Anomalous transport

on the lattice

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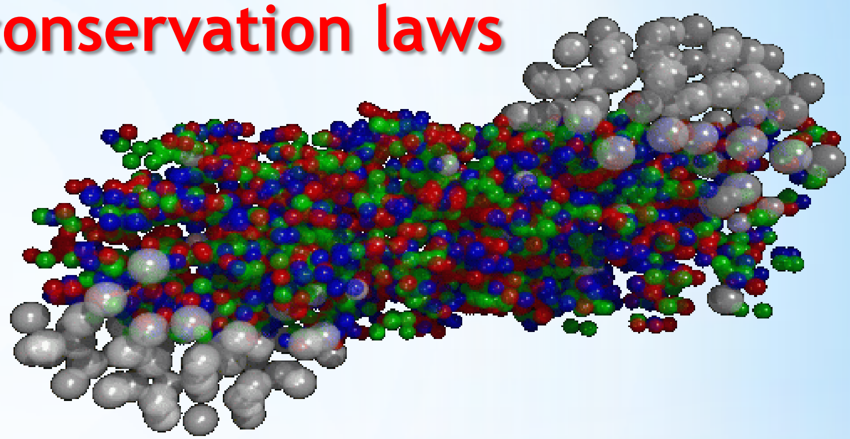
To the memory of my Teacher,
excellent Scientist,
very nice and outstanding Person,
Mikhail Igorevich Polikarpov

"New" hydrodynamics for HIC

Before 2008: classical hydro = conservation laws

- shear/bulk viscosity
- heat conductivity
- conductivity
- …



Essentially classical picture!!!

Quantum effects in hydrodynamics? YES!!!

In massless case – new integral of motion: chirality

"Anomalous" terms in hydrodynamical equations:
macroscopic memory of quantum effects

[Son, Surowka, ArXiv:0906.5044]

Integrate out free massless fermion gas
in arbitrary gauge background.

Very strange gas – can only expand with a speed of light!!!

“New” hydrodynamics: anomalous transport

$$\boxed{j_\mu = n u_\mu + \sigma E_\mu + \sigma_\chi B_\mu + \xi w_\mu}$$

$$\boxed{j_{5\,\mu} = n_5 u_\mu + \sigma'_\chi B_\mu + \xi' w_\mu}$$

$$\boxed{B^\mu = \epsilon^{\mu\nu\alpha\beta} u_\nu F_{\alpha\beta}} \quad \boxed{E^\mu = u_\nu F^{\mu\nu}}$$

$$\boxed{w_\mu = \epsilon_{\mu\nu\alpha\beta}\, u^\nu \partial^\alpha u^\beta}$$

Positivity of entropy production uniquely fixes “magnetic conductivities”!!!

- Insert new equations into some hydro code
- P-violating initial conditions (rotation, B field)
- Experimental consequences?

Anomalous transport: CME, CSE, CVE

Chiral Magnetic Effect
[Kharzeev, Warringa, Fukushima]

$$j_V^i = \sigma_{VV}^{\mathcal{B}} B^i = \frac{N_c e \mu_A}{2\pi^2} B^i$$

Chiral Separation Effect
[Son, Zhitnitsky]

$$j_A^i = \sigma_{AV}^{\mathcal{B}} B^i = \frac{N_c e \mu_V}{2\pi^2} B^i$$

Chiral Vortical Effect
[Erdmenger *et al.*, Banerjee *et al.*]

$$j_V = \sigma_V^{\mathcal{V}} w = \frac{N_c e}{2\pi^2} \mu_A \mu_V w$$

$$j_A = \sigma_A^{\mathcal{V}} w = N_c e \left(\frac{\mu_V^2 + \mu_A^2}{4\pi^2} + \frac{T^2}{12} \right) w$$

Lorenz force $\Rightarrow$ **Coriolis force (Rotating frame)**

T-invariance and absence of dissipation

Dissipative transport (conductivity, viscosity)

- No ground state
- T-noninvariant (but CP)
- Spectral function = anti-Hermitean part of retarded correlator
- Work is performed
- Dissipation of energy
- First k → 0, then w → 0

Anomalous transport (CME, CSE, CVE)

- Ground state
- T-invariant (but not CP!!!)
- Spectral function = Hermitean part of retarded correlator
- No work is performed
- No dissipation of energy
- First w → 0, then k → 0

Anomalous transport: CME, CSE, CVE

Folklore on CME & CSE:

- Transport coefficients are RELATED to anomaly
- and thus protected from:
  - perturbative corrections
  - IR effects (mass etc.)

Check these statements as applied to the lattice

What is measurable? How should one measure?

CVE coefficient is not fixed

Phenomenologically important!!! Lattice can help

CME and CVE: lattice studies

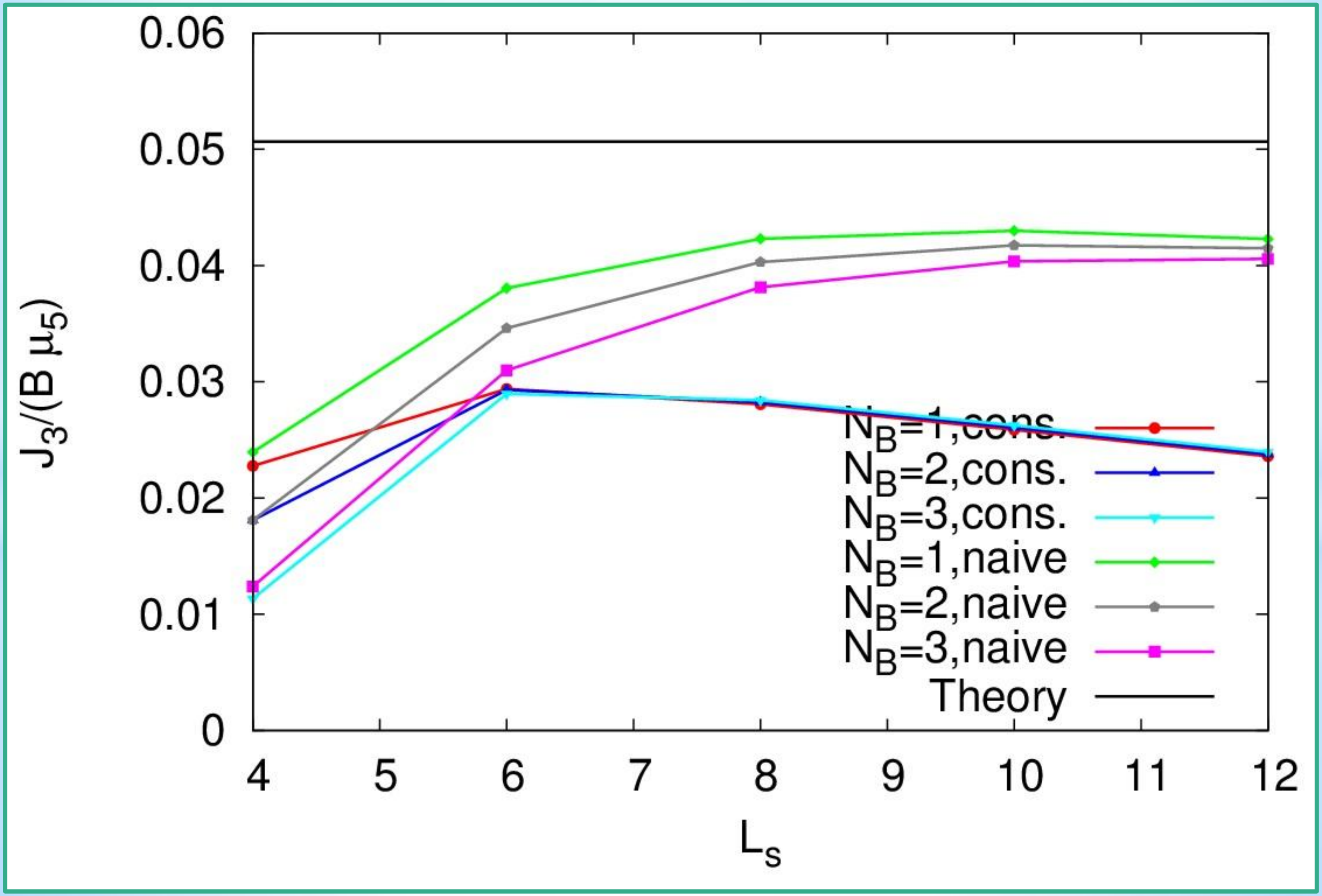
Simplest method: introduce sources in the action

- Constant magnetic field
- Constant μ5 [Yamamoto, 1105.0385]
- Constant axial magnetic field [ITEP Lattice, 1303.6266]
- Rotating lattice??? [Yamamoto, 1303.6292]

"Advanced" method:

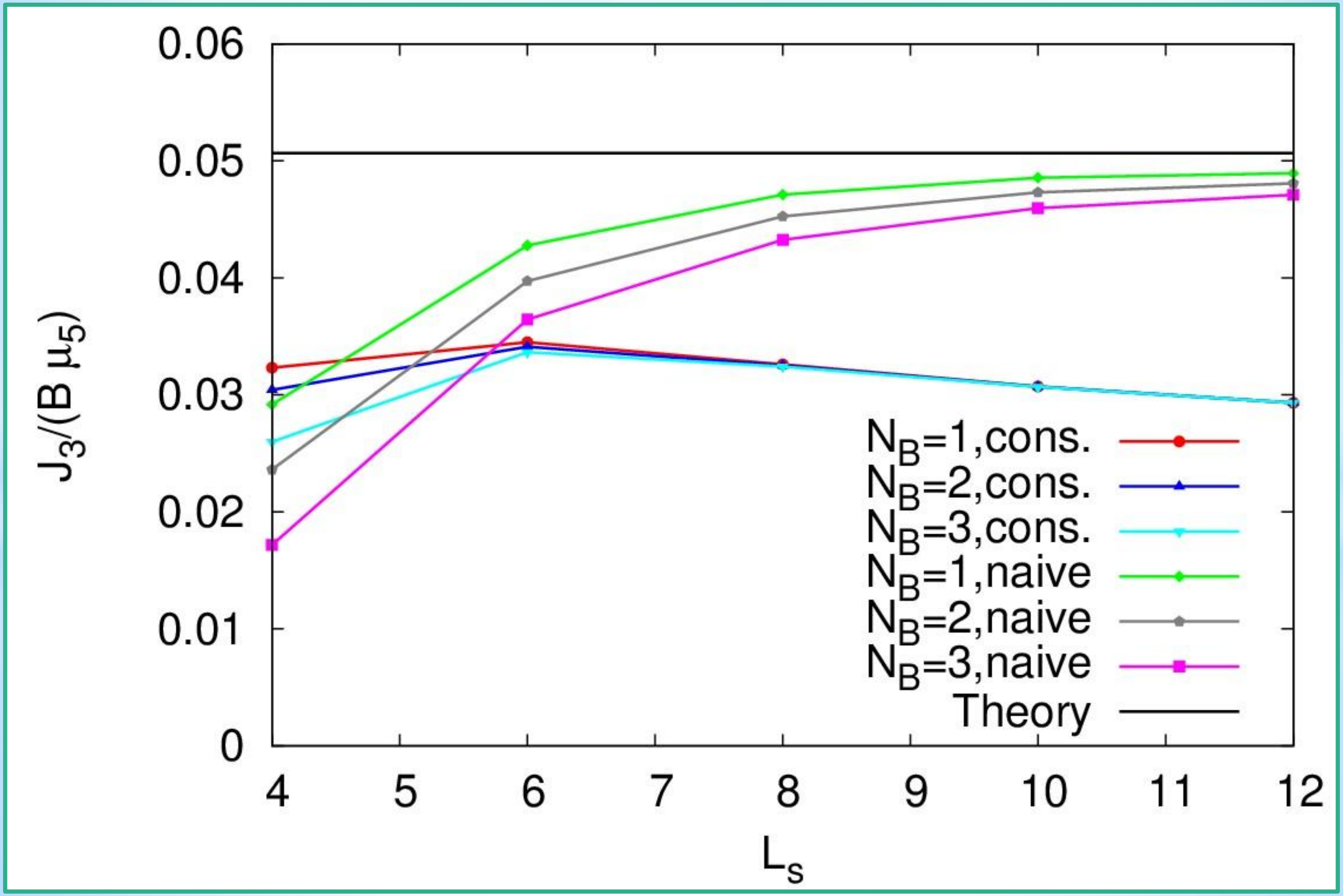
- Measure spatial correlators
- No analytic continuation necessary
- Just Fourier transforms
- BUT: More noise!!!
- Conserved currents/ Energy-momentum tensor not known for overlap

CME with overlap fermions



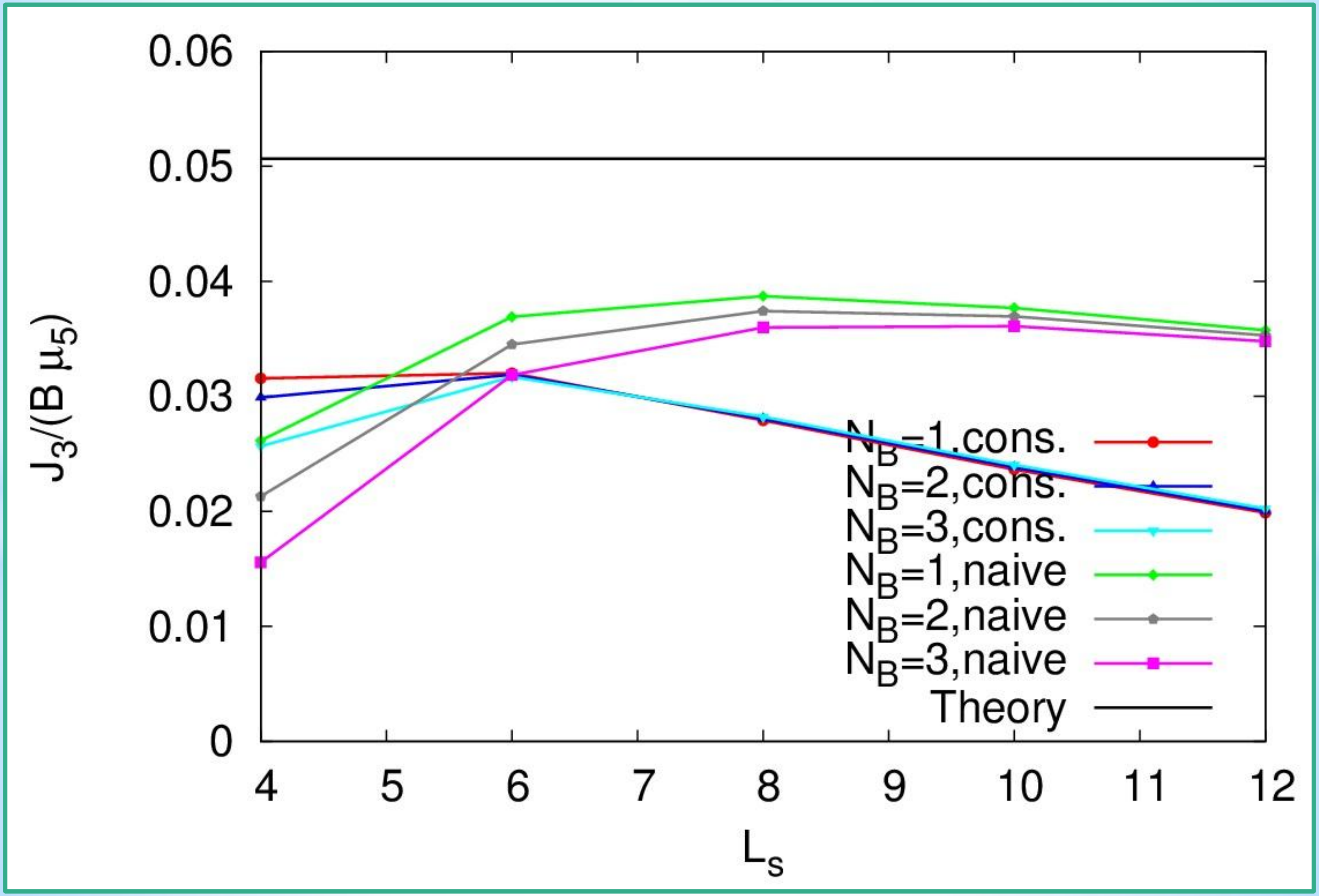
ρ = 1.0, m = 0.05

CME with overlap fermions



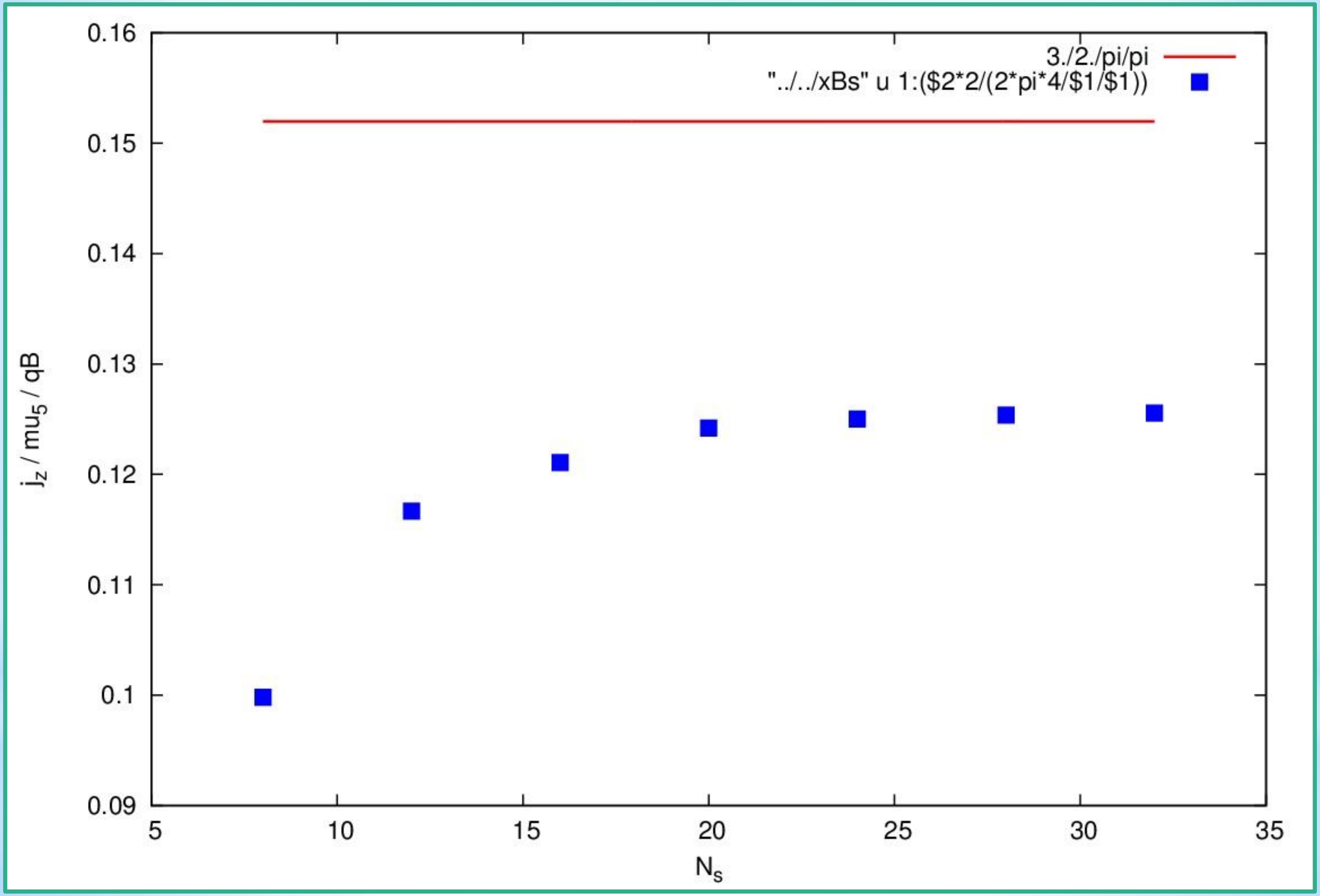
ρ = 1.4, m = 0.01

CME with overlap fermions



ρ = 1.4, m = 0.05

Staggered fermions [G. Endrodi]



