# EMERGENT GRAVITY IN GRAPHENE

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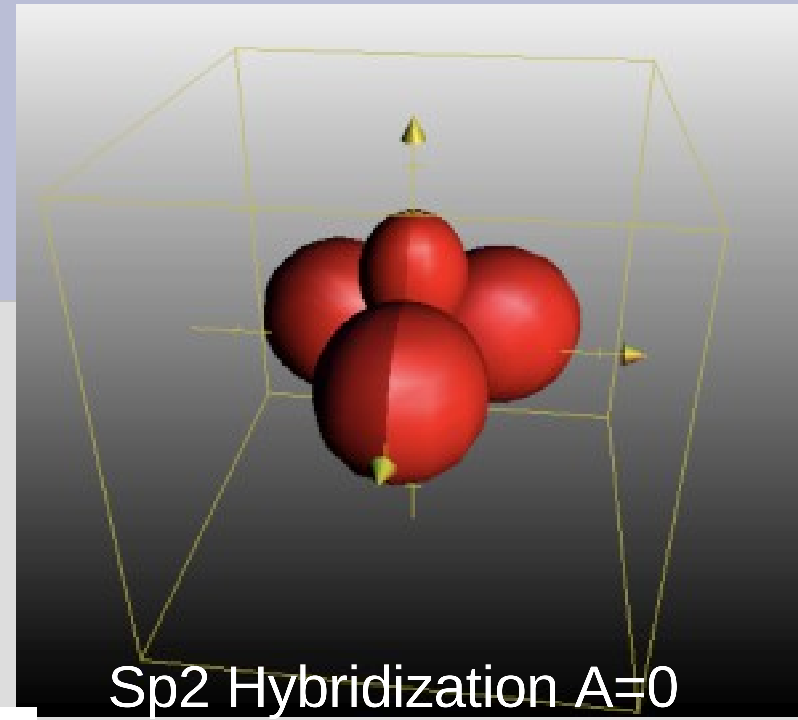
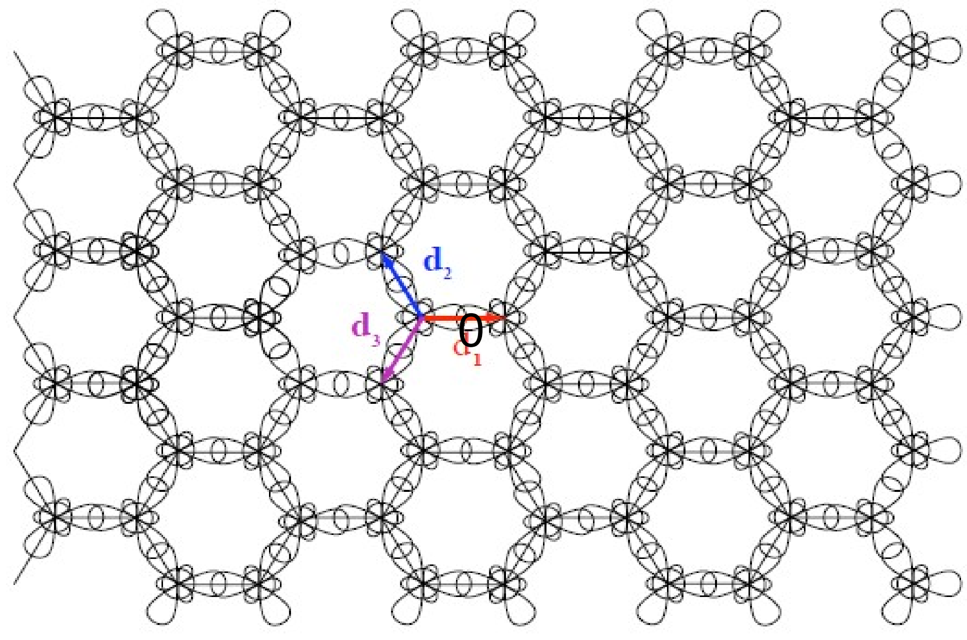
*We reconsider the tight - binding model of monolayer graphene, in which the variations of the hopping parameters are allowed. We demonstrate that the emergent 2D Weitzenbock geometry as well as the emergent U(1) gauge field appear. The emergent gauge field is equal to the linear combination of the components of the zweibein. Therefore, we actually deal with the gauge fixed version of the emergent 2+1 D teleparallel gravity.  In particular, we work out the case, when the variations of the hopping parameters are due to the elastic deformations, and relate the elastic deformations with the emergent zweibein.*

