-factorization calculation vs LHCb

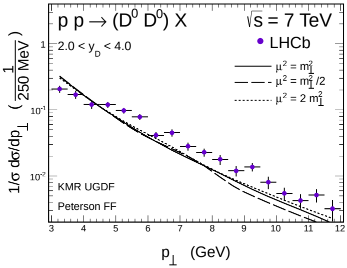
Table: Total cross sections

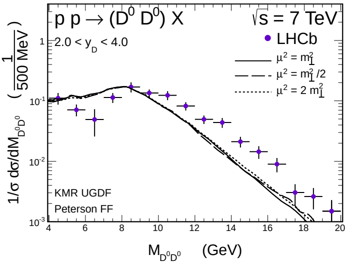
Mode	σ_{tot}^{EXP}	KMR	Jung setA0+	KMS
$D^0 D^0$	$690 \pm 40 \pm 70$	256	101	100
$D^0 D^+$	$520 \pm 80 \pm 70$	204	81	80
$D^0 D_S^+$	$270 \pm 50 \pm 40$	72	29	28
$D^+ D^+$	$80 \pm 10 \pm 10$	41	16	16
$D^+ D_S^+$	$70 \pm 15 \pm 10$	29	12	11
$D_S^+ D_S^+$	–	10	4	4

LHCb acceptance:

$2 < y < 4, \quad 3\ \text{GeV} < p_\perp < 12\ \text{GeV}$

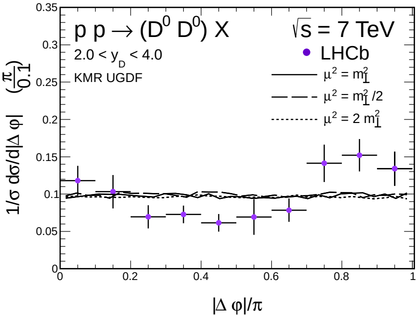
DPS k_t-factorization calculation vs LHCb





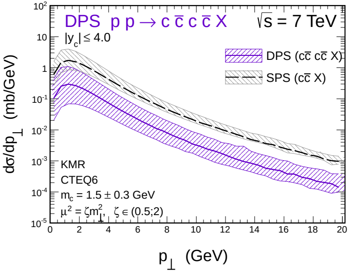
missing SPS contributions (extra gluon splitting) ?

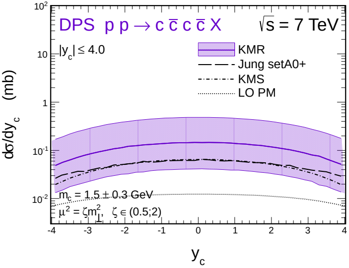
DPS k_t-factorization calculation vs LHCb



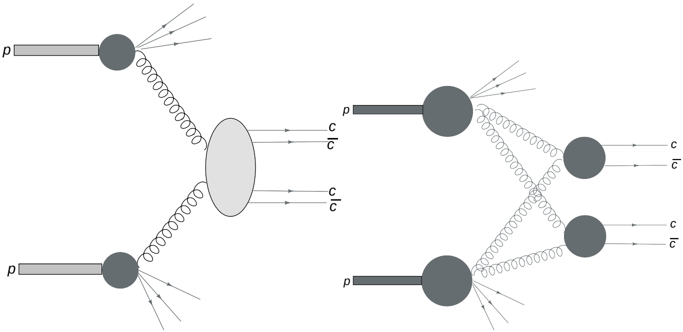
missing SPS contributions ?

One and two pair production, uncertainties





SPS production of $c\bar{c}c\bar{c}$



W. Schäfer, A. Szczurek, arXiv:1203.4129(hep-ph), Phys. Rev. **D85** (2012) 094029.

SPS production of $c\bar{c}c\bar{c}$

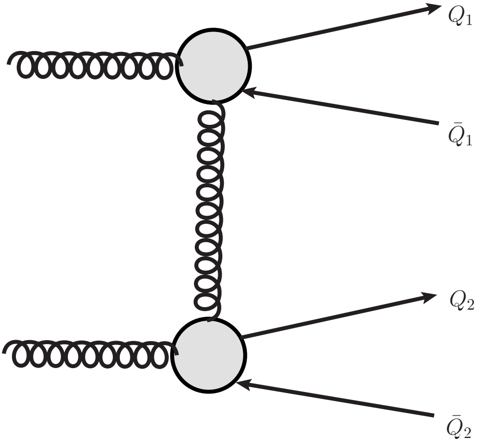


Figure: Subprocess: $gg \rightarrow (c\bar{c})(c\bar{c})$ production.

