# Drell-Yan at small *x* via k_T-cut; treatment of the infrared region

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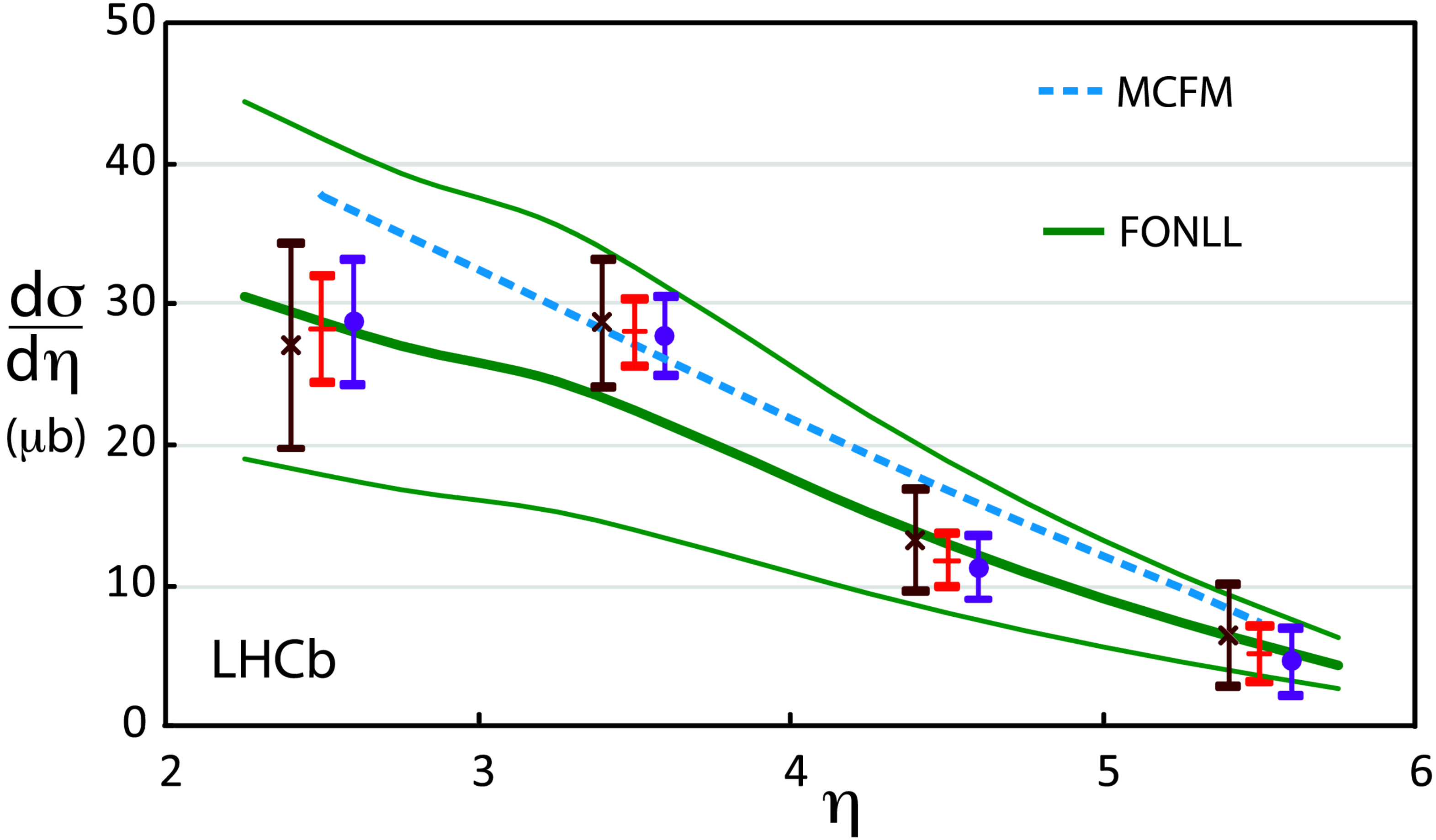
# Motivation – LHC

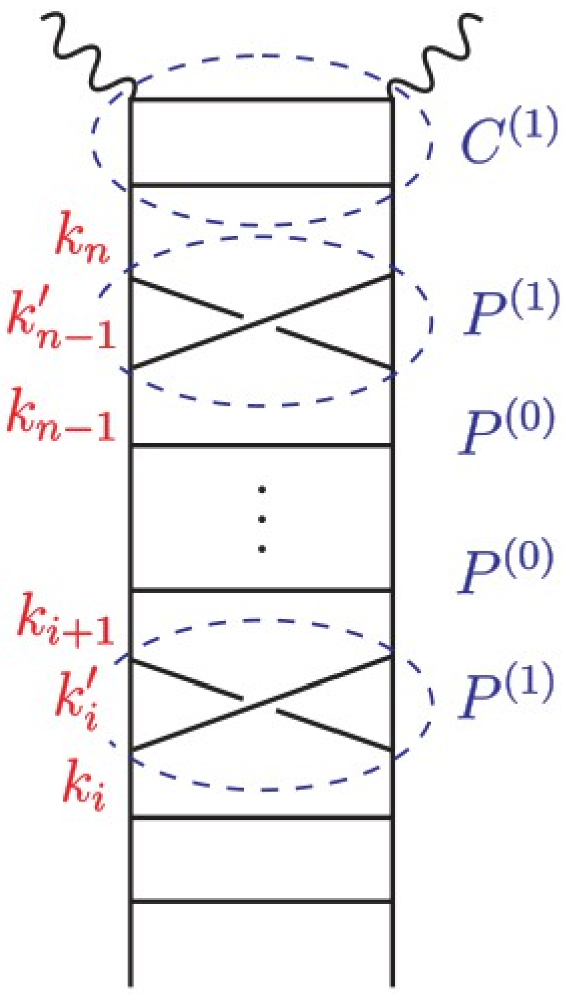
- LHC is a hadron collider at energies of 8 and 13 TeV.
- Parton distributions (quarks and gluons) are not fully determined.
- Inclusive single parton distributions are used in most observables.
- They are used as bulding blocks for double parton distributions  in multiple parton interactions.

