

# Electroweak physics and the discovery of the W/Z bosons

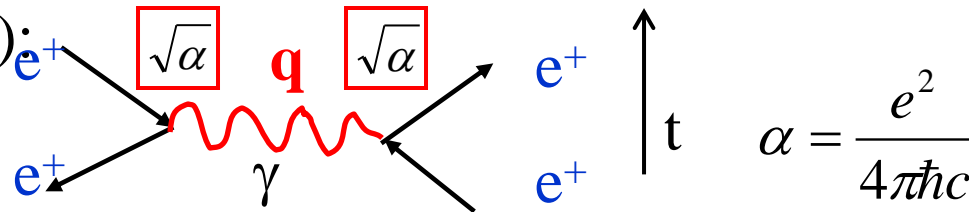
(perhaps the most beautiful, experimentally confirmed, physics theory)

## Facts:


-electromagnetic interactions:

interaction between charged particles mediated by a neutral electromagnetic vector field. Make the theory relativistic and quantized.

The quantized field is the photon with spin 1 and null mass (infinite range):



The mediator (the photon) is neutral and the effect of the electromagnetic potential (Coulomb) is described by the propagator:

  $\Rightarrow \frac{1}{q^2}$

# -The weak interactions:

The first one measured: the beta decay:

$$n \rightarrow p + e^- + \bar{\nu}_e$$

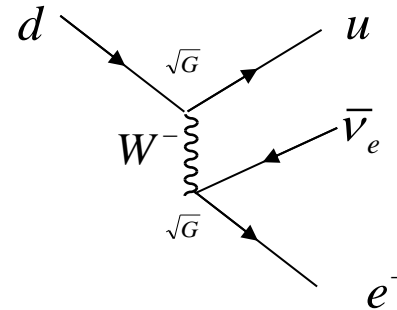
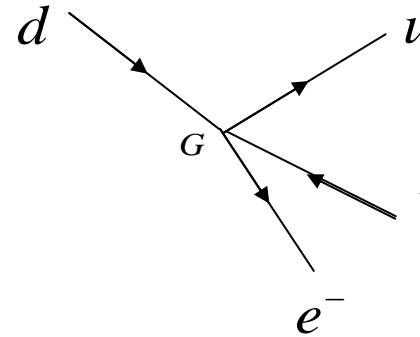
$$d \rightarrow u + e^- + \bar{\nu}_e$$

$${}^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e, {}^6\text{He} \rightarrow {}^6\text{Li} + e^- + \bar{\nu}_e$$

$$M = \frac{G}{\sqrt{2}} \underbrace{\langle u | J_\mu | d \rangle}_{\text{Hadronic weak current (quarks)}} \underbrace{\langle e \bar{\nu} | J^\mu | 0 \rangle}_{\text{leptonic weak current}}$$



The Fermi theory:  
just the product of  
two currents  
multiplied by a  
constant: G  
(contact interaction)



The matrix element M is a constant:

The Phase Space determines the rate :  $N(E)dE = E^2(E_0 - E)^2 dE \Rightarrow N = \int_0^{E_0} N(E)dE = \frac{E_0^5}{30}$

$$\Rightarrow W \propto G^2 E_0^5, E_0 = \text{max electron energy} = m_n - m_p$$

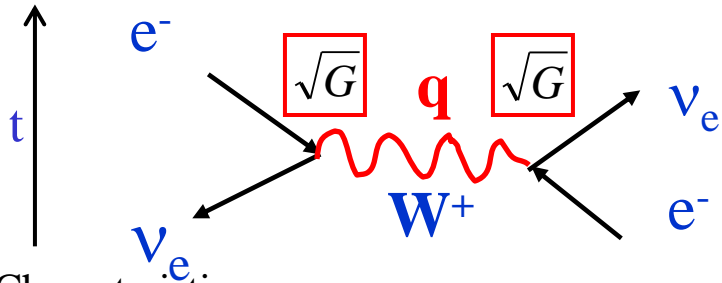
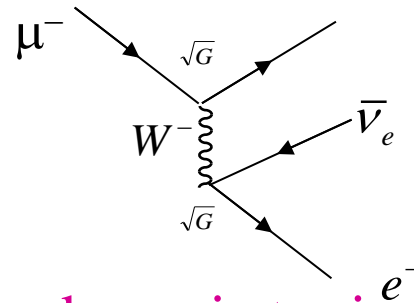
$$W = \frac{G^2}{2\pi^3} \int_0^{p_{\text{max}}} N(p) dp \propto G^2$$

The muon decay is the simplest purely leptonic decay:

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

$$\frac{1}{\Gamma_\mu} \equiv \tau_\mu = \frac{192\pi^3}{G^2 m_\mu^5}$$

The  $m^{-5}$  dependence is typical of any weak decay (neglecting the final particle masses)



Charged weak interaction 2→2

Characteristics:

- fermions: leptons and quarks can interact weakly: universal interaction;
- There is a new neutral particle involved: **the neutrino**;
- The mediator is a spin 1 field but with a mass (short range force)  $\rightarrow \propto \frac{1}{q^2 - M_w^2 + (i\Gamma M_w)}$
- At low interaction energy the intensity of the weak interaction is much lower than the electromagnetic one but it increases with energy (propagator effect).
- Weak interactions violate spatial parity: interacting neutrino is left-handed and antineutrino is right-handed

Weak charged current :

$$J^\mu = \frac{g}{\sqrt{2}} \bar{u}(e) \gamma^\mu \frac{1}{2} (1 - \gamma^5) u(\nu_e)$$

Difference From QED: (V-A) instead of V

Muon lifetime:  $\tau_\mu = 2.197 \cdot 10^{-6}$  s,  $m_\mu = 0.105658$  GeV

$$G = 1.166 \cdot 10^{-5} \text{ GeV}^{-2}$$

with  $\frac{G}{\sqrt{2}} = \frac{g_W^2}{8M_W^2}$ ,  $M_W \approx 80 \text{ GeV} \Rightarrow g_W = 0.66$ ,  $\alpha_W = \frac{g_W^2}{4\pi} = \frac{1}{129}$

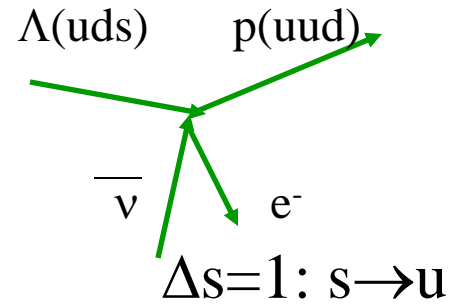
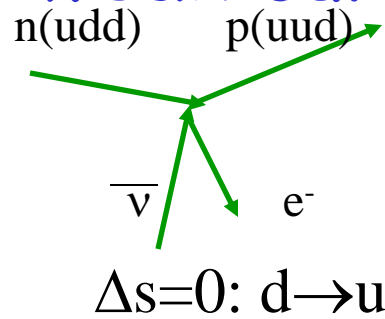
The weak interactions are “weak” because the mediator (W) is very heavy and at low  $Q^2$  it dominates the coupling

Se  $Q^2 \sim M_W^2$  (80 GeV)<sup>2</sup> the weak interaction become comparable to the electromagnetic ones:

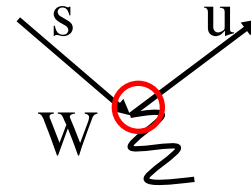
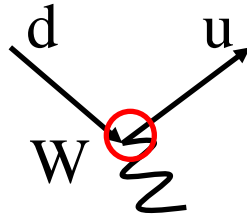
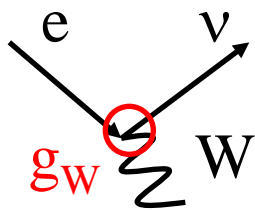
**Unifications?**

# Hadronic weak currents

Experimental evidence of two beta-like decays



Universality of the beta-decay?



Leptonic :  $g_W$     hadronic  $\Delta s=0$ :  $g_W \cos \theta_c$     hadronic  $\Delta s=1 = \Delta Q$ :  $g_W \sin \theta_c$

Cabibbo:  $\sin \theta_C = 0.23$  ( $\theta = 13^\circ$ ) **not predicted**: just an experimental result.

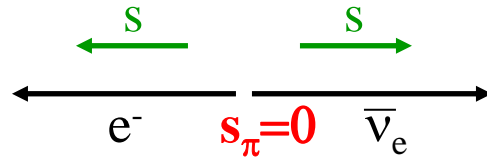
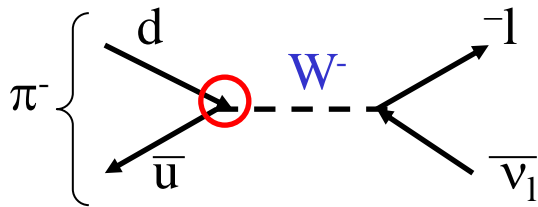
Current still (V-A)  $(1-\gamma_5)$ . But the quark wavefunctions entering the interaction are:

$$\begin{aligned} d' &= d \cos \theta + s \sin \theta & \begin{bmatrix} d' \\ s' \end{bmatrix} &= \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} d \\ s \end{bmatrix} \\ s' &= -d \sin \theta + s \cos \theta \end{aligned}$$

As in the leptonic case, the charged weak interaction makes the transition between elements of a weak doublet:

$$\begin{bmatrix} u \\ d' \end{bmatrix} \text{ equivalent to } \begin{bmatrix} \nu_e \\ e^- \end{bmatrix}$$

# Charged weak interactions of quarks : $\pi^- \rightarrow l^- \nu_l$ decay



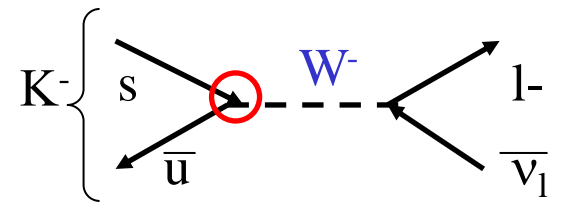
The electron has a “wrong” helicity (+1) disfavoured as:

$$P(\lambda = +1) = 1 - \beta_{e^-} = \frac{2m_e^2}{m_\pi^2 + m_e^2}$$

$$\Rightarrow \Gamma_\pi = \frac{f_\pi^2}{\pi m_\pi^3} \left[ \frac{g_W}{4M_W} \right]^4 m_l^2 [m_\pi^2 - m_l^2]^2 = \frac{f_\pi^2 G^2}{\pi m_\pi^3} m_l^2 [m_\pi^2 - m_l^2]^2$$

ansatz :  $f_\pi = m_\pi \cos \theta_C$

Similarly :  $K^- \rightarrow \mu^- \bar{\nu}_\mu$  (BR = 63.44%)



ansatz :  $f_K = m_K \sin \theta_C$

$$\Rightarrow \Gamma_K = \frac{G^2 f_K^2}{8\pi} \frac{1}{m_K^3} m_\mu^2 [m_K^2 - m_\mu^2]^2$$

from the measured  
 $\pi$  and K lifetimes :  $\tau_\pi, \tau_K$   
( $\tau = 1/\Gamma_{tot}$ )

$$\tan^2 \theta_C = 0.63 \frac{\tau_\pi m_K^3 (m_\pi^2 - m_\mu^2)^2}{\tau_K m_\pi^3 (m_K^2 - m_\mu^2)^2} \Rightarrow \begin{cases} \sin \theta_C = 0.265 \\ \cos \theta_C = 0.964 \end{cases}$$

# Cabibbo Kobayashi Maskawa matrix (CKM)

1973: Cabibbo theory generalization with 3 quark doublets : **at least three quark generations needed to provide a CP violation (i.e an irriducible phase).**

The matrix V is complex: 18 elements, but it is also unitary:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{vmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{vmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$V_{\alpha\beta}^+ V_{\beta\alpha} = \delta_{\alpha\beta} \text{ (9 equations)} \Rightarrow \text{9 elements}$$

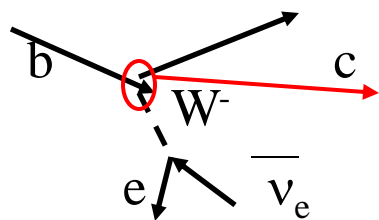
There is an arbitrary phase at each particle:  $9 - 3 \cdot 2 = 3$ , but still a common phase factor remains :  $3 + 1 = \mathbf{4 \text{ independent}}$

**elements (1 phase)**

Standard parametrization (PDG): 3 angles  $\theta_{ij}$  ( $\theta_{12}$  Cabibbo angle) + a phase:  $\delta_{13}$

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \quad \begin{matrix} c_{ij} = \cos \theta_{ij} \\ s_{ij} = \sin \theta_{ij} \end{matrix}$$

N.B. V cannot be predicted theoretically: its elements extracted from the experiments:

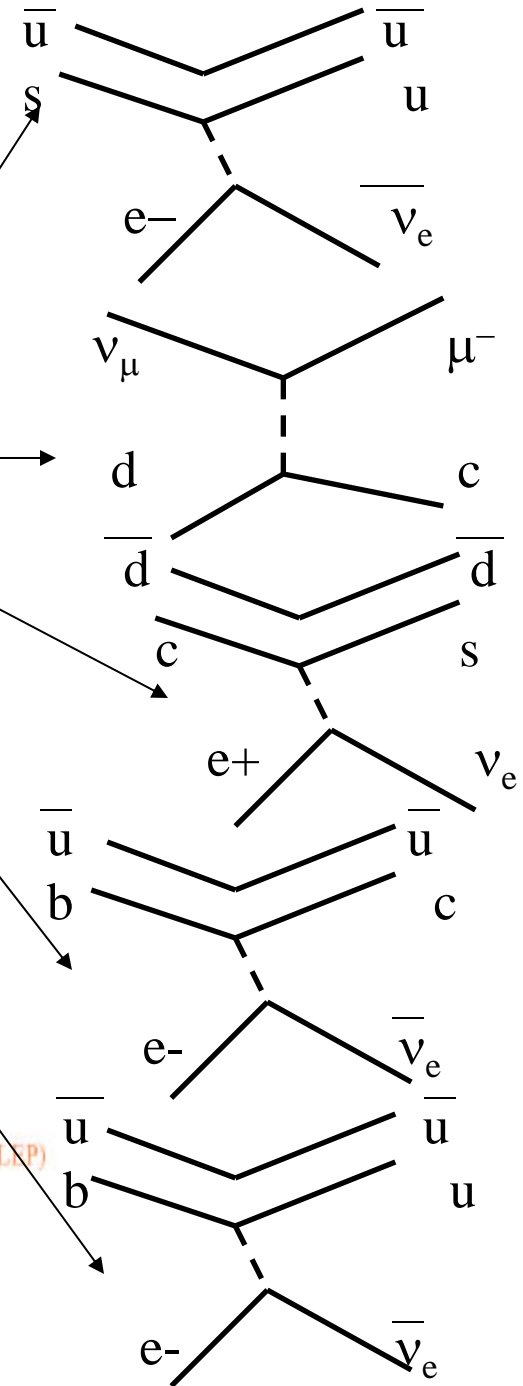


$$\mathbf{V}_{cb} \quad \Gamma(b \rightarrow ce^- \bar{\nu}_e) = \frac{G^2 m_b^5}{192\pi} |V_{cb}|^2 F(m_c / m_b)$$

$F(m_c/m_b)$  is a phase-space factor (=1 if you neglect the final state particle masses)

# CKM

CKM entry	Value	Source
$ V_{ud} $	$0.9740 \pm 0.0005$ $0.9731 \pm 0.0015$ $0.9739 \pm 0.0005$	Nuclear $\beta$ decay $n \rightarrow p e^- \bar{\nu}_e$
$ V_{us} $	$0.2196 \pm 0.0026$	$K \rightarrow \pi e^- \bar{\nu}_e$
$ V_{cd} $	$0.224 \pm 0.016$	$vd \rightarrow cX$
$ V_{cs} $	$1.04 \pm 0.16$ $0.97 \pm 0.11$	$D \rightarrow \bar{K} e^+ \nu_e$ $W^+ \rightarrow c\bar{s}$
$ V_{cb} $	$0.0421 \pm 0.0021$ $0.0414 \pm 0.0011$ $0.0416 \pm 0.0020$	$B \rightarrow D^* l \bar{\nu}_l$ $b \rightarrow c l \bar{\nu}_l$
$ V_{ub} $	$0.0033 \pm 0.0005$ $0.0041 \pm 0.0006$ $0.0036 \pm 0.0005$	$B \rightarrow \rho l \bar{\nu}_l$ $b \rightarrow u l \bar{\nu}_l$
$ V_{tb}  / \sqrt{\sum_q  V_{tq} ^2}$	$0.97^{+0.16}_{-0.12}$	$t \rightarrow bW / qW$



$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9999 \pm 0.0006$$

$$\sum_j (|V_{uj}|^2 + |V_{cj}|^2) = 2.039 \pm 0.025 \quad (\text{LEP})$$

A third doublet is required



# From experiments:

$$V_{CKM} = \begin{pmatrix} 0.97427 & 0.22534 & 0.00351 \\ 0.22520 & 0.97344 & 0.04120 \\ 0.00867 & 0.0404 & 0.999146 \end{pmatrix}$$

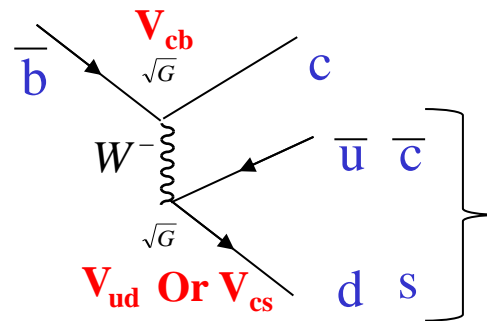
Unitarity puts constraints on rows and columns elements, ex:

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$$

Note:

The matrix is nearly diagonal:

In particular, the off diagonal elements of the 3<sup>rd</sup> row and column are very small: the III generation (t,b) is almost decoupled: The b-quark lifetime is large despite the large mass.



x3 :  
summed up on the three quark colors

Are there additional quark doublets?  
Generalization to n quark generations

$$\frac{n(n-1)}{2} \text{ angles; } \frac{(n-1)(n-2)}{2} \text{ phases.}$$

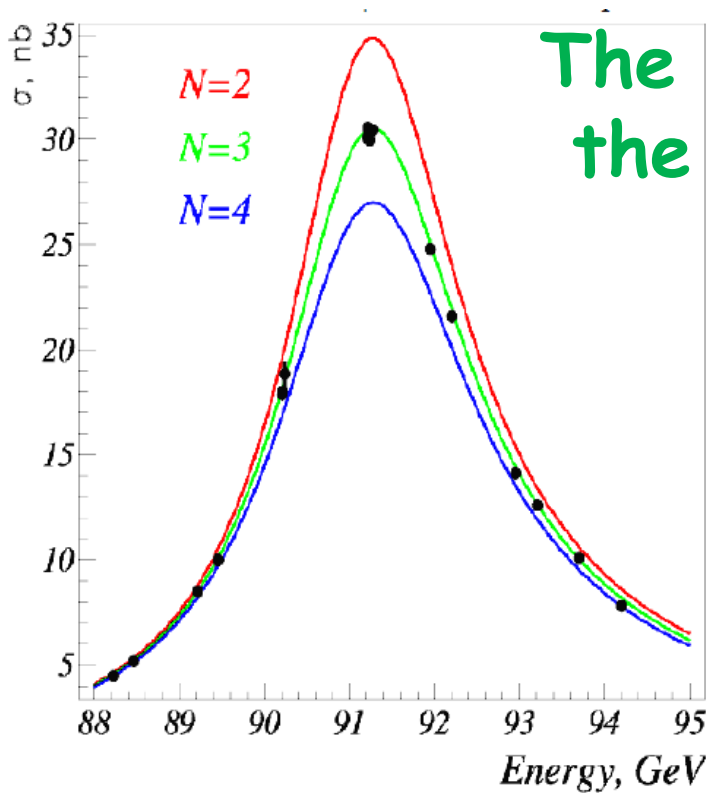


Fig.2. The shape of the  $Z^0$  resonance yields information on the number of light neutrino types [three and only three].

