Elastic scattering of hadrons

I.M. Dremin

Lebedev Physical Institute, Moscow

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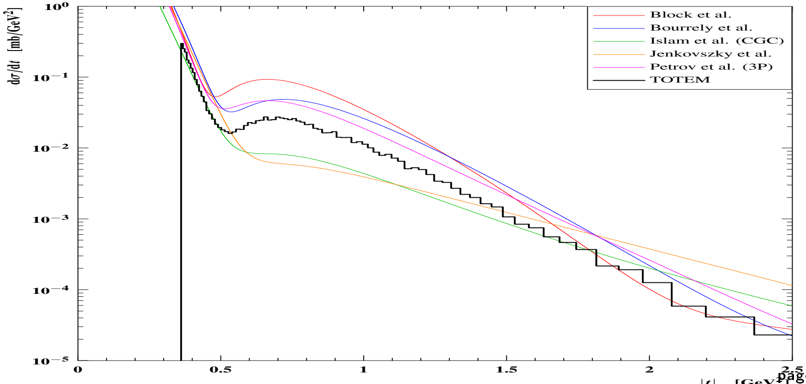
Kinematically simplest process with two variables s and t

The only measurable characteristics

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2}|A|^2 = \frac{1}{16\pi s^2}[(\mathrm{Im}A(s,t))^2 + (\mathrm{Re}A(s,t))^2]$$

Two functions $\mathrm{Im}A(s,t)$, $\mathrm{Re}A(s,t)$ as parts of a single analytic function $A(s,t)$



$s = 4E^2$; $t = -2p^2(1-\cos\theta)$ $(\approx -p^2\theta^2$ *at* $\theta \ll 1)$
Other **FOUR** characteristics: $\sigma_t(s)$, $\sigma_{el}(s)$, $\rho(s,t)$, $B(s,t)$

$$\sigma_t(s) = \frac{\mathrm{Im}A(p,\theta=0)}{s} - \mathrm{optical\ \ theorem}$$

$$\sigma_{el}(s) = \int_{t_{min}}^{0} dt \frac{d\sigma}{dt}(s,t)$$

$$\rho(s,t) = \frac{\mathrm{Re}A(s,t)}{\mathrm{Im}A(s,t)}$$

The diffraction cone

$$\frac{d\sigma}{dt} / \left(\frac{d\sigma}{dt}\right)_{t=0} = e^{Bt} \approx e^{-Bp^2\theta^2} \qquad (B \approx \mathrm{const}(t))$$

The amplitude in the diffraction cone (Gaussian, imaginary)

$$A(s,t) \approx is\sigma_t e^{Bt/2} \approx 4ip^2\sigma_t e^{-Bp^2\theta^2/2}$$

Coulomb-nuclear interference – $\rho(s,0) = \rho_0$.

Theory

The local dispersion relation

$$\rho(s,0)=\rho_0(s)\approx\frac{1}{\sigma_t}\left[\tan\left(\frac{\pi}{2}\frac{d}{d\ln s}\right)\right]\sigma_t\approx\frac{\pi}{2}\frac{d\ln\sigma_t}{d\ln s}$$

The unitarity relation

$$\mathrm{Im}A(p,\theta)=I_2(p,\theta)+F(p,\theta)=$$

$$\frac{1}{32\pi^2}\int\int d\theta_1 d\theta_2\frac{\sin\theta_1\sin\theta_2 A(p,\theta_1)A^*(p,\theta_2)}{\sqrt{[\cos\theta-\cos(\theta_1+\theta_2)][\cos(\theta_1-\theta_2)-\cos\theta]}}+F(p,\theta$$

The region of integration

$$|\theta_1-\theta_2|\le\theta,\qquad \theta\le\theta_1+\theta_2\le 2\pi-\theta.$$

$$\mathrm{Im}a_l(s)=|a_l(s)|^2+F_l(s)-\textit{partial wave representation}$$

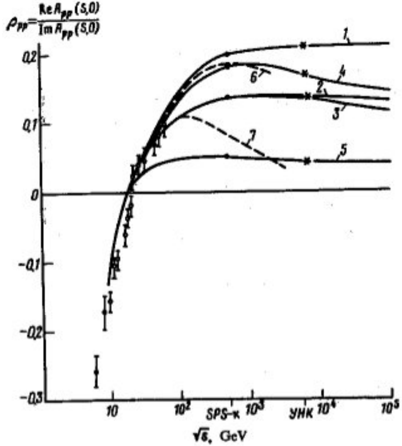
$$\mathrm{Im}h(s,b)\approx|h(s,b)|^2+F(s,b)-\textit{spatial (impact parameter) view}$$