# Factorization scale dependence

- LHCb – Phys.Lett.B694:209-216,2010



# DGLAP evolution



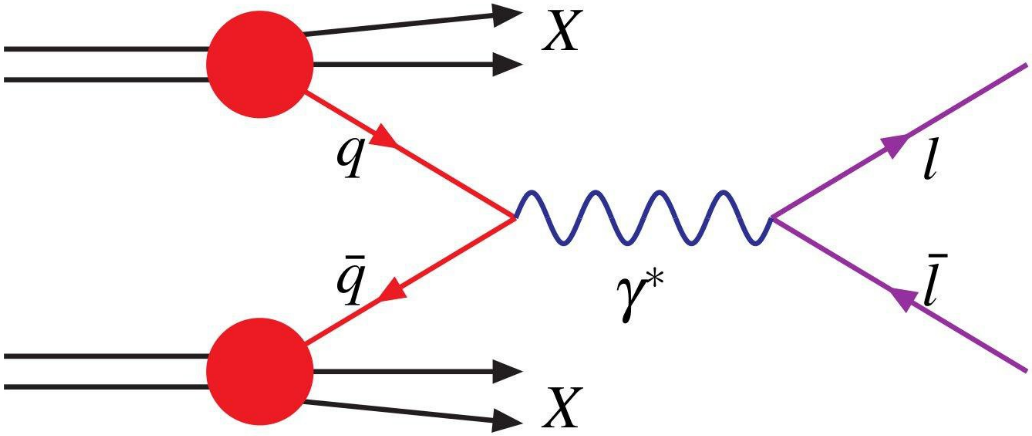

- DGLAP resumms large logarithms in the factorization scale

$$x_i > x_{i+1}$$

$$k_{i,t} \ll k_{i+1,t}$$

- For small *x*, there is a high probability of multiple emissions

# Drell-Yan production




- Drell–Yan cross section

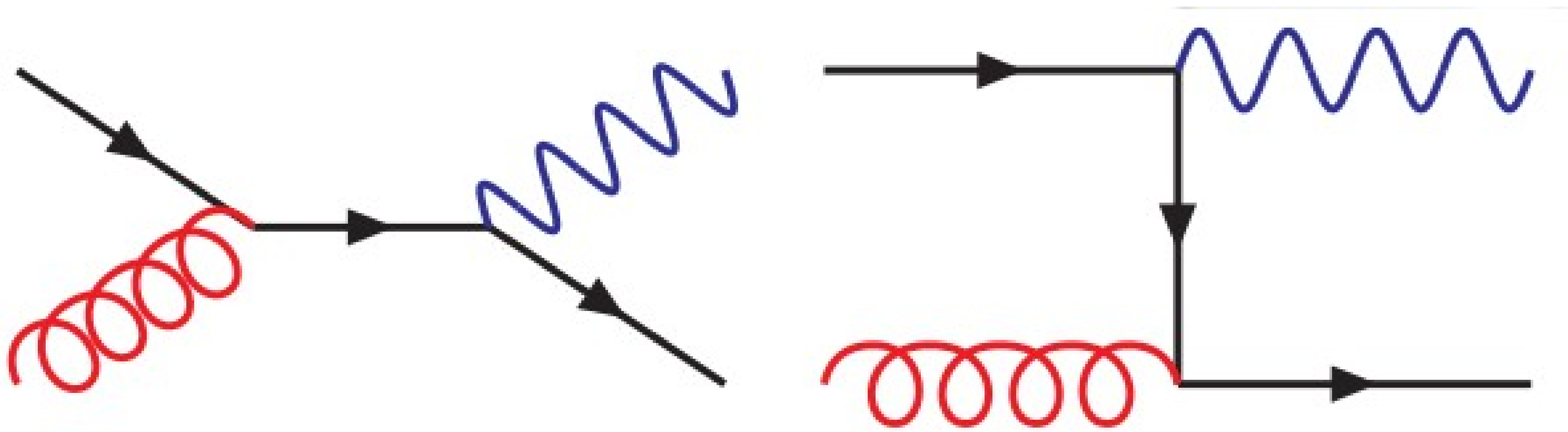
$$d\sigma/d^3p = \int dx_1 dx_2 \,\mathrm{PDF}(x_1, \mu_F)\, |\mathcal{M}(p; \mu_F, \mu_R)|^2 \,\mathrm{PDF}(x_2, \mu_F) \ ,$$

- *x* variables

$$x_{1,2} = \frac{m_{\mathrm{hard}}}{\sqrt{s}} \exp(\pm y)$$

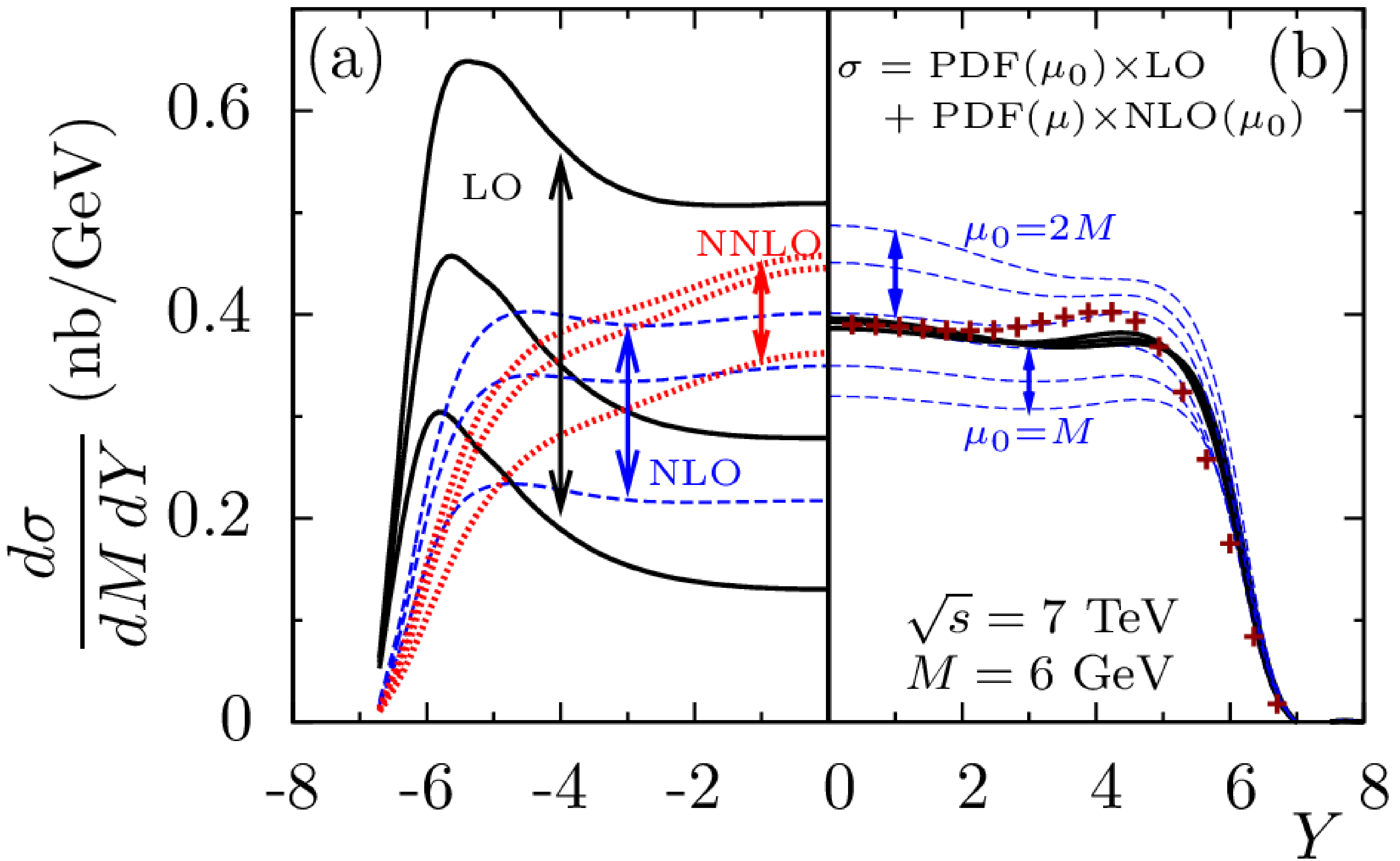
# Choosing the optimal factorization scale for small *x*

- Choose the optimal scale that resumms all contribution from dominant NLO diagrams:



- Resumms $1/t$ divergence.
- Assume gluon flux factor of $1/x$

# DY without k_t cut
# \mu_0 = 1.45 M



# Introducing a k_t cut

- Introduce a **cutoff** on transverse momentum of the lepton pair:

$$k_t < k_0$$

- Properly account for **Sudakov** effects.

# Optimal scale from NLO matrix element

- Introducing the cut in the NLO matrix element:

$$z\left[z^2+(1-z)^2\right]\ln\frac{\mu_0^2}{|t_0|}=z\left[\left((1-z)^2+z^2\right)\left(\ln\frac{M^2}{|t_0|}+\ln\frac{1-z}{z}\right)+\frac{1+3z}{2}(1-z)-\mathcal{I}(z,k_0)\right]$$

- with the cutoff contribution:

$$\mathcal{I}=\Theta(z_{\max}-z)\left[2(1+2z^2-2z)\tanh^{-1}\left(\sqrt{1-\frac{4z}{(1-z)^2}\frac{k_0^2}{M^2}}\right)+\frac{1+3z}{2}\sqrt{(1-z)^2-4z\frac{k_0^2}{M^2}}\right]$$

- and maximum *z* of cutoff:

$$z_{\max}=1+2\frac{k_0^2}{M^2}-2\sqrt{\frac{k_0^2}{M^2}\left(\frac{k_0^2}{M^2}+1\right)}$$

# Accounting for Sudakov

- In the evolution, one must **not allow** emission with **k_t > k_0**.