Impact factors

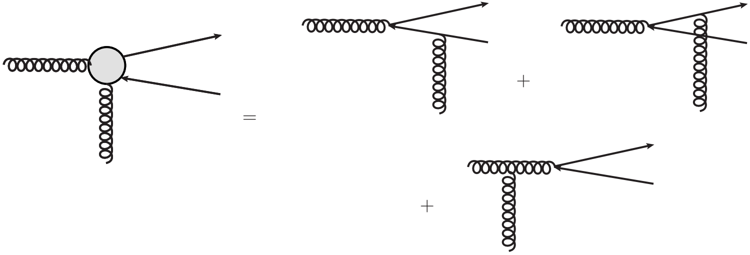


Figure: Coupling of (t-channel) gluon to g, Q, $\bar{Q}$

9 diagrams for the $gg \to c\bar{c}c\bar{c}$ cross section.

gg collisions, momentum representation

The compact cross section formula:

$$d\sigma = \frac{N_c^2 - 1}{N_c^2} \frac{4\pi^2 \alpha_s^2}{[\boldsymbol{q}^2 + \mu_G^2]^2} I(z, \boldsymbol{k}, \boldsymbol{q}) I(u, \boldsymbol{l}, -\boldsymbol{q}) \, dz \frac{d^2\boldsymbol{k}}{(2\pi)^2} du \frac{d^2\boldsymbol{l}}{(2\pi)^2} \frac{d^2\boldsymbol{q}}{(2\pi)^2} . \tag{2}$$

1) 8-dim integration
2) Impact factors are quite complicated.
3) First pair:

$$\boldsymbol{p}_Q = \boldsymbol{k} + z\boldsymbol{q}, \quad \boldsymbol{p}_{\bar{Q}} = -\boldsymbol{k} + (1 - z)\boldsymbol{q} , \tag{3}$$

4) Second pair:

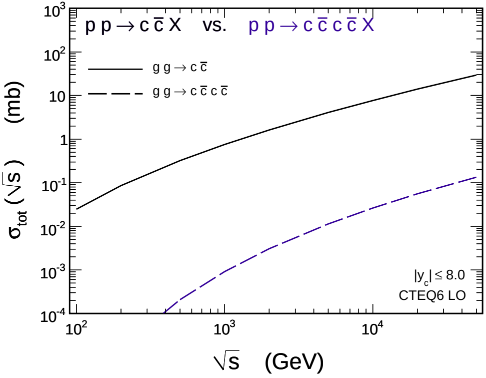
$$\boldsymbol{p}_Q = \boldsymbol{l} - u\boldsymbol{q}, \quad \boldsymbol{p}_{\bar{Q}} = -\boldsymbol{l} - (1 - u)\boldsymbol{q} . \tag{4}$$

$pp \to (Q\bar{Q})(Q\bar{Q})$ inclusive cross section

$$\sigma_{pp\to(Q\bar{Q})(Q\bar{Q})}(W) = \int dx_1 dx_2 \, g(x_1,\mu_F^2)\, g(x_2,\mu_F^2)\, \sigma_{gg\to(Q\bar{Q})(Q\bar{Q})}(\hat{s}^{1/2}) \,, \quad (5)$$