$1s^2 \ 2s^2 \ 2p^2$

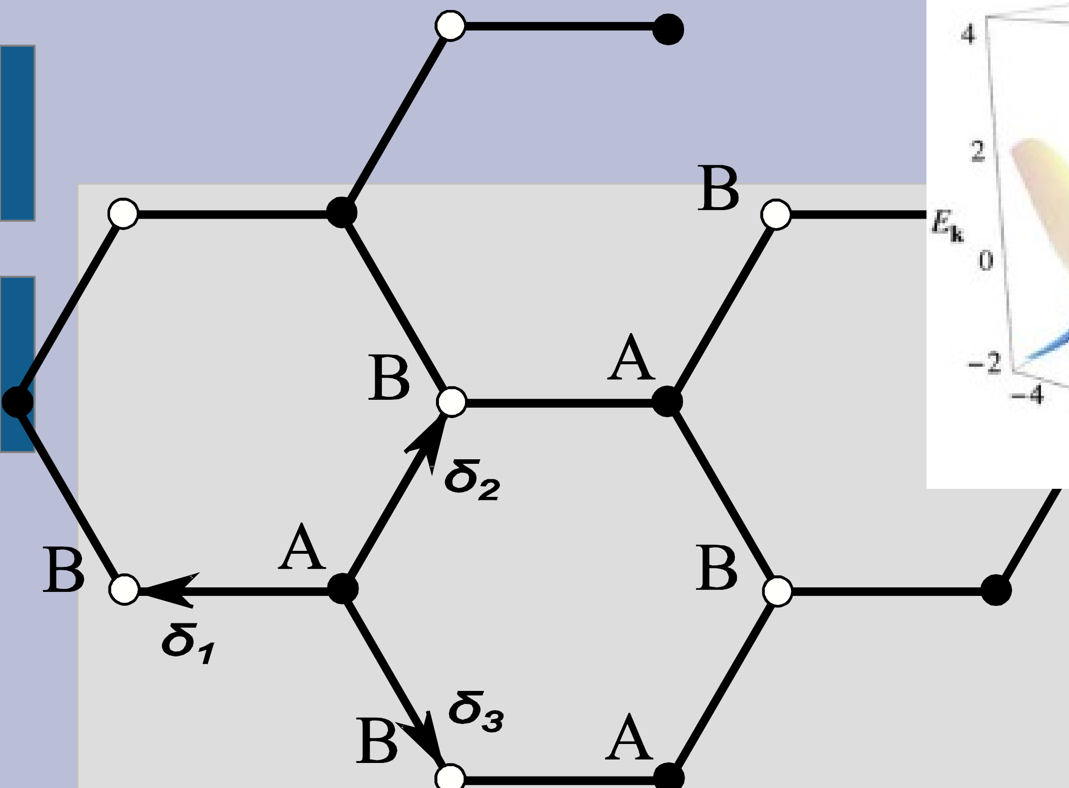
$$|0\rangle = A|s\rangle + \sqrt{1-A^2}|p_z\rangle$$

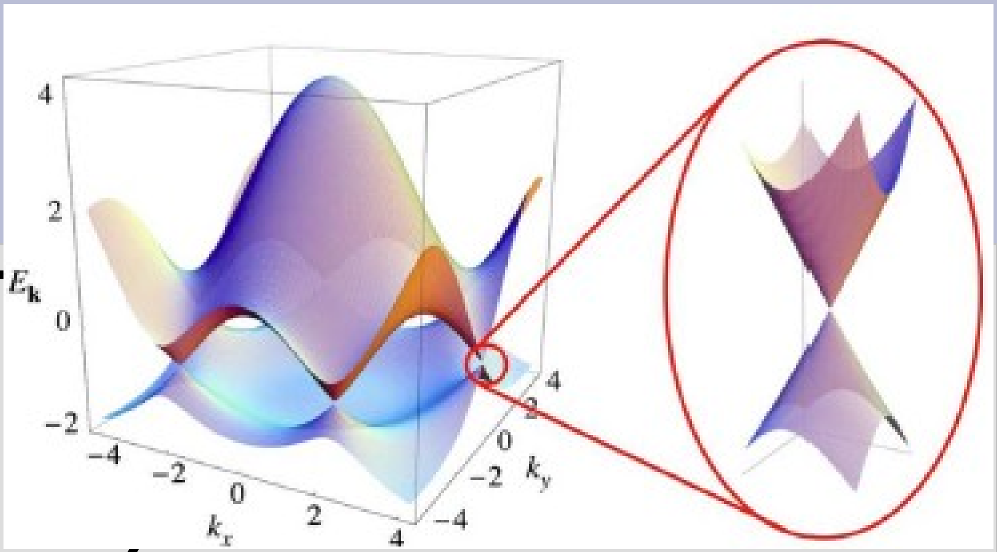
$$
\begin{aligned}
|1\rangle &= \sqrt{(1-A^2)/3}|s\rangle + \sqrt{2/3}|p_x\rangle - (A/\sqrt{3})|p_z\rangle\,, \\
|2\rangle &= \sqrt{(1-A^2)/3}|s\rangle - \sqrt{1/6}|p_x\rangle - \sqrt{1/2}|p_y\rangle - (A/\sqrt{3})|p_z\rangle \\
|3\rangle &= \sqrt{(1-A^2)/3}|s\rangle - \sqrt{1/6}|p_x\rangle + \sqrt{1/2}|p_y\rangle - (A/\sqrt{3})|p_z\rangle
\end{aligned}
$$



