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QCD and jets (Part I)

Darren Price

HASCO Summer School, Göttingen, July 8th—19th 2013

Introduction

This is an overview of certain aspects of QCD, there are far more details than I can contain within these lectures!

Will skip over a lot of details and experimental results!

I am an experimentalist, so there will nonetheless be a bias in these slides toward more experimental aspects and results

In this session:

- What is QCD, and what does it predict?
- What is colour – experimental verification?
- Jets and algorithmic definitions
- Reality of gluons and quarks
- Precision predictions in e+e-
- Electron-proton scattering
- Substructure of the proton and evolution with scale
- Implications for hadron-hadron scattering and the LHC…

Quantum Chromodynamics

What is QCD? Why do we need it?

Before QCD, Quantum Electrodynamics (**QED**) had **great successes** as a quantum theory describing interactions of matter and light.

The 50's saw a **large increase in the number of hadrons** observed in experiments – puzzling to describe in coherent way

Became understood that if [at least] three quarks (u, d, s) existed, these hadrons could be composite, could explain the patterns observed

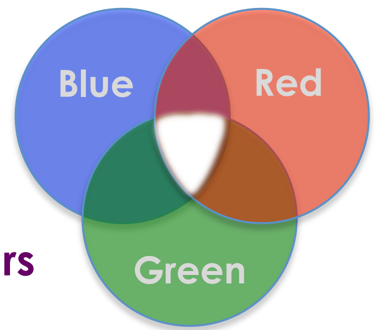
Existence of Ω^{-}(sss) hyperon: three strange quarks with parallel spins first indication that quarks have an additional quantum number…

Further evidence through consideration of Δ^{++}(uuu) baryon

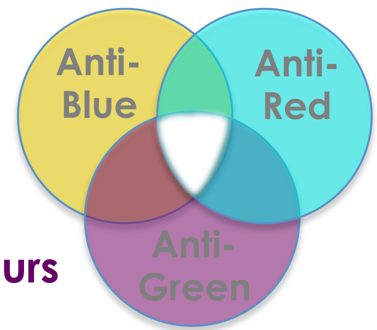
Introduction of a "colour" charge for quarks…

QCD: colour is key

QCD theory predicts *three colour charges*, compared to the one charge of QED [we call these red, green & blue]

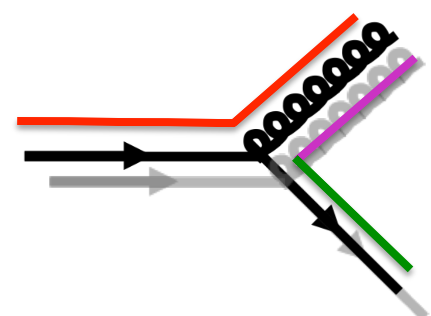


Quark colours



Quark anti-colours

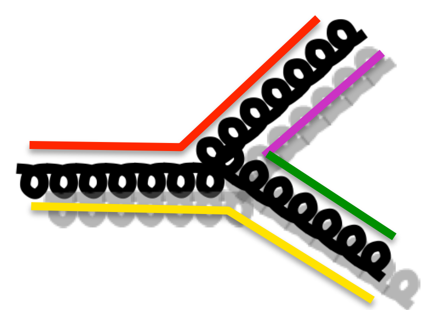
Theory predicts quarks carry one colour charge



Theory also predicted existence of “gluons”, vector gauge bosons that would interact with the quarks (analogous to the photons of QED)

These gluons carry one colour charge and one anti-colour charge (unlike photon, electrically neutral)