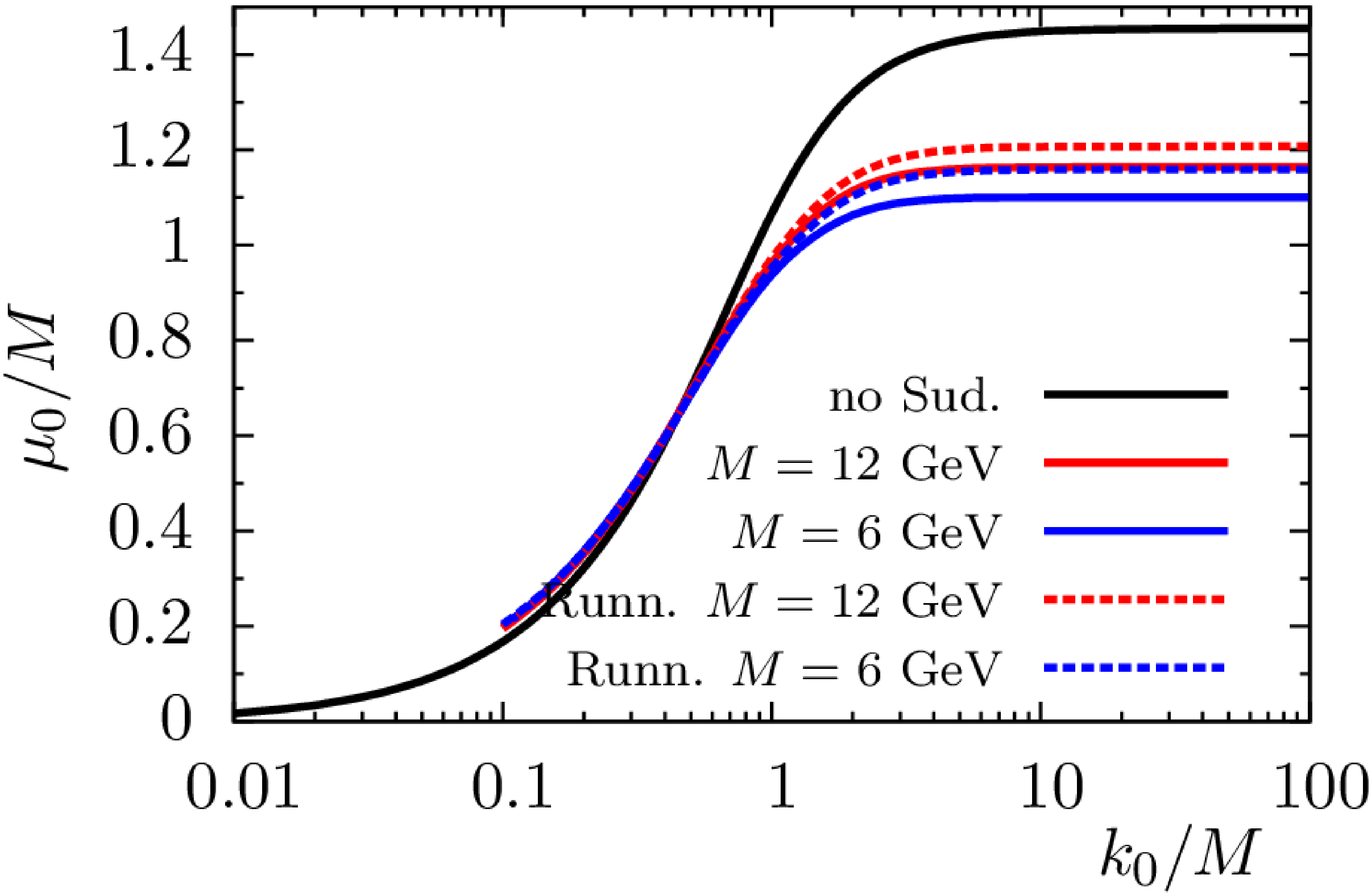
- One does that with the **Sudakov factor**:

$$T_q(k_t^2, \mu^2) \equiv \exp\left( - \int_{k_t^2}^{\mu^2} \frac{\mathrm{d}\kappa_t^2}{\kappa_t^2} \frac{\alpha_S(\kappa_t^2)}{2\pi} \int_0^1 \frac{\mathrm{d}z}{z} \, z \, P_{qq}(z) \Theta(1 - z - \Delta) \right)$$

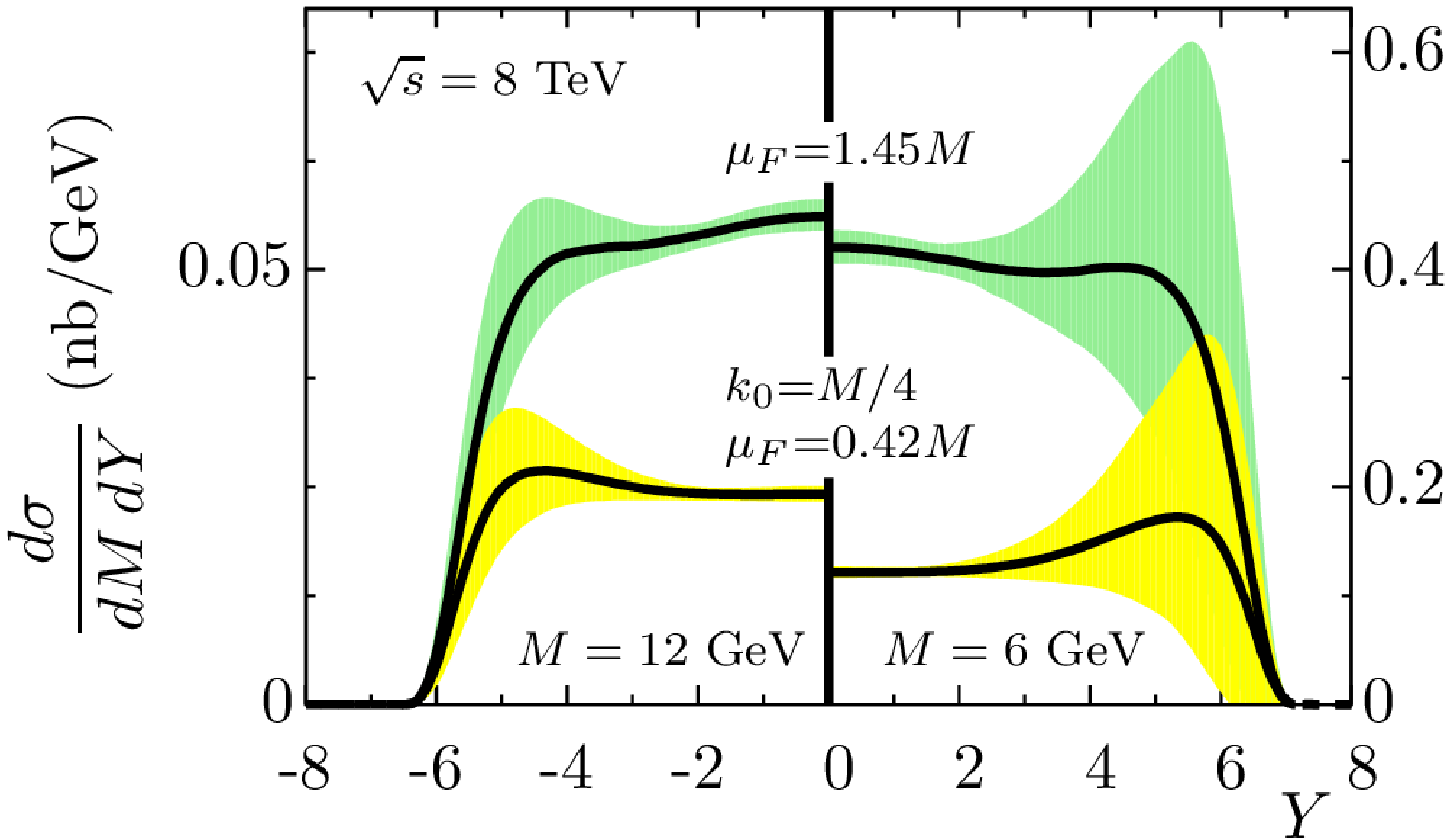
- One for each incoming parton, with infrared cutoff:

$$\Delta \equiv \frac{k_t}{\mu + k_t}$$

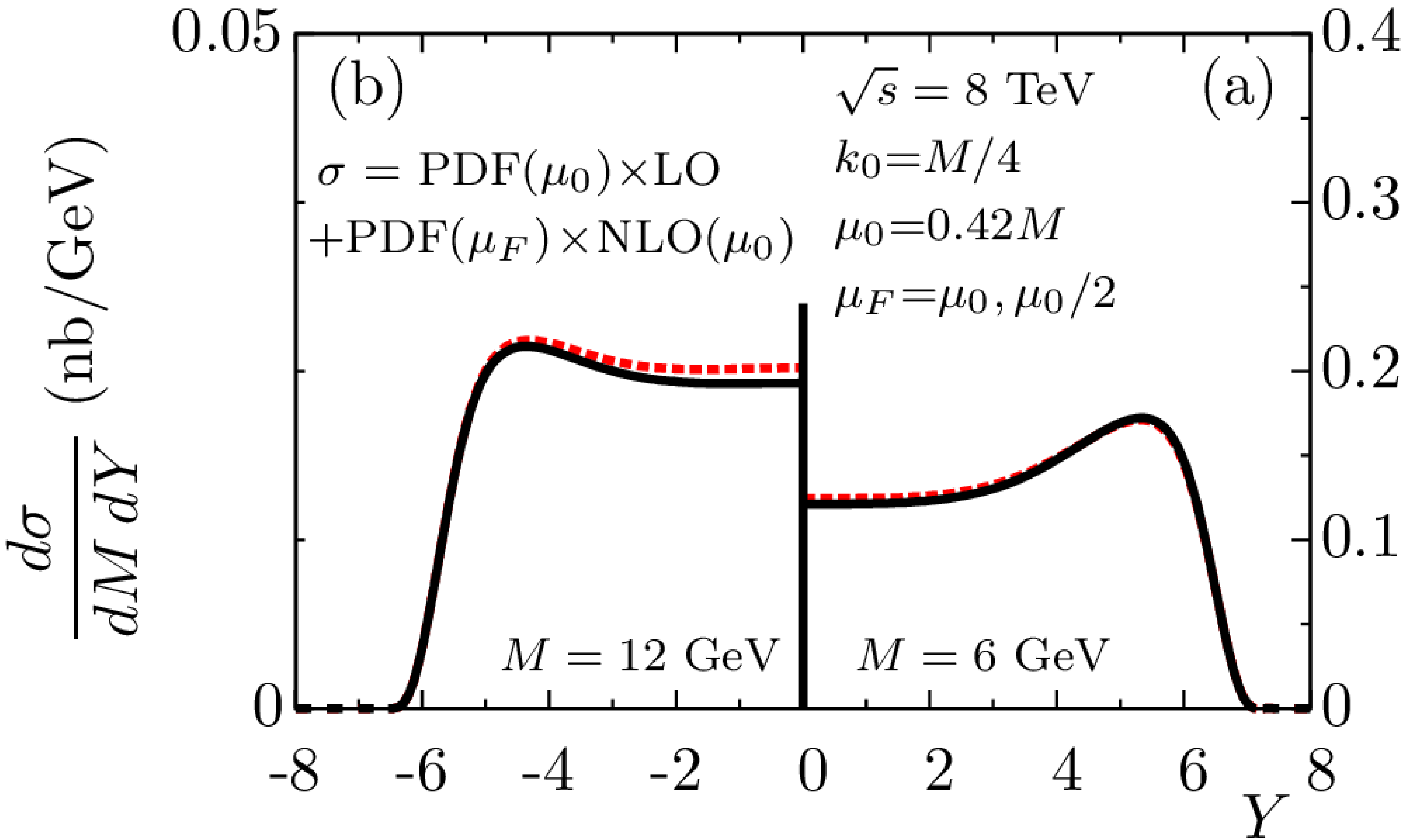
# Sudakov effects



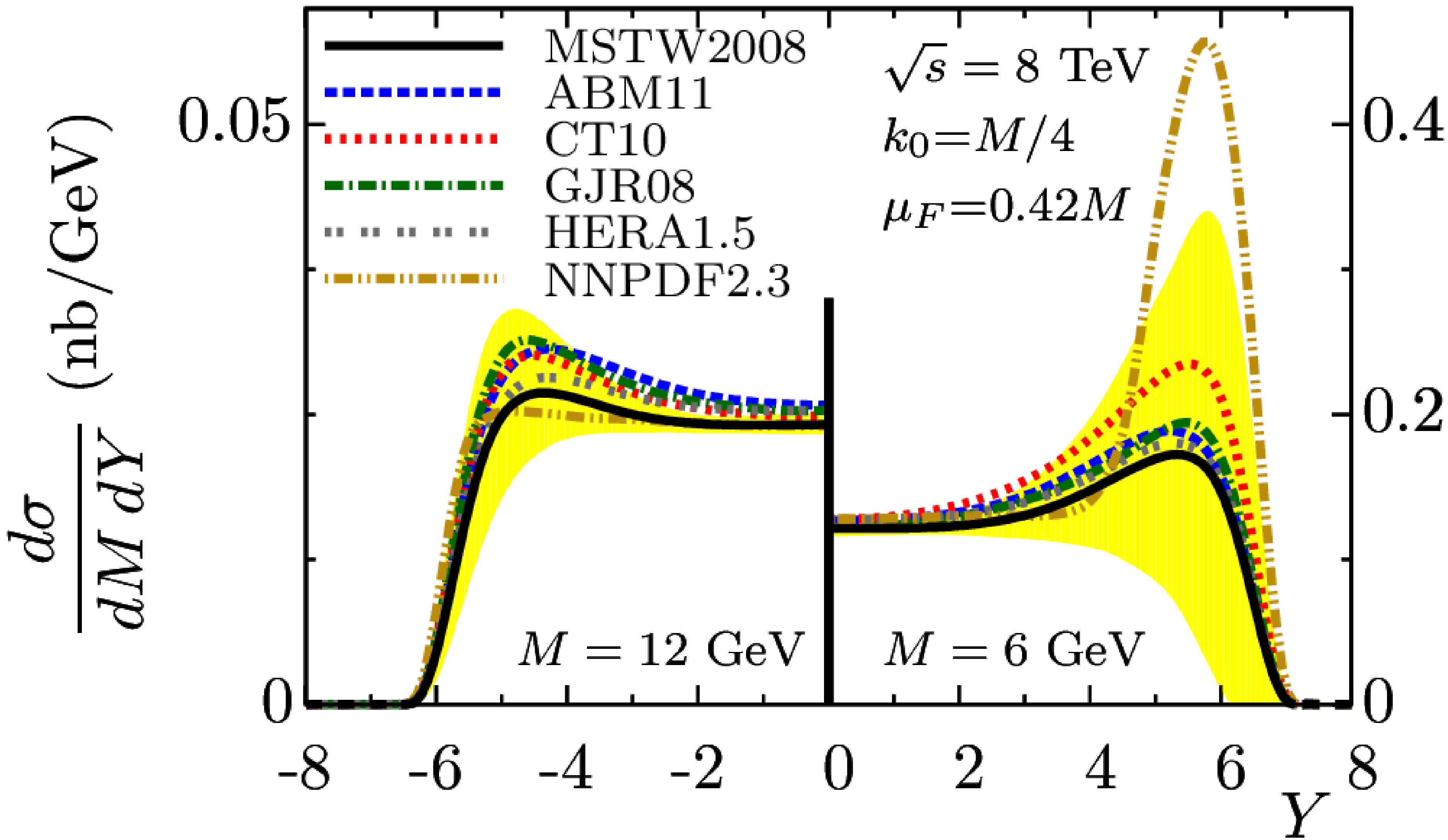
# Predictions and uncertainty



# Factorization scale variation



# Parton distribution functions



# Defining partons in the MSbar scheme

- Coefficient function are not infrared finite at NLO
- Using dimensional regularization:

$$\frac{A_0}{\epsilon} + A_1 + \epsilon A_2 + \cdots$$

- Conventional approach: kills divergence and set N=4:

$$A_1$$

# Physical partons

- DGLAP also changes with dimensional regularization:

$$\frac{A_0}{\epsilon} + B_1 + \epsilon B_2 + \cdots$$

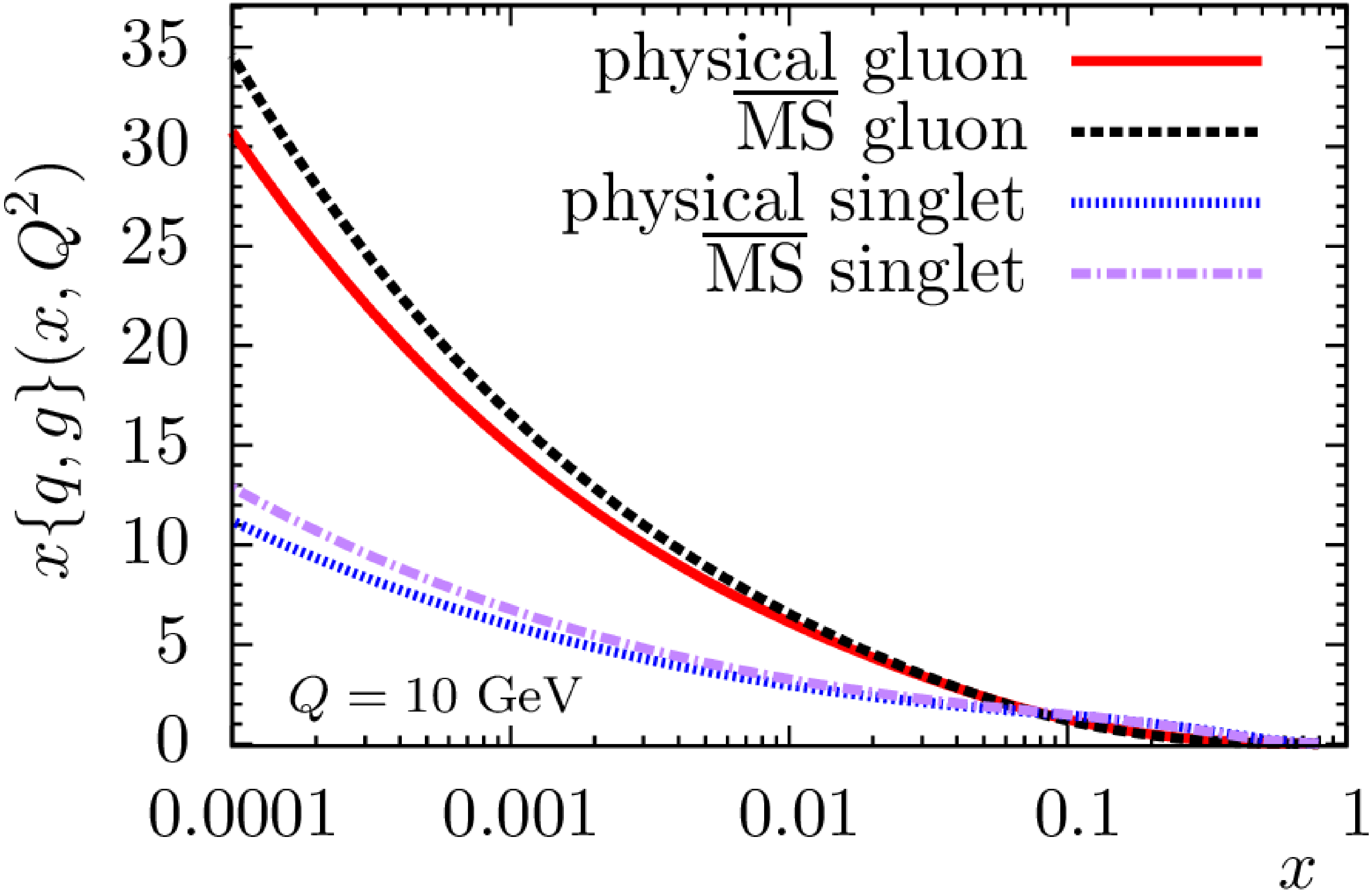
- Subtract DGLAP from coefficient functions and set N=4 (physical scheme):

$$A_1 - B_1$$

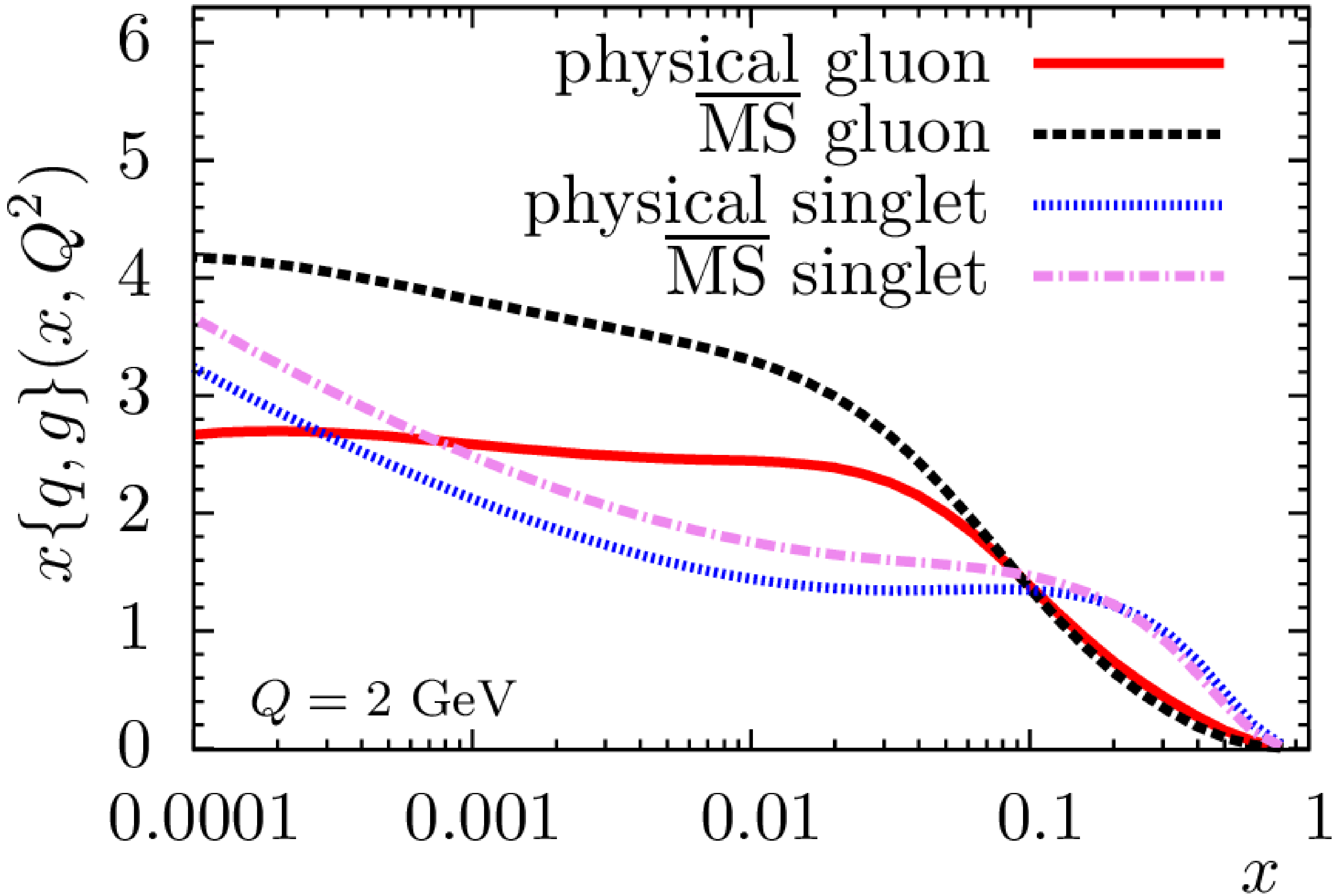
# Redefining partons

- Partons can be redefined using a scheme change.
- The scheme change works for inclusive observables (factorization theorem)
- For exclusive observables this does not work
- Difficult interpretation of the factorization scale
- Convergence may be and issue
- With the scheme change, NLO splitting functions will be redefined as well

