High Energy QCD and Pomeron: today

Eugene Levin

Kaidalov phenomenology WS, July 21-25, Moscow , 2013

Half of century with Pomeron

Eugene Levin

Kaidalov phenomenology WS, July 21-25, Moscow , 2013

Pomeron and its ups and downs (My personal point of view)

Eugene Levin

Kaidalov phenomenology WS, July 21-25, Moscow , 2013

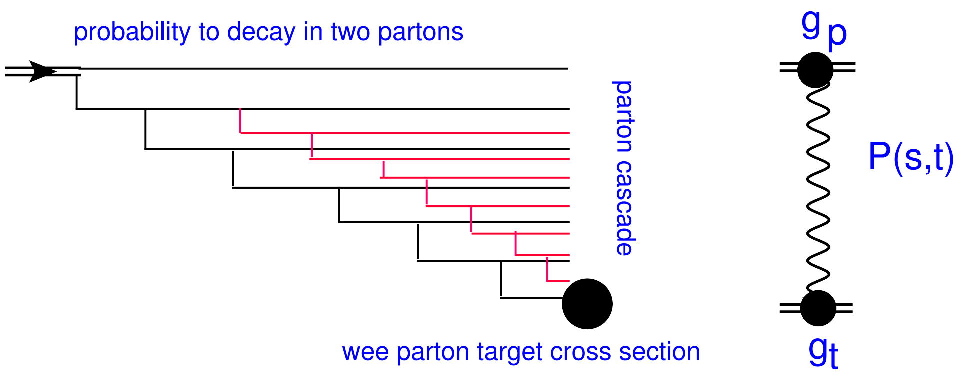
Outline:

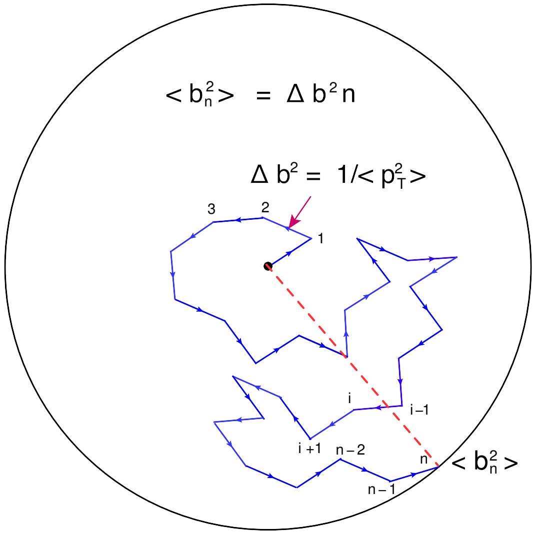
- Brief history of Pomeron (projection on my life);
- Four ideas on Pomeron structure that made me happy;
- Pomeron from HE phenomenology (very short and personal);
- BFKL Pomeron: modeling confinement.

Fifty years with Pomeron:

1966 - 1974 $\longrightarrow$ **Frontier of HE physics**

- **s-channel picture:** **Gribov (1967)**

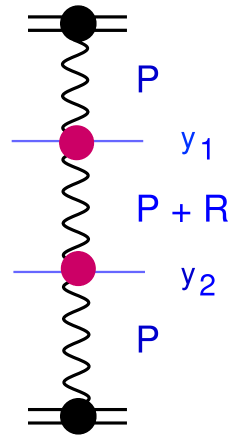




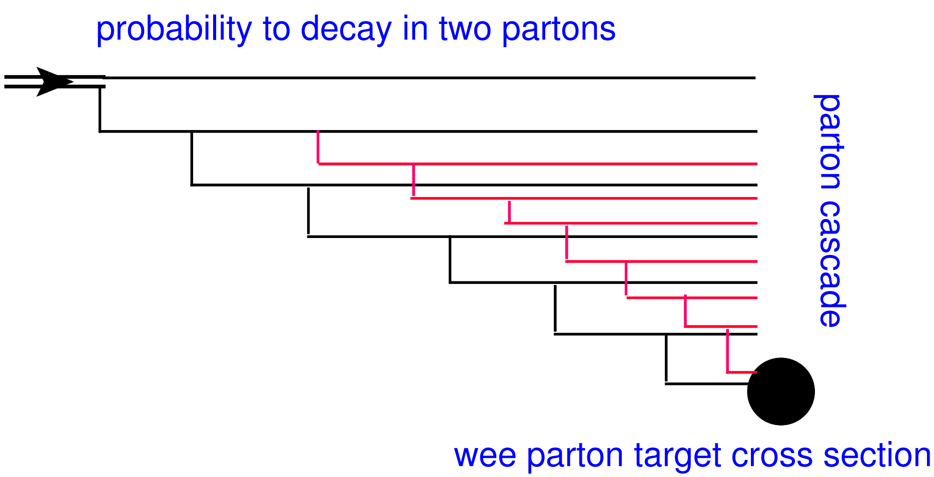
σ = N(wee partons) * σ(wee parton-target interaction)

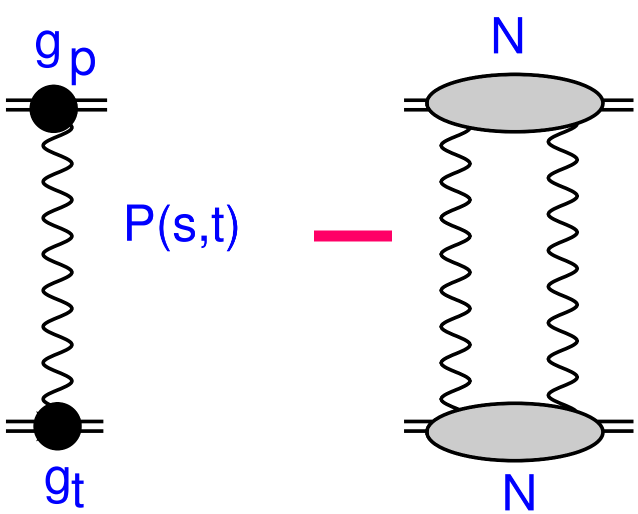
- **Mueller diagrams:** **Mueller, Kancheli (1970-1972)**

$$d\sigma/dy_1 dy_2 \quad =$$

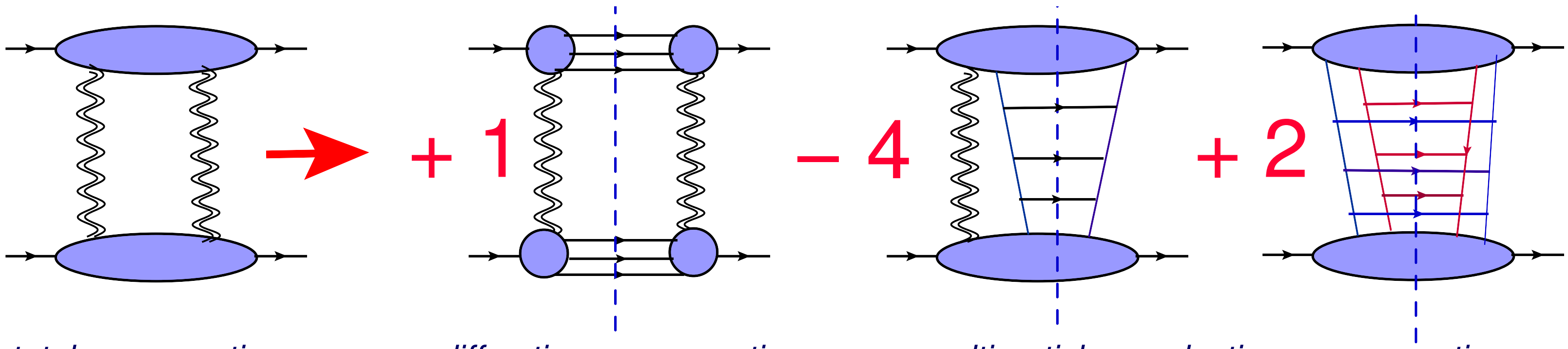


- **Multi Pomeron exchanges and interactions:**





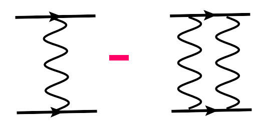
• Abramovsky-Gribov-Kancheli cutting rules (1973):



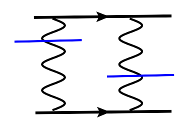
total cross section *diffraction cross section* *multiparticle production cross sections*

multiplicity < n > *multiplicity 2 < n >*

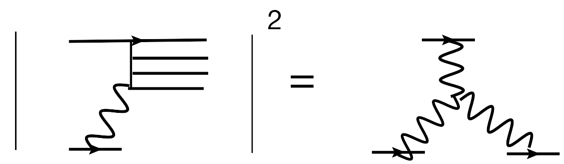
- minima and maxima in elastic cross section



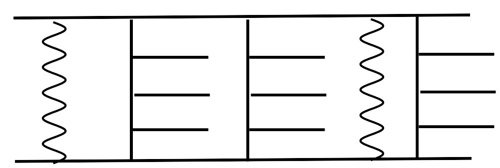
- long range rapidity correlations



- large mass diffraction production



- KNO scaling σ_n/σ_t =F(n/<n>)



- **Gribov Pomeron Calculus** (1967):
  - $Z[\Phi,\Phi^{+}] = \int D\Phi\, D\Phi^{+}\, e^{S}$ **with** $S = S_0 + S_I + S_E$
  - $S_0 = \int dY\, \Phi^{+}(Y)\left\{-\frac{d}{dY} + \Delta + \alpha'_{I\!P}\nabla^2\right\}\Phi(Y,r);$
  - $S_I = G_{3I\!P}\int dY\, \left\{\Phi(Y,r)\,\Phi^{+}(Y,r)\,\Phi^{+}(Y,r) + h.c.\right\}$