Bulk definition of μ_5 !!! Around 20% deviation

CME: “Background field” method

CLAIM: constant magnetic field in finite volume is NOT a small perturbation

↓

“triangle diagram” argument invalid
(Flux is quantized, 0 → 1 is not a perturbation, just like an instanton number)

More advanced argument:

$F = \epsilon_{\mu\nu}\partial_\mu A_\nu$ in a finite volume → $\int d^2 x F = 0$

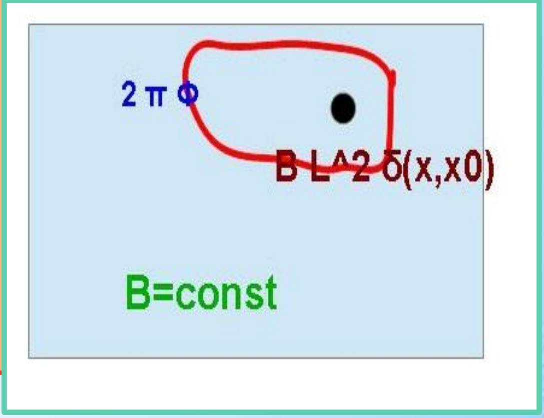
Solution: hide extra flux in the delta-function

$$F = F_0 - F_0 L^2 \delta\left(x, x_0\right)$$

Fermions don't note this singularity if

$$F_0 L^2 = 2\pi\Phi, \quad \Phi \in \mathbb{Z}$$

Flux quantization!



Closer look at CME: analytics

- Partition function of Dirac fermions in a finite Euclidean box
- Anti-periodic BC in time direction, periodic BC in spatial directions
- Gauge field $A_3=\theta$ - source for the current
- Magnetic field in XY plane
- Chiral chemical potential μ_5 in the bulk

Dirac operator:

$$\mathcal{D} = \begin{pmatrix} m & ik_0 - \mu_5 + \sigma_3 (k_3 + \theta) - i\sigma_a \nabla_a \\ ik_0 + \mu_5 - \sigma_3 (k_3 + \theta) + i\sigma_a \nabla_a & m \end{pmatrix},$$

$$\sigma^a \nabla_a = \begin{pmatrix} 0 & \nabla_x + i\nabla_y \\ \nabla_x - i\nabla_y & 0 \end{pmatrix} = \sqrt{2B} \begin{pmatrix} 0 & A^\dagger \\ -A & 0 \end{pmatrix},$$

Closer look at CME: analytics

Creation/annihilation operators in magnetic field:

$$A = \frac{-a_x + i a_y}{\sqrt{2}}, \; a_{x,y} = \frac{1}{\sqrt{B}}\left(\partial_x + \frac{Bx}{2}\right), \; \left[A, A^\dagger\right] = 1$$

Now go to the Landau-level basis:

$$|\psi^R_{n,s}\rangle = \begin{bmatrix} |n\rangle \\ si\,|n-1\rangle \\ 0 \\ 0 \end{bmatrix}, \quad |\psi^L_{n,s}\rangle = \begin{bmatrix} 0 \\ 0 \\ |n\rangle \\ si\,|n-1\rangle \end{bmatrix}$$

$$|\psi^R_0\rangle = \begin{bmatrix} |0\rangle \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad |\psi^L_0\rangle = \begin{bmatrix} 0 \\ 0 \\ |0\rangle \\ 0 \end{bmatrix}$$

Higher Landau levels

(topological) zero modes

$$\mathcal{D}_n = \langle \psi^{L,R}_{n,s} | \mathcal{D} | \psi^{L,R}_{n,s'} \rangle =$$

$$= \begin{pmatrix} m & 0 & ik_0 + \sqrt{2Bn} - \mu_5 & \theta + k_3 \\ 0 & m & \theta + k_3 & ik_0 - \sqrt{2Bn} - \mu_5 \\ ik_0 - \sqrt{2Bn} + \mu_5 & -\theta - k_3 & m & 0 \\ -\theta - k_3 & ik_0 + \sqrt{2Bn} + \mu_5 & 0 & m \end{pmatrix}$$

Closer look at CME: LLL dominance

Dirac operator in the basis of LLL states:

$$\mathcal{D}_0 = \langle \psi_0^{L,R} | \mathcal{D} | \psi_0^{L,R} \rangle = \begin{pmatrix} m & ik_0 - \mu_5 + k_3 + \theta \\ ik_0 + \mu_5 - k_3 - \theta & m \end{pmatrix}$$

Vector current:

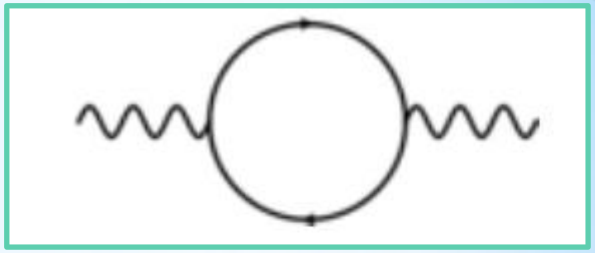
$$j = \frac{B}{2\pi} \frac{\partial}{\partial \theta} \log\det\left(\mathcal{D}_0\right) + \frac{B}{2\pi} \sum_{n=1}^{+\infty} \frac{\partial}{\partial \theta} \log\det\left(\mathcal{D}_n\right)$$

Prefactor comes from LL degeneracy
Only LLL contribution is nonzero!!!

$$j = \frac{T B}{2\pi} \sum_{k_0} \int \frac{dk_3}{2\pi} \frac{2\left(k_3 - \mu_5\right)}{k_0^2 + m^2 + \left(k_3 - \mu_5\right)^2}$$

Dimensional reduction: 2D axial anomaly

Polarization tensor in 2D:



$$j_\mu = \epsilon_{\mu\sigma}\Pi_{\sigma\nu}A_\nu, \quad A_0 \to i\mu_5$$

Proper regularization (vector current conserved):

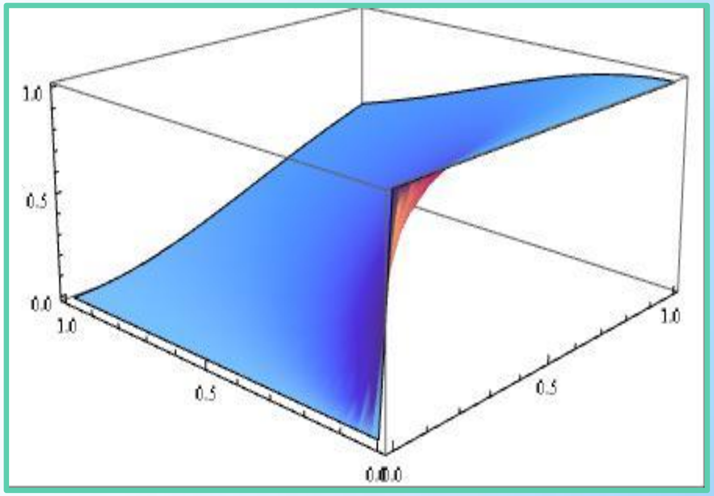
$$\Pi_{\mu\nu} = \frac{1}{\pi}\frac{k^2\delta_{\mu\nu} - k_\mu k_\nu}{k^2}$$

[Chen, hep-th/9902199]

Final answer:

$$j_3(k) = i\Pi_{33}(k)\,\mu_5(k) = \frac{1}{2\pi^2}\,\frac{k_0^2}{k_0^2+k_3^2}\,\mu_5(k)$$

- Value at $k_0=0$, $k_3=0$: NOT DEFINED (without IR regulator)
- First $k_3 \to 0$, then $k_0 \to 0$
- Otherwise zero



Chirality n_5 vs μ_5

μ_5 is not a physical quantity, just Lagrange multiplier

Chirality n_5 is (in principle) observable

Express everything in terms of n_5

To linear order in μ_5 :

