

SUSY Dark Matter Annihilation in the Galactic Halo

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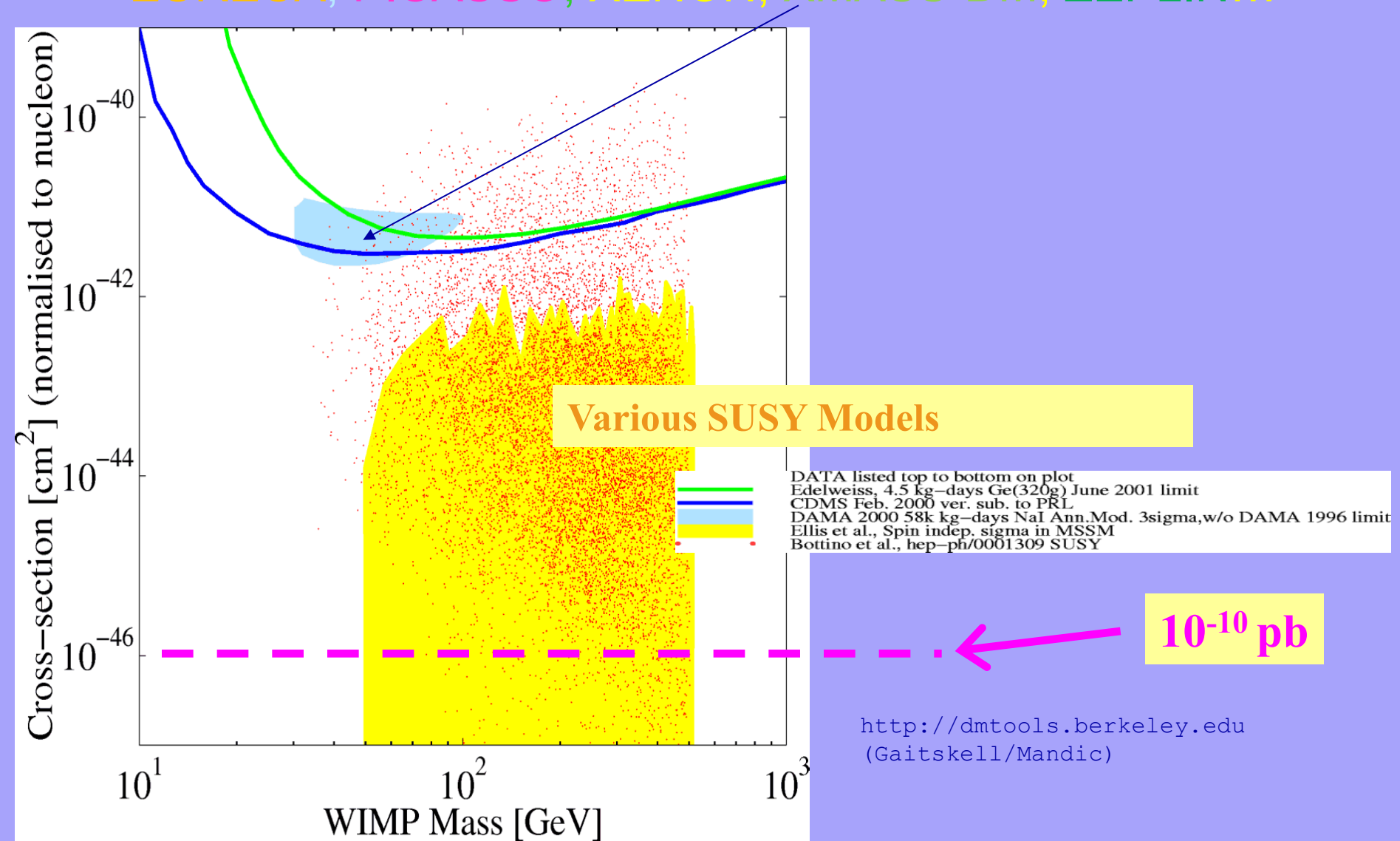
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SUSY neutralino DM detection

- Direct neutralino χ detection
 - ✱ Recoil detectors: DAMA $m_\chi \approx 60$ GeV?, Edelweiss, CDMS, XENON
 - LHC $m_\chi = \mathcal{O}(100)$ GeV?
- Indirect χ detection
 - ✱ $\chi \chi$ annihilation in the galactic haloes
 - ✱ WIMP annihilation in the Sun, Earth, Moon...
 - ...
- ◆ Gamma-rays, positrons, antiprotons, neutrino
 - ✱ Halo structure: DM profile $\rho_{\text{halo}}(r)$ – NFW, Moore, Einasto...
 - ✱ Halo clumpiness: mass fraction of clumps ξ
 - ✱ Minimal mass of clumps M_{min}
 - ✱ Clump distribution function $\xi(M_{\text{cl}}, \rho_{\text{cl}}, r)$

Direct DM detection

Experiments: CDMS, CRESST, CUORE, DAMA, EDELWEISS, EURECA, PICASSO, XENON, XMASS-DM, ZEPLIN...



Neutralino as WIMP

χ is LSP in mSUGRA: MSSM with SUGRA inspired breaking

spin 1/2 Majorana particle: $|\chi\rangle = N_1|B_0\rangle + N_2|W_3\rangle + N_3|H_1\rangle + N_4|H_2\rangle$
 B_0, W_3 – gauginos, H_1, H_2 - higgsinos

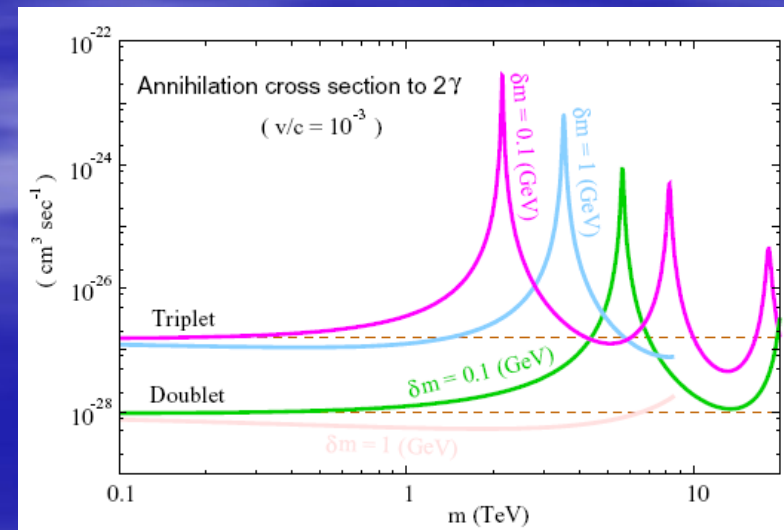
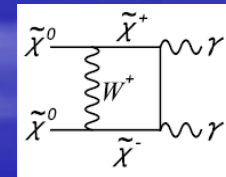
χ is almost pure bino: $(N_1, N_2, N_3, N_4) = (0.95, -0.10, 0.27, -0.09)$

Only 5 parameters:

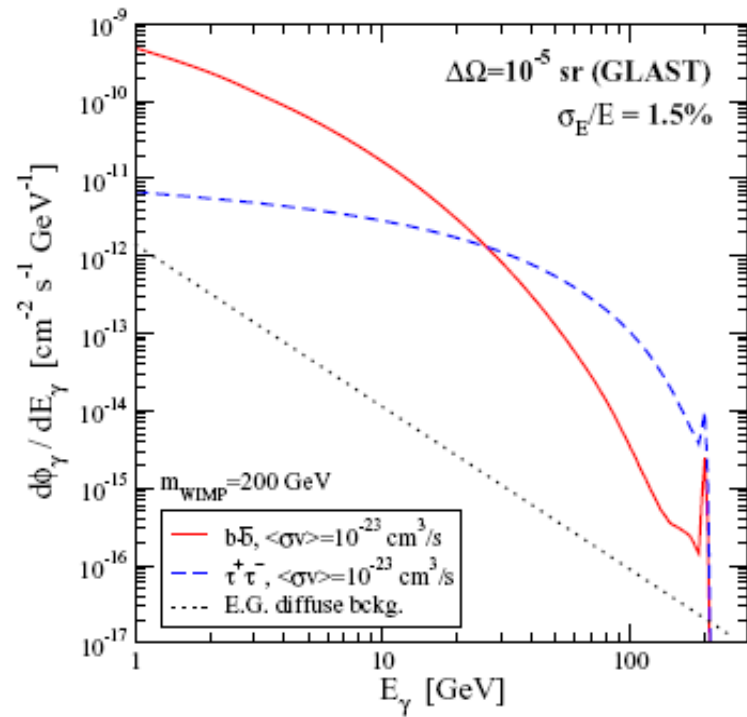
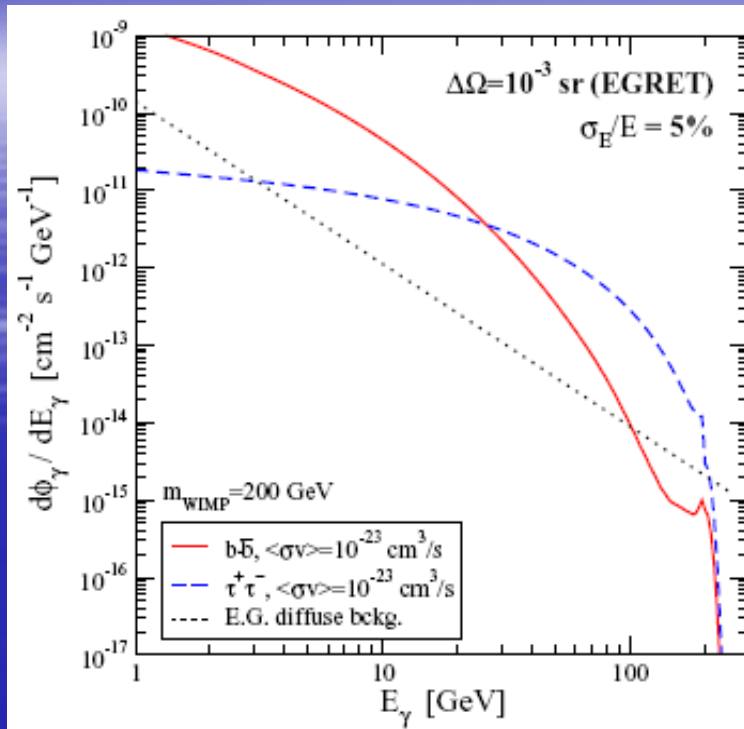
Parameter	Value	Particle	Mass [GeV]
m_0	1500 GeV	$\tilde{\chi}_{1,2,3,4}^0$	64, 113, 194, 229
$m_{1/2}$	170 GeV	$\tilde{\chi}_{1,2}^\pm, \tilde{g}$	110, 230, 516
A_0	$0 \cdot m_0$	$\tilde{u}_{1,2} = \tilde{c}_{1,2}$	1519, 1523
$\tan \beta$	52.2	$\tilde{d}_{1,2} = \tilde{s}_{1,2}$	1522, 1524
$\text{sign } \mu$	+	$\tilde{t}_{1,2}$	906, 1046
		$\tilde{b}_{1,2}$	1039, 1152
$\alpha_s(M_Z)$	0.122	$\tilde{e}_{1,2} = \tilde{\mu}_{1,2}$	1497, 1499
$\alpha_{em}(M_Z)$	0.0078153697	$\tilde{\tau}_{1,2}$	1035, 1288
$1/\alpha_{em}$	127.953	$\tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau$	1495, 1495, 1286
$\sin^2(\theta_W)_{\overline{MS}}$	0.2314	h, H, A, H^\pm	115, 372, 372, 383
m_t	175 GeV	Observable	Value
m_b	4.214 GeV	$Br(b \rightarrow X_s \gamma)$	$3.02 \cdot 10^{-4}$
		Δa_μ	$1.07 \cdot 10^{-9}$
		Ωh^2	0.117

Dominant annihilation into quark pairs

Annihilation to $\gamma\gamma$ is suppressed by a loop factor



Photon spectra of SUSY $\chi\chi$ annihilation



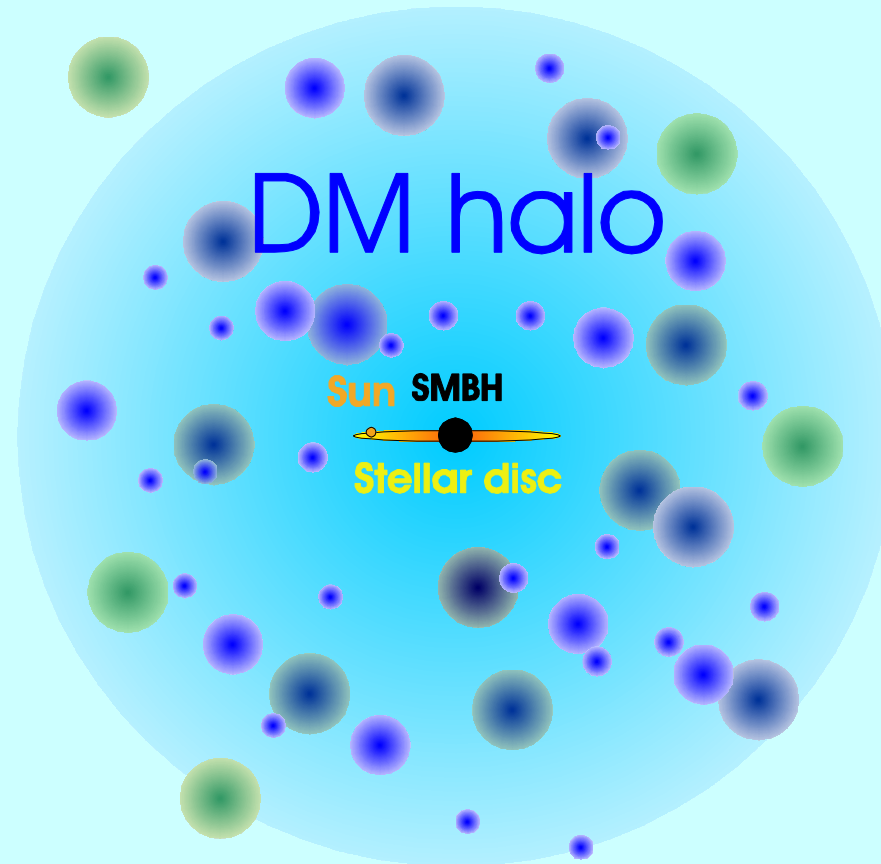
μ	m_1	m_2	m_3	m_A	$m_{\tilde{g}}$	$A_{\tilde{g}_3}$	$\tan\beta$
$30 \div 1200$	$2 \div 1200$	$50 \div 1200$	$m_{\text{LSP}} \div 20000$	$100 \div 10m_{\text{LSP}}$	$(1 \div 10)m_{\text{LSP}}$	$(-3 \div 3)m_{\tilde{g}}$	$1 \div 60$

Table 2: Ranges of the MSSM parameters used to generate the models shown in Figs. 6 and 8. All masses are in GeV, and $m_{\text{LSP}} \equiv \min(\mu, m_1, m_2)$. The quantity $m_{\tilde{g}}$ indicates the following scalar masses (which were independently sampled): $m_{\tilde{Q}_{1,3}}, m_{\tilde{u}_{1,3}}, m_{\tilde{d}_{1,3}}, m_{\tilde{L}_{1,2,3}}, m_{\tilde{e}_{1,2,3}}$. To avoid FCNC constraints, we assumed the squark soft supersymmetry breaking terms of the first two generations to be equal. $A_{\tilde{g}_3}$ stands for the third generation sfermion trilinear terms: those of the first two generations were taken to vanish.

Boosting factor $\eta \sim 100$ is required!

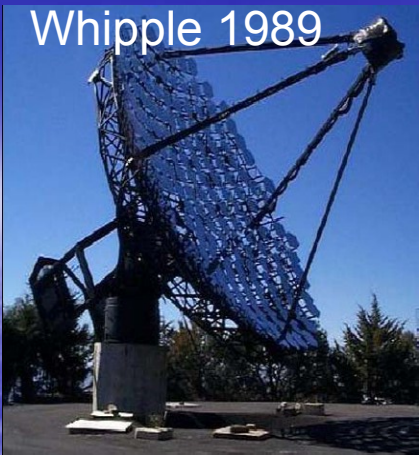
Clumpiness of DM!

Small-scale DM clumps in the Galactic halo



Atmospheric Cherenkov Telescopes

Whipple 1989



CANGAROO III



MAGIC



H.E.S.S. 2002



HEGRA 1997



CELESTE



VERITAS
photomontage



CACTUS Gamma-Ray Excess from Draco

CACTUS: Converted Atmospheric Cherenkov Telescope Using Solar-2

Solar 2 Heliostat Array CACTUS. Barstow, California, effective area $5 \times 10^4 \text{ m}^2$, 144 heliostats, each 42 m^2

Dwarf spheroidal (dSph) galaxies within 100 kpc from the Milky Way center:

Carina, Draco, Ursa Minor and Sextans

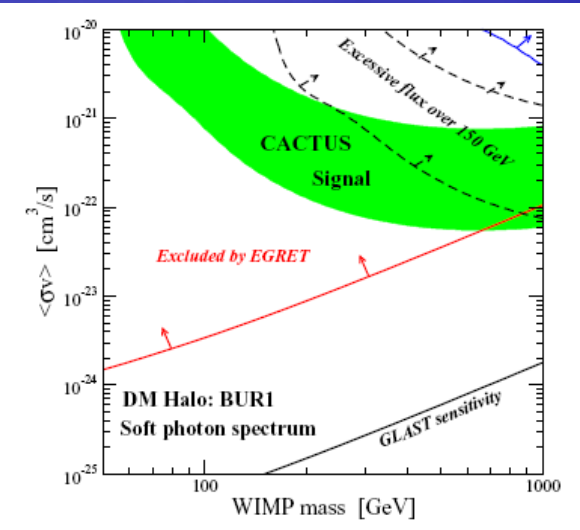
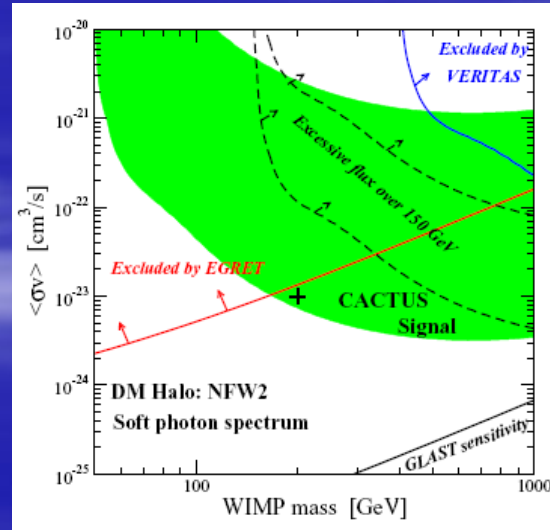
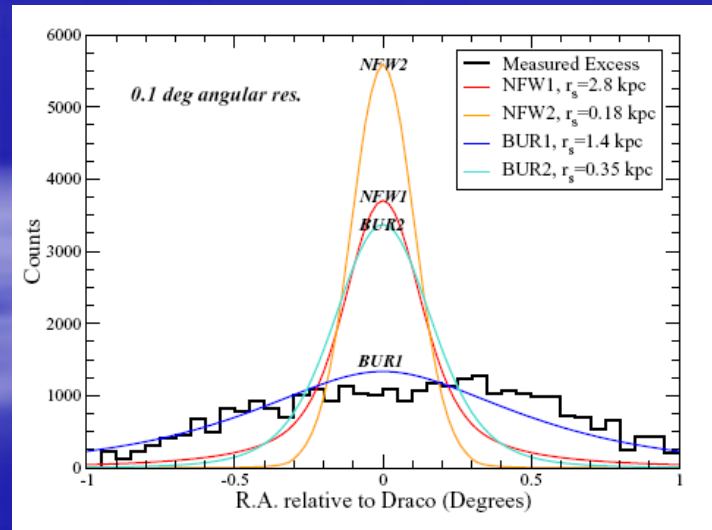
Boosting factor ~ 40 insufficient for detection

