Danish Institute for Advanced Study

CP^3 - Origins
Particle Physics & Origin of Mass

Standard Model vacuum stability from gradient flows

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Outline

- Renormalization group as the gradient flow
- Example: Standard Model
- New consistent perturbative expansion
- Phenomenology: Standard Model vacuum stability analysis revisited
- Conclusions

Renormalization group

- Suppose that physics at a given energy scale μ is specified by dimensionless couplings (g_1,g_2,g_3,...)
- Then the physics at some other scale μ' is the same as at μ if we allow the couplings to "flow" according to:

$$\mu \frac{dg_i(\mu)}{d\mu} = \beta_i[g_i(\mu)]$$

Is there a general principle organizing RG flows?

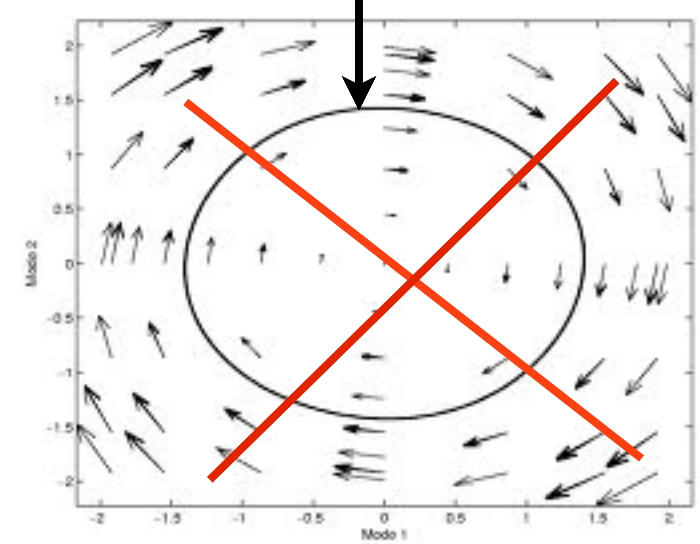
RG flow as the gradient flow

Is there a general principle organizing RG flows?

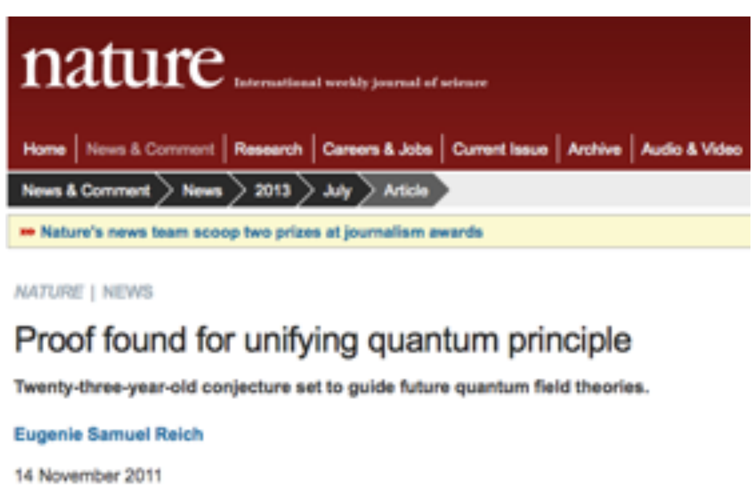
It has been shown that in 2d QFT there exist the function “c” of the couplings of the theory which is non-increasing along the RG trajectory (Zamolodchikov c-theorem).

$$\mu \frac{dc(g_i)}{d\mu} = \beta_i \frac{\partial c}{\partial g_i} < 0 \text{ , where beta functions } \beta_i \equiv \mu \frac{d g_i}{d\mu} \ .$$

→ Constraint on the way RG flow can be realized (e.g. no limit cycles)



Generalization to 4D QFT seems to be found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NATURE | NEWS

Proof found for unifying quantum principle

Twenty-three-year-old conjecture set to guide future quantum field theories.

