Non-extensivity of hadronic systems

Airton Deppman

First KWPPP - Moscow - July, 2013


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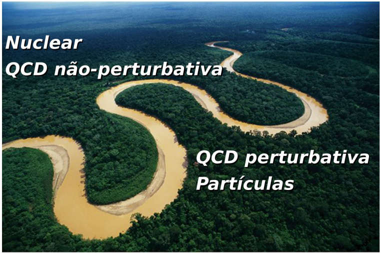
Outline

- HEP and non-perturbative QCD.
- Hagedorn's theory.
- Experimental verification of the theory.
- Non-perturbative-QCD, lattice-QCD and AdS/CFT.
- Generalization of Hagedorn's formalism.
- Non-extensive self-consistent theory.
- Experimental evidences of non-extensive self-consistency.
- Thermodynamical functions and lattice-QCD.
- Conclusions.

The non-perturbative river

QCD is a well-known theory for strong interaction

and works very well for Particle Physics.



In Nuclear Physics it faces some problems: non-perturbative character.

We need a bridge!

Hagedorn's theory

A hadronic system is considered as an ideal gas of hadrons a a temperature T. The partition function is

$$ln[1 + Z(V_0, T)] = \frac{V_0 T}{2\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \int_0^{\infty} \rho(m; n) m^2 K_2 (\beta\, n\, m)\, dm.$$

The bootstrap idea is:
Fireball is "$*$the static equillibrium of a system composed by *fireballs*, which on their turn are . . . (goto $*$)"

The partition function can also be written as

$$Z(V_0, T) = \int_0^E \sigma(E')Z_0(E')dE'$$

According to the bootstrap principle, both forms of partition function must be asymptotically identical with

$$\ln[\sigma(E')] = \ln[\rho(m)]$$

Self-consistency

Both constraints can be satisfied with

$$\sigma(E) \to b\,E^{\alpha-1}\,e^{\beta_o E}$$

$$\rho(m) \to m^{-5/2} e^{\beta_o m}$$

And in this case both expression for the partition function reduce to

$$Z(V_o, T) \sim \left(\frac{1}{\beta - \beta_o)}\right)^{\alpha}$$

with

$$\alpha = \frac{a V_0}{(2\pi\beta_0)^{3/2}}$$

Experimental verification

With

$$\bar{\nu}_k = -\frac{1}{\beta}\frac{\partial ln\, Z(T)_\pm}{\partial \epsilon_k}$$

and using the thermodynamical relations we get

$$\bar{\nu}_k = e^{-\beta_0 \epsilon_k} \Rightarrow ln(\bar{\nu}_k) = -\beta_0 \epsilon_k$$

It is more usefull to write it in terms of the transversal momemtum. Using

$$\epsilon = \sqrt{p_\perp + p_z^2 + m^2}$$

we get, for $p_\perp >> T_0 >> m$,

$$w(p_\perp) \approx C \cdot p_\perp^{3/2} exp\left(-\frac{p_\perp}{T_0}\right)$$

Phase transition

Volume 59B, number 1 PHYSICS LETTERS 13 October 1975

EXPONENTIAL HADRONIC SPECTRUM AND QUARK LIBERATION

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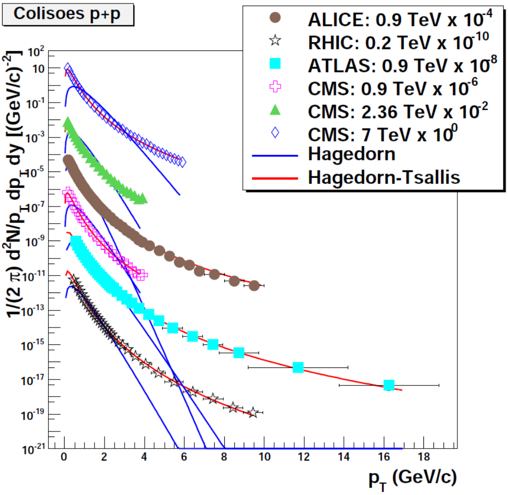
Received 9 June 1975

The exponentially increasing spectrum proposed by Hagedorn is not necessarily connected with a limiting temperature, but it is present in any system which undergoes a second order phase transition. We suggest that the "observed" exponential spectrum is connected to the existence of a different phase of the vacuum in which quarks are not confined.