- **Statistical (probabilistic) interpretation:**

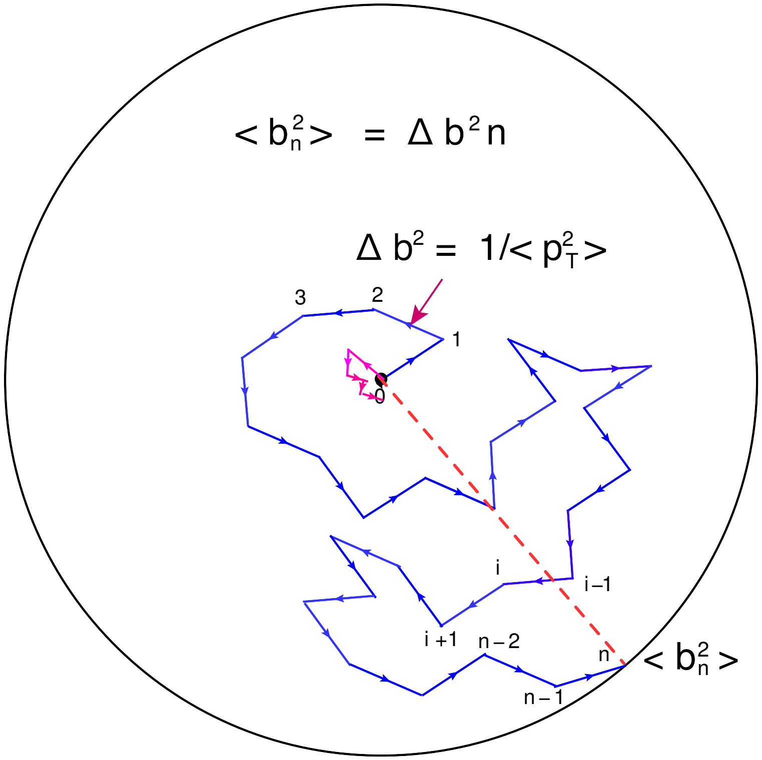
(Grassberger & Sundermeyer (1978), Boreskov (2001))

  - $$-\frac{\partial\, P_n(y,r)}{\partial\, y} + \alpha'_{I\!P}\nabla^2 P_n(y,r) =$$
$$\Gamma(1\to 2)\,\{-n\,P_n + (n-1)\,P_{n-1}\}$$
$$+\ \Gamma(2\to 1)\,\{-n\,(n-1)\,P_n + (n+1)\,n\,P_{n+1}\}$$

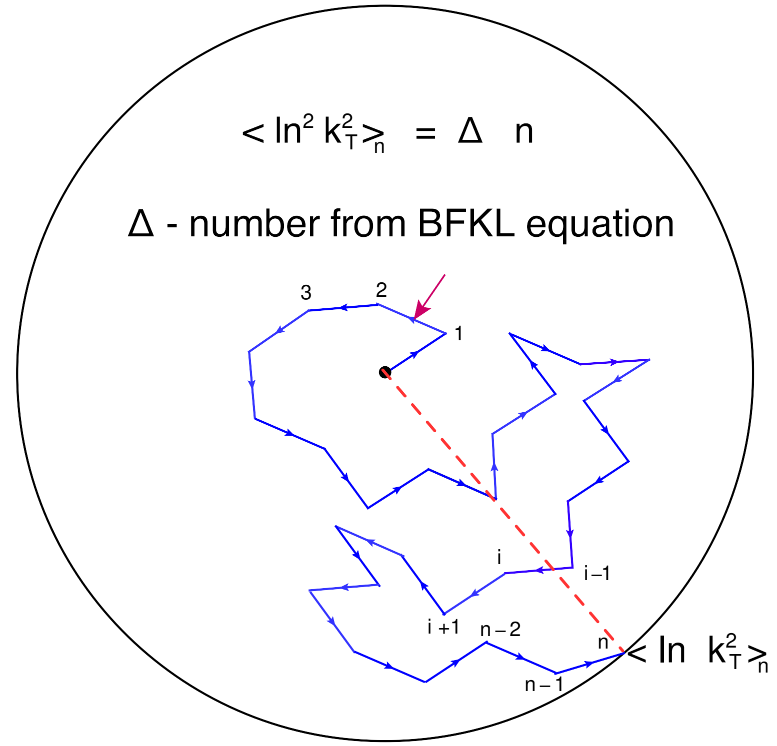
1974 - 1990 $\longrightarrow$ Dark ages

High energy QCD: main ingredients

- **Pomeron $\Longrightarrow$ BFKL Pomeron $\propto s^{\Delta_{BFKL}}$ $\Delta_{BFKL} = 4\ln 2\,\alpha_S$;**
- **BFKL Pomeron: $\sigma = N$(number of gluons)$\times \sigma^{BA}$;**
- **BFKL Pomeron: no Gribov's diffusion, $< b^2 >=$ Const;**
- **BFKL Pomeron: diffusion in $\ln k_T^2$;**
- **BFKL Pomeron calculus;**



$< b_n^2 > \;=\; \Delta\, b^2\, n$
$\Delta\, b^2 = \; 1/ < p_T^2 >$
3
2
1
0
i
i − 1
i + 1
n − 2
n
n − 1
$< b_n^2 >$



$< \ln^2 k_T^2 >_n \;=\; \Delta \;\; n$
Δ - number from BFKL equation
3
2
1
i
i − 1
i + 1
n − 2
n
n − 1
$< \ln \;\; k_T^2 >_n$

1991 - present $\longrightarrow$ Renaissance

- **High energy Pomeron phenomenology with Asher Gotsman and Uri Maor;**
- **Four theoretical approaches that made me happy;**
- **BFKL Pomeron: modeling confinement;**

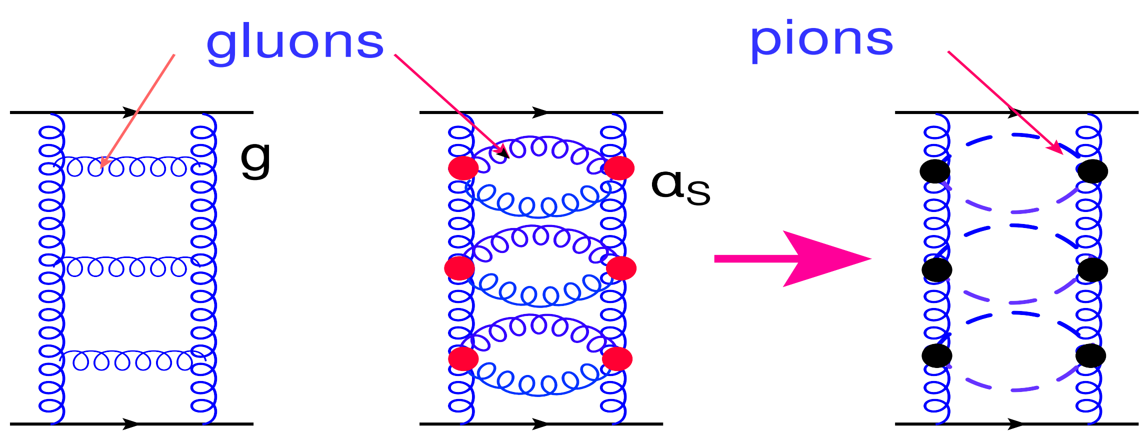
Four theoretical approaches that made me happy

1. **Scale anomaly (breakdown of scale invariance)**

$\Longrightarrow \theta^\mu_\mu \quad \propto \quad g^0 \quad \neq \quad 0$ **(Kharzeev & Levin(1999))**

$\Longrightarrow$ **large mass scale** ($M_0^2 = 4 \div 6\ GeV^2$) **in the QCD vacuum due to semiclassical fluctuations of gluon fields**

$\Longrightarrow$ **Soft Pomeron**



- $\alpha_S F^{\mu\nu,a} F_{\mu\nu,a} \quad \propto \quad \theta^\mu_\mu \quad \sim \quad \alpha_S^0;$
- $< \pi^+\pi^- | \theta^\mu_\mu | 0 > = M^2 \quad \leftarrow$ **chiral Lagrangian**

(Migdal & Shifman (1982))

$$\Pi(0) = i\int dx\,\langle 0|T\left\{\theta^{\mu}_{\mu}(x)\,\theta^{\mu}_{\mu}(0)\right\}|0\rangle = -4\langle 0|\theta^{\mu}_{\mu}|0\rangle = -16\epsilon_{vac} \neq 0$$

- $$\int \frac{\mathrm{dM}^2}{\mathrm{M}^2}\left[\rho^{\mathrm{phys}}_{\theta}(\mathrm{M}^2) - \rho^{\mathrm{pt}}_{\theta}(\mathrm{M}^2)\right] = 16\,|\epsilon_{\mathrm{vac}}|$$

- $$\rho^{\mathrm{phys}}_{\theta}(M^2) \xrightarrow{M^2 > M_0^2} \rho^{\mathrm{pt}}_{\theta}(M^2)$$

- $$\boxed{M_0^2 \simeq 32\pi\left\{\frac{|\epsilon_{\mathrm{vac}}|}{N_f^2 - 1}\right\}^{\frac{1}{2}}}$$

Numerically: $\epsilon_{vac} \approx -(0.24\ GeV)^4$ and $\boxed{M_0^2 = 4 \div 6\ GeV^2}$

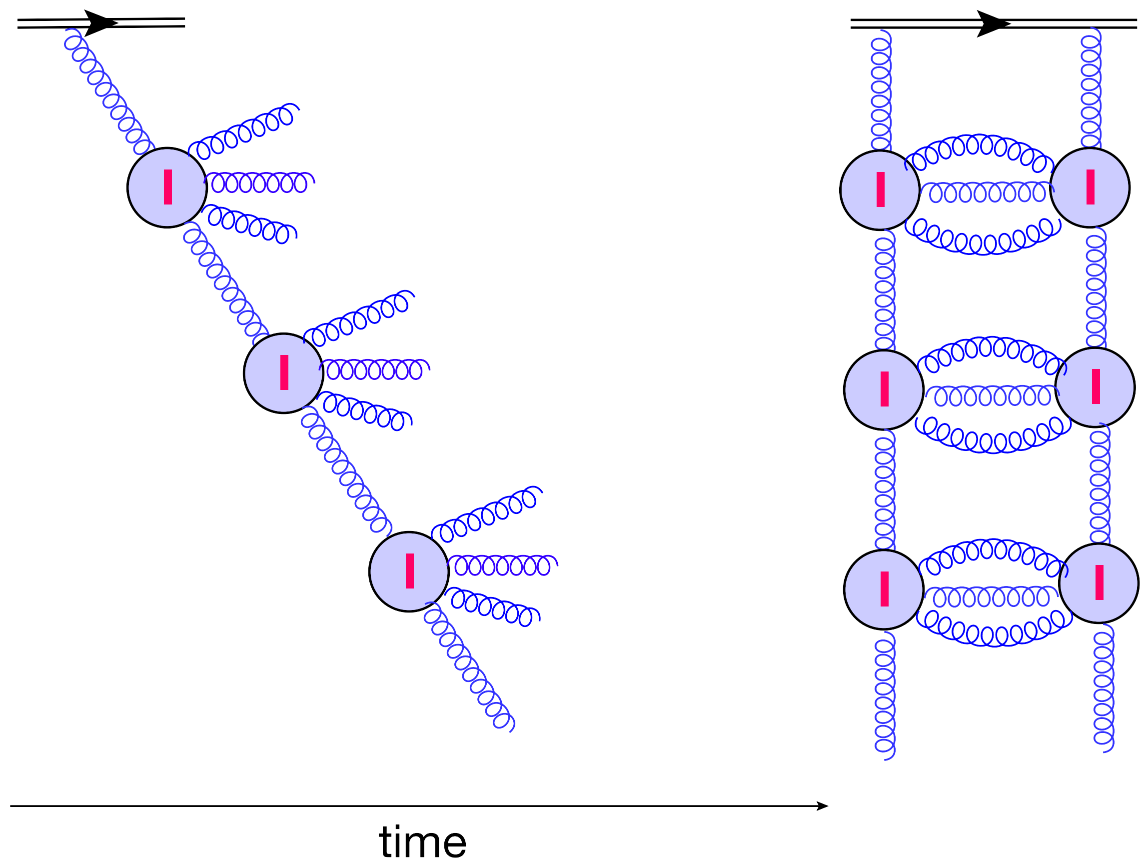
Pomeron intercept

$$\boldsymbol{\Delta \;=\; \frac{1}{48}\,\ln\frac{M_0^2}{4m_\pi^2} \;=\; 0.082 \div 0.09}$$