Eugenie Samuel Reich

14 November 2011

http://www.nature.com/news/proof-found-for-unifying-quantum-principle-1.9352

RG flow as the gradient flow

The stronger condition is that of gradient flow

Assume: $$\frac{\partial c}{\partial g_i} = -\beta^i \,, \qquad \beta^i \equiv \chi^{ij}\beta_j .$$

"χ" is the Zamolodchikov metric needed since gradient of the function is a co-vector and the beta functions are vectors.

$$\Rightarrow \quad \mu \frac{dc(g_i)}{d\mu} = \beta_i \frac{\partial c}{\partial g_i} = -\beta_i \beta^i = -\chi^{ij}\beta_i\beta_j$$

If the metric is positive definite the "c-theorem" follows automatically

Moreover: $$\frac{\partial^2 c}{\partial g_i \partial g_j} = \frac{\partial^2 c}{\partial g_j \partial g_i} \quad \Rightarrow \quad \frac{\partial \beta^j}{\partial g_i} = \frac{\partial \beta^i}{\partial g_j}$$

gives relations between the different beta functions of the theory

4D "c-theorem"

Consider 4D CFT augmented by the set of marginal operators

$$\mathcal{L} = \mathcal{L}_{CFT} + g_i \mathcal{O}^i$$

Promote g=g(x) and work in arbitrary curved background.

Perform conformal transformation of the metric simultaneously with rescaling of the renormalized couplings:

$$\gamma_{\mu\nu} \to e^{2\sigma(x)} \gamma_{\mu\nu}$$
$$g_i(\mu) \to g_i(e^{-\sigma(x)} \mu)$$

Find infinitesimal variation of the generating functional:

$$W \equiv \log \left[\int D\Phi e^{i \int d^4 x L} \right]$$

There is a Weyl anomaly :

Vanish in the flat space and with constant couplings

$$\Delta_\sigma W \equiv \int d^4x\, \sigma(x) \left(2\gamma_{\mu\nu} \frac{\delta W}{\delta \gamma_{\mu\nu}} - \beta_i \frac{\delta W}{\delta g_i} \right) = \sigma \left(aE(\gamma) + \chi^{ij} \partial_\mu g_i \partial_\nu g_j G^{\mu\nu} \right) + \partial_\mu \sigma w^i \, \partial_\nu g_i G^{\mu\nu} + \ldots$$

$$2\gamma_{\mu\nu} \frac{\delta W}{\delta \gamma_{\mu\nu}} \sim T^\mu_\mu \qquad \frac{\delta W}{\delta g_i} \sim O_i$$

$$T^\mu_\mu = \beta_i O_i + \ldots$$

recover the usual scale anomaly in flat space

$$E(\gamma) = R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4R^{\mu\nu} R_{\mu\nu} + R^2 \quad \text{and} \quad G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}\gamma^{\mu\nu} R$$

a, χ^{ij} and ω^i are functions of the renormalized couplings,

The Weyl anomaly is of Abelian nature:

$$\Delta_\sigma \Delta_\tau W = \Delta_\tau \Delta_\sigma W$$ ⟹ Relations between a, χ^{ij} and ω^i

$$\frac{\partial \tilde{a}}{\partial g_i} = \left(-\chi^{ij} + \frac{\partial w^i}{\partial g_j} - \frac{\partial w^j}{\partial g_i} \right) \beta_j \,, \quad \text{where} \quad \tilde{a} \equiv a - w^i \beta_i$$

Multiply by β_i and use:

$$\left[\left(\frac{\partial w^i}{\partial g_j} - \frac{\partial w^j}{\partial g_i} \right) \beta_i \beta_j = 0 \ \text{ because we sum over i and j} \right]$$

⟹ $$\mu \frac{d}{d\mu} \tilde{a} = -\chi^{ij} \beta_i \beta_j$$

Moreover, by explicit calculation function "w" turns out to be exact one-form at the lowest order of perturbation theory

⟹ $$\frac{\partial \tilde{a}}{\partial g_i} = -\beta^i \,, \qquad \beta^i \equiv \chi^{ij} \beta_j \,.$$

Example: Standard Model

SM gauge couplings: top-Yukawa Higgs quartic

$$\alpha_1 = \frac{g_1^2}{(4\pi)^2}, \alpha_2 = \frac{g_2^2}{(4\pi)^2}, \alpha_3 = \frac{g_3^2}{(4\pi)^2}, \alpha_t = \frac{g_t^2}{(4\pi)^2}, \alpha_\lambda = \frac{\lambda^2}{(4\pi)^2} .$$