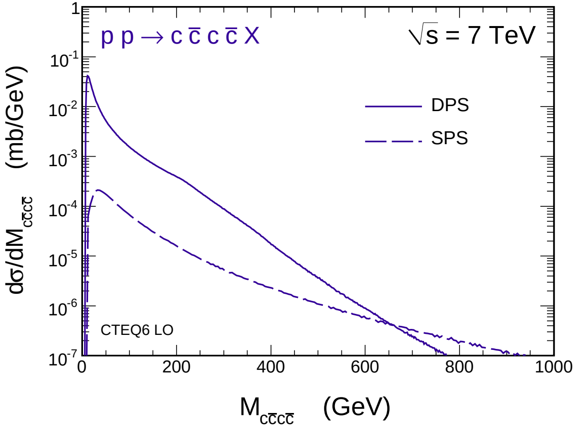
- $\sigma_{gg\to(Q\bar{Q})(Q\bar{Q})}(\hat{s}^{1/2})$ - elementary cross section for $gg \to c\bar{c}c\bar{c}$. Calculated and stored.
- $g(x_1,\mu_F^2), g(x_2,\mu_F^2)$ - collinear gluon distributions from the literature.
- The integral over $\xi_1 = log_{10}(x_1)$ and $\xi_2 = log_{10}(x_2)$ is performed next instead of x_1 and x_2.
- $\hat{s} = x_1 x_2 W^2$.
- $\mu_F^2 = 4m_Q^2$ (or m_Q^2).

pp collisions, $c\bar{c}$ versus $c\bar{c}c\bar{c}$



Only about 1 % at high energies

pp collisions, $c\bar{c}c\bar{c}$ invariant mass distr.



At intermediate invariant masses SPS $\ll$ DPS.

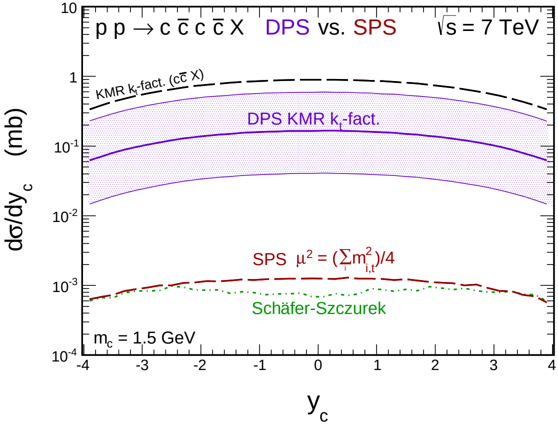
At very large invariant masses SPS $\gg$ DPS.

Exact LO calculation of SPS $c\bar{c}c\bar{c}$ contribution

- Till recently only approximate (high-energy approx.) SPS contribution to $gg \to c\bar{c}c\bar{c}$
- Recently full calculation of leading-order $2 \to 4$ diagrams for SPS $gg \to c\bar{c}c\bar{c}$ and $q\bar{q} \to c\bar{c}c\bar{c}$ (small)
- Automatic calculation with exact LO matrix elements and full phase space integration (van Hameren code, similar to HELAC)
- Generation of unweighted events and building quark/antiquark distributions
- Hadronization with fragmentation functions
- No K-factor (relatively small for $b\bar{b}b\bar{b}$)

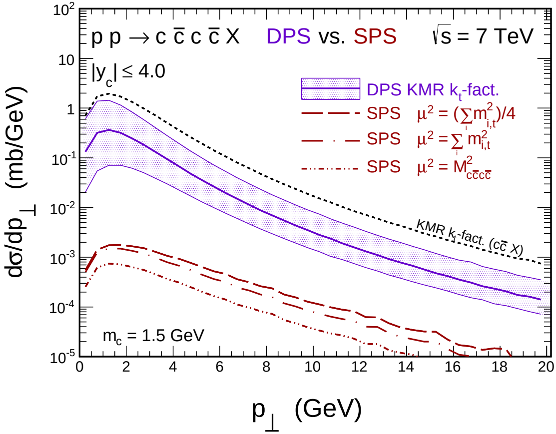
van Hameren, Maciuła, Szczurek, work in preparation

Exact SPS, quark level



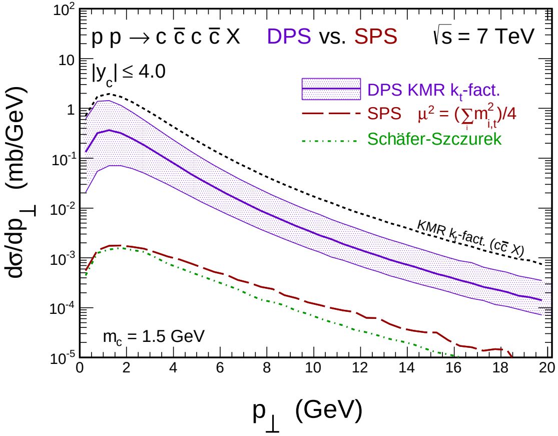
Agreement of high-energy approx. and exact at large quark rapidities