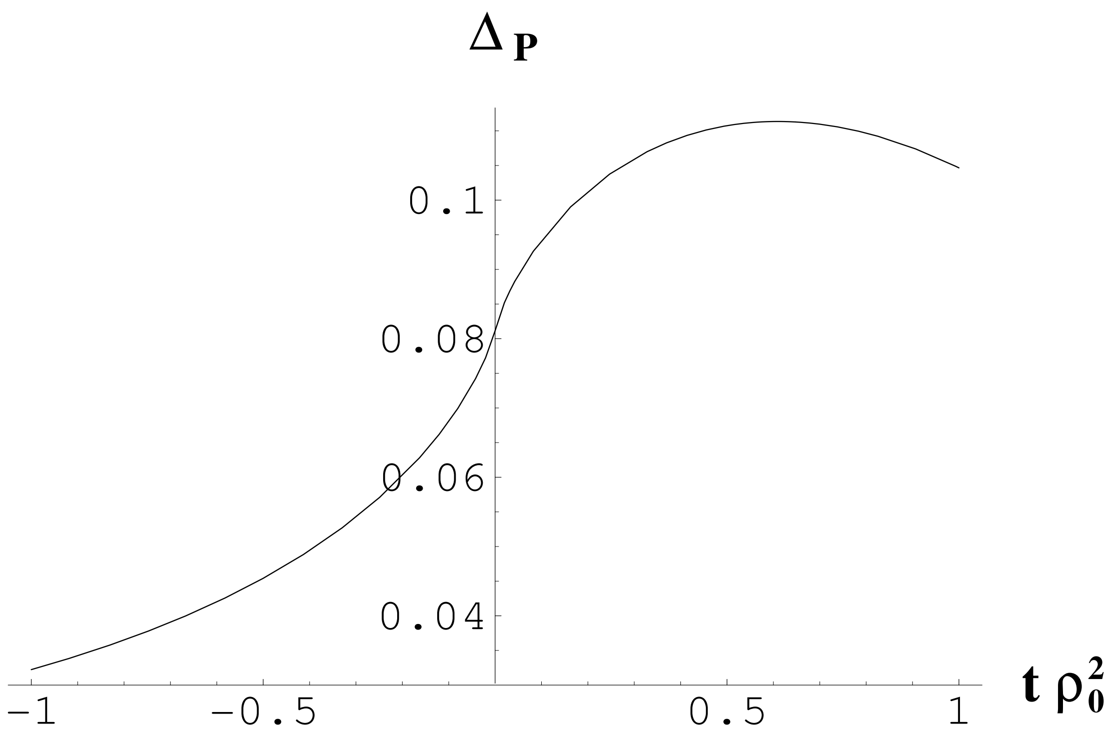
2. $\Longrightarrow$ **classical solutions in QCD**

$\Longrightarrow$ **Instantons** (**Kharzeev , E.L. &Kovchegov (2000)**)

$\Longrightarrow$ **Soft Pomeron**



Soft Pomeron:



Problems:

- **Uncertainties in dimensional parameters $(M_0, \rho_0 \ldots)$**
- **Power - like behaviour at large b**

3. $\Longrightarrow$ **N=4 SYM**

$\Longrightarrow$ **AdS/CFT correspondance**

(Kotikov &Lipatov, Brower,Polchinski,Strassler & C. I. Tan (2006))

$\Longrightarrow$ **Soft Pomeron**

Reminder:

- **Weak coupling and universality for Pomerons** (Gribov. 1970))

$\Delta_{I\!P} = 0$; $g_h =$ **Universal constant**, $G_{3I\!P} = 0$

Pomeron in N=4 SYM:

$$A_{I\!P}(s,b;z,z') = g^2\,(1+i\rho)\,\frac{1}{4\pi}\,\frac{z^2\,z'^2\,\left(z\,z'\,s\right)^{1-\rho}}{\sqrt{u(u+2)}}\,\sqrt{\frac{\rho}{\pi\,\ln\left(s\,z\,z'\right)}}\,\exp\left(-\frac{\ln^2\left(1+u+\sqrt{u(u+2)}\right)}{\rho\,\ln\left(s\,z\,z'\right)}\right)$$

$$u = \frac{(z-z')^2+b^2}{2\,z\,z'};\quad \rho = 2/\sqrt{\lambda};\qquad \lambda = g_{YM} N_c;\ \ g_s = g^2_{YM}/4\pi;\quad R = \alpha^{1/2}_{I\!P}\lambda^{1/4};$$

$$\boxed{\lambda \gg 1;\qquad g_s \ll 1}$$

Glossary $\equiv$ AdS/CFT correspondance:

N=4 SYM		QCD
Reggeized graviton	$\Longrightarrow$	BFKL Pomeron
z	$\Longrightarrow$	r (dipole size)
$1 - 2/\sqrt{\lambda}$	$\Longrightarrow$	ω_{BFKL}(intercept of BFKL Pomeron)
$2/\sqrt{\lambda}$	$\Longrightarrow$	$D_{BFKL}(\omega(\nu) = \omega_{BFKL} - D\nu^2)$

For $\lambda \gg 1$

- $$A\left(s,t=0\right) \;=\; \int dz dz' \int d^2 b \Phi\left(z\right)\Phi\left(z'\right) \times \left(1 \;-\; \exp\left(i A_{(}s,b;z,z'\right)\right)\right)$$

$$2 Im A\left(s,b\right) \;=\; |A\left(s,b\right)|^2 \;+\; O\left(\frac{2}{\sqrt{\lambda}}\right)$$

Pomeron in N=4 SYM: characteristic properties

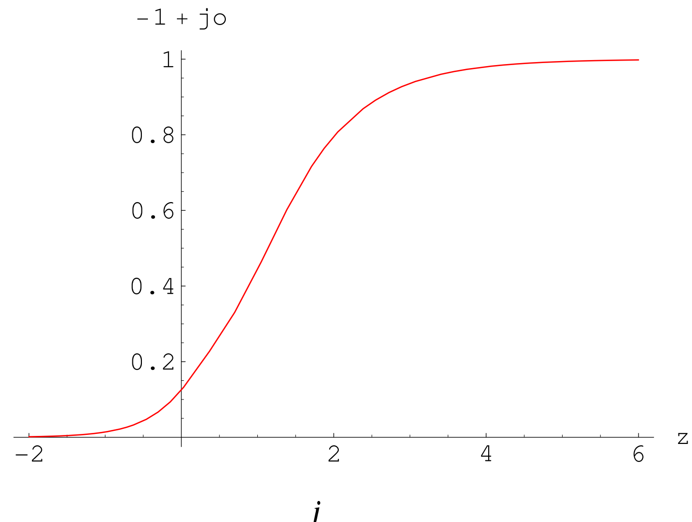
$\lambda \quad \gg \quad 1$

- $\Delta_{I\!P} \Longleftarrow$ **large** $(\sim \ 0.3?)$

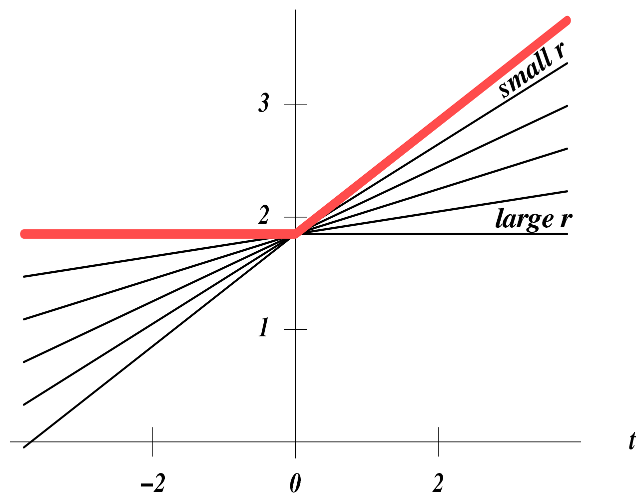
- $\alpha'_{I\!P} = \quad 0$

- **only Good-Walker mechanism for diffraction**

- $G_{3 I\!P} = 0$



- **Kotikov & Lipatov , Stasto, M. Costa et al.**



- **Brower,Polchinski,Strassler & C. I. Tan**

-'s & ?'s

- Pomeron is a standing cut but not a Regge pole (?);
- At large b amplitude falls as a power of b (-);
- Radius of interaction $R \;\propto\; s^{1/3}$ (-);
- Estimates show small cross section for diffraction (?);
- Multiparticle production is small(?);

N=4 SYM and QCD motivated phenomenology

GLM: Gotsman, Levin and Maor (1992 - present)

Assumptions:

1. Pomeron is a Regge pole;
2. $\Delta_{I\!P} = 0.2 \div 0.3$;
3. $\alpha'_{I\!P} = 0$;
4. $\Delta_{I\!P} = 0.2 \div 0.3$;
5. Large Good-Walker component, two channels model;
6. Only $G_{3I\!P}$ as in QCD;
7. $G_{3I\!P}$ is small (in QCD $G_{3I\!P} \propto \bar{\alpha}_S$);