The Z boson total width:  $\Gamma_Z$  and the number of neutrinos (families)

$$P(m) = \frac{1}{2\pi} \frac{\Gamma}{\Gamma^2/4 + (m - m_0)^2}$$

$$\Gamma_Z = \Gamma(e^+e^-) + \Gamma(\mu^+\mu^-) + \Gamma(\tau^+\tau^-) + \Gamma(u\bar{u}) + \Gamma(d\bar{d}) + \Gamma(s\bar{s}) + \Gamma(c\bar{c}) + \Gamma(b\bar{b}) + N_\nu \times \Gamma(\nu\bar{\nu})$$

$N_\nu$  number of neutrino types (with  $m_\nu < M_Z/2$ ) ( $\Gamma(\nu\bar{\nu}) = 166.2 \text{ MeV}$  if  $m_\nu = 0$ )

The Z width is a measure of the neutrinos number

$$N_\nu = 2.984 \pm 0.008 \text{ (PGD LEP data)}$$

**6 leptons+ 6 quarks** spin  $\frac{1}{2}$ , masses from :

0.5 MeV to 175 GeV/c<sup>2</sup> (+ neutrino masses: eV?)

For weak interactions, fermions are organized in doublets (weak isospin 1/2)

Leptons :  $e^-$ ,  $\mu^-$ ,  $\tau^-$  m=0.51, 106, 1777 MeV

$\nu_e$ ,  $\nu_\mu$ ,  $\nu_\tau$  m~ eV?????

Quarks :  $u$ ,  $c$ ,  $t$   
 $u$ ,  $c$ ,  $t$  m ~1, 1500, 175000 MeV

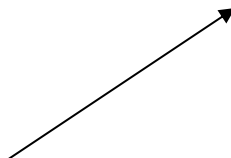
$u$ ,  $c$ ,  $t$

$d$ ,  $s$ ,  $b$

$d$ ,  $s$ ,  $b$  m=1, 170, 5000 MeV

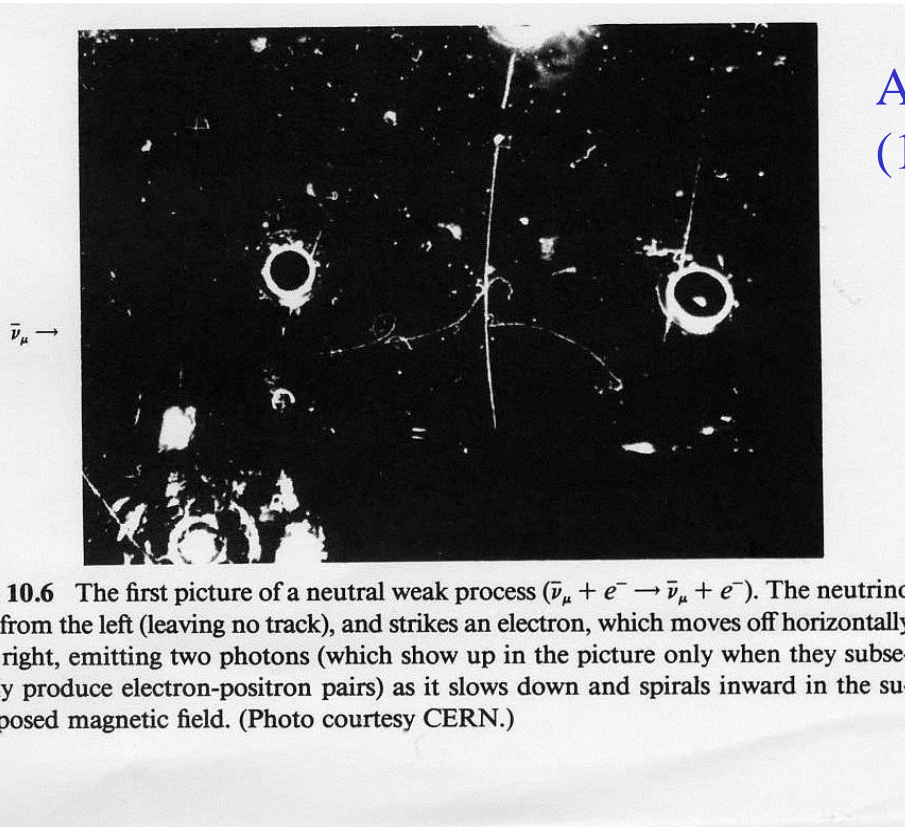
$d$ ,  $s$ ,  $b$

3 “color” charges



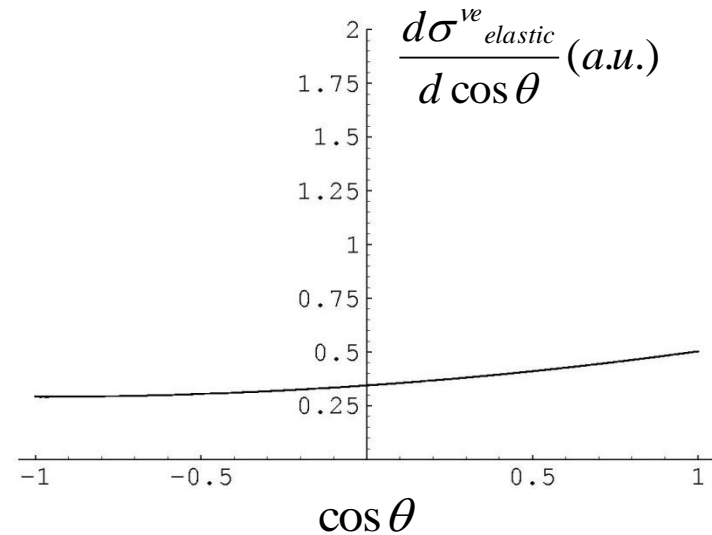
# To build an unified EW theory we need also a weak neutral interaction

A neutral current event in bubble chamber (1973)



**Figure 10.6** The first picture of a neutral weak process ( $\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$ ). The neutrino enters from the left (leaving no track), and strikes an electron, which moves off horizontally to the right, emitting two photons (which show up in the picture only when they subsequently produce electron-positron pairs) as it slows down and spirals inward in the superimposed magnetic field. (Photo courtesy CERN.)

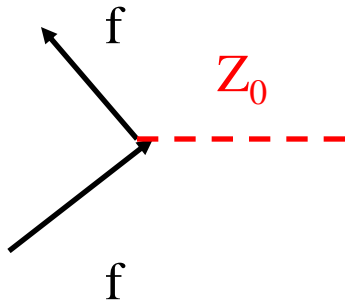
$$\bar{\nu}_\mu e^- \rightarrow \bar{\nu}_\mu e^-$$



Note that in the scattering  $\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-$  would contribute both the neutral current interaction (Z) and the charged current one (W)



# Basics of weak neutral currents



**Mediated by an heavy neutral boson :  $Z_0$**

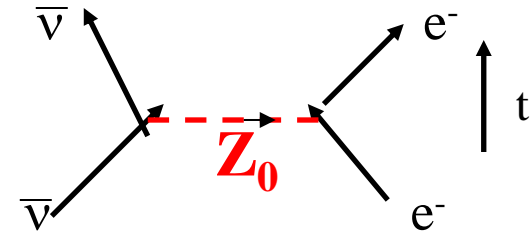
Initial and final fermions are the same:

$$\mu^- \rightarrow \mu^- Z_0; s \rightarrow s Z_0$$

1973: events measured in bubble chamber (Gargamelle)

$\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e$  event with only one electron

$\bar{\nu}_\mu N \rightarrow \bar{\nu}_\mu N; \nu_\mu N \rightarrow \nu_\mu N$  event with only hadrons (no muon)



**Facts:** neutral current cross sections are  $\approx 1/3$  of those equivalent of charged currents

Moreover the current is not pure (V-A):  $(\gamma_\mu(1-\gamma_5))$ : the coefficient of the vector and axial parts are not the same

$$-i \frac{g_Z}{2} \gamma_\mu [C_V^f - C_A^f \gamma_5] \quad \text{The coefficients } C_V \text{ e } C_A \text{ depend on fermion type}$$

**A unified electro weak model has to be built (GWS):** a new parameter :  $\theta_W$

Connecting “ $g_W$ ”, “ $g_Z$ ” and “ $e$ ”:

$$g_e = e; g_W = \frac{g_e}{\sin \theta_W}; g_Z = \frac{g_e}{\sin \theta_W \cos \theta_W}$$

Leptons and quarks organized in weak isospin doublets  $(T, T_3)$ :

$$\begin{matrix} T_3 = 1/2 \\ T_3 = -1/2 \end{matrix} \quad \begin{bmatrix} \nu_e \\ e \end{bmatrix} \quad \begin{bmatrix} \nu_\mu \\ \mu \end{bmatrix} \quad \begin{bmatrix} \nu_\tau \\ \tau \end{bmatrix} \quad \begin{bmatrix} u \\ d \end{bmatrix} \quad \begin{bmatrix} c \\ s \end{bmatrix} \quad \begin{bmatrix} t \\ b \end{bmatrix}$$

# Towards an electroweak unification

There is a general principle in particle quantum field theory:

**the local gauge invariance:**

-For the electrodynamics this implies the invariance of the lagrangian if the particle wave functions are multiplied by a local phase:  $e^{i\theta(x)}$ . This corresponds to a symmetry invariance of the group U(1)

-For the weak interactions the wave function is a doublet ex: (ν, e) and the invariance is for rotations in the space of the weak isotopic spin (same algebra of the ordinary 1/2 spin) : the corresponding symmetry group is SU(2).

For an unified electroweak theory the lagrangian should be invariant for both:

**U(1) x SU(2)**

Electric charge conservation  
(Hypercharge:  $Y=Q-T_3$ )

Weak isospin, T conservation

# Basics The GWS unification model

U(1) implies a isosinglet massless boson  $B_\mu$

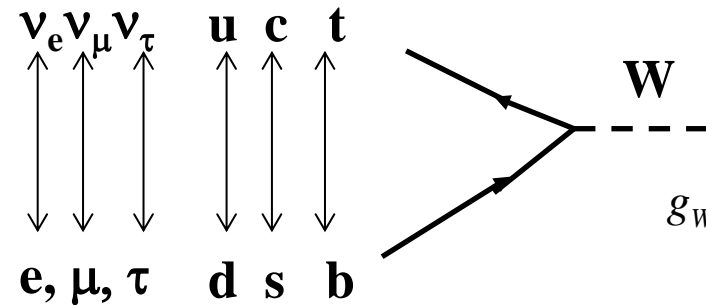
SU(2) implies a isovector triplet:  $\vec{W} = (W_\mu^1, W_\mu^2, W_\mu^3)$

SU(2)xU(1) is broken : the neutral state (B, W<sup>3</sup>) mix to give the physical states

$$A_\mu = B_\mu \cos \theta_W + W_\mu^3 \sin \theta_W$$

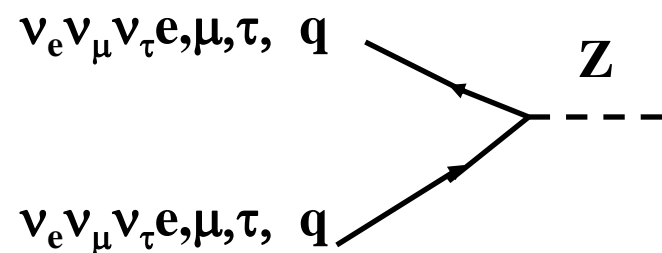
$$Z_\mu = -B_\mu \sin \theta_W + W_\mu^3 \cos \theta_W$$

$A_\mu$  e  $Z_\mu$  are the physical observable states,  
 $\sin \theta_W$  is a free parameter



Universal charged current connects lepton and quark doublets with coupling  $g_W$  (V-A) :  $g_W(1-\gamma_5)$

$$g_W = \frac{e}{\sin \theta_W} \quad \gamma^5 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \frac{1}{2}(1-\gamma^5) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \text{select spinor of given helicity, } \left. \begin{matrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{matrix} \right\}$$



Neutral current connects the same flavour leptons and quarks depending on charge, isospin,  $\sin \theta_W$  and with coupling :

$$g_Z (V-A\gamma_5) \quad g_Z = \frac{e}{\sin \theta_W \cos \theta_W}, V = T_3 - 2q \sin^2 \theta_W, A = T_3$$

# Minimal model

$$g_W = \frac{e}{\sin \theta_W}, g_Z = \frac{e}{\sin \theta_W \cos \theta_W}, \frac{g_W^2}{8M_W^2} = \frac{G_F}{\sqrt{2}}, M_Z = \frac{M_W}{\cos \theta_W}$$

$$\text{Parameter } \rho = \frac{\left(\frac{g_Z^2}{M_Z^2}\right)}{\left(\frac{g_W^2}{M_W^2}\right)} = 1$$

Particles mediating the interactions  $W^\pm, Z, \gamma$  massless so far!!!

An additional scalar field doublet, the Higgs, is required to provide masses to W and Z  
(see Marumi Kado lecture)

The model parameters are 3, ex :  $\alpha, G_F, \sin^2 \theta_W$

Theory is determined  $\alpha = \frac{e^2}{4\pi\hbar c} = \frac{1}{137.035999074(44)}$  (a  $Q^2 = m_e^2$ ) 0.000032 ppm (atomic physics)

$G_F = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}$  (muon decay) 0.5ppm

$\sin^2 \theta_W (M_Z) = 0.23116(12)$  520 ppm

if we know :  $\alpha, G_F, \sin^2 \theta_W$

we can predict :

$$M_W = \left[ \frac{\pi\alpha}{\sqrt{2}G_F} \right]^{\frac{1}{2}} \frac{1}{\sin \theta_W} \approx 78 \text{ GeV}$$

$$M_Z = \frac{M_W}{\cos \theta_W} \approx 89 \text{ GeV}$$



# The search for W/Z bosons

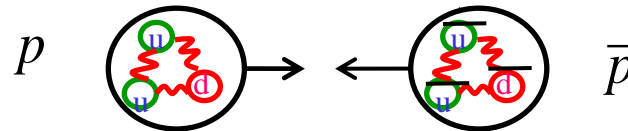
Masses  $\sim 80\text{-}90\text{ GeV}$ .

1980: highest accelerator cms energy  $\sqrt{s}$ , at fixed target (SPS at CERN):

$$\sqrt{s} = \sqrt{2m_N E} \approx 30\text{GeV} (E = 450\text{ GeV})$$

But if you collide head-on:  $\sqrt{s} = 2E \approx 900\text{GeV}$

Use SPS as proton-antiproton Collider (Rubbia- Van Der Meer)



Annihilate quark and antiquark to produce W/Z

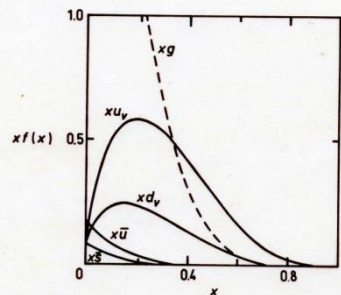
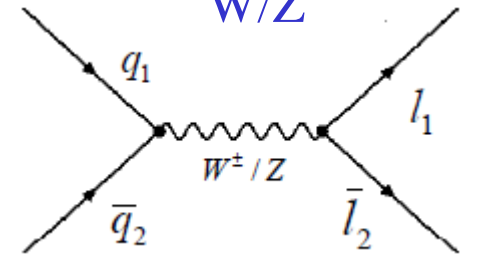


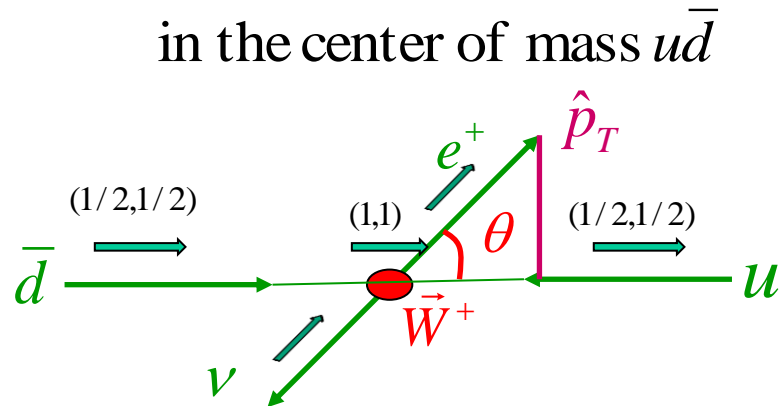
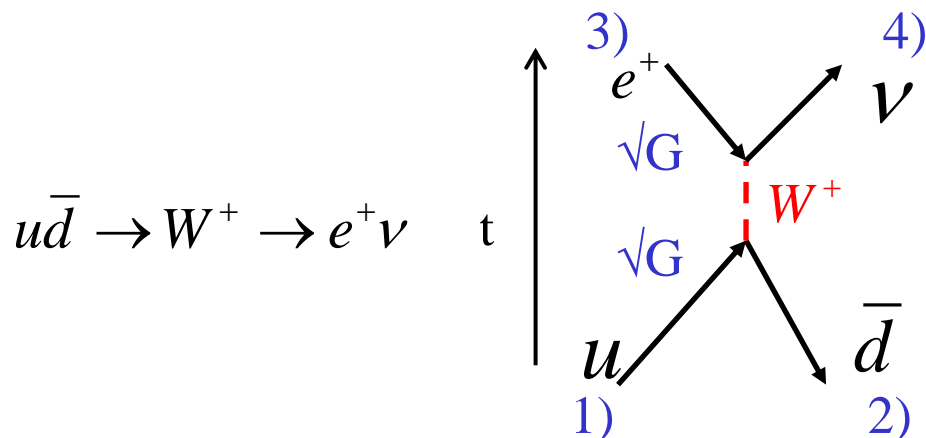
Fig. 2.9. Parton momentum distribution functions for the proton,  $z f(z)$ . (From Gen. Ref. 3.)