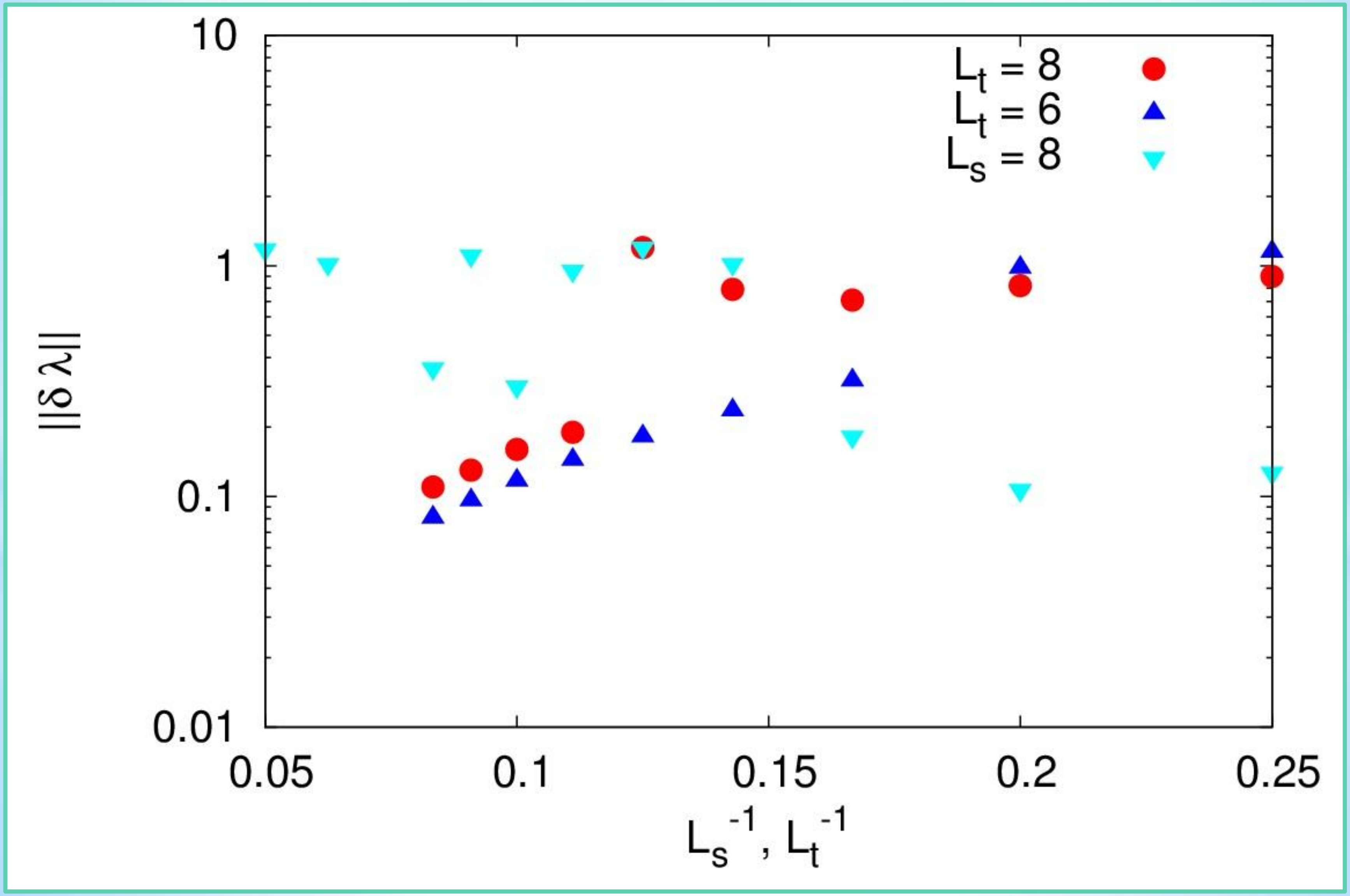
$$\boxed{n_5 = \epsilon_{0\alpha}\Pi_{\alpha\beta}\epsilon_{\beta 0}\mu_5 = \Pi_{33}\mu_5}$$

Singularities of Π_{33} cancel !!!

$$\boxed{j_3 = n_5 B}$$

Note: no non-renormalization for two loops or higher and no dimensional reduction due to 4D gluons!!!

Dimensional reduction with overlap



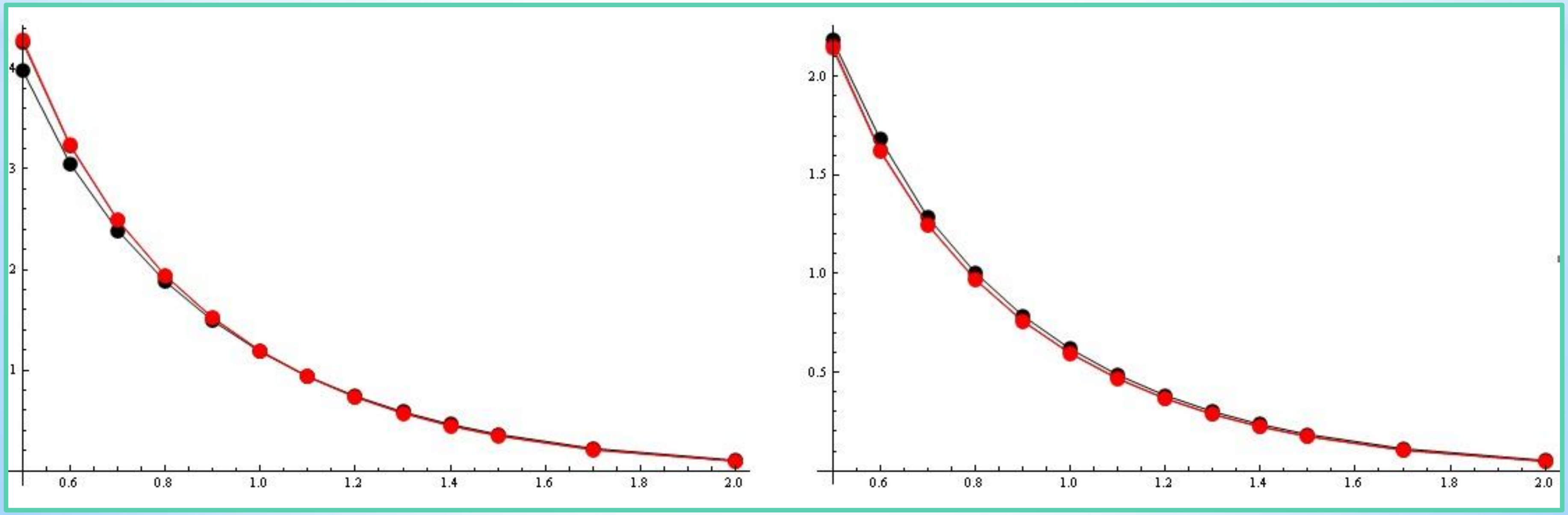
First Lx,Ly →∞ at fixed Lz, Lt, Φ !!!

IR sensitivity: aspect ratio etc.

$$j_3 = \sum_{k_0, k_3} \frac{\partial}{\partial k_3} \det\left(\mathcal{D}\left(k_0, k_3\right)\right)$$

$$k_0 = 2\pi m_0 / L_t, k_3 = 2\pi m_3 / L_3$$

- **L3 ⟶∞, Lt fixed: ZERO (full derivative)**
- **Result depends on the ratio Lt/Lz**



Importance of conserved current

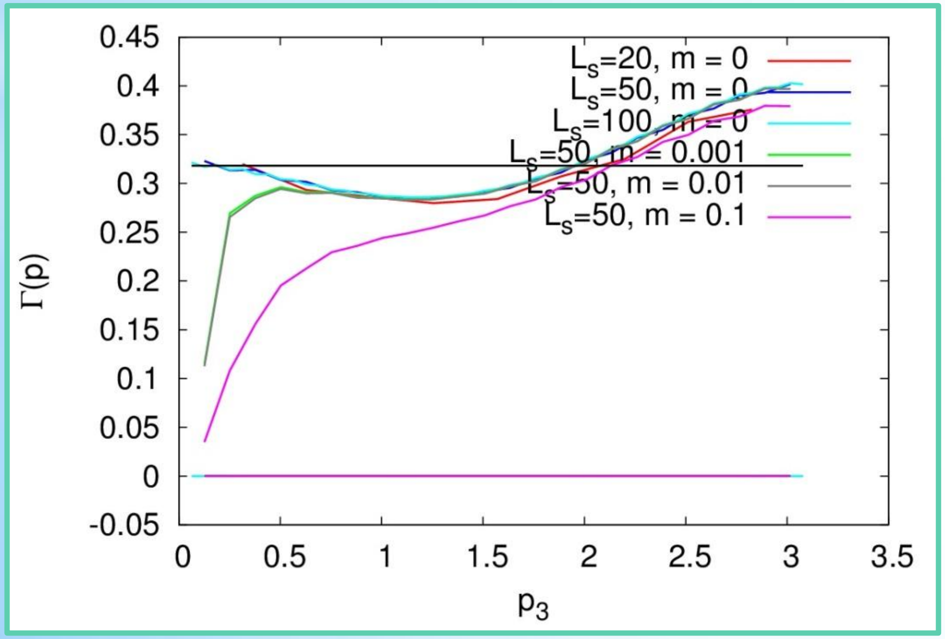
2D axial anomaly: $$\frac{1}{2\pi^2} = \frac{1}{2\pi}\Pi_{\mu\mu}$$

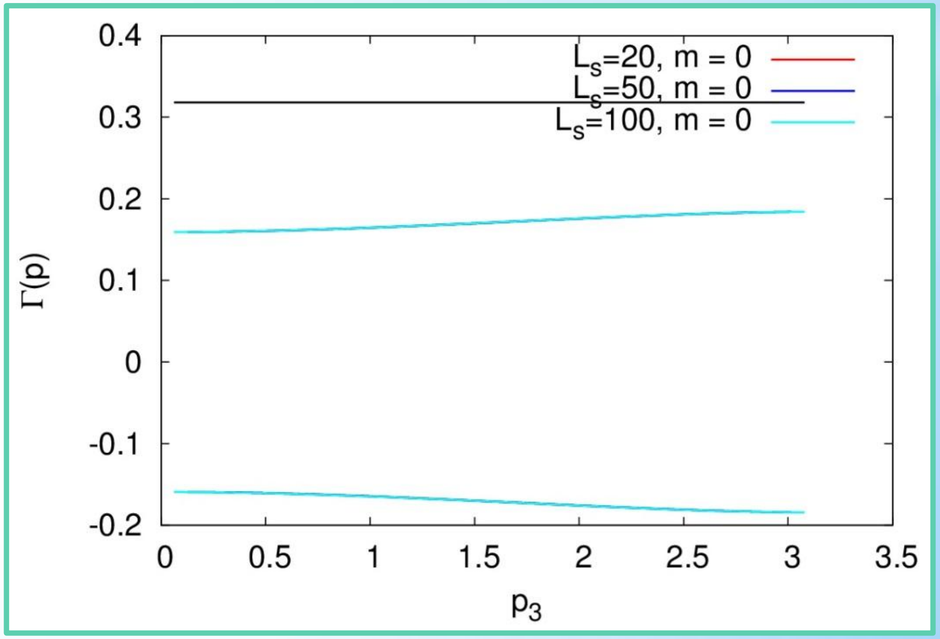
Correct polarization tensor:

$$\frac{\partial^2}{\partial A_{x,\mu}\partial A_{y,\nu}}\det\left(\mathcal{D}_{ov}\right)$$

Naive polarization tensor:

$$\mathrm{Tr}\,\left(\mathcal{D}_{ov}^{-1}\gamma_\mu\mathcal{D}_{ov}^{-1}\gamma_\nu\right)$$