- $$Z[\Phi,\Phi^+] = \int D\Phi\, D\Phi^+\, e^S$$ **with** $$S = S_0 + S_I + S_E\,,$$

Where

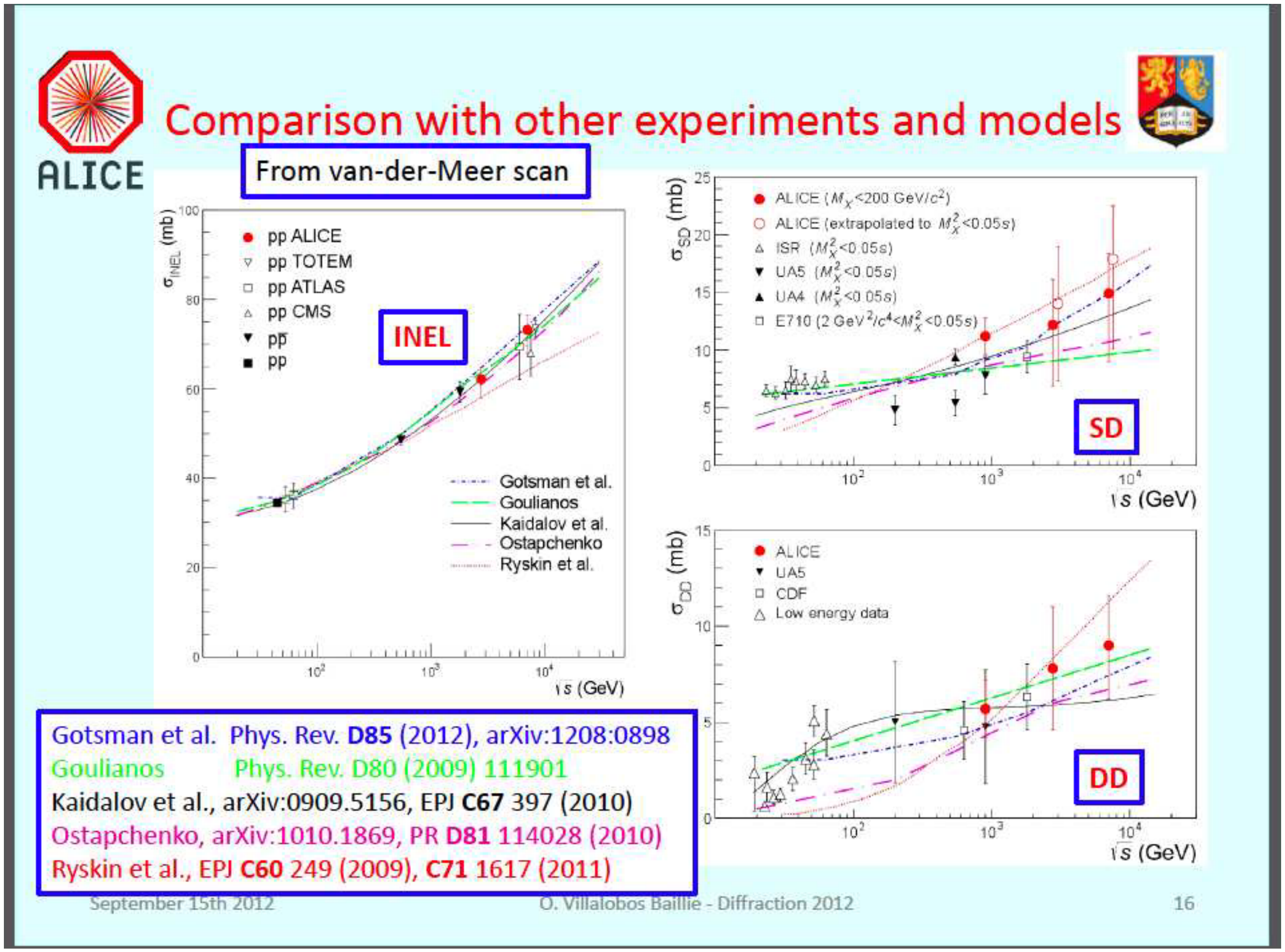
$$S_0 = \int dY\,\Phi^+(Y)\left\{-\frac{d}{dY} + \Delta\right\}\Phi(Y);$$

$$S_I = G_{3I\!P}\int dY\ \left\{\Phi(Y)\,\Phi^+(Y)\,\Phi^+(Y) + h.c.\right\};$$

$$S_E = -\int dY\sum_{i=1}^{2}\left\{\Phi(Y)\,g_i(b) + \Phi^+(Y)\,g_i(b)\right\};$$

+ : **So simple that can be solved analytically;**

- : **It has large almost black GW component;**


ALICE
Comparison with other experiments and models
From van-der-Meer scan
σ_{INEL} (mb)
pp ALICE
pp TOTEM
pp ATLAS
pp CMS
$p\bar{p}$
pp
INEL
Gotsman et al.
Goulianos
Kaidalov et al.
Ostapchenko
Ryskin et al.
$\sqrt{s}$ (GeV)
σ_{SD} (mb)
ALICE (M_X<200 GeV/c^2)
ALICE (extrapolated to M_X^2<0.05s)
ISR (M_X^2<0.05s)
UA5 (M_X^2<0.05s)
UA4 (M_X^2<0.05s)
E710 (2 GeV^2/c^4<M_X^2<0.05s)
SD
σ_{DD} (mb)
ALICE
UA5
CDF
Low energy data
DD
Gotsman et al. Phys. Rev. D85 (2012), arXiv:1208:0898
Goulianos Phys. Rev. D80 (2009) 111901
Kaidalov et al., arXiv:0909.5156, EPJ C67 397 (2010)
Ostapchenko, arXiv:1010.1869, PR D81 114028 (2010)
Ryskin et al., EPJ C60 249 (2009), C71 1617 (2011)
September 15th 2012
O. Villalobos Baillie - Diffraction 2012
16

4. $\Longrightarrow$ **modeling confinement in the BFKL kernel**

$\Longrightarrow$ **correct large b behaviour**

(E.L. & S. Tapia(2013))

$\Longrightarrow$ **Soft Pomeron ?!**

- $$N\left(r_1, r_2; Y, b\right) = \int \frac{d\gamma}{2\,\pi\,i}\,\phi^{(0)}_{in}(\nu)\,e^{\omega(\gamma = \frac{1}{2} + i\nu,\,0)\,Y}$$
$$\times\left\{b_\nu\,(w w^*)^{\frac{1}{2} + i\nu} + b_{-\nu}\,(w w^*)^{\frac{1}{2} - i\nu}\right\} \longrightarrow \frac{r_1\,r_2}{b^2}\,e^{\omega_0\,Y}$$

- $$w\,w^* = \frac{r_1^2\,r_2^2}{\left(\vec{b} - \frac{1}{2}\,(\,\vec{r}_1 - \vec{r}_2)\right)^2\,\left(\vec{b} + \frac{1}{2}\,(\,\vec{r}_1 - \vec{r}_2)\right)^2}$$

$$\boxed{N\left(r_1, r_2; Y, b\right) \leq 1 \quad \text{for} \quad b^2 \leq r_1\,r_2 e^{\omega_0 Y}}$$

Violation of Froissart theorem: **(Kovner & Wiedemann)**

$$\int d^2b\,N\left(r_1, r_2; Y, b\right) \propto s^{\omega_0} \gg Y^2 = ln^2 s$$

Lessons from numerical solutions and theory considerations:

((Kovner & Wiedemann, McLerran and Iancu,Golec-Biernat & Stasto, Gotsman et al, Berger & Stasto,2011)

- **The confinement of quarks and gluon have to be included in the BFKL kernel (to include in the initial conditions is not enough);**

- **Suppressing large sizes of the produced dipoles in the decay** $one\ dipole \rightarrow two\ dipoles$ **we reproduce correct** b**-dependence;**

- **Since at large** b **the amplitude is small we do not need to take into account the non-linear corrections;**

Corrections from confinement have to be included in the kernel of the BFKL equation

- $$\frac{\partial N\left(x_{10}, b, Y\right)}{\partial Y} = \bar{\alpha}_S \int d^2 x_{12} K\left(x_{12}, x_{20} | x_{10}\right) \times \left\{ 2\, N\left(x_{12}, \vec{b} - \tfrac{1}{2}\vec{x}_{02}; Y\right) \;-\; N\left(x_{10}, b; Y\right) \right\}$$

Modified BFKL kernel:

- $$K\left(x_{12}, x_{20} | x_{10}\right) \;=\; \frac{x_{10}^2}{x_{12}^2\, x_{02}^2}\, e^{-B\left(x_{12}^2 + x_{02}^2\right)}$$

Results

1. $N\left(x_{10}, b, Y\right) \xrightarrow{Bb^2 \gg 1} e^{-4B\,b^2} \longleftarrow$ **expected**

2. $N\left(x_{10}, b, Y\right) \xrightarrow{Y \gg 1} e^{\omega_0 Y}$ **with** $\omega_0 = \omega_{\text{BFKL}}$; $\longleftarrow$ **!!!!**

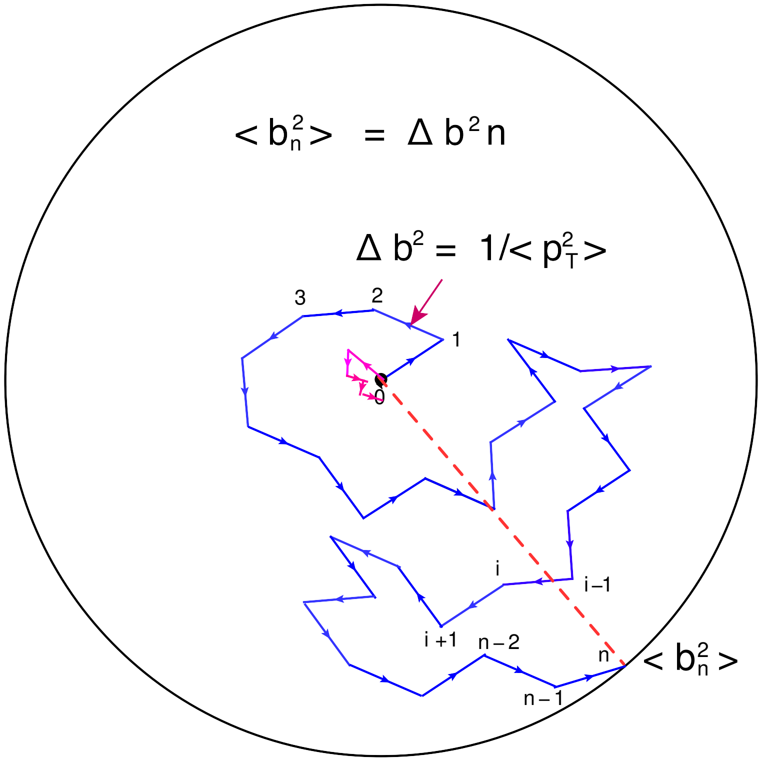
3. $\langle |b^2| \rangle =$ **Constant (Y)** ; $\longleftarrow$ **expected**

