Supersymmetry

HASCO 2012 Göttingen

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This is a lecture about supersymmetry : *beyond the standard model.*

Beyond the Standard Model Physics = "BSM" physics

$\Rightarrow$ Theories that have not been proven experimentally

Best case scenario = only one BSM is right.

So why spend time on this?

Research is about understanding things we do not yet know.

Supersymmetry and exotics (Next week) are various attempts to answer questions that are not answered by the standard model.

1. Why do we need BSM theories?

2. What is SUSY How can SUSY address these questions?

3. How can we tackle these experimentally?

Today- SUSY theory: motivation+constructions, from the symmetry to the physical particle spectrum

Tomorrow- Tommaso Lari (INFN Milano)
SUSY phenomenology + illutration with a few recent LHC results

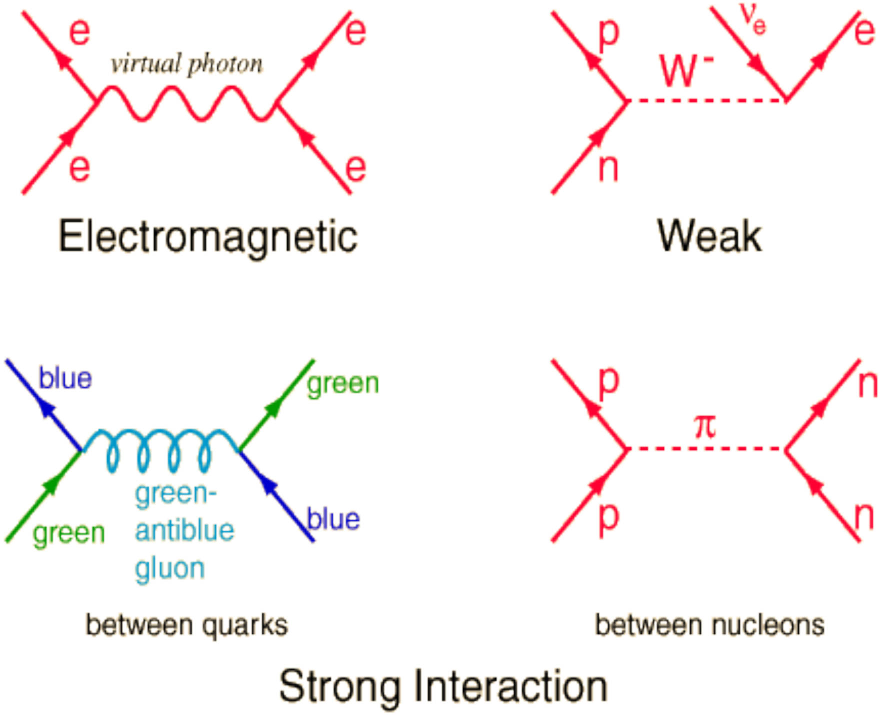
Structure of the SUSY & Exotics Lectures

1. **What questions do we want to answer? WHY**
   - The Planck and GUT Scale
   - The hierarchy problem
   - Dark Matter
   - Sakharov's conditions
   - Other Problems with the Standard Model

2. **Suypersymmetry WHAT**
   - Construction of supersymmetry
   - Particle spectrum
   - SUSY Particle production and decay
   - R-Parity
   - Some experimental searches

Tomorrow: HOW and WHERE …

Where is gravity in the Standard Model?



interaction b/w nucleons is really b/w quarks

Interaction	Relative strength	Range	Exchange	Mass (GeV)	Charge	Spin
Strong	1	short ~ 1 fm	Gluon	0	0	1
Electromagnetic	1/137	infinite ($1/r^2$)	Photon	0	0	1
Weak	10^{-9}	short ~ 10^{-3} fm	W^+,W^-,Z^0	80.4, 91.2	+e,-e,0	1
Gravitational	10^{-38}	infinite ($1/r^2$)	Graviton??	0	0	2

no renormalisable quantum field theory that would fit with Strong and EW

Where is gravity in the Standard Model?

Nowhere! It is not included in the standard model!

It is even worse:
General relativity and quantum mechanics are not compatible.

*In particle physics experiments gravitational interaction is simply too **tiny** to be detectable.*

General relativity and quantum mechanics are incompatible.

• Classical general relativity breaks down at singularities (black holes…)

• How to determine the gravitational field of a particle? since according to Heisenberg's uncertainty principle its location and velocity cannot be known.

• Time has a different meaning in quantum mechanics and general relativity

Planck Scale

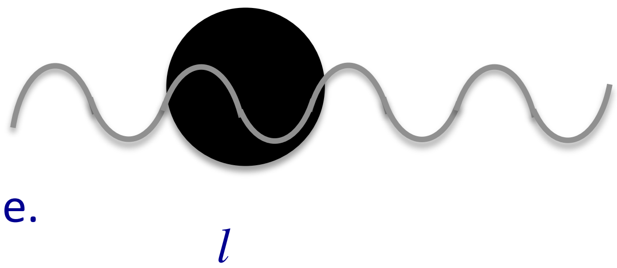
An *energy* or *mass* or *time scale* where both quantum and general relativity effects become important.

Eg.1. Early enough in the Big Bang.

Eg.2. Doing quantum mechanics experiments close to a black hole.

So far did not manage to do either in the lab….

Consider a particle of **very high mass** m .



The Compton wavelength: $\lambda_c = \dfrac{2\pi}{m}$ ~ size l of the particle
or the wavelength of the photon needed to probe that particle.

The particle cannot be localised better than λ_c.

But if λ_c is small enough it is can be within its **Schwarzchild** radius $r_S = 2Gm$ **=> black hole**

$l = \dfrac{2\pi}{m}$ particle inside a black hole => $r_S > \lambda_c \Rightarrow Gm > \dfrac{2\pi}{m}$ or smaller

The condition $\lambda_c < r_S$ starts to be fullfilled for:

$l_P = \sqrt{G}$	Planck length ~10^{-35} m
$m_P = \dfrac{1}{\sqrt{G}}$	Planck mass/ energy ~10^{19} GeV

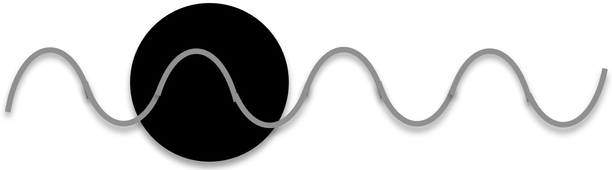
If $m > m_P$, then **it is inside a black hole**
one can never know its position better than l_P.

The Heisenberg uncertainty principle breaks down. ~~$\Delta x \Delta p \sim \hbar$~~ $\Delta x \sim l_P$

Can never distinguish between two locations separated by less than l_P

"Location" loses meaning…

Why is it important to know about the Planck scale?



It shows us the limitations of general gravity and quantum mechanics when put together.

Best case scenario: the laws of physics we know work up to an energy of 10^{19} GeV

But this is not a good scenario for experimental physicists.

If 10^{19} GeV is the only place where known laws of physics they break down, then it is going to be very very very difficult experimentally….

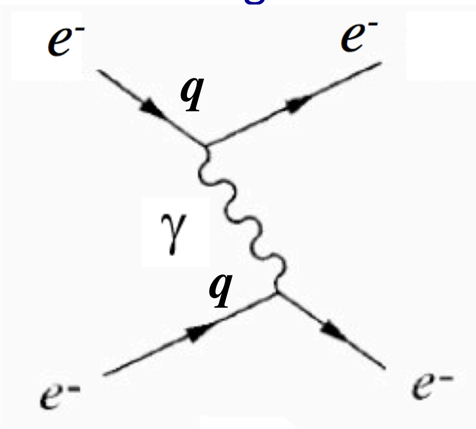
Remember …

- Early enough in the Big Bang.
- Doing quantum mechanics experiments close to a black hole.

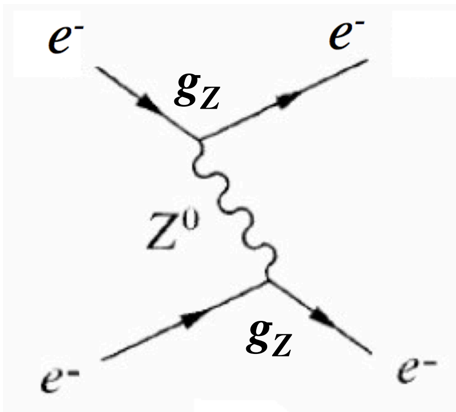
The couplings constants in the Standard Model

Interaction	Relative strength	Range	Exchange
Strong	1	short ~ 1 fm	Gluon
Electromagnetic	1/137	infinite ($1/r^2$)	Photon
Weak	10^{-9}	short ~ 10^{-3} fm	W^+, W^-, Z^0

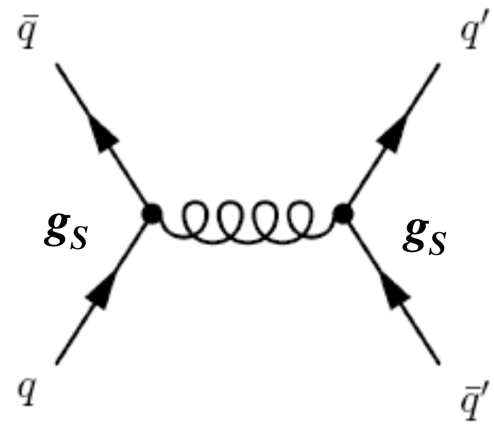
The relative strength is a combination of the value of the coupling constant and the mass of the exchanged boson.



electromagnetic



weak



strong

$q \sim \sqrt{\alpha}$

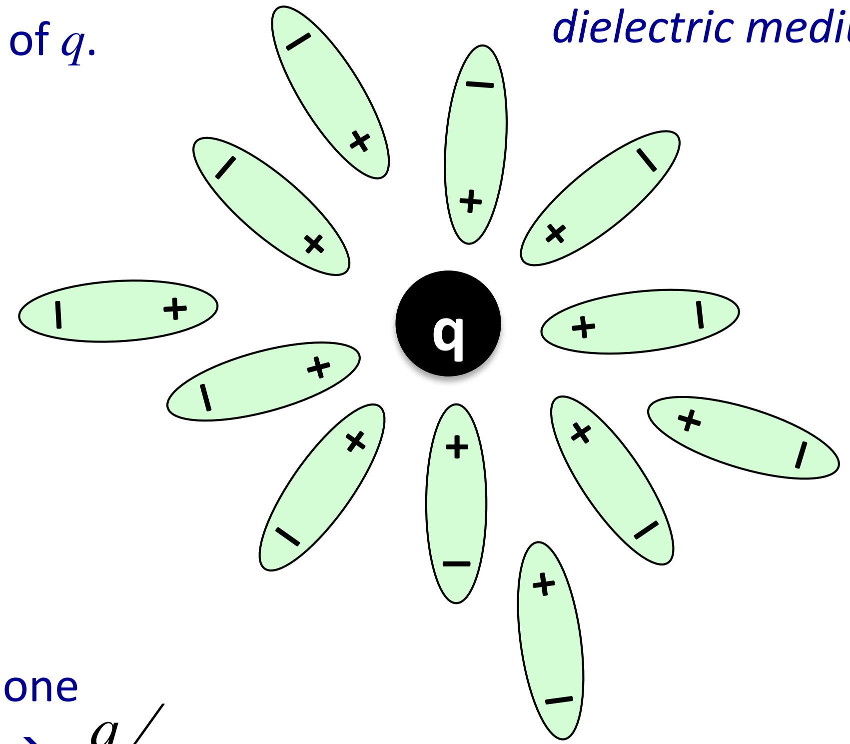
Effective dependence of charge in material

Look for instance at a particle with **negative** charge q in a dielectric.

Assume that the material is composed of molecules which **become polarized** by the electric field of q.

The material produces a dipole field which **reduces** the electric field from the charge q.

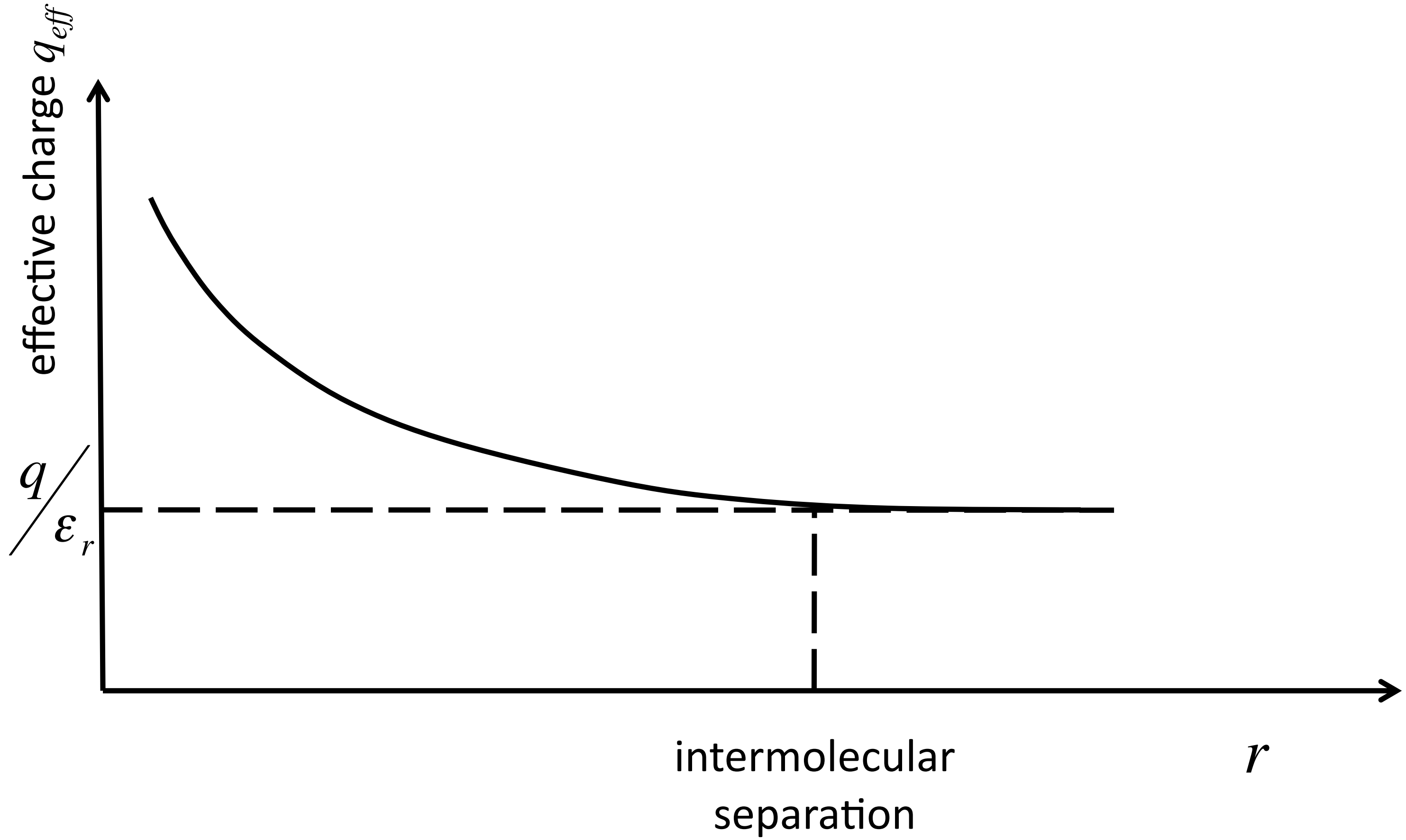
$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\vec{u}_r \rightarrow \frac{q/\varepsilon_r}{4\pi\varepsilon_0 r^2}\vec{u}_r$$



If one measures the charge q in the medium one will in practice measured a screened charge → q/ε_r

The **screening effect** becomes smaller closer to the charge q.

Screening in a Dielectric



The effective charge increases at small distance.

The effective charge increases significantly at distances < the distance b/w molecules

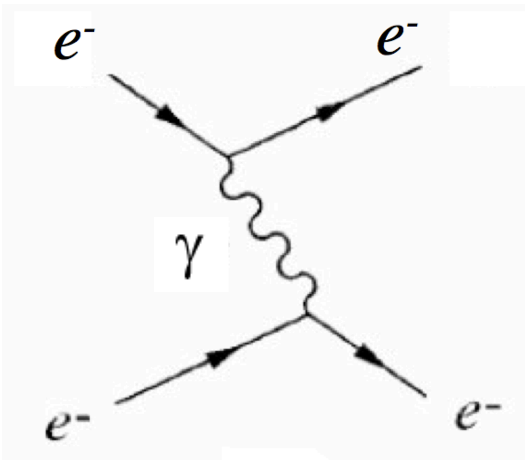
Vacuum Polarisation

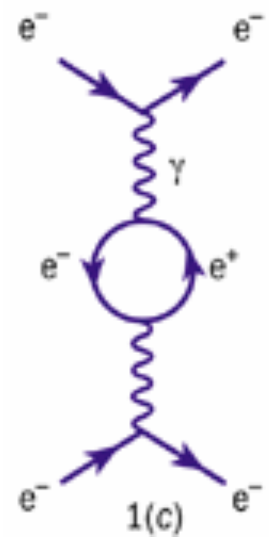
The "vacuum" consists of virtual particles fluctuating into and out of existence.

