Theory of double parton scattering: progress and open questions

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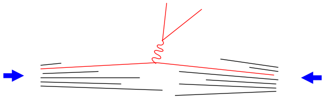
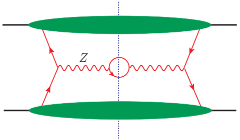
Factorization formulae: single hard scattering

- standard description for hard processes in pp collisions
  example: Z production (followed by decay $Z \to \ell^+ \ell^-$)

$$\frac{d\sigma(pp \to Z + X)}{dx\, d\bar{x}} = f_q(x)\, f_{\bar{q}}(\bar{x})\, \hat{\sigma}(q\bar{q} \to Z)$$

x and $\bar{x}$ measurable, related to Z rapidity, $\hat{\sigma}$ includes $\delta(sx\bar{x} - m_Z^2)$





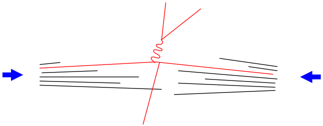
- but this is just tree level, usually not precise enough

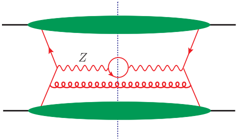
Factorization formulae: single hard scattering

- standard description for hard processes in pp collisions include radiation:

$$\frac{d\sigma(pp \to Z + X)}{dx\, d\bar{x}} = f_q(x)\, f_{\bar{q}}(\bar{x})\, \hat{\sigma}(q\bar{q} \to Z) + \int_x^1 dz \int_{\bar{x}}^1 d\bar{z}\, f_q(z)\, f_{\bar{q}}(\bar{z})\, \hat{\sigma}(q\bar{q} \to Z + g) + \text{further terms}$$

$\hat{\sigma}(q\bar{q} \to Z)$ now includes one-loop corrections





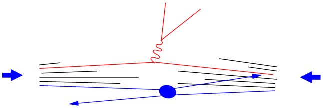
- extra radiation part of unobserved system X
- but this is still oversimplified: “spectator partons” interact as well

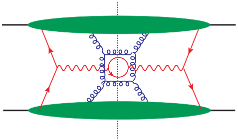
Factorization formulae: single hard scattering

- standard description for hard processes in pp collisions
  inclusive cross section:

$$\frac{d\sigma(pp \to Z + X)}{dx\, d\bar{x}} = f_q(x)\, f_{\bar{q}}(\bar{x})\, \hat{\sigma}(q\bar{q} \to Z)$$

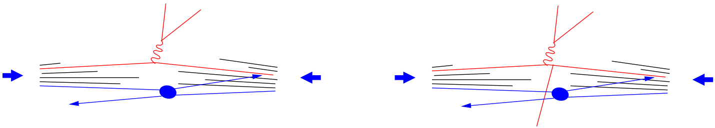
$$+ \int_x^1 dz \int_{\bar{x}}^1 d\bar{z}\, f_q(z)\, f_{\bar{q}}(\bar{z})\, \hat{\sigma}(q\bar{q} \to Z + g) + \text{further terms}$$





- "spectator" interactions produce additional particles
  which are also part of unobserved system X ("underlying event")
- need not calculate this thanks to unitarity
  as long as cross section sufficiently inclusive

Multiparton interactions

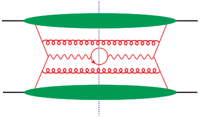


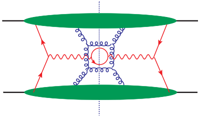
- generically take place in hadron-hadron collisions
- at high c.m. energy several interactions can be hard
  extra interactions enhanced because

  (density of two small-x partons) $\gg$ (density of one small-x parton)
- effects cancel or are suppressed in sufficiently inclusive quantities
  but do affect final state properties
- how can we calculate their effects?
- this talk:
  work within hard-scattering ("DGLAP") factorization
  alternative approach: small-x factorization ("BFKL ladders")