4. **Saturation scale** $Q_s^2 \propto e^{\lambda Y}$ **with** $\lambda = \lambda_{\text{BFKL}} \longleftarrow$ **expected**

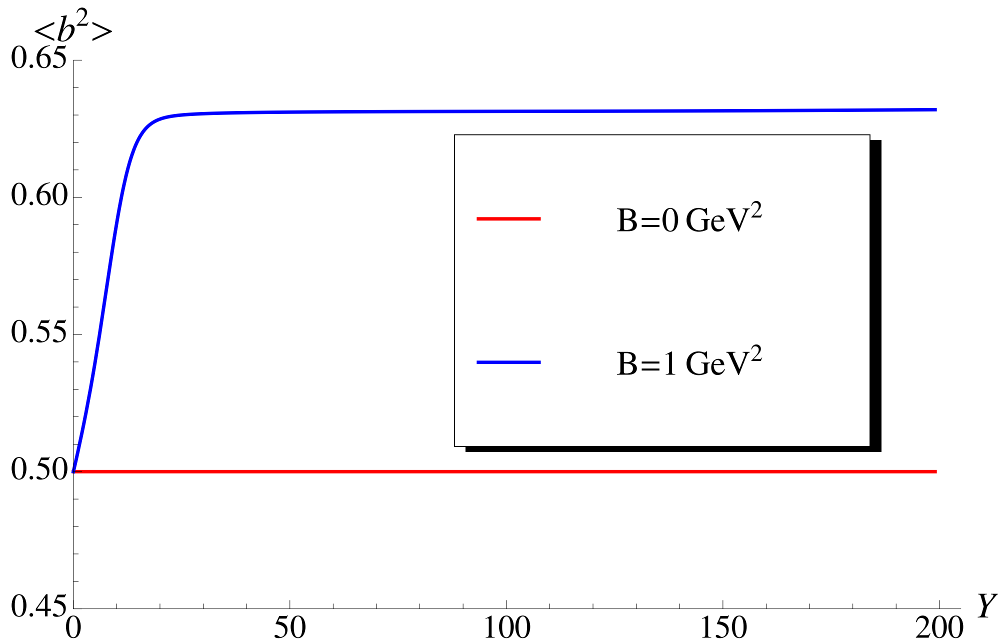
5. **The modified BFKL Pomeron looks similar to the Pomeron in N=4 SYM and high energy phenomenology:** $\Delta_{I\!P} \sim 0.3$; $\alpha'_{I\!P} = 0 \longleftarrow$ **!!!!**

$\langle|b^2|\rangle$ versus Y

Gribov's diffusion: $\Delta b \, p_T \;\sim\; 1$



Numerical calculations



$$\frac{\partial \widehat{\mathcal{N}}\left(x_{01};Y\right)}{\partial Y} = \frac{\partial}{\partial Y}\int d^2b\, b^2\, \mathcal{N}\left(x_{01};Y\right) \qquad \left(\langle|b^2|\rangle = \widehat{\mathcal{N}}\left(x_{01};Y\right)\Big/\mathcal{N}\left(x_{12};Y\right)\right)$$

$$= \bar{\alpha}_S \iint d^2b'\, d^2x_{12}\left(\vec{b'} + \tfrac{1}{2}\vec{x}_{12}\right)^2 \frac{1}{x^2_{12}}\left\{2\,\widetilde{\mathcal{N}}\left(x_{12},\vec{b'};Y\right) - \frac{x_{02}}{x^2_{12}}\widetilde{\mathcal{N}}\left(x_{01},b;Y\right)\right\}$$

$$= \bar{\alpha}_S \int d^2x_{12}\,\frac{1}{x^2_{12}}\left\{2\,\widehat{\mathcal{N}}^{BFKL}\left(x_{12};Y\right) - \frac{x_{01}}{x^2_{12}}\widehat{\mathcal{N}}\left(x_{01};Y\right)\right\} + \tfrac{1}{2}\bar{\alpha}_S \int d^2x_{12}\,\mathcal{N}\left(x_{12};Y\right)$$

$$+ \left\{\tfrac{1}{2}\bar{\alpha}_S \iint d^2b'\, d^2x_{12}\; \vec{b'}\cdot\vec{x}_{12}\,\frac{1}{x^2_{12}}\,2\,\widetilde{N}\left(x_{12},\vec{b'} \equiv \vec{b} - \tfrac{1}{2}\vec{x}_{12};Y\right) = 0\right\}$$

Pomeron intercept ω_0

- **Remnant of conformal symmetry:** $x_{ik} \rightarrow \bar{x}_{ik} = \sqrt{B}\, x_{ik}$

$$\int d^2 x_{12} \frac{x_{10}^2}{x_{12}^2\, x_{02}^2}\, e^{-B\left(x_{12}^2 + x_{02}^2\right)} \rightarrow \int d^2 \bar{x}_{12} \frac{\bar{x}_{10}^2}{\bar{x}_{12}^2\, \bar{x}_{02}^2}\, e^{-\left(\bar{x}_{12}^2 + \bar{x}_{02}^2\right)}$$

- $N\ \left(x_{01}; Y\right) = \int \frac{d\omega}{2\pi i} e^{\omega Y}\, N_\omega\left(x_{01}\right)$
- $\omega N_\omega\left(x_{01}\right) = -\, \bar{\alpha}_S \mathcal{H}\ N_\omega\left(x_{01}\right)$ **or** $E\, N_\omega\left(x_{01}\right) = \mathcal{H}\ N_\omega\left(x_{01}\right)$
- $N_\omega\left(x_{01}\right) \xrightarrow{x_{01}^2 \ll 1/B} N_\nu^{BFKL}\left(x_{01}\right) = \left(\frac{1}{x_{01}^2}\right)^{\frac{1}{2} + i\nu}$ $\quad N_\omega\left(x_{01}\right) \xrightarrow{x_{01}^2 \gg 1/B}$ **Const**
- $E\left(\nu\right) = 2\psi\left(1\right) - \psi\left(-\frac{1}{2} + i\nu\right) - \psi\left(\frac{1}{2} - i\nu\right)$
- $\int d^2 x_{01} N_\omega^*\left(x_{01}\right) N_{\omega'}\left(x_{01}\right) < \infty \qquad \int d^2 x_{01} N_\nu^{BFKL*}\left(x_{01}\right) N_{\nu'}^{BFKL}\left(x_{01}\right) = \delta\left(\nu - \nu'\right)$

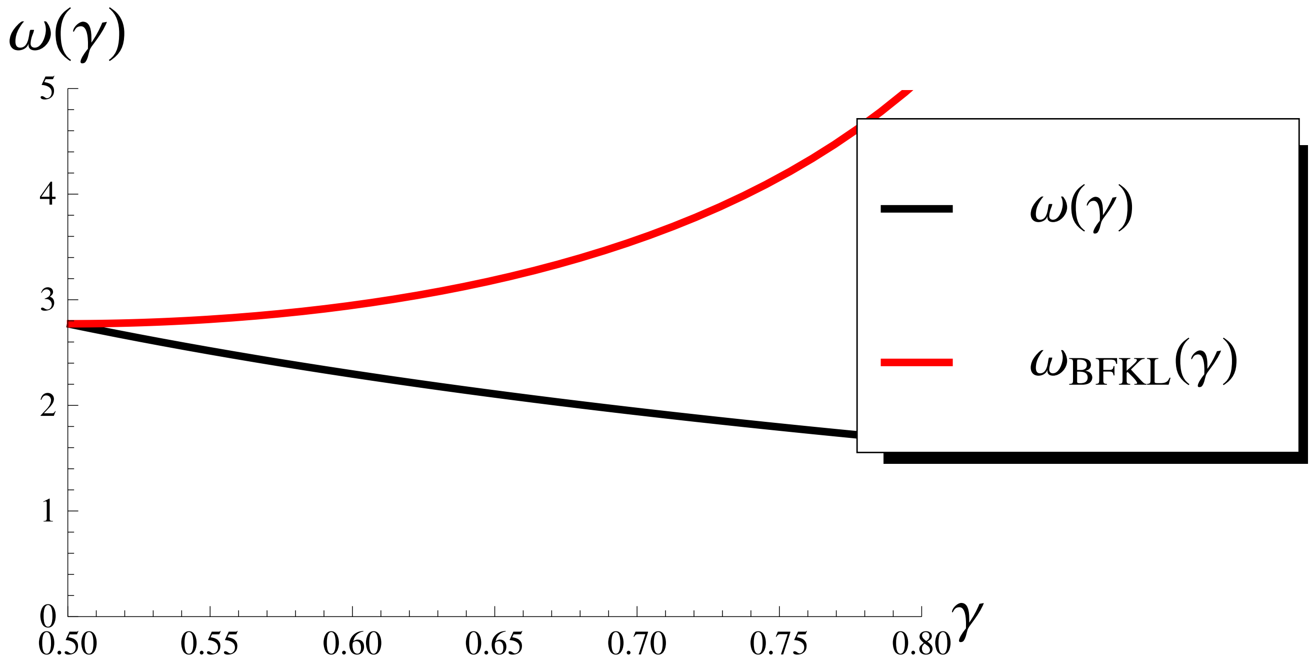
Variational method

- $E_{\text{ground}} \equiv -\omega_0 \leq F[\{N\}] = \dfrac{\left\langle N^*(x_{01}) \middle| \mathcal{H} \middle| N(x_{01}) \right\rangle}{\left\langle N^*(x_{01}) \middle| N(x_{01}) \right\rangle}$

- Our choice: $\{N\} = \{N^{BFKL}\}$
  - $\mathcal{H}\, N^{\text{BFKL}}_{\gamma=-\frac{1}{2}+i\nu}(x_{10}) = \chi(\gamma, x_{10})\, N^{\text{BFKL}}_{\gamma=-\frac{1}{2}+i\nu}(x_{10})$