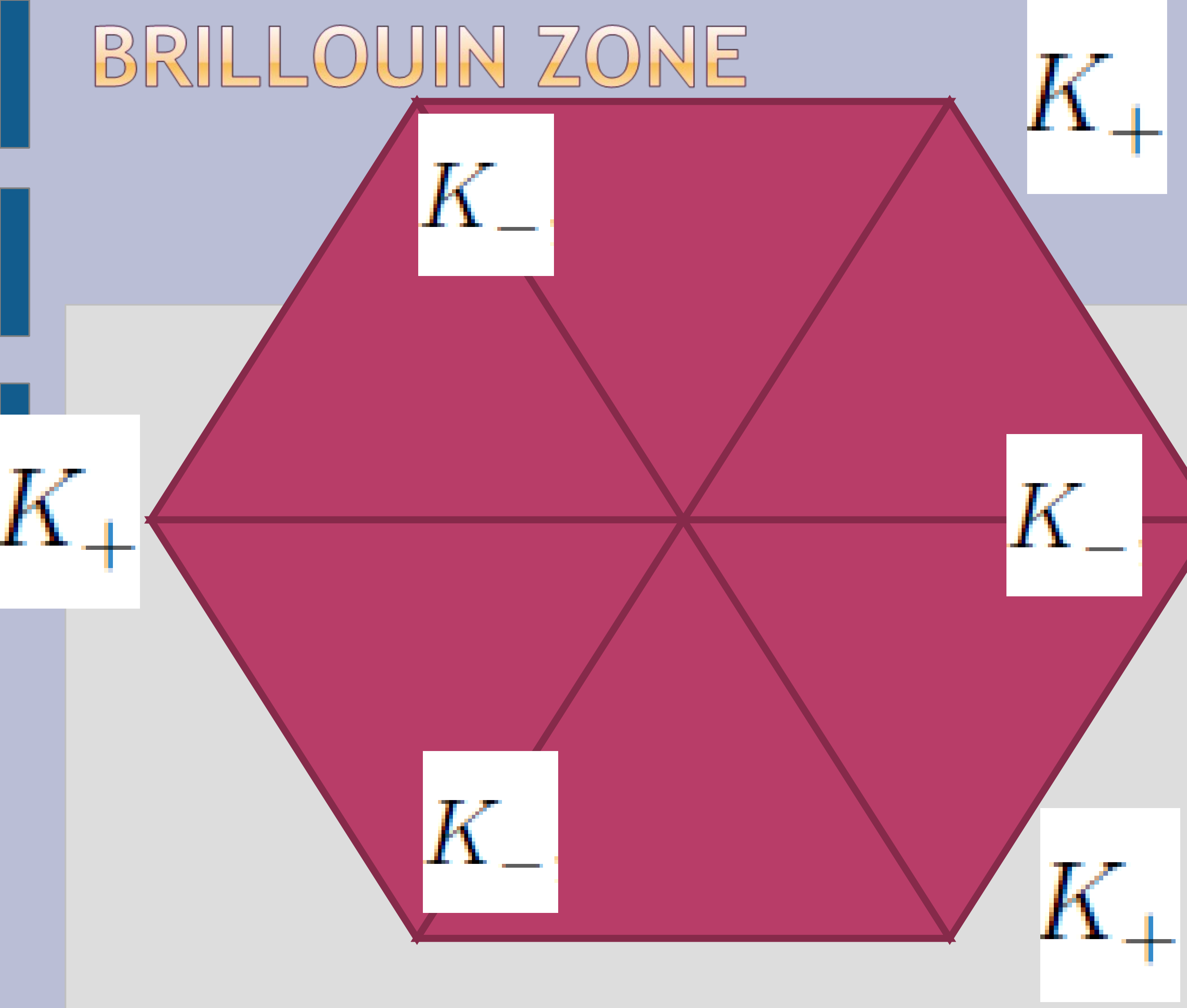
# TIGHT - BINDING MODEL




$$H = -t \sum_{\alpha \in A} \sum_{j=1}^{3} \Big( \psi^\dagger(\mathbf{r}_\alpha)\psi(\mathbf{r}_\alpha + \mathbf{l}_j) + \psi^\dagger(\mathbf{r}_\alpha + \mathbf{l}_j)\psi(\mathbf{r}_\alpha) \Big)$$

