Critical nuclei in the superstrong magnetic fields

M.I.Vysotsky

ITEP

International Moscow Phenomenology Workshop, July 21-25, 2013

based on
S.I.Godunov, B.Machet, M.V.
Phys.Rev. D 85 (2012) 044058;
S.I.Godunov, M.V.
Phys.Rev. D 87 (2013) 124035.

Alexei Borisovich Kaidalov

- Professor (1975, my last student's exam)
- Warrior (ITEP NTS Chairman, nomads from Rosatom, uprising)
- Colleague (3 papers, 2007-2009)
  $B \to \pi\pi$; angle α
  $B \to \pi^0\pi^0$ enhancement by rescattering;
  $C_{+-} > -0.18$
  Belle: $-0.55(9)$; BaBar: $-0.21(9)$
  arXiv 1302.0551, Belle: $-0.33(7)$

  A.B. approach: great experience in hadronic processes.
- Atoms in strong magnetic fields, A.B. + Turbiner

The Coulomb potential in $d = 1$

$$\Phi(\bar{k}) \equiv A_0(\bar{k}) = \frac{4\pi g}{\bar{k}^2} \; ; \;\; \mathbf{\Phi} \equiv \mathbf{A}_0 = D_{00} + D_{00}\Pi_{00}D_{00} + \dots$$

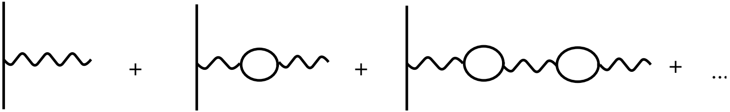


Fig 1. *Modification of the Coulomb potential due to the dressing of the photon propagator.*

Summing the series we get:

$$\mathbf{\Phi}(k) = -\frac{4\pi g}{k^2 + \Pi(k^2)} \, , \;\; \Pi_{\mu\nu} \equiv \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \Pi(k^2), \;\; \Pi \equiv -4g^2 P$$

$$\begin{aligned}
\mathbf{\Phi} &= 4\pi g \int\limits_{-\infty}^{\infty} \frac{e^{ik_\parallel z} dk_\parallel/2\pi}{k_\parallel^2 + 4g^2(k_\parallel^2/2m^2)/(3+k_\parallel^2/2m^2)} = \\
&= \frac{4\pi g}{1+2g^2/3m^2} \int\limits_{-\infty}^{\infty} \left[\frac{1}{k_\parallel^2} + \frac{2g^2/3m^2}{k_\parallel^2 + 6m^2 + 4g^2}\right] e^{ik_\parallel z}\frac{dk_\parallel}{2\pi} = \\
&= \frac{4\pi g}{1+2g^2/3m^2}\left[-\frac{1}{2}|z| + \frac{g^2/3m^2}{\sqrt{6m^2+4g^2}}\exp(-\sqrt{6m^2+4g^2}|z|)\right]
\end{aligned}$$

In the case of heavy fermions ($m \gg g$) the potential is given by the tree level expression; the corrections are suppressed as g^2/m^2.

In the case of light fermions ($m \ll g$):

$$\mathbf{\Phi}(z)\Big|_{m \ll g} = \begin{cases} \pi e^{-2g|z|} & , \quad z \ll \frac{1}{g}\ln\left(\frac{g}{m}\right) \\ -2\pi g\left(\frac{3m^2}{2g^2}\right)|z| & , \quad z \gg \frac{1}{g}\ln\left(\frac{g}{m}\right) \end{cases} .$$

$m = 0$ - Schwinger model; a photon gets mass.

Light fermions provide the continuous transition from $m > g$ to $m = 0$.

The Coulomb potential in $d = 3$ in strong B 1

$$B >> B_0 \equiv m_e^2/e$$

$$\mathbf{\Phi} = \frac{4\pi e}{(k_\parallel^2 + k_\perp^2)\left(1 - \frac{\alpha}{3\pi}\ln\left(\frac{eB}{m^2}\right)\right) + \frac{2e^3 B}{\pi}\exp\left(-\frac{k_\perp^2}{2eB}\right) P\left(\frac{k_\parallel^2}{4m^2}\right)} .$$

P the same, as in $d = 1$!

2 loops: $(e^3 B)^2$?? No, 2 loop corrections are very small (S.I.Godunov, Yad. Fiz. 76, p.955 (2013)).

Physical reason: nullification of higher loops in D=2 massless QED (Schwinger model).