- Gluons thus can self-interact
- Colour charge conserved at all vertices



EM interactions of the quarks

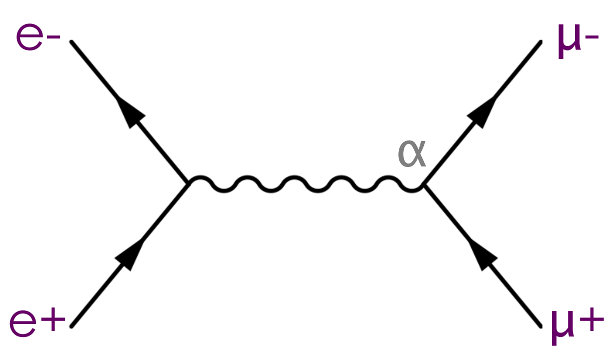
EM interaction couples photon with quark and anti-quark, with strength defined by quark charge:

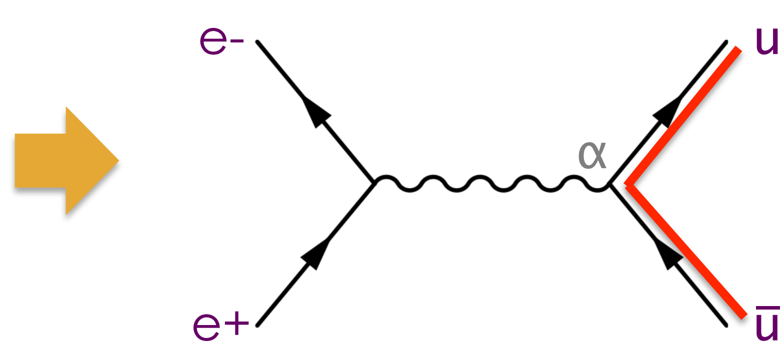
+2/3 for "up-type" quarks (u,c,t)

-1/3 for "down-type" quarks (d, s, b)

EM interactions of the leptons and quarks are similar:

- Coupling to photon cannot change type of fermion, just 4-momentum
- Coupling strength proportional to electric charge of the fermion





$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3Q^2}$$

$$\text{where } Q^2 = 4E^2_{\text{beam}}$$

$$\sigma(e^+e^- \to u\bar{u}) = \frac{4\pi\alpha^2}{3Q^2} \times (3) \times \left(\frac{2}{3}\right)^2$$

A dramatic prediction of QCD

This relationship between the production cross-section of di-muons and quark-anti-quark pairs

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3Q^2} \qquad \sigma(e^+e^- \to u\bar{u}) = \frac{4\pi\alpha^2}{3Q^2} \times (3) \times \left(\frac{2}{3}\right)^2$$

means that

$$\sigma(e^+e^- \to \text{hadrons}) = \sum_q \sigma(e^+e^- \to q\bar{q}) = N_c \sum_q e_q^2 \sigma(e^+e^- \to \mu^+\mu^-)$$

where N_c are the number of colours (3) and e_q is the quark electric charge.