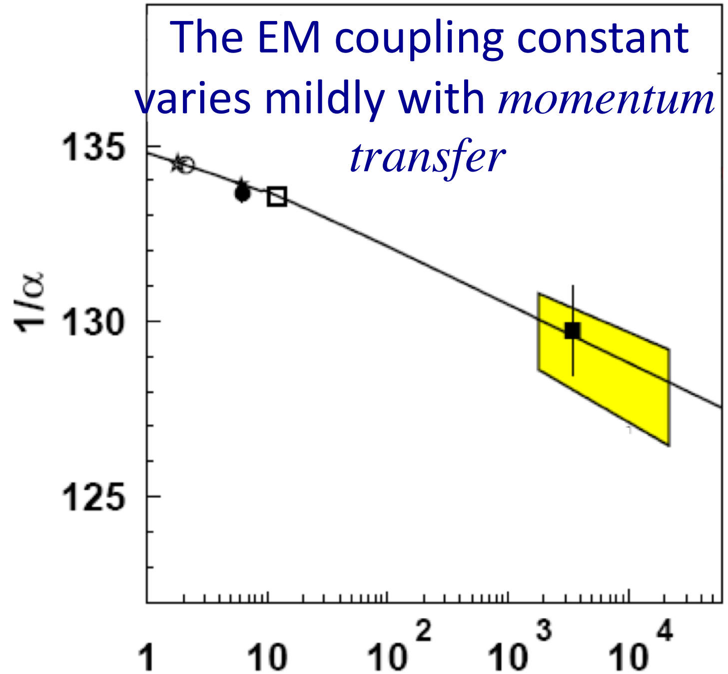
An electron **is surrounded by virtual particles** which act to shield the charge in the same way as the dielectric medium screens the charge.

The lightest charged particles are the *electron* and the *positron*, so the easiest pair of virtual particle-anti-particle to produce is $e^+ e^-$.

The screening decreases at smaller distances r



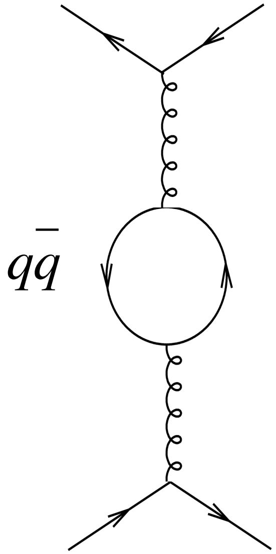


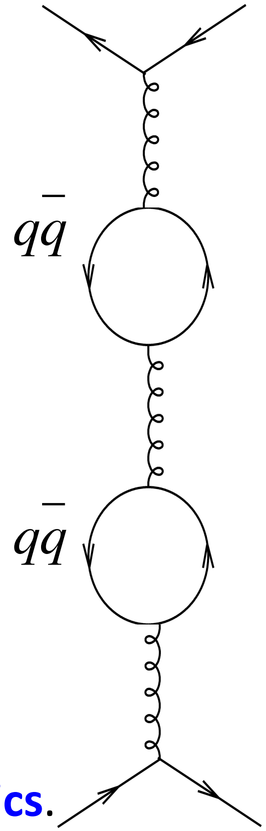


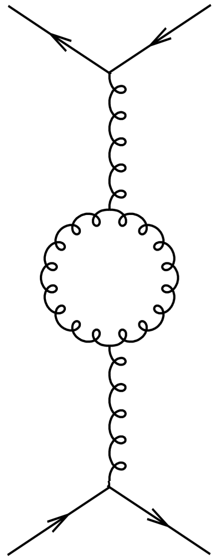
(momentum of exchanged photon)^2 Q^2

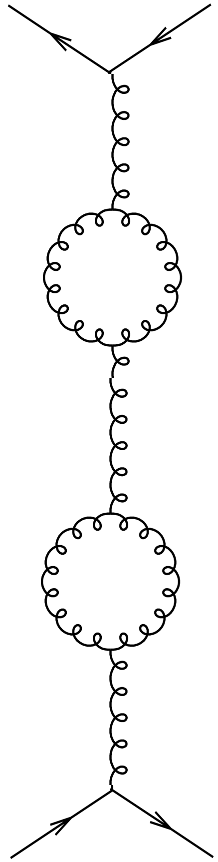
plus additional loops …

What about the Strong Interaction?









Similar phenomena

in Quantum Chromodynamics.

Additional complication due to the gluon-gluon interaction!

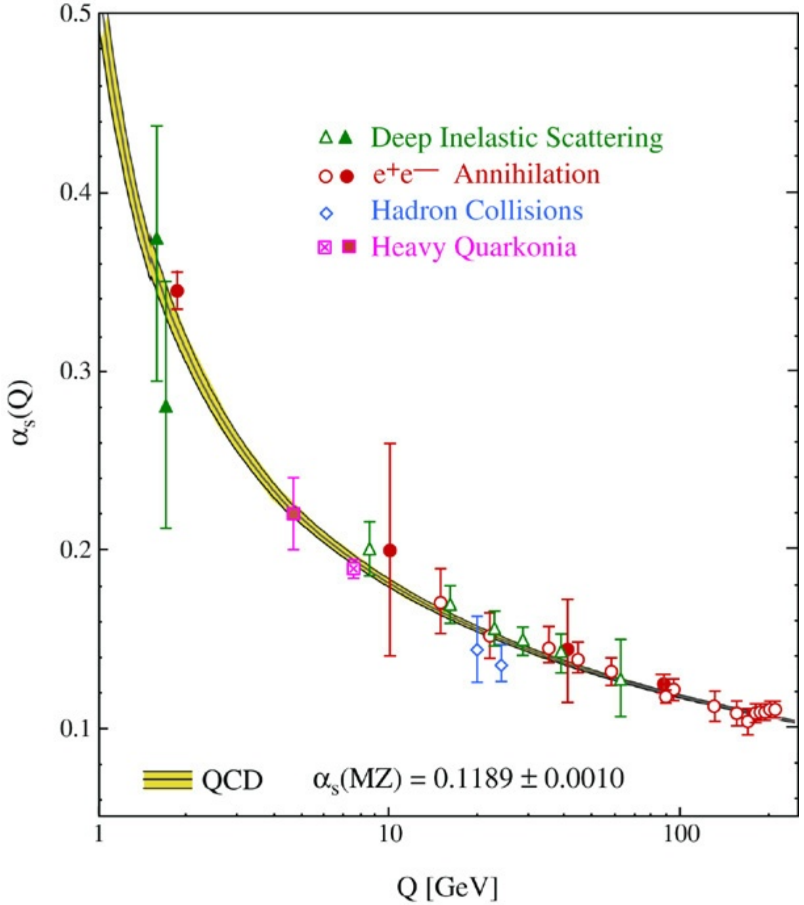
The $q\bar{q}$ loop leads to a screening in the same way as for QED, proportional to the number of quark flavours.

gluon-gluon => <u>opposite and stronger effect</u>, related to the number of gluons.

The total resulting effect => α_s decreases at small distance (opposite of QED…)

Asymptotic Freedom

Coupling constant of the Strong Interaction vs Exchanged Momentum



Measured α_S as function of the exchanged Q

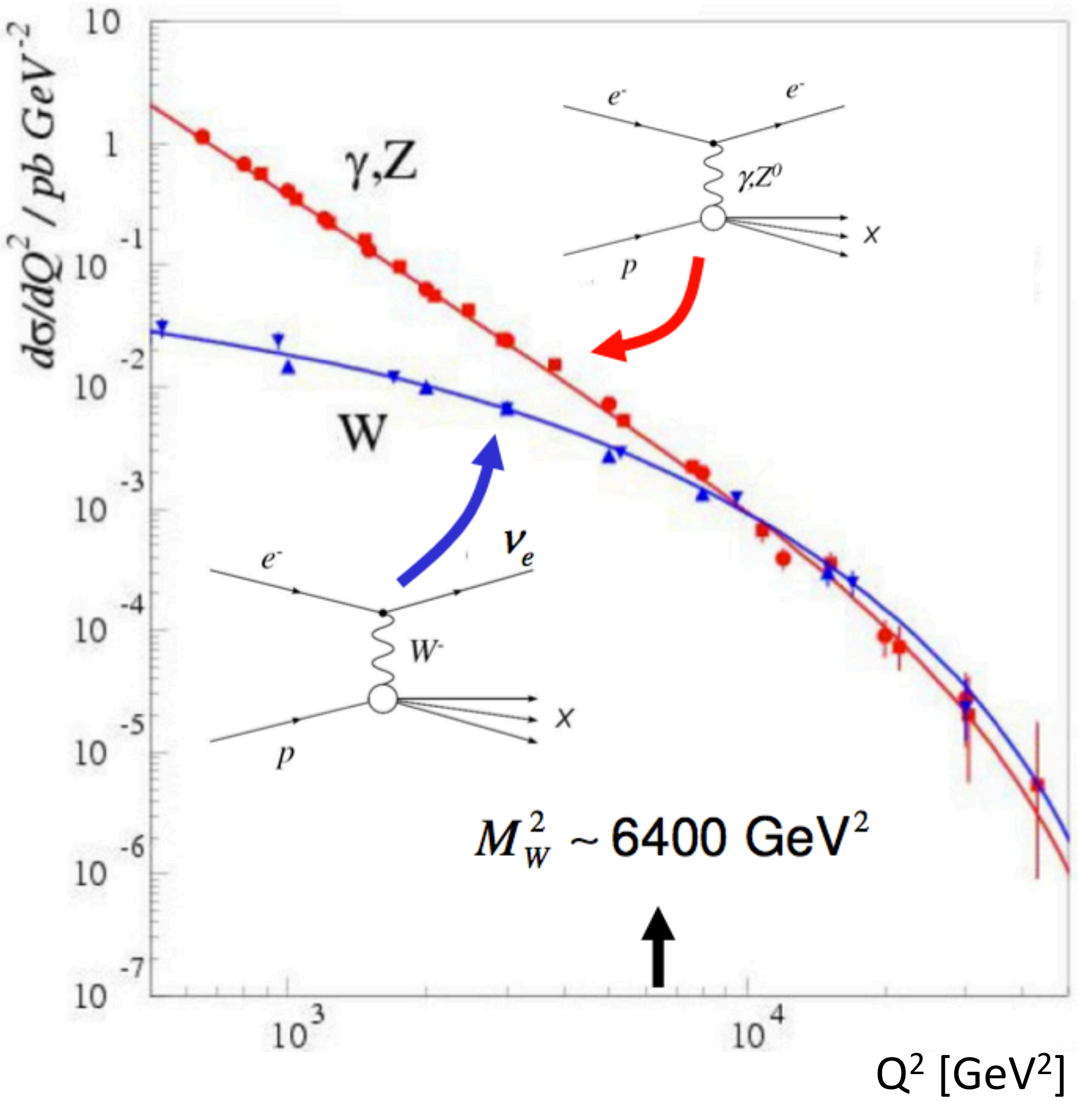
Strong interaction (QCD) is realized by gluon exchange.

The coupling constant $\alpha_S \approx 1$ decreases at small distances.

At $R>\sim 1fm$ $\alpha_S \approx 1$ non-pertubative, hadrons
At $R<< 1fm$ $\alpha_S < 1$ pertubative,
quarks,gluons

Saves perturbation theory and usage of Feynman diagrams for high energy, $Q>\sim 1GeV$

Strength of the Electromagnetic and Weak Forces



As expected for $Q^2 > M^2_W$ the cross sections for the electromagnetic and weak forces becomes of similar strength.

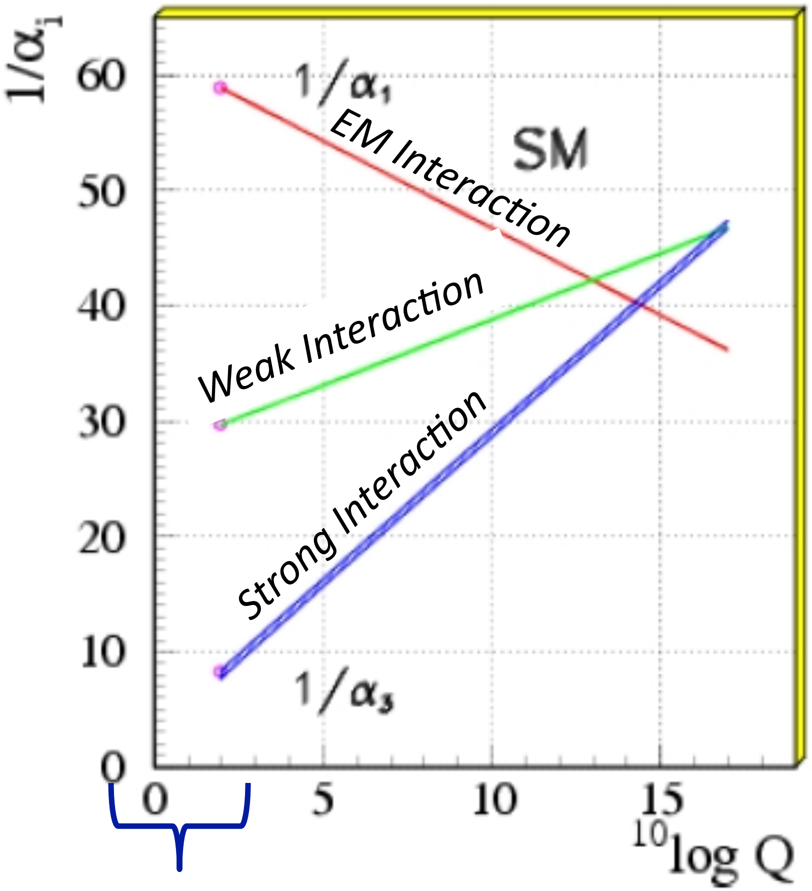
<= Data from the HERA electron-proton collider.

This hints at **the unification of the electromagnetic and the weak interaction.**

$$\approx \frac{|g^2|^2}{|g_W^2|^2}$$

Running of the Coupling Constants

1 / Coupling Constants



The experimental domain: $Q \sim 1000$ GeV

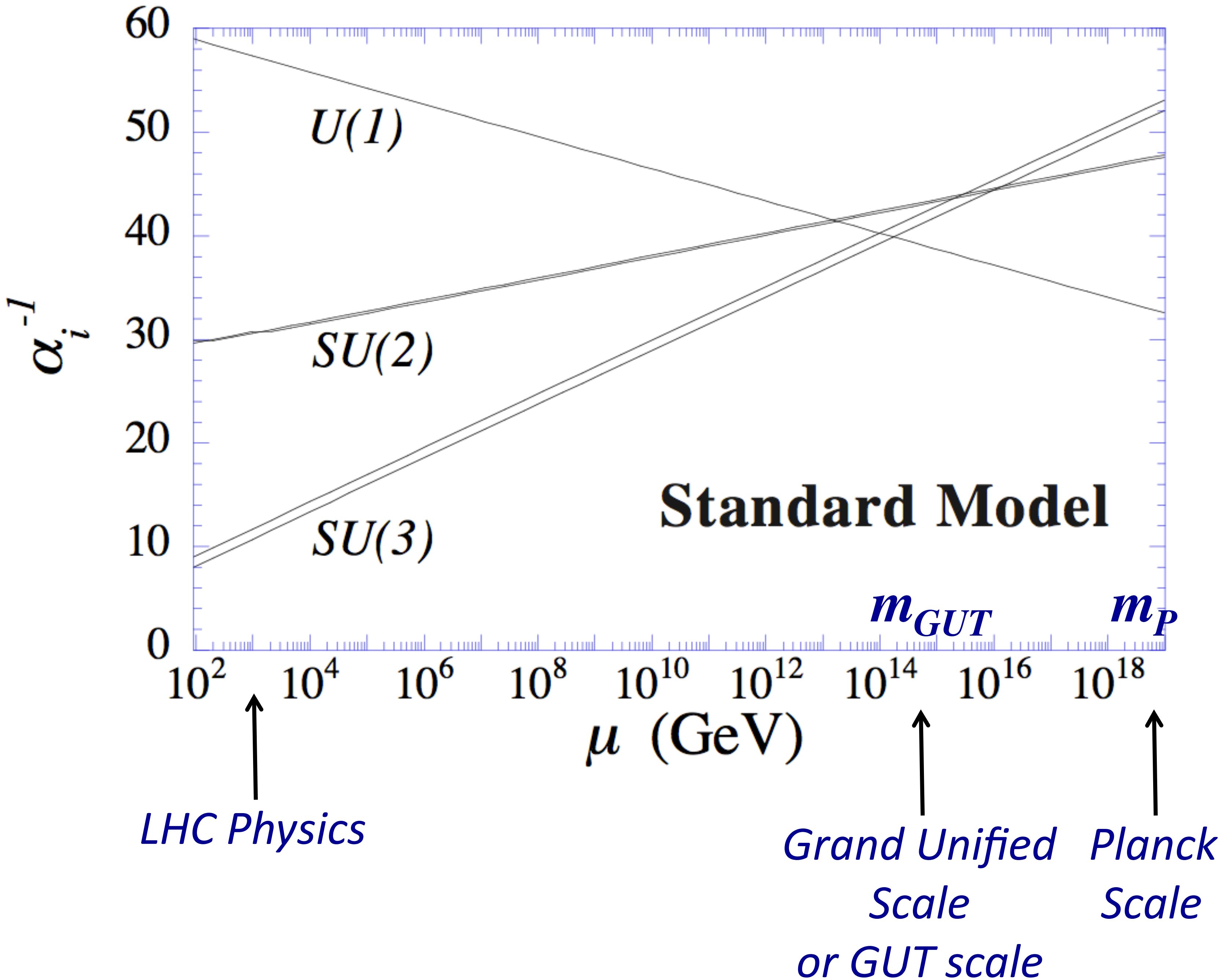
The coupling constants *approximately* converge at high energy.

