Anisotropic pressure in the quark core of a strongly magnetized hybrid star

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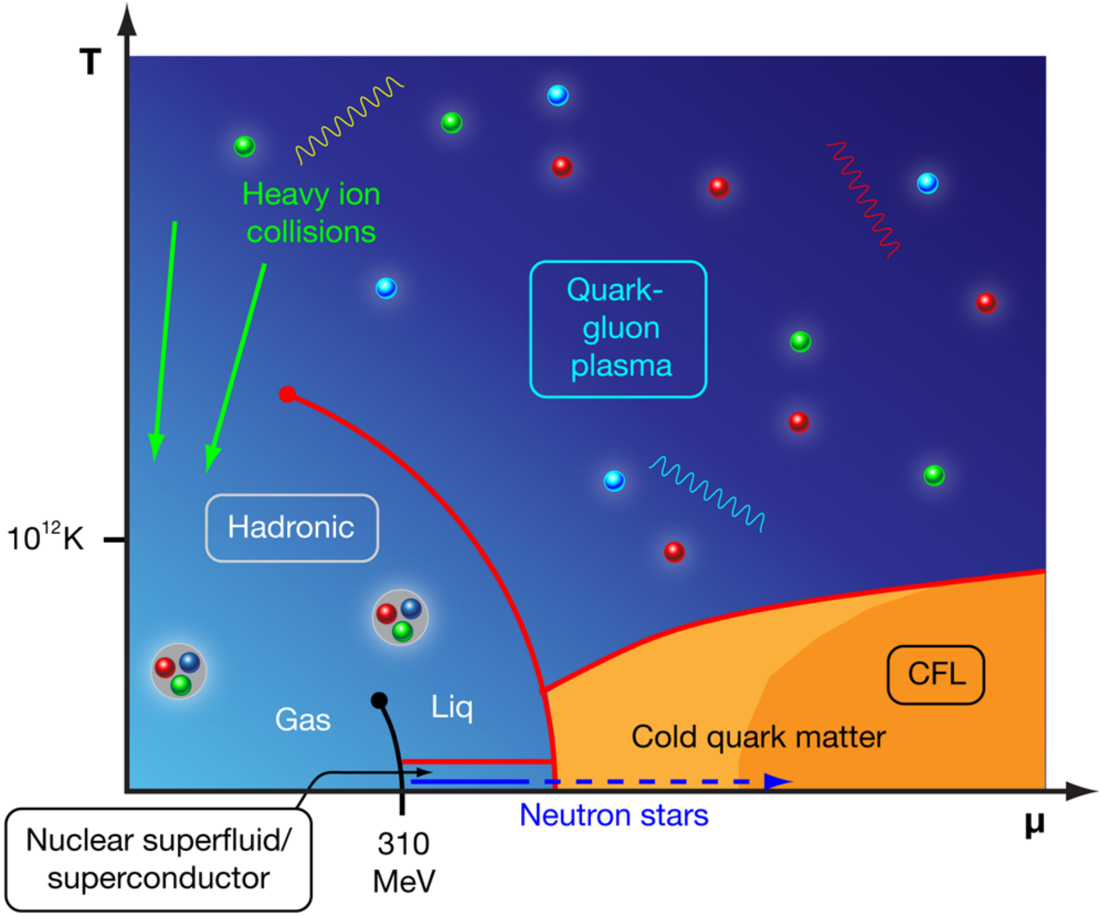
A good name is rather to be chosen than great riches, and loving favour rather than silver and gold.

Proverbs 22:1

Introduction



QCD phase diagram in the plane μ_B-T (from Physics 3, 44 (2010))

Introduction

Quark matter: Strange quark matter consisting of deconfined *u, d, s* quarks.

At zero temperature and pressure, the energy per baryon in strange quark matter (SQM) for a certain range of the model QCD-related parameters can be less than that for the most stable ^{56}Fe nucleus, and, hence, SQM can be the true ground state of matter.

A. Bodmer, Phys. Rev. D 4, 1601 (1971).
E. Witten, Phys. Rev. D 30, 272 (1984).
E. Farhi and R. L. Jaffe, Phys. Rev. D 30, 2379 (1984).

Astrophysical implications. SQM can be encountered in:

1. Strange quark stars, self-bound by strong interactions (in contrast to neutron stars bound by gravitational forces).

2. Hybrid stars: SQM is metastable at zero pressure, but it can appear in the high-density core of a neutron star as a result of the deconfinement phase transition. In this case, the stability of SQM is provided by the gravitational pressure from the outer hadronic layers.

3. Small nuggets, called strangelets (can be found in cosmic rays).

Introduction

Another important aspect: compact stars are endowed with the magnetic field.

Neutron stars observed in nature are magnetized objects with the magnetic field strength at the surface in the range 10^9-10^{13} G.

R.C. Duncan, and C. Thompson (1992): even more strongly magnetized objects, called magnetars, can exist in the universe with the field strength of about 10^{14}-10^{15} G at the surface. Golden candidates for magnetars: soft gamma-ray repeaters and anomalous X-ray pulsars. Nowadays it is believed that magnetars constitute about 10 % of the whole population of neutron stars. Recently it was suggested that a magnetized hybrid star, or a magnetized strange quark star can be a real source of the SGRs or AXPs.

In the interior of a magnetar the magnetic field strength may be even larger, $H \sim 10^{18}$-10^{20} G (from a scalar virial theorem).

Introduction

Possible mechanisms to generate such strong magnetic fields:

- **turbulent dynamo amplification mechanism in a neutron star with fast rotating core - C. Thompson and R.C. Duncan, ApJ 473 (1996) 322;**

- **spontaneous ordering of hadron or quark spins in the dense core of a neutron star:**

  T. Tatsumi, Phys. Lett. B 489, 280 (2000)
  A. A. Isayev and J. Yang, Phys. Rev. C 69, 025801 (2004)
  A. A. Isayev, Phys. Rev. C 74, 057301 (2006).

Strong magnetic fields are generated also in noncentral high-energy heavy-ion collisions with $H \sim 10^{18}$ G (RHIC) or even an order of magnitude larger (LHC).

Worthy to note: In such ultrastrong magnetic fields, because of the breaking of the rotational O(3) symmetry by the magnetic field, the total pressure becomes anisotropic having smaller value along than perpendicular to the field direction. As a result, the equation of state of matter becomes essentially anisotropic in strong magnetic fields.

Introduction

Main objective: **to study the impact of a strong magnetic field on the thermodynamic properties of strange quark matter under the conditions relevant to the cores of magnetars with account of the effects of the pressure anisotropy.**

Related articles:

A.A. Isayev, and J. Yang, J. Phys. G 40, 035105 (2013)
A.A. Isayev, and J. Yang, Phys. Lett. B 707, 163 (2012)
A.A. Isayev, and J. Yang, Phys. Rev. C 84, 065802 (2011)

General Formalism: Magnetized Strange Quark Matter

A theoretical framework to study SQM: the MIT bag model.
In the simplest version of the MIT bag model, quarks are considered as free fermions moving inside a finite region of space called a ”bag”. The effects of the confinement are accomplished by endowing the finite region with a constant energy per unit volume, the bag constant *B*. The bag constant *B* can be also interpreted as the inward pressure - the ”bag pressure”, needed to confine quarks inside the bag.

The Lagrangian density for a relativistic system of noninteracting quarks (*u,d,s*) and leptons (e^-) in an external magnetic field reads

$$L=\sum_{u,d,s,e}\bar{\psi}_i[\gamma^\mu(i\partial_\mu-q_iA_\mu)-m_i]\psi_i-\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu},$$

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu,\quad A_\mu=Hx_1\delta_{\mu 2}\ (\mu=0,1,2,3)$$

Dirac equation for the quark and electron spinors

$$[\gamma^\mu(i\partial_\mu-q_iA_\mu)-m_i]\psi_i=0,\quad i=u,d,s,e$$

General Formalism

The energy spectrum of free relativistic fermions in an external magnetic field has the form

$$\varepsilon_{\nu}^{i} = \sqrt{k_z^2 + m_i^2 + 2\nu\,|q_i|\,H}, \quad \nu = n + \frac{1}{2} - \frac{s}{2}\mathrm{sgn}(q_i)$$

v=0,1,2,… enumerates the Landau levels, *n* is principal quantum number, *s=+1* corresponds to a fermion with spin up, *s* = - *1* to a fermion with spin down.

The lowest Landau level with *v* = 0 is single degenerate and other levels with *v* > 0 are double degenerate. For positively charged particles, the lowest Landau level is occupied by fermions with spin up, and for negatively charged particles by fermions with spin down. As a result, each charged fermion subsystem acquires spin polarization in a magnetic field.