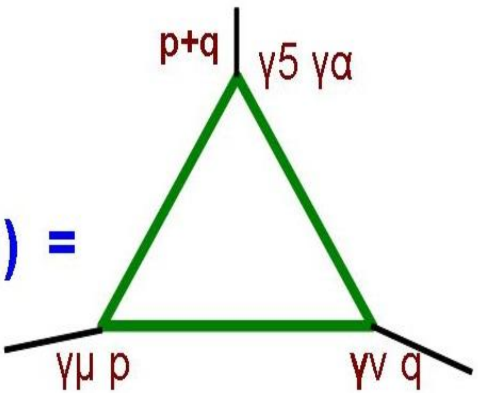


Relation of CME to anomaly

Flow of a massless fermion gas in a classical gauge field and chiral chemical potential

$$j_\mu = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \; \bar{\psi}\gamma_\mu\psi$$
$$\exp\left(-\bar{\psi}\not{D}\psi + A_\nu\bar{\psi}\gamma_\nu\psi + \mu_5\bar{\psi}\gamma_5\gamma_0\psi\right) \sim$$
$$\sim A_\nu\mu_5 \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \; \exp\left(-\bar{\psi}\not{D}\psi\right)$$
$$\color{red}{\bar{\psi}\gamma_\mu\psi \;\; \bar{\psi}\gamma_\nu\psi \;\; \bar{\psi}\gamma_5\gamma_0\psi}$$

$\Gamma^{\mu\nu\alpha}(p,q) =$



In terms of correlators:

$$\sigma^{\mathcal{B}}_{VV} = \lim_{k_z \to 0} \frac{i}{k_z} \langle \, J^x_{\color{red}{V}} \; J^y_{\color{red}{V}} \, \rangle$$

$$\sigma^{\mathcal{B}}_{AV} = \lim_{k_z \to 0} \frac{i}{k_z} \langle \, J^x_{\color{red}{A}} \; J^y_{\color{red}{V}} \, \rangle$$

CME, CVE and axial anomaly

Most general decomposition for VVA correlator [M. Knecht *et al.*, hep-ph/0311100]:

$$\begin{aligned}\mathcal{W}_{\mu\nu\rho}(q_1,q_2) \;=\; -\frac{1}{8\pi^2}\Big\{ & -w_L\left(q_1^2,q_2^2,(q_1+q_2)^2\right)(q_1+q_2)_\rho\,\epsilon_{\mu\nu\alpha\beta}\,q_1^\alpha q_2^\beta \\ & + w_T^{(+)}\left(q_1^2,q_2^2,(q_1+q_2)^2\right)t^{(+)}_{\mu\nu\rho}(q_1,q_2) \\ & + w_T^{(-)}\left(q_1^2,q_2^2,(q_1+q_2)^2\right)t^{(-)}_{\mu\nu\rho}(q_1,q_2) \\ & + \widetilde{w}_T^{(-)}\left(q_1^2,q_2^2,(q_1+q_2)^2\right)\widetilde{t}^{(-)}_{\mu\nu\rho}(q_1,q_2)\Big\},\end{aligned}$$

$$\begin{aligned} t^{(+)}_{\mu\nu\rho}(q_1,q_2) &= q_{1\nu}\,\epsilon_{\mu\rho\alpha\beta}\,q_1^\alpha q_2^\beta - q_{2\mu}\,\epsilon_{\nu\rho\alpha\beta}\,q_1^\alpha q_2^\beta - (q_1\cdot q_2)\,\epsilon_{\mu\nu\rho\alpha}\,(q_1-q_2)^\alpha \\ &\quad + \frac{q_1^2+q_2^2-(q_1+q_2)^2}{(q_1+q_2)^2}\,\epsilon_{\mu\nu\alpha\beta}\,q_1^\alpha q_2^\beta (q_1+q_2)_\rho\,, \\ t^{(-)}_{\mu\nu\rho}(q_1,q_2) &= \left[(q_1-q_2)_\rho - \frac{q_1^2-q_2^2}{(q_1+q_2)^2}(q_1+q_2)_\rho\right]\epsilon_{\mu\nu\alpha\beta}\,q_1^\alpha q_2^\beta \\ \widetilde{t}^{(-)}_{\mu\nu\rho}(q_1,q_2) &= q_{1\nu}\,\epsilon_{\mu\rho\alpha\beta}\,q_1^\alpha q_2^\beta + q_{2\mu}\,\epsilon_{\nu\rho\alpha\beta}\,q_1^\alpha q_2^\beta - (q_1\cdot q_2)\,\epsilon_{\mu\nu\rho\alpha}\,(q_1+q_2)^\alpha\,. \end{aligned}$$

Axial anomaly: $w_L(q_1^2, q_2^2, (q_1+q_2)^2)$

CME ($q_1 = -q_2 = q$): $w_T^{(+)}(q^2, q^2, 0)$

CSE ($q_1=q$, $q_2 = 0$): **IDENTICALLY ZERO!!!**

CME and axial anomaly (continued)

In addition to anomaly non-renormalization, new (perturbative!!!) non-renormalization theorems [M. Knecht *et al.*, hep-ph/0311100] [A. Vainstein, hep-ph/0212231]:

$$\left\{\left[w_T^{(+)} + w_T^{(-)}\right]\left(q_1^2, q_2^2, (q_1+q_2)^2\right) - \left[w_T^{(+)} + w_T^{(-)}\right]\left((q_1+q_2)^2, q_2^2, q_1^2\right)\right\}_{\mathrm{pQCD}} = 0$$

$$\left\{\left[\tilde{w}_T^{(-)} + w_T^{(-)}\right]\left(q_1^2, q_2^2, (q_1+q_2)^2\right) + \left[\tilde{w}_T^{(-)} + w_T^{(-)}\right]\left((q_1+q_2)^2, q_2^2, q_1^2\right)\right\}_{\mathrm{pQCD}} = 0$$

$$\left\{\left[w_T^{(+)} + \tilde{w}_T^{(-)}\right]\left(q_1^2, q_2^2, (q_1+q_2)^2\right) + \left[w_T^{(+)} + \tilde{w}_T^{(-)}\right]\left((q_1+q_2)^2, q_2^2, q_1^2\right)\right\}_{\mathrm{pQCD}} - w_L\left((q_1+q_2)^2, q_2^2, q_1^2\right)$$
$$= -\left\{\frac{2\,(q_2^2 + q_1\cdot q_2)}{q_1^2}\, w_T^{(+)}\left((q_1+q_2)^2, q_2^2, q_1^2\right) - 2\,\frac{q_1\cdot q_2}{q_1^2}\, w_T^{(-)}\left((q_1+q_2)^2, q_2^2, q_1^2\right)\right\}_{\mathrm{pQCD}}$$

Valid only for massless QCD!!!

CME and axial anomaly (continued)

From these relations one can show

$$2w_T^{(+)}\left(q^2, q^2, 0\right) = w_L\left(0, q^2, q^2\right) = \frac{1}{4\pi q^2}$$

And thus CME coefficient is fixed:

$$\Gamma_{\mu\nu 0}\left(q, -q, 0\right) = \frac{1}{2\pi^2}\epsilon_{\mu\nu\sigma 0}\, q^\sigma$$

In terms of correlators:

$$\Gamma_{\mu\nu 0}\left(p, -p\right) = \int\limits_V d^4x_1 d^4x_2 e^{ip(x_1 - x_2)} \langle\, J_\mu\left(x_1\right) J_\nu\left(x_2\right) J_{5\,0}\left(0\right)\,\rangle$$

Naively, one can also use

$$\Gamma_{\mu\nu 0}\left(p, -p\right) = \frac{\partial}{\partial\mu_5}\Pi_{\mu\nu}\left(p\right)$$

Simplifies lattice measurements!!!

CME and axial anomaly (continued)

- CME is related to anomaly (at least) perturbatively in massless QCD
- Probably not the case at nonzero mass
- Nonperturbative contributions could be important (confinement phase)?
- Interesting to test on the lattice
- Relation valid in linear response approximation

➡ Hydrodynamics!!!

Dirac operator with axial gauge fields

First consider coupling to axial gauge field:

$$j_{5\mu} \sim \frac{\partial \mathcal{D}_{ov}[V_\mu, A_\mu]}{\partial A_\mu}$$

Assume local invariance under modified chiral transformations [Kikukawa, Yamada, hep-lat/9808026]:

$$e^{i\gamma_5\theta}\mathcal{D}\left[V_\mu, A_\mu\right]e^{i\gamma_5\theta} = \\ = \mathcal{D}\left[V_\mu, A_\mu + \partial_\mu\theta\right]$$

$$\delta\psi_x = \sum_y \alpha_x \gamma_5 \left(1 - \frac{\mathcal{D}_{ov}}{2}\right)_{xy} \psi_y \quad \delta\bar{\psi}_x = \sum_y \bar{\psi}_y \left(1 - \frac{\mathcal{D}_{ov}}{2}\right)_{xy} \gamma_5 \alpha_y$$

Require $$\delta S = \delta\left(\bar{\psi}\mathcal{D}_{ov}\psi\right) = \sum_x \alpha_x \partial_{x,\mu} j_{5x,\mu}$$

$$\frac{\partial \mathcal{D}_{ov}[V_\mu, A_\mu]}{\partial A_{x,\mu}} = \frac{\partial \mathcal{D}_{ov}[V_\mu, A_\mu]}{\partial V_{x,\mu}} \gamma_5 \left(1 - \mathcal{D}_{ov}\right)$$

(Integrable) equation for *D_{ov}* !!!