# Physical partons at 10 GeV

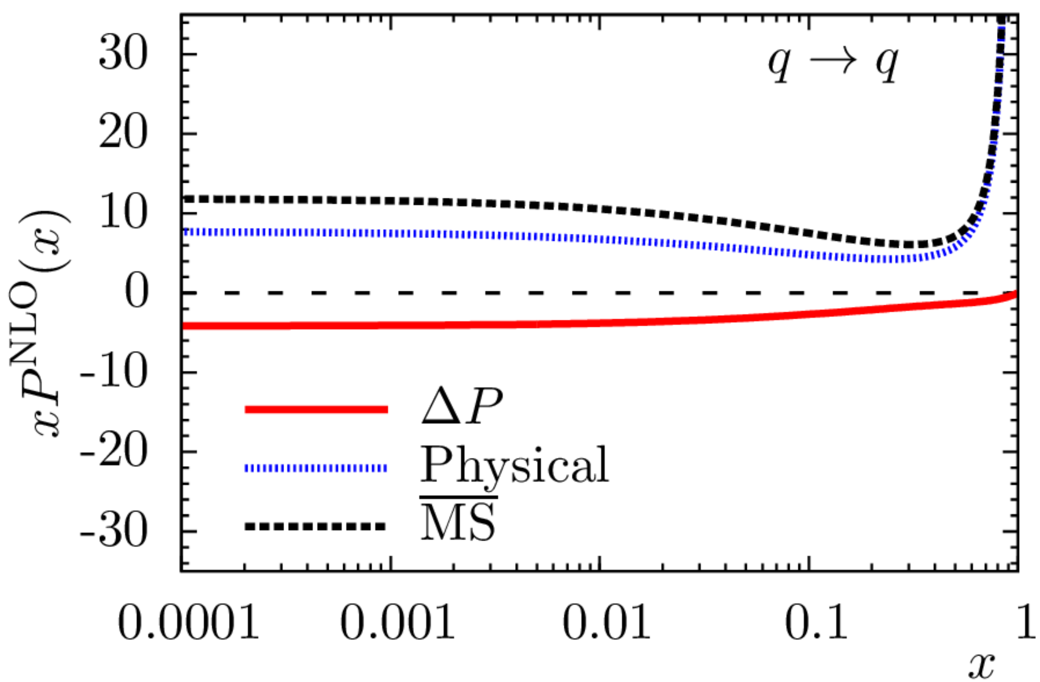


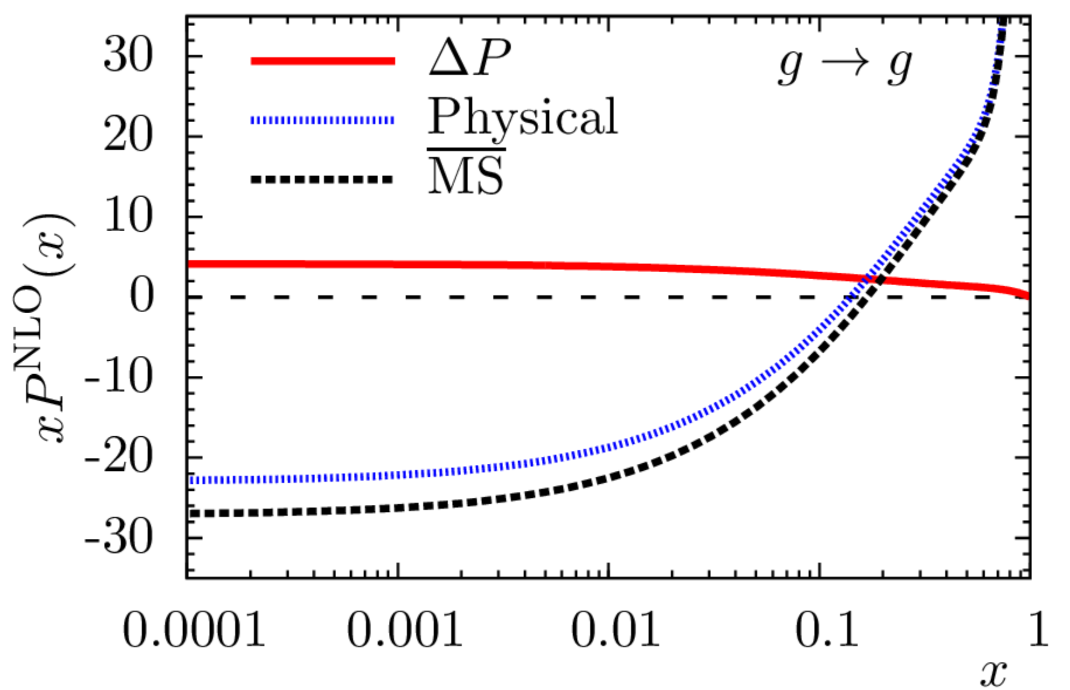
# Physical partons at 2 GeV



# Physical diagonal NLO splitting functions

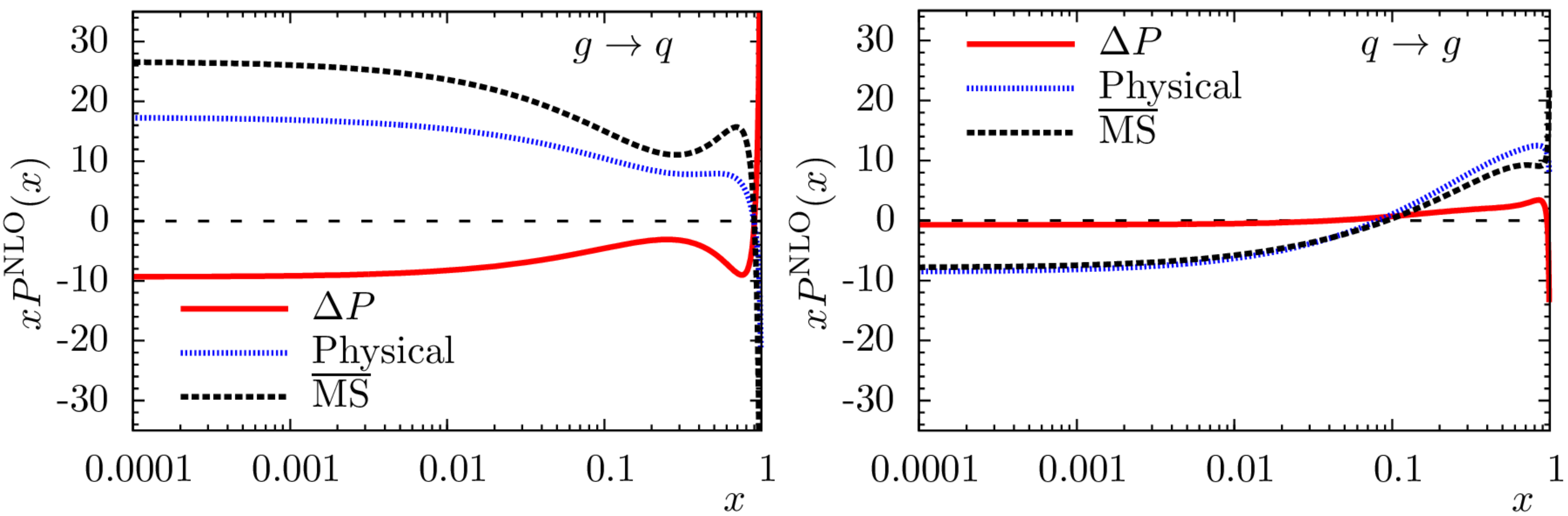
- Opposite sign difference
- Decrease in magnitude at small *x*
- Decrease (for qq) or increase (for gg) at large *x*