Validade da Teoria de Hagedorn

Hagedorn's works nicely up to $\sqrt{s} \approx 10$ GeV. For higher energies:



Other possible bridges

Lattice QCD

trace-anomaly

speed of sound

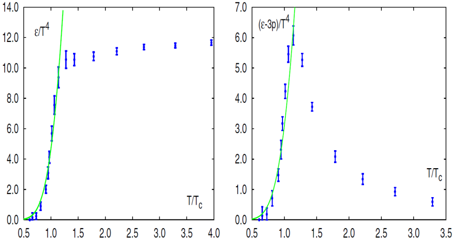
Duality AdS/CFT

viscosity

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

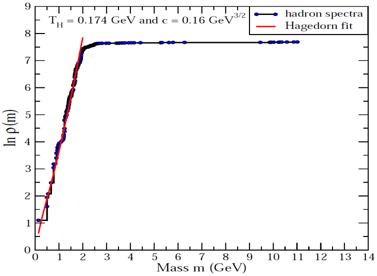
Hadron Resonance Gas Models

F. Karsch et al.: Hadron resonance mass spectrumand lattice QCD thermodynamics



J. Cleymans & D. Worku

EPJC 2003



Mod. Phys. Lett A 2011

J. Noronha-Hostler et al. arXiv:1206.5138 for a recent comparison of different mass spectra.

Hadron Resonance Gas Models - problems

Eur. Phys. J. C (2010) 66: 207–213
DOI 10.1140/epjc/s10052-009-1231-8

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

The speed of sound in hadronic matter

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1 Introduction

The abundant formation of resonances of increasing mass and rotational degrees of freedom is one of the most striking features of strong interaction physics, which has attracted intense theoretical attention for half a century or more. Even before the quark infrastructure of hadrons was known, a self-similar composition scheme, the statistical bootstrap model (SBM), led to an exponentially increasing resonance spectrum [1, 2]. Shortly thereafter the dual resonance model (DRM) provided a description of hadron interactions in

The partition function of an ideal gas of constituents of mass $m(T)$ is in the Boltzmann limit for $SU(N)$ gauge theory given by

$$\ln Z(T) = 2\frac{(N^2-1)V}{2\pi^2}\int_0^\infty dp\, p^2 \exp\left(-\frac{1}{T}\sqrt{p^2+m^2}\right) = 2\frac{(N^2-1)VTm^2}{2\pi^2}K_2\left(\frac{m}{T}\right), \quad (15)$$

where $K_i(x)$ denotes the Hankel function of imaginary argument. The resulting pressure becomes

$$P(T) = T\left(\frac{\partial \ln Z}{\partial V}\right)_T = 2\frac{(N^2-1)T}{2\pi^2}\int_0^\infty dp\, p^2 \exp\left(-\frac{1}{T}\sqrt{p^2+m^2}\right) = 2\frac{(N^2-1)T^2m^2}{2\pi^2}K_2\left(\frac{m}{T}\right) \quad (16)$$

while the energy density is found to be

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Vol. 26, No. 16 (2011) 1197–1209

DOI: 10.1142/S0217732311035584

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THE HAGEDORN TEMPERATURE REVISITED

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increase determines the value of the Hagedorn temperature, $T_{\rm H}$. Recent papers,[16–21] have used the latest results from the Particle Data Group[15] to revisit the original analysis of Hagedorn to update the value of $T_{\rm H}$. This resulted in a surprising wide spread of possible values, with large variations as to whether one considers mesons or baryons with values ranging from $T_{\rm H} = 141$ MeV to $T_{\rm H} = 340$ MeV depending on the parametrization used and on the set of hadrons (mesons or baryons). There thus exists uncertainty as to the value of the Hagedorn temperature. These have two origins:

- sparse information about hadronic resonances certainly above 3 GeV,
- the analytical form of the Hagedorn spectrum, especially the factor multiplying the exponential.