- $\chi(\gamma; \tilde{x}_{12}) = \int_0^1 dt\, \dfrac{t^{\gamma-1}}{1-t} + \int_1^{1/\tilde{x}^2_{12}} dt \dfrac{t^{\gamma-1}}{t-1} - \int_0^{1/\tilde{x}^2_{12}} \dfrac{1}{t} \left[\dfrac{1}{|t-1|} - \dfrac{1}{\sqrt{4\,t^2+1}} \right]$

- $\chi(\gamma; \tilde{x}_{12}) = \chi^{\text{BFKL}}(\gamma) + \operatorname{arccsch}\left(2/\tilde{x}^2_{12}\right) - B(\tilde{x}_{12}; 1-\gamma, 0) - \ln\left(1 - \tilde{x}^2_{01}\right)$



$$\boxed{\omega \quad \geq \quad \omega_{\mathrm{BFKL}}}$$

General argument why $\omega(\gamma) = \omega_{\text{BFKL}}$:

$$E\,\Psi\left(x_{12},\omega\right) \;=\; \mathcal{H}\Psi\left(x_{12},\omega\right)$$

$$\Psi\left(x_{12},\omega\right) \xrightarrow{x_{12}\to 0} \Psi_{BFKL}\left(x_{12},\omega\right) \;=\; \left(x_{12}^2\right)^{-\frac{1}{2}\,\pm\,i\nu}$$

and

$$\Psi\left(x_{12},\omega\right) \xrightarrow{x_{12}\to 1/B} \text{Const}$$

For $\nu \quad \Longrightarrow \quad$ real:

$$\Psi\left(x_{12},\omega\right) \;=\; \left(1/x_{12}^2\right)^{\frac{1}{2}} \sin\left(\nu\ln\left(1/(Bx_{12}^2)\right)+\phi_0\right) \xrightarrow{x_{12}=1/B} \sin\phi_0$$

For $\nu = i\kappa\ (\kappa\ >\ 0)$:

$\left(x_{12}^2\right)^{-\frac{1}{2}\,-\,\kappa}$ and $\left(x_{12}^2\right)^{-\frac{1}{2}\,+\,\kappa}$. Normalization leads that only one survives but it cannot satisfy the IC at $x_{12} = 1/B$,

Semi-classical approach

- $$N\left(\mathcal{Y};l\right) = e^{S(\mathcal{Y},l)} = e^{\omega(\mathcal{Y},l)\mathcal{Y} + (\gamma(\mathcal{Y},l)-1|)l}$$

where $\omega\left(\mathcal{Y},l\right) = \dfrac{\partial S\left(\mathcal{Y};l\right)}{\partial \mathcal{Y}}$; $\gamma\left(\mathcal{Y},l\right) - 1 = \dfrac{\partial S|\left(\mathcal{Y};l\right)}{\partial l}$

with smooth functions $\omega\left(\mathcal{Y},l\right)$ and $\gamma\left(\mathcal{Y},l\right)$ and $\mathcal{Y} = \bar{\alpha}_S Y$.

Equation: $$\omega\left(\mathcal{Y},l\right) - \chi\left(\gamma, x^2_{12}\right) = 0$$

$$F\left(\mathcal{Y}, l, S, \gamma, \omega\right) = 0$$

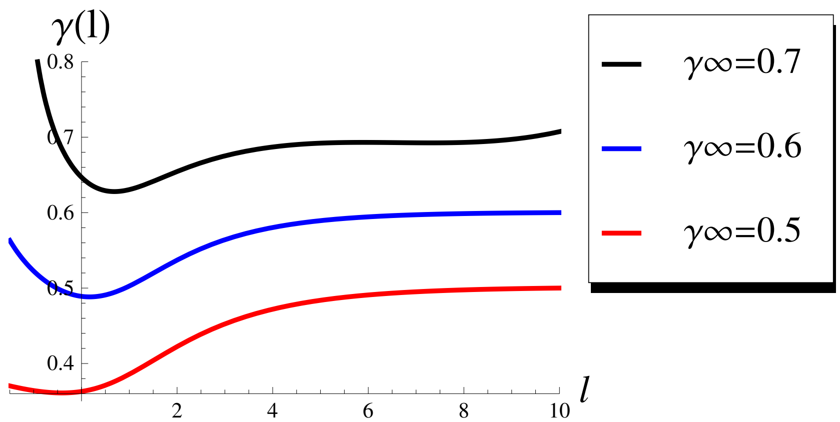
$$(1.)\quad \frac{dl}{d\,t} = F_\gamma = -\frac{d\chi\left(\gamma,0,l\right)}{d\gamma}$$

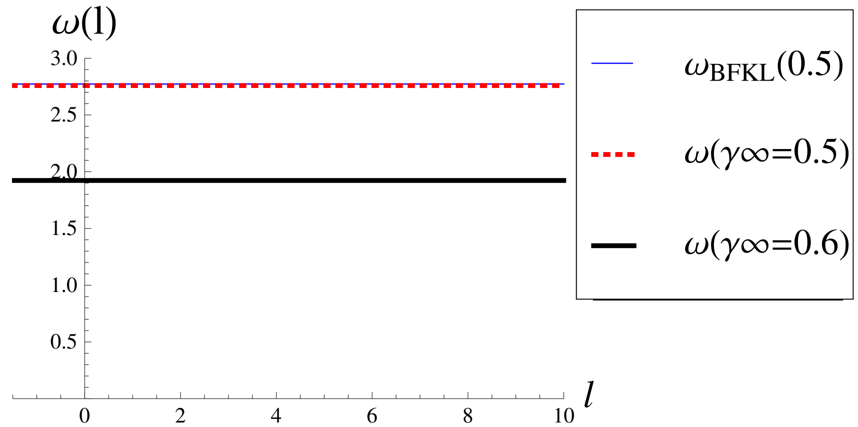
$$(2.)\quad \frac{d\mathcal{Y}\left(t\right)}{d\,t} = F_\omega = 1$$

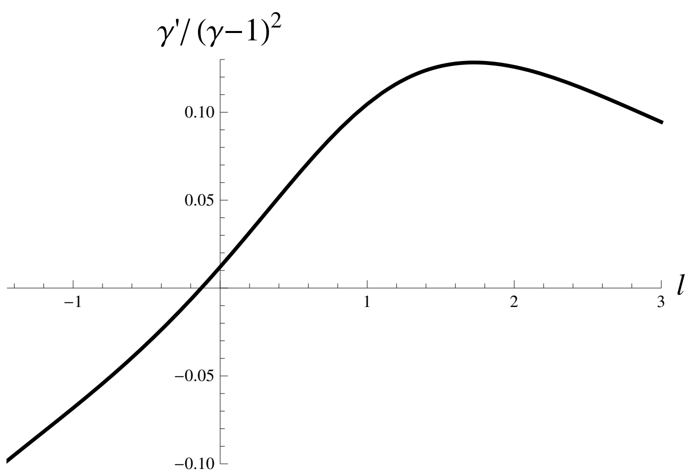
$$(3.)\quad \frac{dS}{d\,t} = \gamma\,F_\gamma + \omega\,F_\omega = -\left(\gamma - 1\right)\frac{\partial\chi\left(\gamma,0,l\right)}{\partial\gamma} + \omega$$

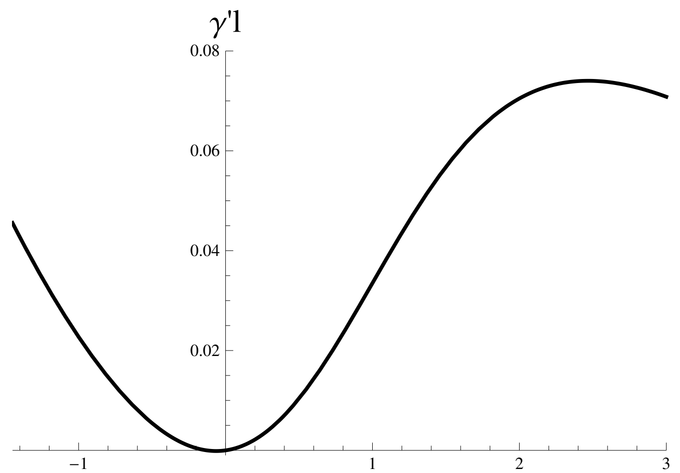
$$(4.)\quad \frac{d\,\gamma}{d\,t} = -(\,F_l + \gamma\,F_S\,) = \frac{\partial\chi\left(\gamma(t),0,l(t)\right)}{\partial\,l}$$

- $$\frac{d\,\gamma}{d\,l} = -\frac{\frac{\partial\chi(\gamma(t),l(t))}{\partial\,l}}{\frac{d\chi(\gamma,l(t))}{d\gamma}}$$









$$\boxed{\omega \quad \leq \quad \omega_{\mathrm{BFKL}}}$$

Diffusion approximation

$$\bar{N}(\mathcal{Y}, l) = e^{\frac{1}{2}l} N(\mathcal{Y}, l) = \int_{\epsilon - i\infty}^{\epsilon + i\infty} \frac{d\omega}{2\pi i} e^{\omega\,\mathcal{Y}}\, \bar{n}(\omega, l)$$

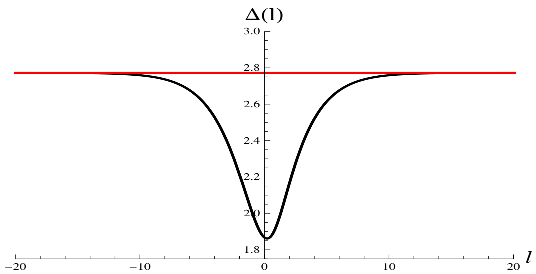
$$= \int_{\epsilon - i\infty}^{\epsilon + i\infty} \frac{d\omega}{2\pi i} \int_{i\epsilon - \infty}^{i\epsilon + \infty} \frac{d\nu}{2\pi i} \bar{n}(\omega, \nu)\; e^{\omega\,\mathcal{Y} + i\nu l}$$

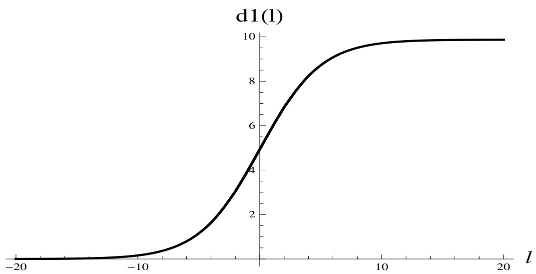
$$\bar{n}\left(\omega, l'\right) = \bar{n}(\omega, l) + \frac{\partial \bar{n}(\omega, l')}{\partial l'}|_{l'=l}\left(l' - l\right) + \frac{1}{2}\frac{\partial^2 \bar{n}\left(\omega, l'\right)}{\partial l'^2}|_{l'=l}(l - l)^2 + \dots$$