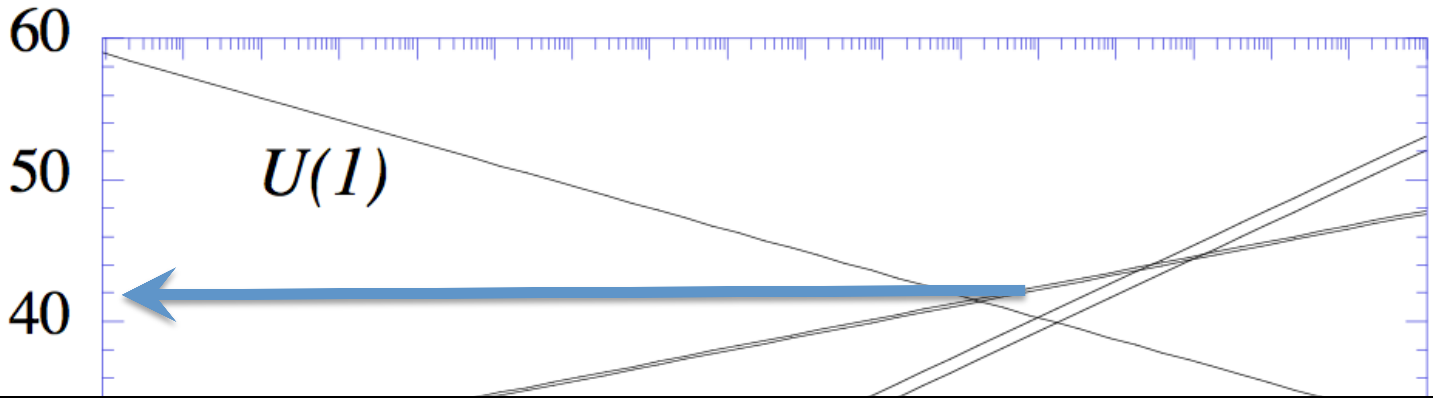
If one makes **a mental experiment at very small distances** (i.e. very high Q), then **screening effects for all 3 forces disappear** and **they become roughly the same!**

Note: The extrapolation is based on the very low energy measurements (many order of magnitudes before convergence)….

The coupling for the EM, Weak and Strong interactions seem to converge at very high energy = very small distances.

Is this evidence that they are different manifestation of a single force?


60
50
40
30
20
10
0
U(1)
SU(2)
SU(3)
Standard Model
α_i^{-1}
m_{GUT}
m_P
10^2
10^4
10^6
10^8
10^{10}
10^{12}
10^{14}
10^{16}
10^{18}
μ (GeV)
LHC Physics
Grand Unified Scale or GUT scale
Planck Scale



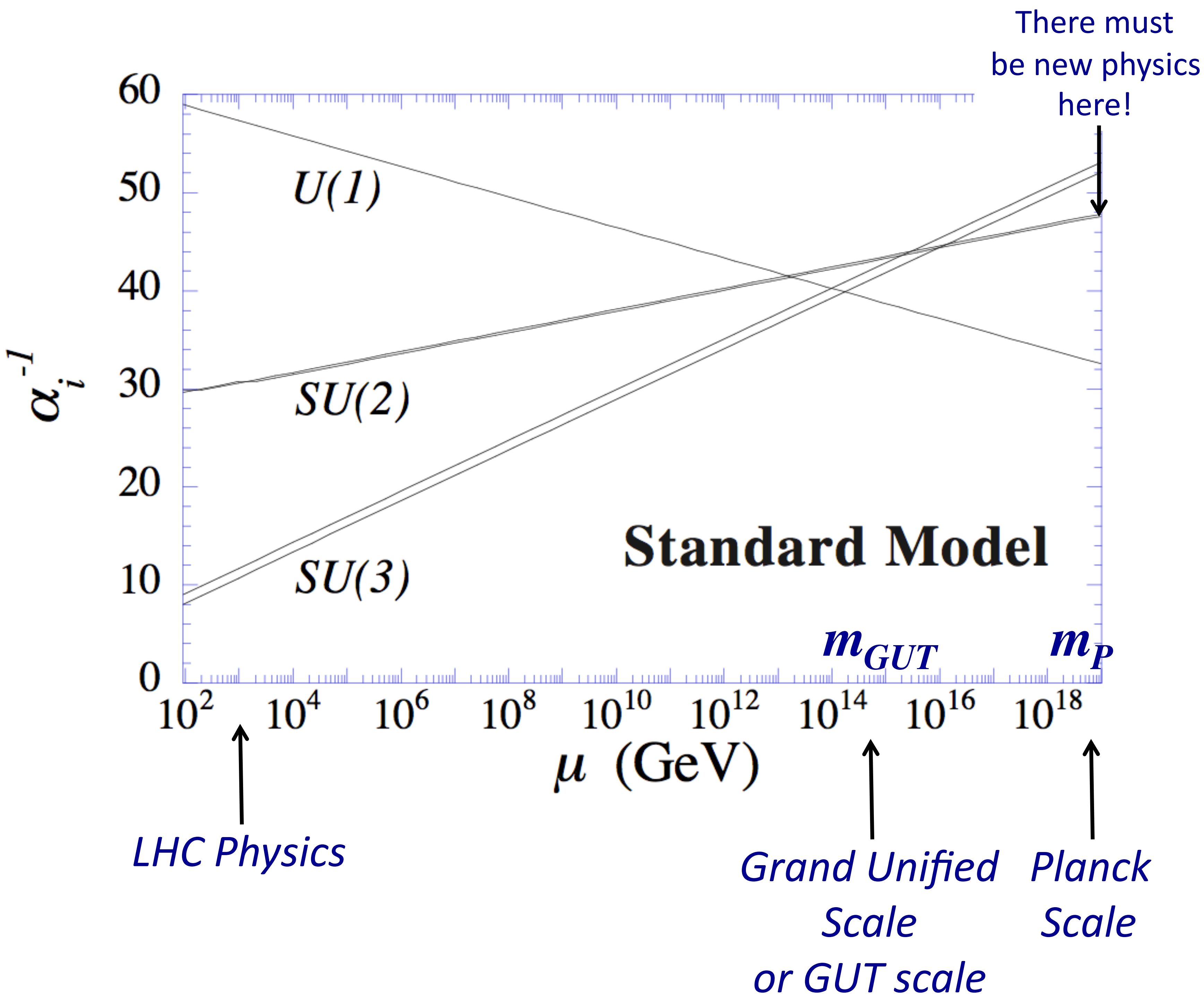
The Answer is 42!

Now in the rest of the lecture let’s try to find the question

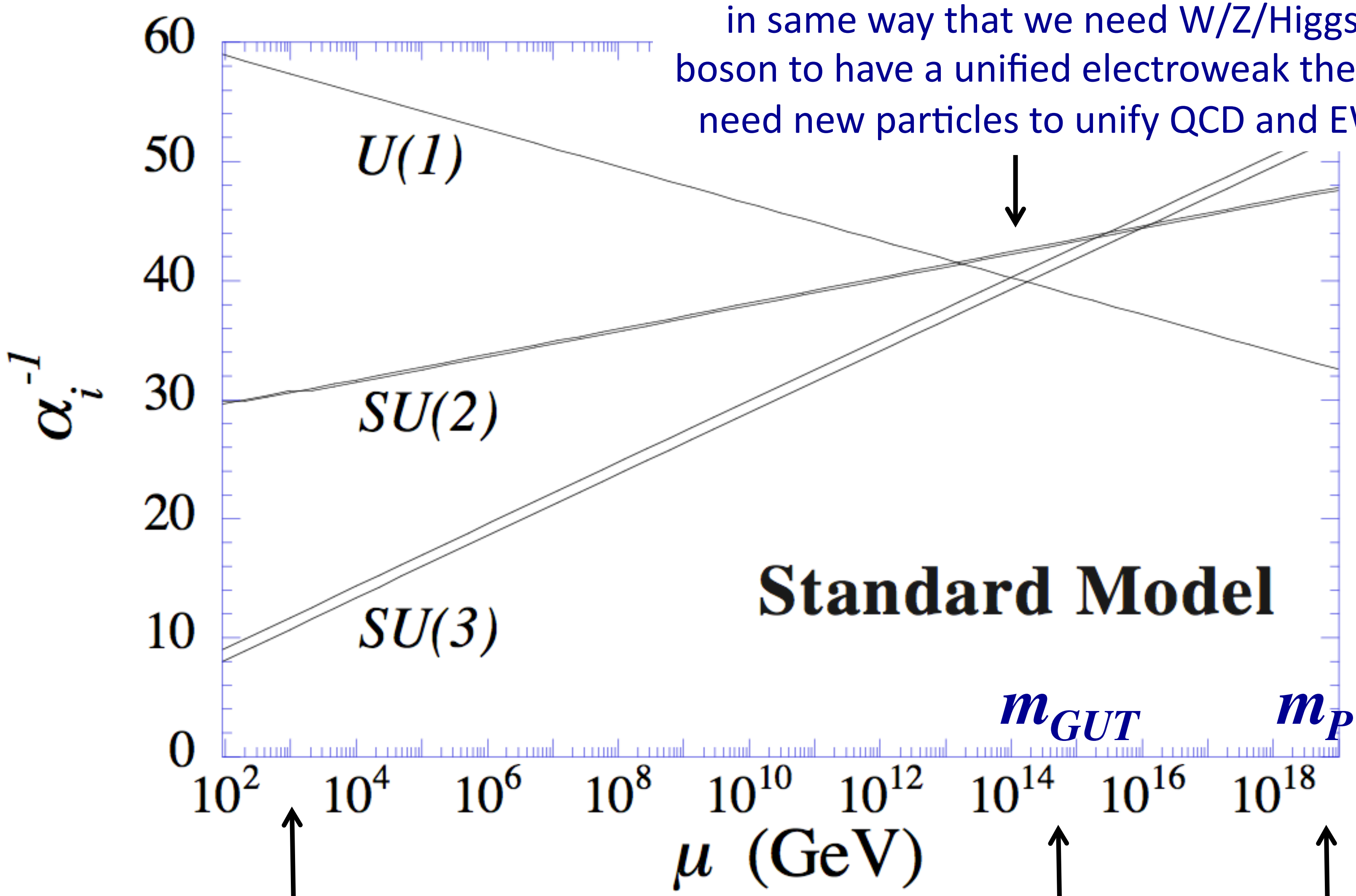
LHC Physics

Grand Unified Scale or GUT scale

Planck Scale


There must
be new physics
here!
60
50
40
30
20
10
0
α_i^{-1}
U(1)
SU(2)
SU(3)
Standard Model
m_{GUT}
m_P
10^2
10^4
10^6
10^8
10^{10}
10^{12}
10^{14}
10^{16}
10^{18}
μ (GeV)
LHC Physics
Grand Unified
Scale
or GUT scale
Planck
Scale

Should be new physics here too
in same way that we need W/Z/Higgs
boson to have a unified electroweak theory
need new particles to unify QCD and EW

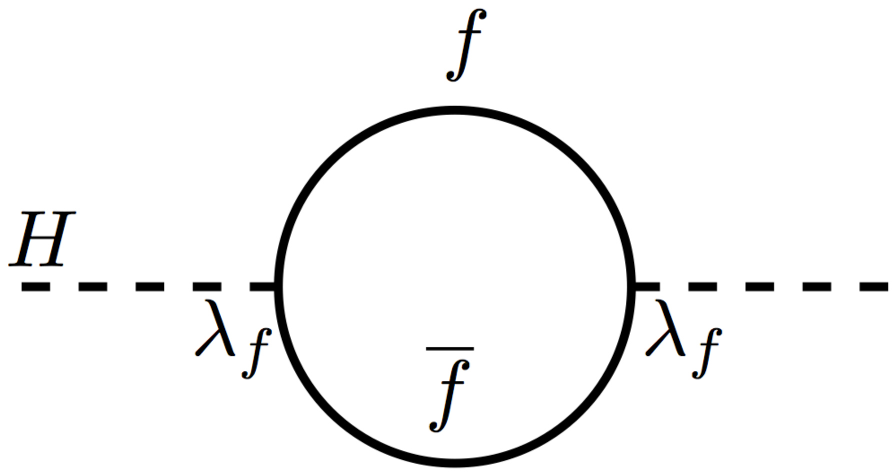


LHC Physics

Grand Unified Scale or GUT scale

Planck Scale

The Hierarchy Problem



We now know from the LHC that the Higgs mass is m_H ~125 GeV

The Higgs mass receives large corrections from fermions:

$$\Delta m_H^2 = \frac{\lambda_f^2}{16\pi^2}\left[-2M_{\mathrm{UV}}^2 + 6m_f^2 \ln\left(M_{\mathrm{UV}}/m_f\right) + \ldots\right]$$

The biggest contribution arises from the top quark which has the highest mass and $\boldsymbol{\lambda_f}$.

M_{UV} is the highest energy scale in the standard model, this is the upper end of an integral

what should we chose for M_{UV}?

The Hierarchy Problem

$$\Delta m_H^2 = \frac{\lambda_f^2}{16\pi^2}\left[-2M_{\mathrm{UV}}^2 + \cdots\right.$$

M_{UV} at most m_P or m_{GUT} ?

If Λ_{UV} at = 10^{16} or 10^{19} GeV then the Higgs mass should be enormous! not 125 GeV...

3 Generic answers to the hierarchy problem

1. Fine tuning (unnatural)
   - All the corrections to the Higgs mass compensate each other to the 30th decimal in a miraculous way.

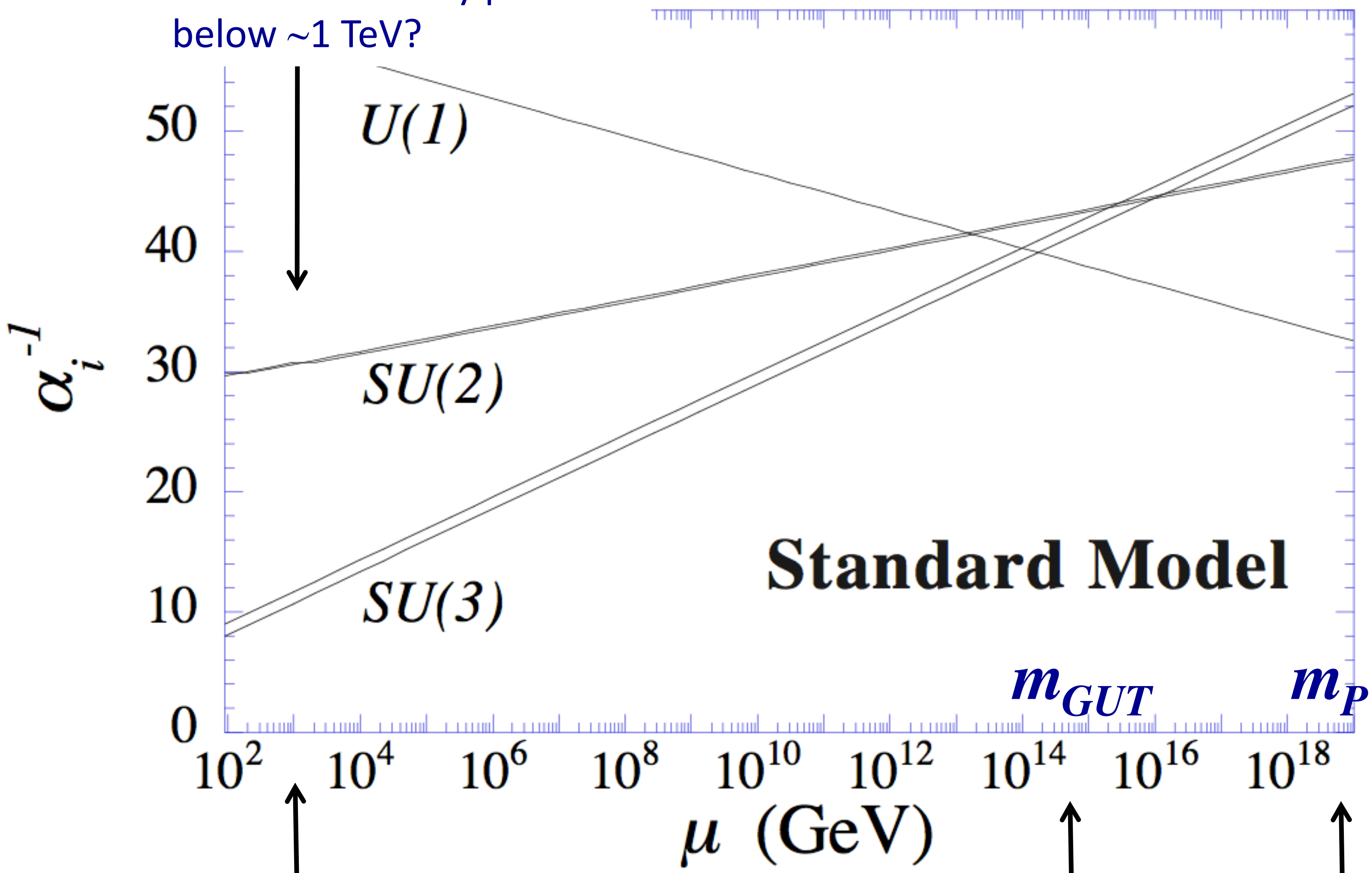
2. **There is a reason for the cancellations of all the terms**
   - New symmetry and new particles kick in well before m_P or m_{GUT} .
   - m_H ~125 GeV tells us something about the scale of new physics
   - $m_{NEW} \sim$ 100 – 1000 GeV

SUPERSYMMETRY
Compositeness
...