Froissart bound

$$\sigma_t\le\frac{\pi}{2m_\pi^2}\ln^2(s/s_0)$$

I.M. Dremin, M.T. Nazirov, Pis'ma ZhETF 37 (**1983**) 163
(JETP Lett. 37 (1983) 198)   (see also arXiv:1304.5345)

Predictions according to the integral dispersion relations



The ratio of real to imaginary part of elastic pp-scattering amplitude at t=0 according to dispersion relations with different assumptions about high energy behavior of the total cross section (different signs at low and high energies!).

OUR GUESSES ABOUT ASYMPTOTICS

$$\sigma_t(s) \leq \frac{\pi}{2m_\pi^2} \ln^2(s/s_0)$$

THE BLACK DISK: $\sigma_t = 2\pi R^2$; $R = R_0 \ln s$; $\frac{\sigma_{el}}{\sigma_t} = \frac{\sigma_{in}}{\sigma_t} = \frac{1}{2}$

$B(s) = \frac{R^2}{4}$; $\rho(s, t = 0) = \frac{\pi}{\ln s}$ None observed in experiment!

THE GRAY DISKS: two parameters - radius+opacity

Gray and Gaussian disks ($X = \sigma_{el}/\sigma_t$; $Z = 4\pi B/\sigma_t$; $\alpha \leq 1$)

Model	$1 - e^{-\Omega}$	σ_t	σ_{el}	B	Z	XZ	X/Z
Gray	$\alpha\theta(R - b)$	$2\pi\alpha R^2$	$\pi\alpha^2 R^2$	$R^2/4$	$1/2\alpha$	$1/4$	α^2
Gauss	$\alpha e^{-b^2/R^2}$	$2\pi\alpha R^2$	$\pi\alpha^2 R^2/2$	$R^2/2$	$1/\alpha$	$1/4$	$\alpha^2/4$

The energy behavior

$\sqrt{s}$, GeV	2.70	4.74	6.27	7.62	13.8	62.5	546	1800	7000
X	0.42	0.27	0.24	0.22	0.18	0.18	0.21	0.23	0.25
Z	0.64	1.09	1.26	1.34	1.45	1.50	1.20	1.08	1.00
XZ	0.27	0.29	0.30	0.30	0.26	0.25	0.26	0.25	0.25
X/Z	0.66	0.25	0.21	0.17	0.16	0.12	0.18	0.21	0.25

The energy evolution of the proton's shape

Purely phenomenological fit (see arXiv:1306.5384)

$$A(s,t) = s\sqrt{16\pi} f(s,t).$$

Fit at ISR in U. Amaldi, K. Schubert Nucl. Phys. B166 (1980) 301

$$f(s,t) = i\alpha[A_1 \exp(\frac{1}{2} b_1 \alpha t) + A_2 \exp(\frac{1}{2} b_2 \alpha t)] - iA_3 \exp(\frac{1}{2} b_3 t),$$

where $\alpha(s)$ is complex and is given by

$$\alpha(s) = [\sigma_t(s)/\sigma_t(23.5\,\mathrm{GeV})](1 - i\rho_0(s)).$$

6 parameters at given s (A_i, b_i, ρ_0 minus normalization at t=0)