General Formalism

The grand thermodynamic potential density for nonmagnetized fermions of *i*th species at finite temperature

$$\Omega_i = -g_i T \int \left\{ \ln\left[1+e^{-(\varepsilon_i-\mu_i)/T}\right] + \ln\left[1+e^{-(\varepsilon_i+\mu_i)/T}\right] \right\} \frac{d^3k}{(2\pi)^3}$$

For magnetized fermions of *i*th species:

$$\int \frac{d^3k}{(2\pi)^3}\ldots \to \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} \int \frac{d^2k_\perp}{(2\pi)^2}\ldots \to 2\pi |q_i| H \frac{1}{(2\pi)^2} \sum_{s=\pm1}\sum_n \int_{-\infty}^{\infty} \frac{dk_z}{2\pi}\ldots = \frac{|q_i| H}{2\pi^2} \sum_{s=\pm1}\sum_n \int_0^{\infty} dk_z \ldots$$

Hence

$$\Omega_i = -T \frac{|q_i| g_i H}{2\pi^2} \sum_{s=\pm1}\sum_n \int_0^{\infty} dk_z \left\{ \ln\left[1+e^{-(\varepsilon^i{}_{n,s}-\mu_i)/T}\right] + \ln\left[1+e^{-(\varepsilon^i{}_{n,s}+\mu_i)/T}\right] \right\}$$

or

$$\Omega_i = -T \frac{|q_i| g_i H}{2\pi^2} \sum_{\nu=0}^{\infty} (2-\delta_{\nu,0}) \int_0^{\infty} dk_z \left\{ \ln\left[1+e^{-(\varepsilon^i{}_\nu-\mu_i)/T}\right] + \ln\left[1+e^{-(\varepsilon^i{}_\nu+\mu_i)/T}\right] \right\}$$

General Formalism

The grand thermodynamic potential density for fermions of *i*th species in the external magnetic field at zero temperature reads

$$\Omega_i = -\frac{|q_i| g_i H}{4\pi^2} \sum_{\nu=0}^{\nu_{\max}^i} (2-\delta_{\nu,0}) \left\{ \mu_i k_{F,\nu}^i - \bar{m}_{i,\nu}^2 \ln \left| \frac{k_{F,\nu}^i + \mu_i}{\bar{m}_{i,\nu}} \right| \right\},$$

$$\bar{m}_{i,\nu} = \sqrt{m_i^2 + 2\nu |q_i| H}, \quad k_{F,\nu}^i = \sqrt{\mu_i^2 - \bar{m}_{i,\nu}^2},$$

$$\nu_{\max}^i = I\left[\frac{\mu_i^2 - m_i^2}{2|q_i| H} \right],$$ $I[...]$ means the integer part of the argument

$$g_i = \begin{cases} 3, & \text{for quarks (\# of colors)} \\ 1, & \text{for electrons} \end{cases}$$

The number density of fermions of *i*th species $\rho_i = -\left(\frac{\partial \Omega_i}{\partial \mu_i} \right)_T$

$$\rho_i = \frac{|q_i| g_i H}{2\pi^2} \sum_{\nu=0}^{\nu_{\max}^i} (2-\delta_{\nu,0}) k_{F,\nu}^i$$

General Formalism

The particle number density can be split into two terms

$$\rho_i = \rho_i^{\uparrow} + \rho_i^{\downarrow}$$

The only difference between the two sums is in the term with *v* = 0, corresponding to spin-up fermions if they are positively charged, and to spin-down fermions, if they are negatively charged.

Spin polarization parameter for fermions of *i*th species at T=0 reads

$$\Pi_i \equiv \frac{\rho_i^{\uparrow} - \rho_i^{\downarrow}}{\rho_i} = \frac{q_i g_i H}{2\pi^2 \rho_i} \sqrt{\mu_i^2 - m_i^2}$$

In a strong enough magnetic field, when only a lowest Landau level is occupied by fermions of *i*th species, a full polarization occurs with $|\Pi_i| = 1$.

General Formalism

Conditions for finding the chemical potentials of all fermion species (and, hence, the corresponding particle number densities):

1. The conservation of the total baryon number

$$\frac{1}{3}(\rho_u + \rho_d + \rho_s) = \rho_B$$

2. Charge neutrality

$$2\rho_u - \rho_d - \rho_s - 3\rho_{e^-} = 0$$

3. Chemical equilibrium with respect to weak processes in the quark core of a hybrid star

$$d \to u + e^- + \bar{\nu}_e, \quad u + e^- \to d + \nu_e$$

$$s \to u + e^- + \bar{\nu}_e, \quad u + e^- \to s + \nu_e$$

$$s + u \leftrightarrow d + u$$

with the conditions

$$\mu_d = \mu_u + \mu_{e^-}, \quad \mu_d = \mu_s$$

General Formalism

The energy density for fermions of *i*th species at zero temperature $E_i = \Omega_i + \mu_i \rho_i$

$$\boldsymbol{E_i = \frac{|q_i|\, g_i H}{4\pi^2} \sum_{\nu=0}^{\nu_{\max}^i} (2-\delta_{\nu,0}) \left\{ \mu_i k_{F,\nu}^i + \bar{m}_{i,\nu}^2 \ln\left| \frac{k_{F,\nu}^i + \mu_i}{\bar{m}_{i,\nu}} \right| \right\}}$$

In the MIT bag model, the total energy density, longitudinal p_l and transverse p_t pressures in quark matter are given by [Phys. Rev. C **82**, 065802 (2010)]

$$E = \sum_i E_i + \frac{H^2}{8\pi} + B,$$

$$p_l = -\sum_i \Omega_i - \frac{H^2}{8\pi} - B, \quad p_t = -\sum_i \Omega_i - HM + \frac{H^2}{8\pi} - B$$

$M = \Sigma_i M_i$ is the total magnetization, M_i is the partial magnetization $M_i = -\left(\frac{\partial \Omega_i}{\partial H}\right)_{\mu_i}$

In a strong enough magnetic field, the quadratic on the magnetic field strength term (the Maxwell term) will be dominating, leading to increasing the transverse pressure and to decreasing the longitudinal pressure. Hence, **there exists a critical magnetic field H_{cr}, at which the longitudinal pressure vanishes, resulting in the longitudinal instability of strange quark matter.**

Model Parameters

Two sets of values for a neutron star core density and bag pressure: ***B* = 76 MeV/fm^3,** ϱ_B **= 3**ϱ_0 **and *B* = 90 MeV/fm^3,** ϱ_B **= 4**ϱ_0 ($\varrho_0 = 0.16$ fm^{-3} being the nuclear saturation density).

Concerning the core density: in principle, sufficient to produce deconfinement.

Concerning the bag pressure: slightly larger than the upper bound $B \simeq 75$ MeV/fm^3 from the absolute stability window [Phys. Rev. D 83, 043009 (2011)].

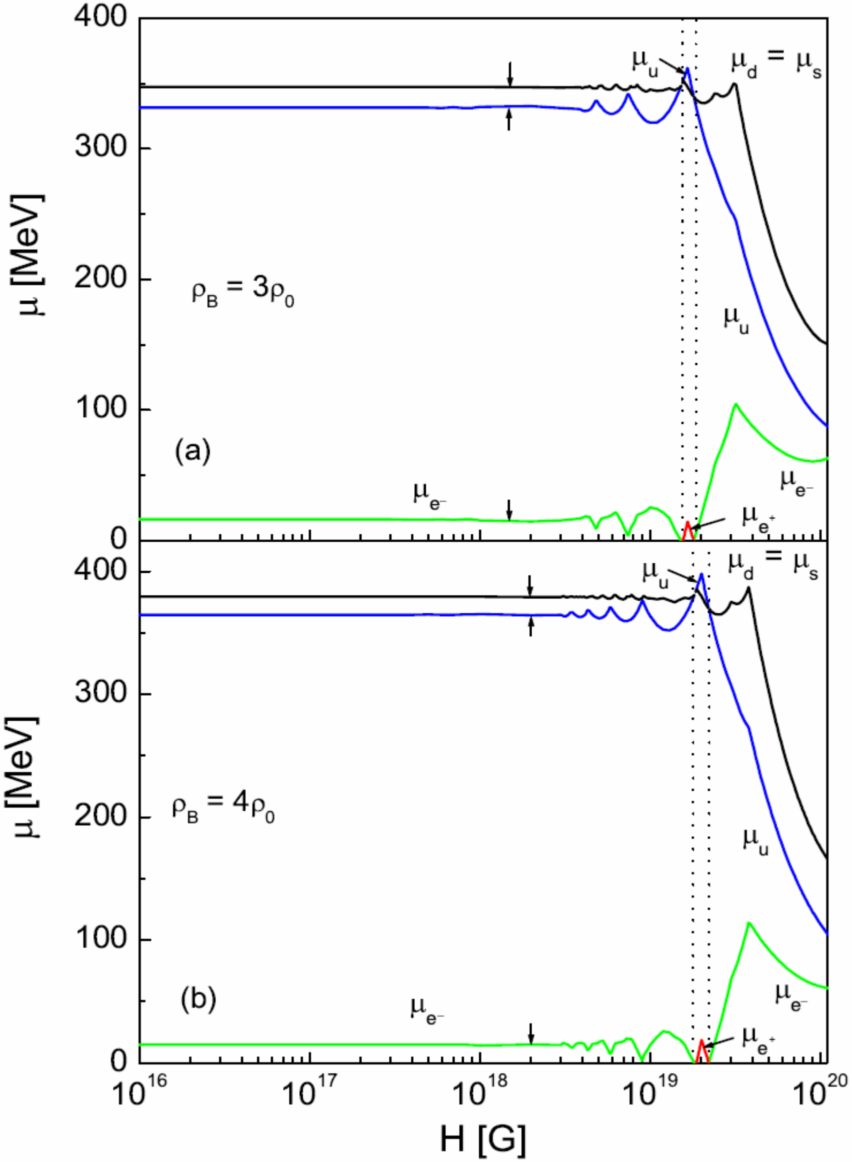
The absolute stability window: at zero external pressure and temperature

$$\left.\frac{E}{A}\right|_{MSQM} < \left.\frac{E}{A}\right|_{^{56}Fe} \approx 930 \text{ MeV}$$

Hence, in the astrophysical context, it is assumed a scenario in which strange quark matter can be formed in the core of a strongly magnetized neutron star and is stabilized by the gravitational pressure from the outer hadronic layers.