Introducing non-extensivity: Bediaga's generalization

Bediaga:

$$x_{ij} = (1 + (q-1)\beta\epsilon_{ij})^{-q/(q-1)}$$

Partition function:

$$logZ = -\sum_{ij} log(1 - x_{ij}) + \sum_{i'j'} log(1 + x_{i'j'})$$

and ...

$$w(p_\perp) \approx const \cdot p_\perp \int_0^\infty dp_L (1 + (q-1)\beta\sqrt{p_\perp^2 + p_L^2 + m_0^2})^{-\frac{q}{q-1}}$$

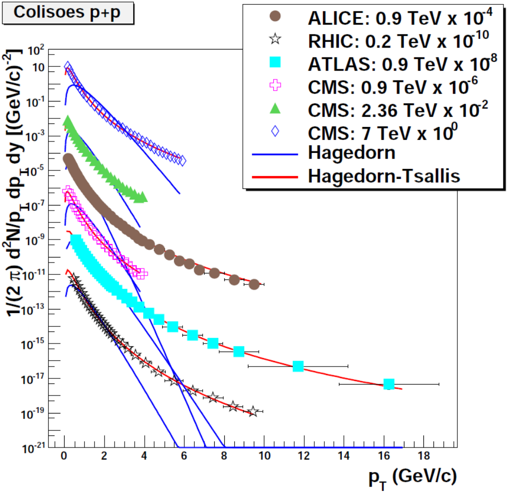
Transverse momentum distribution (C. Beck):

$$\frac{1}{\sigma}\frac{d\sigma}{dp_\perp} = c\ (2(q-1))^{-1/2} B\left(\frac{1}{2}, \frac{q}{q-1} - \frac{1}{2}\right) u^{3/2}(1+(q-1)u)^{-\frac{q}{q-1}+\frac{1}{2}}$$

with

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

Comparison of Bediaga's formula with experiment



Self-consistency in the non-extensive thermodynamics

$$Z_q(V_o,T)=\int_0^\infty \sigma(E)[1+(q-1)\beta E]^{-\frac{q}{(q-1)}}\,dE$$

and

$$\ln[1+Z_q(V_o,T)]=\frac{V_o}{2\pi^2}\sum_{n=1}^\infty\frac{1}{n}\int_0^\infty dm\int_0^\infty dp\,p^2\rho(n;m)\times[1+(q-1)\beta\sqrt{p^2+m^2}]^{-\frac{nq}{(q-1)}}\,,$$

The bootstrap principle

$$Z_q(V_o,T)=\int_0^\infty \sigma(E)[1+(q-1)\beta E]^{-\frac{q}{(q-1)}}\,dE = \exp\left\{\frac{V_o}{2\pi^2\beta^{3/2}}\int_0^\infty dm\,m^{3/2}\rho(m)[1+(q-1)\beta m]^{-\frac{1}{q-1}}\right\}-1$$

At the same time we must have

$$\ln[\sigma(E)]=\ln[\rho(m)]$$

Mass spectru and density of states

The self-consistency principle is satisfied if

$$m^{3/2}\rho(m) = \frac{\gamma}{m}\left[1+(q_o-1)\beta_o m\right]^{\frac{1}{q_o-1}} = \frac{\gamma}{m}[1+(q'_o-1)m]^{\frac{\beta_o}{q'_o-1}}$$

and

$$\sigma(E) = bE^a\left[1+(q'_o-1)E\right]^{\frac{\beta_o}{q'_o-1}}$$

Using properties of $\Gamma(z)$ function it results that for $(q'_o-1)\to 0$,

$$Z_q(V_o,T) \to b\Gamma(a+1)\left(\frac{1}{\beta-\beta_o}\right)^{a+1}$$

Then both expression for the partition function Z_q converge if

$$a+1=\alpha=\frac{\gamma V_o}{2\pi^2\beta^{3/2}}$$

Limiting temperature: β_o and entropic index: q_o.
A. Deppman, Physica A 391 (2012) 6380–6385.

Consistency with thermodynamical functions

Tsallis → Thermodynamical relations → Bediaga formula?