  $\rightarrow$ several talks at this meeting

Multiparton interactions





- assumed factorization formula, example: production of $Z+$ 2 jets

$$\frac{d\sigma(pp \to Z + 2 \text{ jets} + X)}{dx_1\, d\bar{x}_1\, dx_2\, d\bar{x}_2} = \text{single hard scattering}$$

$$+ \frac{1}{C} \int d^2\boldsymbol{b}\, F_{qg}(x_1, x_2, \boldsymbol{b})\, F_{\bar{q}g}(\bar{x}_1, \bar{x}_2, \boldsymbol{b})\, \hat{\sigma}(q\bar{q} \to Z)\, \hat{\sigma}(gg \to 2 \text{ jets})$$

$$+ \text{other subprocesses} + \text{higher orders} + \dots$$

F_{qg} = double parton distribution (DPD)
$\boldsymbol{b}$ = transverse distance between two partons
C = combinatorial factor

- X includes further radiation from each hard scattering at higher orders and particles from further "spectator" interactions
- also have contribution from triple hard scattering
  e.g. $q\bar{q} \to Z$, $gg \to \text{jet} + X$, $gg \to \text{jet} + X$

Multiparton interactions

- standard factorization formulae are for inclusive cross sections
- requires proper counting: an event with "3 jets" counts several times in $\sigma(pp \to Z + 2 \text{ jets} + X)$

  → M. Seymour, A. Siodmok 2013, talks yesterday + today

- computation of "exclusive cross sections" in general more complicated

  example: $pp \to Z + \text{exactly } 2 \text{ jets} + X$ with no further jet above $p_{T\text{cut}}$

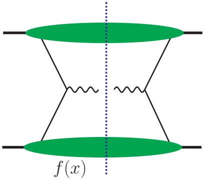
$$\begin{aligned}\sigma = {} & \sigma_{\text{single hard}}(pp \to Z + 2 \text{ jets} + X) - \sigma_{\text{single hard}}(pp \to Z + 3 \text{ jets} + X) \\ & + \sigma_{\text{double hard}}(pp \to Z + 2 \text{ jets} + X) - \sigma_{\text{double hard}}(pp \to Z + 3 \text{ jets} + X) \\ & - \sigma_{\text{triple hard}}(pp \to Z + 2 \text{ jets} + 2 \text{ jets} + X) - \ldots\end{aligned}$$

  where "jets" are required to have $p_T > p_{T\text{cut}}$

- if $p_{T\text{cut}} \ll$ other hard scales (e.g. m_Z) then
  - hardest scale for approximations is $p_{T\text{cut}}$, not m_Z
  - must resum Sudakov logarithms $\ln(p_{T\text{cut}}/m_Z)$ to all orders

Cross sections for definite transverse momenta

- standard factorization formulae have $\int$ over total transv. momentum produced in hard scattering
- example: Z production



- two possibilities to compute for measured q_T of Z
  both can be extended to double hard scattering
  needed for $d\sigma/dq_T$ and for $\sigma(q_T > q_{T\text{cut}})$

Cross sections for definite transverse momenta

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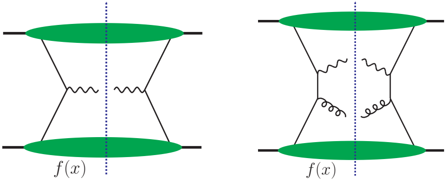
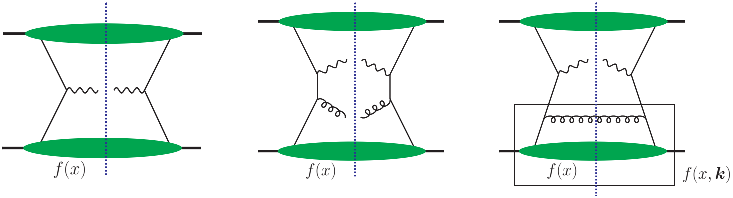
- example: Z production



- collinear factorization: compute emission of recoiling parton(s)
  - need $q_T \gg \Lambda$ since q_T is now a hard scale
  - for $q_T \ll m_Z$ find large Sudakov logs in q_T/m_Z $\rightsquigarrow$ must resum

Cross sections for definite transverse momenta

- standard factorization formulae have $\int$ over total transv. momentum produced in hard scattering
- example: Z production