Exact SPS, quark level



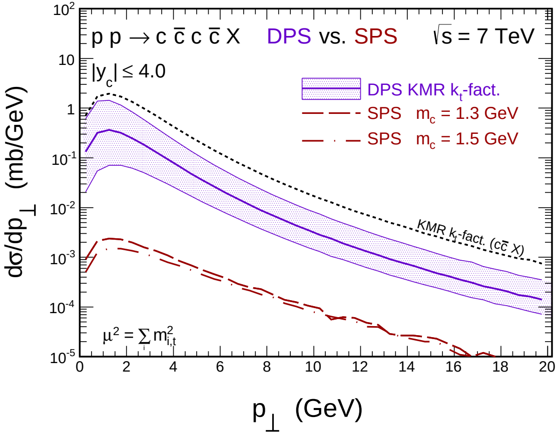
dependence onf renormalization and factorization scale of SPS

Exact SPS, quark level



similar shape of SPS and DPS

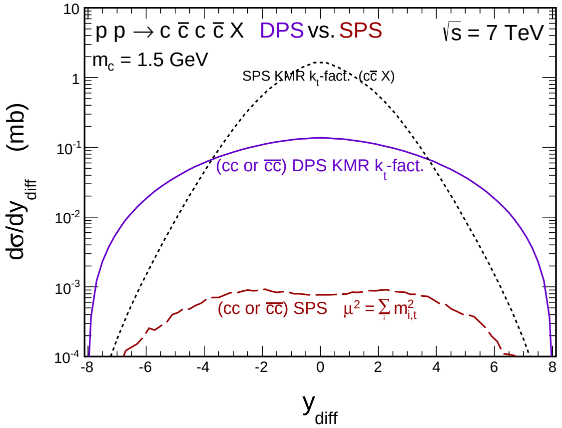
Exact SPS, quark level



weak dependence on the quark mass for SPS

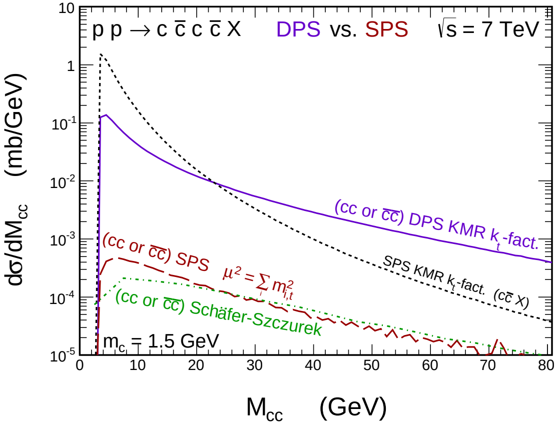
Exact SPS, quark level

Rapidity distance between two c quarks



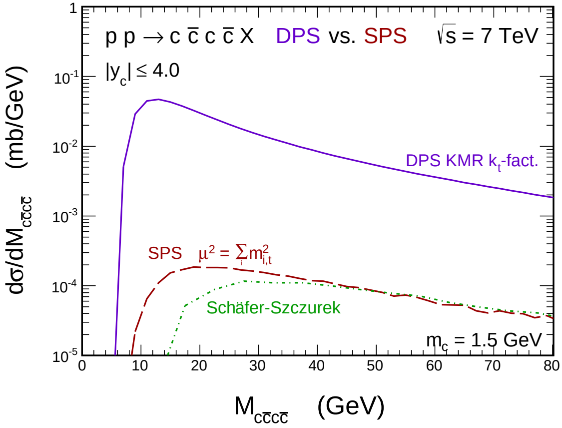
Difference of high-energy approx. and exact at small rapidity distances