Draco heliocentric distance $75.8 \pm 0.7 \pm 5.4 \text{ kpc}$, $R=0.5 \text{ kpc}$

Threshold: 50 GeV

Background: γ -like hadronic EASs, CR electrons, diffuse extragalactic background $>50 \text{ GeV}$

Excess $\sim 20\%$ of the background $>50 \text{ GeV}$

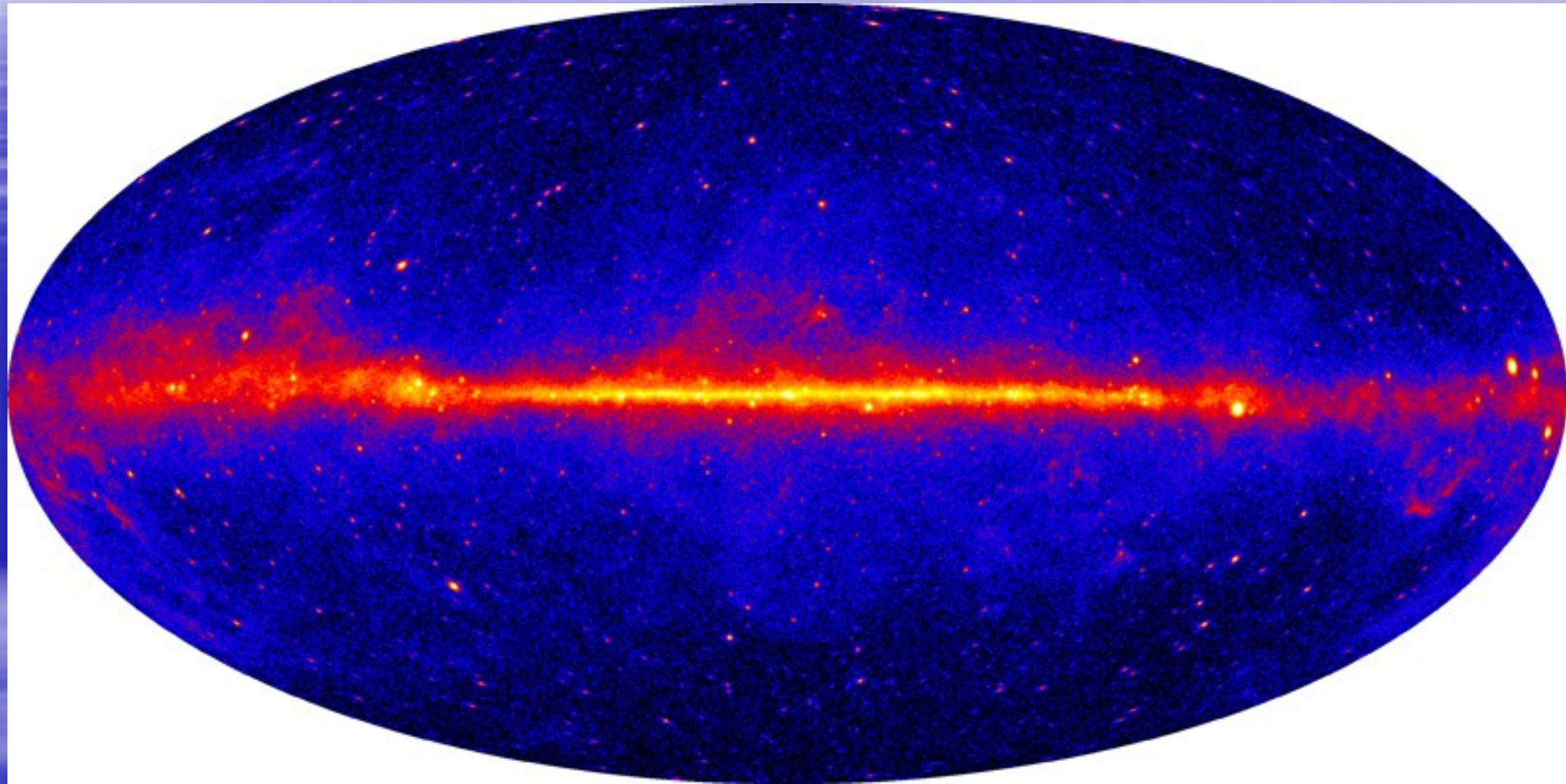


angular resolution $\Delta\theta=1^\circ?$

Profumo & Kamionkowski, 2006

Opportunity for Fermi (formerly GLAST)?

All-sky view from Fermi



<http://fermi.gsfc.nasa.gov/ssc/>



Project Columbia supercomputer (NASA)

The image shows a dense, interconnected network of particles from the Millennium Simulation. The particles are represented as small dots, with a color gradient from dark purple to bright yellow. The network forms a complex, web-like structure with many small loops and branches. A horizontal scale bar is located at the top left, consisting of a white line with vertical end caps, labeled "1 Gpc/h".

1 Gpc/h

Millennium Simulation

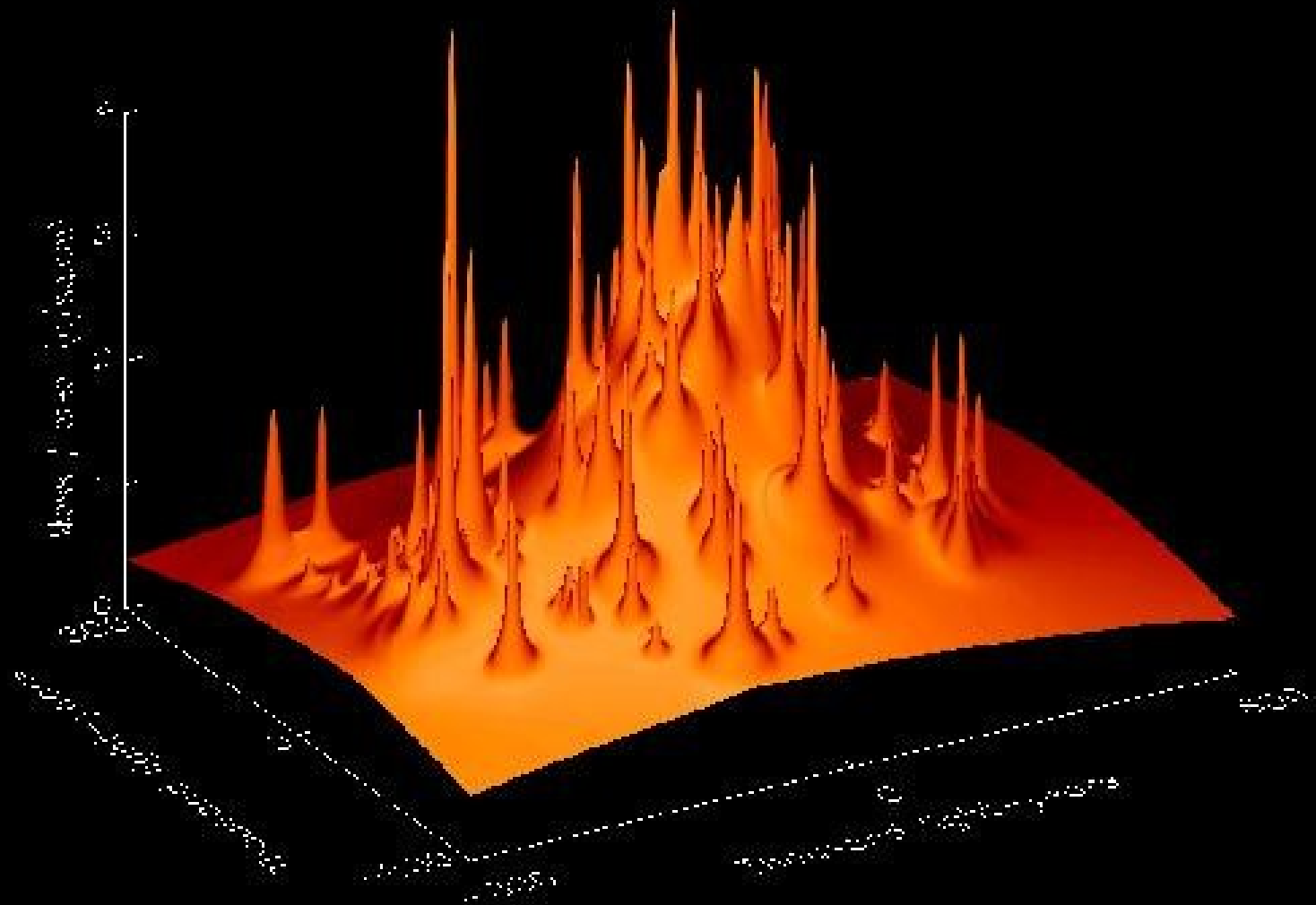
10,077,696,000 particles

($z = 0$)