50% momentum carried by quarks and 50% by gluons. Average momentum budget :

$$\sqrt{\hat{s}} = \sqrt{x_1 x_2 s} \xrightarrow{\text{balanced momenta } x_1 = x_2 = x} x\sqrt{s}$$

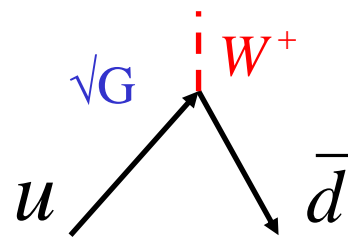
$$\xrightarrow{x=1/6} \frac{900}{6} = 150\text{GeV}$$

# W-measurement



Angular distribution  $\frac{d\sigma}{d\cos\theta} \propto (1 + \cos\theta)^2$  (angular momentum conservation effect)

Inclusive cross section for W production  $u\bar{d} \rightarrow W^+$



Rough order of magnitude of cross section: “G” :G has dimension  $\text{GeV}^{-2}$

We need a cross section:  $\text{cm}^2$

a length can be expressed in  $\text{GeV}^{-1}$  :

$$\lambda_c = \hbar/mc \text{ if } mc^2 = 1\text{GeV}, 1\text{GeV}^{-1} \approx 0.2 \cdot 10^{-13} \text{cm}; 1\text{GeV}^{-2} \approx 4 \cdot 10^{-28} \text{cm}^2$$

$$\sigma(u\bar{d} \rightarrow W^+) \approx (?)G \approx 10^{-5} \text{ GeV}^2 \approx 10^{-33} \text{ cm}^2 = 1 \text{ nb}$$

Better calculation: take into account PDF's, available phase space, angular distribution,...

$$p\bar{p}, \sqrt{s} = 630 \text{ GeV}, \sigma(W^\pm) \approx 3 \text{ nb (ud)}$$

$$p\bar{p}, \sqrt{s} = 2 \text{ TeV}, \sigma(W^\pm) \approx 20 \text{ nb (ud)}$$

$$pp, \sqrt{s} = 7 \text{ TeV}, \sigma(W^+) \approx 56 \text{ nb}$$

$$pp, \sqrt{s} = 7 \text{ TeV}, \sigma(W^-) \approx 40 \text{ nb}$$

$$pp, \sqrt{s} = 14 \text{ TeV}, \sigma(W^\pm) \approx 150 \text{ nb (} u\bar{d} + \bar{d}u \text{)}$$

Measure the leptonic decay:  $W \rightarrow e\nu$

further 0.1 factor (leptonic BR)

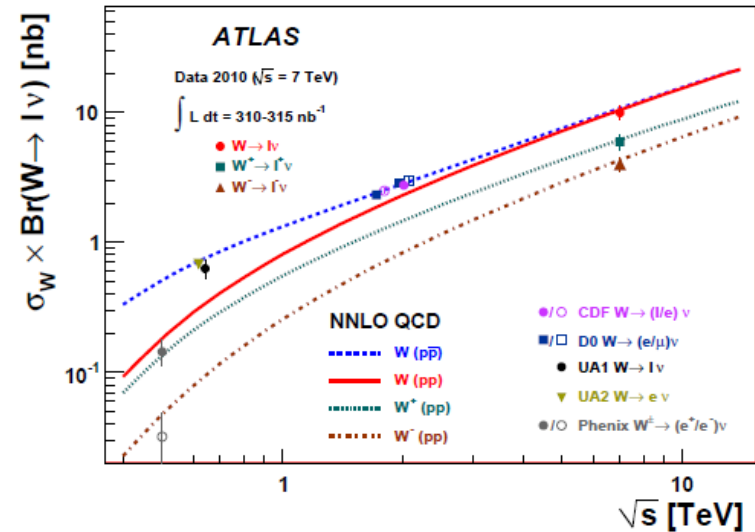


Fig. 12: The measured values of  $\sigma_W \cdot \text{BR}(W \rightarrow \ell\nu)$  for  $W^+$ ,  $W^-$  and for their sum compared to the theoretical predictions based on NNLO QCD calculations (see text). Results are shown for the combined electron-muon results. The predictions are shown for both proton-proton ( $W^+$ ,  $W^-$  and their sum) and proton-antiproton colliders ( $W$ ) as a function of  $\sqrt{s}$ . In addition, previous measurements at proton-antiproton and proton-proton colliders are shown. The data points at the various energies are staggered to improve readability. The CDF and D0 measurements are shown for both Tevatron collider energies,  $\sqrt{s} = 1.8 \text{ TeV}$  and  $\sqrt{s} = 1.96 \text{ TeV}$ . All data points are displayed with their total uncertainty. The theoretical uncertainties are not shown.

Questions:

- How to realize a p-pbar collider
- At a collider the collision rate is :

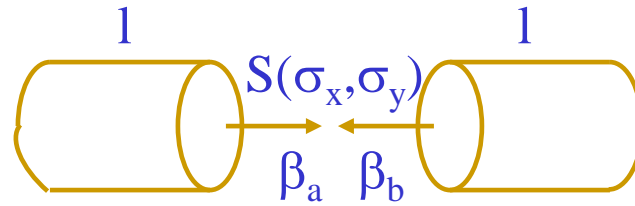
$$R = \sigma \cdot L, L \equiv \text{luminosity} : \text{cm}^{-2} \text{s}^{-1}; \Rightarrow \text{if } \sigma = 10^{-33} \text{ cm}^2 \text{ and } R = 1 \text{ s}^{-1}$$

$$\Rightarrow L = 10^{33} \text{ cm}^{-2} \text{s}^{-1}$$

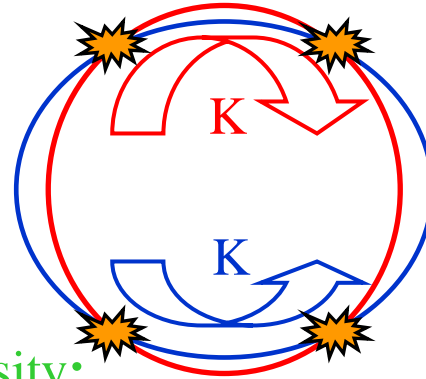
L depends on the accelerator and is proportional to the number of colliding particles:  
how to get a sufficient number of antiprotons??

Note that the difference of crosssections  $p\bar{p}$  and  $pp$  tend to vanish as energy increases

The colliding beams are structured in bunches of particles



$$L = \frac{n_a n_b}{4\pi\sigma_x \sigma_y} K \cdot f$$



We introduce also an integrated luminosity:

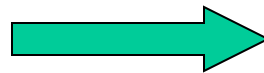
$$N = \sigma \int L dt \quad \left( \int L dt : \text{dimension cm}^{-2} \right)$$

Ex.  $n_a = n_b = 10^{10}$

$K=2$

$\sigma_x = \sigma_y = 1 \text{ mm}$

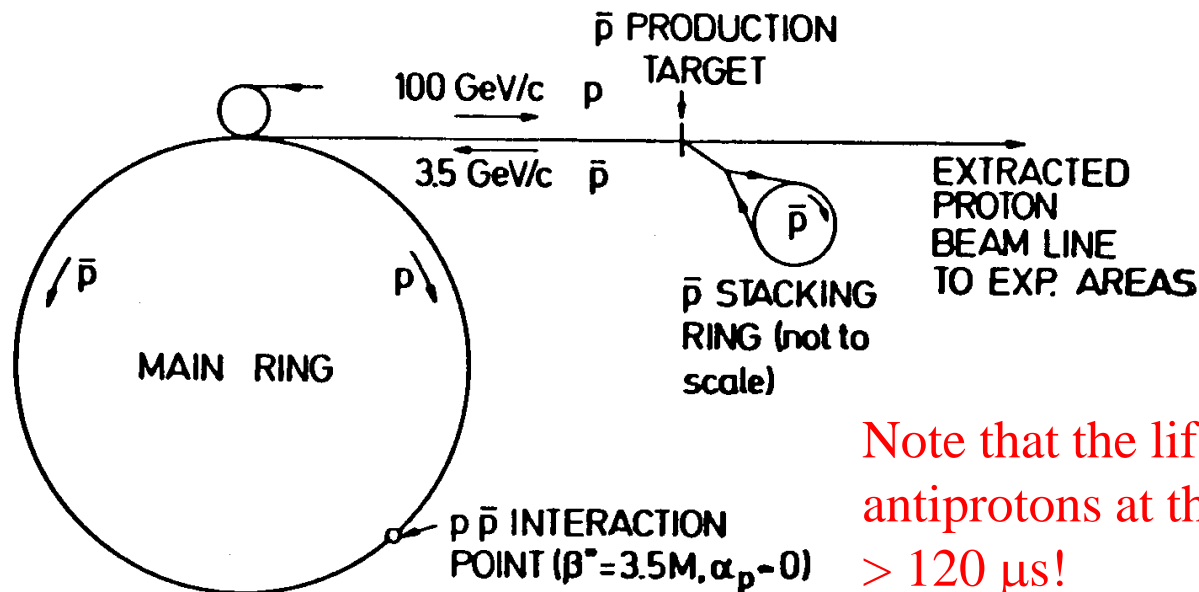
$f=43\text{KHz}$



$L \sim 10^{30} \text{ cm}^{-2} \text{ s}^{-1}$

# The proposed PBAR-P collider

Scheme to transform a fixed target accelerator into a p-pbar collider : C. Rubbia, D.Cline e P. Mac Intyre for the Main Ring of 450 GeV at Fermilab in 1974.



Note that the lifetime of antiprotons at that time was only  $> 120 \mu\text{s}$ !

Fig. 5. General layout of the  $p\bar{p}$  colliding scheme, from Ref. [9]. Protons ( $100 \text{ GeV}/c$ ) are periodically extracted in short bursts and produce  $3.5 \text{ GeV}/c$  antiprotons, which are accumulated and cooled in the small stacking ring. Then  $\bar{p}$ 's are reinjected in an RF bucket of the main ring and accelerated to top energy. They collide head on against a bunch filled with protons of equal energy and rotating in the opposite direction.

# The PBAR-P COLLIDER OF THE CERN SpS

Need to build:

- an antiproton source
- a system to compact antiprotons both in angle and momentum:

**the stochastic cooling**

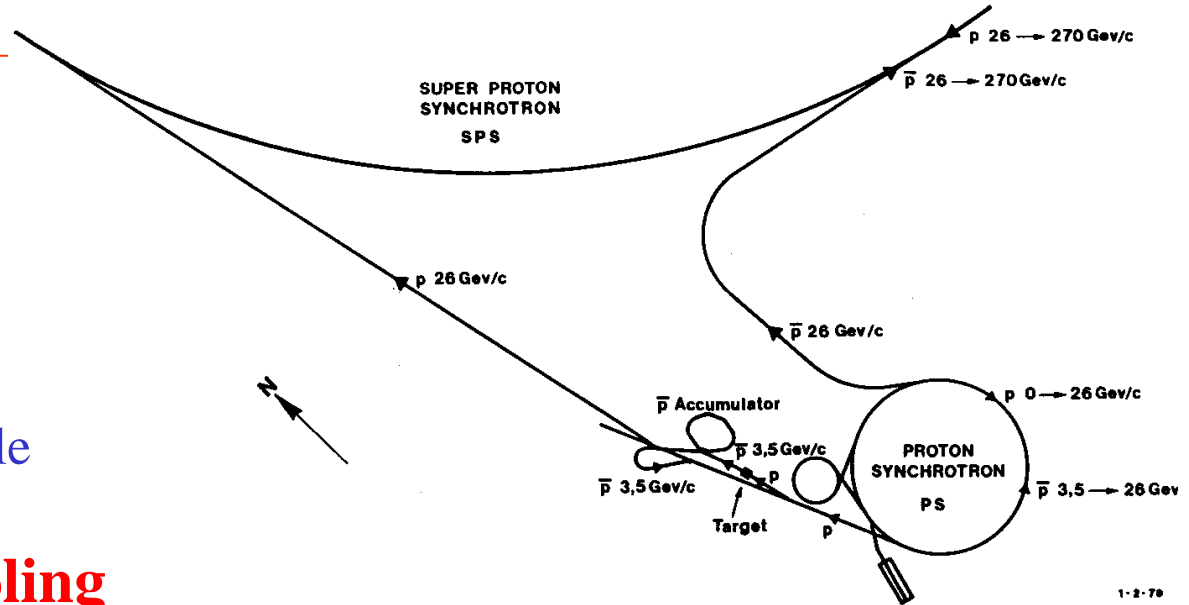


Fig. 1. Overall layout of the  $p\bar{p}$  project.

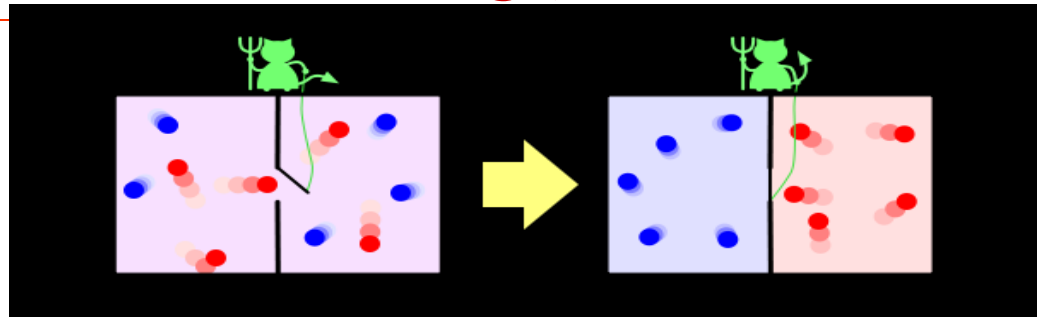
*Linac 50 MeV → Booster 800 MeV → PS 26 GeV*

→ Target :  $\frac{\bar{p}}{p} \approx 10^{-6}$  → magnetic lens →

→ AA + AC ( $\approx 10^{12} \bar{p}/\text{day}$ ) → SpS 315 GeV

Moel et al., Physics Reports 58, No.2 (1980), p.73.

# Stochastic cooling (Maxwell's demon)



Two pick-up measure the transverse and longitudinal deviation of particles from the ideal orbit. A correction signal (kicker) is applied, in average after an appropriate delay on the orbit of the particles

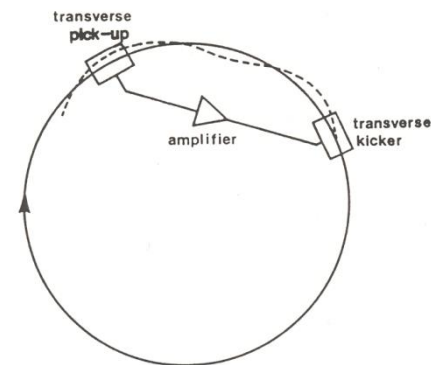


FIG. 2. Cooling of the horizontal betatron oscillation of a single particle.

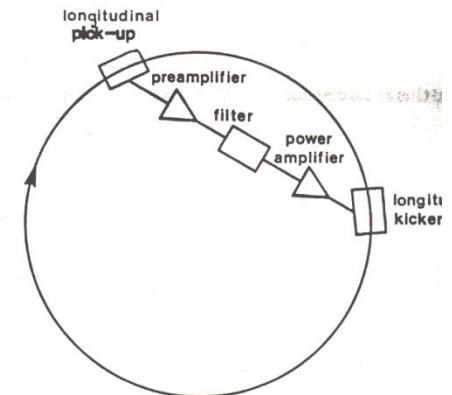


FIG. 7. Filter cooling.

D. Mohl, Stochastic Cooling for Beginners, CERN 84-15, 1984, p.97  
S. van der Meer, Stochastic Cooling and the Accumulation of Antiprotons, Rev. Mod. Physics, Vol 57, No.3, part 1, July 1985.

# Integrated luminosity at the SPS collider

$$\sigma(\bar{p}p \rightarrow Z(Z \rightarrow e^+e^-) + X) \sim 0.05 \text{ nb}$$
$$\sigma(\bar{p}p \rightarrow W^+(W^+ \rightarrow e^+\nu) + X) \sim 0.50 \text{ nb}$$

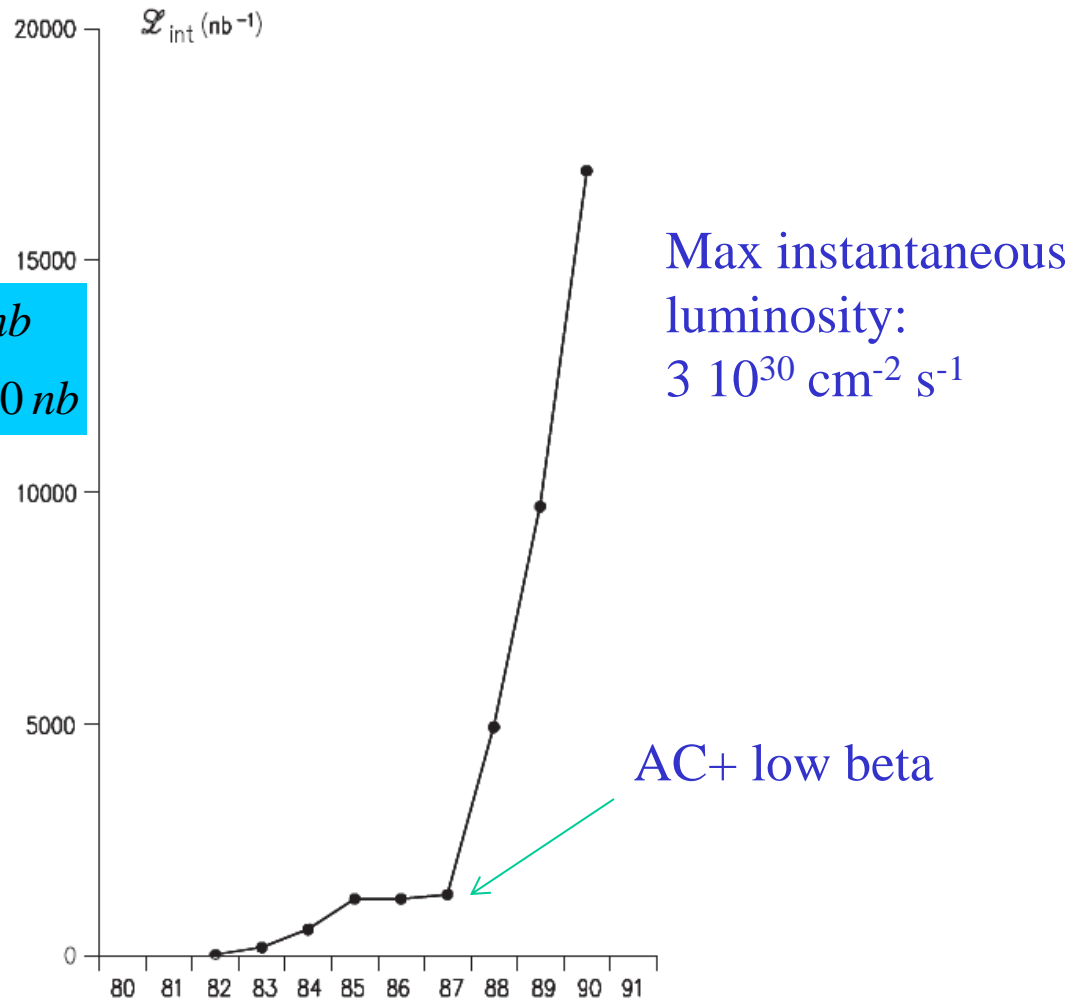


Fig. 8. Integrated luminosity of the SPS Collider, from 1982 (first year of routine operation) to 1990 (last full operation). 1980 was the year of AA running-in, 1981 of Collider and detector tests. The luminosity integrated over 1982 and 1983 appears tiny, but sufficed to detect the  $W$  and  $Z$  and bring the Nobel prize 1984 to CERN. The break in 1986 was due to the repair of UA1 and the beginning of AC installation. AC running-in was completed in 1987, with only a short Collider run at the end of the year. From 1988 onwards, the effect of the AC and the improvements made to the SPS came to bear.