Exact SPS, quark level



full and approximate aproaches coincide at large M_{cc}

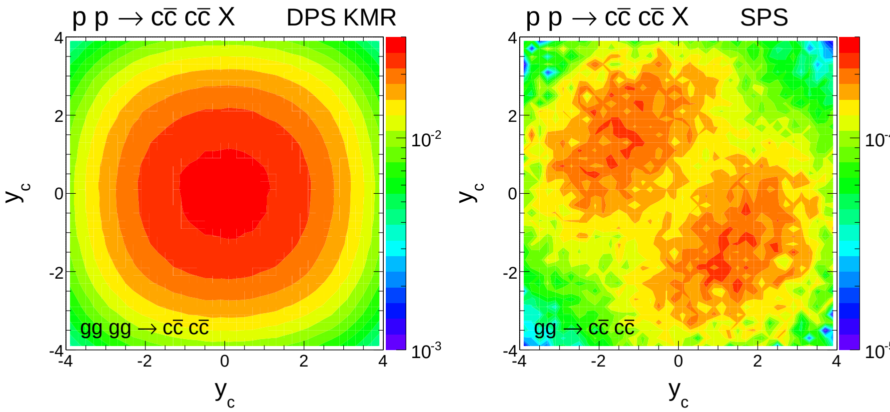
Exact SPS, quark level



full and approximate aproaches coincide at large M_{cc}

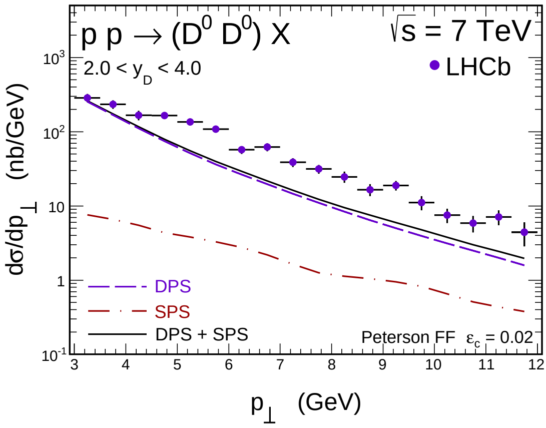
Exact SPS, quark level

Rapidity correlations for DPS and SPS

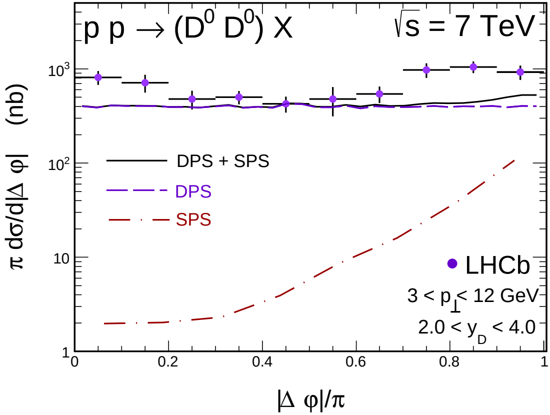


decorrelation for DPS and anticorrelation for SPS

Exact SPS, D meson level

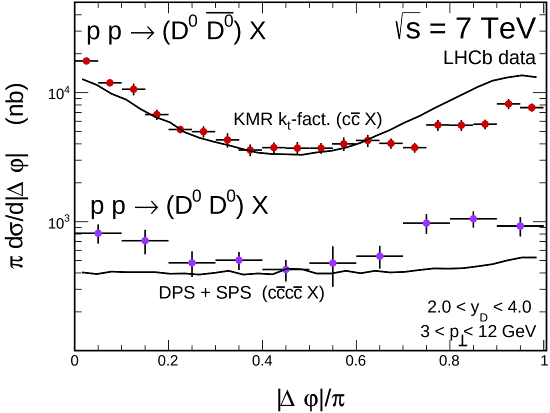


Exact SPS, D meson level

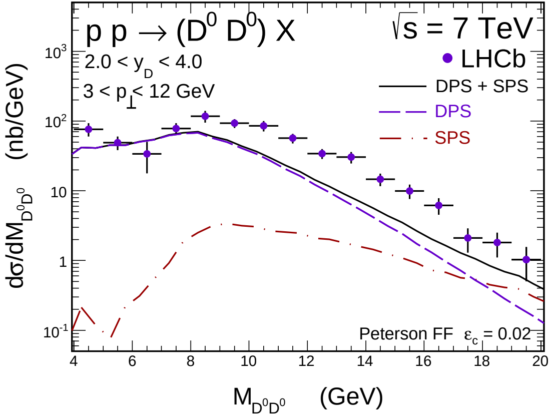


Exact SPS, D meson level

colorfblue$D^0 D^0$ versus $D^0 \bar{D}^0$ correlations



Exact SPS, D meson level

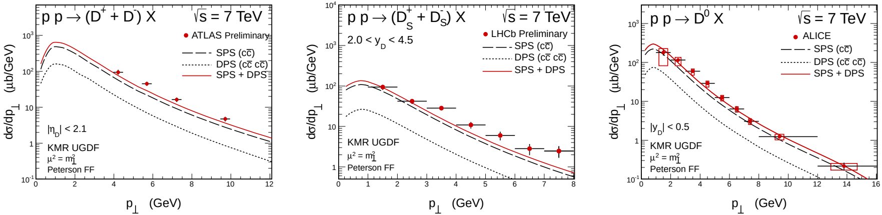


DPS contribution to inclusive D meson distributions

Let us consider for example transverse momentum distribution

$$
\begin{aligned}
\frac{d\sigma_{inc}^{D_i,DPS}}{dp_t} = & P_{D_i}(1 - P_{D_i}) \frac{d\sigma^D}{dp_{1,t}}|_{p_{1,t}=p_t}(-2.1 < \eta_1 < 2.1, -\infty < \eta_2 < \infty) \\
& + P_{D_i}(1 - P_{D_i}) \frac{d\sigma^D}{dp_{2,t}}|_{p_{2,t}=p_t}(-\infty < \eta_1 < \infty, -2.1 < \eta_2 < 2.1) \\
& + P_{D_i} P_{D_i} \frac{d\sigma^D}{dp_{1,t}}|_{p_{1,t}=p_t}(-2.1 < \eta_1 < 2.1, -\infty < \eta_2 < \infty) \\
& + P_{D_i} P_{D_i} \frac{d\sigma^D}{dp_{2,t}}|_{p_{2,t}=p_t}(-\infty < \eta_1 < \infty, -2.1 < \eta_2 < 2.1). \quad (6)
\end{aligned}
$$

DPS contribution to inclusive D meson distributions



even slightly larger(!)

Comments on two-gluon correlations

- So far $\sigma_{eff} = const.$
- In general $\sigma_{eff} = \sigma_{eff}(x_1, x_2, x_1', x_2', \mu_1^2, \mu_2^2)$.
- Correlations were discussed:
  Flensburg, Gustafson, Lönnblad, Ster
  (Mueller dipole cascade model)
  Blok, Dokshitzer, Frankfurt, Strikman