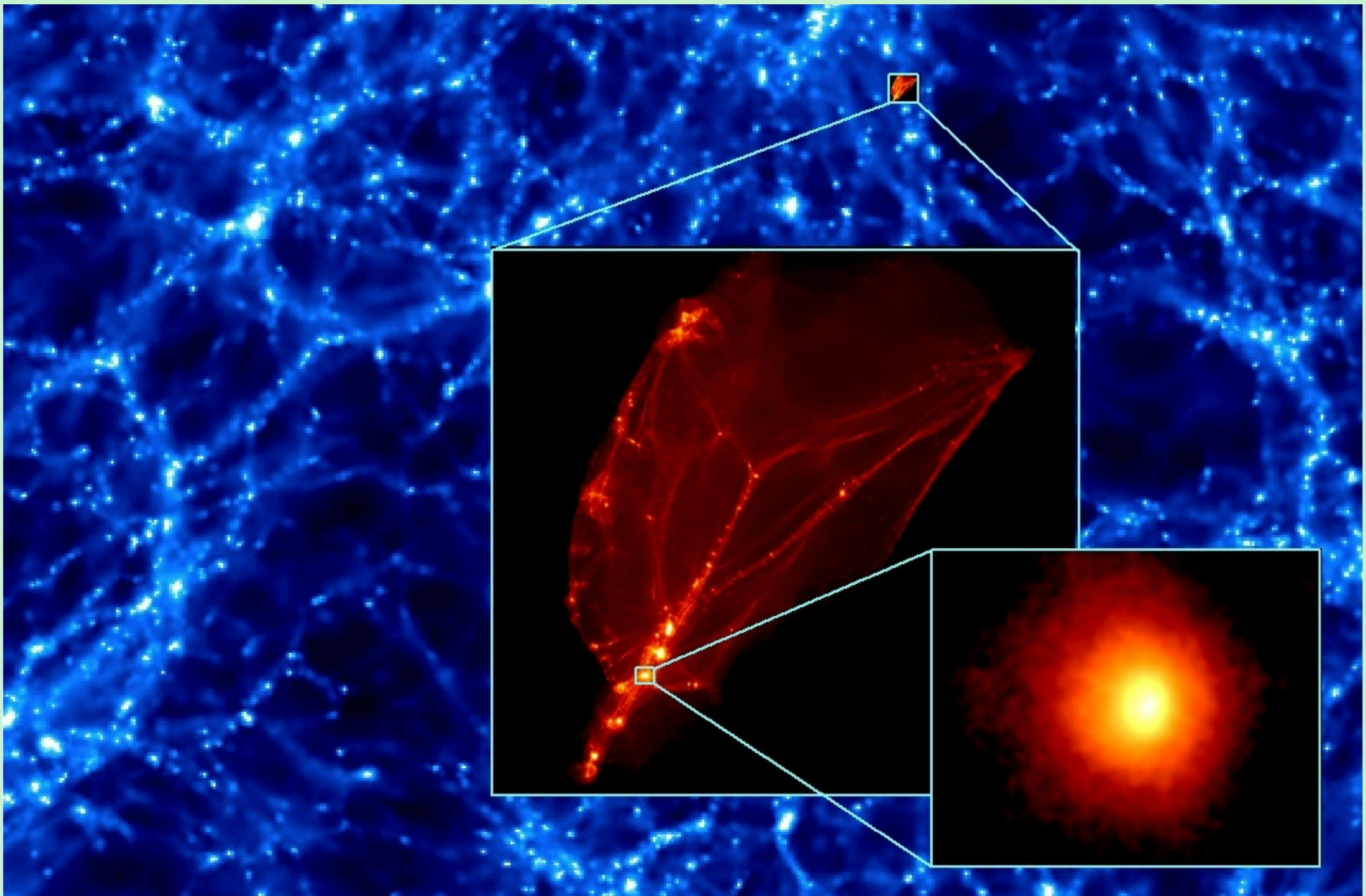
$z=0.0$

80 kpc

The image displays a simulated galaxy at redshift $z=0.0$. It features a vast field of stars, with a significant concentration in the center, forming a bright, diffuse core. The stars are represented as small, glowing points of light, with some larger and brighter than others, suggesting a distribution of stellar masses. The overall color palette is dominated by warm, golden-yellow and orange tones, typical of a stellar population. In the bottom-left corner, a white horizontal scale bar is shown, labeled "80 kpc", indicating the physical size of the simulated system.



Mass reconstruction of the cluster. Note the large, smooth distribution of (apparently invisible) matter.



3 kpc

60 pc

0.024 pc

$N=62 \cdot 10^6$, $m=1.2 \cdot 10^{-10} M_{\odot}$ $z=350 \rightarrow z=26$, grav. softening of 10^{-2} pc

★ Clumps of Minimal Masses

$$M_{\min} \sim 10^{-12} M_{\odot}$$

Zybin Vysotsky Gurevich 99

$$M_{\min} \sim (10^{-7} - 10^{-6}) M_{\odot}$$

Schwarz Hofmann Stocker 01

Kinetic decoupling

$$\frac{1}{\tau_{rel}} \simeq H(t), \quad n_0(\omega) = \frac{1}{2\pi^2} \frac{\omega^2}{e^{\omega/T} + 1}$$

Energy relaxation time

$$\frac{1}{\tau_{rel}} = \frac{1}{E_k} \frac{dE_k}{dt} = \frac{40}{2E_k m_{\chi}} \int d\Omega \int d\omega n_0(\omega) \left(\frac{d\sigma_{el}}{d\Omega} \right)_{fL\chi} (\delta p)^2$$

$$t_d = 3 \cdot 10^{-5} \left(\frac{m_{\chi}}{100 \text{ GeV}} \right)^{-1/2} \left(\frac{\tilde{M}}{1 \text{ TeV}} \right)^{-2} \left(\frac{g_*}{10} \right)^{-3/4} \text{ s}$$

$$T_d = 150 \left(\frac{m_{\chi}}{100 \text{ GeV}} \right)^{1/4} \left(\frac{\tilde{M}}{1 \text{ TeV}} \right) \left(\frac{g_*}{10} \right)^{1/8} \text{ MeV}$$

★ Diffusion cutoff of the mass spectrum

Diffusion equation

$$\frac{\partial \delta(\vec{x}, t)}{\partial t} = \frac{D(t)}{a^2(t)} \Delta_{\vec{x}} \delta(\vec{x}, t)$$

$$\delta_{\vec{k}}(t) = \delta_{\vec{k}}(t_f) \exp \left\{ -k^2 C \tilde{M}^4 \left(t^{5/2} - t_f^{5/2} \right) \right\}$$

$$M_D = \frac{4\pi}{3} \rho_{\chi}(t_d) \lambda_D^3(t_d) = 10^{-13} \left(\frac{m_{\chi}}{100 \text{ GeV}} \right)^{-15/8} \left(\frac{\tilde{M}}{1 \text{ TeV}} \right)^{-3/2} \left(\frac{g_*}{10} \right)^{-15/16} M_{\odot}$$

★ Free streaming cutoff of the mass spectrum

$$\vec{x} = \vec{f}(\vec{q}, \vec{v}_d, t) = \vec{q} + \int_{t_d}^t \frac{\vec{v}(t') dt'}{a(t')} = \vec{q} + g(t) \vec{v}_d$$

$$\begin{aligned} n(\vec{x}, t) &= \int d^3 v_d \phi(\vec{v}_d) \sum_{\vec{q}_*} n(\vec{q}_*, t_d) \left| \frac{D\vec{f}}{D\vec{q}} \right|_{\vec{q}=\vec{q}_*} \\ &= \int d^3 v_d \phi(\vec{v}_d) \int d^3 q n(\vec{q}, t_d) \delta^{(3)}(\vec{x} - \vec{f}(\vec{q}, \vec{v}_d, t)) \end{aligned}$$

$$n_{\vec{k}}(t) = n_{\vec{k}}(t_d) e^{-\frac{1}{2} k^2 g^2(t) \frac{T_d}{m\chi}}, \quad g(t) = a(t_d) \int_{t_d}^t \frac{dt'}{a^2(t')}$$

★ Minimal mass of the clump

$$M_{\min} = \frac{\pi^{1/4}}{2^{19/4} 3^{1/4}} \frac{\rho_{\text{eq}}^{1/4} t_d^{3/2}}{G^{3/4}} \left(\frac{T_d}{m_\chi} \right)^{3/2} \ln^3 \left\{ \frac{24}{\pi G \rho_{\text{eq}} t_d^2} \right\}$$
$$= 10^{-8} \left(\frac{m_\chi}{100 \text{ GeV}} \right)^{-15/8} \left(\frac{\tilde{M}}{1 \text{ TeV}} \right)^{-3/2} \left(\frac{g_*}{10} \right)^{-15/16} M_\odot$$