The current quark masses are: $\boldsymbol{m_u = m_d}$ **= 5 MeV, and** $\boldsymbol{m_s}$ **= 150 MeV**

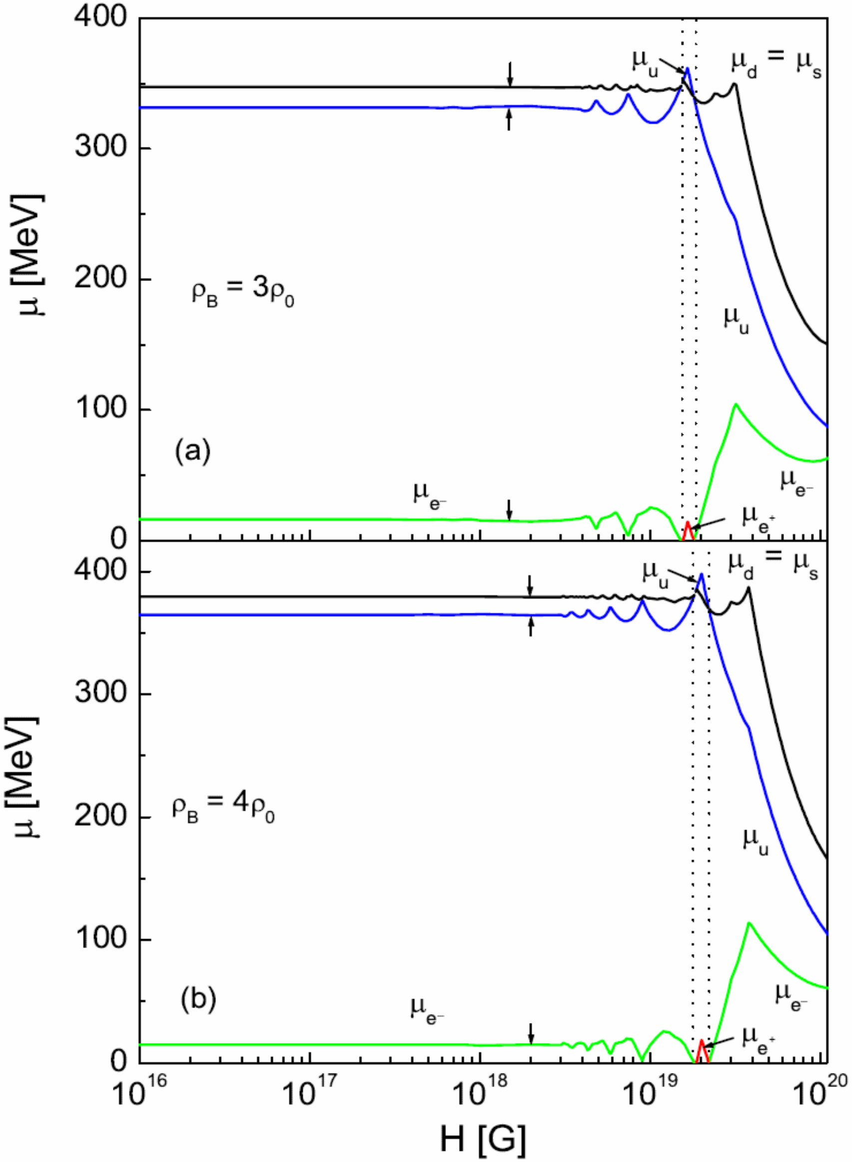
Numerical Results: Chemical Potentials



Peculiarities:

- chemical potentials of fermions, first, stay practically constant under increasing *H*;
- the apparent Landau oscillations of the chemical potentials appear beginning from *H* ~ $3\cdot 10^{18}-4\cdot 10^{18}$ G, depending on the total baryon number density;
- At *H* > ~$4 \cdot 10^{19}$ G, the quark chemical potentials decrease with the magnetic field;

Numerical Results: Chemical Potentials



- in a narrow interval near H ~ 2 · 10[19] G (between the vertical dotted lines) $\mu_u > \mu_d = \mu_s$. Hence, for such magnetic fields, according to

$$\mu_d = \mu_u + \mu_{e^-}$$

μ_e would be negative, $\mu_e < 0$. If to recall that

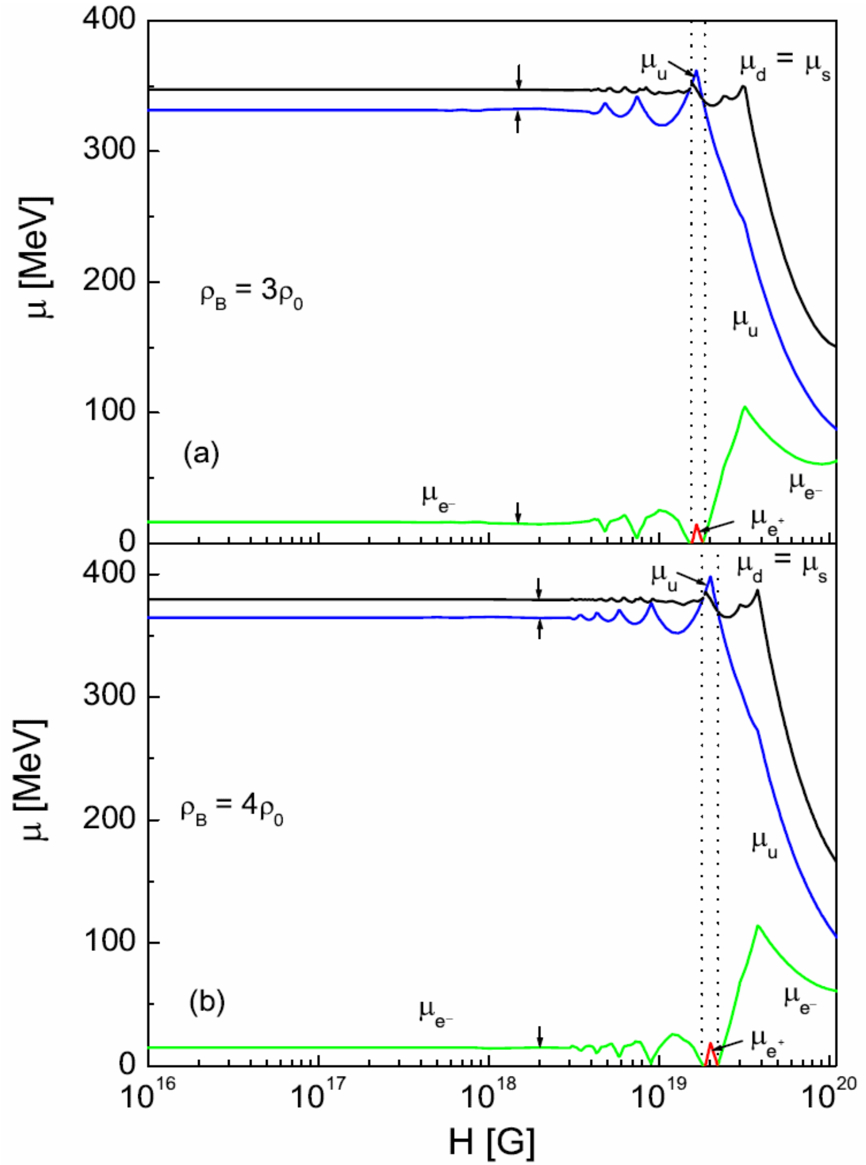
$$\rho_{e^-} = \frac{|q_e|H}{2\pi^2}\sum_{\nu=0}^{\nu_{\max}^i}(2-\delta_{\nu,0})\int_0^\infty dk_z\left(\frac{1}{e^{\beta(\varepsilon_\nu^e-\mu_{e^-})}+1}-\frac{1}{e^{\beta(\varepsilon_\nu^e+\mu_{e^-})}+1}\right)$$

then in the T=0 limit, $\varrho_e < 0$ at $\mu_e < 0$, contrary to the constraint $\varrho_e >= 0$. This means that in this interval on *H* electrons are missing and, hence, the following weak β^- processes are impossible

$$d \to u + e^- + \bar{\nu}_e, \quad u + e^- \to d + \nu_e$$

$$s \to u + e^- + \bar{\nu}_e, \quad u + e^- \to s + \nu_e$$

Numerical Results: Chemical Potentials



However, for such magnetic fields, the following weak β^+ processes become allowable

$$u \to d + e^+ + \nu_e, \quad d + e^+ \to u + \bar{\nu}_e$$

$$u \to s + e^+ + \nu_e, \quad s + e^+ \to u + \bar{\nu}_e$$

For this interval on *H* charge neutrality and chemical equilibrium conditions should read

$$2\rho_u - \rho_d - \rho_s + 3\rho_{e^+} = 0$$

$$\mu_u = \mu_d + \mu_{e^+}, \quad \mu_d = \mu_s$$

Solutions of these equations, together with the total baryon number conservation condition, are shown by the curves between the dotted lines (for positrons by the red curves). **Thus, as a matter of principle, in strongly magnetized SQM at zero temperature, subject to the total baryon number conservation, charge neutrality and chemical equilibrium conditions, positrons can appear in a certain narrow interval on *H*, replacing electrons.**