- Mostly correlation in longitudinal momenta were studied.
- Do correlation between $D^0 D^0$ mesons in azimuthal angle reflect correlations between initial two-gluons? Perhaps.
- Or we observe SPS -- DPS mixing ? Perhaps.
  Manohar, Waalewijn

Conclusions

- k_t-factorization as well as collinear NLO give slightly too small cross section compared to recent data on D meson production.
  Something missing ?
- Many small subleading contributions (single and double diffraction, exclusive $c\bar{c}$, photon induced processes).
  Not sufficient!
- Huge contribution of double-parton scattering for $pp \to (c\bar{c})(c\bar{c})X$.
  The best laboratory to study MPI.
- Especially large cross section for cc or $\bar{c}\bar{c}$ with large rapidity distance between them.
- Especially large cross section for large $p_{t,cc}$.
- Idea: look at $D^0 D^0$ (or $\bar{D}^0 \bar{D}^0$) correlations (LHCb)
  ATLAS and CMS: at the edges of main detectors,
  ALICE: large $p_{t,DD}$

Conclusions

- Relatively small contribution of single-parton scattering for $pp \to (c\bar{c})(c\bar{c})X$.
- SPS $\ll$ DPS at intermediate invariant masses of $c\bar{c}c\bar{c}$.
- SPS $\gg$ DPS at extremely large invariant masses of $c\bar{c}c\bar{c}$.
- Enhancement of large rapidity-distance region of SPS by BFKL ladders.
- Result in k_t factorization for the same flavour charmed mesons almost consistent with recent LHCb data
- ATLAS, CMS and ALICE should join the studies.
- $c\bar{c}c\bar{c}$ production -- fantastic tool for studying gluon-gluon correlations, also in transverse momenta.
- A detailed comparison of DPS and SPS for nonphotonic electrons would be useful (CMS, ALICE?).
- More refined approaches needed (Diehl et al.).