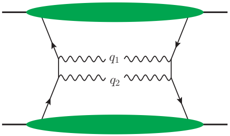


- for $q_T \ll m_Z$ have TMD factorization
  (only worked out for prod'n of color singlet particles)
  - use TMD $f(x, \boldsymbol{k})$ and $q\bar{q} \to Z$ without parton emission
  - for $q_T \gg \Lambda$ compute

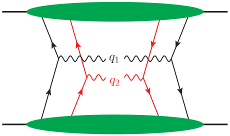
    $$f(x, \boldsymbol{k}) = \text{hard scattering} \otimes \text{collinear dist'n}$$
  - resummation of Sudakov logs with Collins-Soper evolution equation

Single versus double hard scattering: power behavior

- example: ZZ production, transverse Z momenta $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$



$$\frac{d\sigma}{d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} \sim \frac{1}{m_Z^4\, |\boldsymbol{q}_1 + \boldsymbol{q}_2|^2}$$



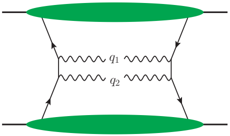
$$\frac{d\sigma}{d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} \sim \frac{\Lambda^2}{m_Z^4\, |\boldsymbol{q}_1|^2\, |\boldsymbol{q}_2|^2}$$

Λ^2 from $\int d^2\boldsymbol{b}\, F(\ldots,\boldsymbol{b}) F(\ldots,\boldsymbol{b})$

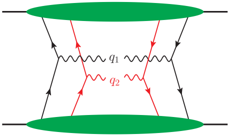
- no power suppression of double scatt. for $\boldsymbol{q}_i^2 \sim \Lambda^2$
  in addition double scatt. enhanced by small-x effects

Single versus double hard scattering: power behavior

- example: ZZ production, transverse Z momenta $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$



$$\frac{d\sigma}{d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} \sim \frac{1}{m_Z^4\, |\boldsymbol{q}_1 + \boldsymbol{q}_2|^2}$$



$$\frac{d\sigma}{d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} \sim \frac{\Lambda^2}{m_Z^4\, |\boldsymbol{q}_1|^2\, |\boldsymbol{q}_2|^2}$$

Λ^2 from $\int d^2\boldsymbol{b}\, F(\ldots,\boldsymbol{b}) F(\ldots,\boldsymbol{b})$

$$\int \frac{d^2(\boldsymbol{q}_1+\boldsymbol{q}_2)}{|\boldsymbol{q}_1+\boldsymbol{q}_2|^2} \sim 1 \qquad \int d^2\boldsymbol{q}_1 \sim m_Z^2$$

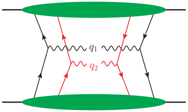
$$\Rightarrow\ \sigma \sim \frac{1}{m_Z^2}$$

$$\int \frac{d^2\boldsymbol{q}_i}{\boldsymbol{q}_i^2} \sim 1$$

$$\Rightarrow\ \sigma \sim \frac{\Lambda^2}{m_Z^4}$$

- $\int$ all transv. mom. $\Rightarrow$ double hard scattering power suppressed
  because has smaller transverse phase space than single hard scatt.

Double parton scattering: pocket formula



- if two-parton density factorizes as

$$F(x_1, x_2, \boldsymbol{b}) = f(x_1)\, f(x_2)\, G(\boldsymbol{b})$$

  where $f(x_i)$ = usual PDF

- if assume same $G(\boldsymbol{b})$ for all parton types
  then cross sect. formula turns into

$$\frac{d\sigma_{\text{double}}}{dx_1\, d\bar{x}_1\;dx_2\, d\bar{x}_2} = \frac{1}{C}\, \frac{d\sigma_1}{dx_1\, d\bar{x}_1}\; \frac{d\sigma_2}{dx_2\, d\bar{x}_2}\; \frac{1}{\sigma_{\text{eff}}}$$

  with $1/\sigma_{\text{eff}} = \int d^2\boldsymbol{b}\; G(\boldsymbol{b})^2$

  $\rightsquigarrow$ scatters are completely independent

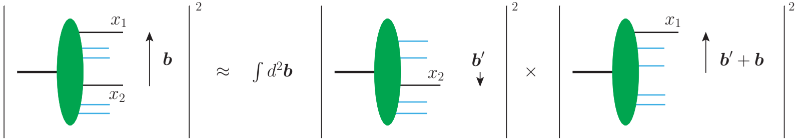
- derivation works including higher-order corrections to hard scattering
- pocket formula fails if any of the above assumptions is invalid
  or if further terms must be added to original expression of cross sect.
  (will encounter such terms later)