# Physical nondiagonal NLO splitting functions

- Larger change (decrease in magnitude) for g → q

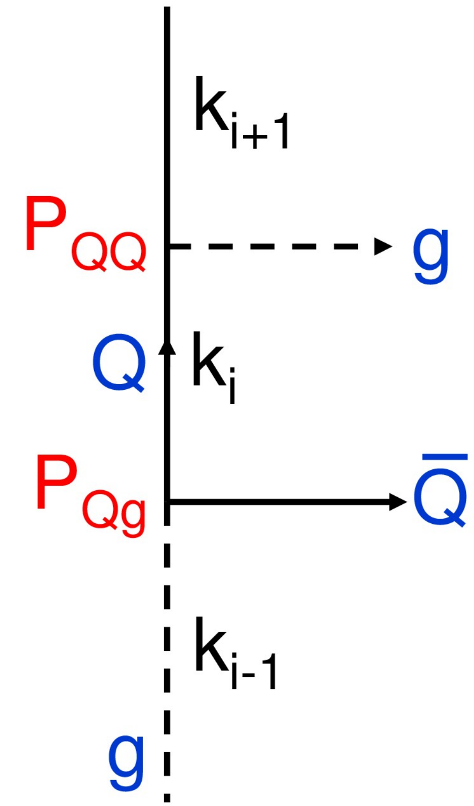


# Heavy quarks evolution

- Usually quarks are taken as massless.
- GM-VFNS (General mass – variable flavour number scheme)

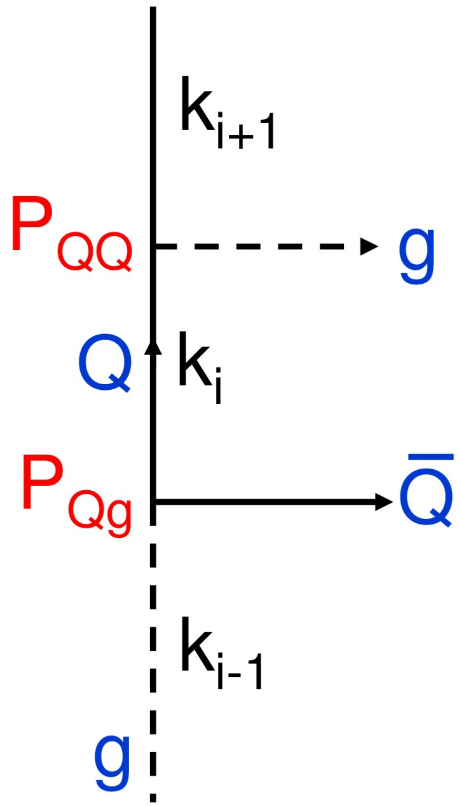
  → quarks as massless for scale bigger than quark mass

  → zero contribution for scale smaller than the quark mass



# Heavy quarks evolution

- GM-VFNS is justified at LO
- Produces kinks at quark masses.
- We want a smooth behaviour
- Only one step needs to be corrected




$$\cdots \int \frac{dk_{i-1}^2}{k_{i-1}^2} \int \frac{dk_i^2 \; k_i^2}{(k_i^2 + m_{\mathtt{Q}}^2)^2} \int \frac{dk_{i+1}^2}{k_{i+1}^2} \cdots$$

# g → Q splitting function

$$P_{\mathtt{Q}g}(z,Q^2) = T_R\left([z^2+(1-z)^2]\frac{Q^2}{m_{\mathtt{Q}}^2+Q^2} + \frac{2m_{\mathtt{Q}}^2Q^2z(1-z)}{(Q^2+m_{\mathtt{Q}}^2)^2}\right)\Theta\left(Q^2-\frac{zm_{\mathtt{Q}}^2}{1-z}\right)$$

- Color factor, *z*, scale, quark mass
- No divergence!
- Correct high scale limit
- Split flip contribution
- Step function (putting the emitted heavy quark on shell)

# Conclusions

- New global analyses are in need.
- Taking into account

  → physical partons

  → heavy quark effects
- From this, many opportunities to work on refining the factorization to better describe gluons and quarks

  → optimal scales

# Thank you

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- Thanks the IPPP at the Durham University for hospitality.

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# Other splitting functions

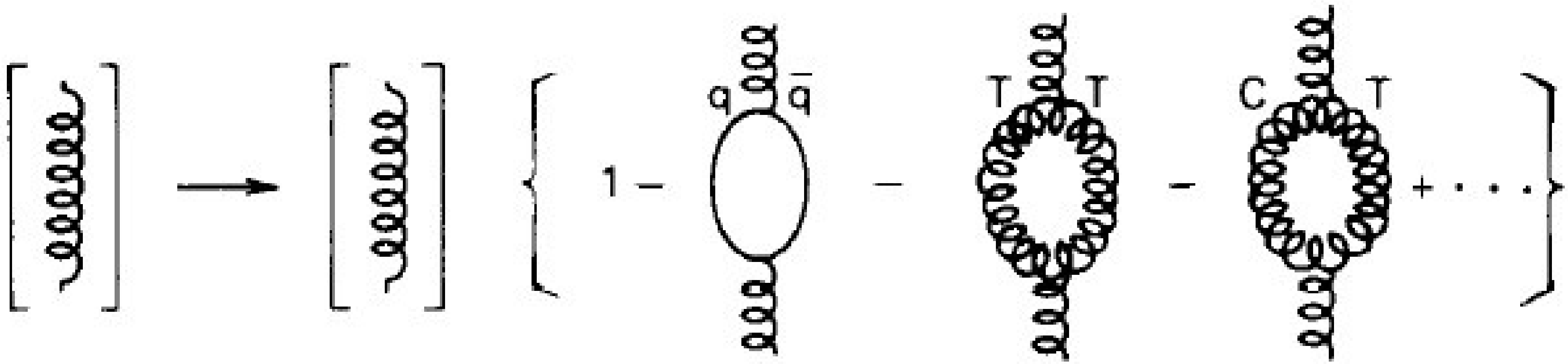
- Momentum conservation
- Intrinsic heavy flavour

$$\delta P_{gg} = -\delta(1-z)\sum_{\mathtt{Q}}\int_0^{z_{\mathtt{Q}}} P_{\mathtt{Q}g}(z', Q^2)dz'$$

$$P_{\mathtt{QQ}}(z, Q^2) = C_F\left(\frac{1+z^2}{1-z}\frac{Q^2}{m_{\mathtt{Q}}^2+Q^2} + \frac{z(1-3z)}{1-z}\frac{Q^2 m_{\mathtt{Q}}^2}{(Q^2+m_{\mathtt{Q}}^2)^2}\right)$$

$$P_{g\mathtt{Q}}(z, Q^2) = C_F\left(\frac{1+(1-z)^2}{z}\frac{Q^2}{m_{\mathtt{Q}}^2+Q^2} + \frac{z^2+z-2}{z}\frac{Q^2 m_{\mathtt{Q}}^2}{(Q^2+m_{\mathtt{Q}}^2)^2}\right)\Theta\left(Q^2 - \frac{z m_{\mathtt{Q}}^2}{1-z}\right)$$

# Running of the coupling




- Conventional approach
- Active number of light (zero mass) flavours

  = 3 for scales between strange and charm masses

  = 4 for scales between charm and bottom masses

  = 5 etc...