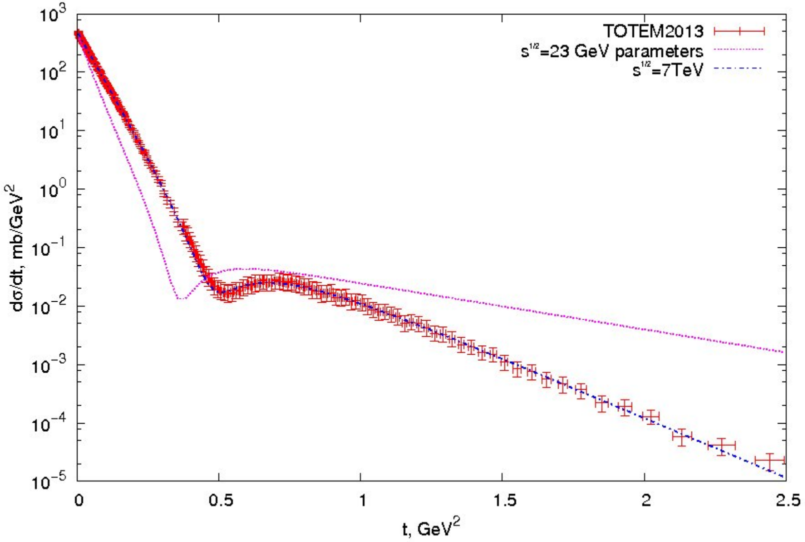


Рис.: Fit of the TOTEM data – dash-dotted curve. Dotted curve is calculated with parameters used at 23.5 GeV and with $\rho_0 = 0.14$

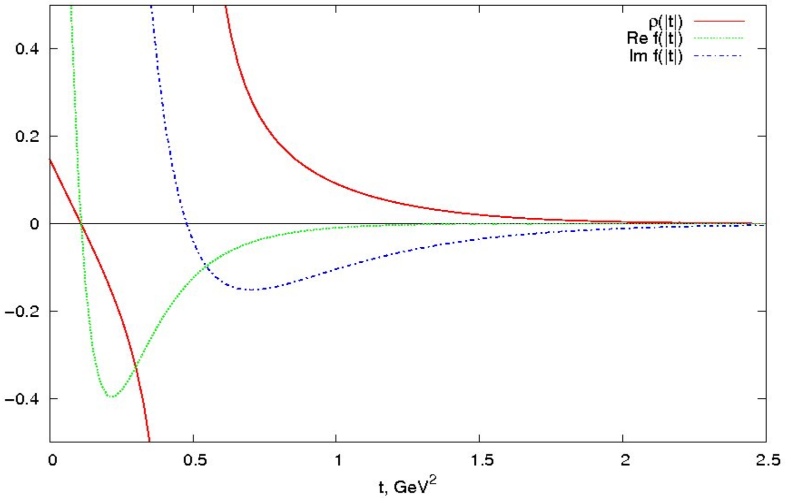


Рис.: Real (dotted curve) and imaginary (dash-dotted curve) parts of the amplitude and their ratio (solid curve). Similar curves are usually predicted by other models which describe the dip as zero of $\mathrm{Im}A(t)$.

The energy evolution of the impact parameter picture

$$i\Gamma(s,b) = \frac{1}{\sqrt{\pi}} \int_0^\infty dqqf(s,t) J_0(qb). \qquad (1)$$

$$2\Re\Gamma(s,b) = |\Gamma(s,b)|^2 + G(s,b), \qquad (2)$$

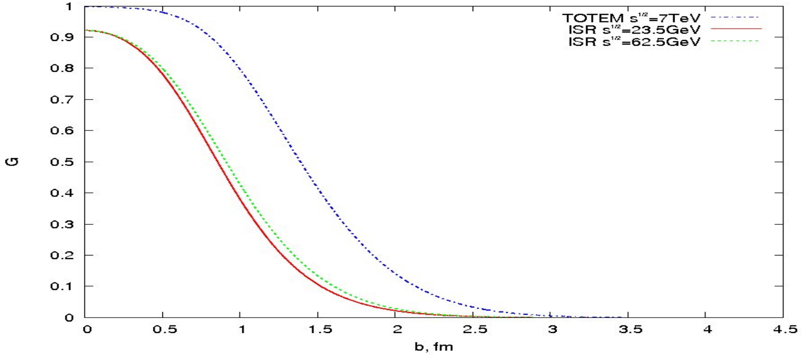


Рис.: The overlap functions at 23.5 GeV (solid curve), 62.5 GeV (dotted curve) and 7 TeV (dash-dotted curve)

$$\Delta G(b) = G(s_1, b) - G(s_2, b) \quad (\sqrt{s_1} = 7\,\mathrm{TeV},\ \sqrt{s_2} = 23.5\,\mathrm{GeV})$$