# BRILLOUIN ZONE



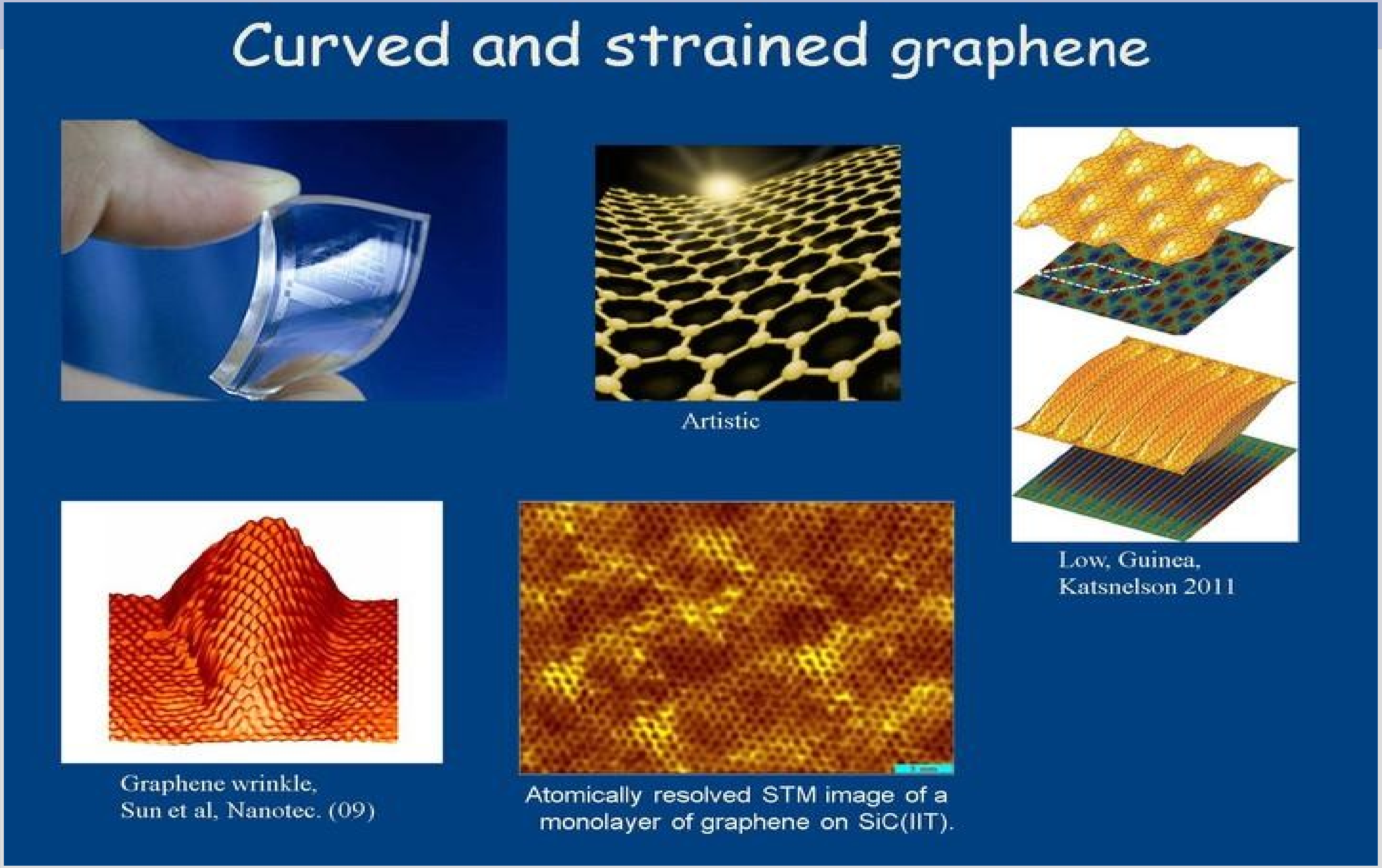

$$\begin{pmatrix} \psi_A^+ \\ \psi_B^+ \\ \psi_A^- \\ \psi_B^- \end{pmatrix}$$

# EFFECTIVE FIELD MODEL

$$H = \sum_{\pm} \int d^2x [\Psi^{\pm}(\mathbf{x})]^{\dagger} \mathbf{H}_{\pm} \Psi^{\pm}(\mathbf{x})$$