# Running equation

$$\frac{d}{d\ln Q^2}\left(\frac{\alpha_s}{4\pi}\right) = -\beta_0\left(\frac{\alpha_s}{4\pi}\right)^2 - \beta_1\left(\frac{\alpha_s}{4\pi}\right)^3$$

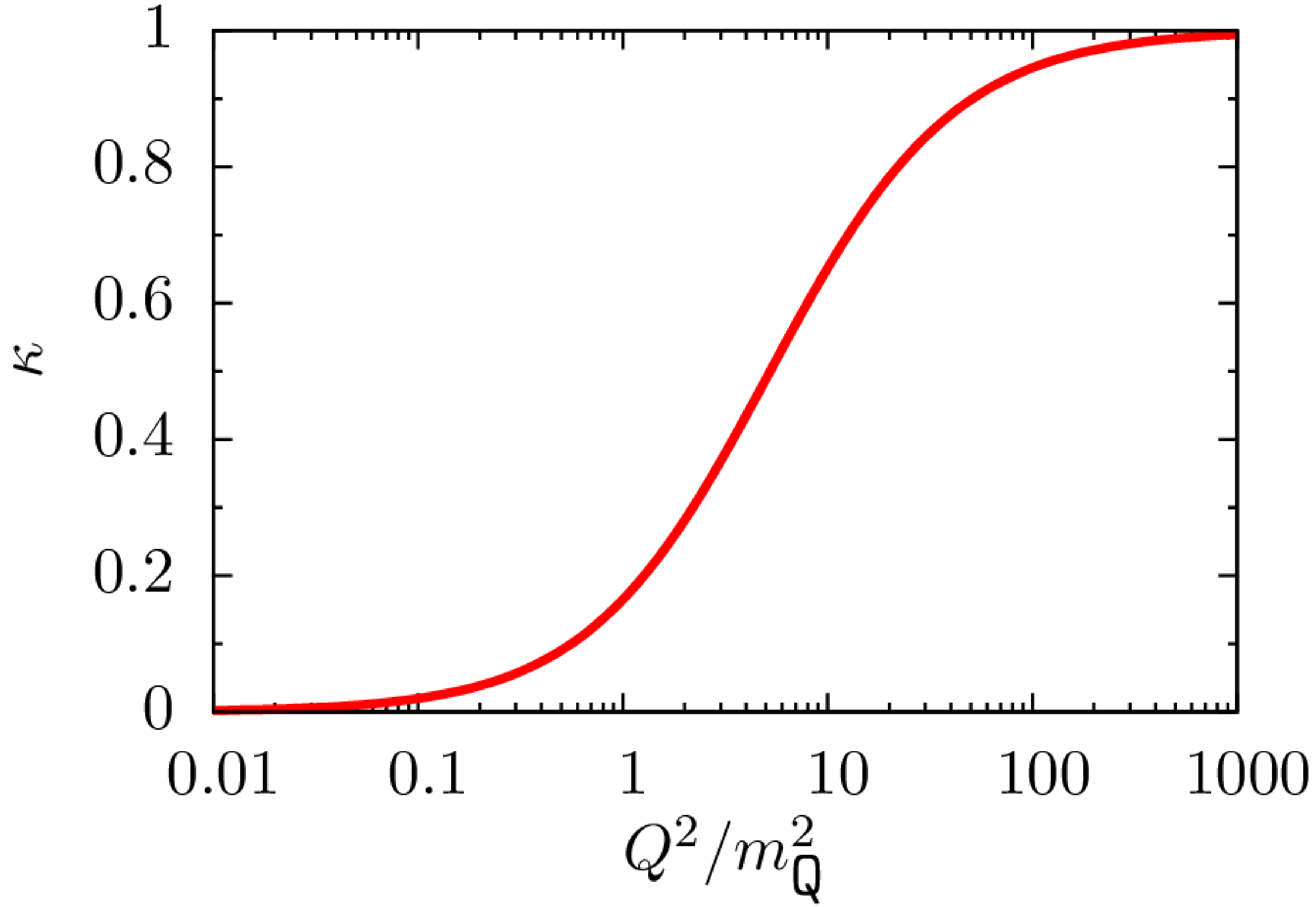
$$\beta_0(n_f) = 11 - \frac{2}{3}n_f, \qquad \beta_1(n_f) = 102 - \frac{38}{3}n_f$$

(Our work) Smooth transition: multiply each unit in n_F by:

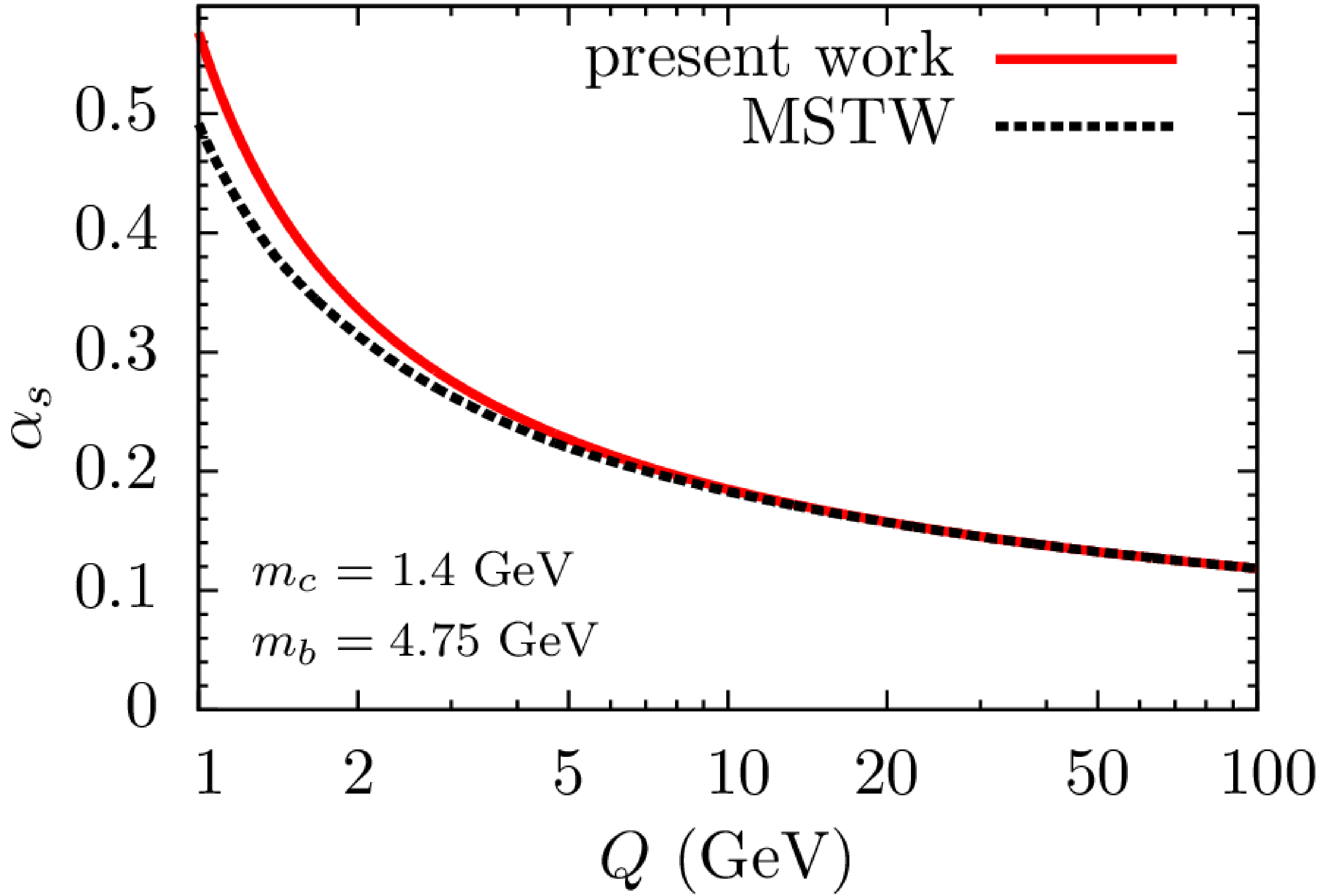
$$r = m_Q^2/Q^2$$

$$\kappa(r) = \left[1 - 6r + 12\frac{r^2}{\sqrt{1+4r}}\ln\frac{\sqrt{1+4r}+1}{\sqrt{1+4r}-1}\right]$$

# Smooth running



# Change in the running



# Ratio of change