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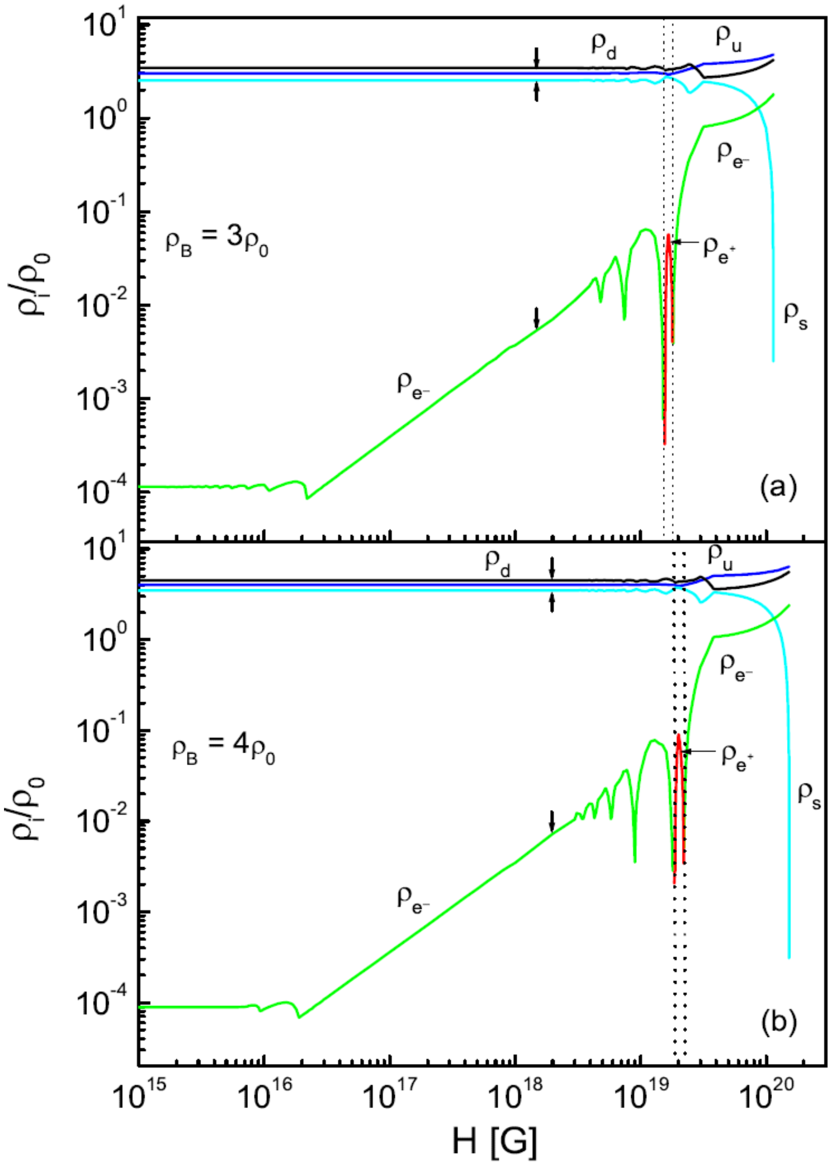
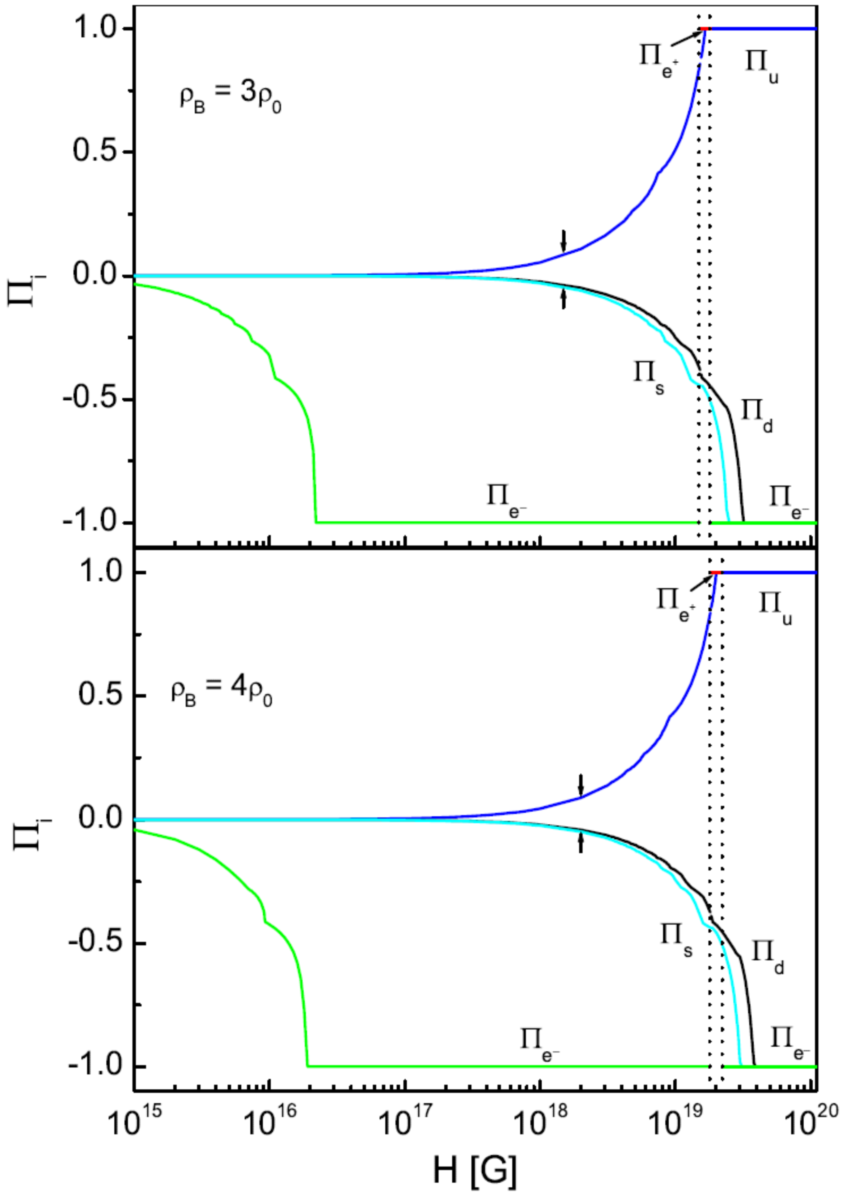
Numerical Results: Particle Abundances



- The number densities of *u* and *d* quarks are quite close to each other for all magnetic fields under consideration;
- The electron number density begins quite rapidly to increase at $H \approx 2.2 \cdot 10^{16}$ G for $\varrho_B = 3\varrho_0$ and at $H \approx 1.9 \cdot 10^{16}$ G for $\varrho_B = 4\varrho_0$. In the narrow interval near $H \sim 2 \cdot 10^{19}$ G electrons are replaced by positrons, and beyond this interval electrons appear again with the number density increasing with *H*.
- The s quark content of SQM stays practically constant till the field strength $H \approx 4.1 \cdot 10^{18}$ G at $\varrho_B = 3\varrho_0$ and $H \approx 3.8 \cdot 10^{18}$ G at $\varrho_B = 4\varrho_0$, beyond which the s quark number density experiences visible Landau oscillations. Then, beginning from the field strength $H \approx 3.2 \cdot 10^{19}$ G at $\varrho_B = 3\varrho_0$ and $H \approx 3.9 \cdot 10^{19}$ G at $\varrho_B = 4\varrho_0$, the s quark content rapidly decreases. SQM loses its strangeness and turns into two-flavor quark matter in the magnetic fields slightly larger than 10^{20} G.