★ **Density perturbation spectrum** $\delta(\vec{r}) = (\rho(\vec{r}) - \bar{\rho}) / \bar{\rho}$

Power spectrum $P(k)$:

$$\langle \delta_{\vec{k}}^* \delta_{\vec{k}'} \rangle = (2\pi)^3 P(k) \delta_D^{(3)}(\vec{k} - \vec{k}'), \quad \delta_{\vec{k}} = \int \delta(\vec{r}) e^{i\vec{k}\vec{r}} d^3r$$

Transfer function $T(k)$ $P_{\text{eq}}(k) = P_p(k) T^2(k)$

Moments of spectrum $P(k)$

$$\sigma_{(j)}^2 = \frac{1}{2\pi^2} \int_0^\infty k^2 dk P(k) k^{2j}, \quad \langle T_{ij} T_{ji} \rangle = \frac{2}{3} s_{(2)}^2 = \frac{2}{3} (4\pi)^2 G^2 \bar{\rho}^2 \sigma_{(0)}^2$$

Power-law spectrum $P_{\text{eq}}(k) \propto k^n$, $\sigma_{\text{eq}}(M) \propto M^{-(n+3)/6}$

Effective power-law index $n = -3 - 6 \frac{\partial \ln \sigma_{\text{eq}}(M)}{\partial \ln M}$, $n_p \approx 1$, $n_p \approx 1 \pm 0.1$

$$\sigma_{\text{eq}}(M) \simeq \frac{2 \cdot 10^{-4}}{\sqrt{f_s(\Omega_\Lambda)}} \left[\ln \left(\frac{k}{k_{\text{eq}}} \right) \right]^{3/2} \left(\frac{k}{k_{h0}} \right)^{(n_p-1)/2}$$

★ Core of a Dark Matter Clump

$$\phi(\vec{r}, t) = \phi_0 + \left. \frac{\partial \phi}{\partial r^i} \right|_0 r^i + \frac{1}{6} \Phi_{ll}|_0 \delta_{ij} r^i r^j + \frac{1}{2} T_{ij}|_0 r^i r^j + \dots$$

$$\Phi_{ij} = \frac{\partial^2 \phi(\vec{r})}{\partial r^i \partial r^j}, \quad T_{ij} = \Phi_{ij} - \frac{1}{3} \Phi_{ll} \delta_{ij}$$

Peak height

$$\nu = \delta_{\text{eq}} / \sigma_{\text{eq}}(M)$$

Tidal velocity

$$\frac{dv_{tid,i}}{dt} = -T_{ij}(t)r^j$$

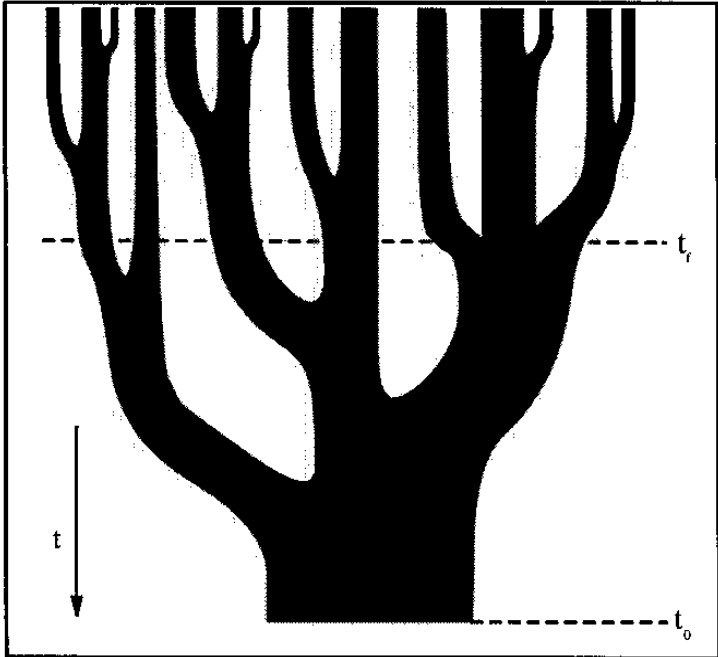
$$\Delta E \simeq \Delta V$$

$$\Delta E \simeq \int d^3r \rho_{\text{int}}(t_s) v_{tid}^2(t_s) / 2, \quad \Delta V \simeq \frac{GM M_c}{R}$$

Clump core radius

$$\frac{R_c}{R} \simeq \frac{\pi 2^{5/3} 3^{13/3}}{5^3} G \rho_{\text{eq}} t_{\text{eq}}^2 \frac{f^2}{\nu^2}(\delta_{\text{eq}}) \simeq 0.3 \nu^{-2} f^2(\delta_{\text{eq}})$$

Press-Schechter formalism:



'Merger tree'

Mass function of unconfined clumps

$$\xi_{\text{PS}}(t) \frac{dM}{M} = \frac{2\delta_c}{\sqrt{2\pi}\sigma_{\text{eq}}^2 D(t)} \frac{d\sigma_{\text{eq}}}{dM} \exp\left[-\frac{\delta_c^2}{2\sigma_{\text{eq}}^2 D^2(t)}\right] dM,$$

$$\delta_c = 3(12\pi)^{2/3}/20 \approx 1.686$$

HIERARHICAL STRUCTURING

Press & Schechter 1974
Lacey & Cole 1993

★ Tidal destruction of clumps in hierarchical model

Clump destruction at $\Delta E \geq |E| \sim GM^2/2R, \quad \delta(M, t_f) = \delta_c$

Number density of unconfined (free) clumps

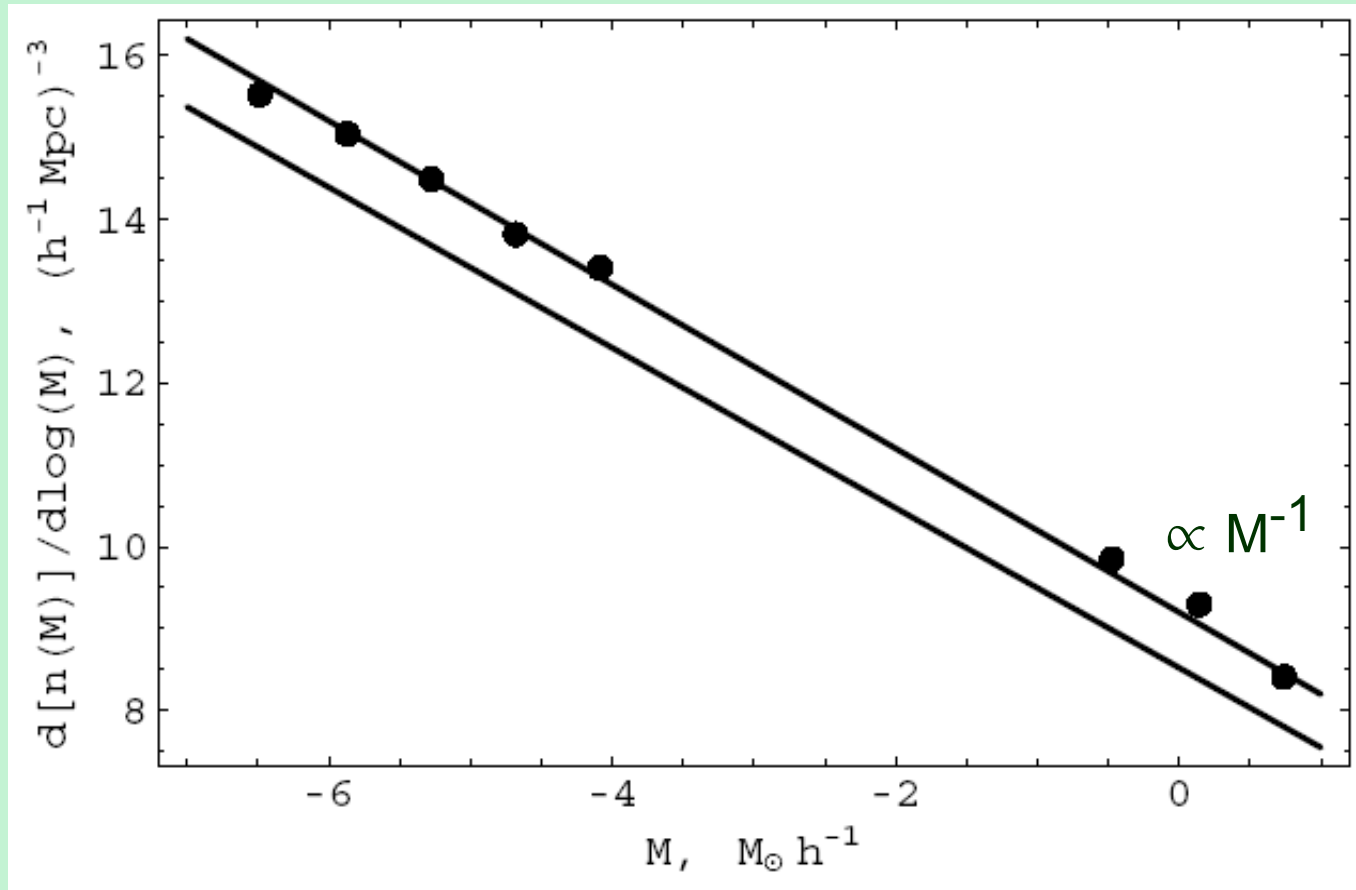
$$\phi_{PS} = \left(\frac{2}{\pi}\right)^{1/2} \frac{\rho}{M D_g(t) \sigma_{eq}^2} \frac{d\sigma_{eq}}{dM} \exp\left[\frac{-\delta_c^2}{2D_g(t)^2 \sigma_{eq}^2}\right] dM$$