3. **We made a mistake when we computed the Planck mass**
   - Something we do not know about gravity and quantum mechanics
   - Planck mass is much lower than we think.

New theories of gravity
Extra dimensions...

New symmetry and new particles to solve the hiearchy problem below ~1 TeV?



LHC Physics

Now there is hope!

Grand Unified Scale or GUT scale

Planck Scale

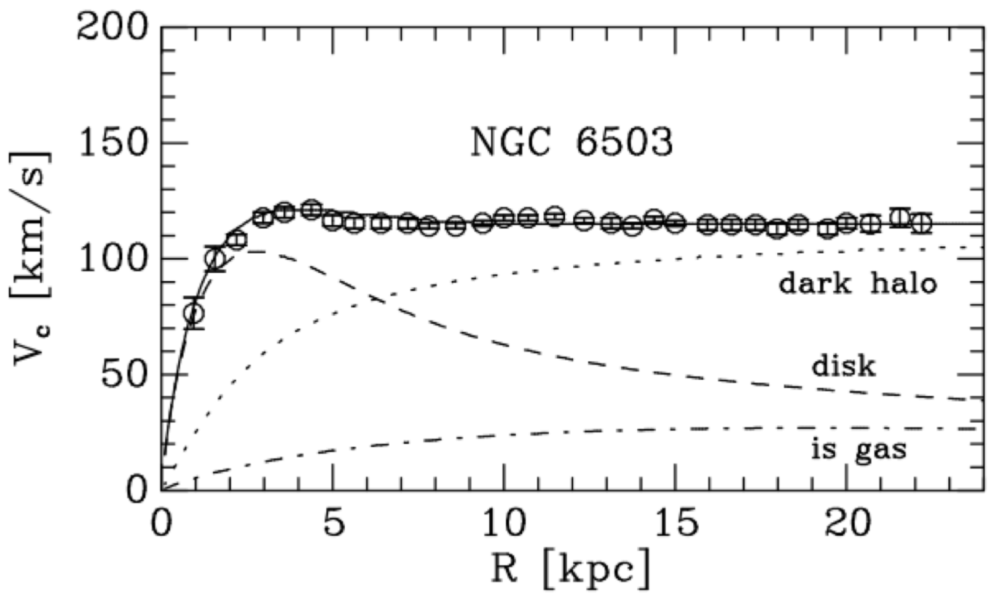
Dark Matter Problem

- *Rotation curves of galaxies*

$$v = \sqrt{\frac{GM(R)}{R}}$$

- *Gravitational lensing*

- *WMAP / Planck*
  - *Allows to weigh the Universe*



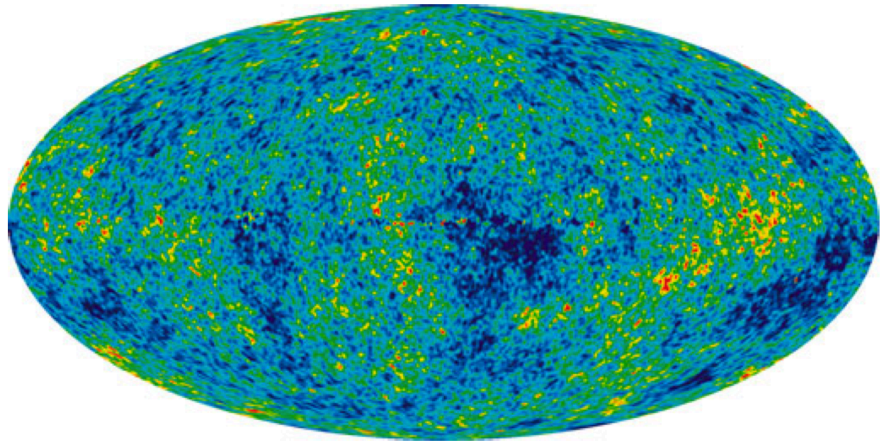
<u>Conclusions</u>

1) Dark Energy (75%)

- *responsible for accelerated expansion of the Universe- no clue what it is. <u>Repulsive effect.</u>*

2) Dark Matter (20%)

- *<u>"normal" gravitational</u> behaviour, best candidate is a weakly interacting **particle** χ with mass 100-1000 GeV*



Cosmic microwave background temperature measured with WMAP

Created during the big bang and left over "relic" observed now: must be stable

Relic Dark Matter Density

(1) Universe is dense and hot, all particles are in thermal equilibrium.

$f\bar{f} \leftrightarrow \chi\chi$

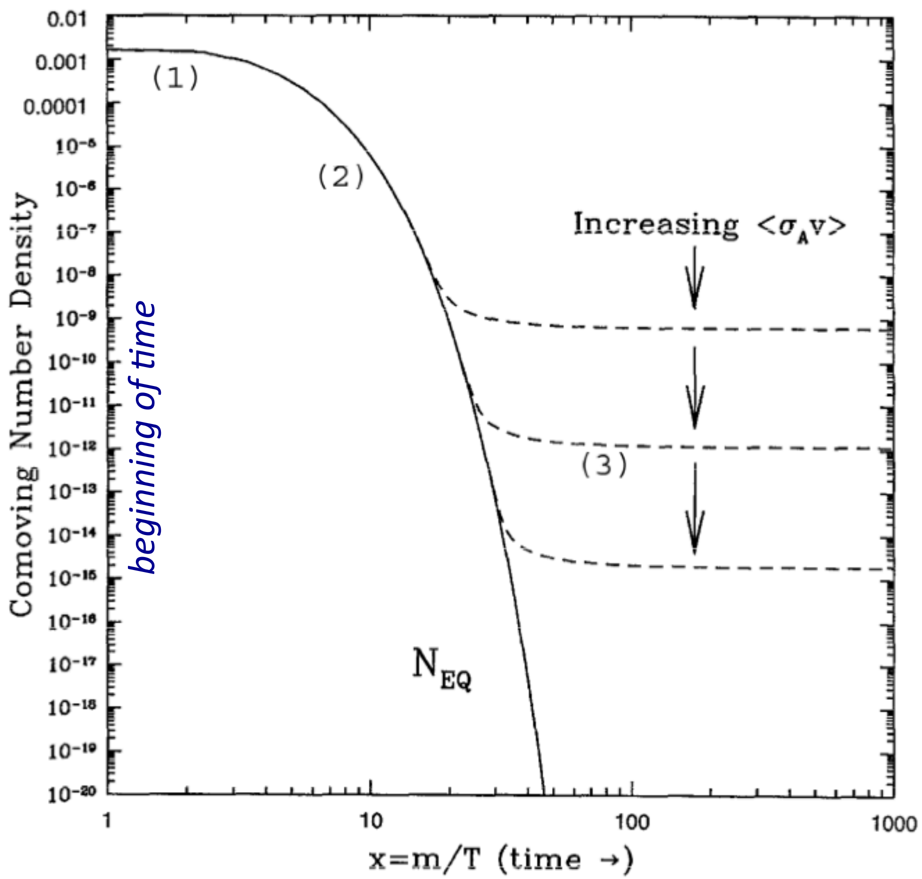
(2) The Universe is still at equilibrium but expands and cool to a temperature $T < m_\chi$. The χ density is Boltzmann suppressed, falling as: $N_\chi \propto e^{-m_\chi/T}$

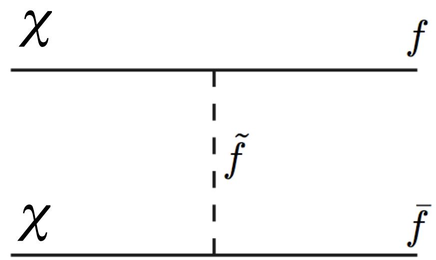
$f\bar{f} \leftarrow \chi\chi$ takes over

The higher the annihilation cross section $<\sigma_A v>$, the longer $f\bar{f} \leftarrow \chi\chi$ continues

(3) The Universe is so cool and diluted that **equilibrium is no longer maintained**
$f\bar{f} \leftarrow \chi\chi$ almost stops and we are left

with an almost constant relic dark matter density.





This cross section determines how much dark matter left today in the Universe

$$\Omega_{\rm DM} \sim \langle \sigma_A v \rangle^{-1}$$

The WIMP Miracle

Using *dimensional analysis* one can show that:

$$\sigma_A v = k \frac{4\pi\alpha_1^2}{m_\chi^2} \; (1 \text{ or } v^2)$$

1 or v^2 depending on the angular momentum in $\chi\chi \to f\bar{f}$

α_1 is the **coupling constant** associated to the weak interaction in the SM.

k = is just a **measure of deviation** from this simplistic dimensional analysis.

For a given value of m_χ vary k in 0.5 - 2

Best candidate for Dark Matter is a **Weakly Interacting Massive Particle** (WIMP) in the mass range 100-1000 GeV (must be electrically neutral).

This naturally yields the observed current amount of dark matter.

The Standard Model does not contain a WIMP -> predicts a new particle below 1 TeV!

Recommended reading
"**Collider Physics and Cosmology**" J.L. Feng http://arxiv.org/abs/0801.1334

Sakharov's conditions

Observation: **The Universe does not contain large amounts of anti-matter**

Therefore during the big bang microscopical processes between particles must have slightly favoured matter over anti-matter.

To generate the visible matter / anti-matter asymmetry in the Universe

- Baryon Number B must be violated
- C- and CP- symmetries must be violated
- Out of equilibrium

Baryon number conserved in all particle physics experiments so far and is a result of symmetries in the SM.

C- and **CP- violations** are present in the Standard Model, but not large enough to explain the large asymmetry we see.

Minimal conclusion:
Additional sources of CP- violations beyond the Standard Model are needed.

Flavour physics and general architecture of the standard model

Table 1. The fermionic particle content of the Standard Model. Here we've put primes on the neutrinos in the same spirit of putting primes on the down-quarks in the quark doublets, indicating that the mass eigenstates are rotated by the MNS and CKM matrices, respectively. The subscripts g, r, b refer to colors.

$$
\begin{array}{cccccc}
\begin{pmatrix} \nu'_e \\ e \end{pmatrix}_L^{-1/2} & \begin{pmatrix} \nu'_\mu \\ \mu \end{pmatrix}_L^{-1/2} & \begin{pmatrix} \nu'_\tau \\ \tau \end{pmatrix}_L^{-1/2} & e_R^{-1} & \mu_R^{-1} & \tau_R^{-1} \\
\begin{pmatrix} u \\ d' \end{pmatrix}_{L,g}^{1/6} & \begin{pmatrix} c \\ s' \end{pmatrix}_{L,g}^{1/6} & \begin{pmatrix} t \\ b' \end{pmatrix}_{L,g}^{1/6} & u_{R,g}^{2/3} \; d_{R,g}^{-1/3} & c_{R,g}^{2/3} \; s_{R,g}^{-1/3} & t_{R,g}^{2/3} \; b_{R,g}^{-1/3} \\
\begin{pmatrix} u \\ d' \end{pmatrix}_{L,r}^{1/6} & \begin{pmatrix} c \\ s' \end{pmatrix}_{L,r}^{1/6} & \begin{pmatrix} t \\ b' \end{pmatrix}_{L,r}^{1/6} & u_{R,r}^{2/3} \; d_{R,r}^{-1/3} & c_{R,r}^{2/3} \; s_{R,r}^{-1/3} & t_{R,r}^{2/3} \; b_{R,r}^{-1/3} \\
\begin{pmatrix} u \\ d' \end{pmatrix}_{L,b}^{1/6} & \begin{pmatrix} c \\ s' \end{pmatrix}_{L,b}^{1/6} & \begin{pmatrix} t \\ b' \end{pmatrix}_{L,b}^{1/6} & u_{R,b}^{2/3} \; d_{R,b}^{-1/3} & c_{R,b}^{2/3} \; s_{R,b}^{-1/3} & t_{R,b}^{2/3} \; b_{R,b}^{-1/3}
\end{array}
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\end{array}
$$

Is it not ugly?

Flavour physics and general architecture of the standard model

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$$
\begin{array}{cccccc}
\begin{pmatrix} \nu'_e \\ e \end{pmatrix}_L^{-1/2} & \begin{pmatrix} \nu'_\mu \\ \mu \end{pmatrix}_L^{-1/2} & \begin{pmatrix} \nu'_\tau \\ \tau \end{pmatrix}_L^{-1/2} & e_R^{-1} & \mu_R^{-1} & \tau_R^{-1} \\
\begin{pmatrix} u \\ d' \end{pmatrix}_{L,g}^{1/6} & \begin{pmatrix} c \\ s' \end{pmatrix}_{L,g}^{1/6} & \begin{pmatrix} t \\ b' \end{pmatrix}_{L,g}^{1/6} & \begin{matrix} u_{R,g}^{2/3} \\ d_{R,g}^{-1/3} \end{matrix} & \begin{matrix} c_{R,g}^{2/3} \\ s_{R,g}^{-1/3} \end{matrix} & \begin{matrix} t_{R,g}^{2/3} \\ b_{R,g}^{-1/3} \end{matrix} \\
\begin{pmatrix} u \\ d' \end{pmatrix}_{L,r}^{1/6} & \begin{pmatrix} c \\ s' \end{pmatrix}_{L,r}^{1/6} & \begin{pmatrix} t \\ b' \end{pmatrix}_{L,r}^{1/6} & \begin{matrix} u_{R,r}^{2/3} \\ d_{R,r}^{-1/3} \end{matrix} & \begin{matrix} c_{R,r}^{2/3} \\ s_{R,r}^{-1/3} \end{matrix} & \begin{matrix} t_{R,r}^{2/3} \\ b_{R,r}^{-1/3} \end{matrix} \\
\begin{pmatrix} u \\ d' \end{pmatrix}_{L,b}^{1/6} & \begin{pmatrix} c \\ s' \end{pmatrix}_{L,b}^{1/6} & \begin{pmatrix} t \\ b' \end{pmatrix}_{L,b}^{1/6} & \begin{matrix} u_{R,b}^{2/3} \\ d_{R,b}^{-1/3} \end{matrix} & \begin{matrix} c_{R,b}^{2/3} \\ s_{R,b}^{-1/3} \end{matrix} & \begin{matrix} t_{R,b}^{2/3} \\ b_{R,b}^{-1/3} \end{matrix}
\end{array}
$$

Is it not ugly?

The second generation led the Nobel Laureate I.I. Rabi to ask "who ordered muon?"

Flavour physics and general architecture of the standard model

- Why this assignment of quantum numbers?

- Why are hypercharges quantized in units of 1/6?

These specific values are **crucial to guarantee the cancellation of anomalies** which would jeopardize the gauge invariance (will come back to that later)

- Conspire for the neutrality of matter

Q(e) + 2Q(u) + Q(d)=0 (hydrogen)
Q(u)+2 Q(d)=0 (neutron)

Verified experimentally to a very high degree of accuracy.

- The proton and electron a priori have no connection!
  Yet they have exactly opposite charges!

There must be an underlying connection! (for theorist this is Grand Unification theories…)

- Where do the flavour matrices (CKM and MNS) come from?