Metric in the coupling constants space:

$$\chi = \mathrm{diag}\left(\frac{1}{\alpha_1^2}, \frac{3}{\alpha_2^2}, \frac{8}{\alpha_3^2}, \frac{2}{\alpha_t}, 4\right) .$$

4D analogue of the c-function is called a-function

$$\frac{\partial \tilde{a}}{\partial g_i} = -\beta^i , \qquad \beta^i \equiv \chi^{ij}\beta_j .$$

Reminder: $$\frac{\partial^2 c}{\partial g_i \partial g_j} = \frac{\partial^2 c}{\partial g_j \partial g_i} \quad \Rightarrow \quad \frac{\partial \beta^j}{\partial g_i} = \frac{\partial \beta^i}{\partial g_j}$$

gives relations between the different beta functions of the theory

$$\beta^i \equiv \chi^{ij} \beta_j .$$

SM ⇒

$$2\frac{\partial}{\partial \alpha_t}\beta_\lambda = \frac{\partial}{\partial \alpha_\lambda}\left(\frac{\beta_t}{\alpha_t}\right) + O\left(\alpha_i^2\right) \tag{9}$$

$$4\frac{\partial}{\partial \alpha_1}\beta_\lambda = \frac{\partial}{\partial \alpha_\lambda}\left(\frac{\beta_1}{\alpha_1^2}\right) + O\left(\alpha_i^2\right) \tag{10}$$

$$\frac{4}{3}\frac{\partial}{\partial \alpha_2}\beta_\lambda = \frac{\partial}{\partial \alpha_\lambda}\left(\frac{\beta_2}{\alpha_2^2}\right) + O\left(\alpha_i^2\right) \tag{11}$$

$$2\frac{\partial}{\partial \alpha_1}\left(\frac{\beta_t}{\alpha_t}\right) = \frac{\partial}{\partial \alpha_t}\left(\frac{\beta_1}{\alpha_1^2}\right) + O\left(\alpha_i^2\right) \tag{12}$$

$$\frac{2}{3}\frac{\partial}{\partial \alpha_2}\left(\frac{\beta_t}{\alpha_t}\right) = \frac{\partial}{\partial \alpha_t}\left(\frac{\beta_2}{\alpha_2^2}\right) + O\left(\alpha_i^2\right) \tag{13}$$

$$\frac{1}{4}\frac{\partial}{\partial \alpha_3}\left(\frac{\beta_t}{\alpha_t}\right) = \frac{\partial}{\partial \alpha_t}\left(\frac{\beta_3}{\alpha_3^2}\right) + O\left(\alpha_i^2\right) \tag{14}$$

$$\frac{1}{3}\frac{\partial}{\partial \alpha_2}\left(\frac{\beta_1}{\alpha_1^2}\right) = \frac{\partial}{\partial \alpha_1}\left(\frac{\beta_2}{\alpha_2^2}\right) + O\left(\alpha_i^2\right) \tag{15}$$

$$\frac{1}{8}\frac{\partial}{\partial \alpha_3}\left(\frac{\beta_1}{\alpha_1^2}\right) = \frac{\partial}{\partial \alpha_1}\left(\frac{\beta_3}{\alpha_3^2}\right) + O\left(\alpha_i^2\right) \tag{16}$$

$$\frac{3}{8}\frac{\partial}{\partial \alpha_3}\left(\frac{\beta_2}{\alpha_2^2}\right) = \frac{\partial}{\partial \alpha_2}\left(\frac{\beta_3}{\alpha_3^2}\right) + O\left(\alpha_i^2\right) \tag{17}$$

SM beta functions in a “3-2-1” ordering scheme

3-loops

$$\beta_1 = 2\alpha_1^2\Big\{\frac{1}{12} + \frac{10n_G}{9} + \Big(\frac{1}{4} + \frac{95n_G}{54}\Big)\alpha_1 + \underbrace{\Big(\frac{3}{4} + \frac{n_G}{2}\Big)\alpha_2}_{\text{Eq. (15)}} + \underbrace{\frac{22n_G}{9}\alpha_3}_{\text{Eq. (16)}} + \Big(\frac{163}{1152} - \frac{145n_G}{81} - \frac{5225n_G^2}{1458}\Big)\alpha_1^2$$