$$\begin{aligned} \mathbf{H}_{-} &= -\sigma^3 \, v_F \, \delta_a^k(\mathbf{x}) \sigma^a \partial_k \\ \mathbf{H}_{+} &= -\sigma^2 \left( \sigma^3 \, v_F \, \delta_a^k(\mathbf{x}) \sigma^a \partial_k \right) \sigma^2 \end{aligned}$$

From M.Vozmediano, talk at
KITP Conference: Fundamental Aspects of Graphene and Other Carbon Allotropes (Jan 9-13, 2012)



# Previous suppositions

## 1. Strain induces the U(1) gauge field.

Ken-ichi Sasaki, Yoshiyuki Kawazoe, and Riichiro Saito, Local Energy Gap in Deformed Carbon Nanotubes, Progress of Theoretical Physics, Vol. 113, No. 3, March 2005

## 2. Strain induces Riemannian Gravity

Gauge fields in graphene

M. A. H. Vozmediano, M. I. Katsnelson, F. Guinea

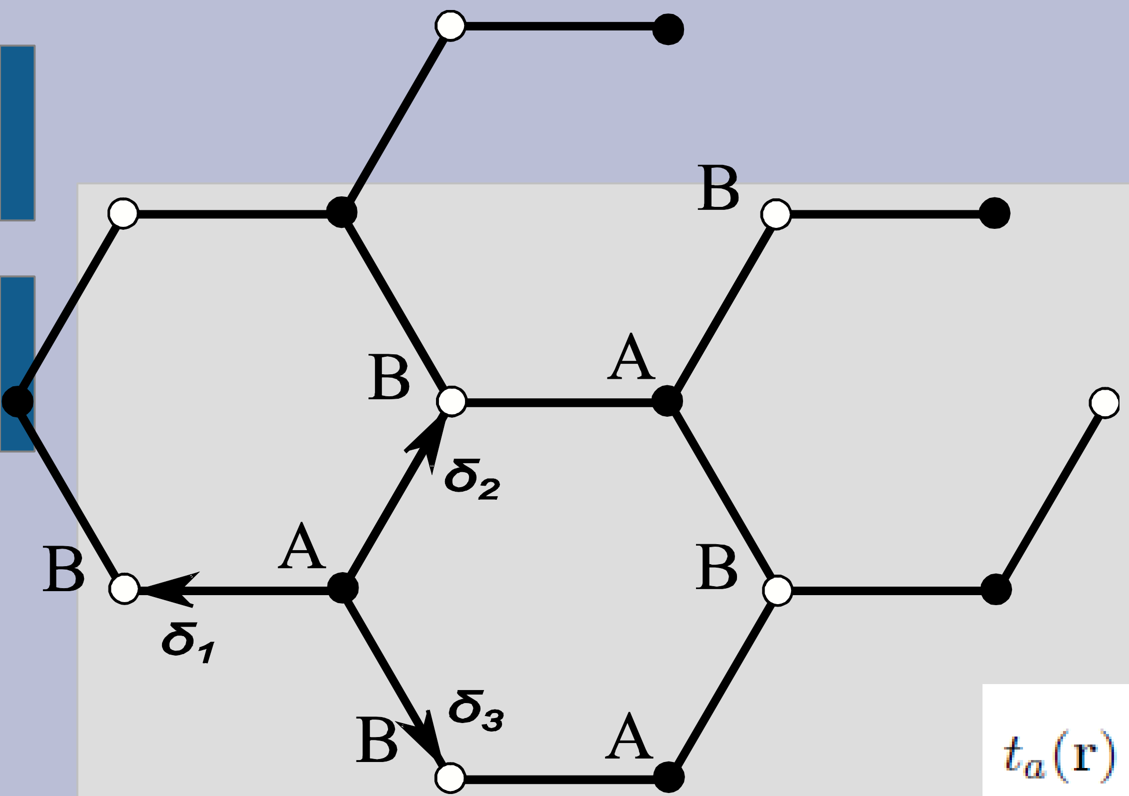
Physics Reports 496, 109 (2010)

## 3. Strain induces the U(1) gauge field and the space – dependent fermi - velocity

Fernando de Juan, Mauricio Sturla, and Maria A.H. Vozmediano, Phys. Rev

Lett. 108, 227205 (2012)