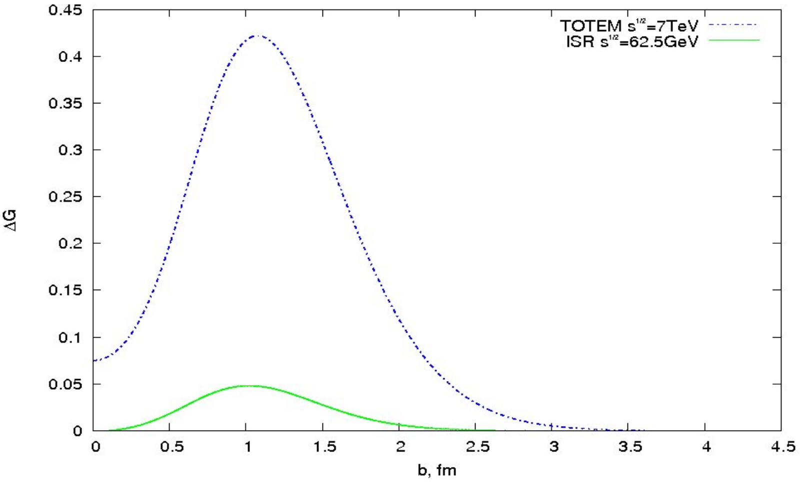


Рис.: The difference between the overlap functions. Dash-dotted curve is for 7 TeV and 23.5 GeV energies, solid curve is for 62.5 GeV and 23.5 GeV energies. The parton density at periphery increases!

THEORETICAL APPROACHES (MODELS)

1. Geometric picture and eikonal

The impact parameter (**b**) representation

$$A(s,t=-q^2)=\frac{2s}{i}\int d^2be^{i\mathbf{qb}}(e^{2i\delta(s,\mathbf{b})}-1)=2is\int d^2be^{i\mathbf{qb}}(1-e^{-\Omega(s,\mathbf{b})})$$

Two or three regions of the internal hadron structure.
Heisenberg relation: large b (external regions) - small $|t|$, small b (internal regions) - large $|t|$. 15-20 parameters!
E.g., the diffraction profile function is

$$\Gamma(s,b)=1-\Omega(s,b)=g(s)\left[\frac{1}{1+e^{(b-r)/a}}+\frac{1}{1+e^{(-b+r)/a}}-1\right]$$

and special shapes for internal regions. UNITARIZATION!

2. Electromagnetic analogies

Use of electromagnetic (and parton density) form factors.
The droplet model.

3. Reggeon exchanges

$$\Omega(s, \mathbf{b}) = S(s)F(\mathbf{b}^2) + (\mathrm{non-leading\ \ terms})$$

$S(s)$ is crossing symmetric and reproduces Pomeron trajectory

$$S(s) = \frac{s^c}{(\ln s)^{c'}} + \frac{u^c}{(\ln u)^{c'}}$$

$F(\mathbf{b}^2)$ is the Bessel transform of "Pomeron vertices"

$$F(t) = f|G(t)|^2 \frac{a^2 + t}{a^2 - t}$$

$G(t)$ is the proton "nuclear form factor"parametrized like the electromagnetic form factor with two poles

$$G(t) = \frac{1}{(1 - t/m_1^2)(1 - t/m_2^2)}$$

or (one chooses/adds) the exponential form factors like

$$F(s, t) = \hat{s}^{\epsilon_1} e^{B(s)t}$$

4. QCD-inspired approaches

Gluons and quarks as active partons. Similar form factors.