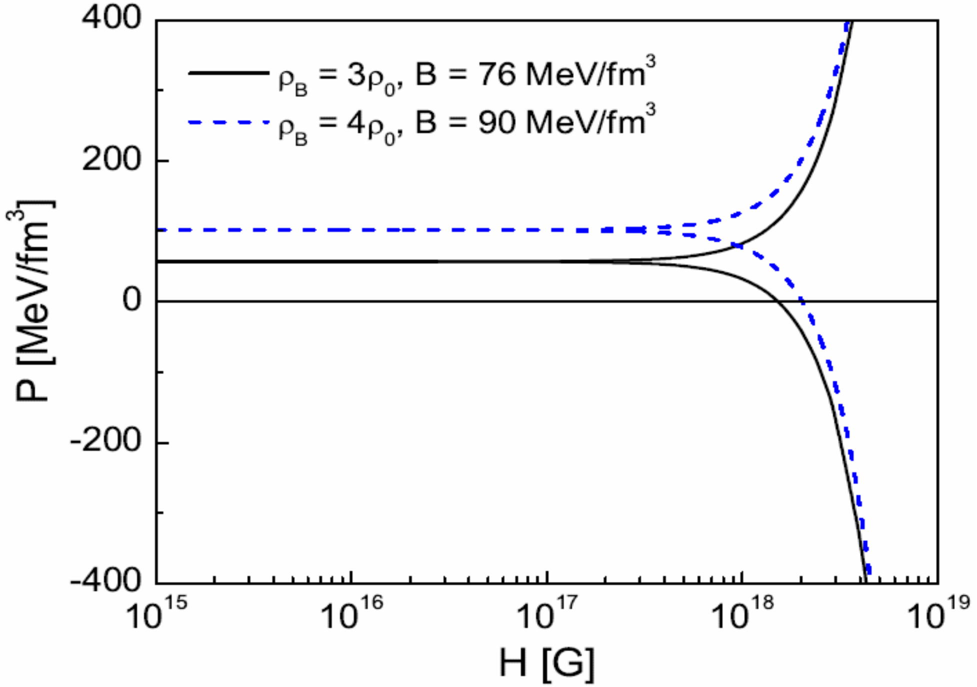
Numerical Results: Spin Polarizations



- The magnitude of the spin polarization parameter Π_i increases with H till it is saturated at the respective saturation field H_i^s. E.g., at $\varrho_B = 4\varrho_0$: $H_e^s \approx 1.9 \cdot 10^{16}$ G for e^-, $H_u^s \approx 2.0 \cdot 10^{19}$ G for u quarks, $H_s^s \approx 3.1 \cdot 10^{19}$ G for s quarks and $H_d^s \approx 3.9 \cdot 10^{19}$ G for d quarks.

- Positrons occur already fully polarized, and *u* quarks become totally polarized just in this range of the magnetic field strengths where positrons appear.

Anisotropic pressure vs. *H*



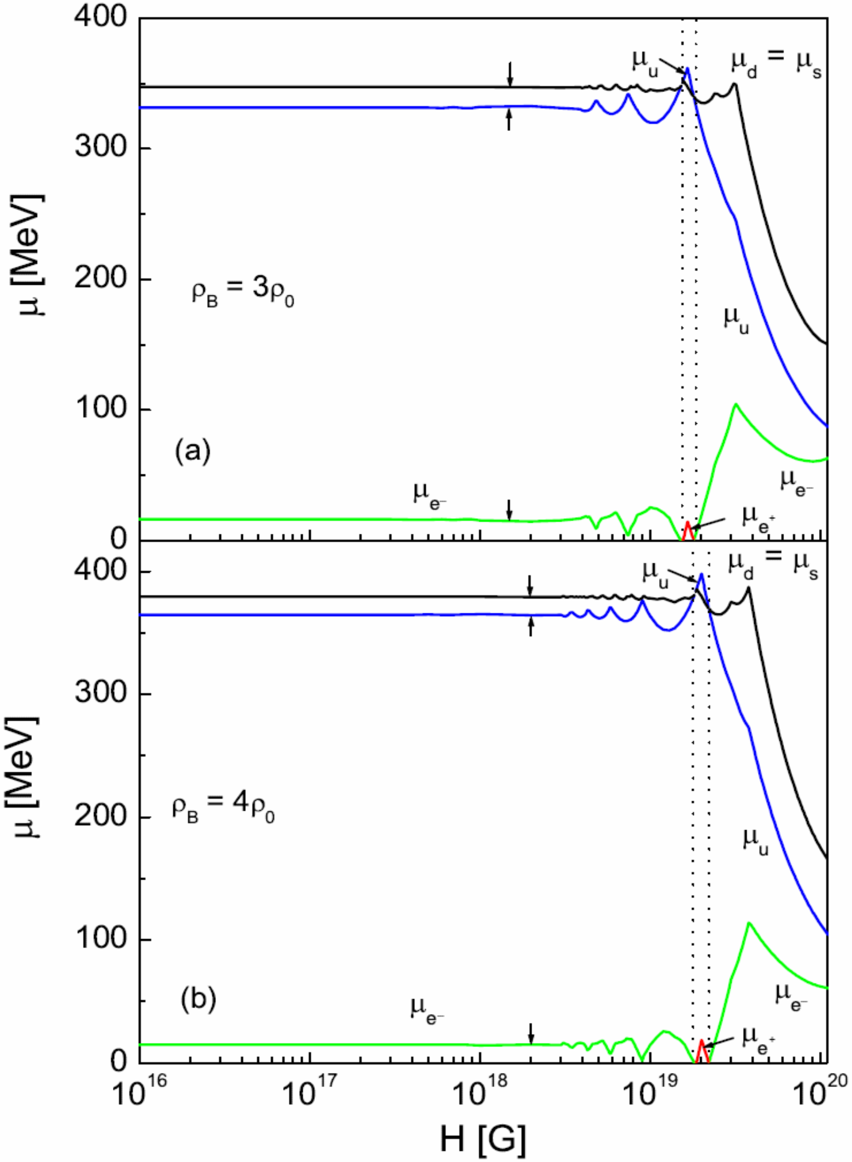
$H_{cr} \approx 1.5 \cdot 10^{18}$ G for $\varrho_B = 3\varrho_0$, B = 76 MeV/fm^3,

$H_{cr} \approx 2 \cdot 10^{18}$ G for $\varrho_B = 4\varrho_0$, B = 90 MeV/fm^3.

Transverse (ascending branches) and longitudinal (descending branches) pressures in magnetized SQM at zero temperature vs. *H*

- **Isotropic regime:** the transverse and longitudinal pressures, first, stay practically constant and indistinguishable from each other.
- **Anisotropic regime:** Beyond some threshold magnetic field H_{th}, the transverse pressure p_t increases with *H* while the longitudinal pressure p_l decreases with it.
- In the critical magnetic field H_{cr}, the longitudinal pressure p_l vanishes, resulting in the appearance of the longitudinal instability in SQM.

Chemical Potentials and Critical Field H_{cr}

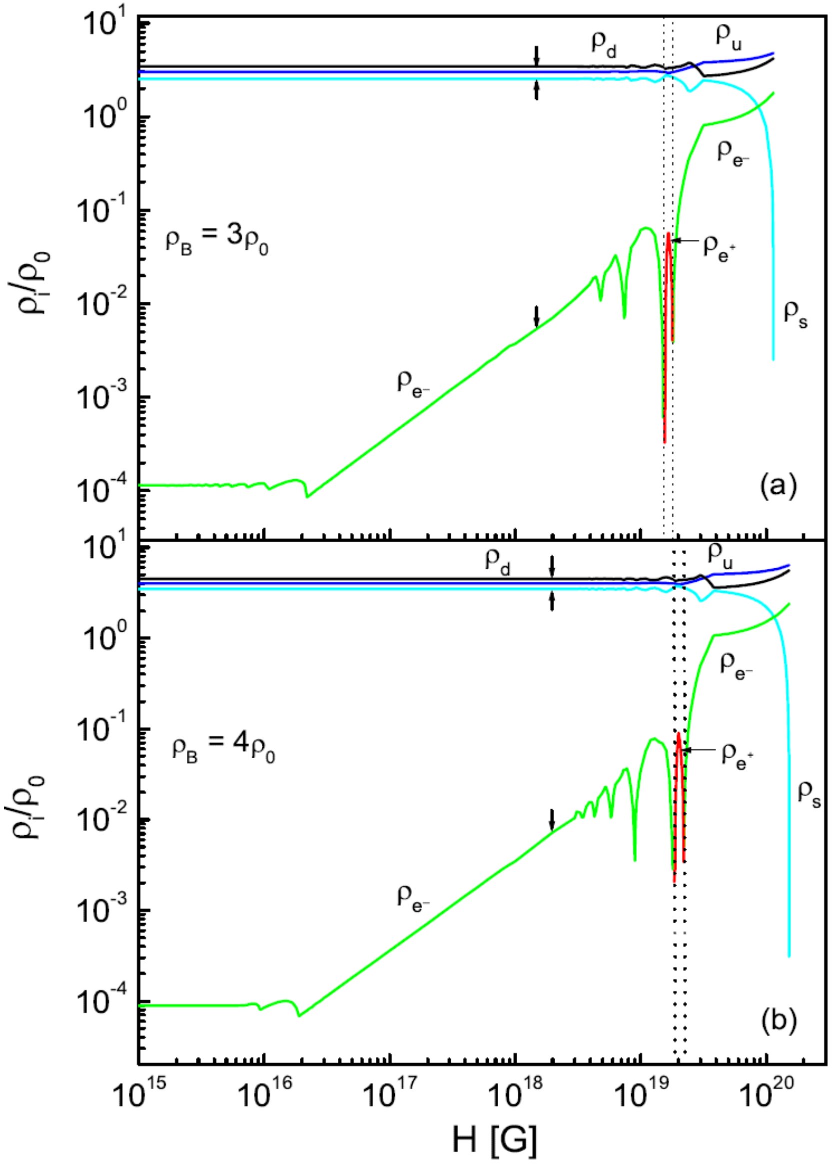


The vertical arrows indicate the points corresponding to the critical field H_{cr}.

- The chemical potentials of quarks and electrons stay practically unchanged before the appearance of the longitudinal instability. The significant changes in the chemical potentials occur only in the fields $H > H_{cr}$.