# SPSC Collider story

Year	Collision Energy (GeV)	Peak luminosity ( $\text{cm}^{-2} \text{s}^{-1}$ )	Integrated luminosity ( $\text{cm}^{-2}$ )
1981	546	$\sim 10^{27}$	$2.0 \times 10^{32}$
1982	546	$5 \times 10^{28}$	$2.8 \times 10^{34}$
1983	546	$1.7 \times 10^{29}$	$1.5 \times 10^{35}$
1984-85	630	$3.9 \times 10^{29}$	$1.0 \times 10^{36}$
1987-90	630	$\sim 2 \times 10^{30}$	$1.6 \times 10^{37}$

← W discovery

← Z discovery

**1991: END OPERATIONS**

# Two detectors at the SpSC to measure $W/Z$

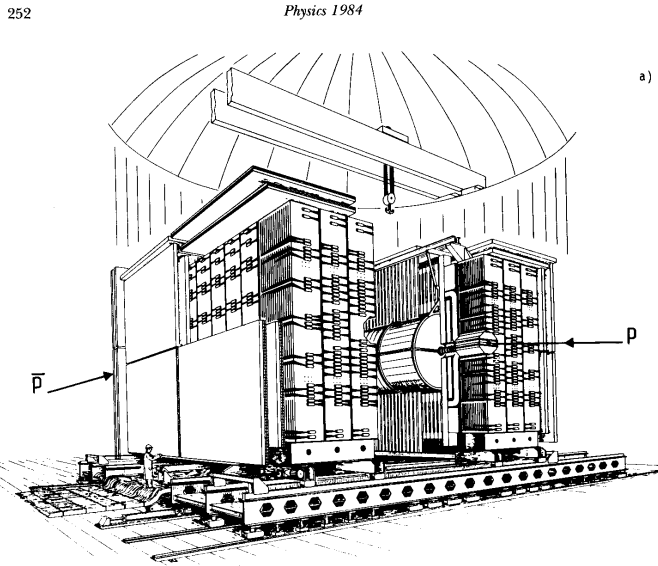
Measure the leptonic decays of

$$W \rightarrow e\nu_e, \mu\nu_\mu, Z \rightarrow e^+e^-, \mu^+\mu^-$$

**UA1**, calorimeters and central dipolar magnetic field + muon detection

**UA2**, calorimeters, no central magnetic field

Calorimeter with projective towers



ie 122B, number 5,6

PHYSICS LETTERS

17 Marc

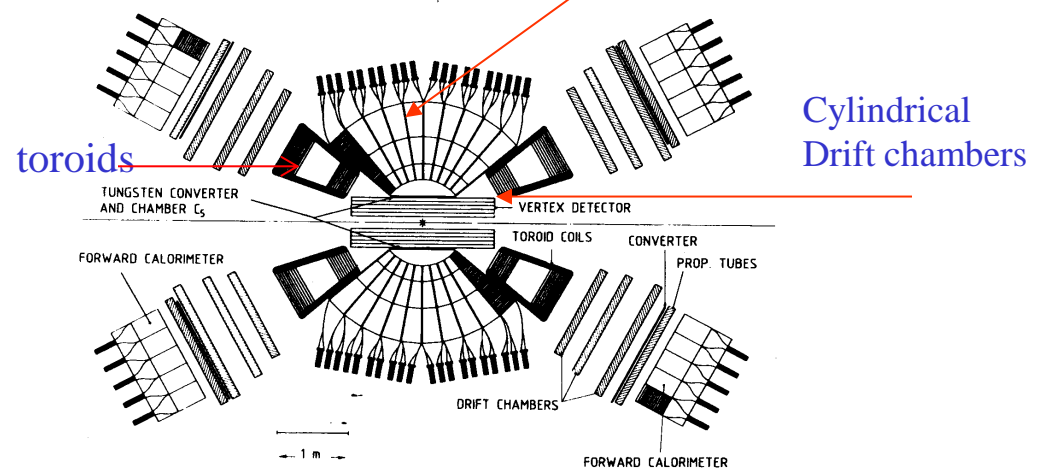
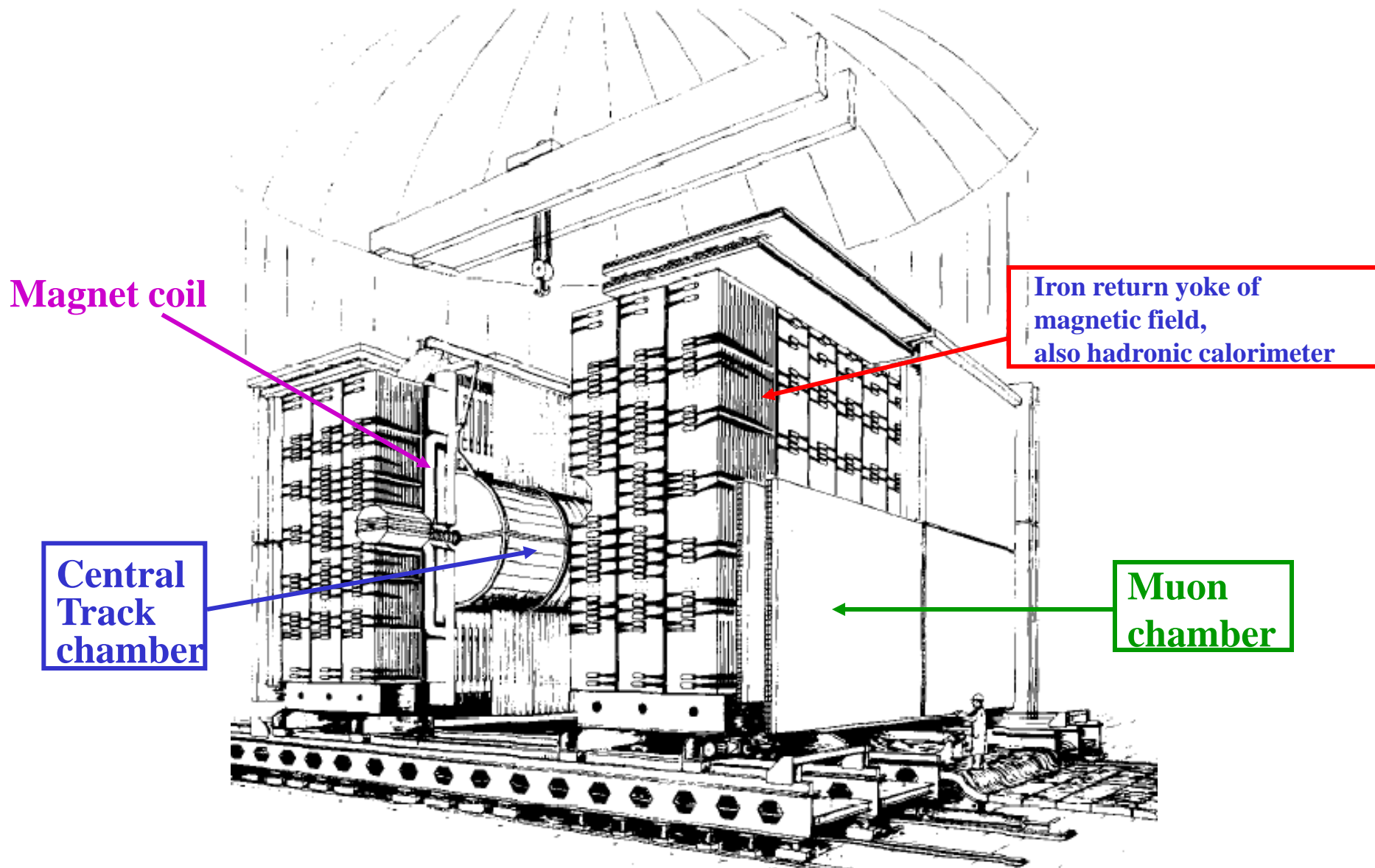


Fig. 1. The UA2 detector: Schematic cross section in the vertical plane containing the beam.

# The detector UA1

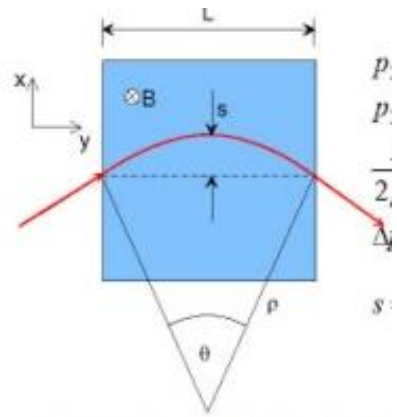
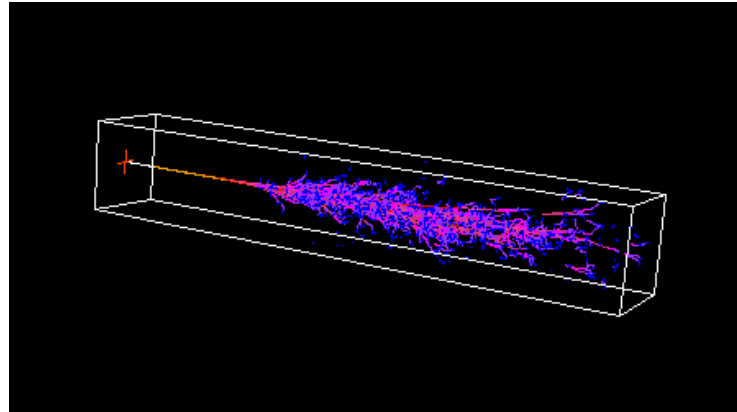


(in the figure the two halves of the dipolar magnet are open)

# Electromagnetic calorimeters to measure electrons and magnetic spectrometer for muons

Calorimeter: typical energy resolution (at that time) of an electromagnetic calorimeter: (UA2)

$$\frac{\Delta E}{E} = \frac{0,15}{\sqrt{E(\text{GeV})}} = 1,5\% \text{ at } E = 100 \text{ GeV} \text{ (ATLAS : } \frac{\Delta E}{E} = \frac{0,10}{\sqrt{E(\text{GeV})}} \text{)}$$

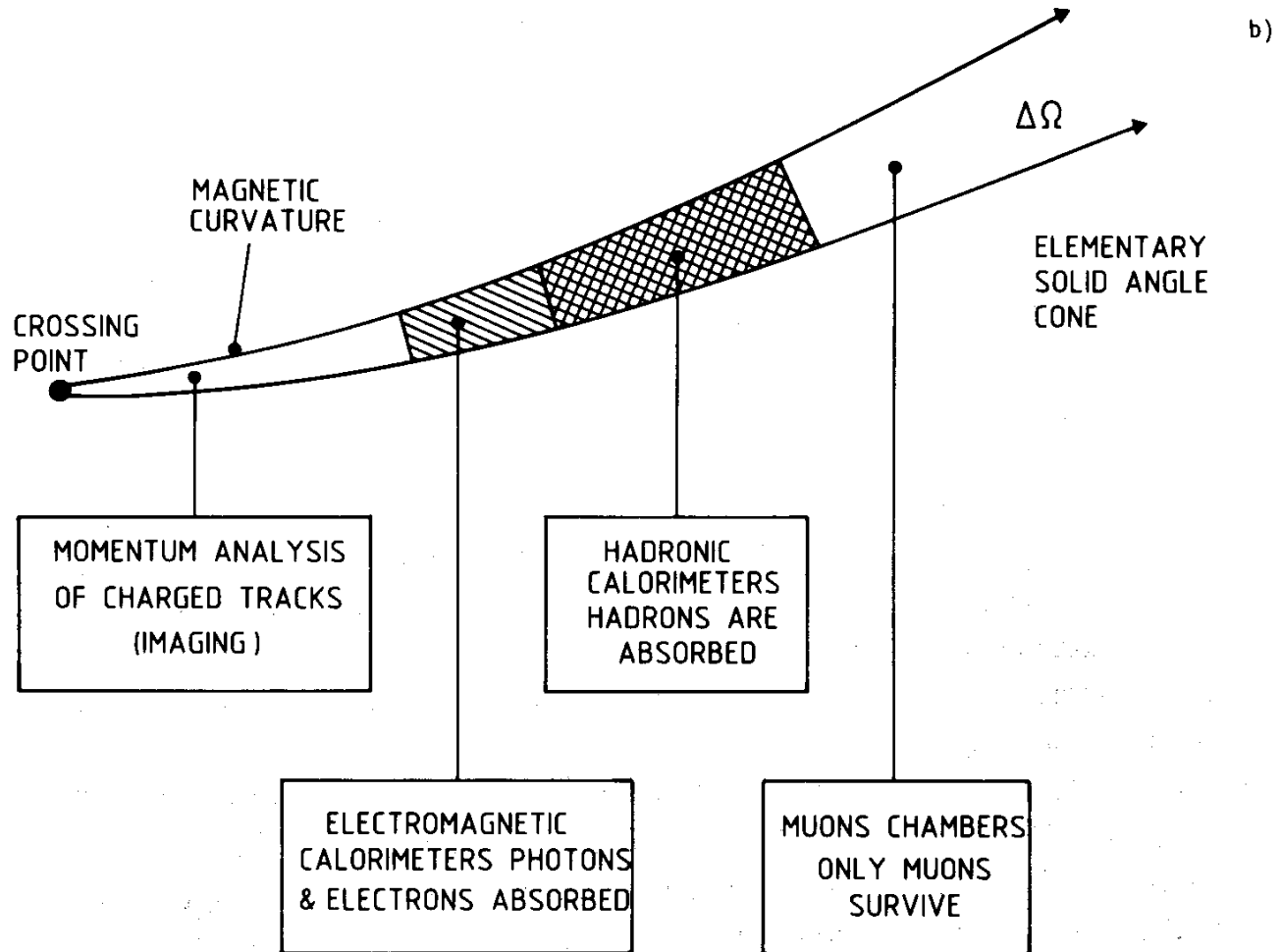


Magnetic spectrometer

$$\frac{\Delta p}{p} = \frac{\Delta E}{E} = 10^{-3} E(\text{GeV}) \text{ at } E = 100 \text{ GeV}, \frac{\Delta E}{E} = 10\%$$

$$\approx 10^{-4} \text{ (at LHC)}$$

# Measurements in UA1



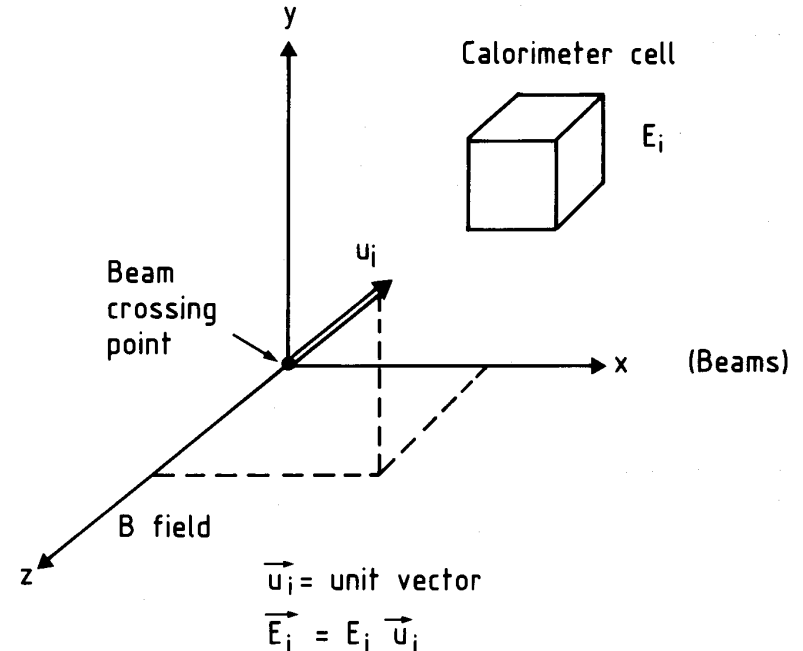
*Fig. 8b.* The schematic functions of each of the elementary solid-angle elements constituting the detector structure.

# Neutrino transverse energy

Momentum balance is possible only for transverse component: in fact a large fraction of the longitudinal momentum is lost (with large fluctuations) in the vacuum tube of the beams.

More complete ( $4\pi$ , hermetic) and accurate is the calorimeter coverage, better is the measurement of the missing transverse momentum.

## CONSTRUCTION OF ENERGY VECTORS

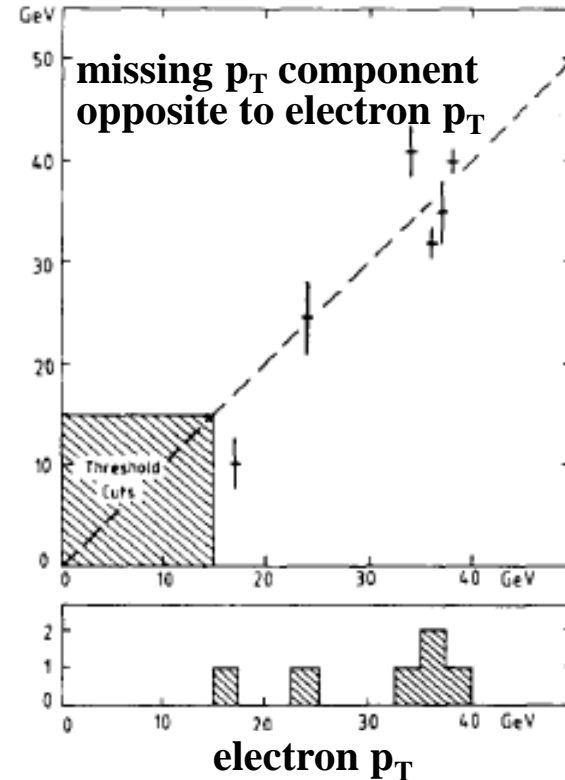
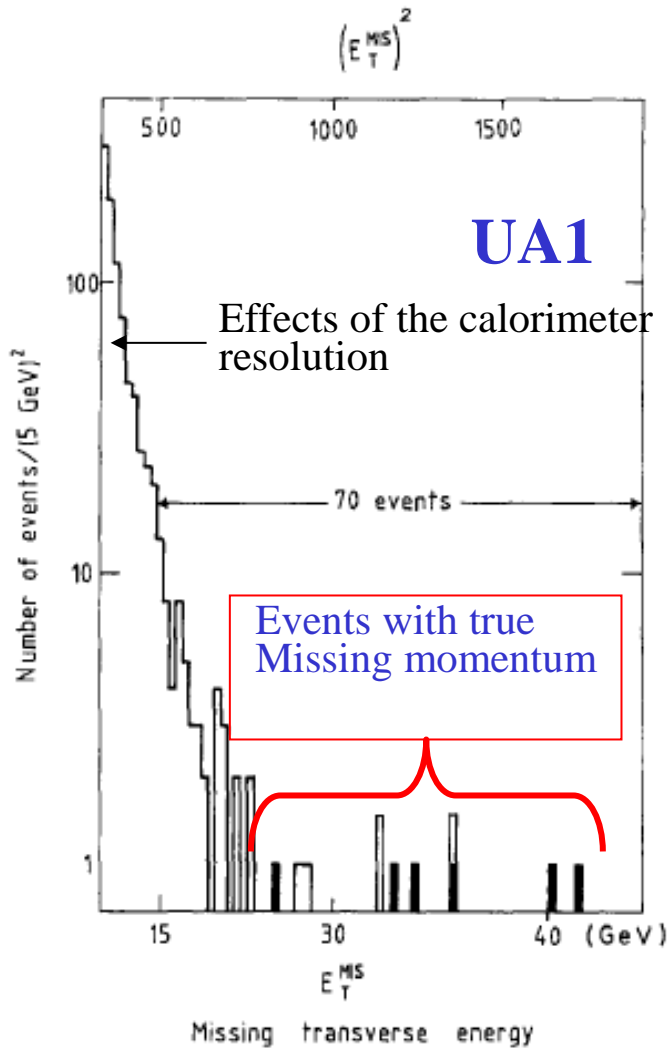


Momentum conservation  $\rightarrow \sum_i \vec{E}_i = 0$   
(ideal detector)

$$\vec{E}_{t,\nu} = -\sum \vec{E}_{t,cells(i)}$$

$$\sum_i |\vec{E}_{t,i}| \equiv H_t = \text{"event temperature"}$$

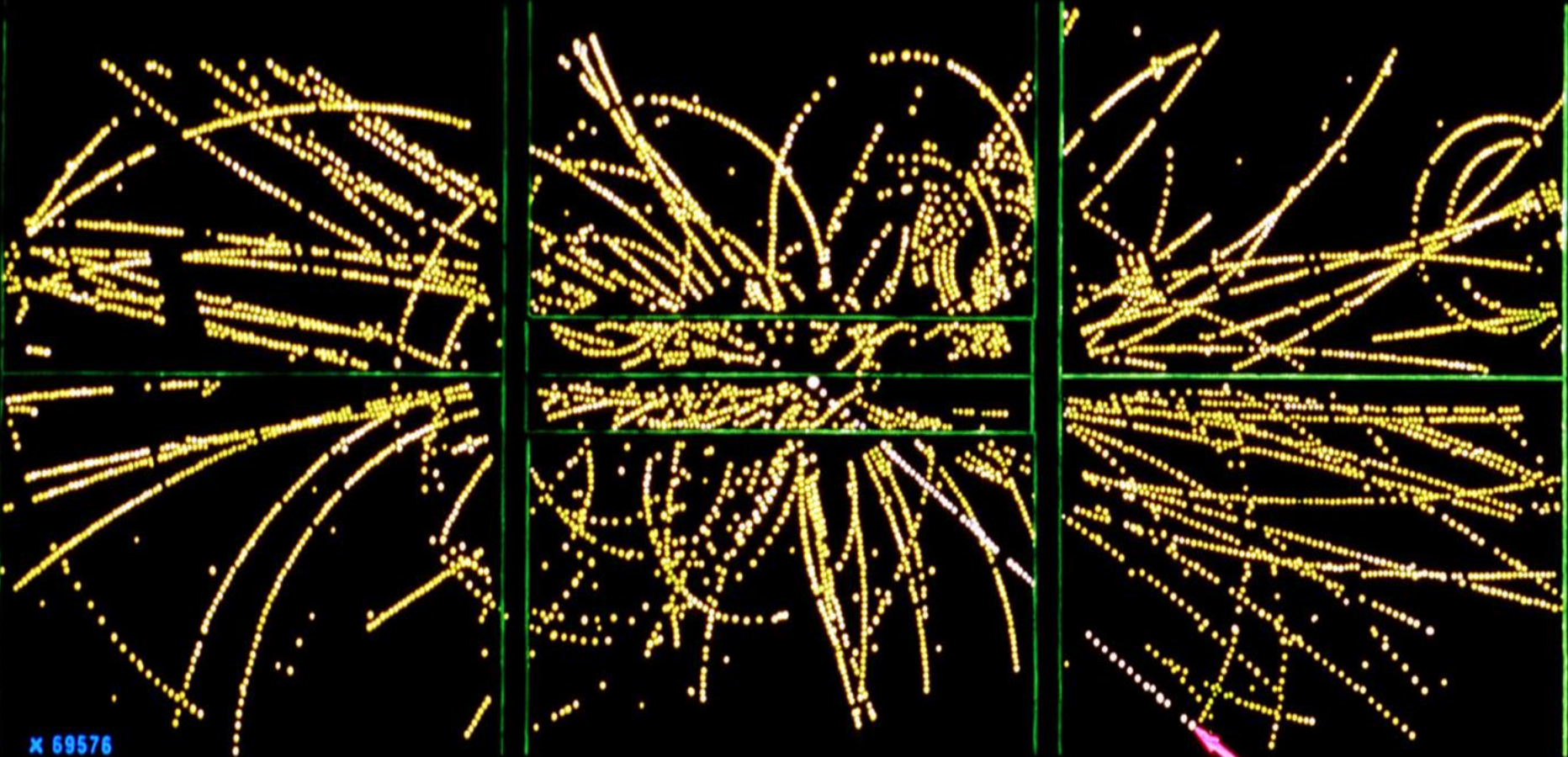
# First $W$ 's measured in UA1



Six events with a large electron  $p_T$  electron balanced by large missing  $p_T$  consistent with  $W \rightarrow e n$  decays

(CERN seminar 20 January, 1983)

EVENT 2958. 1279.