Parton correlations

- if neglect correlations between two partons

$$F(x_1, x_2, \boldsymbol{b}) = \int d^2\boldsymbol{b}'\ f(x_1, \boldsymbol{b}' + \boldsymbol{b})\, f(x_2, \boldsymbol{b}')$$

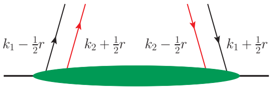
where $f(x_i, \boldsymbol{b})$ = impact parameter dependent single-parton density



- $f(x, \boldsymbol{b}) \xrightarrow{\text{Fourier trf}}$ generalized parton distribution at zero skewness studied in exclusive processes (e.g. at HERA) and on lattice

- with $F(x_1, x_2, \boldsymbol{b}) \xrightarrow{\text{Fourier trf}}$ generalized two-parton distribution have simple multiplicative structure Blok et al. 2011

$$F(x_1, x_2, \boldsymbol{r}) = f(x_1, \boldsymbol{r})\, f(x_2, -\boldsymbol{r})$$



Parton correlations

- if neglect correlations between two partons

$$F(x_1, x_2, \boldsymbol{b}) = \int d^2\boldsymbol{b}' \; f(x_1, \boldsymbol{b}' + \boldsymbol{b}) \, f(x_2, \boldsymbol{b}')$$

where $f(x_i, \boldsymbol{b})$ = impact parameter dependent single-parton density

- if neglect correlations between x and $\boldsymbol{b}$ of single parton

$$f(x_i, \boldsymbol{b}) = f(x_i) F(\boldsymbol{b})$$

with same $F(\boldsymbol{b})$ for all partons

then $G(\boldsymbol{b}) = \int d^2\boldsymbol{b}' \; F(\boldsymbol{b}' + \boldsymbol{b}) \, F(\boldsymbol{b}')$

Parton correlations

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then $G(\boldsymbol{b}) = \int d^2\boldsymbol{b}'\ F(\boldsymbol{b}' + \boldsymbol{b})\, F(\boldsymbol{b}')$

- for Gaussian $F(\boldsymbol{b})$ with average $\langle \boldsymbol{b}^2 \rangle$

$$\sigma_{\text{eff}} = 4\pi \langle \boldsymbol{b}^2 \rangle = 41\,\text{mb}\ \times \langle \boldsymbol{b}^2 \rangle / (0.57\,\text{fm})^2$$

determinations of $\langle \boldsymbol{b}^2 \rangle$ range from $\sim (0.57\,\text{fm} - 0.67\,\text{fm})^2$

is $\gg \sigma_{\text{eff}} \sim 10$ to $20\,\text{mb}$ from experimental extractions

if $F(\boldsymbol{b})$ is Fourier trf. of dipole then $41\,\text{mb} \to 36\,\text{mb}$

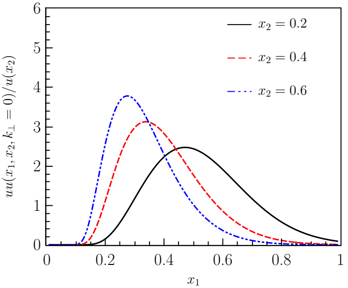
complete independence between two partons is disfavored
or something is systematically wrong with σ_{eff} extractions

cf. Calucci, Treleani 1999; Frankfurt, Strikman, Weiss 2003, 2004

Correlations involving x

- $F(x_1, x_2, \boldsymbol{b}) = f(x_1)\, f(x_2)\, G(\boldsymbol{b})$ cannot hold for all x_1, x_2
- most obvious: energy conservation $\Rightarrow \; x_1 + x_2 \leq 1$
  often used: $F(x_1, x_2, \boldsymbol{b}) = f(x_1)\, f(x_2)\, (1 - x_1 - x_2)^n\, G(\boldsymbol{b})$
  to suppress region of large $x_1 + x_2$
- significant x_1 – x_2 correlations found in constituent quark model