$$+ \Big(\frac{87}{64} - \frac{7n_G}{72}\Big)\alpha_1\alpha_2 - \frac{137n_G}{162}\alpha_1\alpha_3 + \Big(\frac{3401}{384} + \frac{83n_G}{36} - \frac{11n_G^2}{18}\Big)\alpha_2^2 + \Big(\frac{1375n_G}{54} - \frac{242n_G^2}{81}\Big)\alpha_3^2 - \frac{n_G}{6}$$

$$+ \alpha_t\Big[\underbrace{-\frac{17}{12}}_{\text{Eq. (12)}} - \frac{2827}{576}\alpha_1 - \frac{785}{64}\alpha_2 - \frac{29}{6}\alpha_3 + \Big(\frac{113}{32} + \frac{101n_t}{16}\Big)\alpha_t\Big] + \underbrace{\alpha_\lambda\Big(\frac{3}{4}\alpha_1 + \frac{3}{4}\alpha_2 - \frac{3}{2}\alpha_\lambda\Big)}_{\text{Eq. (10)}}\Big\},$$

$$\beta_2 = 2\alpha_2^2\Big\{-\frac{43}{12} + \frac{2n_G}{3} + \underbrace{\Big(\frac{1}{4} + \frac{n_G}{6}\Big)\alpha_1}_{\text{Eq. (15)}} + \Big(-\frac{259}{12} + \frac{49n_G}{6}\Big)\alpha_2 + \underbrace{2n_G\alpha_3}_{\text{Eq. (17)}} + \Big(\frac{163}{1152} - \frac{35n_G}{54} - \frac{55n_G^2}{162}\Big)\alpha_1^2$$

$$+ \Big(\frac{187}{64} + \frac{13n_G}{24}\Big)\alpha_1\alpha_2 - \frac{n_G}{18}\alpha_1\alpha_3 + \Big(-\frac{667111}{3456} + \frac{3206n_G}{27} - \frac{415n_G^2}{54}\Big)\alpha_2^2$$

$$+ \frac{13n_G}{2}\alpha_2\alpha_3 + \Big(\frac{125n_G}{6} - \frac{22n_G^2}{9}\Big)\alpha_3^2$$

$$+ \alpha_t\Big[\underbrace{-\frac{3}{4}}_{\text{Eq. (13)}} - \frac{593}{192}\alpha_1 - \frac{729}{64}\alpha_2 - \frac{7}{2}\alpha_3 + \Big(\frac{57}{32} + \frac{45n_t}{16}\Big)\alpha_t\Big] + \underbrace{\alpha_\lambda\Big(\frac{1}{4}\alpha_1 + \frac{3}{4}\alpha_2 - \frac{3}{2}\alpha_\lambda\Big)}_{\text{Eq. (11)}}\Big\},$$

$$\beta_3 = 2\alpha_3^2\Big\{-\frac{11}{2} + \frac{2n_G}{3} + \underbrace{\frac{11n_G}{36}\alpha_1}_{\text{Eq. (16)}} + \underbrace{\frac{3n_G}{4}\alpha_2}_{\text{Eq. (17)}} + \Big(-51 + \frac{38n_G}{3}\Big)\alpha_3 + \Big(-\frac{65n_G}{432} - \frac{605n_G^2}{972}\Big)\alpha_1^2$$

$$- \frac{n_G}{48}\alpha_1\alpha_2 + \frac{77n_G}{54}\alpha_1\alpha_3 + \Big(\frac{241n_G}{48} - \frac{11n_G^2}{12}\Big)\alpha_2^2 + \frac{7n_G}{2}\alpha_2\alpha_3$$

$$+ \Big(-\frac{2857}{4} + \frac{5033n_G}{18} - \frac{325n_G^2}{27}\Big)\alpha_3^2 + \alpha_t\Big[\underbrace{-1}_{\text{Eq. (14)}} - \frac{101}{48}\alpha_1 - \frac{93}{16}\alpha_2 - 20\alpha_3 + \Big(\frac{9}{4} + \frac{21n_t}{4}\Big)\alpha_t\Big]\Big\},$$