Quarks spin = 1/2		
Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.003	2/3
d down	0.006	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	175	2/3
b bottom	4.3	–1/3

Leptons spin = 1/2		
Flavor	Mass GeV/c^2	Electric charge
ν_e electron neutrino	$<1\times10^{-8}$	0
e electron	0.000511	–1
ν_μ muon neutrino	<0.0002	0
μ muon	0.106	–1
ν_τ tau neutrino	<0.02	0
τ tau	1.7771	–1

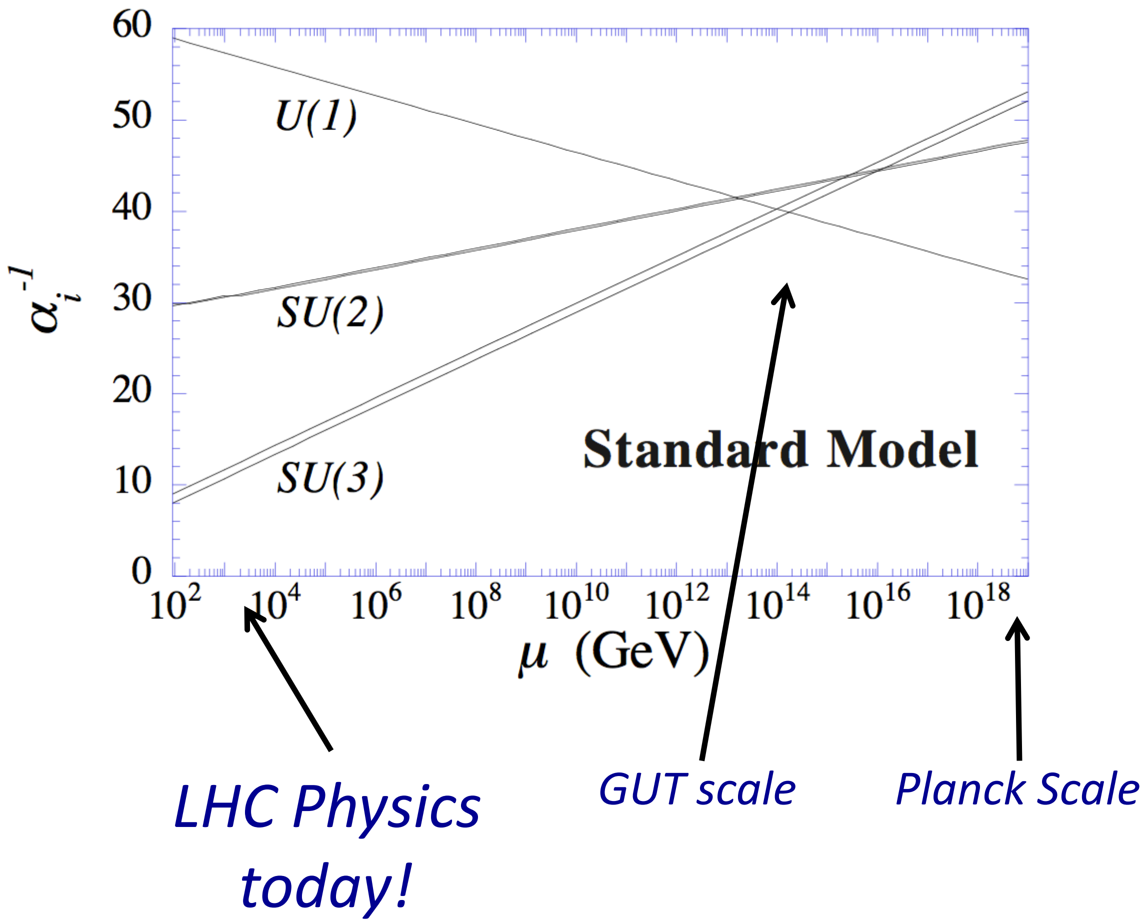
The masses of the fermions stretches over 13 order of magnitudes!

We understand how particles get a mass, by interacting with the Higgs field but …

We cannot predict the values of the Yukawa couplings to the Higgs (which set the masses)

CONCLUSION SO FAR

- The Standard Model is an incomplete theory.
- There must be new physics / particles at an energy scale of ~1 TeV



Structure of the SUSY & Exotics Lectures

1. **What questions do we want to answer?** **WHY**
    - The Planck and GUT Scale
    - The hierarchy problem
    - Dark Matter
    - Sakharov's conditions
    - Other Problems with the Standard Model

2. **Suypersymmetry** **WHAT**
    - Construction of supersymmetry
    - Particle spectrum
    - SUSY Particle production and decay
    - R-Parity
    - Some experimental searches

Tomorrow: **HOW and WHERE …**

Analogy with electrodynamics

See Hitoshi Murayama
http://www.arxiv.org/abs/hep-ph/0002232v2

The reason why supersymmetry is needed is similar to the reason why the positron is needed.

An electron in vacuum has a Coulomb electric field around it with energy:

$$\Delta E_{\text{Coulomb}} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_e}$$

To avoid the **Coulomb self-energy to become infinite** we are forced to define a **size** of the electron r_e.

Since this energy is present for any electrons it has to be considered **as part of an electron rest energy**.

$$(m_e c^2)_{obs} = (m_e c^2)_{bare} + \Delta E_{\text{Coulomb}}$$

Experimentally we know that the size of the electron must be small $r_e < 10^{-17}$ cm, this leads $\Delta E_{\text{Coulomb}} \sim 10$ GeV. The bare mass must be negative:

$$0.511 = -9999.489 + 10000 \text{ MeV}$$

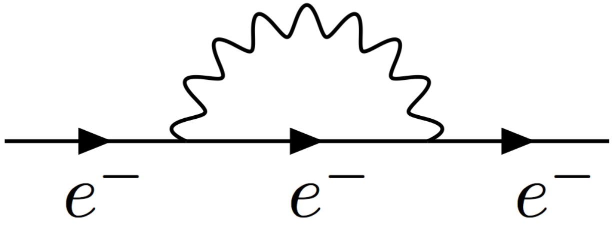
Requires a ridiculous **fine-tuning** to give the observed electron mass.

Avoid this by requiring $\Delta E_{\text{Coulomb}} < m_e c^2$ but this leads to $r_e > 10^{-13}$ cm.

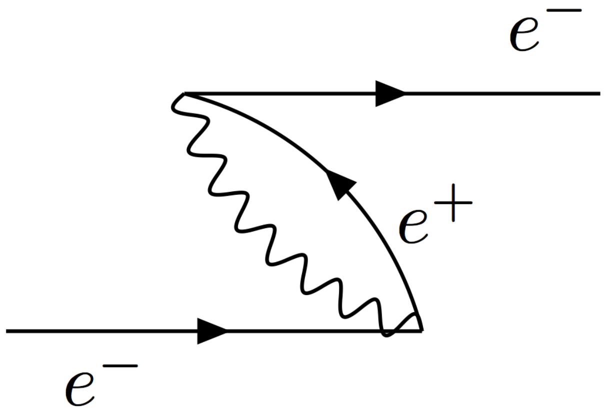
$\Rightarrow$ Applying the simple model above to $r_e < 10^{-13}$ cm leads to absurd results.

Analogy with electrodynamics (2)

We can depict the electron self-interaction by:



But the **positron exists** and the world is quantum mechanical so the vacuum can fluctuate to give an **electron-positron** during a a time: $\Delta t \sim \hbar/\Delta E \sim \hbar/(2m_e c^2)$



during that time the electron can cover a distance:

$$d \sim c\Delta t \sim \hbar c/(2m_e c^2) = 200 \times 10^{-13}\ \text{cm}$$

below this scale must take into account vacuum fluctuations.

Provides a contribution to the electron self-energy that cancels with the previous divergent term.

$$\Delta E_{\text{pair}} = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_e}$$

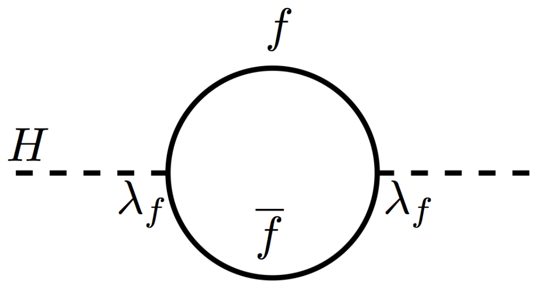
Leading remaining term:

$$(m_e c^2)_{obs} = (m_e c^2)_{bare}\left[1 + \frac{3\alpha}{4\pi}\log\frac{\hbar}{m_e c r_e}\right]$$

Only 9% correction event at m_P. Proportional to bare mass

⇒ no correction if the particle is massless

Higgs Boson Self-Energy and Top-Quark Loops

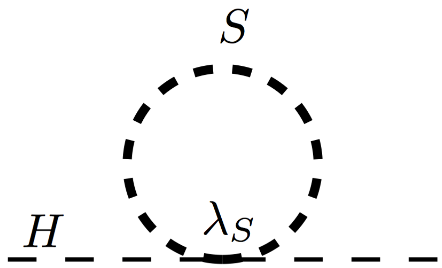


fermion loop

The biggest contributor to the loop corrections to the Higgs mass is **the top quark due to its high mass** (hence its large coupling to Higgs)

$$\Delta\mu^2_{\rm top} = -6\frac{h_t^2}{4\pi^2}\frac{1}{r_H^2}$$

distance at which SM has problems:
$r_H \sim 10^{-17}\text{cm} \sim 1$ TeV



scalar loop

If we invent the **superpartner of the top quark**, the **scalar top** quark or "**stop**"(same quantum numbers as top but spin 0)

$$\Delta\mu^2_{\rm stop} = +6\frac{h_t^2}{4\pi^2}\frac{1}{r_H^2}$$

We are left with the top / stop correction to the Higgs mass:

$$\Delta\mu^2_{\rm top} + \Delta\mu^2_{\rm top} = -6\frac{h_t^2}{4\pi^2}(m^2_{\tilde t} - m^2_t)\log\frac{1}{r_H^2 m^2_{\tilde t}}$$

Ideally one would have $m_{\tilde t} = m_t$
in the same way that the positron has the same mass as the electron.
Can be overcome if the two masses not too different, will come back to that later.

We need a conspiracy of **new spin zero particles** that would precisely cancel the divergences from the standard model fermions.

$$\Delta m_H^2 = -\frac{\lambda_f^2}{16\pi^2}(2M_{\mathrm{UV}}^2) + \ldots$$ **Dirac fermions**

$$\Delta m_H^2 = +\frac{\lambda_S}{16\pi^2}M_{\mathrm{UV}}^2 + \ldots$$ **Scalar**

fermion and boson loops give **opposite contributions**.

⇒ **Conspiracy in physics is called a symmetry**

We need a symmetry between fermions and bosons = **SUPERSYMMETRY**

A symmetry between fermions and bosons in fact guarantees that for each standard model term there will be an exactly compensating supersymmetric term.

Mathematical beauty?

The idea of the positron arose when putting together quantum mechanics and relativity in form of the **Dirac equation** to describe the fermions.

Quantum mechanical equation **invariant under the Poincare group**.

⇒Describe more with less: both *positrons* and *electrons* described by the **same equation**

Supersymmetry is the only non-trivial extension of the Poincare group

If history repeats itself, this could be very fruitful.

This beauty argument alone does not tells us whether Supersymmetry should exist at the electroweak scale, *it could also be a symmetry that rules at GUT for instance*.

In this lecture consider only **weak scale supersymmtry** (= that **solves the hierarchy problem**)

Note that if the hierarchy problem was solved by something else SUSY could still be relevant at much higher energies (but not for the LHC ☹)

Supersymmetry Schematics

For details and to work through notations see:
S. Martin hep-ph/1205.4076v1 (adopted here)
also
S. Martin hep-ph/9709356 A. Bilal hep-th/0101055v1

A supersymmetry transformation turns fermion states into boson states and vice versa.
We consider Q, $Q\dagger$ the operator that generates such transformations

$$Q|fermion\rangle = |boson\rangle \qquad Q|boson\rangle = |fermion\rangle$$

$$Q^{\dagger}|fermion\rangle = |boson\rangle \qquad Q^{\dagger}|boson\rangle = |fermion\rangle$$

We have to think the **fermions as 4 component Dirac spinors** which represent the **fermion** and **anti-fermion** and **left-handed** and **right-handed** *helicities*.

Q is an anticommuting spinor.

Q_{α} operates on the first two components, the **left handed** part.
and $Q^{\dagger}_{\beta}$ operates on the **right handed** part (the last two components).

$$\begin{pmatrix} \xi_{\alpha} \\ \chi_{\beta} \end{pmatrix} = \begin{pmatrix} e_L \\ e_R \end{pmatrix}$$

From now on remove the subscript α,β.

Theories that fullfill these requirements are restricted in 4 dimensions with chiral fermions.
Q carried helicity L and spin ½ $\quad Q^{\dagger}$ carries helicity R and spin ½.
They must satisfy …

Schematics of Supersymmetry Algebra

$\{Q, Q^\dagger\} = P^\mu$ — P^μ generator of space-time translations

$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$

$[P^\mu, Q] = [P^\mu, Q^\dagger] = 0$

$[T^a, Q] = [T^a, Q^\dagger] = 0$ — T^a generator of gauge transformations

A single particle state is then represented by a **super-multiplet**.

A **super multiplet** contains both **fermions and bosons**, that are **superpartners** of each other.

$|\Omega'\rangle$ is a superpartner of $|\Omega\rangle$ means that $|\Omega'\rangle$ can be obtained by a combination of Q and $Q^\dagger$ operating on $|\Omega\rangle$.

Q and $Q^\dagger$ commute with P^μ. Therefore they commute with P^2.
$\Rightarrow$ **superpartners must have the same mass.**

Q and $Q^\dagger$ commute with T^a.
$\Rightarrow$ **superpartners have the same quantum numbers.**

There must be the same number of Bosons and Fermions in a super-multiplet

S is the spin, we consider the operator $(-1)^{2S}$.

$$(-1)^{2S}\left|fermion\right\rangle = -\left|fermion\right\rangle$$

$$(-1)^{2S}\left|boson\right\rangle = \left|boson\right\rangle$$

$(-1)^{2S}$ must anticommute with Q and Q†:

$$(-1)^{2S}Q\left|fermion\right\rangle = (-1)^{2S}\left|boson\right\rangle = \left|boson\right\rangle$$

$$Q(-1)^{2S}\left|fermion\right\rangle = -Q\left|fermion\right\rangle = -\left|boson\right\rangle$$

Consider the sub-space of fermions and bosons $\left|i\right\rangle$ with the same momentum p^μ .
Any transformed state obtained by applying Q and Q† on $\left|i\right\rangle$ has still the same momentum.

If one writes $(-1)^{2S}$ in that subspace on a basis of fermions and bosons, it must be diagonal with *ones* and *minus ones* on the diagonal for each boson and fermion.

example: if we have a basis made of 2 bosons and 2 fermions $(B_1,\ B_2,\ F_1,\ F_2)$

$$(-1)^{2S} \equiv \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

⇒The trace of the $(-1)^{2S}$ operator is equal to
Number of Bosons – Number of Fermions = $\boldsymbol{n_B - n_F}$ in the **supermultiplet**.

A priori we don't yet know how many fermions and bosons in the supermultiplet.

Number of Bosons and Fermions in a super-multiplet

$$Tr[(-1)^{2S}] = n_B - n_F$$

now let's calculate that trace:

$$Tr[(-1)^{2S}] = \sum_i \langle i|(-1)^{2S})|i\rangle$$

$$p^\mu Tr[(-1)^{2S}] = p^\mu \sum_i \langle i|(-1)^{2S}|i\rangle = \sum_i \langle i|(-1)^{2S} P^\mu|i\rangle$$

we know that the SUSY generators must fullfill: $\{Q,Q^\dagger\} = P^\mu$ hence:

$$= \sum_i \langle i|(-1)^{2S} QQ^\dagger|i\rangle + \sum_i \langle i|(-1)^{2S} Q^\dagger Q|i\rangle$$

insert the completeness relationship:

$$= \sum_i \langle i|(-1)^{2S} QQ^\dagger|i\rangle + \sum_j \sum_i \langle i|(-1)^{2S} Q^\dagger| j\rangle\langle j|Q|i\rangle$$

$$= \sum_i \langle i|(-1)^{2S} QQ^\dagger|i\rangle + \sum_i \sum_j \langle j|Q|i\rangle\langle i|(-1)^{2S} Q^\dagger| j\rangle$$

$$= \sum_i \langle i|(-1)^{2S} QQ^\dagger|i\rangle + \sum_j \langle j|Q(-1)^{2S} Q^\dagger| j\rangle$$

anticommutation relationship Q and $(-1)^{2S}$

$$= \sum_i \langle i|(-1)^{2S} QQ^\dagger|i\rangle - \sum_i \sum_j \langle j|(-1)^{2S} QQ^\dagger| j\rangle = 0$$

$$\Rightarrow p^\mu Tr[(-1)^{2S}] = p^\mu (n_B - n_F) = 0$$

$$\Rightarrow n_B = n_F$$

$n_B = n_F$ **must be our guideline to construct super multiplets.**

A standard model fermion has **two helicity states**, hence if we put one SM fermion in a supermultiplet we have n_F=2.

So we must have n_B=2 with two corresponding spin-0 fields.

⇒ ***One scalar (=spin 0) field for each helicity states***

quarks, leptons and Higgs fit here

These are the so called ***matter supermultiplets (also called chiral or scalar)***

A standard model vector boson (e.g. photon) with spin 1 has two helicity states (massless, before EW symmetry breaking)

⇒ n_B=2

The superpartner of the gauge boson the multiplet must be a fermionic state: **the gaugino**, with two helicity states

⇒ n_F=2

gauge bosons fit here

This is called a ***gaugino or vector super-multiplet.***

Theories of extended supersymmetry have more than one pair of generators Q and $Q^\dagger$. Consider only one pair ie. “N=1 supersymmetry” in these lectures.

Back to Standard Model Fermions

Example the electron $e_L^-,\ e_R^-$ must be part of a matter supermultiplet.

There are two scalar states one associated to the ***LH electron*** *and one to the* ***RH electron****.*

$\tilde{e}_L, \tilde{e}_R$ *these particles have* ***spin zero****, the indices* R,L *refers to which of the LH or RH electrons they are the partner of. (since they have spin zero, they have no helicity…)*

Situation is the same for the squarks.

For the neutrinos, usually neglect their mass: no RH handed neutrinos, only: $\nu_e, \tilde{\nu}_e$

SUSY particles are denoted with a ~ on top of the standard model particle denomination.