Rinaldi, Scopetta, Vento: arXiv:1302.6462
talk on Thursday



plot shows $\int d^2\boldsymbol{b}\, F_{uu}(x_1, x_2, \boldsymbol{b}) / f_u(x_2)$
is x_2 independent if factorization holds

- unknown: size of correlations when one or both of x_1, x_2 small

Correlations involving x and $\boldsymbol{b}$

- have some knowledge of single-parton distribution $f(x, \boldsymbol{b})$ from studies of parton distributions (exclusive processes, lattice, theory)
  - HERA results on $\gamma p \to J/\Psi\, p$ give

    $$\langle \boldsymbol{b}^2 \rangle \propto \text{const} + 4\alpha' \log(1/x)$$

    with $\alpha' \approx 0.15\,\mathrm{GeV}^{-2} = (0.08\,\mathrm{fm})^2$ for gluons at $x \sim 10^{-3}$
    $\to$ weak but nonzero correlation between x and $\boldsymbol{b}$
  - lattice simulations $\to$ strong decrease of $\langle \boldsymbol{b}^2 \rangle$ with x above ~ 0.1 seen by comparing moments $\int dx\, x^{n-1} f(x, \boldsymbol{b})$ for $n = 0, 1, 2$

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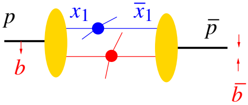
  - lattice simulations $\to$ strong decrease of $\langle \boldsymbol{b}^2 \rangle$ with x above ~ 0.1 seen by comparing moments $\int dx\, x^{n-1} f(x,\boldsymbol{b})$ for $n = 0, 1, 2$
- expect similar correlations between x_i and $\boldsymbol{b}$ in two-parton dist's even if $F(x_1,x_2,\boldsymbol{b}) = \int d^2\boldsymbol{b}'\; f(x_1,\boldsymbol{b}'+\boldsymbol{b})\, f(x_2,\boldsymbol{b}')$ does not hold
- if interaction 1 produces high-mass system

  $\to$ have large $x_1, \bar{x}_1$

  $\to$ smaller $\boldsymbol{b}$, more central collision

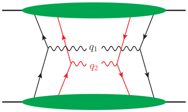
  $\to$ secondary interactions enhanced



Frankfurt, Strikman, Weiss 2003

study in Pythia 8: Corke, Sjöstrand 2011 $\to$ tunes 4C, 4C*

Spin correlations

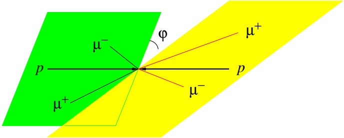


- polarizations of two partons can be correlated even in unpolarized target — already pointed out by Mekhfi (1985)
  - quarks: longitudinal and transverse pol.
  - gluons: longitudinal and linear pol.
- how important are spin correlations inside proton?
  - large effects expected in valence quark region
    study in bag model: Chang, Manohar, Waalewijn 2012, talk on Thursday
  - how fast do correlations decrease with evolution scale?
    T. Kasemets, talk on Thursday
- can be included in factorization formula

$$\text{e.g.} \quad F_{\bar{q}g}\, F_{\bar{q}g}\, \sigma(q\bar{q} \to Z)\, \sigma(gg \to 2 \text{ jets}) + F_{\Delta\bar{q}\Delta g}\, F_{\Delta\bar{q}\Delta g}\, \Delta\sigma(q\bar{q} \to Z)\, \Delta\sigma(gg \to 2 \text{ jets})$$

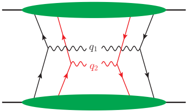
- if spin correlations are large → large effects for rate and final state distributions of double hard scattering

Spin correlations



- detailed calc'n for gauge boson pair production followed by leptonic decay
  T. Kasemets, MD 2012; see also A. Manohar, W. Waalewijn 2011
- longitudinal quark spin correlations
  ⇝ overall rate and distribution of lepton rapidities and p_T
- transverse quark spin correlations
  ⇝ azimuthal correlation between lepton planes
  ⇝ two hard scatters are not independent
- expect similar effects for other processes (esp. for jets)
- note: independent scattering planes sometimes assumed as criterion to characterize double parton scattering