2-loops

$$\beta_t = 2\alpha_t\Big\{\frac{9}{4}\alpha_t - \underbrace{4\alpha_3}_{\text{Eq. (14)}} - \underbrace{\frac{17}{24}\alpha_1}_{\text{Eq. (12)}} - \underbrace{\frac{9}{8}\alpha_2}_{\text{Eq. (13)}} + \underbrace{3\alpha_\lambda^2 - 6\alpha_t\alpha_\lambda}_{\text{Eq. (9)}} - 6\alpha_t^2 + 18\alpha_3\alpha_t$$

$$+ \alpha_3^2\Big(-\frac{202}{3} + \frac{40n_G}{9}\Big) + \alpha_t\Big(\frac{131}{32}\alpha_1 + \frac{225}{32}\alpha_2\Big) + \frac{1187}{432}\alpha_1^2 - \frac{3}{8}\alpha_1\alpha_2 + \frac{19}{18}\alpha_1\alpha_3 - \frac{23}{8}\alpha_2^2 + \frac{9}{2}\alpha_3\alpha_2\Big\},$$

1-loop

$$\beta_\lambda = \underbrace{\frac{9}{16}\alpha_2^2 - \frac{9}{2}\alpha_\lambda\alpha_2}_{\text{Eq. (11)}} + \underbrace{\frac{3}{16}\alpha_1^2 - \frac{3}{2}\alpha_\lambda\alpha_1}_{\text{Eq. (10)}} + \underbrace{\frac{3}{8}\alpha_1\alpha_2}_{\text{Eqs. (10–11)}} + 12\alpha_\lambda^2 + \underbrace{6\alpha_\lambda\alpha_t - 3\alpha_t^2}_{\text{Eq. (9)}}.$$

“3-2-1” ordering scheme corresponds to truncation of the a-function to the order α^3

$$-\tilde{a} = \ldots + \underbrace{\frac{9}{4}\alpha_2^2\alpha_\lambda - 9\alpha_\lambda^2\alpha_2}_{Eq.\ (11)} + \underbrace{\frac{3}{4}\alpha_1^2\alpha_\lambda - 3\alpha_\lambda^2\alpha_1}_{Eq.\ (10)} + \underbrace{\frac{3}{2}\alpha_1\alpha_2\alpha_\lambda}_{Eqs.\ (10\text{--}11)} + 16\alpha_\lambda^3 + \underbrace{12\alpha_\lambda^2\alpha_t - 12\alpha_t^2\alpha_\lambda}_{Eq.\ (9)} + \ldots$$

- Truncating a-function at the order α^4 corresponds to the “4-3-2” ordering scheme, at the order α^5 to the “5-4-3” ordering scheme and so on...
- Consistent perturbative expansion must be adopted at the level of the a-function NOT at the level of the beta functions

Different from the conventional loop expansion scheme which would calculate all beta functions to 3-loops and therefore would correspond to the “3-3-3” counting