Dirac operator with chiral chemical potential

In terms of $\tilde{\mathcal{D}}_{ov} = \frac{2\mathcal{D}_{ov}}{2-\mathcal{D}_{ov}}$ or $G_{ov} = \mathcal{D}_{ov}^{-1}$

$$\frac{\partial \tilde{\mathcal{D}}_{ov}[V_\mu, A_\mu]}{\partial A_{x,\mu}} = \frac{\partial \tilde{\mathcal{D}}_{ov}[V_\mu, A_\mu]}{\partial V_{x,\mu}} \gamma_5$$

$$\frac{\partial G_{ov}[V_\mu, A_\mu]}{\partial A_{x,\mu}} = \frac{\partial G_{ov}[V_\mu, A_\mu]}{\partial V_{x,\mu}} \gamma_5$$

Solution is very similar to continuum:

$$\tilde{\mathcal{D}}_{ov}\left[V_\mu, A_\mu\right] = P_+ \tilde{\mathcal{D}}_{ov}\left[V_\mu + A_\mu\right] P_- + P_- \tilde{\mathcal{D}}_{ov}\left[V_\mu - A_\mu\right] P_+$$

Finally, Dirac operator with chiral chemical potential:

$$\tilde{\mathcal{D}}_{ov}\left(\mu_5\right) = P_+ \tilde{\mathcal{D}}_{ov}\left(\mu = +\mu_5\right) P_- + P_- \tilde{\mathcal{D}}_{ov}\left(\mu = -\mu_5\right) P_+$$

$$\mathcal{D}_{ov}\left(\mu_5\right) = 2\tilde{\mathcal{D}}_{ov}\left(\mu_5\right) / \left(2 + \tilde{\mathcal{D}}_{ov}\left(\mu_5\right)\right)$$

Conserved current for overlap

$$j_\mu(x) = \frac{\partial}{\partial\theta_\mu(x)} \det\left(\mathcal{D}_{ov}\right)$$

Generic expression for the conserved current

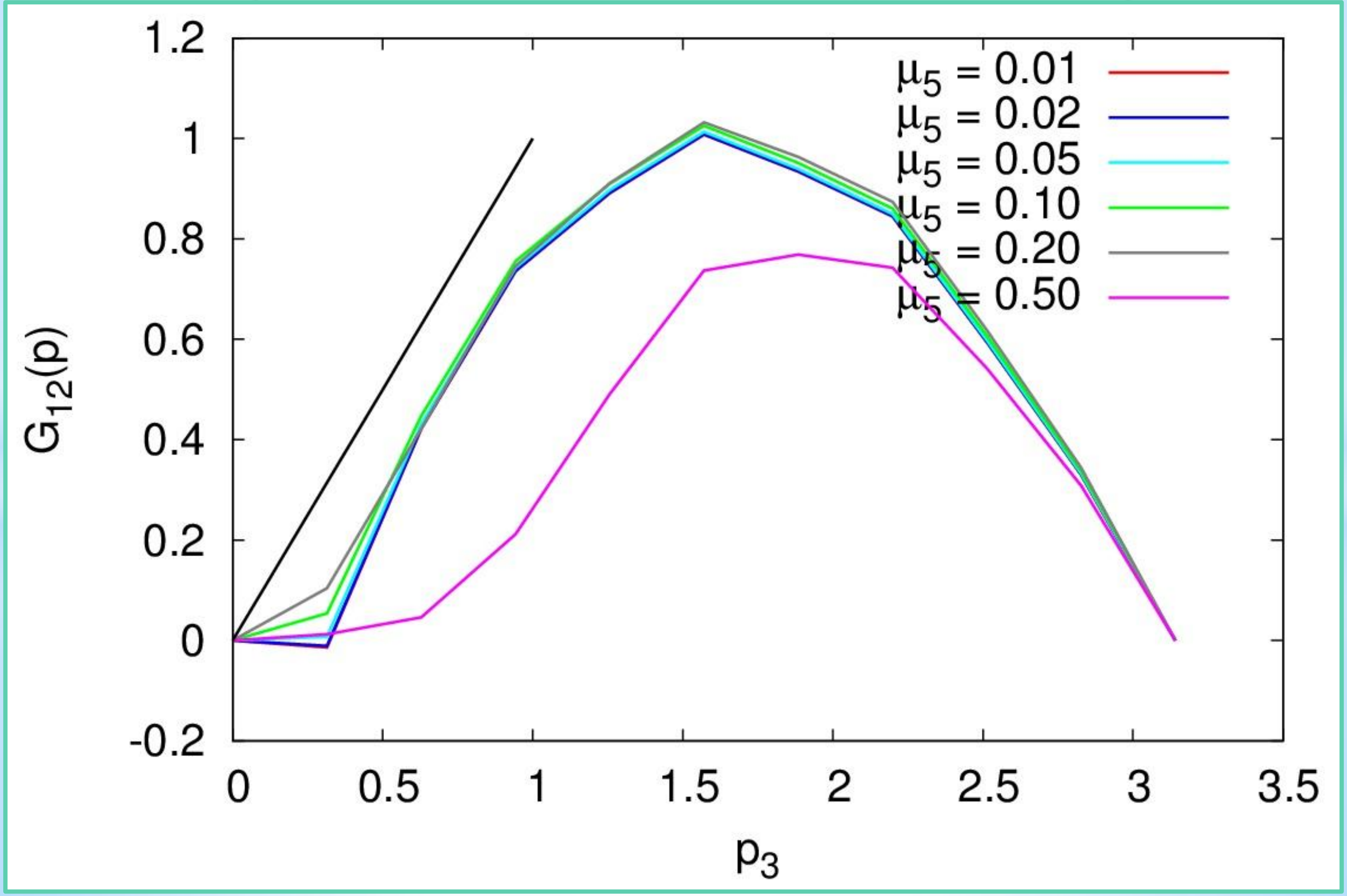
$$\det\left(\mathcal{D}_{ov}\right) = \det\left(\delta_{kl} + \langle L_k|\,\gamma_5\,|R_l\rangle \mathrm{sign}\;\left(\mathrm{Re}\,\lambda_l\right)\right)$$

$$j_\mu(x) = \mathcal{D}^{-1}_{ov\,lk}\left(\frac{\partial\langle L_k|}{\partial\theta_\mu(x)}\gamma_5\,|R_l\rangle + \langle L_k|\,\gamma_5\frac{\partial\,|R_l\rangle}{\partial\theta_\mu(x)}\right)\mathrm{sign}\;\left(\mathrm{Re}\,\lambda_l\right) + \\ +2\langle L_k|\,\gamma_5\,|R_l\rangle\delta\left(\mathrm{Re}\,\lambda_l\right)\frac{\partial\mathrm{Re}\,\lambda_l}{\partial\theta_\mu(x)}$$

$$\partial_\theta\,|R_i\rangle = \sum_{j\neq i}\frac{|R_j\rangle\langle L_j|\,\partial_\theta\hat{A}\,|R_i\rangle}{\lambda_i-\lambda_j} \qquad \partial_\theta\langle L_i| = \sum_{j\neq i}\frac{\langle L_i|\,\partial_\theta\hat{A}\,|R_j\rangle\langle L_j|}{\lambda_i-\lambda_j}$$

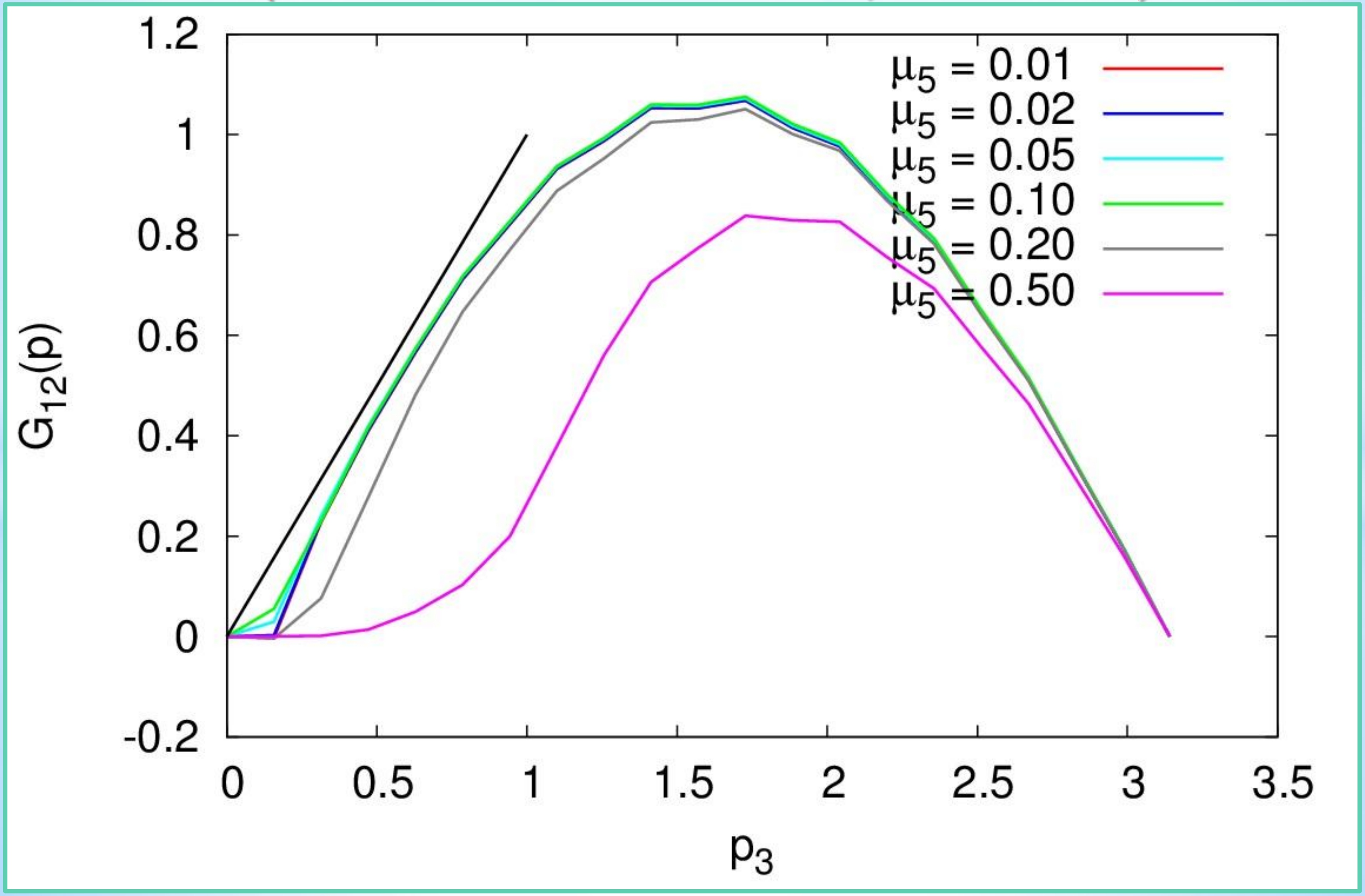
Eigenvalues of D_W in practice never cross zero…