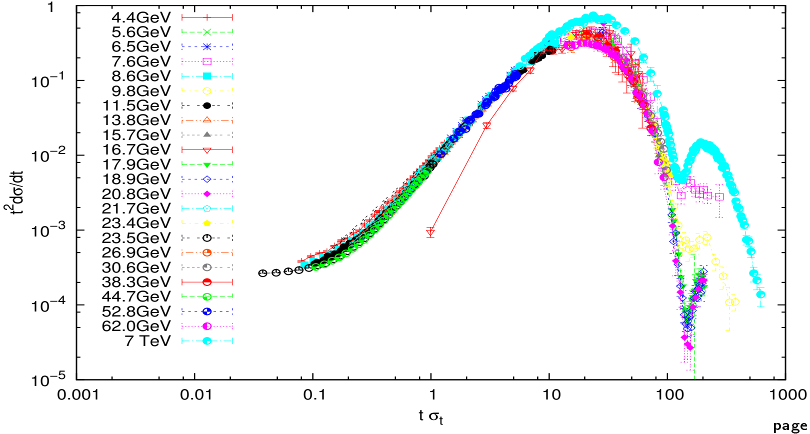
All approaches are rather successful in the diffraction cone.

Scaling laws in the diffraction cone (arXiv:1212.3313)

$$\frac{\pi}{2}\left[\frac{\partial \ln \mathrm{Im}A(s,t)}{\partial \ln s} - 1\right] = \rho_0 \left[1 + \frac{\partial \ln \mathrm{Im}A(s,t)}{\partial \ln t}\right]$$

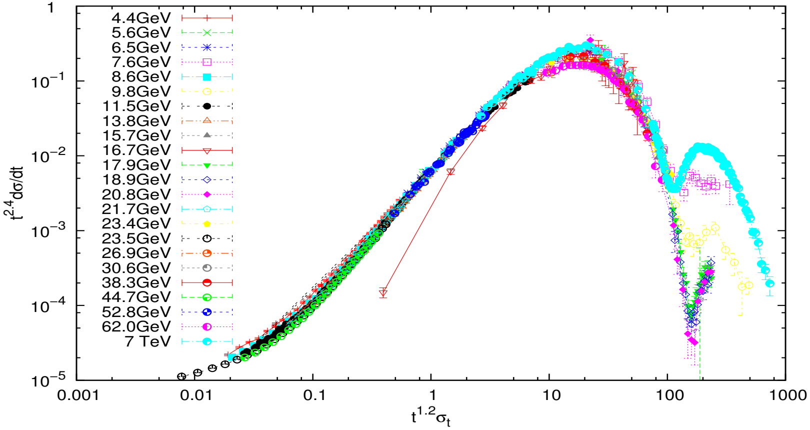
$$\frac{\partial \ln \mathrm{Im}A(s,t)}{\partial \ln \sigma_t} - \frac{\partial \ln \mathrm{Im}A(s,t)}{\partial \ln t} = 1 + \frac{d \ln s}{d \ln \sigma_t}.$$

$$t^2 d\sigma/dt = \Phi(t\sigma_t) - \textit{geometric scaling}$$



$$\rho(s,t) = \rho(s,0)\left[1 + \frac{1}{a}\frac{\partial \ln \mathrm{Im} A(s,t)}{\partial \ln |t|}\right].$$

$$t^{2a} d\sigma/dt = \omega(t^a \sigma_t), \qquad a = 1.2.$$



The value of a accounts for different energy behavior of B and σ_t - violation of geometric scaling.

THREE REGIONS: the diffraction cone, the Orear regime, the hard parton scattering

$$\mathrm{Im}A(p,\theta) = I_2(p,\theta) + F(p,\theta) =$$

$$\frac{1}{32\pi^2}\int\int d\theta_1 d\theta_2 \frac{\sin\theta_1 \sin\theta_2 A(p,\theta_1)A^*(p,\theta_2)}{\sqrt{[\cos\theta - \cos(\theta_1+\theta_2)][\cos(\theta_1-\theta_2) - \cos\theta]}} + F(p,\theta$$

$$|\theta_1 - \theta_2| \leq \theta, \qquad \theta \leq \theta_1 + \theta_2 \leq 2\pi - \theta$$

I_2 - two-particle intermediate states (σ_{el}), F - inelastic ones (σ_{inel}). For angles θ outside the diffraction cone one amplitude in I_2 is at small angles and another at large ones: **linear** integral equation

$$\mathrm{Im}A(p,\theta) = \frac{p\sigma_t}{4\pi\sqrt{2\pi B}}\int_{-\infty}^{+\infty} d\theta_1 f_\rho e^{-Bp^2(\theta-\theta_1)^2/2}\mathrm{Im}A(p,\theta_1) + F(p,\theta).$$

$f_\rho = 1 + \rho_0\rho(\theta_1)$.