The Coulomb potential in $d = 3$ in strong B 2

$$\mathbf{\Phi}(z) =$$

$$= 4\pi e \int \frac{e^{ik_\parallel z} dk_\parallel d^2 k_\perp/(2\pi)^3}{k_\parallel^2 + k_\perp^2 + \frac{2e^3 B}{\pi}\exp(-k_\perp^2/(2eB))(k_\parallel^2/2m_e^2)/(3 + k_\parallel^2/2m_e^2)} \ ,$$

$$\mathbf{\Phi}(z) = \frac{e}{|z|}\left[1 - e^{-\sqrt{6m_e^2}|z|} + e^{-\sqrt{(2/\pi)e^3 B + 6m_e^2}|z|}\right] \ .$$

For the magnetic fields $B \ll 3\pi m^2/e^3$ the potential is the Coulomb up to small power suppressed terms:

$$\mathbf{\Phi}(z)\Bigg|_{\ e^3 B \ll m_e^2} = \frac{e}{|z|}\left[1 + O\left(\frac{e^3 B}{m_e^2}\right)\right]$$

in full accordance with the $d = 1$ case, where g^2 plays the role of $e^3 B$.

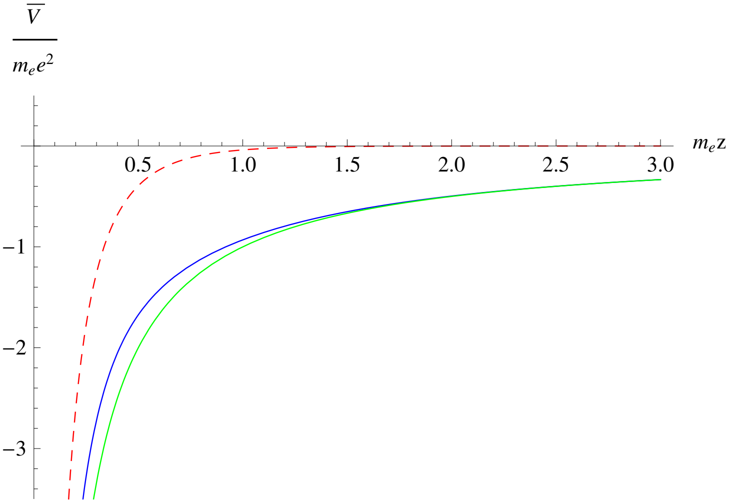
The Coulomb potential in $d = 3$ in strong B 3

In the opposite case of the superstrong magnetic fields $B \gg 3\pi m_e^2/e^3$ we get:

$$\mathbf{\Phi}(z) = \begin{cases} \frac{e}{|z|} e^{(-\sqrt{(2/\pi)e^3 B}|z|)}, \frac{1}{\sqrt{(2/\pi)e^3 B}} \ln\left(\sqrt{\frac{e^3 B}{3\pi m_e^2}}\right) > |z| > \frac{1}{\sqrt{eB}} \\ \frac{e}{|z|}(1 - e^{(-\sqrt{6m_e^2}|z|)}), \frac{1}{m_e} > |z| > \frac{1}{\sqrt{(2/\pi)e^3 B}} \ln\left(\sqrt{\frac{e^3 B}{3\pi m_e^2}}\right) \\ \frac{e}{|z|} \quad , \qquad |z| > \frac{1}{m_e} \end{cases},$$

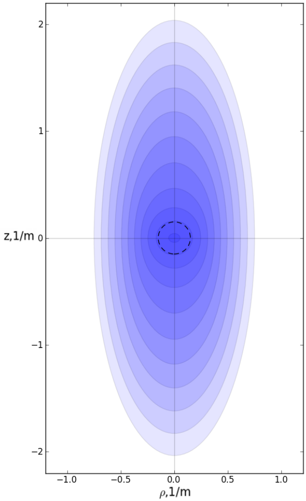
$$V(z) = -e\mathbf{\Phi}(z)$$

The screened Coulomb potential along B



The modified Coulomb potential at $B = 10^{17}$ G (blue) and its long distance (green) and short distance (red) asymptotics.

D=3 equipotential lines



Hydrogen spectrum, even states

Karnakov - Popov equation with screening (Machet, Vysotsky):

$$\ln\left(\frac{B}{m_e^2 e^3 + \dfrac{e^6}{3\pi}B}\right) = \lambda + 2\ln\lambda + 2\psi\left(1 - \frac{1}{\lambda}\right) + \ln 2 + 4\gamma + \psi(1+|m|)$$

$E = -(m_e e^4/2)\lambda^2$, for $B \to \infty$: $\lambda^{gr} \to 11.2$, $E^{gr} \to -1.7 KeV$

Freezing of the ground level was discovered by Shabad and Usov (2007, 2008).