Matter Supermultiplets in the Minimal Supersymmetric Standard Model

Names		spin 0	spin 1/2	$SU(3)_C, SU(2)_L, U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L \;\; \widetilde{d}_L)$	$(u_L \;\; d_L)$	$(\,\mathbf{3},\ \mathbf{2}\,,\ \frac{1}{6})$
(×3 families)	$\overline{u}$	$\widetilde{u}^*_R$	$u^\dagger_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	$\overline{d}$	$\widetilde{d}^*_R$	$d^\dagger_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu} \;\; \widetilde{e}_L)$	$(\nu \;\; e_L)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$
(×3 families)	$\overline{e}$	$\widetilde{e}^*_R$	$e^\dagger_R$	$(\,\mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	H_u	$(H_u^+ \;\; H_u^0)$	$(\widetilde{H}_u^+ \;\; \widetilde{H}_u^0)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ +\frac{1}{2})$
	H_d	$(H_d^0 \;\; H_d^-)$	$(\widetilde{H}_d^0 \;\; \widetilde{H}_d^-)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$

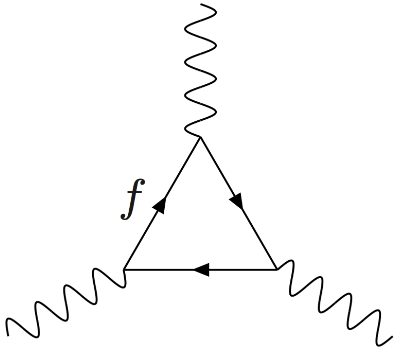
The scalar quarks are often called “squarks” and the scalar leptons are often called “sleptons”:

Scalar fermions are given their name with a “s” in front:
top quark → stop (s)quark, the “stop”…
electron → selectron, sneutrino…

The superpartners of the SM bosons are appended the suffix ***–ino.***

Higgs boson → Higgsino, W → Wino, B → Bino

How Many Higgs Supermultiplets?

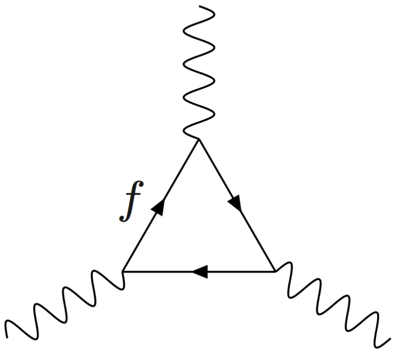


$$\sum_{SM\ fermions} Y_f^3 = 0$$

anomaly cancellation
Miracle* of the standard model

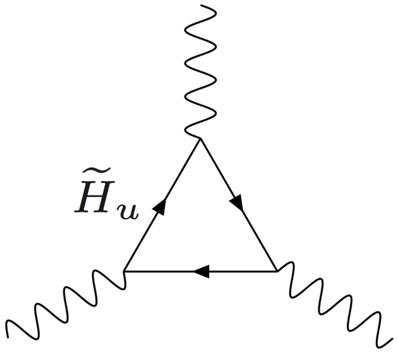
*Miracle:
More likely the standard model
is the low energy remnant of
a higher order gauge group
eg. SO(10) at GUT scale

How Many Higgs Supermultiplets?



$$\sum_{SM\ fermions} Y_f^3 = 0$$

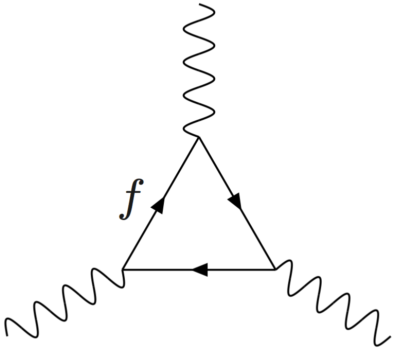
anomaly cancellation



$$\left(\frac{1}{2}\right)^3$$

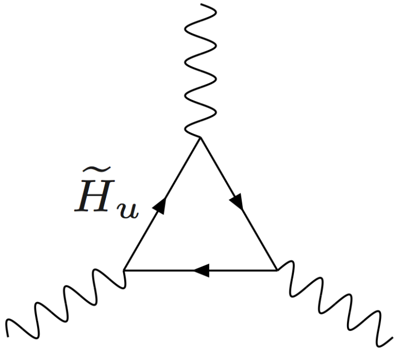
Now in SUSY we got at least
one new fermion, the Higgsino

How Many Higgs Supermultiplets?



$$\sum_{SM\ fermions} Y_f^3 = 0$$

anomaly cancellation
Miracle of the standard model



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Now in SUSY we got at least
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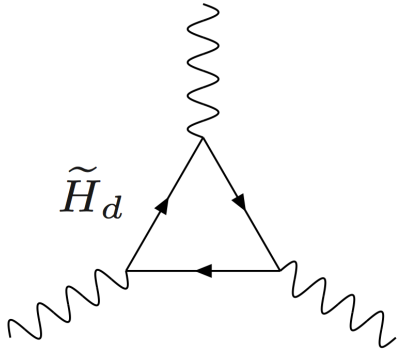


$$\left(-\frac{1}{2}\right)^3$$

Need a Higgsino with
Y=-1/2 to avoid anomalies

This new anomaly cancels if and only if both the $\tilde{H}_u$ and $\tilde{H}_d$ Higgsinos exist.

The masses of the up-quarks (u,c,t) arise from coupling with H_u

The masses of the down-quarks (d,s,b) arise from coupling with H_d

2 Higgs doublets needed!

The masses of the squarks and sleptons is another story, will come back to that point.

Matter Supermultiplets in the Minimal Supersymmetric Standard Model

Names		spin 0	spin 1/2	$SU(3)_C, SU(2)_L, U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L \;\; \widetilde{d}_L)$	$(u_L \;\; d_L)$	$(\,\mathbf{3},\, \mathbf{2}\,,\, \frac{1}{6})$
(×3 families)	$\overline{u}$	$\widetilde{u}^*_R$	$u^\dagger_R$	$(\,\overline{\mathbf{3}},\, \mathbf{1},\, -\frac{2}{3})$
	$\overline{d}$	$\widetilde{d}^*_R$	$d^\dagger_R$	$(\,\overline{\mathbf{3}},\, \mathbf{1},\, \frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu} \;\; \widetilde{e}_L)$	$(\nu \;\; e_L)$	$(\,\mathbf{1},\, \mathbf{2}\,,\, -\frac{1}{2})$
(×3 families)	$\overline{e}$	$\widetilde{e}^*_R$	$e^\dagger_R$	$(\,\mathbf{1},\, \mathbf{1},\, 1)$
Higgs, higgsinos	H_u	$(H_u^+ \;\; H_u^0)$	$(\widetilde{H}_u^+ \;\; \widetilde{H}_u^0)$	$(\,\mathbf{1},\, \mathbf{2}\,,\, +\frac{1}{2})$
	H_d	$(H_d^0 \;\; H_d^-)$	$(\widetilde{H}_d^0 \;\; \widetilde{H}_d^-)$	$(\,\mathbf{1},\, \mathbf{2}\,,\, -\frac{1}{2})$

leads to 5 physical Higgs bosons in MSSM …

Gauge Supermultiplets in the Minimal Supersymmetric Standard Model

We have to do this before electroweak symmetry breaking

Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\mathbf{8},\ \mathbf{1},\ 0)$
winos, W bosons	$\widetilde{W}^{\pm}\ \ \widetilde{W}^{0}$	$W^{\pm}\ \ W^{0}$	$(\mathbf{1},\ \mathbf{3},\ 0)$
bino, B boson	$\widetilde{B}^{0}$	B^{0}	$(\mathbf{1},\ \mathbf{1},\ 0)$

We add the suffix **–ino** to the name of the Standard Model gauge bosons.

Gauge bosons => gauginos

The gluinos are spin ½ colored octets. They also correspond to a physical mass state.

The **Bino** and **Winos** are however not mass eigenstates, mix with Higgsinos and with each other.

Supersymmetry Must be a Broken Symmetry

Remember?

Q and $Q^{\dagger}$ commute with P^{μ}. Therefore they commute with P^2.
$\Rightarrow$ **superpartners must have the same mass.**

The electron has a mass of 0.511MeV so the **selectron** should be a particle of spin zero and mass 0.511 MeV.

This particle would have been found a long time ago, should also exist abundantly in cosmic rays (the way many other particles were discovered, eg. the positron).

$\Rightarrow$ **Supersymmetry must be a broken symmetry** in the vacuum state we are in.

Remember standard model fermions are **massless before the EW symmetry breaking**...

$\Rightarrow$ The SUSY particles do not get their mass from the Higgs, but from some other unknown mechanism, will be discussed later.

$$\Delta\mu^2_{\rm top} + \Delta\mu^2_{\rm top} = -6\frac{h_t^2}{4\pi^2}(m^2_{\tilde{t}} - m^2_t)\log\frac{1}{r^2_H m^2_{\tilde{t}}}$$

SOFT SUSY Breaking

only logarithmic divergence $\text{Log}\,(1/r^2_{\rm H})$

$$(m_e c^2)_{obs} = (m_e c^2)_{bare}\left[1 + \frac{3\alpha}{4\pi}\log\frac{\hbar}{m_e c r_e}\right]$$

Soft Supersymmetry Breaking

Only certain terms are allowed in the Lagrangian so that SUSY is broken, but in *a **soft** way so that it still functions to solve the hiearchy problem*

$$L_{MSSM} = L_{SUSY} + L_{soft}$$

Purely supersymmetric entirely determined by **gauge couplings** and **standard model Yukawa couplings**

Allows eg. for different masses between SM and SUSY particles.

Specific to our vacuum state

Should be predicted by a theory of supersymmetry breaking (difficult problem, many ideas…)

SOFT SUSY Breaking

- Only certain terms (digrams/ interactions) are allowed
- The mass parameters cannot be much larger than ~1 TeV

Soft Supersymmetry Breaking

$$
\begin{aligned}
L_{soft} \quad = \quad & -\frac{1}{2}\left(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B} + \text{c.c.}\right) \\
& -\left(\widetilde{\overline{u}}\,\mathbf{a_u}\,\widetilde{Q}H_u - \widetilde{\overline{d}}\,\mathbf{a_d}\,\widetilde{Q}H_d - \widetilde{\overline{e}}\,\mathbf{a_e}\,\widetilde{L}H_d + \text{c.c.}\right) \\
& -\widetilde{Q}^\dagger\,\mathbf{m_Q^2}\,\widetilde{Q} - \widetilde{L}^\dagger\,\mathbf{m_L^2}\,\widetilde{L} - \widetilde{\overline{u}}\,\mathbf{m_{\overline{u}}^2}\,\widetilde{\overline{u}}^\dagger - \widetilde{\overline{d}}\,\mathbf{m_{\overline{d}}^2}\,\widetilde{\overline{d}}^\dagger - \widetilde{\overline{e}}\,\mathbf{m_{\overline{e}}^2}\,\widetilde{\overline{e}}^\dagger \\
& -m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (bH_uH_d + \text{c.c.})\,.
\end{aligned}
$$

Ouch!

Since we do not know how / why Supersymmetry is broken then simply add all the terms that are allowed by Gauge symmetry and that only softly breaks SUSY.

Note: many of the above are matrices…

⇒ **The Minimal Supersymmetric Standard Model has >120 parameters.**

For experimentalists it is crucial to reduce that number, several schemes to do that

The Soft SUSY Breaking Terms (1)

$$\mathcal{L}_{\text{soft}}^{\text{MSSM}} = \boxed{-\frac{1}{2}\left(M_3 \widetilde{g}\widetilde{g} + M_2 \widetilde{W}\widetilde{W} + M_1 \widetilde{B}\widetilde{B} + \text{c.c.}\right)}$$

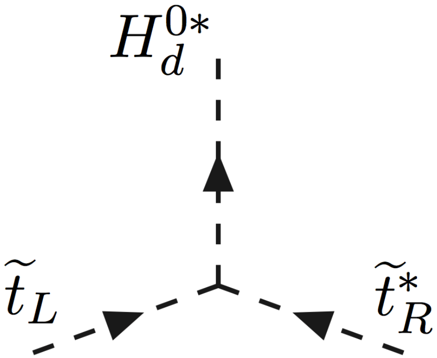
Gaugino masses: bino, wino, gluino: M_1, M_2, M_3

The Soft SUSY Breaking Terms (2)

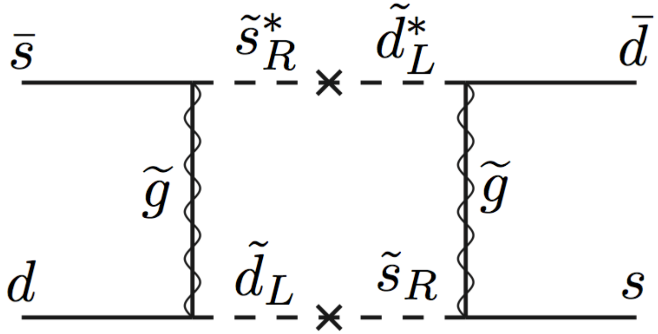
$$\mathcal{L}_{\text{soft}}^{\text{MSSM}} = -\frac{1}{2}\left(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B} + \text{c.c.}\right)$$
$$-\left(\widetilde{\overline{u}}\,\mathbf{a_u}\,\widetilde{Q}H_u - \widetilde{\overline{d}}\,\mathbf{a_d}\,\widetilde{Q}H_d - \widetilde{\overline{e}}\,\mathbf{a_e}\,\widetilde{L}H_d + \text{c.c.}\right)$$

"Trilinear couplings", they couple

- a member of RH supermultiplet (u, d, e)
- a member of LH supermultiplet (Q,L)
- a Higgs doublet (H_u, H_d)



example of diagram arising from this type of coupling, can turn LH quark into a RH handed one.



another example contribution to $K^0 \leftrightarrow \overline{K}^0$ mixing

The Soft SUSY Breaking Terms (3)

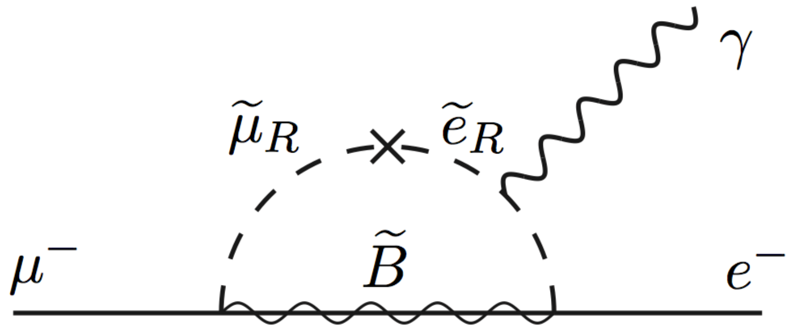
$$\mathcal{L}_{\text{soft}}^{\text{MSSM}} = -\frac{1}{2}\left(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B} + \text{c.c.}\right)$$
$$-\left(\widetilde{\overline{u}}\,\mathbf{a_u}\,\widetilde{Q}H_u - \widetilde{\overline{d}}\,\mathbf{a_d}\,\widetilde{Q}H_d - \widetilde{\overline{e}}\,\mathbf{a_e}\,\widetilde{L}H_d + \text{c.c.}\right)$$
$$-\widetilde{Q}^\dagger\,\mathbf{m_Q^2}\,\widetilde{Q} - \widetilde{L}^\dagger\,\mathbf{m_L^2}\,\widetilde{L} - \widetilde{\overline{u}}\,\mathbf{m_u^2}\,\widetilde{\overline{u}}^\dagger - \widetilde{\overline{d}}\,\mathbf{m_d^2}\,\widetilde{\overline{d}}^\dagger - \widetilde{\overline{e}}\,\mathbf{m_e^2}\,\widetilde{\overline{e}}^\dagger$$

Squark and slepton mass terms

- for the LH matter supermultiplets Q, L
- for the RH matter supermultiplets u,d,e

$\mathbf{m_Q}, \mathbf{m_L}, \mathbf{m_u}, \mathbf{m_d}, \mathbf{m_e}$ are 3x3 matrices in **flavour**

Apriori we do not know whether the flavour eigenstates correspond to EW eigenstates, **there could be some flavour mixing.**



$$\mu^- \rightarrow e^- \gamma$$

not observed experimentally
will be able to use this to
constraint these mixing matrices

The Soft SUSY Breaking Terms (3)

$$\mathcal{L}_{\text{soft}}^{\text{MSSM}} = -\frac{1}{2}\left(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B} + \text{c.c.}\right)$$

$$-\left(\widetilde{\overline{u}}\,\mathbf{a_u}\,\widetilde{Q}H_u - \widetilde{\overline{d}}\,\mathbf{a_d}\,\widetilde{Q}H_d - \widetilde{\overline{e}}\,\mathbf{a_e}\,\widetilde{L}H_d + \text{c.c.}\right)$$

$$-\widetilde{Q}^\dagger\,\mathbf{m_Q^2}\,\widetilde{Q} - \widetilde{L}^\dagger\,\mathbf{m_L^2}\,\widetilde{L} - \widetilde{\overline{u}}\,\mathbf{m_{\overline{u}}^2}\,\widetilde{\overline{u}}^\dagger - \widetilde{\overline{d}}\,\mathbf{m_{\overline{d}}^2}\,\widetilde{\overline{d}}^\dagger - \widetilde{\overline{e}}\,\mathbf{m_{\overline{e}}^2}\,\widetilde{\overline{e}}^\dagger$$

$$-m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (bH_uH_d + \text{c.c.})\,.$$