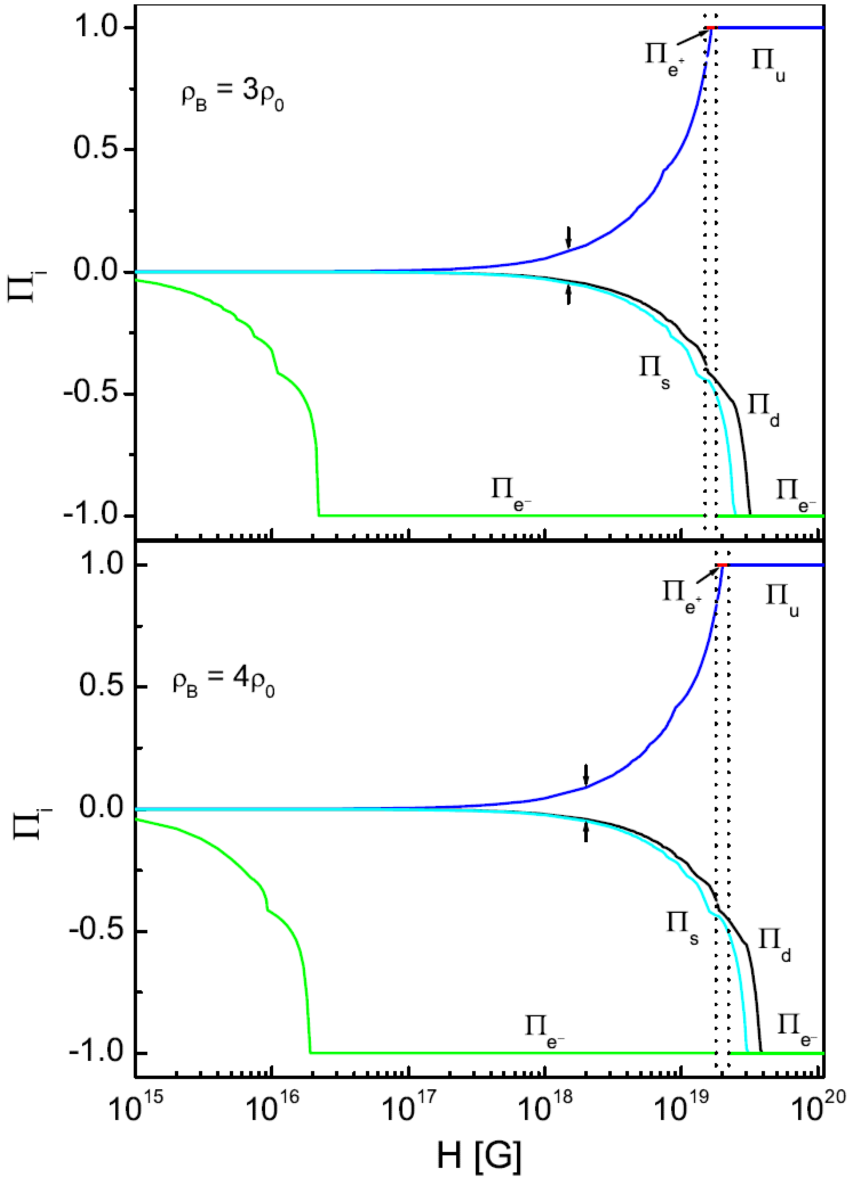
- The longitudinal instability precludes the appearance of positrons for which the fields $H > 10^{19}$ G are necessary.

Particle Abundances and Critical Field H_{cr}



- Till the critical field H_{cr}, the content of quark species stays practically constant while the electron fraction remains quite small, $\varrho_e/\varrho_0 < 10^{-2}$.
- There is no room for the significant drop of the strange quark content in a strong magnetic field which occurs in the fields $H \sim 10^{20}$ G.

Spin Polarizations and Critical Field H_{cr}

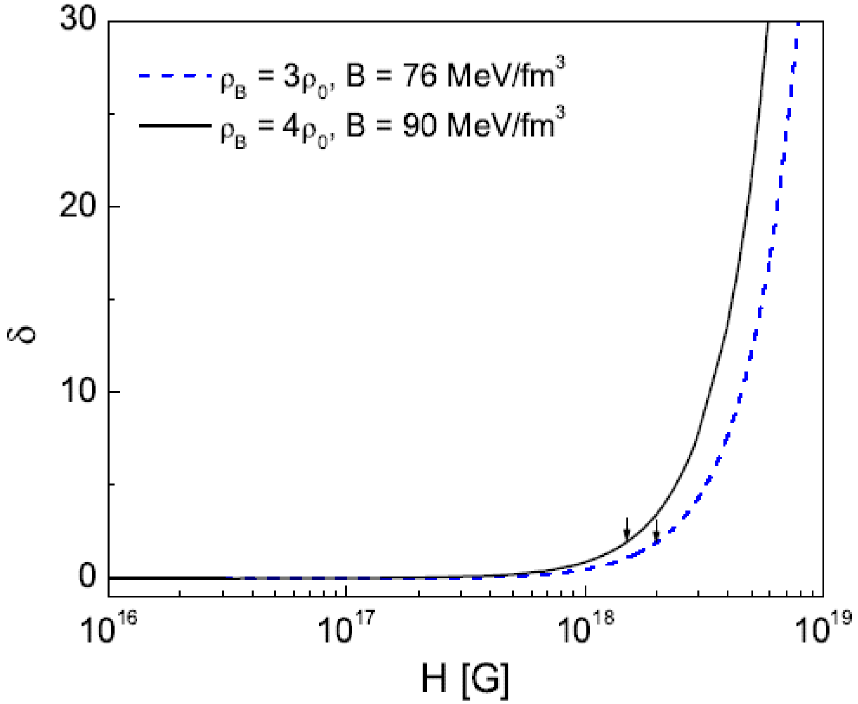


- The full polarization in a strong magnetic field can be achieved only for electrons.
- For various quark species, the spin polarization remains quite moderate up to the critical magnetic field H_{cr}. E.g., at $\varrho_B = 4\varrho_0, H = H_{cr}$ we have $\Pi_u \approx 0.08$, $\Pi_d \approx -0.04$, $\Pi_s \approx -0.05$;

Therefore, the occurrence of a field-induced fully polarized state in SQM is prevented by the appearance of the longitudinal instability in the critical magnetic field.

Isotropic vs. anisotropic regime

$$\delta = \frac{p_t - p_l}{p_0}$$



$H_{th} \approx 3.4\cdot10^{17}$ G at $\rho_B=3\rho_0$,
B = 76 MeV/fm^3

$H_{th} \approx 4.5\cdot10^{17}$ G at $\rho_B=4\rho_0$,
B = 90 MeV/fm^3

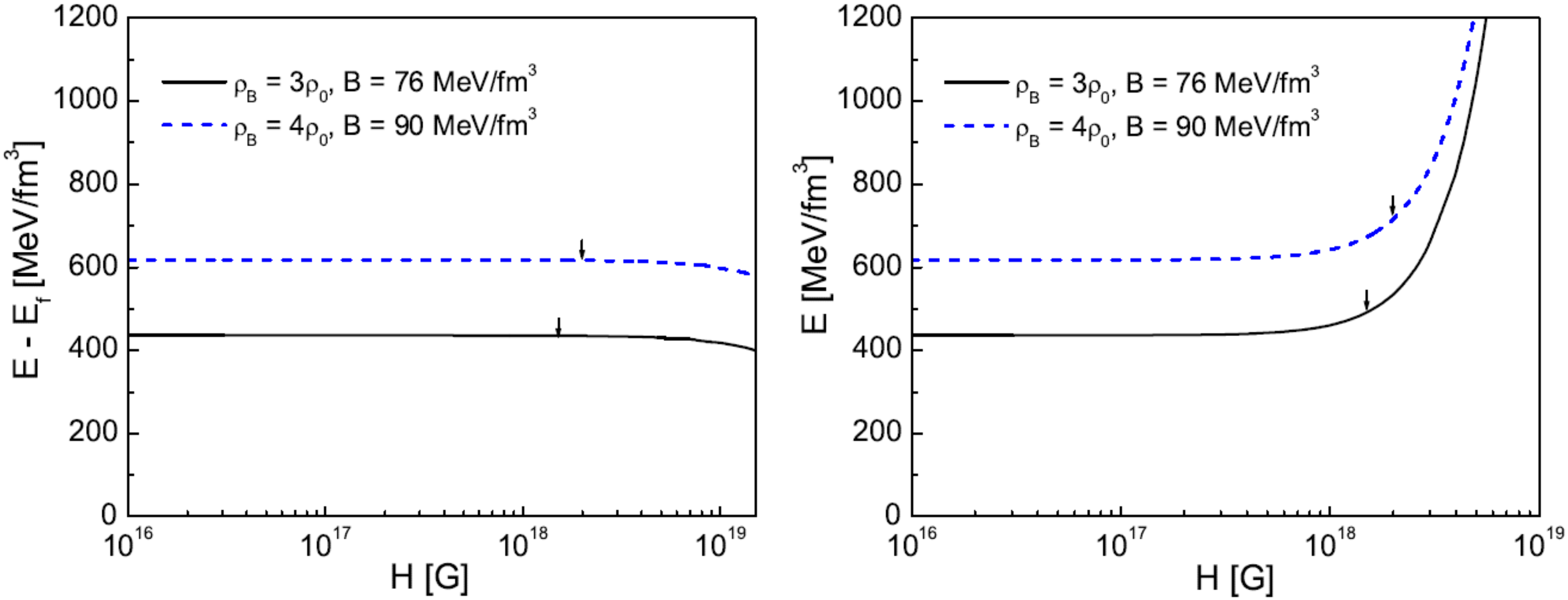
The normalized difference between the transverse and longitudinal pressures vs. H.