J.M. Conroy, H.G. Miller and A.R. Plastino
Phys. Lett A 374 (2010) 4581 → No!

$$n_i = \left(1 + [1 + (q-1)\beta(E-\mu)]^{(1/(q-1)}\right)^{-q}$$

But for $E \to \infty$ it reduces to Bediaga formula!

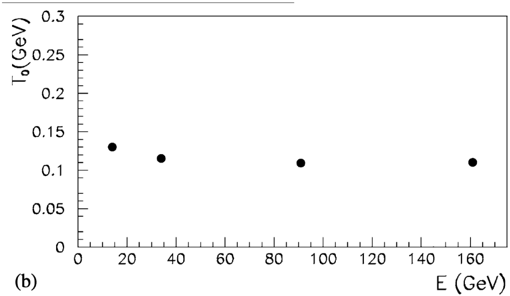
Also: J Cleymans e D Worku: JPG 39 (2012) 025006.

$$\frac{dN}{dp_T} = gV\frac{dN}{dy}\frac{p_T m_T coshy}{4\pi^2}\left[1 + (q-1)\frac{m_T coshy - \mu}{T}\right]^{-q/(q-1)}$$

However STAR, PHENIX, ALICE, CMS still use

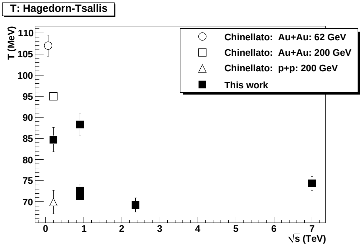
$$\frac{dN}{dp_T} = p_T\frac{dN}{dy}\frac{(n-1)(n-2)}{nC[nC + m_o(n-2)]}\left(1 + \frac{m_T - m_o}{nC}\right)^{-n}$$

Evidences from e^+e^- collisions

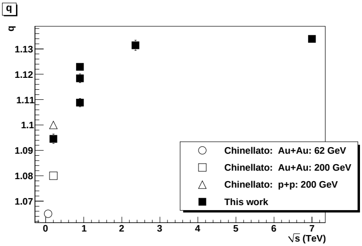


Bediaga (2000).

Evidences from pp collisions

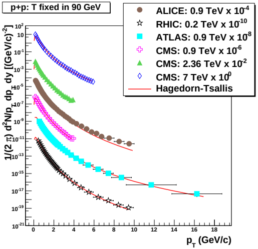


(a)

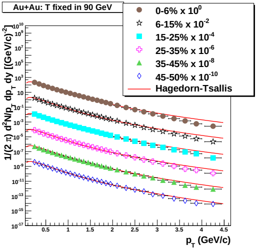


(b)

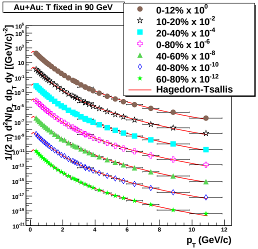
Evidences from AA collisions



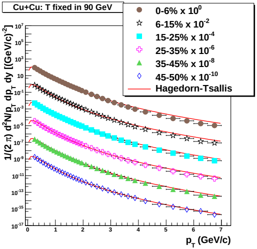
(c)



(d)



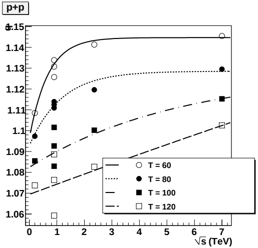
(e)



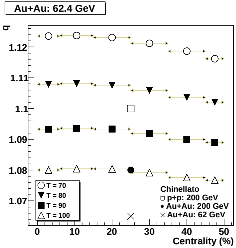
(f)

q as a function of energy and of centrality

arxiv:1208.2952



(g)



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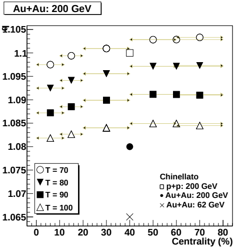
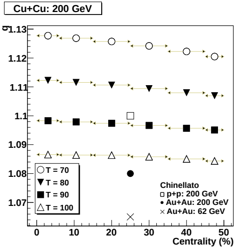
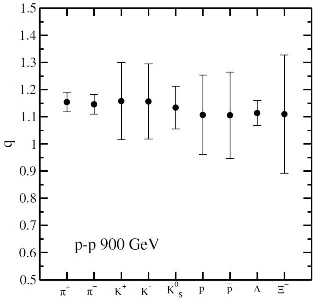