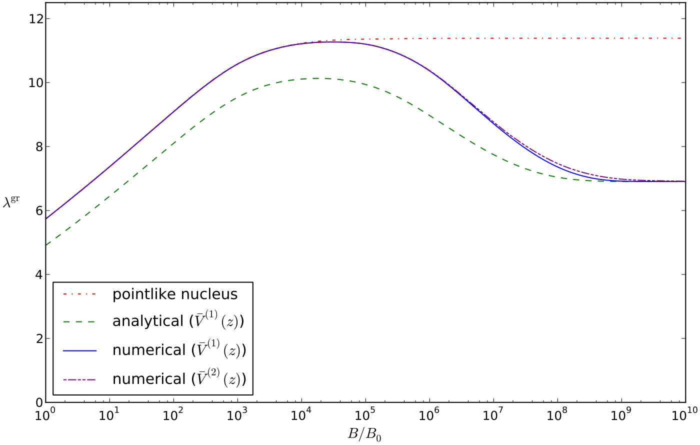
Effective potential

An electron moves adiabatically along the magnetic field in an (averaged over fast transverse motion) potential

$$V(z) \approx -\frac{e^2}{\sqrt{z^2 + a_H^2}} \left[1 - e^{-\sqrt{6m_e^2}|z|} + e^{-\sqrt{(2/\pi)e^3 B + 6m_e^2}|z|} \right] \quad ,$$

$a_H \equiv 1/\sqrt{eB}$ - Landau radius.
When the magnetic field grows a_H becomes smaller than the proton charge radius $R = 0.9$fm and a_H should be substituted by R, that's why the ground level goes up.

Hydrogen ground level



The nonrelativistic problem is solved analytically.

$Z_{critical}$

When Z grows, the ground level goes down and at $Z \approx 170$ it reaches lower continuum, $\varepsilon_0 = -m_e$. Two e^+e^- pairs are produced from vacuum. Electrons with the opposite spins occupy the ground level, while positrons are emitted to infinity.
The magnetic field squeezes the electron wave function in transverse plain making the Coulomb problem one dimensional. So, criticality is reached at smaller Z.
Without screening: V.N. Oraevskii, A.I. Rez, V.B. Semikoz, 1977.
The bispinor which describes an electron on LLL is:

$$\psi_e = \begin{pmatrix} \varphi_e \\ \chi_e \end{pmatrix} \ ,$$

$$\varphi_e = \begin{pmatrix} 0 \\ g(z)\exp\left(-\rho^2/4a_H^2\right) \end{pmatrix} \ , \chi_e = \begin{pmatrix} 0 \\ if(z)\exp\left(-\rho^2/4a_H^2\right) \end{pmatrix} \ .$$

Dirac equation

Dirac equations for functions $f(z)$ and $g(z)$ look like:

$$g_z - (\varepsilon + m_e - \bar{V})f = 0 \quad ,$$
$$f_z + (\varepsilon - m_e - \bar{V})g = 0 \quad ,$$

where $g_z \equiv dg/dz$, $f_z \equiv df/dz$. They describe the electron motion in the effective potential $\bar{V}(z)$:

$$\bar{V}(z) = \frac{1}{a_H^2} \int\limits_0^\infty V(\sqrt{\rho^2 + z^2}) \exp\left(-\frac{\rho^2}{2a_H^2}\right) \rho d\rho$$

Z_{cr} without screening

ORS (1977):

$$\frac{B}{B_0} = 2(Z_{cr}e^2)^2 \exp\left(-\gamma + \frac{\pi - 2\arg\Gamma(1 + 2iZ_{cr}e^2)}{Z_{cr}e^2}\right).$$

The analytical expression is obtained matching the solution of Dirac equation for the Coulomb potential at large distances ($z >> a_H$) and shallow well solution at small distances ($V \equiv Ze^2/z >> 2m_e$).

Screening deforms the Coulomb potential at large distances $a_H < z < 1/m_e$ at which shallow well solution is not valid, so we do not manage to find the analytical solution in case of screening.

Dirac equation with screening

Following V.S.Popov, we reduce the D.eq. to the Schr.eq:

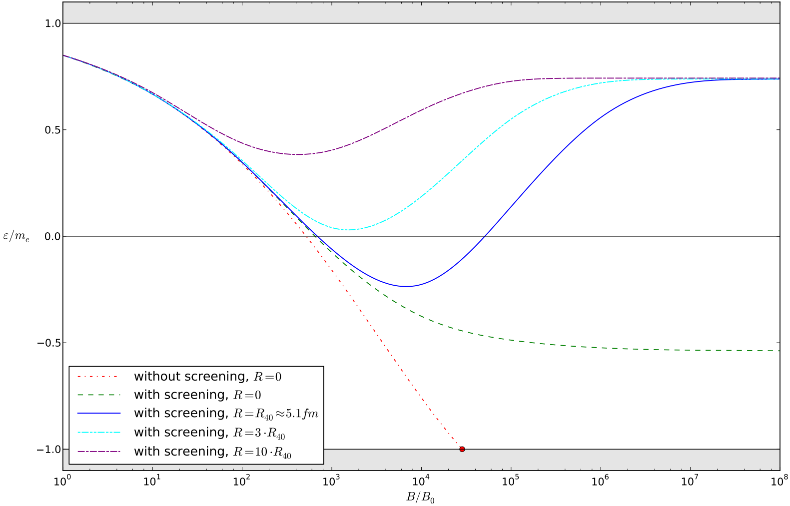
$$\frac{d^2\chi}{dz^2} + 2m_e(E-U)\chi = 0 \ ,$$