Phenomenological applications: SM vacuum stability analysis

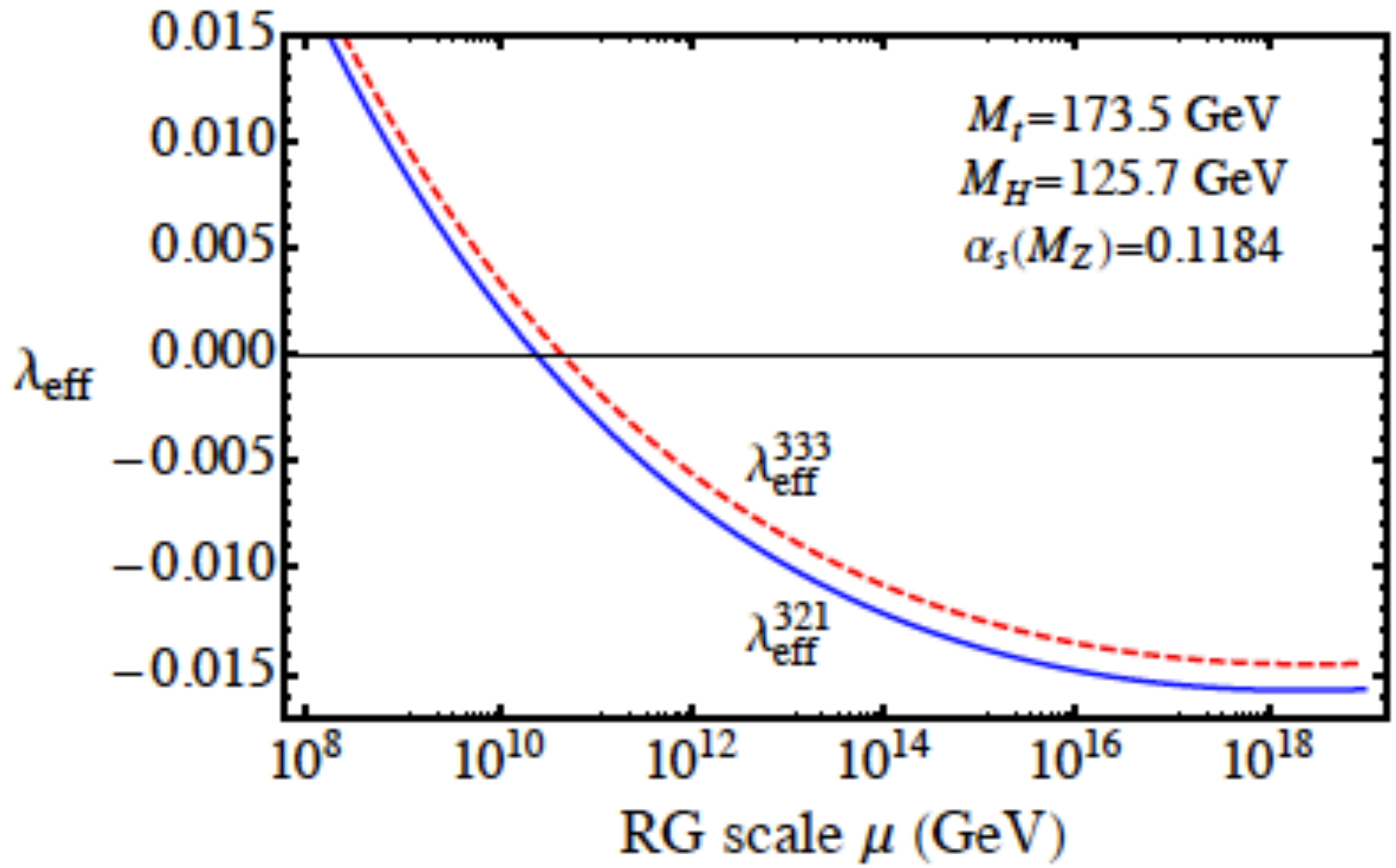
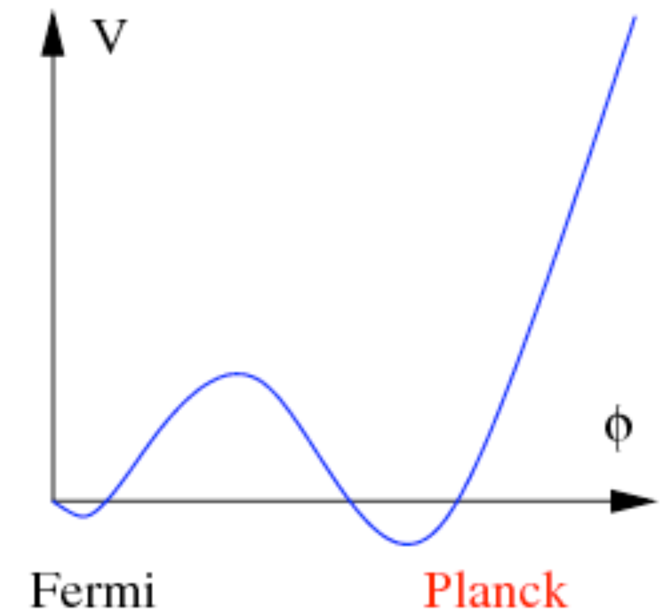