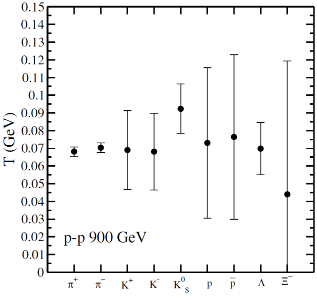


q and T as a function of m

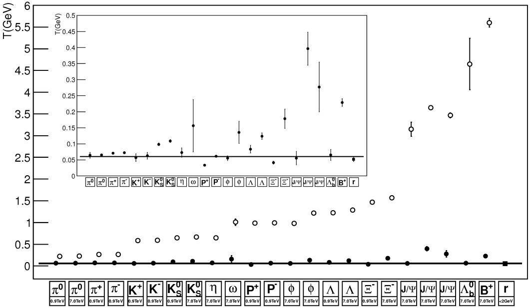
J Cleymans e D Worku: J. Phys. G: Nucl. Part. Phys. 39 (2012) 025006.

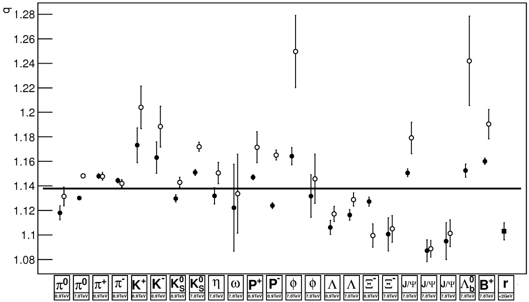


(k)



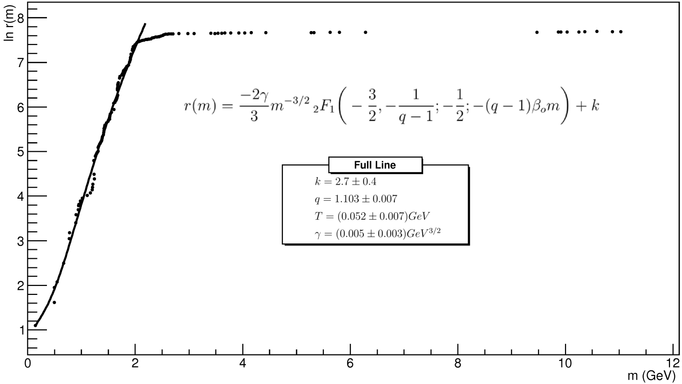
(l)





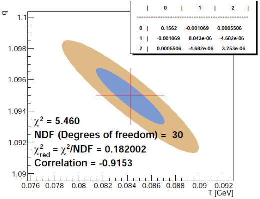
$T_o = (60 \pm 7)$ MeV $\quad q_o = 1.103 \pm 0.007$