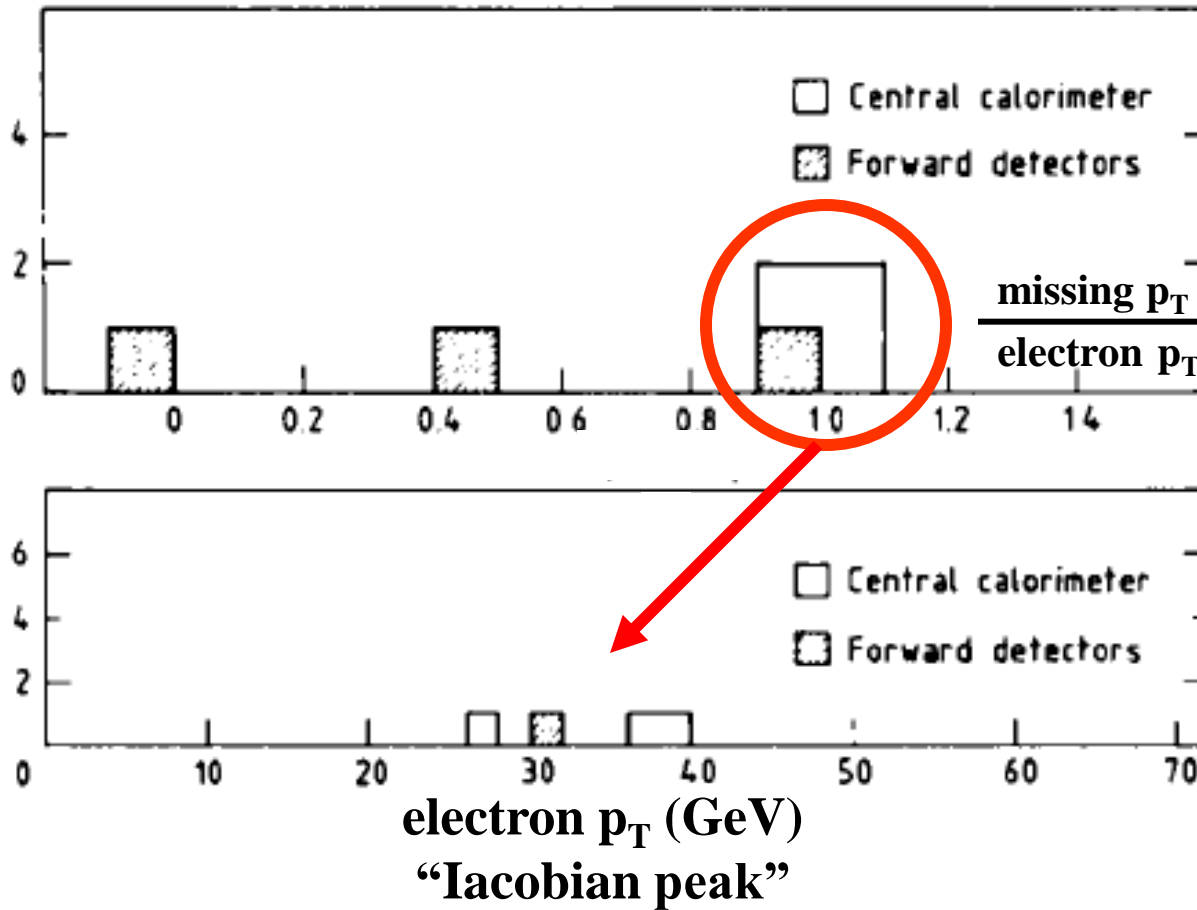


Large  $p_T$  electron from  $W \rightarrow e\nu$



# UA2: result presented at CERN on January 1983

Six events with an electron with  $p_T > 15$  GeV



# UA2 first and second generation

Phys. Lett. 122B, number 5,6

PHYSICS LETTERS

17 March 1984

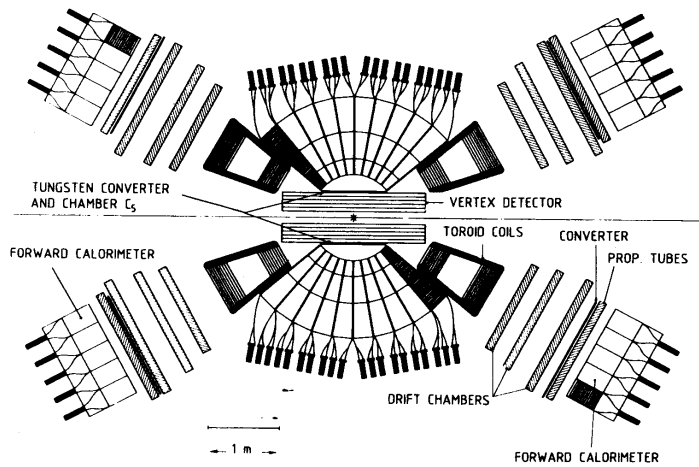
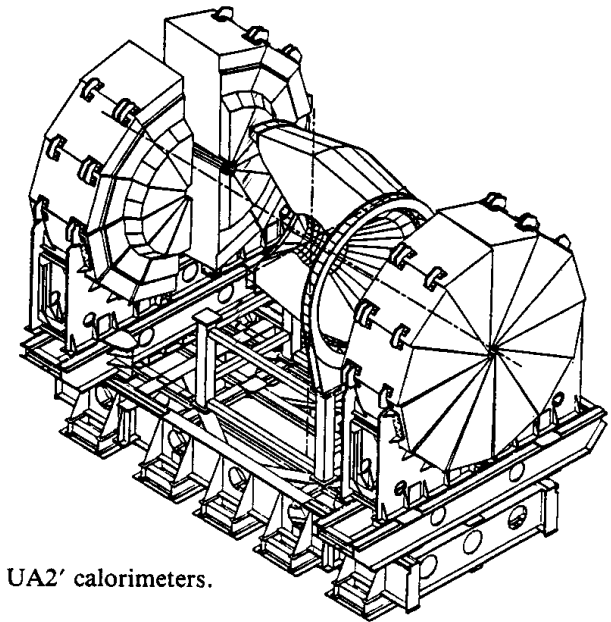
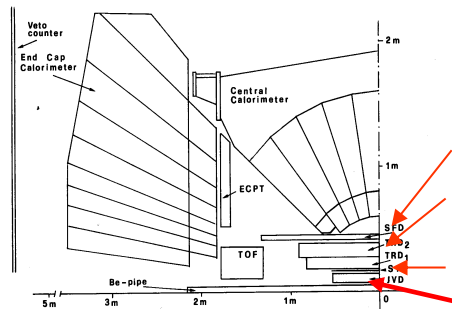


Fig. 1. The UA2 detector: Schematic cross section in the vertical plane containing the beam.



the UA2' calorimeters.



Scintillating fibres

Silicon detector  
Drift chamber vertex chamber

## Particle signature in UA2

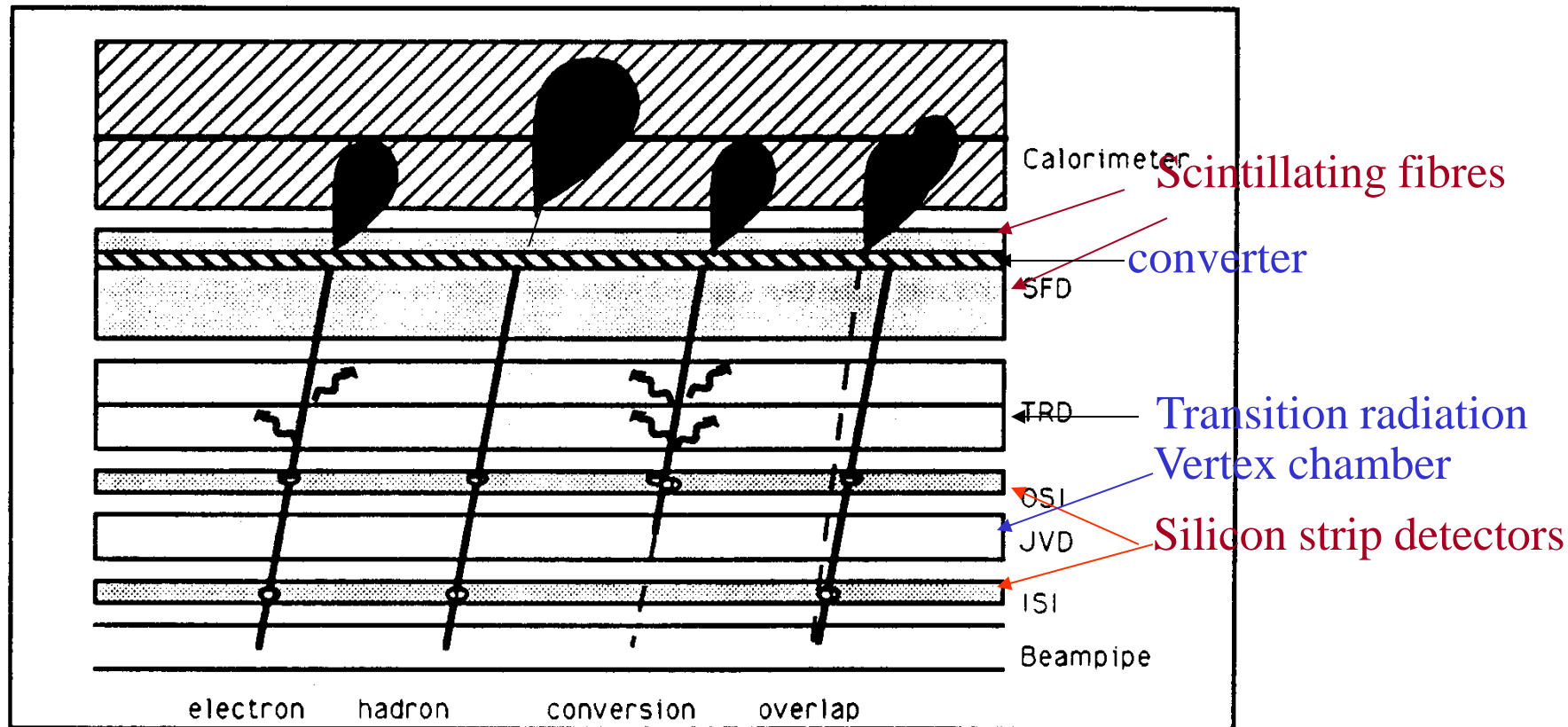


Fig.1 Particle Identification with the Central Detector

# How can we measure W and Z?

Easy for Z's: leptonic channels:

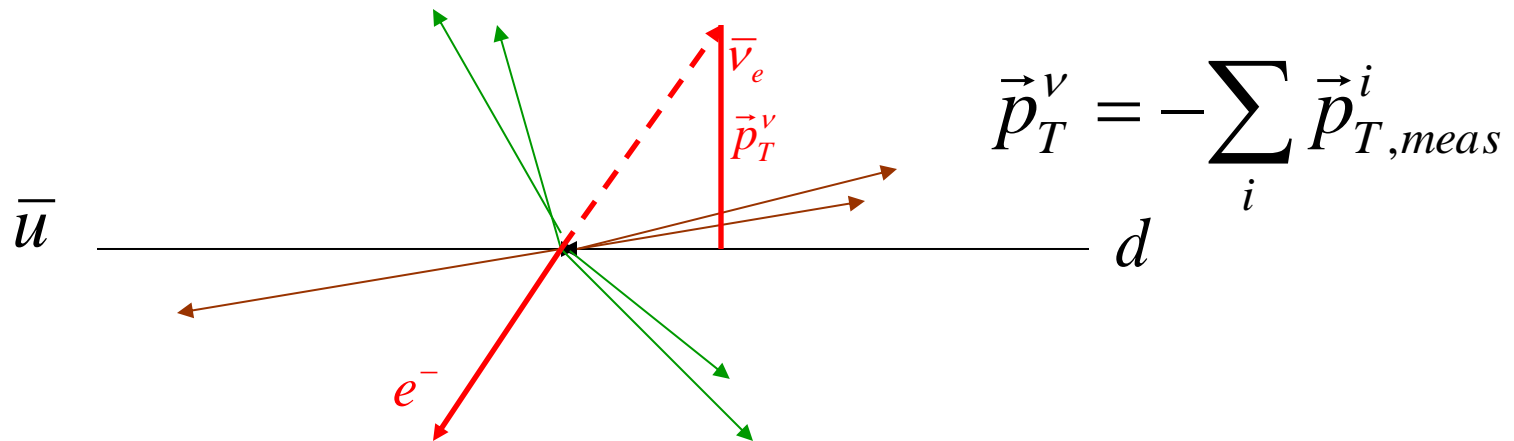
$$Z \rightarrow e^+e^-, \mu^+\mu^-, \tau^+\tau^- \text{ (B.R. } \sim 3.3\%)$$

⇒ Both particles from the decay can be measured

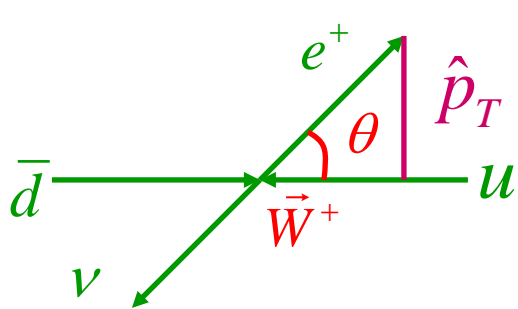
⇒ and the invariant mass can be calculated

Leptonic decay channels of W (B.R. 11%)  $W^- \rightarrow e^- \bar{\nu}_e, \mu^- \bar{\nu}_\mu, \tau^- \bar{\nu}_\tau$

Only the charged lepton is directly measured, of the neutrino, is only measured the **transverse momentum as missing momentum.**



# The annihilation process



$$\hat{p}_T^2 = \frac{\hat{s} \cdot \sin^2 \hat{\theta}}{4}$$

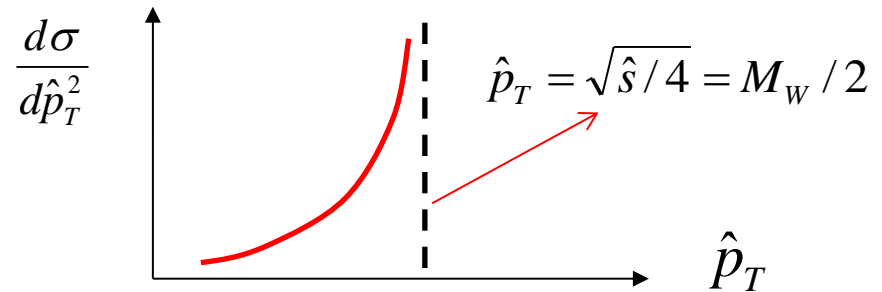
$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d \cos \hat{\theta}} = K(1 + \cos \hat{\theta})^2 \text{ (note: it is null at } \theta = \pi, \text{ helicity)}$$

$$\cos \hat{\theta} = \left(1 - \frac{4\hat{p}_T^2}{\hat{s}}\right)^{1/2} \Rightarrow \frac{d \cos \hat{\theta}}{d\hat{p}_T^2} = -\frac{2}{\hat{s}} \left[1 - \frac{4\hat{p}_T^2}{\hat{s}}\right]^{-1/2} = \frac{-2}{\hat{s} \cos \hat{\theta}} \Rightarrow \frac{d\sigma}{d\hat{p}_T^2} = K \frac{(1 + \cos^2 \hat{\theta})}{\cos \hat{\theta}}$$

Note: the linear term in  $\cos \theta$  vanish: opposite contribute at  $\theta$  and  $(\pi - \theta)$  but the same  $p_T$

$$\frac{d\sigma}{d\hat{p}_T^2} = K \frac{(1 - 2\hat{p}_T^2 / \hat{s})}{(1 - 4\hat{p}_T^2 / \hat{s})^{1/2}}$$

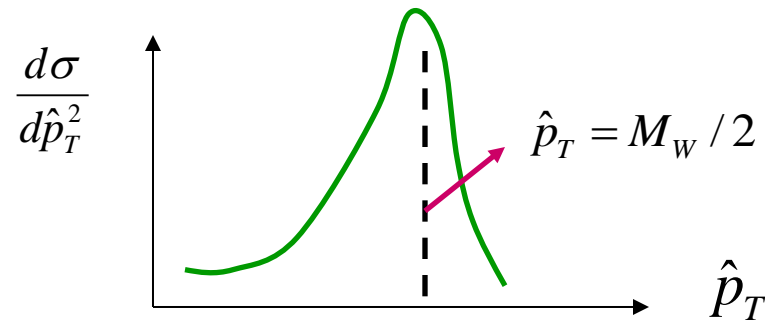
diverges at  $\hat{\theta} = \frac{\pi}{2}$ , or  $\hat{p}_T = \frac{\sqrt{\hat{s}}}{2} \sim M_W / 2$



**Jacobian peak**

# The Iacobian peak

N.B.  $\hat{p}_T = p_T^{lab}$  In the lab divergence “diluted” by the fact that  $\frac{d\sigma}{d\hat{p}_T^2}$  must be convoluted with the resonance shape (BW) which depends on  $\hat{S}=x_1x_2s$ , moreover  $p_T^W$  is not null + experimental effects(resolution).