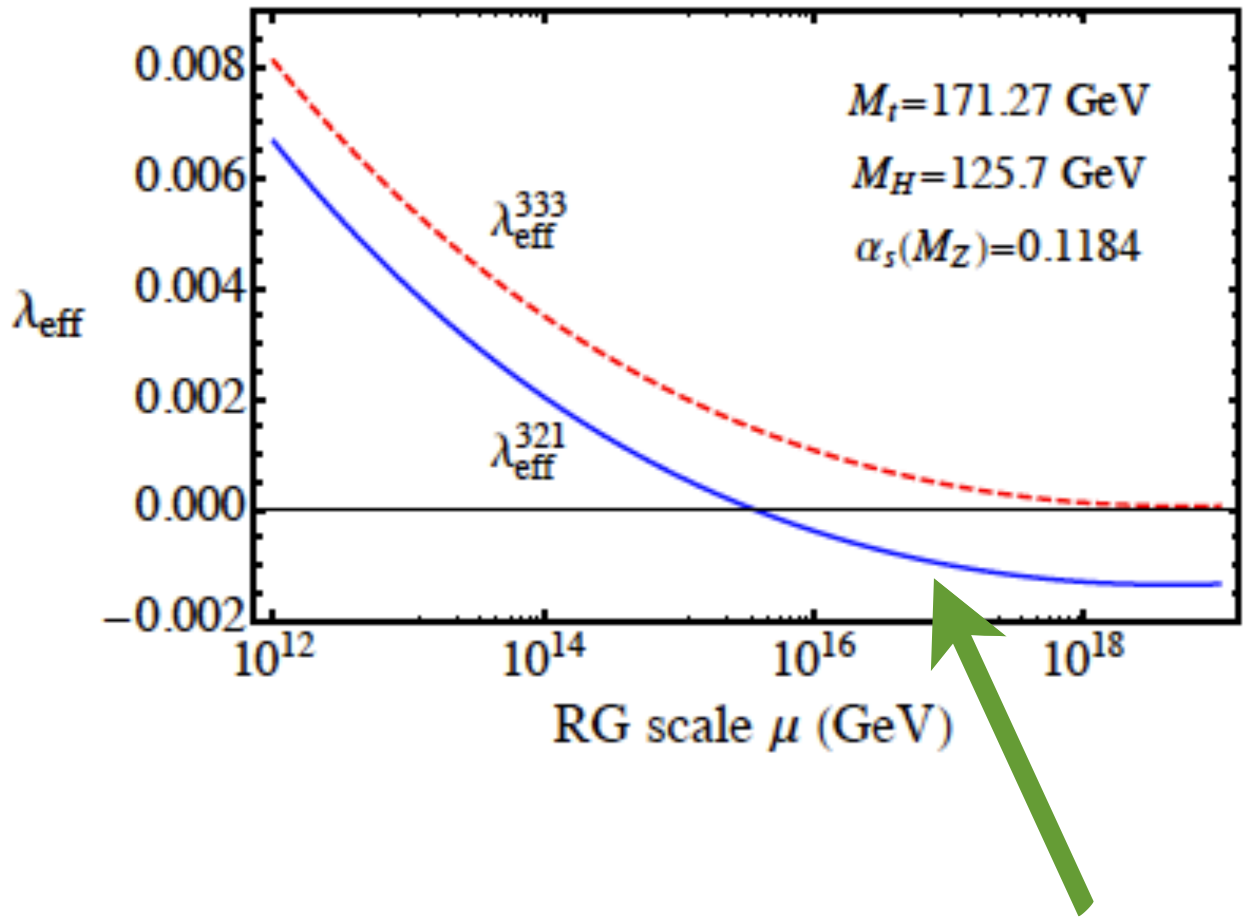
FIG. 1.b RG evolution of the effective standard model Higgs quartic coupling. We indicate with λ_{eff}^{333} (λ_{eff}^{321}) the evolution of the λ_{eff} coupling according to the 3-3-3 (3-2-1) scheme.



$$V_{\text{eff}} = \frac{\lambda_{\text{eff}}(\mu)}{4}\phi^4$$

SM vacuum stability analysis

Evolution of the Higgs quartic is very sensitive to the top mass and therefore precision in theoretical predictions is required



Need lower top mass $M_t \approx 171.05$ GeV to lift the blue 3-2-1 curve to the red one