Three-point function with free overlap (conserved current, Ls = 20)



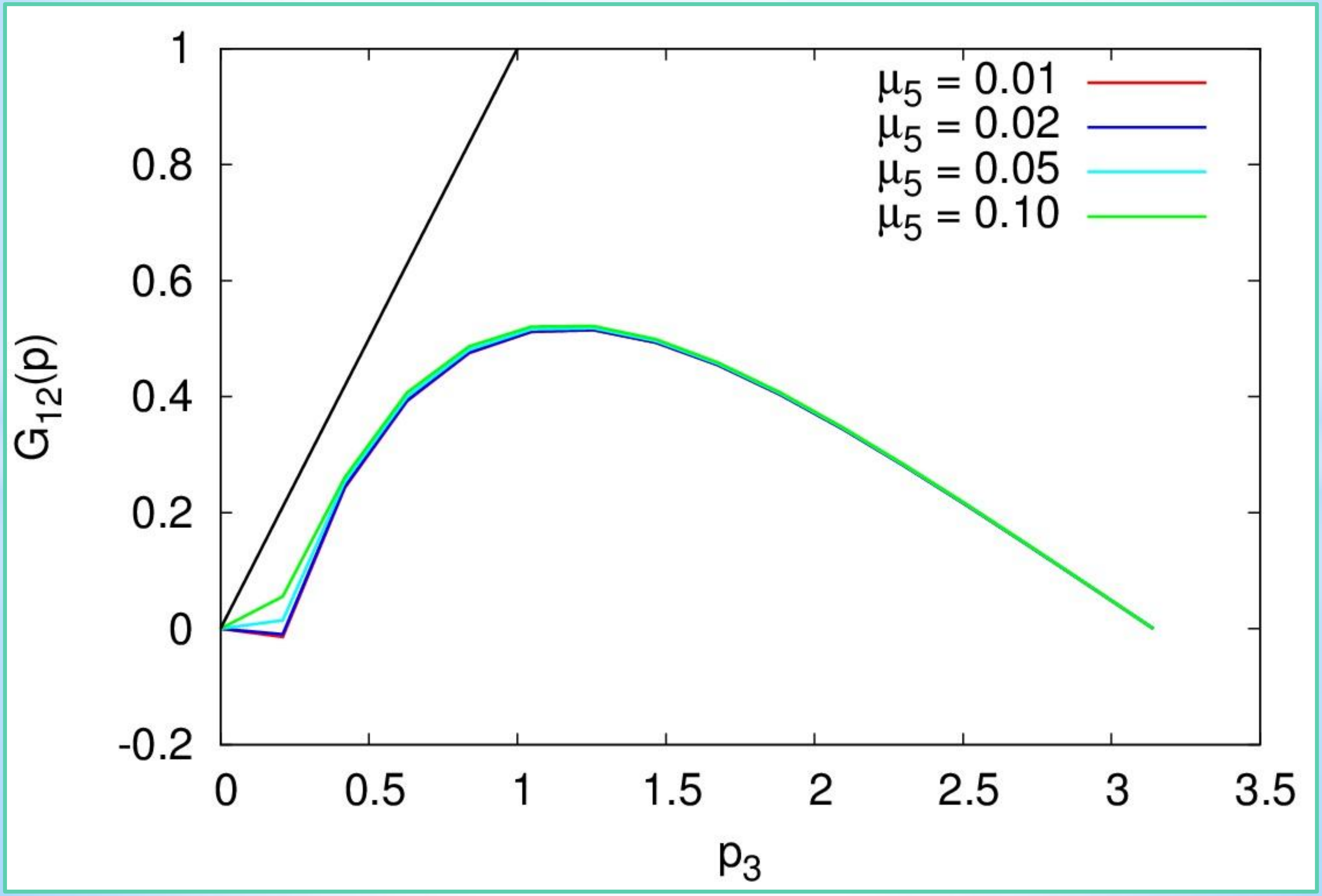
μ_5 is in Dirac-Wilson, still a correct coupling in the IR

Three-point function with free overlap
(conserved current, Ls = 40)

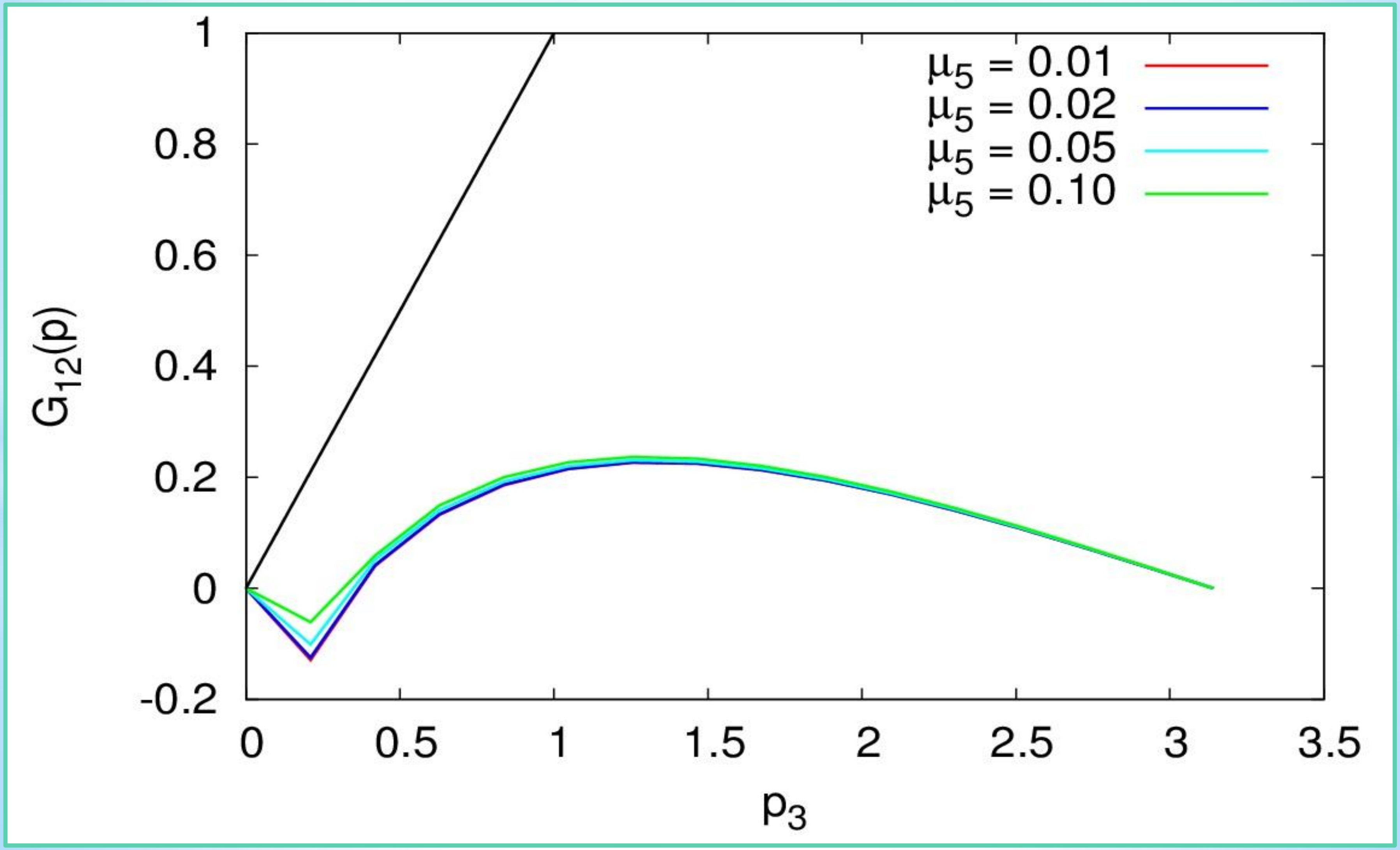


μ_5 is in Dirac-Wilson, still a correct coupling in the IR

Three-point function with massless Wilson-Dirac
(conserved current, Ls = 30)



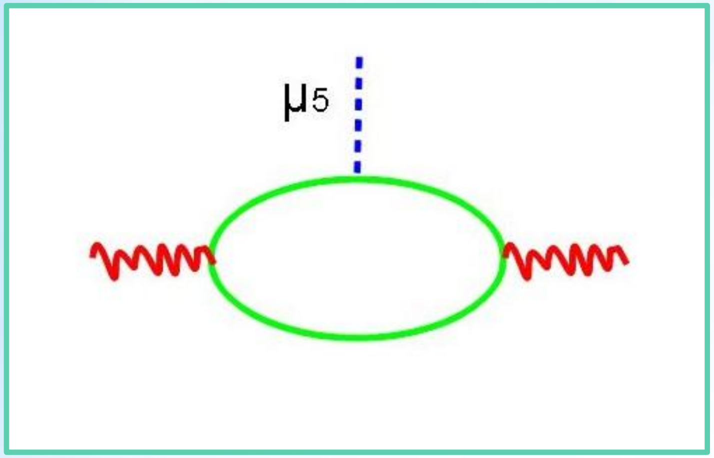
Three-point function with massless overlap
(naive current, Ls = 30)