Higgs mass terms

Particle Content of the MSSM

after EW symmetry breaking

Names	Spin	P_R	Gauge Eigenstates	Mass Eigenstates
Higgs bosons	0	+1	$H_u^0\ H_d^0\ H_u^+\ H_d^-$	$h^0\ H^0\ A^0\ H^\pm$
squarks	0	−1	$\widetilde{u}_L\ \widetilde{u}_R\ \widetilde{d}_L\ \widetilde{d}_R$	(same)
			$\widetilde{s}_L\ \widetilde{s}_R\ \widetilde{c}_L\ \widetilde{c}_R$	(same)
			$\widetilde{t}_L\ \widetilde{t}_R\ \widetilde{b}_L\ \widetilde{b}_R$	$\widetilde{t}_1\ \widetilde{t}_2\ \widetilde{b}_1\ \widetilde{b}_2$
sleptons	0	−1	$\widetilde{e}_L\ \widetilde{e}_R\ \widetilde{\nu}_e$	(same)
			$\widetilde{\mu}_L\ \widetilde{\mu}_R\ \widetilde{\nu}_\mu$	(same)
			$\widetilde{\tau}_L\ \widetilde{\tau}_R\ \widetilde{\nu}_\tau$	$\widetilde{\tau}_1\ \widetilde{\tau}_2\ \widetilde{\nu}_\tau$
neutralinos	1/2	−1	$\widetilde{B}^0\ \widetilde{W}^0\ \widetilde{H}_u^0\ \widetilde{H}_d^0$	$\widetilde{N}_1\ \widetilde{N}_2\ \widetilde{N}_3\ \widetilde{N}_4$
charginos	1/2	−1	$\widetilde{W}^\pm\ \widetilde{H}_u^+\ \widetilde{H}_d^-$	$\widetilde{C}_1^\pm\ \widetilde{C}_2^\pm$
gluino	1/2	−1	$\widetilde{g}$	(same)
goldstino (gravitino)	1/2 (3/2)	−1	$\widetilde{G}$	(same)

also often denoted

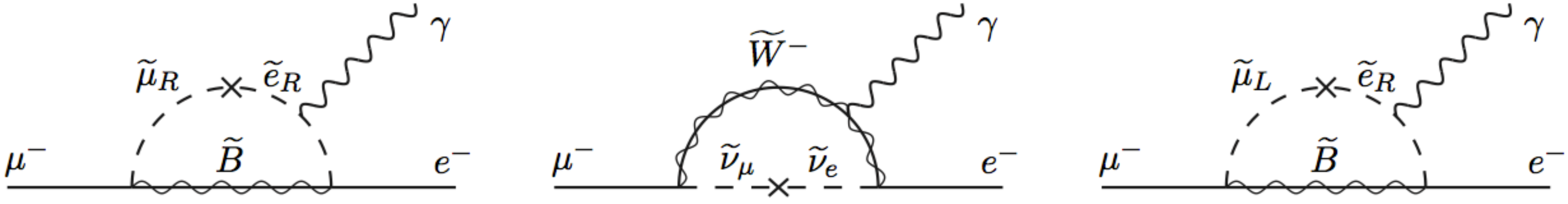
$\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0,$

$\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm$

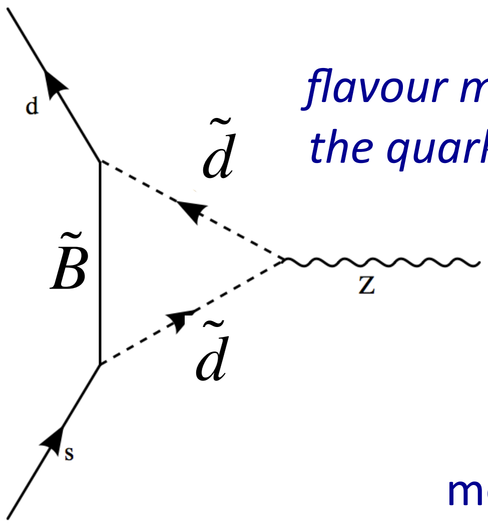
index: increase with mass

Mass eigenstates are not necessarily the same as the gauge eigenstates

SUSY Mass Spectrum: (I) Superpartners of the Fermions



$\mu^- \rightarrow e^- \gamma$ diagrams. From experiments we know these are ~0

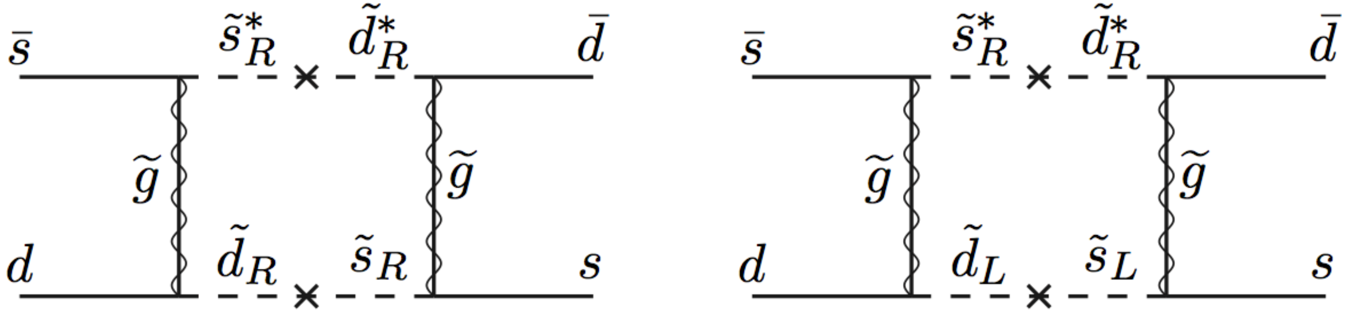


flavour mixing in the quark sector

$s \rightarrow d + Z$

Flavour Changing Neutral Current (FCNC) is not allowed a tree level in SM.
Very strong experimental constraints on this type of interaction.

more contributions to $K^0 \leftrightarrow \overline{K}^0$ mixing



SUSY Mass Spectrum: (I) Superpartners of the Fermions

Due to the very strong experimental constraints on:
- *flavour changing neutral currents*
- *flavour conservation*
- *CP-violation*

We can assume that the mass matrices are diagonal as well as the trilinear coupling matrices.

Even if it is not true it must be a <u>very good approximation.</u>

If Supersymmetry is verified in Nature, it would be good if it provided additional sources of CP violation=> Remember Sakharov's conditions.

For Supersymmetry it is easy to provide more CP-violation than the Standard Model, so that is a good thing.

For the rest of these lectures: **SUSY conserves quark and lepton flavours, no CP violation**.

Also most common assumptions made by experimentalists. (pMSSM or phenoMSSM)

SUSY Mass Spectrum: (II) Superpartners of the Bosons

Names	Spin	P_R	Gauge Eigenstates	Mass Eigenstates
Higgs bosons	0	+1	$H_u^0\ H_d^0\ H_u^+\ H_d^-$	$h^0\ H^0\ A^0\ H^\pm$
squarks	0	−1	$\widetilde{u}_L\ \widetilde{u}_R\ \widetilde{d}_L\ \widetilde{d}_R$	(same)
			$\widetilde{s}_L\ \widetilde{s}_R\ \widetilde{c}_L\ \widetilde{c}_R$	(same)
			$\widetilde{t}_L\ \widetilde{t}_R\ \widetilde{b}_L\ \widetilde{b}_R$	$\widetilde{t}_1\ \widetilde{t}_2\ \widetilde{b}_1\ \widetilde{b}_2$
sleptons	0	−1	$\widetilde{e}_L\ \widetilde{e}_R\ \widetilde{\nu}_e$	(same)
			$\widetilde{\mu}_L\ \widetilde{\mu}_R\ \widetilde{\nu}_\mu$	(same)
			$\widetilde{\tau}_L\ \widetilde{\tau}_R\ \widetilde{\nu}_\tau$	$\widetilde{\tau}_1\ \widetilde{\tau}_2\ \widetilde{\nu}_\tau$

Absence of large extra CP-violation, FCNC and flavour mixing
⇒ RH and LH squarks and sleptons are also the mass eigenstates
(those you are looking for in your experiment)

Not sufficient for the stau, stop and sbottom

$$\tilde{\tau}_L,\ \tilde{\tau}_R,\ \tilde{t}_L,\ \tilde{t}_R,\ \tilde{b}_L,\ \tilde{b}_R$$

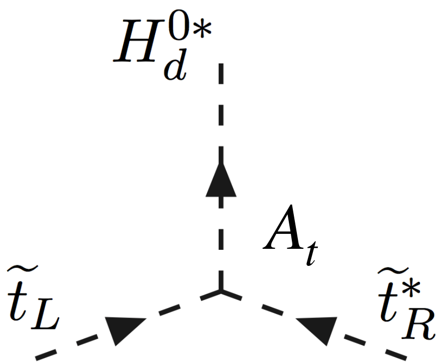
there is something else going on.

SUSY Partners of the heaviest fermions $\tilde{\tau}_L,\ \tilde{\tau}_R,\ \tilde{t}_L,\ \tilde{t}_R,\ \tilde{b}_L,\ \tilde{b}_R$

Look at the stop quark as an example

(A_t related to a_u on slide 59)

$$m_{\tilde{t}}^2 = \begin{pmatrix} m_{Q_3}^2 + m_t^2 + \Delta_L & m_t[A_t - \mu/\tan\beta] \\ m_t[A_t - \mu/\tan\beta] & m_{u_3}^2 + m_t^2 + \Delta_R \end{pmatrix}$$



this process makes the $\tilde{t}_L,\ \tilde{t}_R$ to mix!

The **off-diagonal terms** can be large for **the heavy fermions** since due to large couplings with the Higgs.

The mass eigenstates: $\tilde{t}_1,\ \tilde{t}_2$

are mixes of the gauge eigenstates $\tilde{t}_L,\ \tilde{t}_R$

Denote with index $1, 2$ the **mass eigenstates**, these are the particles you could expect to produce in your high energy collider.

Mass states:
$\tilde{\tau}_1,\ \tilde{\tau}_1,\ \tilde{t}_1,\ \tilde{t}_2,\ \tilde{b}_1,\ \tilde{b}_2$

$$m_{\tilde{t}_1} < m_{\tilde{t}_2}$$

SUSY Mass Spectrum: (II) Superpartners of the Bosons

Names	Spin	P_R	Gauge Eigenstates	Mass Eigenstates
Higgs bosons	0	+1	$H_u^0\ H_d^0\ H_u^+\ H_d^-$	$h^0\ H^0\ A^0\ H^\pm$
squarks	0	−1	$\widetilde{u}_L\ \widetilde{u}_R\ \widetilde{d}_L\ \widetilde{d}_R$	(same)
			$\widetilde{s}_L\ \widetilde{s}_R\ \widetilde{c}_L\ \widetilde{c}_R$	(same)
			$\widetilde{t}_L\ \widetilde{t}_R\ \widetilde{b}_L\ \widetilde{b}_R$	$\widetilde{t}_1\ \widetilde{t}_2\ \widetilde{b}_1\ \widetilde{b}_2$
sleptons	0	−1	$\widetilde{e}_L\ \widetilde{e}_R\ \widetilde{\nu}_e$	(same)
			$\widetilde{\mu}_L\ \widetilde{\mu}_R\ \widetilde{\nu}_\mu$	(same)
			$\widetilde{\tau}_L\ \widetilde{\tau}_R\ \widetilde{\nu}_\tau$	$\widetilde{\tau}_1\ \widetilde{\tau}_2\ \widetilde{\nu}_\tau$
neutralinos	1/2	−1	$\widetilde{B}^0\ \widetilde{W}^0\ \widetilde{H}_u^0\ \widetilde{H}_d^0$	$\widetilde{N}_1\ \widetilde{N}_2\ \widetilde{N}_3\ \widetilde{N}_4$
charginos	1/2	−1	$\widetilde{W}^\pm\ \widetilde{H}_u^+\ \widetilde{H}_d^-$	$\widetilde{C}_1^\pm\ \widetilde{C}_2^\pm$
gluino	1/2	−1	$\widetilde{g}$	(same)
goldstino (gravitino)	1/2 (3/2)	−1	$\widetilde{G}$	(same)

potentially large mixing in the Higgsino/ gaugino sector

also often denoted

$\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0,$

$\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm$

index: increase with mass

Mass of the Gauginos and Higgsinos

mass eigenstates | gauge eigenstates

$$\begin{pmatrix} \tilde{N}_1 \\ \tilde{N}_2 \\ \tilde{N}_3 \\ \tilde{N}_4 \end{pmatrix} = M_{\tilde{N}} \begin{pmatrix} \tilde{B}^0 \\ \tilde{W}^0 \\ \tilde{H}_d^0 \\ \tilde{H}_u^0 \end{pmatrix}$$

The Neutralinos

they are the physical states we are looking for in the experiment

The Neutralino Mixing Matrix

mass eigenstates | gauge eigenstates

$$\begin{pmatrix} C_1^+ \\ C_2^+ \end{pmatrix} = M_{\tilde{C}} \begin{pmatrix} \tilde{H}_u^+ \\ \tilde{W}^+ \end{pmatrix}$$

The Charginos

they are the physical states we are looking for in the experiment

The Chargino Mixing Matrix

notation index: increase with mass $m_{\tilde{N}_1} < m_{\tilde{N}_2} < m_{\tilde{N}_3} < m_{\tilde{N}_4}$ $\quad m_{C_1^+} < m_{C_2^+}$

Mass of the Gauginos and Higgsinos

The Higgsinos and electroweak gauginos mix

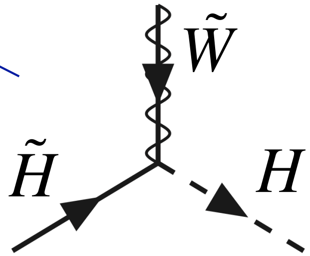
The neutral Higgsinos $(\tilde{H}_u^0, \tilde{H}_d^0)$ mix with the neutral gauginos $(\tilde{B}, \tilde{W}^0)$

The charged Higgsinos $(\tilde{H}_u^+, \tilde{H}_d^-)$ mix with the charged gauginos $(\tilde{W}^+, \tilde{W}^-)$

$$\mathbf{M}_{\widetilde{N}} = \begin{pmatrix} M_1 & 0 & -g'v_d/\sqrt{2} & g'v_u/\sqrt{2} \\ 0 & M_2 & gv_d/\sqrt{2} & -gv_u/\sqrt{2} \\ -g'v_d/\sqrt{2} & gv_d/\sqrt{2} & 0 & -\mu \\ g'v_u/\sqrt{2} & -gv_u/\sqrt{2} & -\mu & 0 \end{pmatrix}$$

Arise from couplings between gaugino, Higgs bosons and Higgsinos.

Bino, Wino masses: M_1, M_2
Higgsino mass μ



$$M_{\tilde{C}} = \begin{pmatrix} M_2 & gv_u \\ gv_d & \mu \end{pmatrix}$$

SUSY Mass Spectrum: (II) Superpartners of the Bosons

Names	Spin	P_R	Gauge Eigenstates	Mass Eigenstates
Higgs bosons	0	+1	$H_u^0\ H_d^0\ H_u^+\ H_d^-$	$h^0\ H^0\ A^0\ H^\pm$
squarks	0	−1	$\widetilde{u}_L\ \widetilde{u}_R\ \widetilde{d}_L\ \widetilde{d}_R$	(same)
			$\widetilde{s}_L\ \widetilde{s}_R\ \widetilde{c}_L\ \widetilde{c}_R$	(same)
			$\widetilde{t}_L\ \widetilde{t}_R\ \widetilde{b}_L\ \widetilde{b}_R$	$\widetilde{t}_1\ \widetilde{t}_2\ \widetilde{b}_1\ \widetilde{b}_2$
sleptons	0	−1	$\widetilde{e}_L\ \widetilde{e}_R\ \widetilde{\nu}_e$	(same)
			$\widetilde{\mu}_L\ \widetilde{\mu}_R\ \widetilde{\nu}_\mu$	(same)
			$\widetilde{\tau}_L\ \widetilde{\tau}_R\ \widetilde{\nu}_\tau$	$\widetilde{\tau}_1\ \widetilde{\tau}_2\ \widetilde{\nu}_\tau$
neutralinos	1/2	−1	$\widetilde{B}^0\ \widetilde{W}^0\ \widetilde{H}_u^0\ \widetilde{H}_d^0$	$\widetilde{N}_1\ \widetilde{N}_2\ \widetilde{N}_3\ \widetilde{N}_4$
charginos	1/2	−1	$\widetilde{W}^\pm\ \widetilde{H}_u^+\ \widetilde{H}_d^-$	$\widetilde{C}_1^\pm\ \widetilde{C}_2^\pm$
gluino	1/2	−1	$\widetilde{g}$	(same)
goldstino (gravitino)	1/2 (3/2)	−1	$\widetilde{G}$	(same)

also often denoted

$\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0,$

$\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm$

index: increase with mass

Now we have the SUSY particle spectrum

Remember

- The mass parameters cannot be much larger than ~1 TeV

Should be able to produce some at the Large Hadron Collider

Two Major Branches of SUSY Phenomenology

R-Parity conserving SUSY (proton is stable)

- SUSY Particles must be produced in pairs
- One SUSY Particle must always contains exactly one SUSY particle in its decay products.
- The **Lightest SUSY Particle** cannot possibly decay "LSP"