Hadron mass spectrum

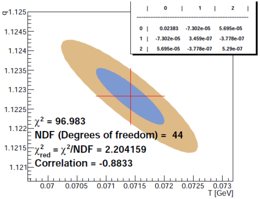


L. Marques, E. Andrade and AD, Phys. Rev. D 87, 114022 (2013)

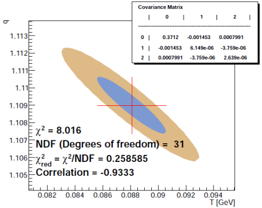
Correlation between T and q

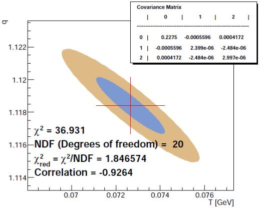


(a) RHIC: 200 GeV



(b) ALICE: 900 GeV





Wilk e Wlodarczyk: Cent. Eur. J. Phys. 10 (2012) 568-575

$$T_{eff} = T_o - (q-1)c$$

T_H = (192±15) MeV   c =-(950±10) MeV

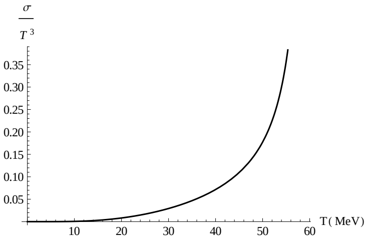
Thermodynamical functions

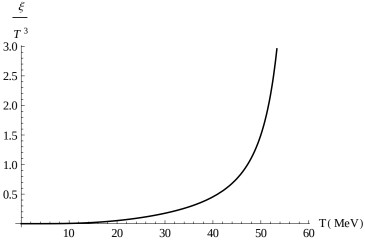
$$\frac{1}{V}\ln[Z(V,\beta)] = \frac{\gamma(q-1)}{2\pi^2\beta^{3/2}}\left(\frac{1}{(\beta-\beta_o)M(q-1)}\right)^{\frac{1}{q-1}} \times {}_2F_1\left[\frac{1}{q-1},\frac{1}{q-1},\frac{q}{q-1},\frac{-1}{(q-1)(\beta-\beta_o)M}\right]$$

pressure: $p = \frac{T}{V}\ln[Z(V,\beta)]$ entropy density: $s = \frac{\partial p}{\partial T}$

energy density: $\varepsilon = \frac{T^2}{V}\frac{\partial}{\partial T}\ln[Z(V,\beta)]$

trace anomaly: $a(T) = \frac{\varepsilon - 3p}{T^2} = T\frac{\partial}{\partial T}\left(\frac{p}{T^4}\right)$





Map between non-extensive and extensive quantities

Mapa entre T e τ:

$\tau(T_o) = \tau_o$

$\tau(0) = 0$

$\tau(T_1 + T_2) = \tau(T_1) + \tau(T_2)$

A função que satisfaz essas condições é:

$$\tau(T) = kT = \frac{\tau_o}{T_o} T$$

From here we get the following relations:

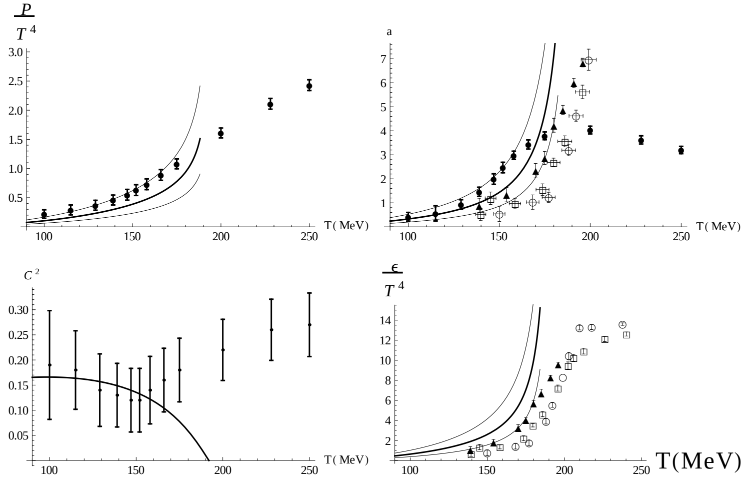
$$\frac{p}{T^4} = k^{-3} \frac{k^{-1} p}{(k^{-1} T)^4} = k^{-3} \frac{\pi}{\tau^4}$$

$$s = \sigma$$

$$\varepsilon = k^{-1} \epsilon$$

$$a = k^{-3} \alpha \,,$$

Comparison to lattice-QCD



Conclusions

- It is possible to obtain a self-consistent theory for fireballs in the non-extensive thermodynamics.
- Self-consistency leads to a limiting effective temperature, T_o, and a limiting entropic index, q_o.
- Experimental data for p_T-distributions give support for the existence of T_o and q_o.
- The mass-spectrum formula describes very well the known hadronic states (mesons and barions).
- It is possible to find a connection between extensive and non-extensive thermodynamics functions.
- Thermodynamics functions resulting from the non-extensive self-consistent theory are in agreement with lattice-QCD results.

(THAN)q