# TIGHT - BINDING MODEL




$$t_a(\mathbf{r}) = t(1 - \Delta_a(\mathbf{r})), \quad |\Delta_a| \ll 1$$

$$H = -\sum_{\alpha \in A} \sum_{j=1}^{3} t_j(\mathbf{r}_\alpha) \Big( \psi^\dagger(\mathbf{r}_\alpha)\psi(\mathbf{r}_\alpha + \mathbf{l}_j) + \psi^\dagger(\mathbf{r}_\alpha + \mathbf{l}_j)\psi(\mathbf{r}_\alpha) \Big)$$

$$\mathbf{l}_1 = (-a, 0), \qquad \mathbf{l}_2 = (a/2, a\sqrt{3}/2), \qquad \mathbf{l}_3 = (a/2, -a\sqrt{3}/2)$$

$$\mathbf{m}_1 = -\mathbf{l}_1 + \mathbf{l}_2, \qquad \mathbf{m}_3 = -\mathbf{l}_3 + \mathbf{l}_1, \qquad \mathbf{m}_2 = -\mathbf{l}_2 + \mathbf{l}_3 = -\mathbf{m}_1 - \mathbf{m}_3$$

$$H = \int \frac{d^2k}{\Omega} \frac{d^2k'}{\Omega} \psi^\dagger(\mathbf{k}') \hat{V}(\mathbf{k}', \mathbf{k}) \psi(\mathbf{k}),$$

$$\hat{V}(\mathbf{k}', \mathbf{k}) = -\sum_{j=1}^{3} t_j(\mathbf{k}' - \mathbf{k}) \begin{pmatrix} 0 & e^{-i\mathbf{l}_j \mathbf{k}} \\ e^{i\mathbf{l}_j \mathbf{k}'} & 0 \end{pmatrix}$$

**First, we consider the situation, when the three hopping parameters t_a are different, but do not depend on the position in coordinate space.**

$$E(k) = \pm |t_1 + t_2 e^{i\mathbf{m}_1(x)\mathbf{k}} + t_3 e^{-i\mathbf{m}_3(x)\mathbf{k}}| \qquad \mathbf{k} = \mathbf{k}'.$$

*Emergent $U(1)$ field.*

$$\sum_{j=1}^{3} t_j \begin{pmatrix} 0 & e^{-i\mathrm{l}_j K^{\pm}} \\ e^{i\mathrm{l}_j K^{\pm}} & 0 \end{pmatrix} = 0$$

$$K_{\pm}^{(0)} = \mp \tfrac{4\pi}{9} \mathbf{m}_2$$

$$K^{\pm} = \pm \frac{1}{a^2}\Big[\frac{\phi_1 - \phi_3}{3}\left(-\mathrm{l}_1\right) + \frac{\phi_1 + \phi_3}{\sqrt{3}}\left(-\frac{\mathrm{m}_2}{\sqrt{3}}\right)\Big]$$

$$\begin{aligned}
\phi_1 &= \frac{\pi}{2} + \arcsin\frac{-t_3^2 + t_2^2 + t_1^2}{2t_2t_1} \approx \frac{2\pi}{3} + \frac{1}{\sqrt{3}}(2\Delta_3 - \Delta_1 - \Delta_2) \\
\phi_3 &= \frac{\pi}{2} + \arcsin\frac{-t_2^2 + t_3^2 + t_1^2}{2t_3t_1} \approx \frac{2\pi}{3} + \frac{1}{\sqrt{3}}(2\Delta_2 - \Delta_1 - \Delta_3)
\end{aligned}$$

$$\begin{aligned}
K^{\pm} &\approx \pm\frac{1}{a^2}\Big[\frac{\Delta_3 - \Delta_2}{\sqrt{3}}\left(-\mathrm{l}_1\right) \\
&\quad + \left(\frac{4\pi}{3\sqrt{3}} + \frac{\Delta_3 + \Delta_2 - 2\Delta_1}{3}\right)\left(-\frac{\mathrm{m}_2}{\sqrt{3}}\right)\Big]
\end{aligned}$$

$$\begin{aligned}
\mathbf{A}_1 &= \frac{1}{a}\,\frac{\Delta_3 - \Delta_2}{\sqrt{3}} \\
\mathbf{A}_2 &= \frac{1}{a}\,\frac{\Delta_3 + \Delta_2 - 2\Delta_1}{3}
\end{aligned}$$