Press, Schechter, 1974

Energy increase during one fly-by

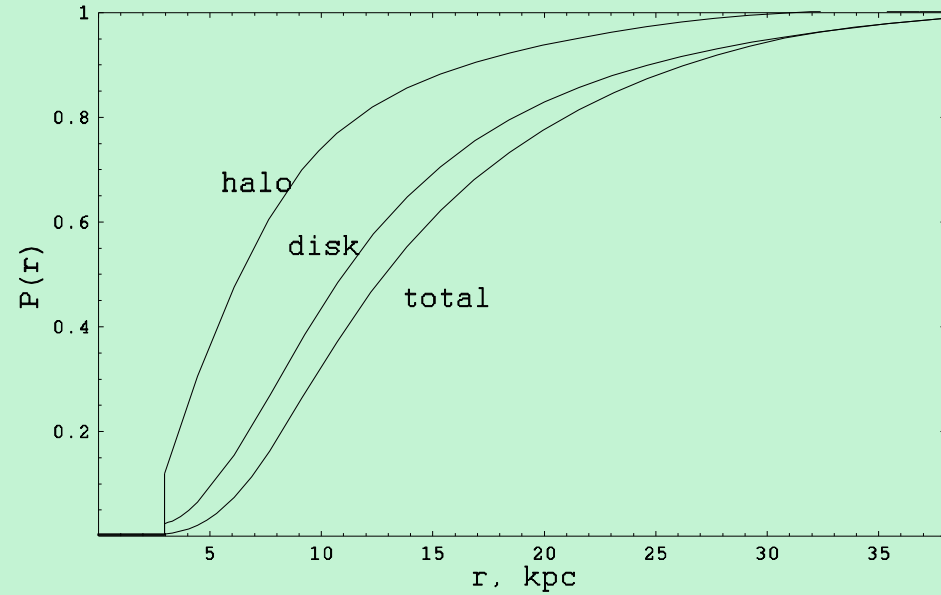
$$\Delta E = \frac{1}{2} \int d^3r \rho(r) (v_x - \tilde{v}_x)^2$$

Mass function of small-scale DM clumps

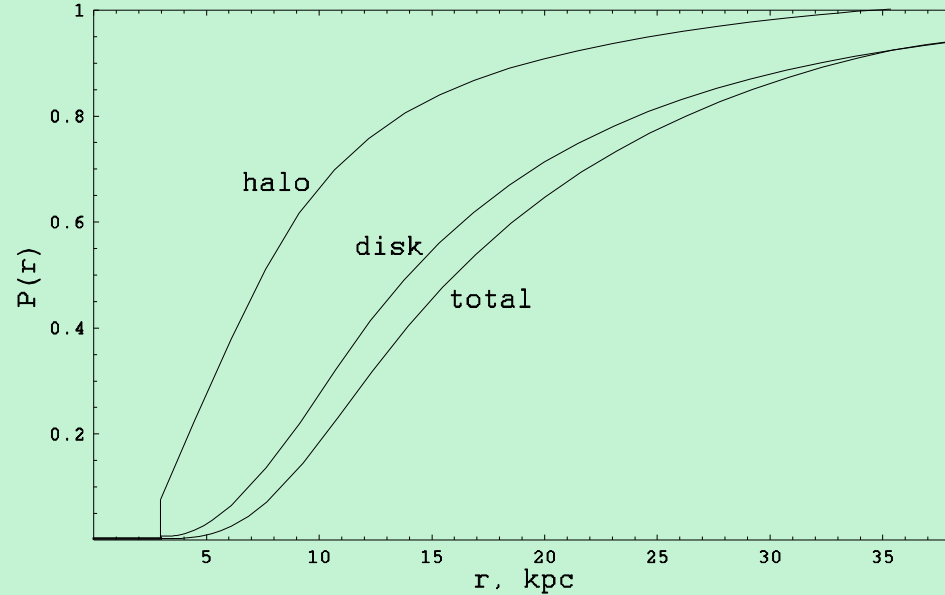


Lower line – model calculation **Berezinsky, Dokuchaev & Eroshenko 2003**
Dots - numerical simulations **Diemand, Moore & Stadel 2005**

Fraction of survived clumps



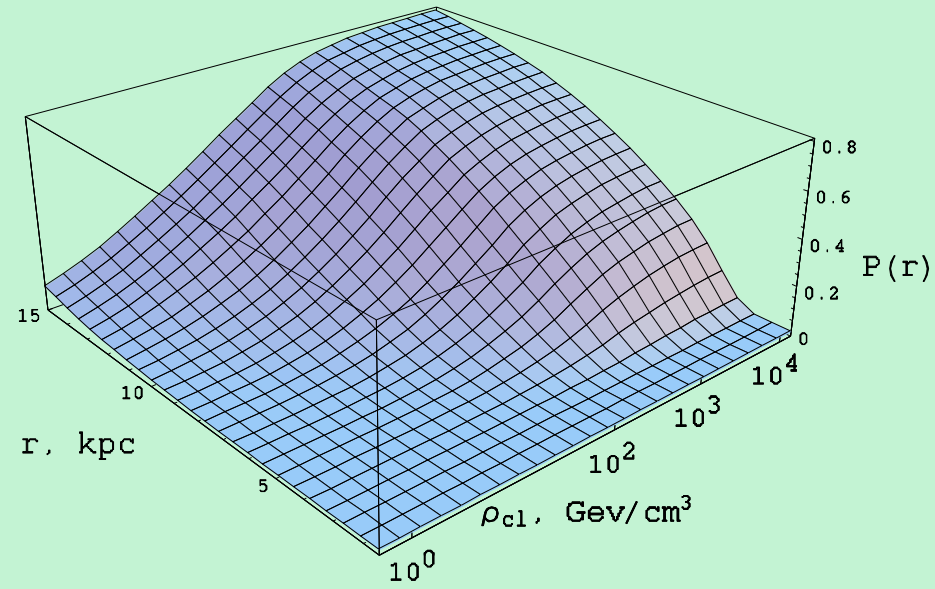
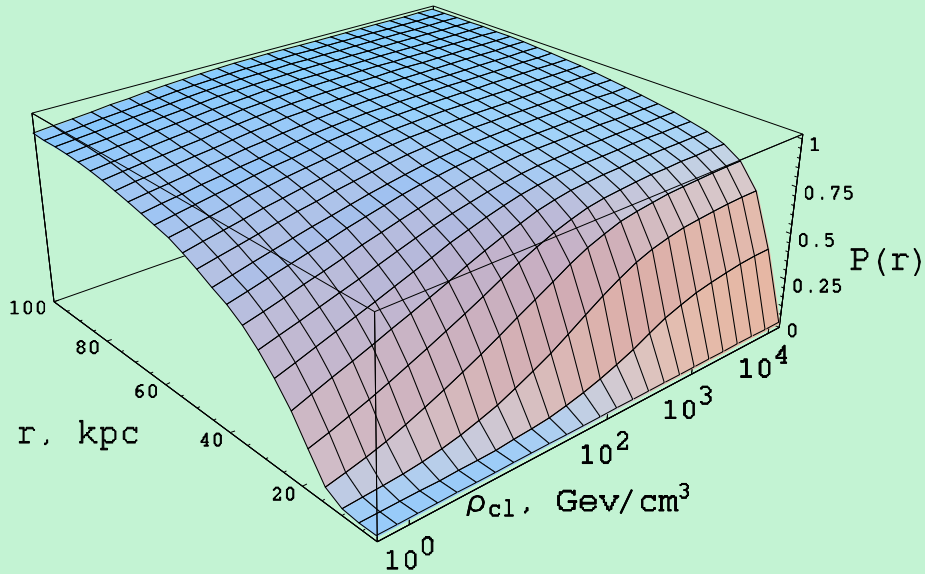
$$M_{\text{cl}} = 10^{-8} M_{\odot}$$



$$M_{\text{cl}} = 10^{-3} M_{\odot}$$

Tidal destruction of clumps in the Galactic bulge, disc, halo and the resulting total fraction of survived clumps with $M_{\text{cl}} = 10^{-8} M_{\odot}$ and $10^{-3} M_{\odot}$