Analytic solution if $F(p,\theta) \ll \mathrm{Im}A(p,\theta)$ and $f_\rho \approx$const outside the diffraction cone!

(I.V. Andreev, I.M. Dremin JETP Lett. 6 (**1967**) 262)

The proof of the assumption about the small overlap function.

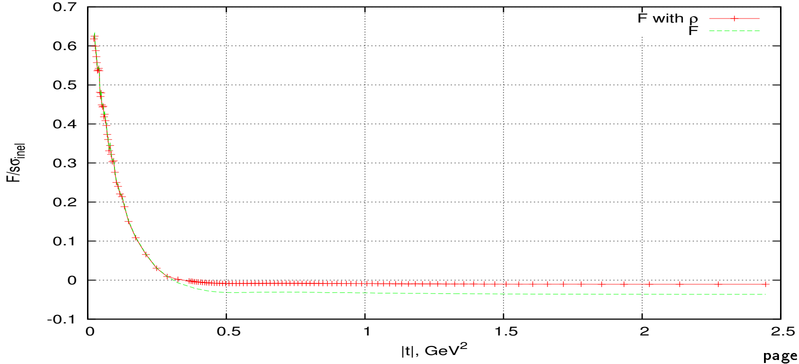
$F(p,\theta)$ computed from experimental data is negligible outside cone:

$$F(p,\theta) = \ 16p^2\left(\pi\frac{d\sigma}{dt}/(1+\rho^2)\right)^{1/2} -$$

$$\frac{8p^4 f_\rho}{\pi}\int_{-1}^{1} dz_2 \int_{z_1^-}^{z_1^+} dz_1 \left[\frac{d\sigma}{dt_1}\cdot\frac{d\sigma}{dt_2}\right]^{1/2} K^{-1/2}(z,z_1,z_2),$$

$z_i = \cos\theta_i; \quad K(z,z_1,z_2) = 1 - z^2 - z_1^2 - z_2^2 + 2zz_1z_2,$

$z_1^{\pm} = zz_2 \pm [(1-z^2)(1-z_2^2)]^{1/2}$



The elastic differential cross section **outside** the diffraction cone contains the exponentially decreasing with θ (or $\sqrt{|t|}$) term (Orear regime!) with imposed on it damped oscillations:

$$\frac{d\sigma}{p_1 dt} = \left(e^{-\sqrt{2B|t| \ln \frac{4\pi B}{\sigma_t f_\rho}}} + p_2 e^{-\sqrt{2\pi B|t|}} \cos(\sqrt{2\pi B|t|} - \phi)\right)^2 .$$

The experimentally measured values of the diffraction cone slope B and the total cross section σ_t determine mostly the shape of the elastic differential cross section in the Orear region of transition from the diffraction peak to large angle parton scattering. The value of $Z = 4\pi B/\sigma_t$ is so close to 1 that the fit is extremely sensitive to f_ρ. Thus, it becomes possible for the first time to estimate the ratio ρ outside the diffraction cone from fits of experimental data.

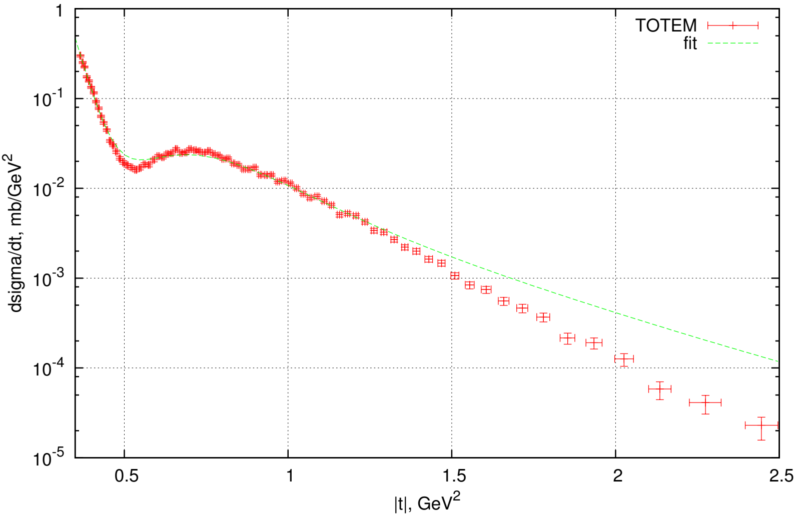
At the LHC, its average value is negative and equal to -2.1!