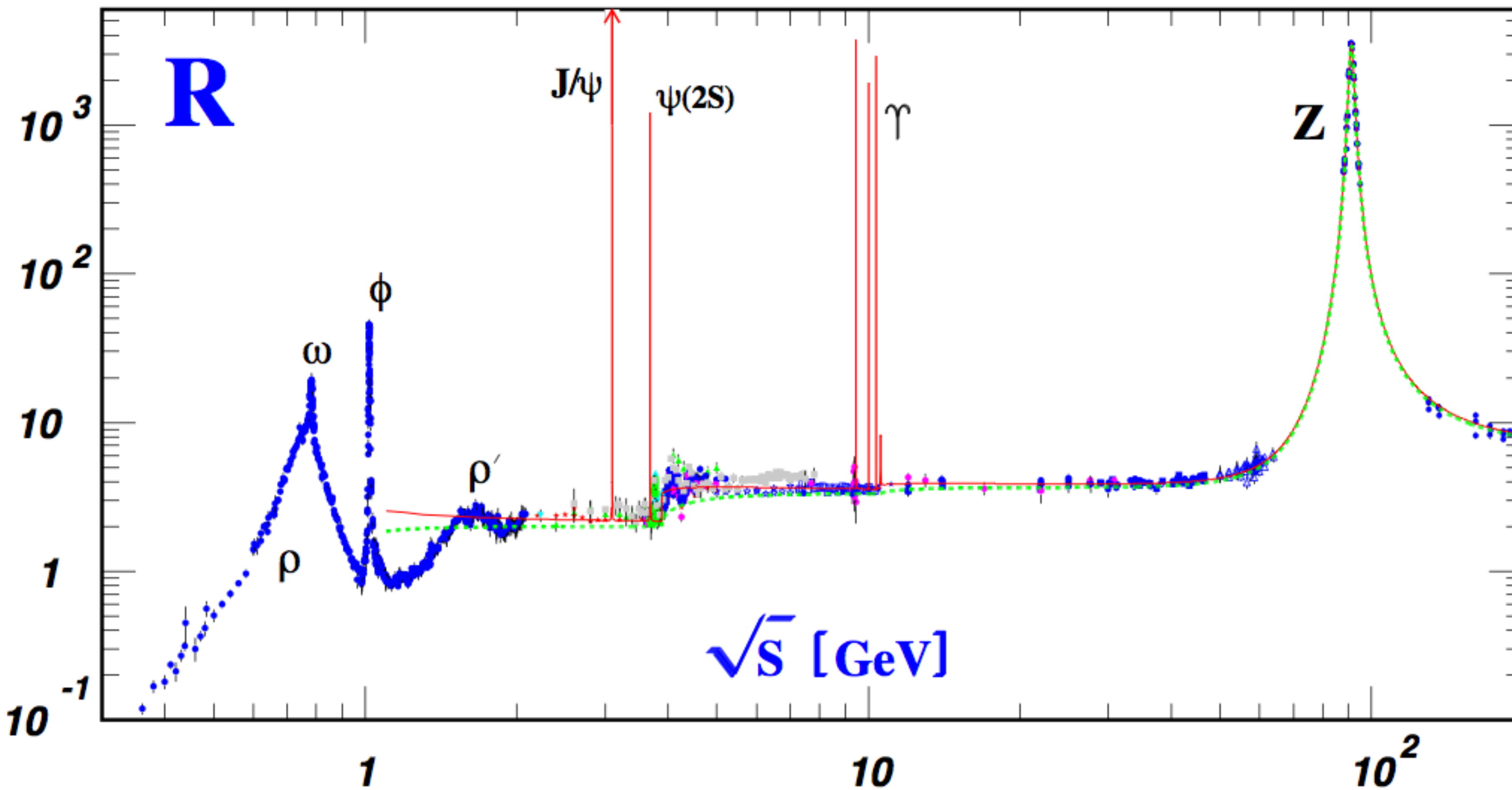
This leads to a powerful prediction of QCD that can be tested through e^+e^- collisions:

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum_q e_q^2$$

A dramatic prediction of QCD

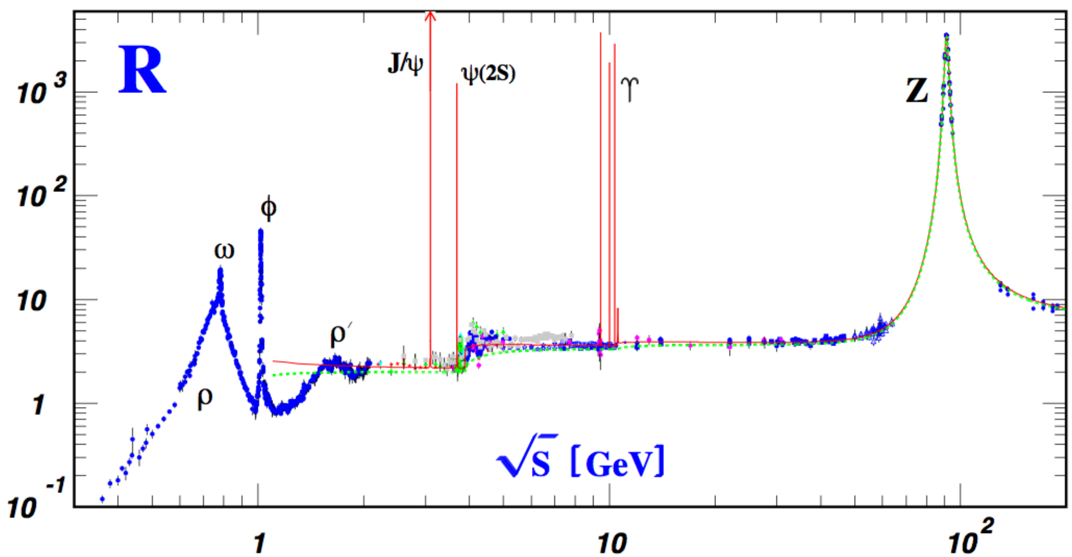
Measuring the total production rate of hadrons then acts as a measure of the number of quarks, their flavours, and their colours!