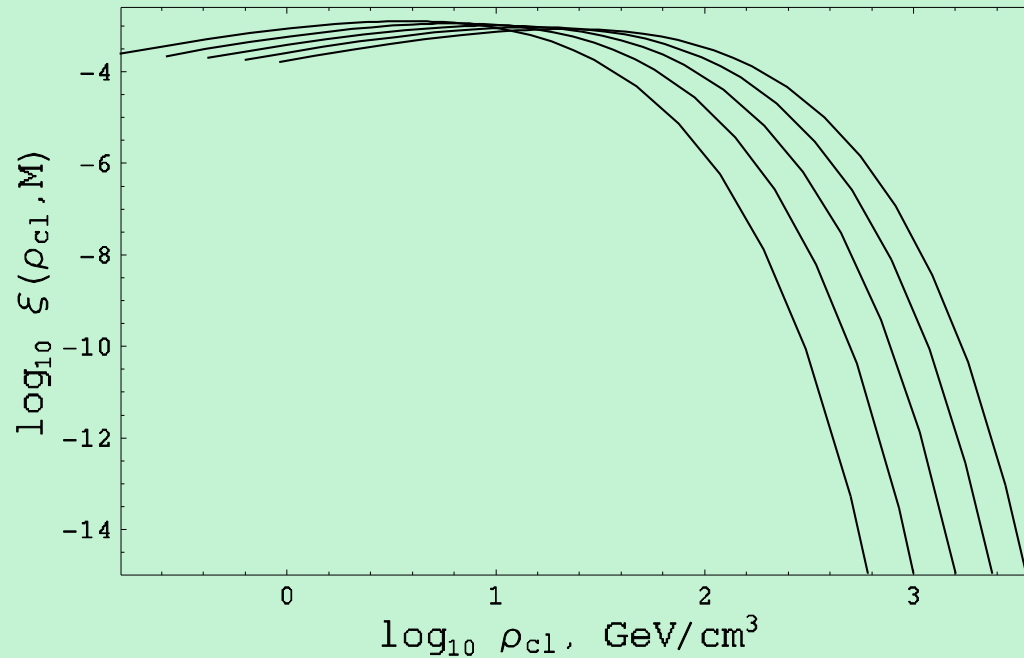
Survived fraction of clumps $P(r)$ in the Galaxy



Radial galactocentric distance r in kpc

Mean internal density of clump ρ_{cl} in GeV/cm^3

Clump distribution function



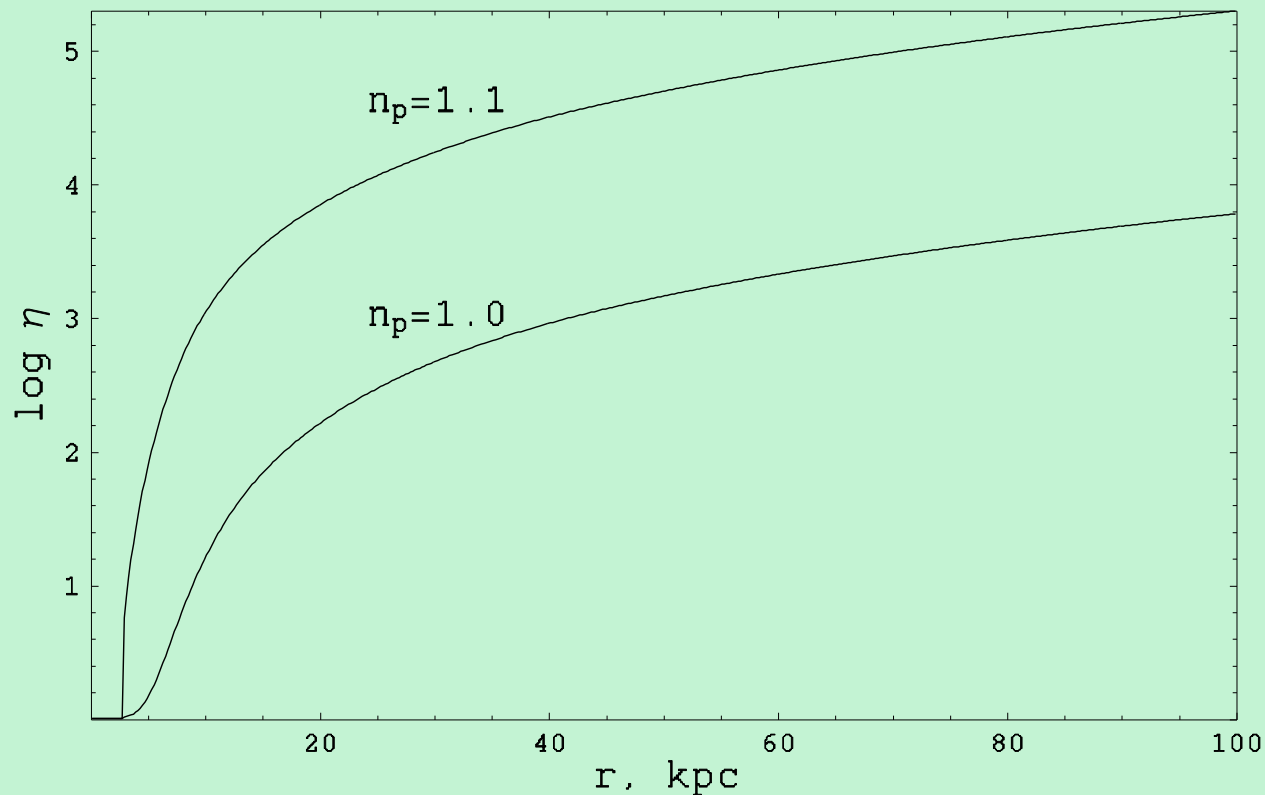
$$\xi(M, \nu) \frac{dM}{M} d\nu \simeq \frac{y(\nu)}{(2\pi)^{1/2}} e^{-\nu^2/2} \frac{d \log \sigma(M)}{dM} dM d\nu$$

Mean internal density of clump ρ_{cl} in GeV/cm^3

Right sides of curves correspond to

$M_{cl}/M_{\odot} = 10^{-8}, 10^{-6}, 10^{-4}, 10^{-2}, 1$ from up to down

Annihilation enhancement in clumps



**Local annihilation enhancement factor (boosting) $\eta(r)$
in isothermal spherical symmetric halo**

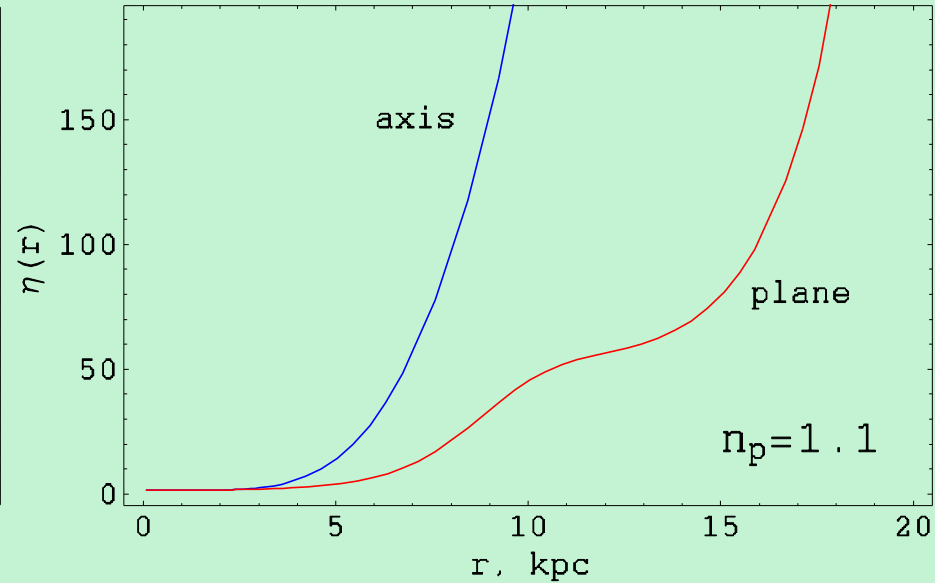
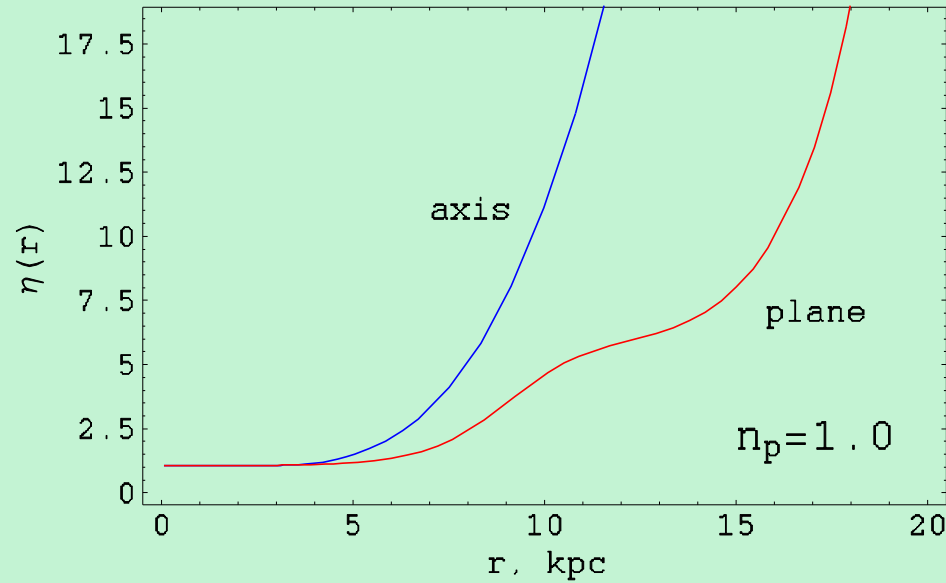
$$dI_{\text{dif}} = \frac{\langle \sigma_{\text{ann}} v \rangle}{4\pi l^2} \frac{\rho_{\text{DM}}^2(r)}{m_\chi^2} d^3r$$

$$\bar{\rho}_{\text{cl,P}}(r, n_p) = \frac{S(x_c, \beta)}{\xi_{\text{P}}(r, n_p)} \int P(r, \rho_{\text{cl}}) \rho_{\text{cl}} \xi(\rho_{\text{cl}}) d\rho_{\text{cl}}$$

$$\eta(r) = \frac{I_{\text{cl}} + I_{\text{dif}}}{I_{\text{dif}}} = 1 + \xi_{\text{P}}(r, n_p) \frac{\bar{\rho}_{\text{cl,P}}(r, n_p)}{\rho_\chi(r)}$$