The criterion for the transition from the isotropic to the anisotropic regime:

$$\delta \approx 0.1$$

Anisotropic regime enters at the threshold field H_{th} with 10^{17} G<H_{th}<10^{18} G. The maximum normalized splitting of the pressures, corresponding to the critical field H_{cr} is $\delta \sim 2$.

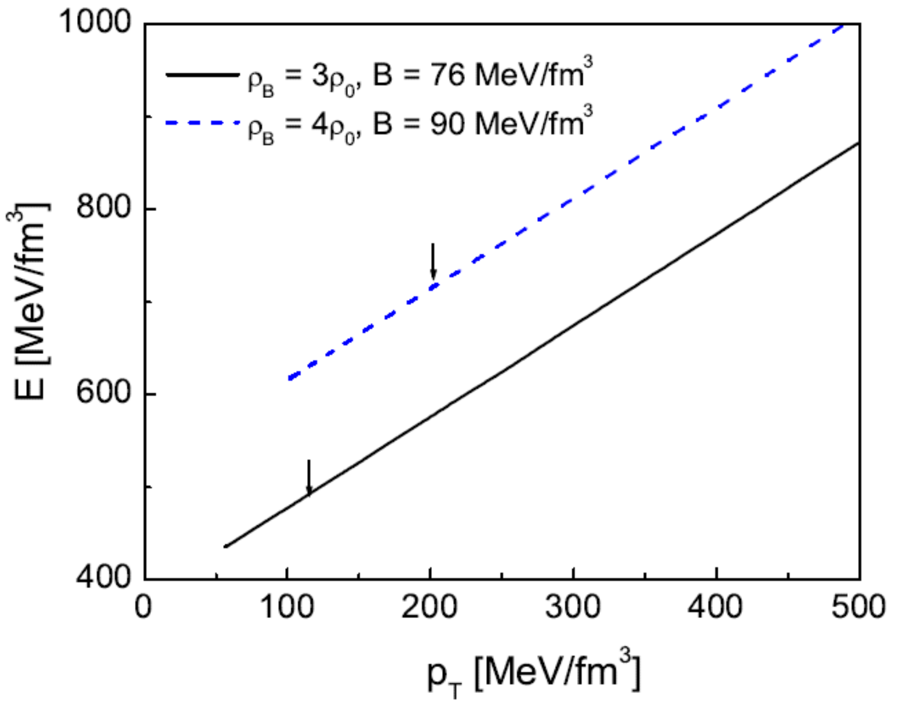
Energy density of magnetized SQM vs. H

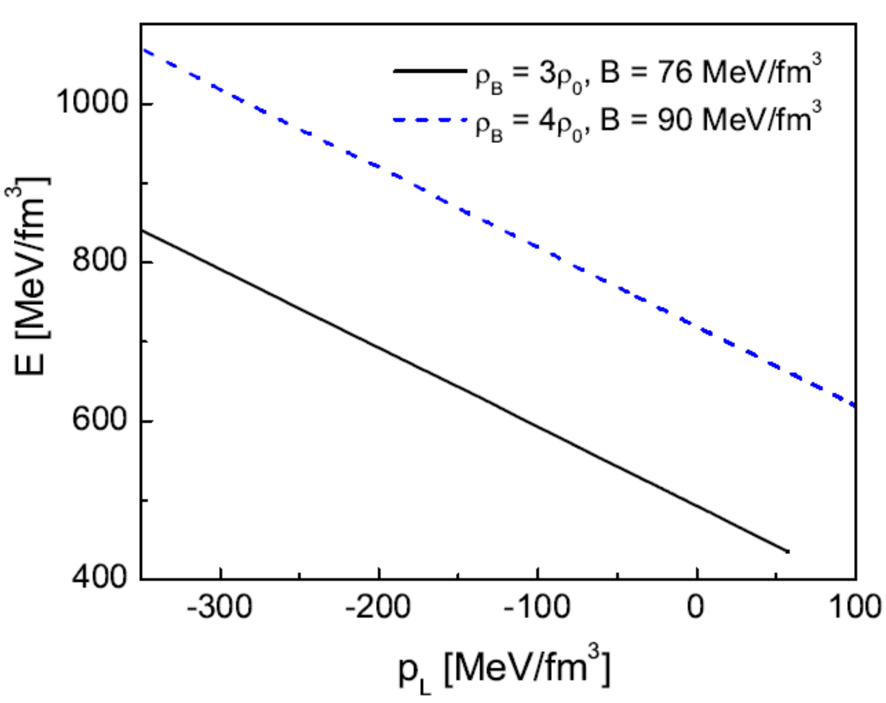


The energy density without (left)/with (right) magnetic field energy density E_f=$H^2/8\pi$ contribution vs. H

Because of Landau diamagnetism, the energy density of magnetized SQM without the pure magnetic field contribution (Maxwell term) decreases with *H*. However, the overall effect of the magnetic field, with account of the Maxwell term, is to increase the energy density. Nevertheless, this effect of the magnetic field is, in fact, insignificant because the magnetic field in the quark core is bound from above by the critical magnetic field H_{cr}.

Anisotropic EoS of SQM in strong *H*





The energy density of SQM as a function of the transverse pressure p_T (left), and the longitudinal pressure p_L(right).

Because of the pressure anisotropy, the EoS of SQM in a strong magnetic field is also anisotropic: the energy density is the increasing function of p_t while it decreases with p_l.

SQM in inhomogeneous magnetic field

In reality, magnetic field varies from the core to the surface. This variation can be modeled by the dependence H(ϱ_B):

$$H(\rho_B) = H_s + (H_c - H_s)\left[1 - e^{-\beta\left(\frac{\rho_B}{\rho_0}\right)^{\gamma}}\right] \qquad (1)$$

or

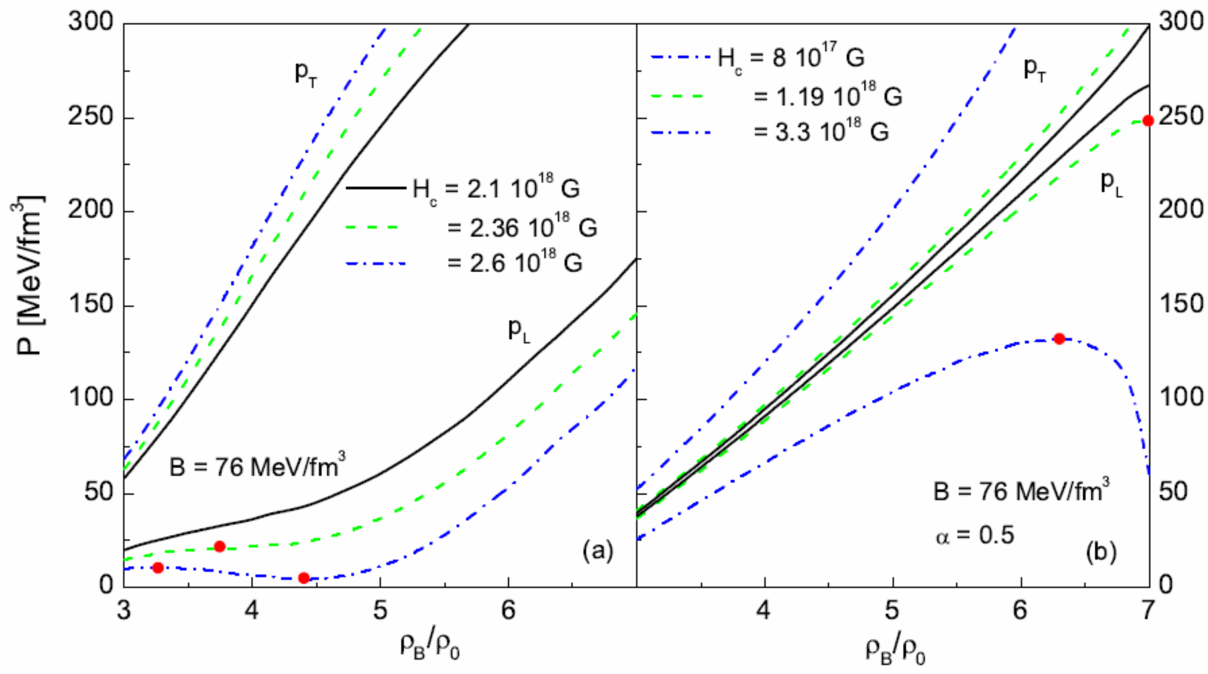
$$H(\rho_B) = H_s + (H_c - H_s)\left[1 - \left(1 - \frac{\rho_B}{\rho_c}\right)^{\alpha}\right] \qquad (2)$$

H_c and H_s – central and surface magnetic field strengths, ρ_c – central density

In numerical calculations: $H_s = 10^{15}$ G, $\rho_c = 7\rho_0$, The exponential parametrization with: **(1) β=0.02, γ=3;** The power parametrization with: (2) α=0.5

Further: assume that the deconfinement phase transition happens at $\varrho_B=3\varrho_0$ and hence the baryon density for SQM changes in the range: $3\varrho_0 <= \varrho_B <= 7\varrho_0$