**Iacobian peak**

The position of the Iacobian peak provides also the W-mass

# The transverse mass

Define:  $m_T^2(e, \nu) = \left[ |\vec{p}_T^e| + |\vec{p}_T^\nu| \right]^2 - (\vec{p}_T^e + \vec{p}_T^\nu)^2 = 2|\vec{p}_T^e||\vec{p}_T^\nu|(1 - \cos \phi_{e\nu})$

$$0 \leq m_T \leq M_W; \text{ se } p_T^W = 0, \vec{p}_T^e = -\vec{p}_T^\nu \Rightarrow m_T = 2|\vec{p}_T^e| = 2|\vec{p}_T^\nu|$$

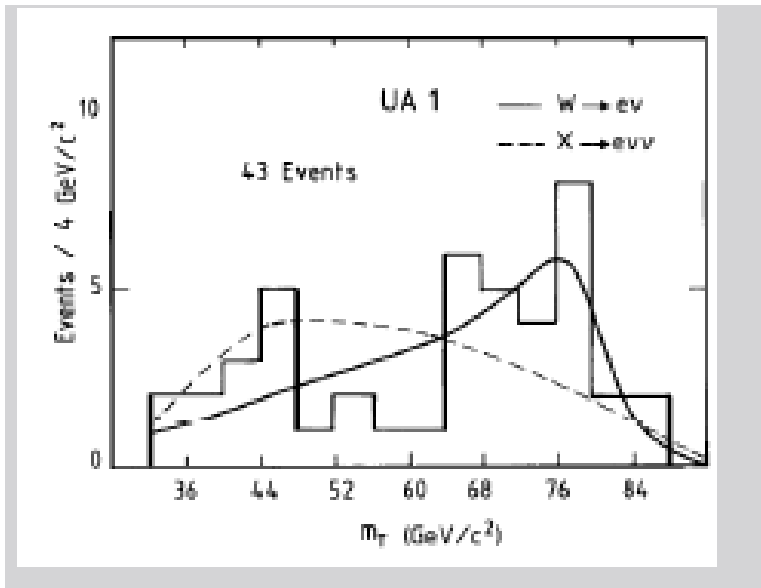
The  $m_T$  distribution is less sensitive than that in  $p_{Te}, p_{T\nu}$ , to the transverse motion of W  
you have corrections  $\propto \beta_{TW}^2$  not to  $\beta_{TW}$

Similarly with the distribution in  $p_{Te}$

$$\frac{d\sigma}{dm_T^2} = \frac{|V_{qq'}|^2}{4\pi} \left[ \frac{GM_W^2}{\sqrt{2}} \right]^2 \frac{1}{(\hat{s} - M_W^2) + (\Gamma_W M_W)^2} \frac{2 - m_T^2/\hat{s}}{(1 - m_T^2/\hat{s})^{1/2}}$$

Jacobian peak at  $\hat{s} = M_W^2 \sim m_T^2$

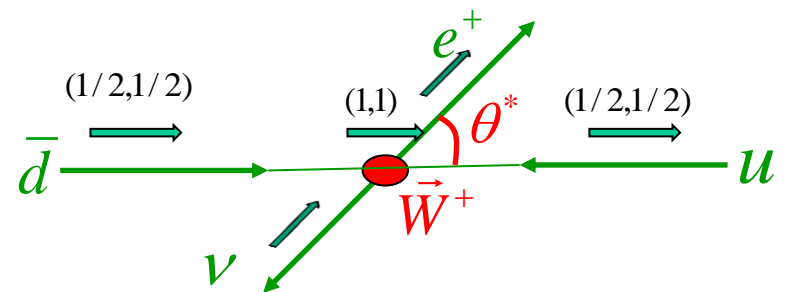
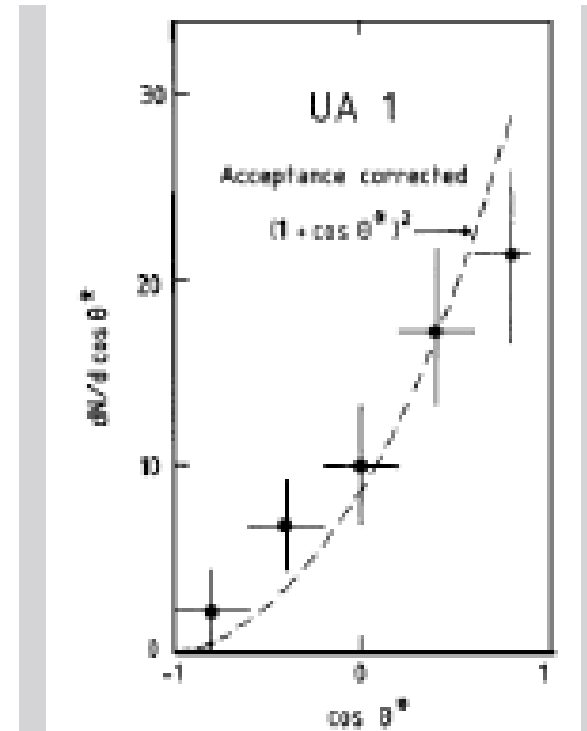
# The UA1 Iacobian Peak



$$M_W = 80.9 \pm 1.5 \text{ GeV}$$

$$\text{PDG} : M_W = 80.385 \pm 0.015 \text{ GeV},$$

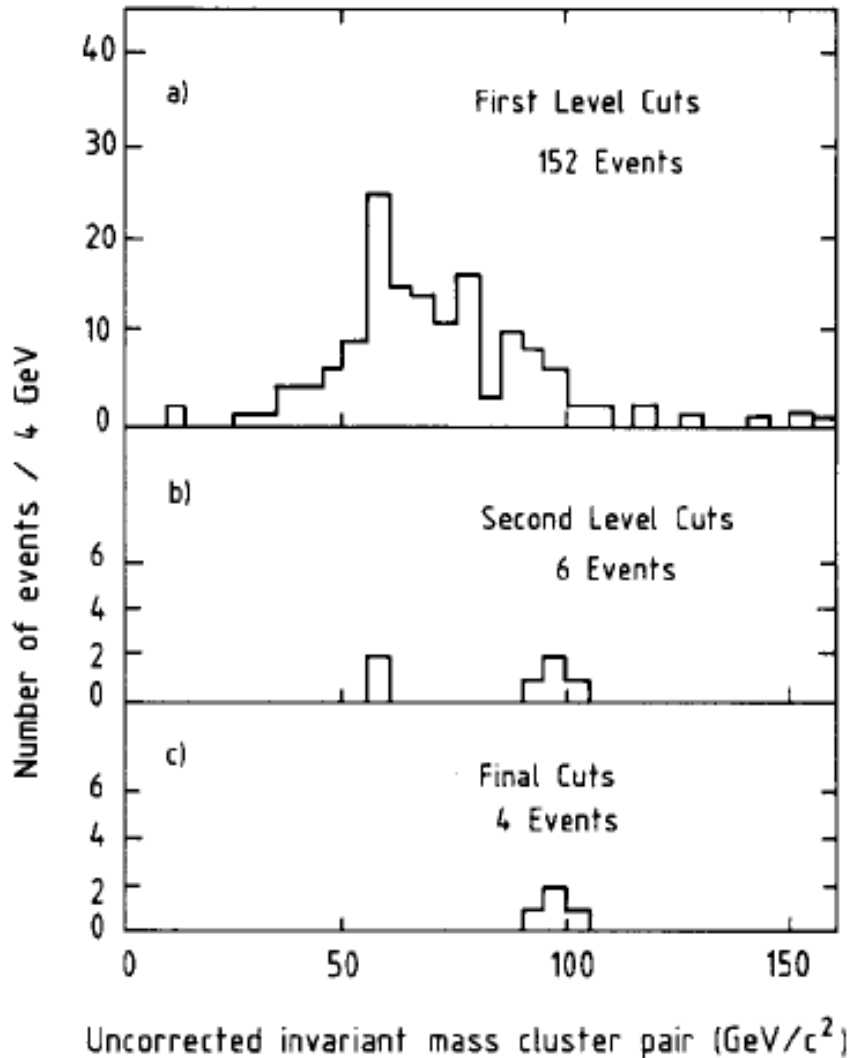
$$\Gamma_W = 2.085 \pm 0.042 \text{ GeV}$$





# UA1: observation of $Z \rightarrow e^+ e^-$

(May 1983)

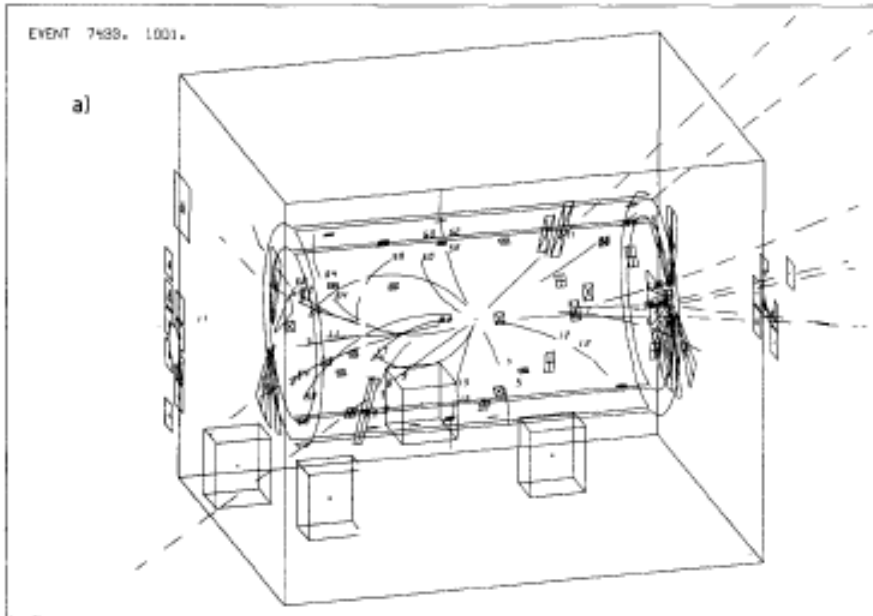


**Two localized deposits of energy in the electromagnetic calorimeter (electrons or photons)**

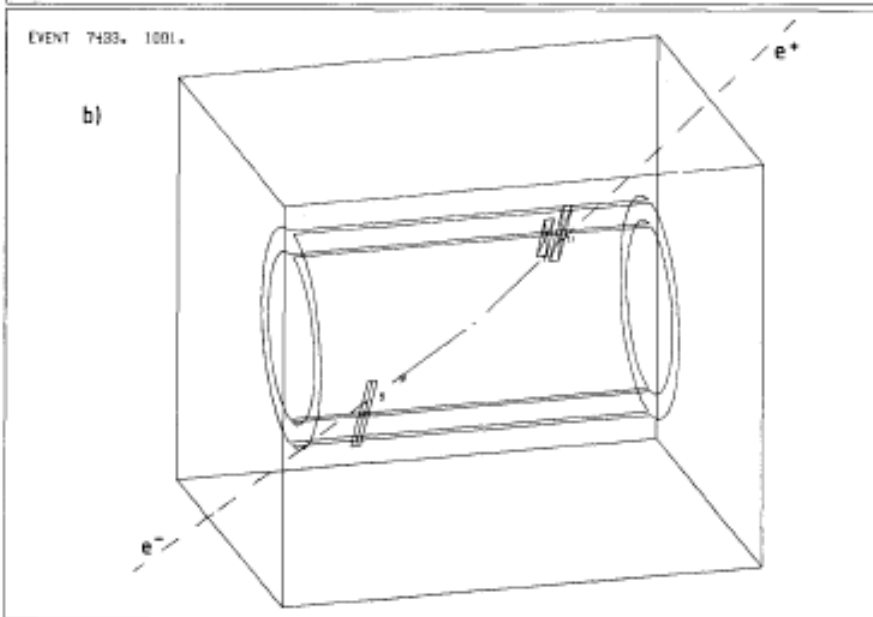
**Isolated charged tracks with  $p_T > 7$  GeV  
At least one should point to the electromagnetic cluster**

**Both tracks with  $p_T > 7$  GeV  
Point to an electromagnetic cluster**

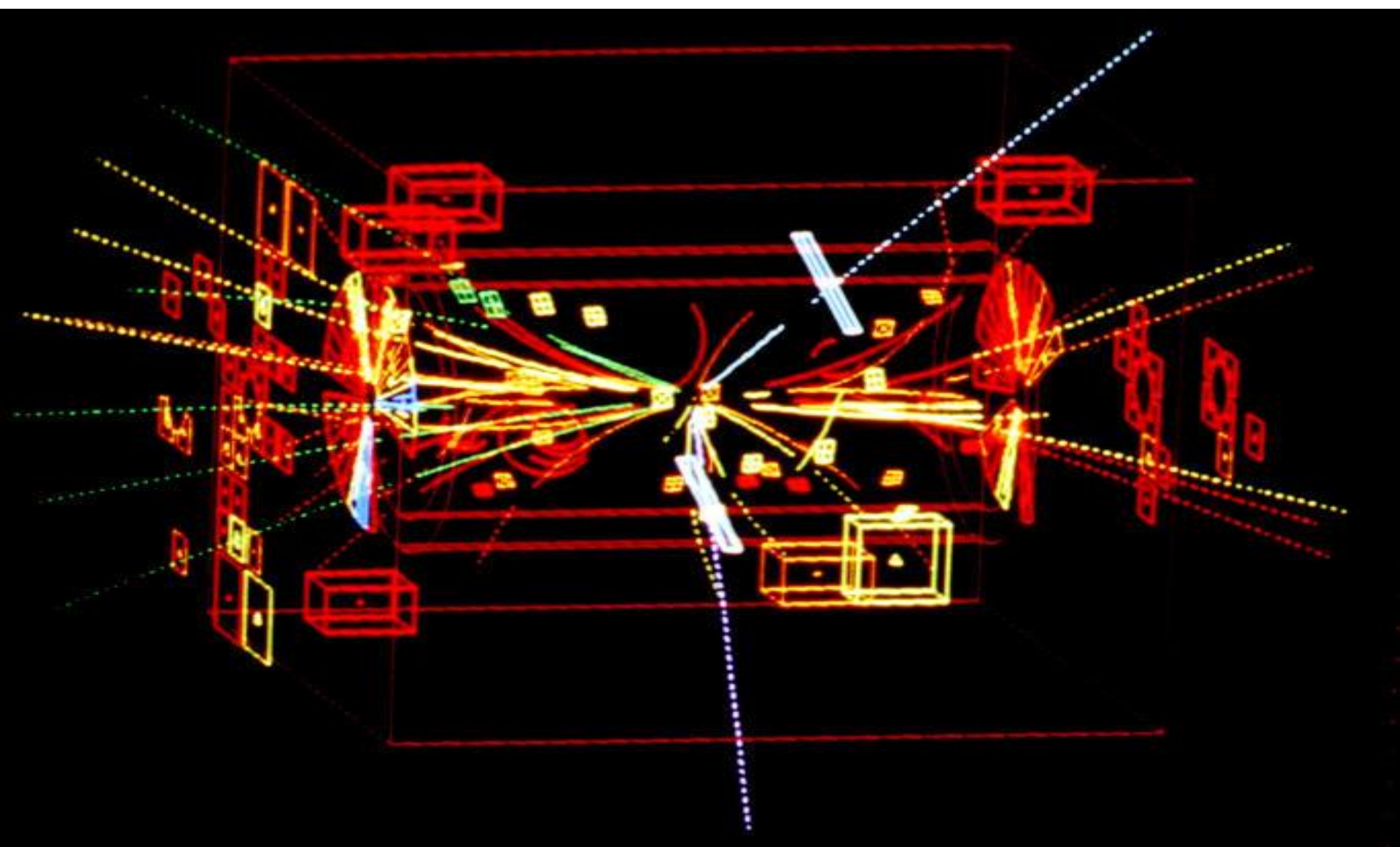
# UA1 $Z \rightarrow e^+ e^-$ event



**Event with all tracks and  
Electromagnetic deposits**

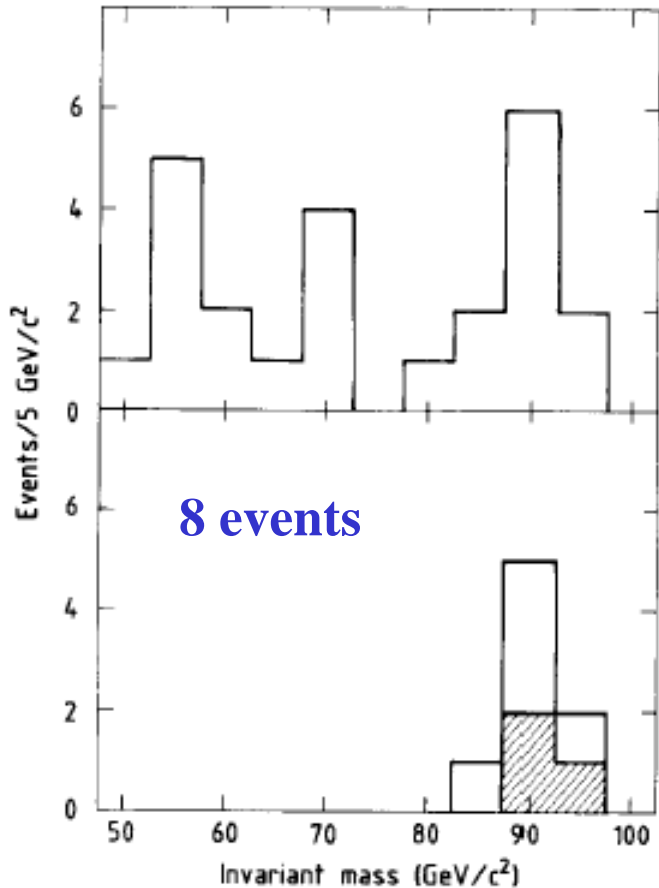


**Require tracks with  $p_T > 2$  GeV**




# UA2: observation of $Z \rightarrow e^+ e^-$

June 1983)



**Two localized electromagnetic clusters  
with  $p_T > 25$  GeV (electrons or photons)**

**Require at least a charged track pointing  
to the electromagnetic cluster**

 Track identified as an isolated electron  
pointing to both energy clusters

$$m_Z = 91.9 \pm 1.3 \pm 1.4 \text{ GeV}$$

(stat) (syst)

# The discovery of W e Z



## The Nobel Prize in Physics 1984

"for their decisive contributions to the large project, which led to the discovery of the field particles W and Z, communicators of weak interaction"