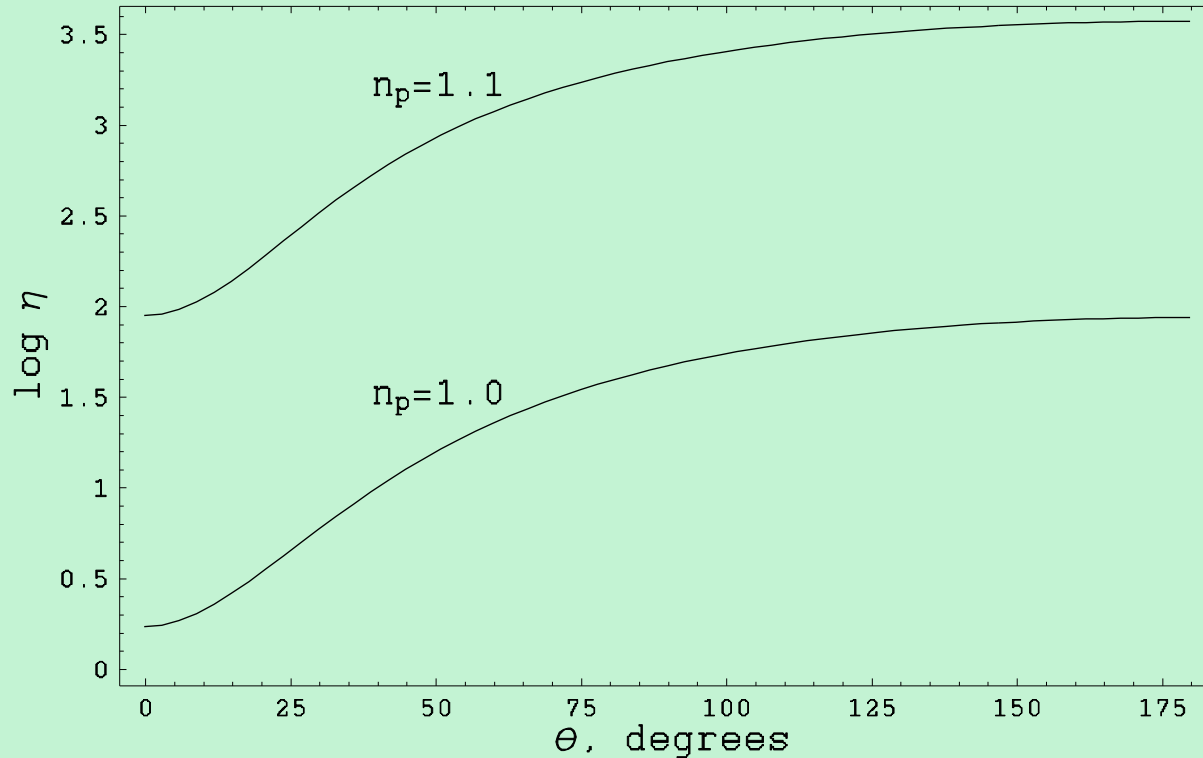
$$\xi_{\text{P}}(r, n_p) = \int P(r, \rho_{\text{cl}}) \xi(\rho_{\text{cl}}) d\rho_{\text{cl}},$$

Local boosting in the Galactic plane and along z-axis



Local annihilation enhancement factor (boosting) $\eta(r)$
in NFW halo with ring at $r=14$ kpc

Integrated along the line of sight (observed) enhancement factor $\eta(\theta)$



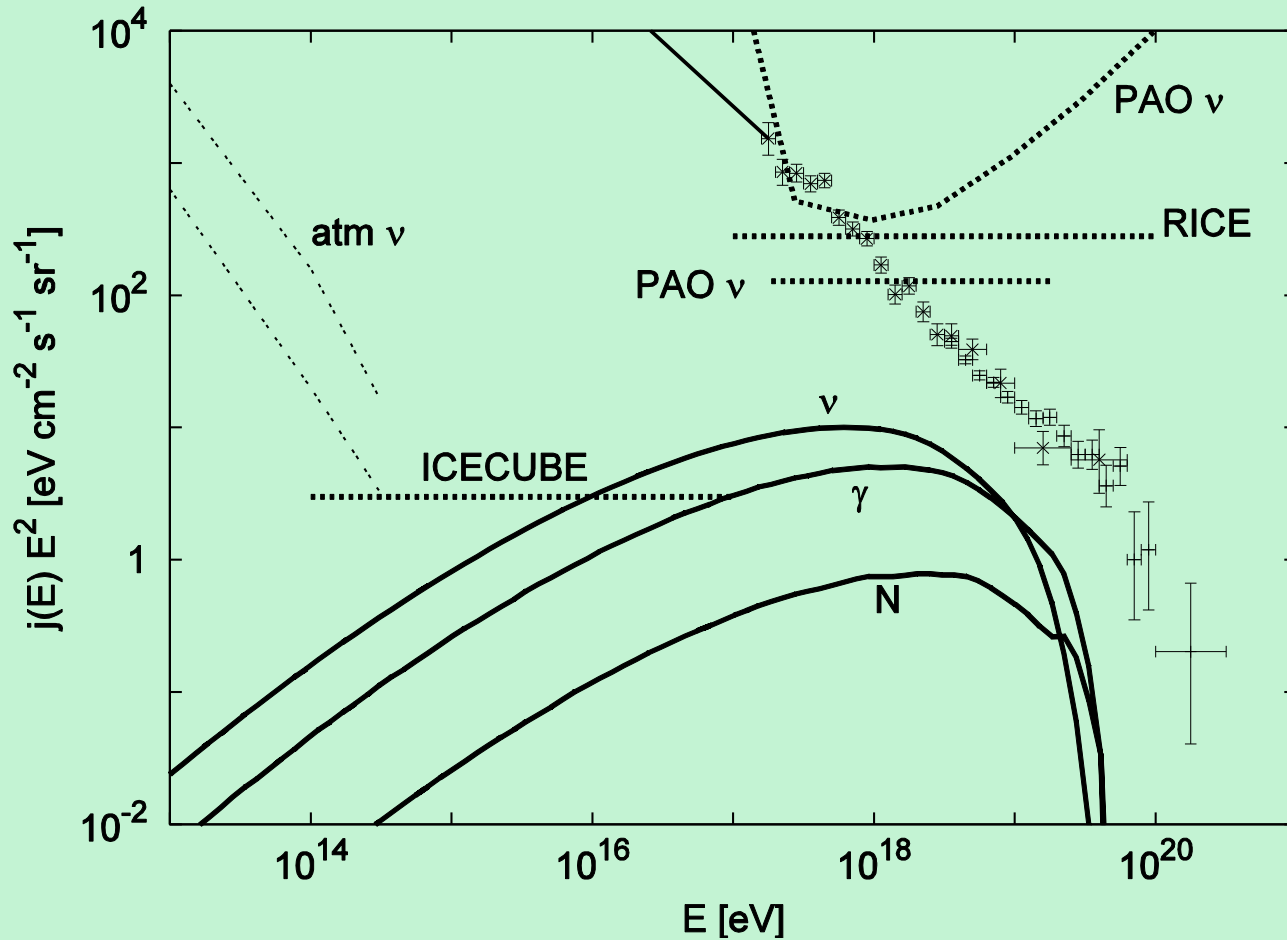
The case of isothermal spherically symmetric halo model

$$\eta(\theta, \phi) = 1 + \frac{\int \rho_{\chi}(r) dr \xi_{\text{P}}(r) \bar{\rho}_{\text{cl,P}}(r, n_p)}{\int \rho_{\chi}^2(r) dr}$$

$$\bar{\rho}_{\text{cl,P}}(r, n_p) = \frac{S(x_e, \beta)}{\xi_{\text{P}}(r, n_p)} \int P(r, \rho_{\text{cl}}) \rho_{\text{cl}} \xi(\rho_{\text{cl}}) d\rho_{\text{cl}}$$

$$\xi_{\text{P}}(r, n_p) = \int P(r, \rho_{\text{cl}}) \xi(\rho_{\text{cl}}) d\rho_{\text{cl}},$$

Annihilations of superheavy DM in superdense clumps



Fluxes $I_i(E)$ of photons, nucleons and neutrinos from neutralino annihilations in the Galactic halo for neutralino with $m_\chi \sim 10^{11}$ GeV

★ **Clumps with M_{\min} give the dominant contribution to DM annihilation**

In the case of $n_p = 1$, $\nu \sim 2.5$

$$M_{\min} \sim 10^{-8} M_{\odot}$$

$$R \simeq 3.6 \cdot 10^{15} \text{ cm}, \quad R_c \simeq 1.8 \cdot 10^{14} \text{ cm}$$

$$\bar{\rho}_{\text{int}} \simeq 2.5 \cdot 10^{-22} \text{ g cm}^{-3}$$

Halo mass fraction in these clumps

$$\xi_{\text{int}} \sim 0.002$$

Mean number density in the halo

$$n_{\text{cl}} \sim 25 \text{ pc}^{-3}$$

DM annihilation enhancement

$$\eta \sim 10 - 10^2$$