Do we approach the black disk limit $Z \to 0.5$?

In Orear slope the decrease of Z must be compensated by the decrease of $f_\rho = 1 + \rho_0 \rho(t)$ but $\rho_0 \propto \ln^{-1} s$! Is it possible that $\rho(t)$ in Orear region increases in modulus being negative?

Fit at 7 TeV (dip+Orear in $0.3 < |t| < 1.5$ GeV^2)

(arXiv:1202.2016)



PROBLEM: The real part outside the diffraction cone

At $t = 0$, it is known from Coulomb-nuclear interference experimentally (at lower than LHC energies) and from dispersion relations theoretically. ρ_0 at LHC may be about 0.13 - 0.14.

No experimental results for $\rho(t)$ are available. However, it can be calculated (Martin formula) if the imaginary part is known:

$$\rho(t) = \rho_0 \left[1 + \frac{t(d\mathrm{Im}A(t)/dt)}{\mathrm{Im}A(t)} \right]$$

Then the equation for $\rho(t)$ follows from the unitarity condition

$$\frac{dv}{dx} = -\frac{v}{x} - \frac{2}{x^2} \left(\frac{Ze^{-v^2} - 1}{\rho^2(t=0)} - 1 \right)$$

$x = \sqrt{2B|t|}, \;\; v = \sqrt{\ln(Z/f_\rho)}, \;\; \rho(t) = (Ze^{-v^2} - 1)/\rho(t=0)$

where v is the solution of the equation.

Asymptotics at $|t| \to \infty$ $\rho \to (Z-1)/\rho(t=0)$.

Then $f_\rho \to Z$ and $\ln(Z/f_\rho) \to 0$!

Prediction: the drastic changes are expected in this region of $|t|$!

(arXiv:1204.4866)

Experimentally observed $|t|^{-8}$-regime in pp-scattering.

Kancheli talk

The dimensional counting

For n partons participating in a single hard scattering

$$A_1(s,t) \propto \left(\frac{s_0}{s}\right)^{\frac{n}{2}-2} f_1(s/t)$$

The coherent scattering

Three gluons coherently exchanged between three pairs of quarks.
Multi-Pomeron exchange with one large-p_T Pomeron.

Conclusions

- The black disk limit is still far away.
- The evolution of the impact parameter overlap function with energy shows steady increase of the contribution of the peripheral regions of protons (ISR $\rightarrow$ S$p\bar{p}$S $\rightarrow$ LHC).
  The parton density at the periphery increases!
  Inelastic diffraction?
- Most theoretical models describe the diffraction peak but fail outside it.
- Scaling laws in the diffraction cone are predicted by the local dispersion relations + Martin formula but comparison with experiment requires some modification of the latter because it shows that geometric scaling is not valid.

- At intermediate angles between the diffraction cone and hard parton scattering region the unitarity condition predicts the Orear regime with exponential decrease in angles and imposed on it damped oscillations.
- The experimental data on elastic *pp* differential cross section at low and high ($\sqrt{s}$=7 TeV) energies have been fitted in this region with well described position of the dip and Orear slope.
- The fit by the "unitarity formula" allows for the first time at 7 TeV to estimate the ratio $\rho(s,t)$ far from forward direction t=0. It happened to be about -2.
- Controversial forms of $\rho(s,t)$ for different models.
  The common feature is the pole at the dip!
  The unitarity condition does not require the pole!
  The estimate of $\rho(t)$ in the unitarity condition is attempted.
- The overlap function is small and negative in the Orear region. That confirms the assumption used in solving the unitarity equation. Important corollary: the phases of inelastic amplitudes are crucial in any model of inelastic processes.