$$\mathbf{A}^b = -\frac{2}{3a^2}\epsilon^{ba}\sum_j \Delta_j \mathrm{l}_j^a$$

## *Emergent zweibein: expansion around the true Fermi - point*

$$\mathbf{k} = K_{\pm} + \mathbf{q}$$

$$\hat{V}_{\pm} = (\pm\sigma^1 \mathbf{f}_2 + \sigma^2 \mathbf{f}_1)\mathbf{q}$$

$$\mathbf{f}_a^k = v_F\left(\delta_a^k - \frac{2}{3a^2}\sum_j \Delta_j \left[\mathbf{l}_j^a \mathbf{l}_j^k - \frac{a}{2}\mathbf{l}_j^d K^{dak}\right]\right)$$

$$K^{ijk} = -\frac{4}{3a^3}\sum_b \mathbf{l}_b^i \mathbf{l}_b^j \mathbf{l}_b^k, \quad K^{111} = -K^{122} = -K^{221} = -K^{212} = 1$$

$$\mathbf{f}_a^i = v_F\left(\delta_a^i - \begin{bmatrix} \Delta_1 & \frac{(\Delta_2 - \Delta_3)}{\sqrt{3}} \\ \frac{(\Delta_2 - \Delta_3)}{\sqrt{3}} & \frac{1}{3}(2\Delta_2 + 2\Delta_3 - \Delta_1) \end{bmatrix}\right)$$

$$\mathbf{A}^i = -\frac{1}{2v_F a}\epsilon^{ik} K^{kjb} \mathbf{f}_b^j$$

$$\Psi_\pm(\mathbf{Q}) = \psi(K^{(0)}_\pm + \mathbf{Q})$$

$$H = \int \frac{d^2\mathbf{Q}}{\Omega} \Psi^\dagger(\mathbf{Q}) \hat{V}_\pm(\mathbf{Q}) \Psi(\mathbf{Q})$$

$$\hat{V}_\pm \;\; = \;\; -i\sigma^3 \Big[ (\mp\sigma^2 \mathbf{f}_2 + \sigma^1 \mathbf{f}_1) \Big( \mathbf{Q} \mp \mathbf{A} \Big) \Big]$$

$$\mathbf{Q} \;\; \to \;\; -i\nabla$$

$$H = \sum_\pm \int d^2x [\Psi^\pm(\mathbf{x})]^\dagger \mathbf{H}_\pm \Psi^\pm(\mathbf{x})$$

$$\begin{aligned} \mathbf{H}_- \;\; &= \;\; -\sigma^3 \, \mathbf{f}^k_a \sigma^a [\partial_k + i\mathbf{A}_k], \quad a = 1,2; k = 1,2; \\ \mathbf{H}_+ \;\; &= \;\; -\sigma^2 \Big( \sigma^3 \, \mathbf{f}^k_a \sigma^a [\partial_k - i\mathbf{A}_k] \Big) \sigma^2. \end{aligned}$$

# Inhomogenious hopping parameters

$$H = \int \frac{d^2\mathbf{Q}}{\Omega}\frac{d^2\mathbf{Q}'}{\Omega}\Psi^\dagger(\mathbf{Q}')\hat{V}_\pm(\mathbf{Q}',\mathbf{Q})\Psi(\mathbf{Q}),$$

$$\hat{V}_\pm(\mathbf{Q},\mathbf{Q}') \quad = \quad -i\sigma^3\Big[(\mp\sigma^2\mathbf{f}_2 + \sigma^1\mathbf{f}_1)\Big(\frac{\mathbf{Q}+\mathbf{Q}'}{2} \mp \mathbf{A}\Big) - (\sigma^1\mathbf{f}_1 \mp \sigma^2\mathbf{f}_2)\sigma^3\frac{\mathbf{Q}-\mathbf{Q}'}{2}\Big]$$

$$H = \sum_\pm \int d^2x [\Psi^\pm(\mathbf{x})]^\dagger \mathbf{H}_\pm \Psi^\pm(\mathbf{x})$$

$$\begin{aligned}
\mathbf{H}_- \quad &= \quad -\sigma^3\, \mathbf{f}_a^k(\mathbf{x})\sigma^a \circ [\partial_k + i(\mathbf{A}_k(\mathbf{x}) + \tilde{\mathbf{A}}_k(\mathbf{x}))] \\
\mathbf{H}_+ \quad &= \quad -\sigma^2\Big(\sigma^3\, \mathbf{f}_a^k(\mathbf{x})\sigma^a \circ [\partial_k - i(\mathbf{A}_k(\mathbf{x}) + \tilde{\mathbf{A}}_k)]\Big)\sigma^2
\end{aligned}$$

$$\tilde{\mathbf{A}}_a(\mathbf{x}) \quad = \quad \frac{1}{2v_F}\nabla_i \mathbf{f}_b^i(\mathbf{x})\epsilon_{ba}$$

$$\mathbf{f}_a^k \circ i\partial_k = \frac{i}{2}\Big(\mathbf{f}_a^k\overrightarrow{\partial_k} - \overleftarrow{\partial_k}\mathbf{f}_a^k\Big)$$