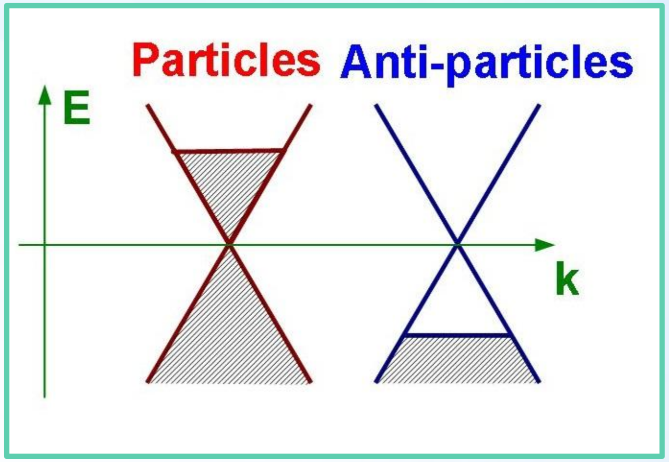


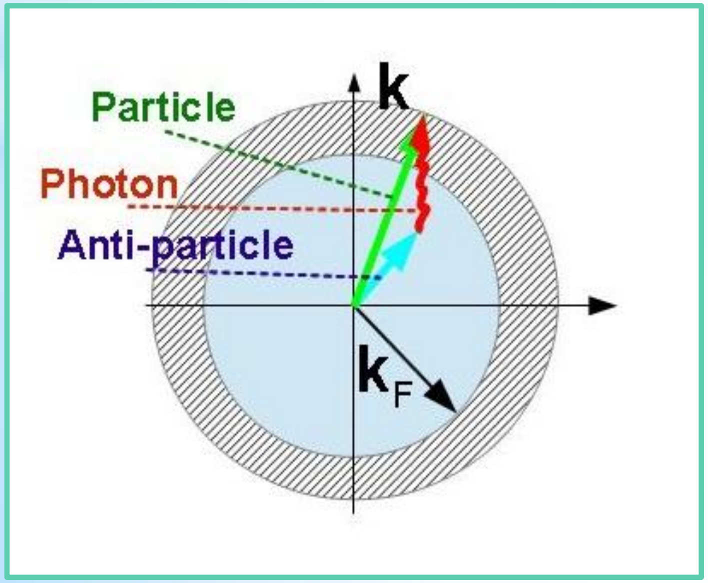
Conserved current is very important!!!

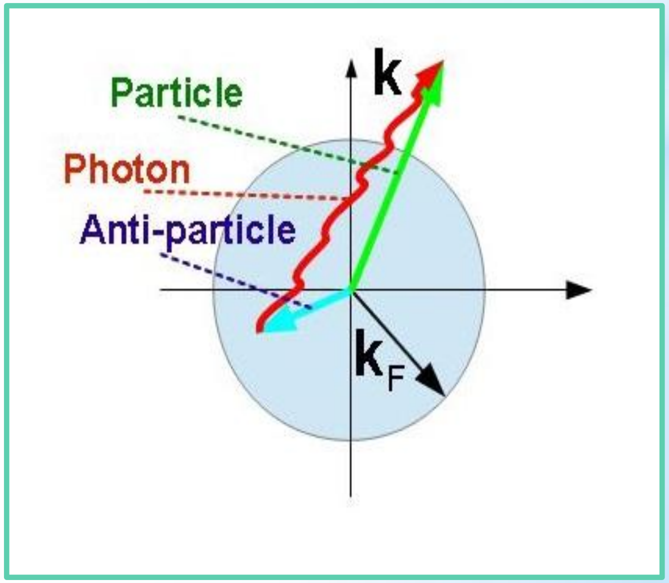
Fermi surface singularity

Almost correct, but what is at small p_3???









Full phase space is available only at $|p|>2|k_F|$

Chiral Vortical Effect

Linear response of currents to "slow" rotation:

$$[g_{\alpha\beta}] = \begin{pmatrix} -\sqrt{1-\frac{r^2\omega^2}{c^2}} & 0 & r^2\omega & 0 \\ 0 & 1 & 0 & 0 \\ r^2\omega & 0 & r^2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\sigma_V^{\mathcal{V}} = \lim_{k_z \to 0} \frac{i}{k_z} \langle J_V^x T^{0y} \rangle$$

$$\sigma_A^{\mathcal{V}} = \lim_{k_z \to 0} \frac{i}{k_z} \langle J_A^x T^{0y} \rangle$$

In terms of correlators

$$j_V = \sigma_V^{\mathcal{V}} w = \frac{N_c e}{2\pi^2} \mu_A \mu_V w$$

Subject to
PT corrections!!!

$$j_A = \sigma_A^{\mathcal{V}} w = N_c e \left(\frac{\mu_V^2 + \mu_A^2}{4\pi^2} + \frac{T^2}{12} \right) w$$

Lattice studies of CVE

A naïve method [Yamamoto, 1303.6292]:

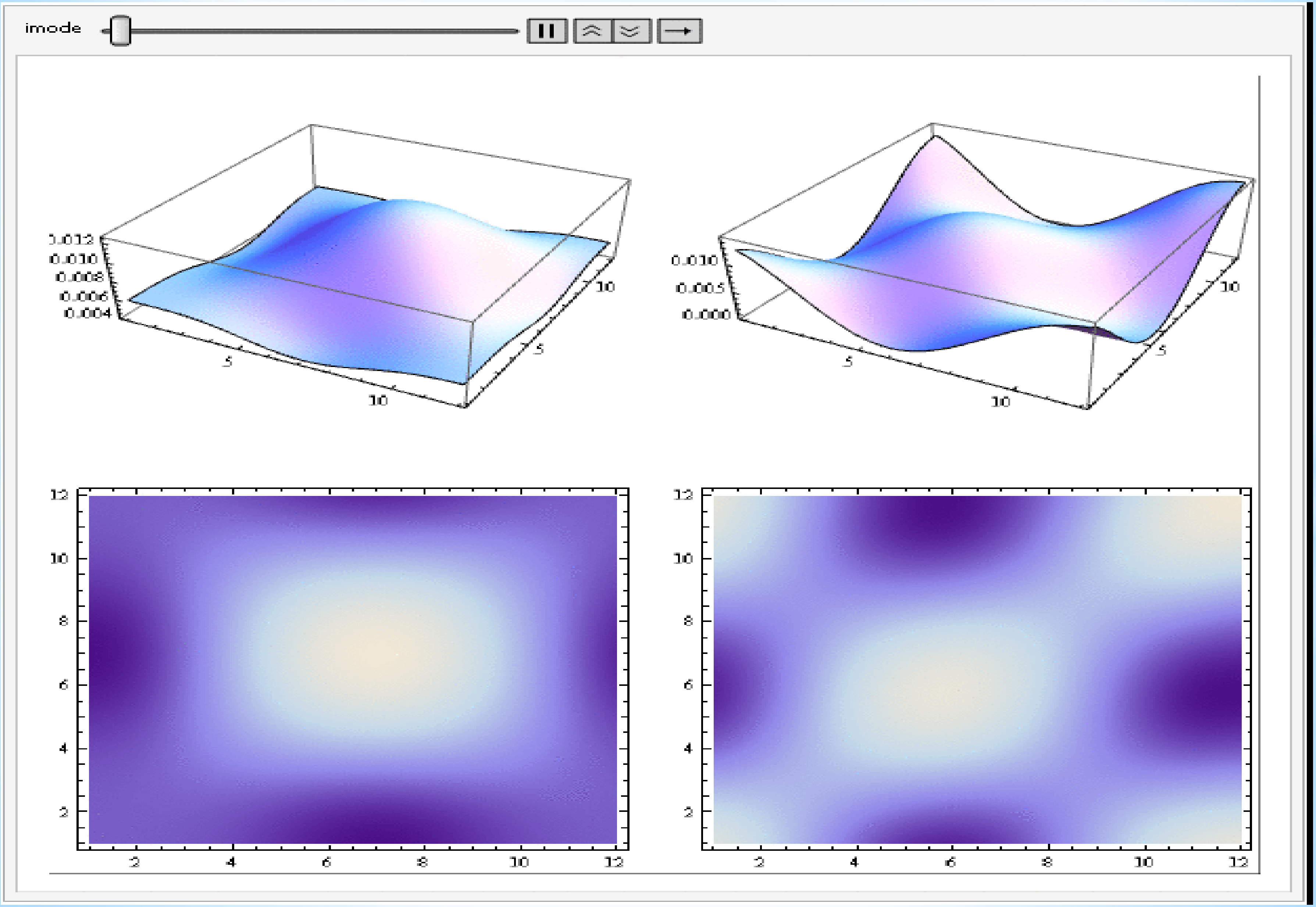
- Analytic continuation of rotating frame metric
- Lattice simulations with distorted lattice
- <u>Physical interpretation is unclear!!!</u>
- By virtue of Hopf theorem:
  only vortex-anti-vortex pairs allowed on torus!!!

More advanced method
[Landsteiner, Chernodub & ITEP Lattice,]:

- Axial magnetic field = source for axial current
- T_{0y} = Energy flow along axial m.f.

Measure energy flow in the background axial magnetic field

Dirac eigenmodes in axial magnetic field



Dirac eigenmodes in axial magnetic field

Landau levels for vector magnetic field:

- Rotational symmetry
- Flux-conserving singularity not visible

Dirac modes in axial magnetic field:

- Rotational symmetry broken
- Wave functions are localized on the boundary (where gauge field is singular)

⬇

“Conservation of complexity”:
Constant axial magnetic field in finite volume is pathological

Conclusions

- Measure spatial correlators + Fourier transform
- External magnetic field: limit $k0 \to 0$ required after $k3 \to 0$, analytic continuation???
- External fields/chemical potential are not compatible with perturbative diagrammatics
- Static field limit not well defined
- Result depends on IR regulators
- Axial magnetic field: does not cure the problems of rotating plasma on a torus

Backup slides

Chemical potential for anomalous charges

Chemical potential for conserved charge (e.g. Q):

$$\hat{H} \to \hat{H} - \mu Q$$

Non-compact gauge transform

$$\Psi(\tau + \beta) = \pm e^{\mu\beta}\,\Psi$$

In the action $\Longleftrightarrow$ **Via boundary conditions**

For anomalous charge:

$$A_\mu \to A_\mu + \partial_\mu \theta$$

General gauge transform

$$S \to S + \int d^D x\, \partial_\mu \theta\, j_\mu = S - \int d^D x\, \theta\, \partial_\mu j_\mu$$

BUT the current is not conserved!!!

$$\partial_\mu j_\mu \sim F\tilde{F} \sim \partial_\mu K^\mu$$

Topological charge density

$$K^\mu = \epsilon^{\mu\nu\alpha\beta} A_\nu \partial_\alpha A_\beta$$

Chern-Simons current