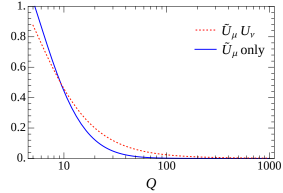
Color structure



- quark lines in amplitude and its conjugate can couple to color singlet or octet:

$$^1F \to (\bar{q}_2 \mathbb{1} q_2)\,(\bar{q}_1 \mathbb{1} q_1) \qquad\qquad ^8F \to (\bar{q}_2\, t^a q_2)\,(\bar{q}_1\, t^a q_1)$$

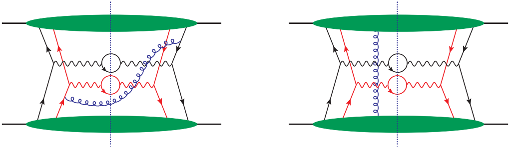
- ^8F describes color correlation between quarks 1 and 2 is essentially unknown (no probability interpretation as a guide)
- for two-gluon dist's more color structures: 1, 8_S, 8_A, 10, $\overline{10}$, 27
- for k_T integrated distributions:
  color correlations suppressed by Sudakov logarithms
  but not necessarily negligible for moderately hard scales



← Manohar, Waalewijn, arXiv:1202.3794
talk on Thursday

Double parton scattering: towards factorization

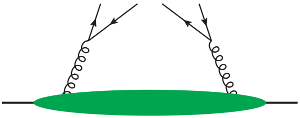
- no complete proof of factorization yet
  several elements worked out four double Drell-Yan process
  MD, Ostermeier, Schäfer 2011; Mahorhar, Waalewijn 2012
- major issue: soft gluon exchange between hard-scattering processes and between spectators



  - with soft-gluon approximation → Sudakov logarithms, describe using Wilson lines
  - in Glauber region $(l_T^2 \gg l^+ l^-)$: open problem
    → D. Ostermeier, talk this afternoon
  - soft gluon exchange $\overset{?}{\leftrightarrow}$ color reconnection in Monte Carlos

Behavior at small interparton distance

- for $\boldsymbol{b} \ll 1/\Lambda$ in perturbative region $F(x_1, x_2, \boldsymbol{b})$ is dominated by graphs with splitting of single parton



- find strong correlations in x_1, x_2, spin and color between two partons
  e.g. 100% correlation for longitudinal pol. of q and $\bar{q}$

- can compute short-distance behavior:

$$F(x_1, x_2, \boldsymbol{b}) \sim \frac{1}{\boldsymbol{b}^2} \text{ splitting fct} \otimes \text{usual PDF}$$

Scale evolution

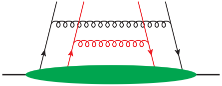
consider only distributions for partons without color correlation

- if define DPD from renormalized twist-two operators $\mathcal{O}$ in analogy with usual PDFs

$$F(x_1, x_2, \boldsymbol{b}; \mu) \sim \langle p | \mathcal{O}_1(\boldsymbol{0}; \mu)\, \mathcal{O}_2(\boldsymbol{b}; \mu) | p \rangle \qquad f(x; \mu) \sim \langle p | \mathcal{O}(\boldsymbol{0}; \mu) | p \rangle$$

$\Rightarrow$ for $F(x_i, \boldsymbol{b})$ at $\boldsymbol{b} \neq \boldsymbol{0}$ have

separate DGLAP evolution for partons 1 and 2



$$\frac{d}{d \log \mu} F(x_i, \boldsymbol{b}) = P \otimes_{x_1} F + P \otimes_{x_2} F$$

μ dep'ce of DPD $\leftrightarrow$ μ dep'ce of hard-scattering procs. at higher order

if $\boldsymbol{b}$ dep'ce does not factorize must compute evolution for each $\boldsymbol{b}$