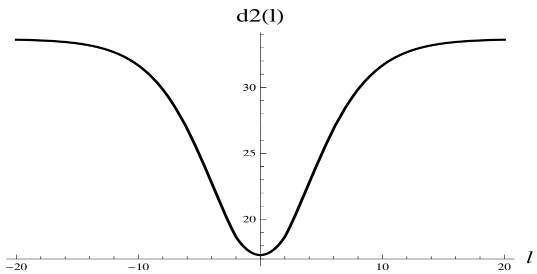
$$\Delta(l) = \chi\left(\frac{1}{2}, e^{\frac{1}{2}l}\right); \quad d1(l) = -\,i\frac{\partial \chi\left(\frac{1}{2} + i\nu, e^{\frac{1}{2}l}\right)}{\partial \nu}|_{\nu=0};$$

$$d2(l) = -\frac{\partial^2 \chi\left(\frac{1}{2} + i\nu, e^{\frac{1}{2}l}\right)^2}{\partial \nu}\;|_{\nu=0}$$

$$(\omega - \Delta(l))\,\bar{n}(\omega, l) - d1(l)\,\frac{\partial \bar{n}(\omega, l)}{\partial l} - \frac{1}{2} d2(l)\,\frac{\partial^2 \bar{n}(\omega, l)}{\partial l^2} = 0$$



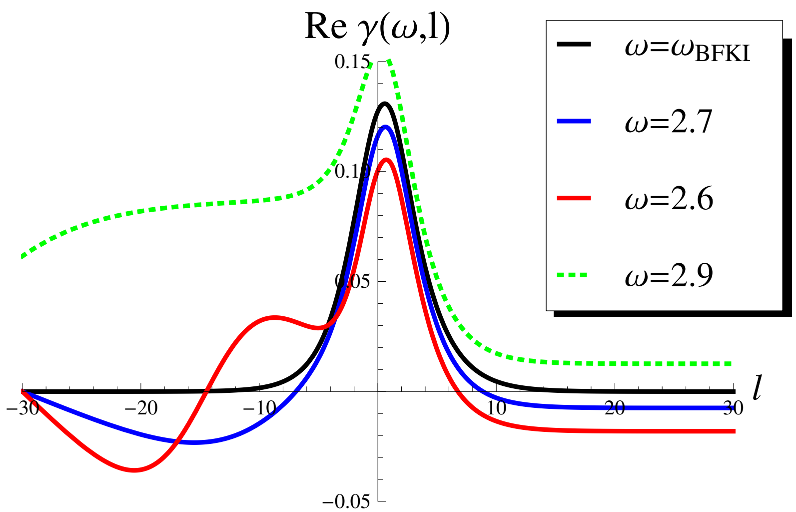


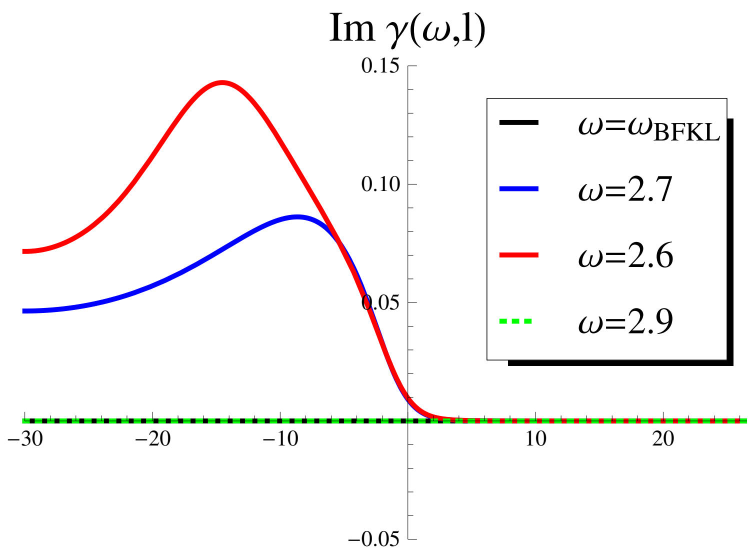


$$\bar{n}\left(\omega,l\right) = \exp\left(\phi\left(\omega,l\right)\right); \quad \text{and} \quad \gamma = \frac{\partial\phi(\omega,l)}{\partial l}$$

$$\left(\omega - \Delta\left(l\right)\right) = d1\left(l\right)\gamma\left(\omega,l\right) + \frac{1}{2}d2\left(l\right)\left(\frac{\partial\gamma(\omega,l)}{\partial l} + \gamma^{2}\left(\omega,l\right)\right)$$

$$\gamma\left(\omega,l\right) \xrightarrow{|l|\gg 1} \sqrt{\left(\omega - \omega_{\mathrm{BFKL}}\right)/\left(2D_{0}\right)}$$





$$\boxed{\omega \quad \leq \quad \omega_{\mathrm{BFKL}}}$$

Numerical calculations of Y dependence

Two problems:

1. **The kernel is not Fredholm type**

- $$\int d^2x_{01}d^2x_{12}\,K\left(x_{12},x_{02}|x_{01}\right)\Longrightarrow\infty$$

2. **The kernel is singular at $x_{12}\ \rightarrow\ x_{01}$**

Checks:

1. Dependence on x_{min} and x_{max};

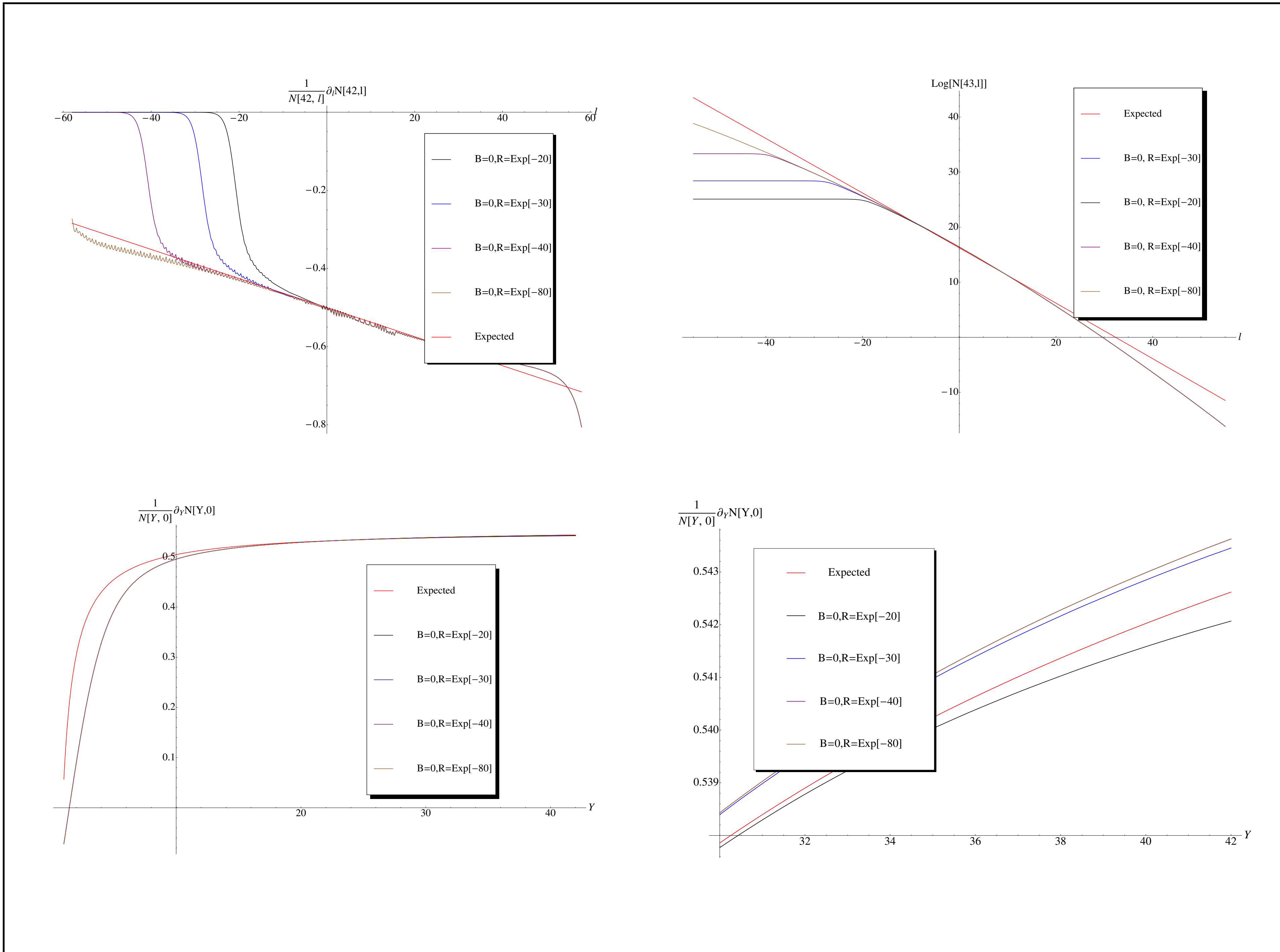
2. Numerical solution to the BFKL equation coincide with the analytic one;