⇒ **Dark Matter Candidate**

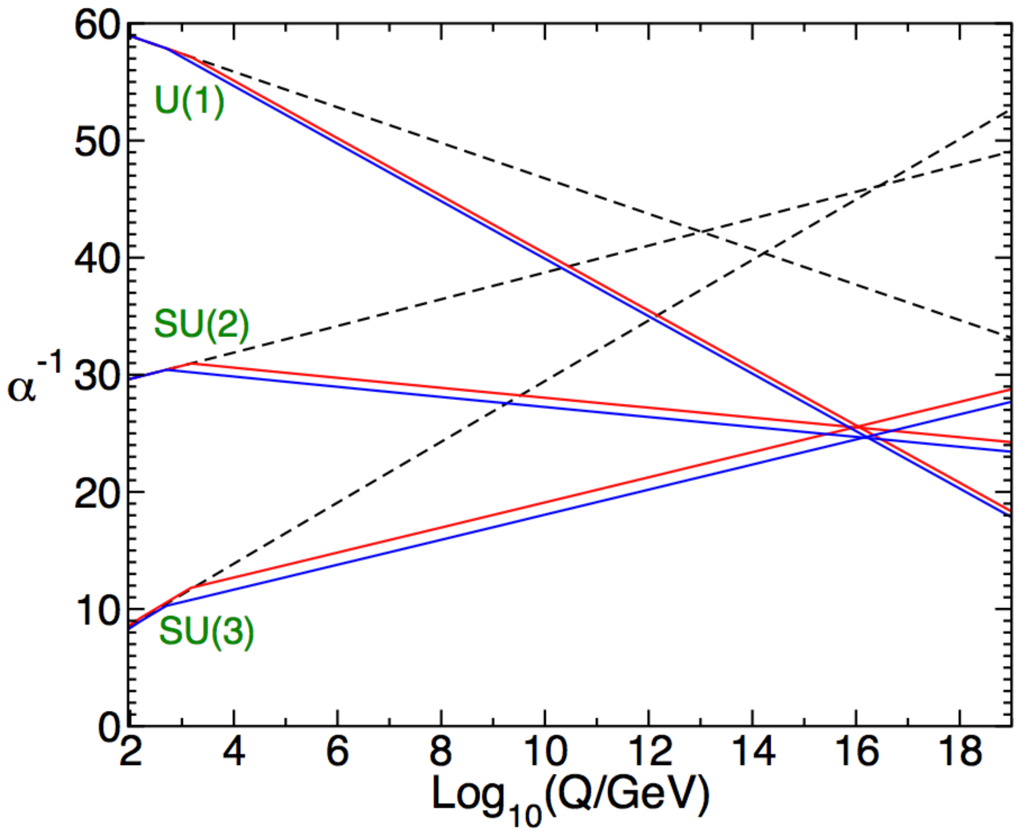
R-Parity Violating SUSY "RPV"
proton is not stable, but lifetime is large

- Single SUSY Particles can be produced
- One SUSY Particle can decay entirely to SM particles alone.
- Rich phenomenology, long-lived particles
- The **Lightest SUSY Particle** can decay "LSP"
- The lifetime of the LSP can be almost anything

⇒ **No Dark Matter** ☹

What does SUSY do to solve questions beyond the SM?

- Solves the hierarchy problem
- Convergence of the coupling constants
- Can provide more CP violation
- Plausible Dark Matter candidate in models with R-parity conservation

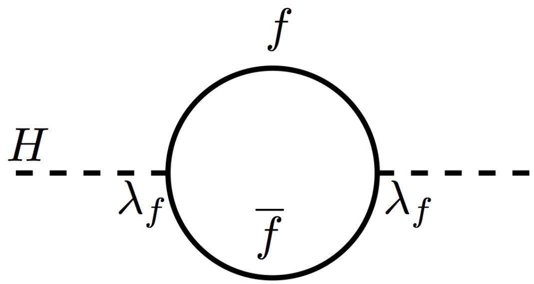


The "Best" candidate for dark matter is the lightest neutralino: $\tilde{N}_1 \; (\tilde{\chi}_1^0)$

The End

Hierarchy Problem Revisited

The Higgs mass is sensitive to the highest mass in the theory.

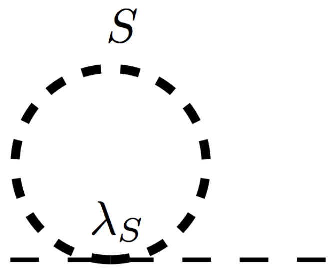


$$\Delta m_H^2 = \frac{\lambda_f^2}{16\pi^2}\left[-2M_{\rm UV}^2 + 6m_f^2 \ln\left(M_{\rm UV}/m_f\right) + \ldots\right]$$

fermion loop

Scalar loop corrections to the Higgs mass also lead to divergent corrections:

$$\Delta m_H^2 = \frac{\lambda_S}{16\pi^2}\left[M_{\rm UV}^2 - 2m_S^2 \ln\left(M_{\rm UV}/m_S\right) + \ldots\right]$$

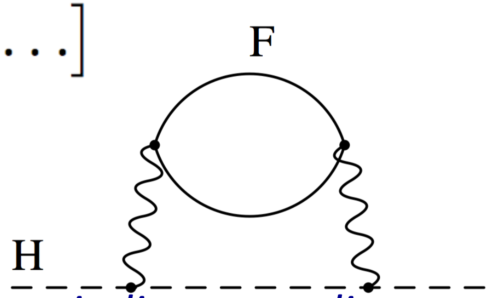


scalar loop

Even if it does not directly couple to any scalar particles it can still couple indirectly to any heavy particle via gauge interactions:

$$\Delta m_H^2 = x\left(\frac{g^2}{16\pi^2}\right)^2\left[kM_{\rm UV}^2 + 48m_F^2 \ln(M_{\rm UV}/m_F) + \ldots\right]$$

k depends on the cut-off procedure, and x is a group-theory factor

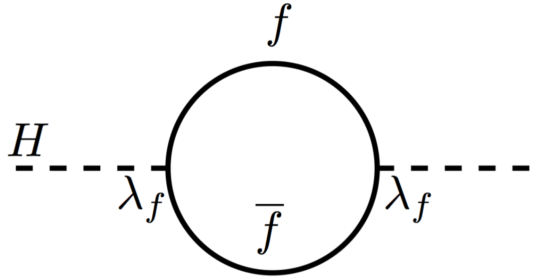


indirect coupling to a heavy Fermion

> Any direct or indirect connection of the Higgs to a heavy sector leads to quadratic divergences, $M^2{}_{UV}$ or $m^2{}_F$

Hierarch Problem Revisited (2)

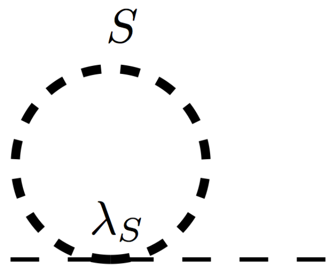
The Higgs mass is sensitive to the highest mass in the theory.



$$\Delta m_H^2 = \frac{\lambda_f^2}{16\pi^2}\left[-2M_{\mathrm{UV}}^2 + 6m_f^2 \ln\left(M_{\mathrm{UV}}/m_f\right) + \ldots\right]$$

fermion loop

Scalar loop corrections to the Higgs mass also lead to divergent corrections:



$$\Delta m_H^2 = \frac{\lambda_S}{16\pi^2}\left[M_{\mathrm{UV}}^2 - 2m_S^2 \ln\left(M_{\mathrm{UV}}/m_S\right) + \ldots\right]$$

scalar loop

Note that these two contributions have opposite signs in front of M^2_{UV} and m^2_f

One Last Piece before we can do Collider Phenomenology

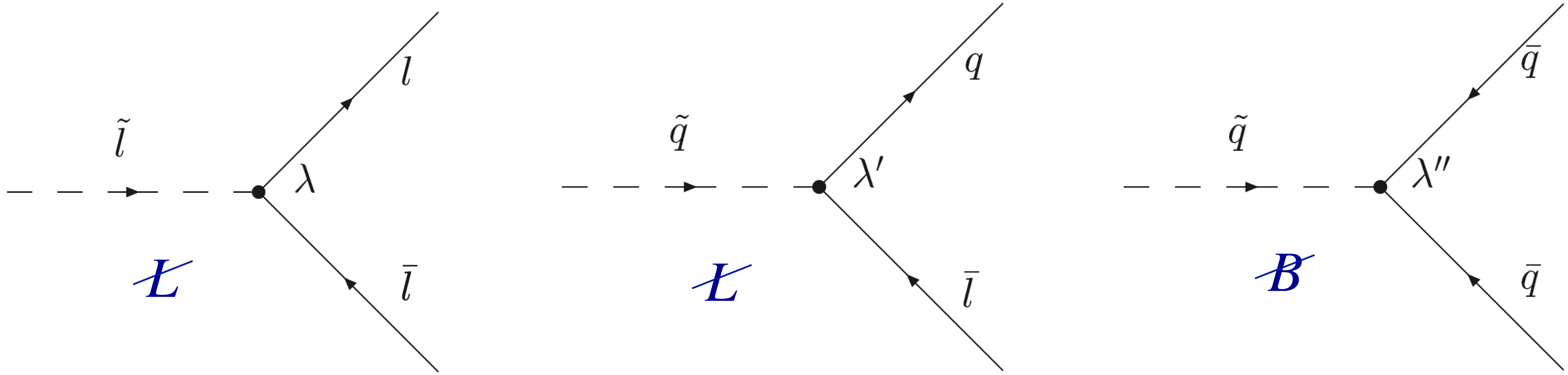
The following soft SUSY breaking terms are also allowed

$$W_{\not{R}_p} = \mu_i H_u L_i + \frac{1}{2}\lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \frac{1}{2}\lambda''_{ijk} U_i^c D_j^c D_k^c$$

These terms explicitely violate **Baryon number** B or **Lepton number** L conservation.

Explicit $\not{B}$ and $\not{L}$ terms are forbidden in the standard model by gauge invariance.

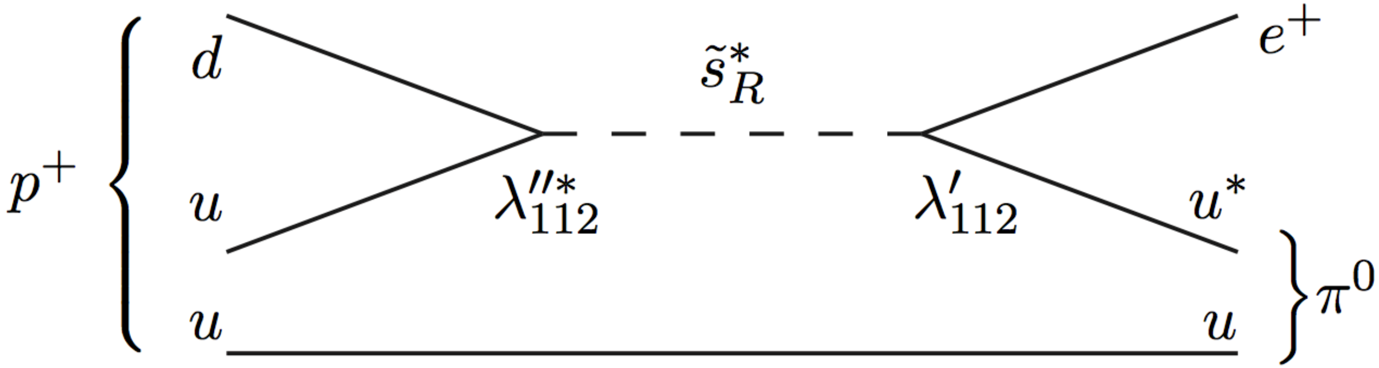
Such terms are allowed in SUSY!



See for a detailed review R.Barbier et al., http://arxiv.org/abs/hep-ph/0406039/

One Last Piece before we can do Collider Phenomenology (2)

These terms are problematic as they can allow for instance the fast proton decay.



Experimentally we know that the life-time of the proton is > 10^{32} years

Many more decay channels / diagram could be allowed (many terms in the previous page)

Cannot simply impose **Baryon number** B or **Lepton number** L conservation.

$\Rightarrow$ Postulate a new discrete symmetry "R-Parity"

R-Parity

R-parity = +1 for Standard Model Particles
= -1 for SUSY particles

$$R = (-1)^{3(B-L)+2S}$$

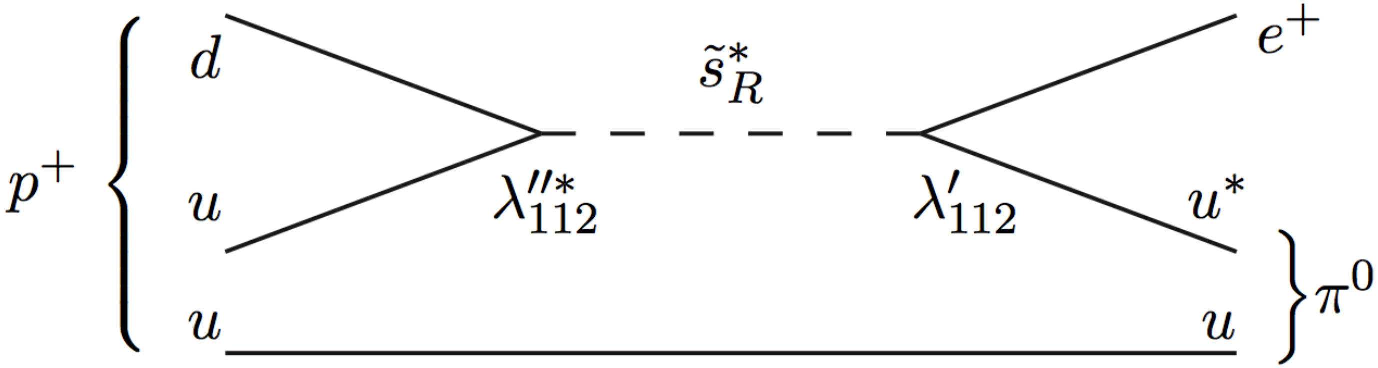
R is a multiplicative quantum number

Require **R-parity conservation** instead of **Baryon number** B or **Lepton number** L

$\Rightarrow$ Looser condition than in the Standard Model

If R-parity is conserved the proton decays can be avoided.

forbidden by R-parity conservation:



R-Parity

R-parity = +1 for Standard Model Particles
= -1 for SUSY particles

$$R = (-1)^{3(B-L)+2S}$$

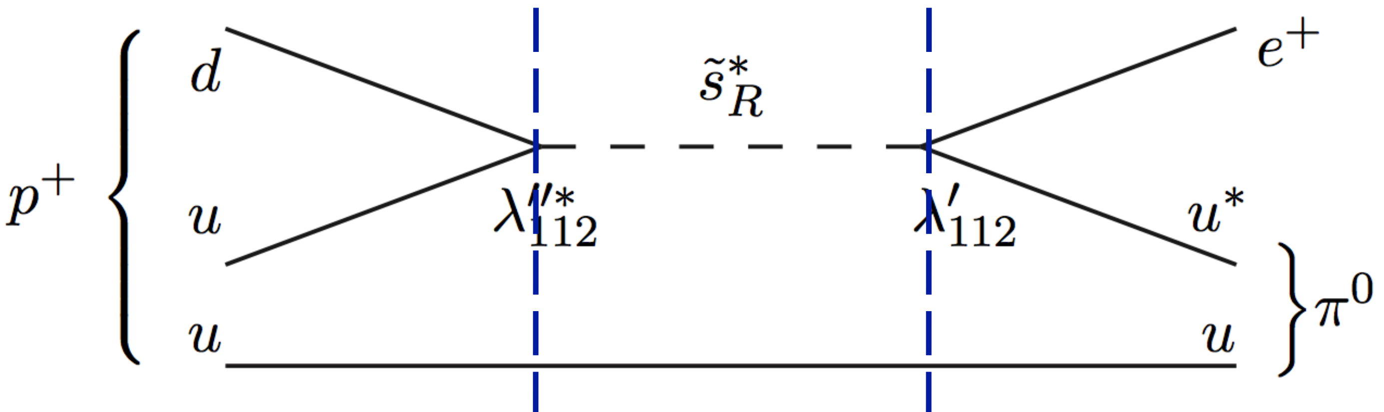
R is a multiplicative quantum number

Require **R-parity conservation** instead of **Baryon number** B or **Lepton number** L

$\Rightarrow$ Looser condition than in the Standard Model

If R-parity is conserved the proton decays can be avoided.

forbidden by R-parity conservation:



R=(+1)x(+1)x(+1)=+1 R=(+1)x(-1)=-1 R=(+1)x(+1)x(+1)=+1

Consequence of R-Parity Conservation

R-Parity conserving SUSY (proton is stable)

- SUSY Particles must be produced in pairs

- One SUSY Particle must always contains exactly one SUSY particle in its decay products.

- The **Lightest SUSY Particle** cannot possibly decay "LSP"

$\Rightarrow$ **Dark Matter Candidate**

$$SM + SM \rightarrow SUSY + SUSY$$
$$R = (+1) \times (+1) = (-1) \times (-1)$$

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$$\begin{array}{lllll} & SUSY & \rightarrow SUSY & + SM & + SM + \ldots \\ R = & (-1) & = (-1) & \times (+1) & \times (+1) \end{array}$$

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⇒ **Dark Matter Candidate**

$$LSP \not\to SUSY + SM + SM + \ldots$$
$$R = (-1) = (-1) \times (+1) \times (+1)$$

not possible because no lighter SUSY particle

$$LSP \not\to SM + SM + \ldots$$
$$R = (-1) \neq (+1) \times (+1)$$

not possible because R-parity must be conserved