# the effective Hamiltonian near to the Fermi point $K_-$

$$H=\sum_{\pm}\int d^2x[\Psi^{\pm}(\mathbf{x})]^\dagger\mathbf{H}_\pm\Psi^{\pm}(\mathbf{x})$$

$$\mathbf{H}_+(\mathbf{A})=\sigma^2\mathbf{H}_-(-\mathbf{A})\sigma^2$$

$$\begin{aligned} H &= -\frac{i}{2}\int e\,d^2x\Big(\bar\Psi_-(\mathbf{x})\mathbf{e}^k_a\sigma^a D_k\Psi_-(\mathbf{x}) - [D_k^\dagger\bar\Psi_-(\mathbf{x})]\mathbf{e}^k_a\sigma^a\Psi_-(\mathbf{x})\Big) \\ &= \int d^2x\,\bar\Psi_-(\mathbf{x})\mathcal{H}\Psi_-(\mathbf{x}) \end{aligned}$$

$$\bar\Psi=-i\Psi^\dagger\sigma^3,$$

$$\mathcal{H}=i\sigma^3\mathbf{H}_- = -ie\,\mathbf{e}^k_a\sigma^a\circ[\partial_k+i\mathbf{A}_k],$$

$$\mathbf{e}^i_a = \mathrm{f}^i_a/e, \quad e=[\det \mathrm{f}]^{1/2}=v_F(1-\frac{1}{3}(\Delta_2+\Delta_3+\Delta_1))$$

$$\mathcal{S}=\int d^4x|\det e|\,e^\mu_a\,\bar\psi\sigma^a(p_\mu-\mathcal{A}_\mu)\psi+\dots$$

# In the presence of strain

$$\begin{aligned} y_k(x) &= x_k + u_k(x), \quad k = 1,2 \\ y_3(x) &= u_3(x) \end{aligned}$$

$$u_{ik} = \frac{1}{2}\Big(\partial_i u_k + \partial_k u_i + \partial_i u_a \partial_k u_a\Big), \quad a = 1,2,3, \quad i,k = 1,2.$$

$$t_a(\mathbf{r}) = t[1 - \beta u_{ik}(\mathbf{r}) l_a^i l_a^k]$$

$$\begin{aligned} H &= -\frac{i}{2}\int e\, d^2x \Big(\bar{\Psi}_-(\mathbf{x}) \mathrm{e}_a^k \sigma^a D_k \Psi_-(\mathbf{x}) - [D_k^\dagger \bar{\Psi}_-(\mathbf{x})] \mathrm{e}_a^k \sigma^a \Psi_-(\mathbf{x})\Big) \\ &= \int d^2x \bar{\Psi}_-(\mathbf{x}) \mathcal{H} \Psi_-(\mathbf{x}) \end{aligned}$$

$$\bar{\Psi} = -i\Psi^\dagger \sigma^3,$$

$$\mathcal{H} = i\sigma^3 \mathbf{H}_- = -ie\, \mathrm{e}_a^k \sigma^a \circ [\partial_k + i\mathbf{A}_k],$$

$$\mathrm{e}_a^i = \mathrm{f}_a^i / e, \quad e = [\det \mathrm{f}]^{1/2} = v_F\left(1 - \frac{1}{3}(\Delta_2 + \Delta_3 + \Delta_1)\right)$$

$$\mathrm{f}_a^i = v_F \left(\delta_a^i - \beta \begin{bmatrix} u_{11} & u_{21} \\ u_{12} & u_{22} \end{bmatrix}\right)$$

$$\mathbf{A}_1 = \frac{1}{2a}(\mathrm{e}_2^1 + \mathrm{e}_1^2), \quad \mathbf{A}_2 = \frac{1}{2a}(\mathrm{e}_1^1 - \mathrm{e}_2^2)$$

# CONCLUSIONS

1. the varying hopping parameters for monolayer graphene give rise to the varying 2D zweibein e.
2. The other existing field (the 2D gauge potential A) is expressed through e.
3. The varying 2D Weitzenbock geometry defined by e appears.
4. The field A[e] gives the terms of the action that are not invariant under the 2D diffeomorphisms.
5. Formally the considered action may be treated as the action for the 2D Weitzenbock geometry if it is considered as a gauge fixed version of the action for the invariant theory.