Scale evolution

consider only distributions for partons without color correlation

- if define DPD from renormalized twist-two operators $\mathcal{O}$ in analogy with usual PDFs

$$F(x_1, x_2, \boldsymbol{b}; \mu) \sim \langle p | \mathcal{O}_1(\boldsymbol{0}; \mu)\, \mathcal{O}_2(\boldsymbol{b}; \mu) | p \rangle \qquad f(x; \mu) \sim \langle p | \mathcal{O}(\boldsymbol{0}; \mu) | p \rangle$$

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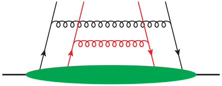
separate DGLAP evolution for partons 1 and 2



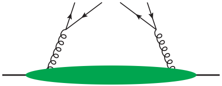
$$\frac{d}{d \log \mu} F(x_i, \boldsymbol{b}) = P \otimes_{x_1} F + P \otimes_{x_2} F$$

μ dep'ce of DPD $\leftrightarrow$ μ dep'ce of hard-scattering procs. at higher order

- $\int d^2\boldsymbol{b}\, F(x_i, \boldsymbol{b})$:

extra term from $2 \to 4$ parton transition

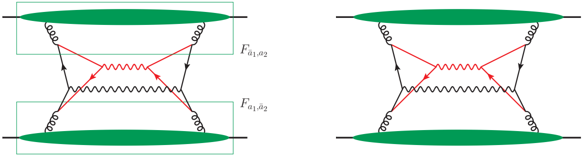
since $F(x_i, \boldsymbol{b}) \sim 1/\boldsymbol{b}^2$



Kirschner 1979; Shelest, Snigirev, Zinovev 1982

Gaunt, Stirling 2009; Ceccopieri 2011

Deeper problems with the splitting graphs



- contribution from splitting graphs in cross section gives divergent integrals $\int d^2\boldsymbol{b}\, F(x_1, x_2, \boldsymbol{b})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{b}) \sim \int d\boldsymbol{b}^2/\boldsymbol{b}^4$
- double counting problem between double scattering with splitting and single scattering at loop level

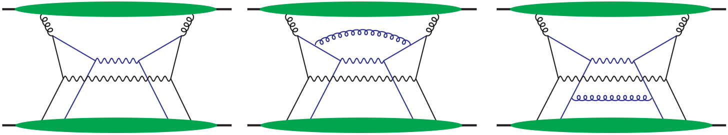
MD, Ostermeier, Schäfer 2011; Gaunt, Stirling 2011; Gaunt 2012
Blok, Dokshitzer, Frankfurt, Strikman 2011; Ryskin, Snigirev 2011, 2012
same problem for jets: Cacciari, Salam, Sapeta 2009

- need consistent separation of physics at small and large $\boldsymbol{b}$

"What is double parton scattering?"

solution will also determine evolution equations for DPDs

Deeper problems with the splitting graphs



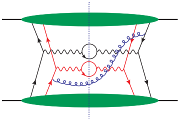
- also have graphs with short-distance splitting on one side but not on the other — B. Blok et al. 2011-13; J. Gaunt 2012
  two talks on Thursday
- nomenclature:

2 vs. 1	Gaunt, Stirling	
$3 \to 4$	Blok et al.	(s channel counting, for double dijets)
4×2	Diehl, Ostermeier, Schäfer	(t channel counting)

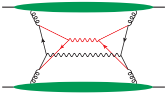
Summary

- double hard scattering $\leftrightarrow$ high-multiplicity final states
  $\rightsquigarrow$ even theoretical control of single hard scattering non-trivial
  $\rightsquigarrow$ carefully consider choice of observables:
  theoretical control $\leftrightarrow$ sensitivity to multiple scattering
- various two-parton correlations can affect
  rate and kinematic distributions of double parton interactions
  - correlations in $\boldsymbol{b}$ dep'ce, between x_1, x_2, and between x_1, x_2 and $\boldsymbol{b}$
  - correlations in spin and color
    $\rightsquigarrow$ new DPDs, not included in usual double scattering formula

  problematic to use templates that assume
  "double hard scattering = two independent single scatterings"
- soft gluon exchange: partially understood



- at small $\boldsymbol{b}$ DPDs are dominated by splitting graphs



  - give strong correlations in x_1, x_2, spin and color
  - open problem: separate dynamics at small and large $\boldsymbol{b}$