Carlo Rubbia

🕒 1/2 of the prize

Italy

CERN  
Geneva, Switzerland

b. 1934



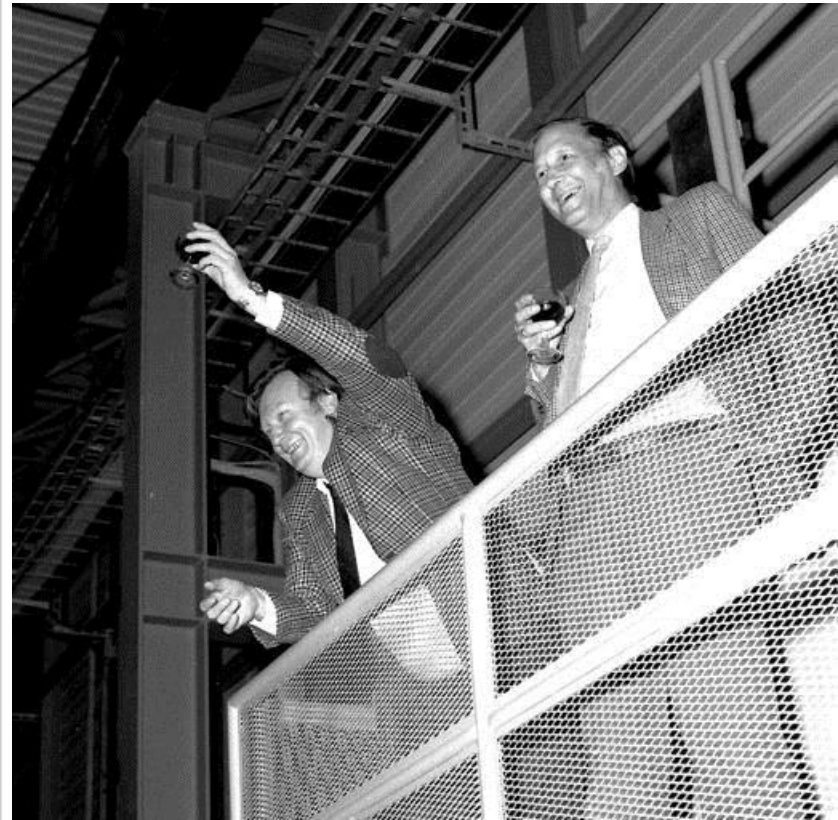
Simon van der Meer

🕒 1/2 of the prize

the Netherlands

CERN  
Geneva, Switzerland

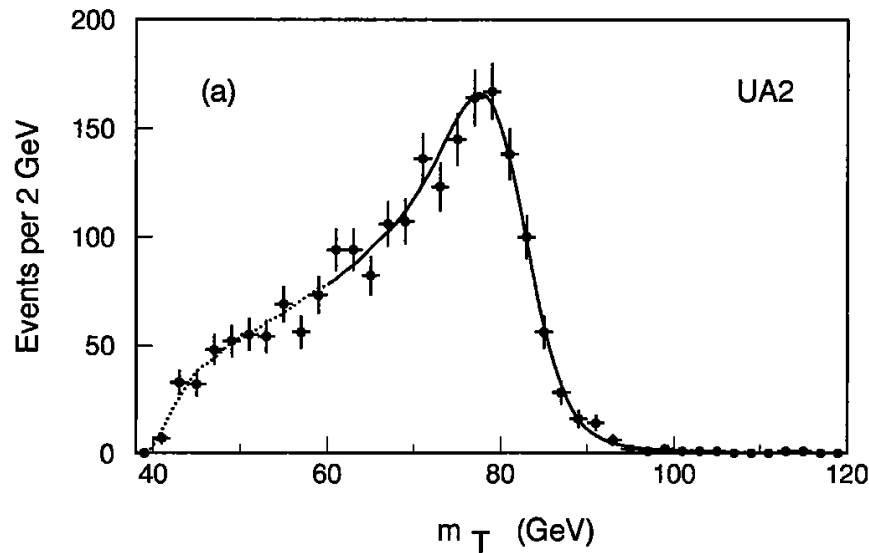
b. 1925



"for their decisive contributions to the large project, which led to the discovery of the field particles W and Z, communicators of the weak interaction"

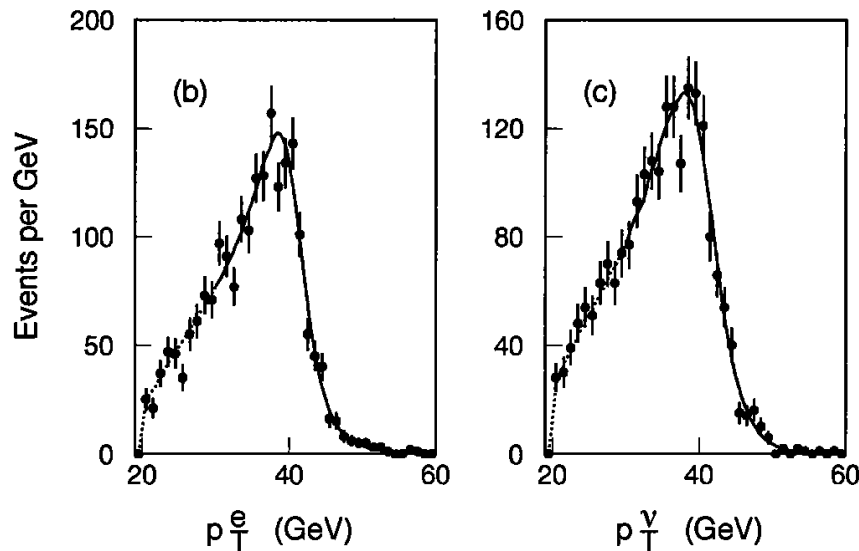
# UA2 final results of the W mass ( $13 \text{ pb}^{-1}$ )

*Phys. Lett. B 276 (1992) 354-364*



	$m_W(\text{GeV})$	$\Gamma_W(\text{GeV})$
$m_T$	$80.84 \pm 0.22$	2.1 (fixed)
fit	$80.83 \pm 0.23$	$2.2 \pm 0.4$
$p_T^e$	$80.86 \pm 0.29$	2.1 (fixed)
fit	$80.79 \pm 0.30$	$2.8 \pm 0.6$
$p_T^\nu$	$80.73 \pm 0.32$	2.1 (fixed)
fit	$80.70 \pm 0.34$	$2.3 \pm 0.7$

$\pm 0.17(\text{sys}) \pm 0.81(\text{scale})$



$$m_W/m_Z = 0.8813 \pm 0.0036(\text{stat}) \pm 0.0019(\text{syst})$$

Re scaled with  $M_Z$  measured at LEP  
(to divide out the energy scale error)

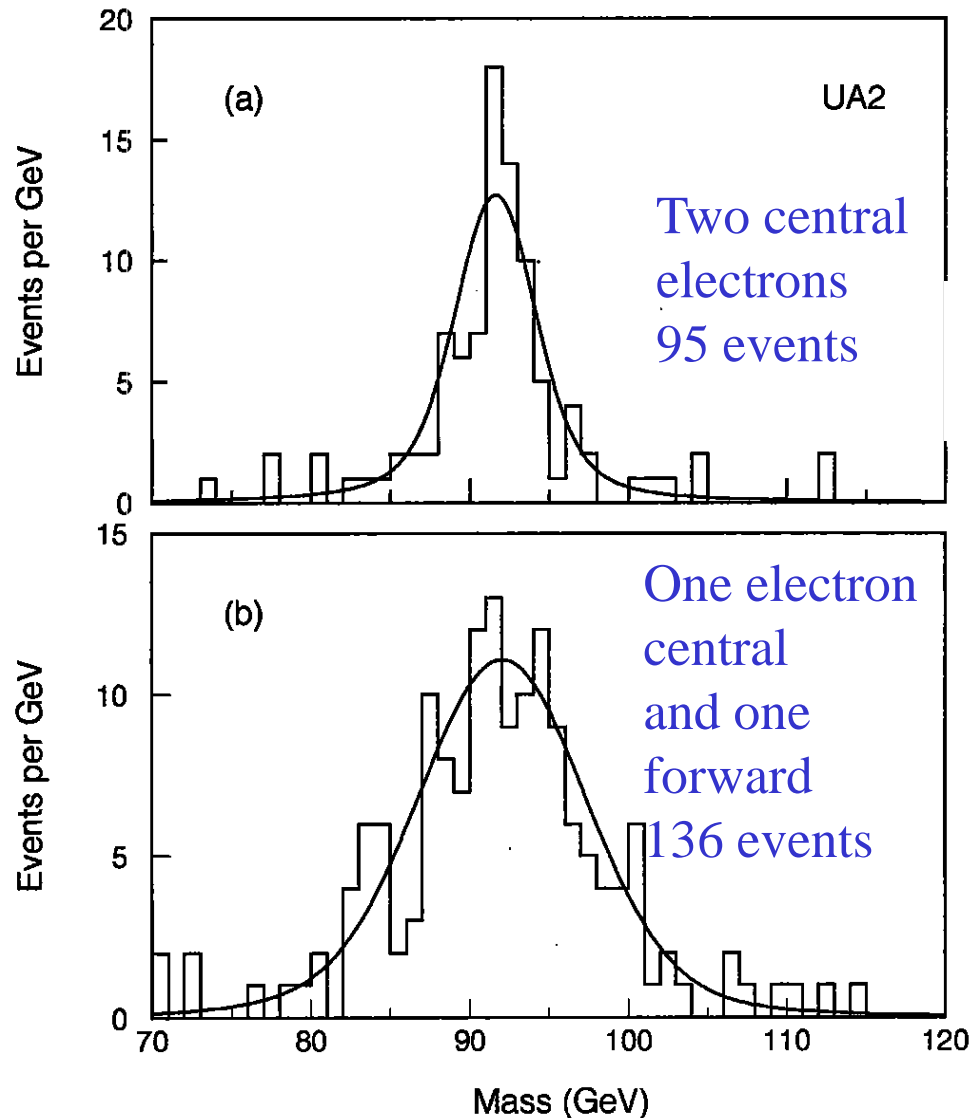
$$M_Z = 91.195 \pm 0.021 \text{ GeV:}$$

$$m_W = 80.35 \pm 0.33(\text{stat}) \pm 0.17(\text{syst}) \text{ GeV.}$$

$$\sin^2 \theta_W \equiv 1 - m_W^2/m_Z^2,$$

$$\sin^2 \theta_W = 0.2234 \pm 0.0064 \pm 0.0033.$$

Figure 4: Fits for  $m_W$  to (a) the  $m_T$  spectrum, (b) the  $p_T^e$  spectrum and (c) the  $p_T^\nu$  spectrum. The points show the data, while the curves show the fit results with the solid portions indicating the ranges over which the fits are performed.



Fit max likelihood with relativistic BW convoluted with the resolution  $\sigma_e$  and weighted with the partonic luminosity  $e^{-\beta m'}$

Input :  $m_{ee}$ ,  $\sigma$ , output :  $m_Z$ ,  $\Gamma_Z$

Probability density function:

$$f(m_{ee}, \sigma, m_Z, \Gamma_Z) \propto \int dm' \frac{m'^2 e^{-\beta m'}}{(m'^2 - m_Z^2)^2 + m'^4 \Gamma_Z^2 / m_Z^2} e^{-(m_{ee} - m')^2 / 2\sigma^2}$$

	$m_Z$ (GeV)	$\Gamma_Z$ (GeV)
central	$91.65 \pm 0.34$	2.5 (fixed)
sample	$91.67 \pm 0.37$	$3.2 \pm 0.8$
$p_T$ -constrained	$92.10 \pm 0.48$	2.5 (fixed)
sample	$92.15 \pm 0.52$	$3.8 \pm 1.1$

$\pm 0.12(\text{sys}) \pm 0.92(\text{scal})$

QCD background <1%

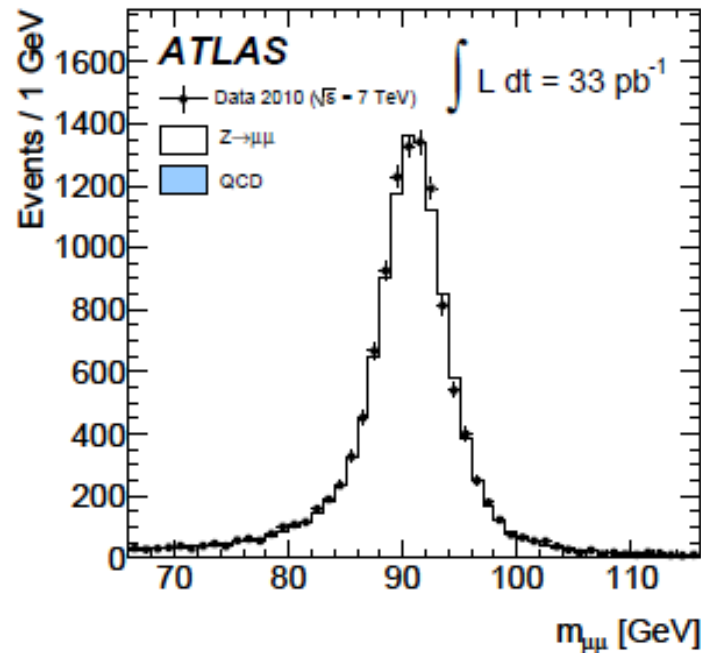
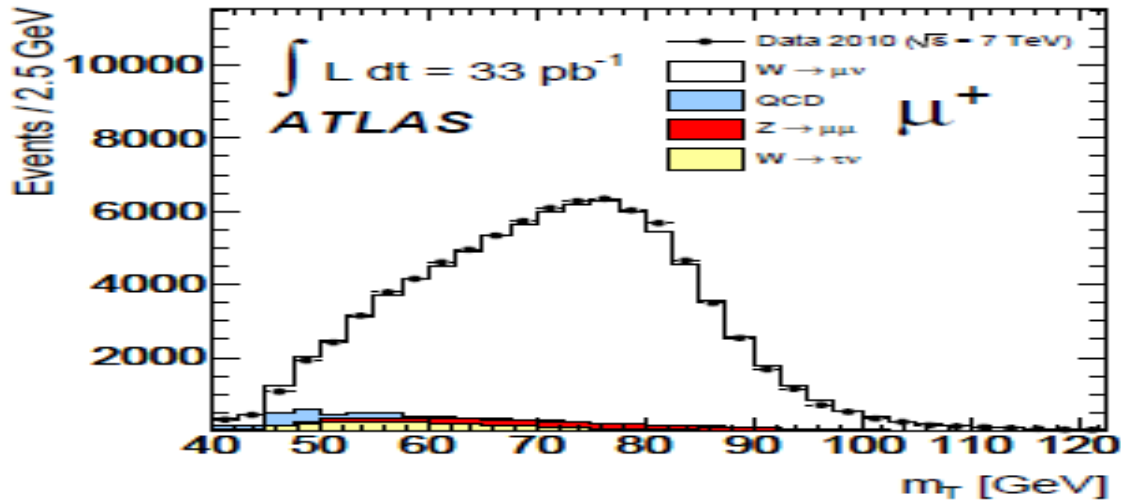
PDG :  $M_Z = 91.1896 \pm 0.0021 \text{ GeV}$ ,

$\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$

Figure 1: Fits for  $m_Z$  to (a) the central sample and (b) the pt-constrained sample.

The curves show the fits, while the histograms show the data.

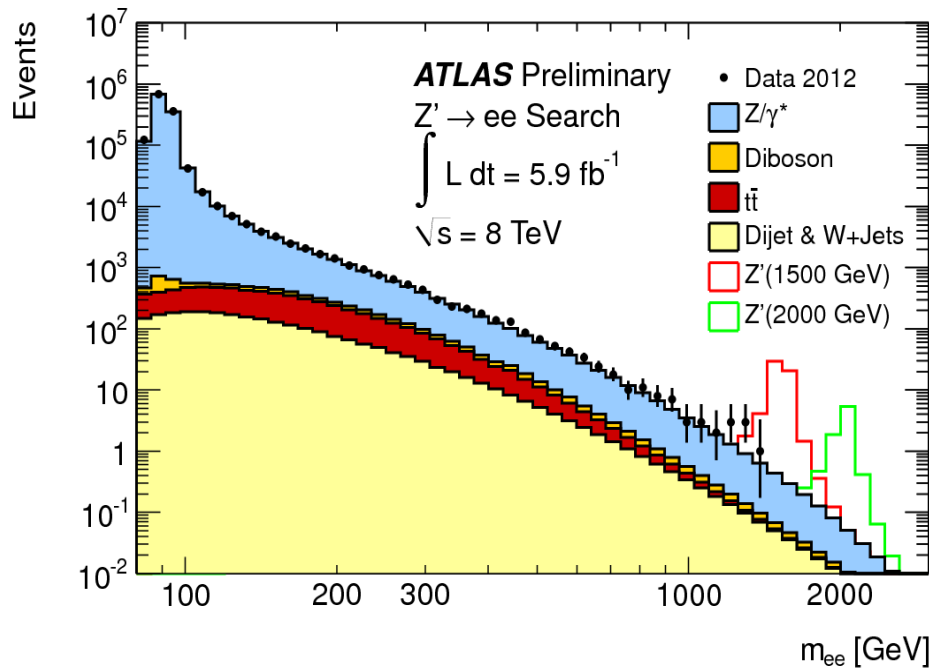
# W/Z at LHC



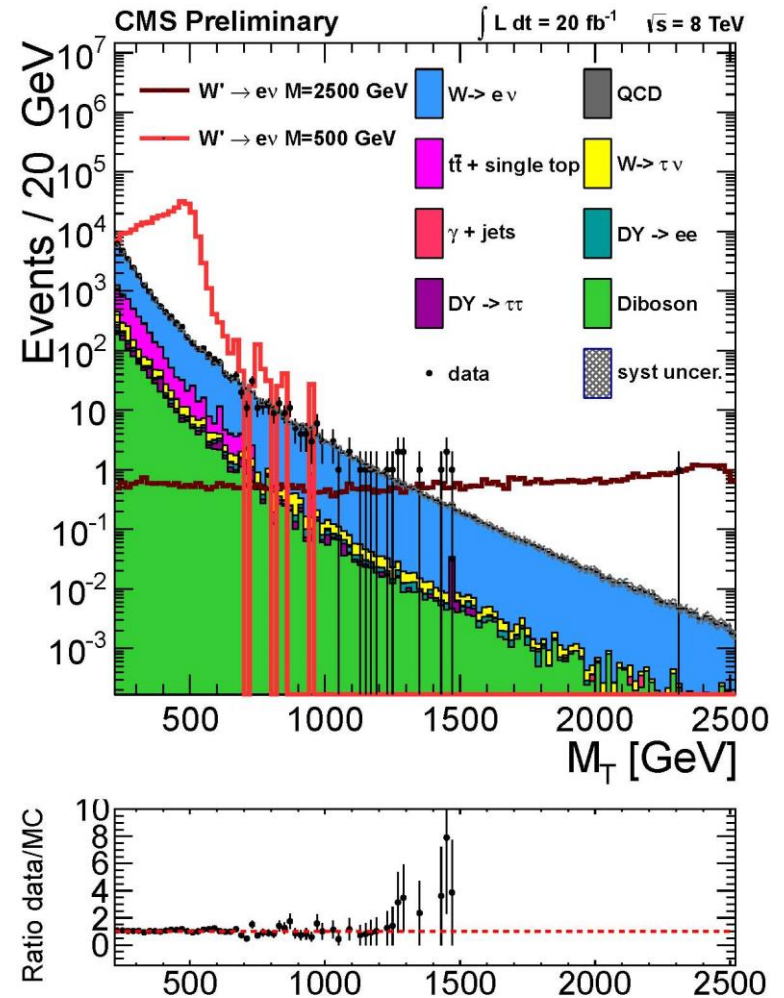
At an integrated  
luminosity comparable  
with that of the ppbar  
Collider  
~30 times more Z's



# New W/Z bosons ??(U(1)',SU(2)')



$Z'$  with same couplings to fermions as  $Z$  excluded at 95% CL for  $M_{Z'} < 2.39$  TeV



$W'$  with same couplings to fermion as  $W$  (SSM) excluded at 95% CL for  $M_{W'} < 3.20$  TeV

# Why it is important to measure precisely $M_W$

In the Standard Model the relationship between fundamental constants

$$g_W^2 = \frac{e^2}{\sin^2 \theta_W}; g_W^2 / 4\pi = \frac{\alpha}{\sin^2 \theta_W};$$

$$\frac{G}{\sqrt{2}} = \frac{g_W^2}{8M_W^2}; M_W = \sqrt{\frac{\pi\alpha}{\sqrt{2}G \sin^2 \theta_W}}$$

$$g_Z^2 = \frac{e^2}{\sin^2 \theta_W \cos^2 \theta_W}; g_Z^2 / 4\pi = \frac{\alpha}{\sin^2 \theta_W \cos^2 \theta_W};$$

$$M_W = M_Z \cos \theta_W$$

Only 3 parameters are independent, ex:

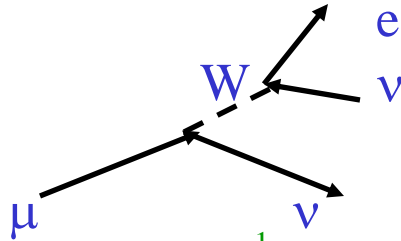
$$\alpha, \left[ \frac{\Delta\alpha}{\alpha} \right] = 0.0007 \cdot 10^{-6} \text{ (at } Q^2 = 0 \text{) from atomic energy levels;}$$

$$G, \left[ \frac{\Delta G}{G} \right] = 05 \cdot 10^{-6} \text{ (from the decay } \mu \rightarrow e \nu \nu \text{);}$$

$$M_W, \left[ \frac{\Delta M_W}{M_W} \right] = 19 \cdot 10^{-6}; \sin^2 \theta_W, \left[ \frac{\Delta \sin^2 \theta_W}{\sin^2 \theta_W} \right] = 52 \cdot 10^{-6}$$

But relations at tree level are modified by radiative corrections:

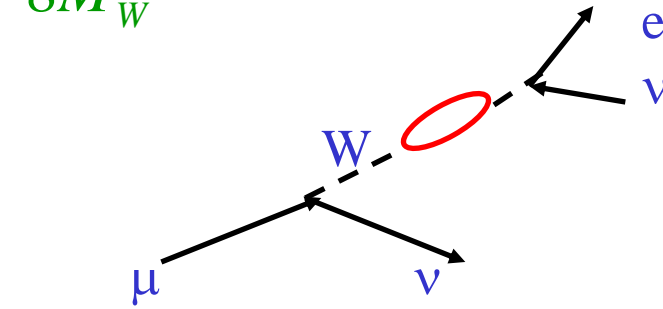
$$\frac{G}{\sqrt{2}} = \frac{g_W^2}{8M_W^2} \rightarrow \frac{G}{\sqrt{2}} = \frac{g_W^2}{8M_W^2} (1 - \Delta r)$$



$$M_W = \frac{(\pi\alpha / \sqrt{2}G_F)^{1/2}}{\sin \theta_W} \equiv \frac{A}{\sin \theta_W}$$

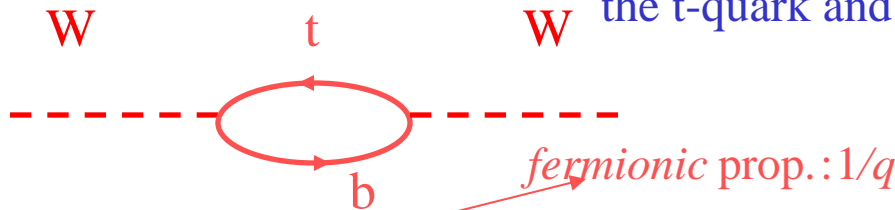
$$A = 37.2810 \pm 0.0003 \text{ GeV}$$

$$\Delta r \sim \Delta\alpha + \frac{G}{8\sqrt{2}\pi^2} \left( -3 \cot^2 \theta_W m_t^2 + \frac{11}{3} M_W^2 \ln \frac{M_H^2}{M_W^2} \right) + \dots \quad \Delta\alpha \cong 1 - \frac{\alpha}{\alpha(M_Z^2)} \approx 0.06$$

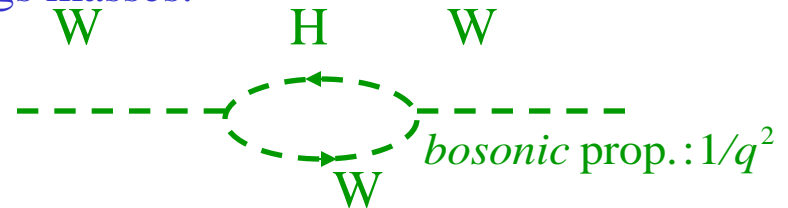


$$\Rightarrow M_W = \frac{A}{\sin \theta_W \sqrt{1 - \Delta r}}$$

Corrections to  $M_W$  come from loops with t-quark and Higgs and then they are sensitive to the t-quark and Higgs masses.



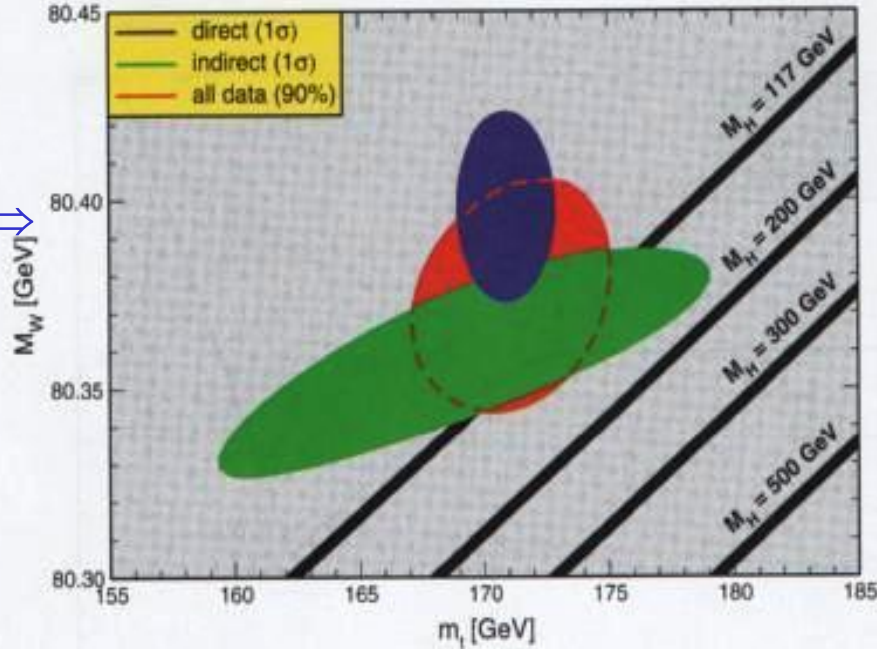
$$\delta M^2 \sim \int d^4 q / q^2 \sim \int q^3 dq / q^2 \sim \int^m q dq \sim m^2$$



$$\delta M^2 \sim \int d^4 q / (q^2)^2 \sim \int q^3 dq / q^4 \sim \int^M dq / q \sim \ln(M)$$

# Higgs mass from top and W masses (prior to the Higgs discovery)

A displacement of  
5 GeV of the top mass  $\Rightarrow$   
22 MeV displacement  
of  $M_W$  (positive)



A displacement of  
100 To 1000 GeV  
of the Higgs mass  
 $\Rightarrow$   
130 MeV displacement  
of  $M_W$  (negative)

**Figure 10.3:** One-standard-deviation (39.35%) region in  $M_W$  as a function of  $m_t$  for the direct and indirect data, and the 90% CL region ( $\Delta\chi^2 = 4.605$ ) allowed by all data. The SM prediction as a function of  $M_H$  is also indicated. The widths of the  $M_H$  bands reflect the theoretical uncertainty from  $\alpha(M_Z)$ .

$M_W$  vs  $M_{\text{top}}$  provides predictions on the Higgs mass:  
**Low Higgs masses favoured (and found).**

# *The Higgs mechanism to provide mass to particles*

*see the Marumi Kado lecture*

Based on the vacuum expectation value of a field (the Higgs)  
different than zero

**With the Higgs discovery everything is understood?**

# Higgs field and energy of the vacuum

$$V(\phi) = \mu^2 \phi^+ \phi + \lambda (\phi^+ \phi)^2$$

Minimum of V:

$$\frac{\partial V}{\partial(\phi^+ \phi)} = 0 \Rightarrow \mu^2 + 2\lambda(\phi^+ \phi) = 0 \Rightarrow$$

$$\Rightarrow \text{minimum if } \mu^2 < 0 \text{ at } \phi^+ \phi = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2}$$

The value of the potential at minimum is then:

$$V_0 = \frac{\lambda v^4}{4} \quad \text{with}$$

$$v = \frac{2M_W}{g_W} = \frac{1}{\sqrt{2}\sqrt{G_F}} \sim 246 \text{ GeV} \Rightarrow V_0 \sim 0.9 \cdot 10^9 \lambda \text{ GeV}^4$$

$$\approx \frac{1 \rho}{m^3} \text{ the total one (dark matter and energy)}$$

$$\approx 100 \text{ the visible one:}$$

$$\text{total energy density} \approx 10^{-4} \frac{\text{GeV}}{\text{cm}^3}$$

$$1 \text{ GeV}^{-1} = 0.2 \cdot 10^{-13} \text{ cm} \Rightarrow 1 \text{ GeV}^3 = 1.3 \cdot 10^{41} \text{ cm}^{-3}$$

$$\lambda = m_H^2 / v^2,$$

$$m_H = 125 \text{ GeV}, v = 246 \text{ GeV (from } G_F)$$

$$\Rightarrow \lambda \approx 0.25$$

If  $\lambda \sim 0.25$  energy of the Higgs field:

$$V_0 \sim 3 \cdot 10^{49} \text{ GeV} / \text{cm}^3$$

**53 orders of magnitude larger than that observed**

Of course we can add a constant term to cancel  $V_0$

but this term is to be calibrated  $1/10^{53}$  !!! (Hint for new physics???)

From AA(antiproton  
accumulator) to Z

(seminar of C. Rubbia at CERN, 1983)

From Z/W to Higgs (2012)

(triumph of the EW model)

From Higgs to ???

**The answer (we hope) with the new  
LHC run: 2015**

Back up slides



Fenomenologically there is a hierarchy among the CKM matrix:

Diagonal elements are  $\approx 1$ ,

$$|V_{us}| \approx |V_{ud}| \approx 0.2,$$

$$|V_{ts}| \approx 10^{-2},$$

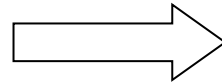
$|V_{ub}| \approx |V_{td}| \approx 10^{-3}$  then, with great accuracy the independent elements are :

$$s_{12} = |V_{us}|, s_{13} = |V_{ub}|, s_{23} = |V_{cb}|, \delta$$

Wolfenstein parametrizations. Define :

$$s_{12} = \lambda, \quad s_{13} e^{i\delta} = A\lambda^3(\rho - i\eta)$$

$$s_{23} = A\lambda^2$$



$$\eta = \frac{s_{13}}{s_{12}s_{23}} \sin \delta,$$

$$\rho = \frac{s_{13}}{s_{12}s_{23}} \cos \delta,$$

By substitution in the standard parametrization we obtain (to  $\lambda^4$ ):

$$V_{CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3[1 - \rho - i\eta] & -A\lambda^2 & 1 \end{pmatrix}$$

We exploit the following (out of 9) unitarity relation:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \Rightarrow 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} + \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} = 0$$

This relation defines a triangle in the complex plane (unitarity triangle)

Redefine:

$$\bar{\rho} = \rho \left(1 - \frac{\lambda^2}{2}\right), \quad \bar{\eta} = \eta \left(1 - \frac{\lambda^2}{2}\right) \longrightarrow 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = \bar{\rho} + i\bar{\eta} + O(\lambda^4)$$

$$\sin 2\alpha = \frac{2\bar{\eta}(\bar{\eta}^2 + \bar{\rho}^2 - \bar{\rho})}{(\bar{\eta}^2 + \bar{\rho}^2)((1 - \bar{\rho}^2) - \bar{\eta}^2)}$$

$$\sin 2\beta = \frac{2\bar{\eta}(1 - \bar{\rho}^2)}{(1 - \bar{\rho}^2) + \bar{\eta}^2}, \quad CA = R_b = \left(1 - \frac{\lambda^2}{2}\right) \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right|$$

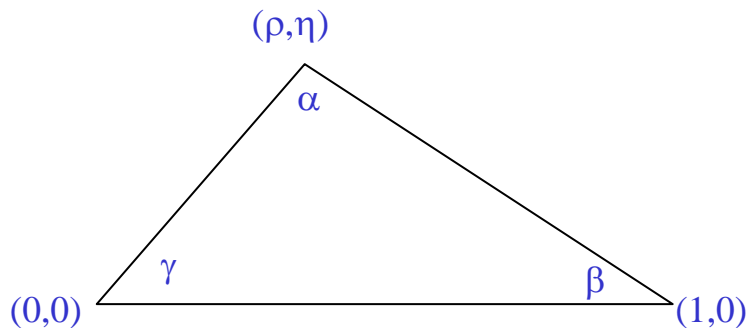
$$\sin 2(\gamma = \delta) = \frac{2\bar{\rho}\bar{\eta}}{\bar{\eta}^2 + \bar{\rho}^2}, \quad BA = R_t = \frac{1}{\lambda} \left| \frac{V_{td}}{V_{cb}} \right|$$

$$V_{td} = |V_{td}| e^{-i\beta},$$

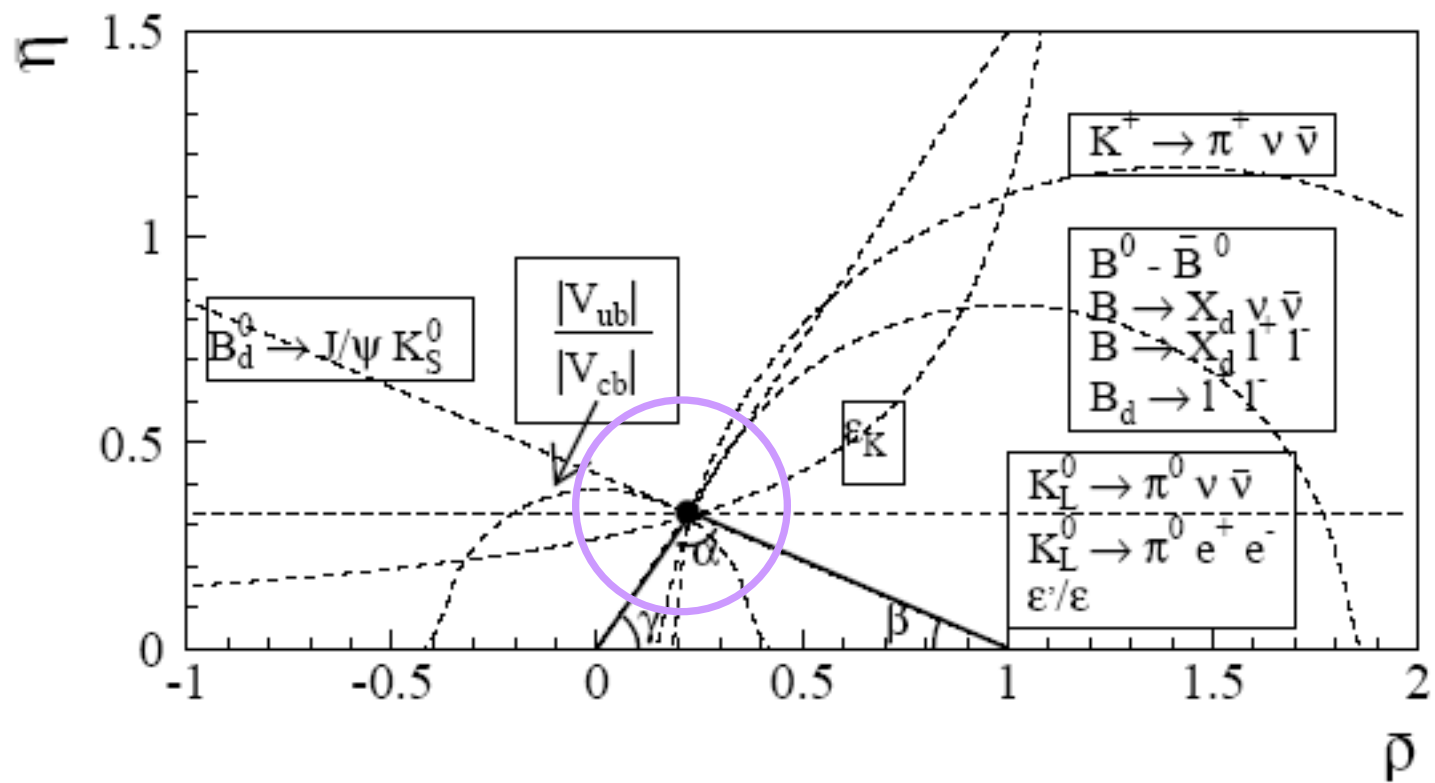
$$V_{ub} = |V_{ub}| e^{-i\gamma},$$

$$R_b e^{i\gamma} + R_t e^{-i\beta} = 1$$

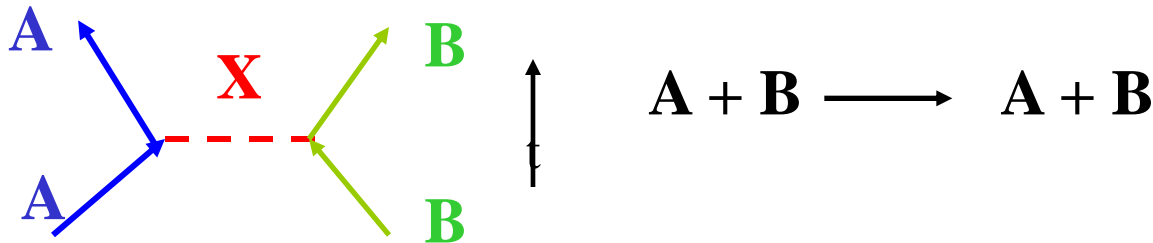
Is the vertex of the triangle:



From the unitarity triangle, with  $|V_{us}|$  e  $|V_{cb}|$ , we can extract the full CKM matrix,  
 Moreover if the experimental results provide deviation from the triangle, would indicate new physics



# Range of elementary forces



We can parametrize the process saying that A emits X

$$A(M_A, \vec{0}) \rightarrow A(E_A, \vec{p}) + X(E_X, -\vec{p})$$

$$\text{with } E_A = \sqrt{M_A^2 + p^2}, \quad E_X = \sqrt{M_X^2 + p^2}$$

The final-initial energy  $\Delta E$  can be written as:

$$\Delta E = E_X + E_A - M_A = \sqrt{M_X^2 + p^2} + \sqrt{M_A^2 + p^2} - M_A > M_X$$

Therefore from the uncertainty principle the process can occur in a time  $\tau$  :

$$\tau \approx \frac{\hbar}{\Delta E} \leq \frac{\hbar}{M_X}$$

The maximum propagation distance of the particle X, R, can be:

$$R = c \cdot \tau \leq \frac{\hbar c}{M_X} \text{ (range)}$$

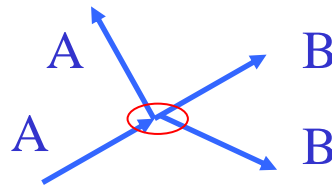
If  $M_X=0$  (the photon)  $R \longrightarrow \infty$ , but also  $\Delta E \longrightarrow 0$  and  $\tau \longrightarrow \infty$ :

The virtuality time goes to infinity: **the photon is real**

In the case of the weak interactions  $M_X=M_Z=90 \text{ GeV}$ :

$$R \leq \frac{\hbar c}{M_Z} = \frac{0.197 \cdot \text{GeV} \cdot \text{fm}}{M_Z} \sim 2 \cdot 10^{-3} \text{ fm}$$

If the momentum of the particle, p, of particle A (or B) e' is such as the De Broglie wave length  $\lambda_B \gg R$ , we can approximate as a "contact interaction" (Fermi theory):



## Questions

- Why you don't need a proton cooling (LHC)?
- it is convenient to measure

$$Z \rightarrow e^+ e^-,$$

*measuring electrons with a spectrometer or a calorimeter?*

$$da : M_W^2 = \frac{\pi\alpha}{G \sin^2 \theta_W \sqrt{2}} \rightarrow \sin^2 \theta_W \cos^2 \theta_W = \left(1 - \frac{M_W^2}{M_Z^2}\right) \frac{M_W^2}{M_Z^2} = \frac{\pi\alpha(M_Z)}{\sqrt{2}GM_Z^2(1-\Delta r)}$$