Anisotropic pressure of SQM in inhomogeneous *H*

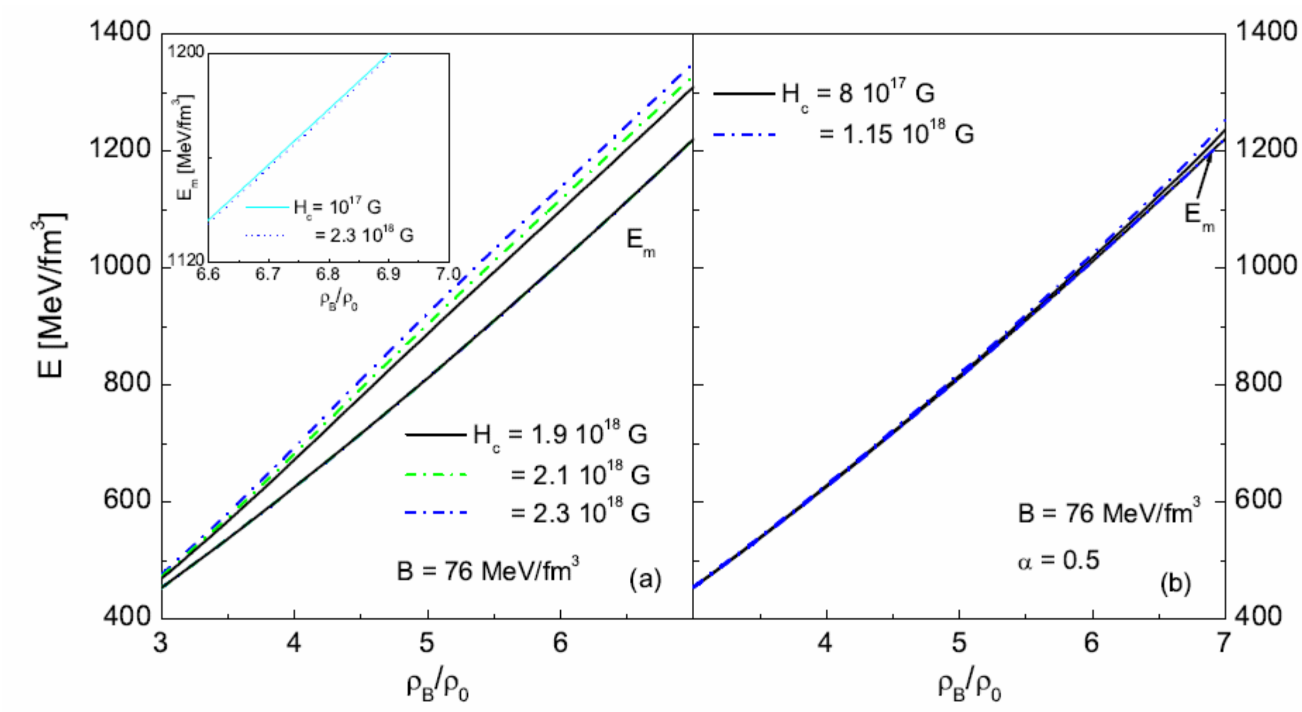


Transverse (the upper 3 curves) and longitudinal (the lower 3 curves) pressures in magnetized SQM vs. ρ_B at T=0 and variable central field H_c. The full dots correspond to the points where $p_L'(\rho_B)=0$.

States of MSQM with the central magnetic field, characterized by the appearance $p_L'(\rho_B)<0$, are unstable: instability is developed along the magnetic field

The onset of the longitudinal instability is associated with vanishing $p_L'(\rho_B)$ when it happens first under varying the central field.

Energy density of magnetized SQM vs. ρ_B

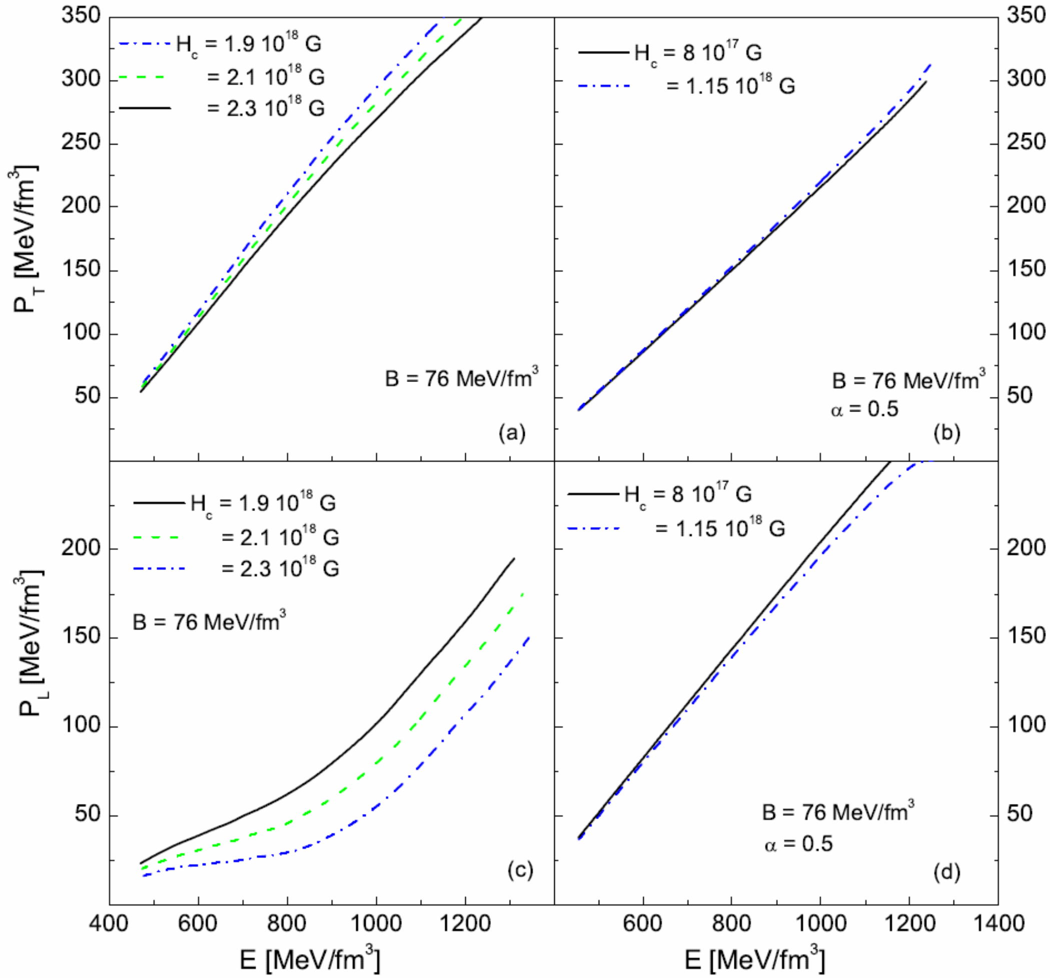


The energy density E (upper curves) and its matter part contribution $E_m = E - H^2/8\pi$ (the most lower curve) vs. ρ_B.

The relative role of the matter E_m and magnetic field E_f contributions to $E = E_m + E_f$: the matter part dominates over the field part at such ρ_B and H

Insert in the left panel: because of the Landau diamagnetism, the matter part contribution is smaller in the case of the stronger central magnetic field. But the overall effect of the stronger central magnetic field is to increase the total energy density E

Anisotropic EoS of magnetized SQM



Because of the pressure anisotropy in strongly magnetized SQM, the equation of state of the system is also highly anisotropic.

Conclusions

In summary, we have considered the impact of strong magnetic fields up to 10^{20} G on the thermodynamic properties of SQM at zero temperature under additional constraints of total baryon number conservation, charge neutrality and chemical equilibrium within the framework of the MIT bag model. The effects of the pressure anisotropy, exhibited in the difference between the pressures along and perpendicular to the field direction, should be accounted for in such ultrastrong magnetic fields.

Homogeneous magnetic field:

The longitudinal pressure vanishes in the critical field H_{cr}, which is somewhat larger than 10^{18} G, depending on the total baryon number density and bag pressure. As a result, the longitudinal instability occurs in strange quark matter, which sets the upper bound on the magnetic field strength in the core of a hybrid star.

The longitudinal instability precludes:

- a significant drop in the content of *s* quarks, which, otherwise, could happen at $H \sim 10^{20}$ G;
- the appearance of positrons in weak processes in a narrow interval near $H \sim 2 \cdot 10^{19}$ G (replacing electrons)

Conclusions

Only electrons can reach the state of full polarization, that is not true for quarks of all flavors, whose polarization remains mild even for magnetic fields near H_{cr}.

The EoS of SQM in strong magnetic fields $H > H_{th}$ (10^{17} G $< H_{th} < 10^{18}$ G) becomes essentially anisotropic. The longitudinal and transverse pressures as well as the anisotropic EoS of magnetized SQM have been determined at the total baryon number densities and magnetic field strengths relevant to the interiors of magnetars.

Inhomogeneous magnetic field (variable with the baryon density):

The onset of the longitudinal instability is associated with vanishing $p_L'(\rho_B)$.
The central magnetic field in the magnetar is bound from above by the value at which vanishing of the derivative $p_L'(\rho_B)$ occurs first somewhere in hybrid star's core.