SM vacuum stability analysis

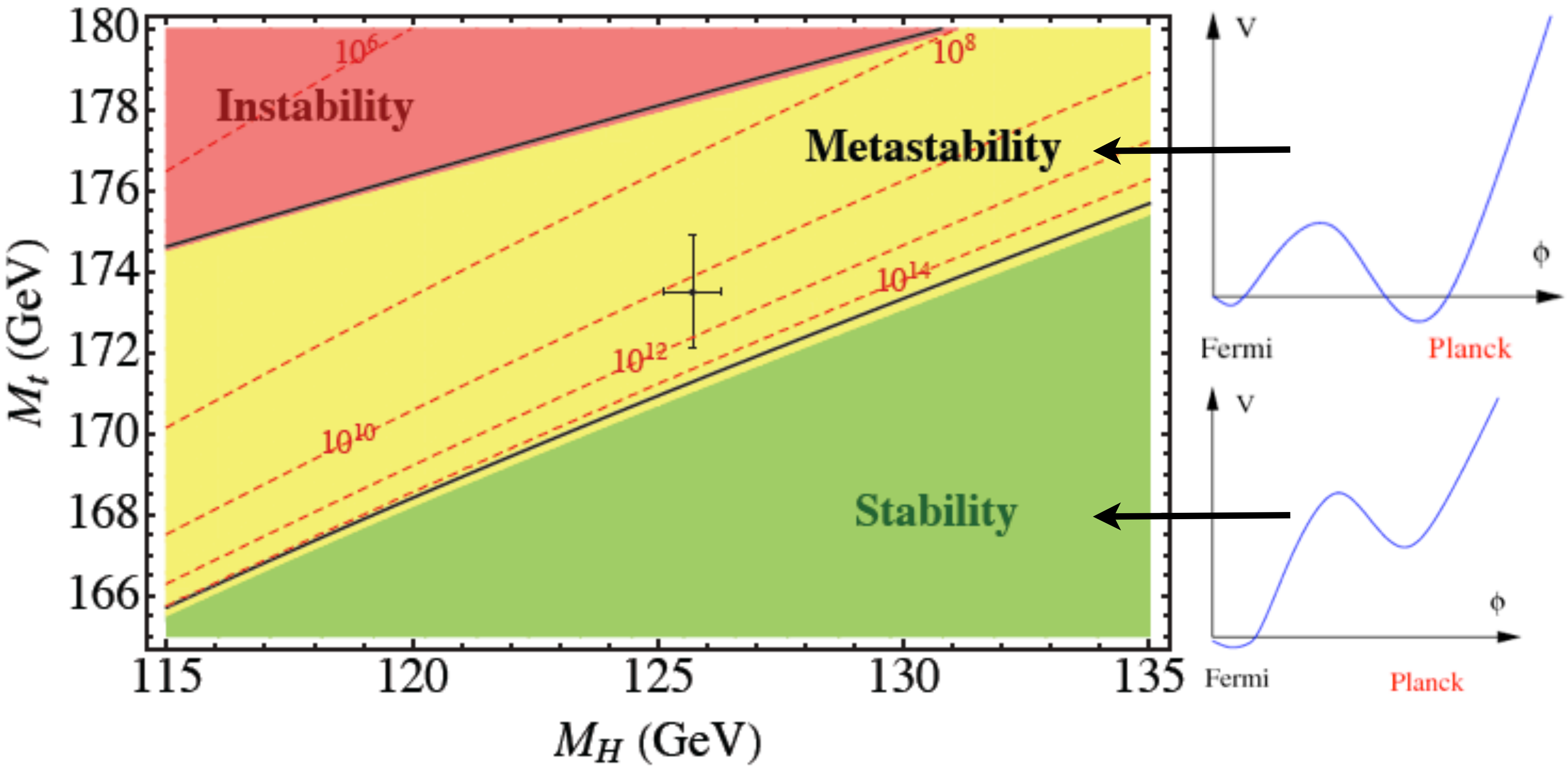


FIG. 2. Standard model stability analysis based on the effective standard model Higgs quartic coupling. The red region indicates instability, the yellow metastability and the green absolute stability following the 3-2-1 counting. For comparison, the black lines indicate the bounds from the 3-3-3 counting. The point with error bars shows the experimental values of the top [23] and Higgs [24] masses. The red dashed lines show the value in GeV at which $\lambda_{\text{eff}}^{321}$ crosses zero.

Conclusions

- We introduced a counting scheme for organization of the perturbative expansion preserving the Weyl symmetry
- Perturbative truncations has to be made at the level of the a-function through which the various beta functions of the theory are linked
- As a phenomenological application we investigated the SM vacuum stability. Within the new counting scheme, sizable numerical corrections to the conventional *loop expansion scheme* appear when analyzing the possible existence of the new Higgs vacuum at high energies