3. Independence on value of the regulator R;

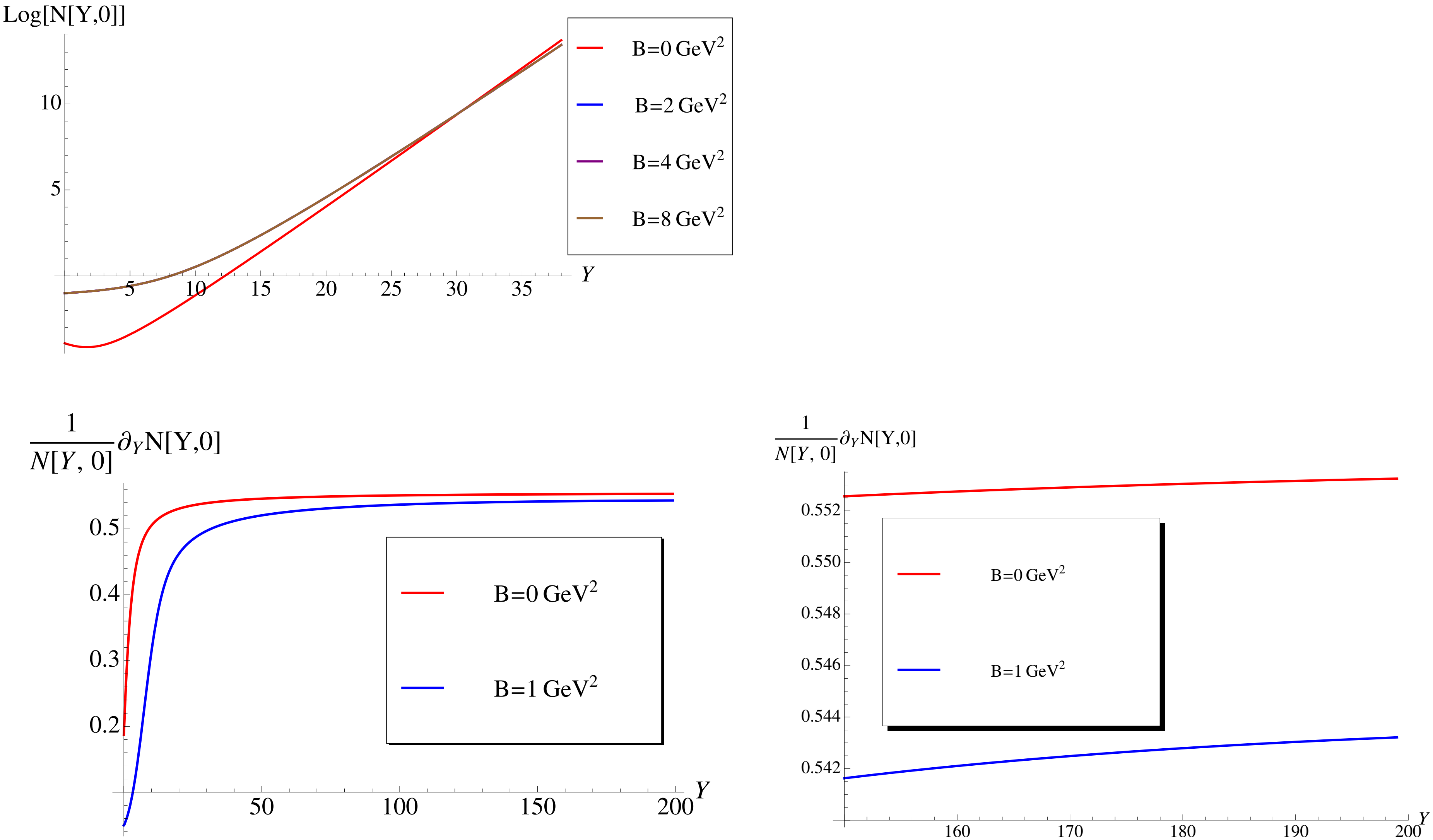
$$\int d^2 x_{13} K^B_R\left(x_{13}, x_{32} | x_{12}\right) \mathcal{N}\left(x_{12}; Y\right) \equiv$$

$$\int d^2 x_{13} \frac{e^{-B\left(x^2_{13} + x^2_{32}\right)}}{x^2_{32} + R^2} \left\{ 2\,\mathcal{N}\left(x_{13}; Y\right) - 2\,\frac{x^2_{12}}{x^2_{13} + x^2_{23} + 2\,R^2} \mathcal{N}\left(x_{12}; Y\right) \right\}$$

4. Independence on value of B;



$\frac{1}{N[42,l]}\partial_l N[42,l]$
B=0,R=Exp[−20]
B=0,R=Exp[−30]
B=0,R=Exp[−40]
B=0,R=Exp[−80]
Expected
Log[N[43,l]]
Expected
B=0, R=Exp[−30]
B=0, R=Exp[−20]
B=0, R=Exp[−40]
B=0, R=Exp[−80]
$\frac{1}{N[Y,0]}\partial_Y N[Y,0]$
Expected
B=0,R=Exp[−20]
B=0,R=Exp[−30]
B=0,R=Exp[−40]
B=0,R=Exp[−80]
$\frac{1}{N[Y,0]}\partial_Y N[Y,0]$
Expected
B=0,R=Exp[−20]
B=0,R=Exp[−30]
B=0,R=Exp[−40]
B=0,R=Exp[−80]



$$\omega = \omega_{\mathrm{BFKL}}$$

Saturation momentum

Saturation momentum can be found from linear equation:

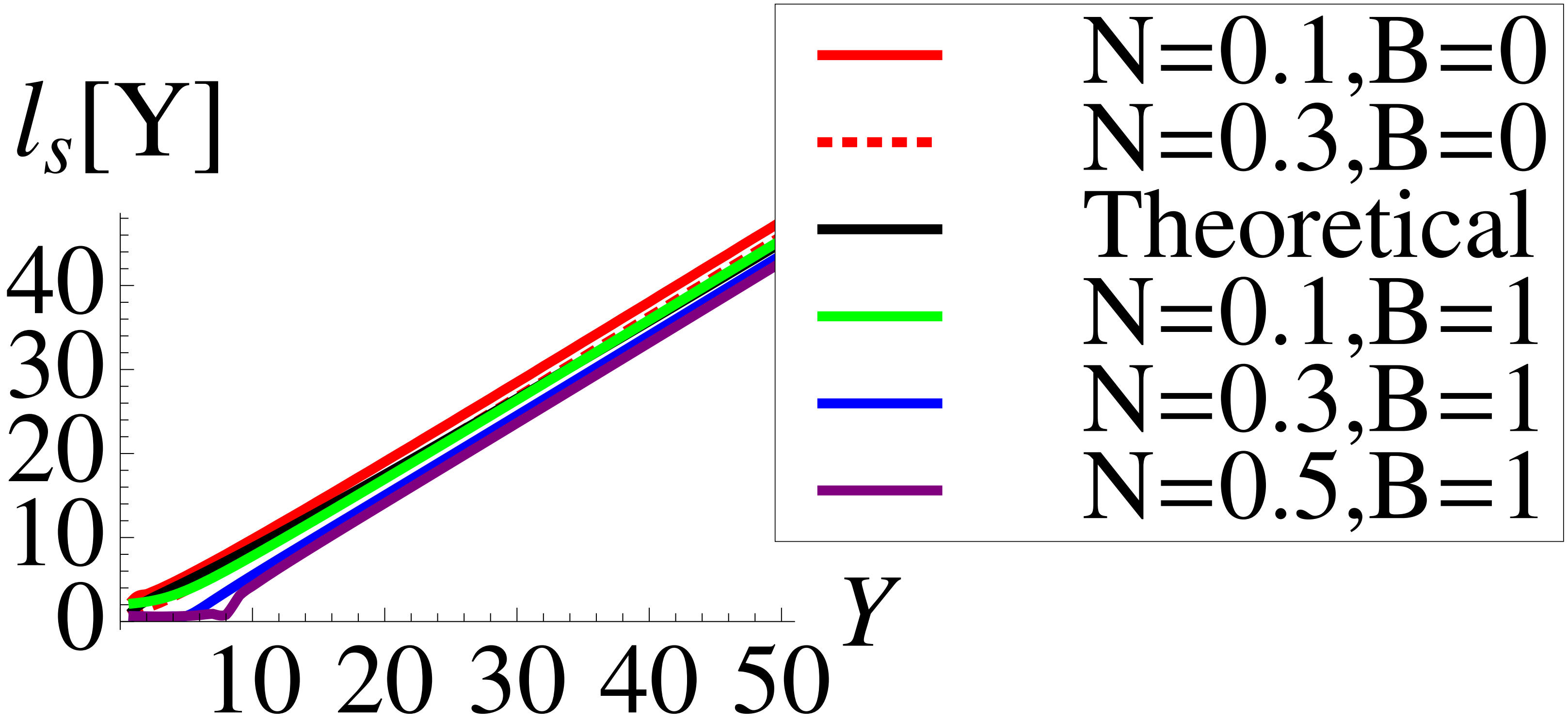
- $\mathcal{N}^{BFKL}\left(\frac{2}{Q_s\left(Y\right)};Y\right) = \mathcal{N}_0 \leq 1$ **where** $\mathcal{N}_0 = $ **Const**

For BFKL theoretical prediction:

$$l_s\left(Y\right) \equiv \ln\left(Q^2_s(Y)\Big/Q^2_s(Y_0)\right) =$$

$$\frac{\omega(\gamma_{cr})}{1-\gamma_{cr}}\left(Y - Y_0\right) - \frac{3}{2(1-\gamma_{cr})}\ln(Y/Y_0)$$

$$- \frac{3}{(1-\gamma_{cr})^2}\sqrt{\frac{2\,\pi}{\omega''(\gamma_{cr})}}\left(\frac{1}{\sqrt{Y}} - \frac{1}{\sqrt{Y_0}}\right)$$



- $$l_s[Y] = \ln\left(Q_s^2/B\right)$$

Next steps:

- Check that different models for large b dependence in the BFKL kernel lead to $\omega = \omega_{\text{BFKL}}$
  - $K\left(x_{12}, x_{20} | x_{10}\right) = \frac{x_{10}^2}{x_{12}^2\, x_{02}}\, e^{-\mu(x_{12} + x_{02})}$;
  - BFKL in gauge theories with the Higgs mechanism of mass generation (E.L.,L.Lipatov and M.Siddikov)

$$E\phi\left(\kappa\right) = \underbrace{\frac{\kappa+1}{\sqrt{\kappa}\sqrt{\kappa+4}} \ln \frac{\sqrt{\kappa+4}+\sqrt{\kappa}}{\sqrt{\kappa+4}-\sqrt{\kappa}}\phi\left(\kappa\right)}_{\text{kinetic energy}}$$

$$- \int_0^\infty \frac{d\kappa' \phi\left(\kappa'\right)}{\sqrt{(\kappa-\kappa')^2 \,+\, 2(\kappa+\kappa')+1}} + \underbrace{\frac{N_c^2+1}{2N_c^2}\frac{1}{\kappa+1}\int_0^\infty \frac{\phi\left(\kappa'\right)\, d\kappa'}{\kappa'+1}}_{\text{contact term}}$$

- Build Pomeron calculus, based on modified BFKL Pomerons;
- Obtain non-linear equations for amplitude;

Hope:

- **The global features of the BFKL Pomeron does not depend on confinement;**
- **We will be able to build the self-consistent theoretical CGC/saturation approach with correct b behaviour of the scattering amplitude;**

a posse ad esse (from being possible to being actual)

THANK YOU