$$E = \frac{\varepsilon^2 - m_e^2}{2m_e} \ , \quad U = \frac{\varepsilon}{m_e}\bar{V} - \frac{1}{2m_e}\bar{V}^2 + \frac{\bar{V}''}{4m_e(\varepsilon + m_e - \bar{V})} +$$

$$\frac{3/8(\bar{V}')^2}{m_e(\varepsilon + m_e - \bar{V})^2} \ .$$

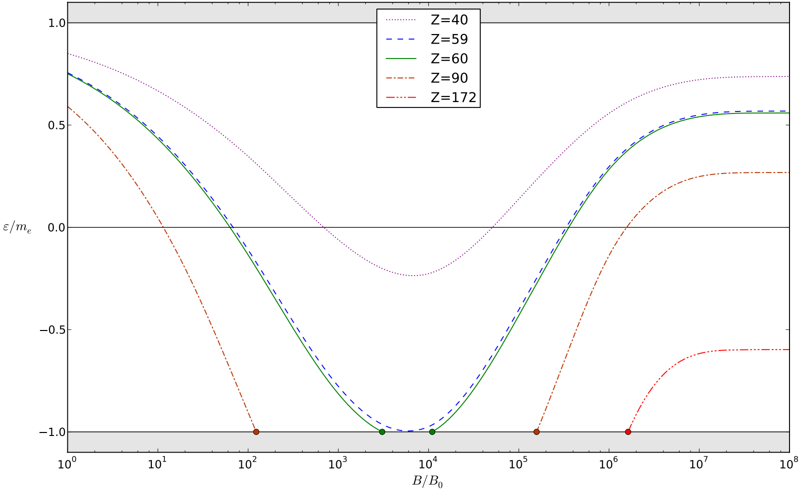
Relativistic "corrections" are very big, $\sim B/B_0, (B/B_0)^{3/2}$, and screening occurs at $B/B_0 \approx 3\pi/e^2 \approx 10^3$ - so, the nonrelativistic potential is greatly deformed while the enegy levels originating from LLL depend on B weakly (freezing). The numerical calculations are very complicated (S.I.Godunov).

Z=40



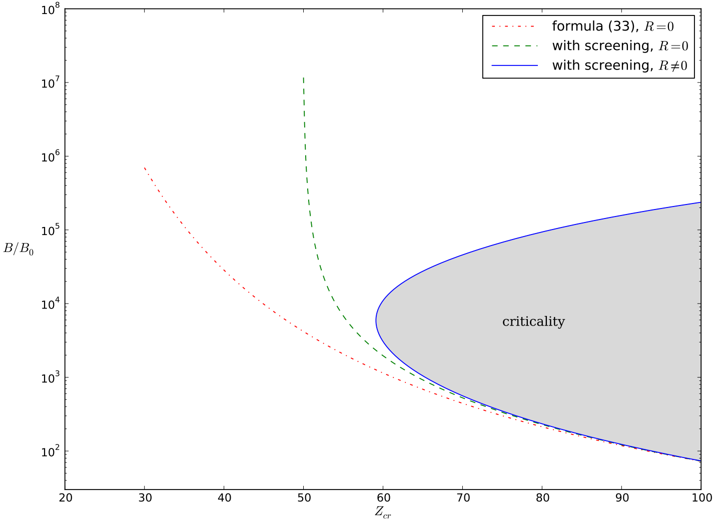
The dependence of the ground state energy on the magnetic field.

Z= 40 - 172



Z=40
Z=59
Z=60
Z=90
Z=172
1.0
0.5
0.0
−0.5
−1.0
ε/m_e
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7
10^8
B/B_0

$Z_{cr}(B)$



The ions with $Z \geq 210$ remain critical regardless of the value of B.

Conclusions

- The energy of the ground level of a hydrogen atom in a superstrong B with the account of the magnetic screening of the Coulomb field is found; the finite proton size pushes the ground level up;

- The Dirac equation is solved numerically for hydrogen-like ions in the superstrong B with the account of the magnetic screening and finite size of the nucleus; the domain of ions criticality is determined;

- $B_0 \approx 4 \cdot 10^{13}$ Gauss: magnetars (10^{15} Gauss), condensed matter.