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum_q e_q^2$$



A dramatic prediction of QCD

Prediction that R should increase in discrete steps, related to quark invariant masses. Size of steps related to charge of quark.



$$R = 3\left[\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2\right] = 2$$

for u, d, s

When only u, d, s quarks were known, rise of R>2 was seen as a problem.

Introduction of charm quark and subsequent discovery of charm bound states a success of QCD

$$R = 2 + 3\left[\left(\frac{2}{3}\right)^2\right] = \frac{10}{3}$$

for u, d, s, c

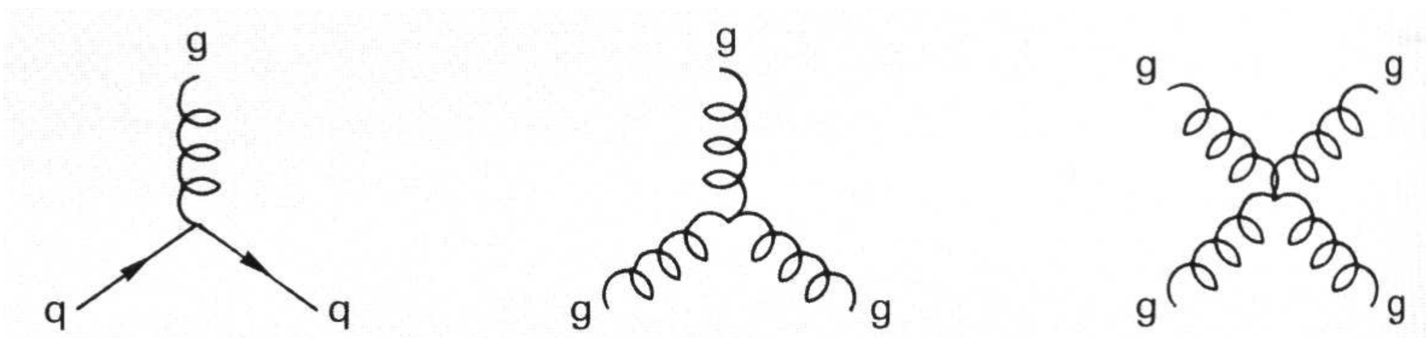
$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum_q e_q^2$$

Worthwhile to recall that originally the quarks, gluons and colour were considered by many to be just a useful mathematical apparatus.
This was the first hint that these were physically meaningful phenomena.

Why don't we see free quarks & gluons?

QCD seems to have successes predicting quarks/gluons/colour

Why then don't we see free quarks and gluons in our detectors?
A key factor is the gluon self-interaction discussed earlier
This is a distinctive feature of QCD theory, differing from QED, leading to the following allowed vertices:



Leads to an "anti-screening" of colour charge (compare with screening of electric charge in QED)
A quark can emit gluons, which can subsequently split into a quark pair or gluon pair – original quark colour enhanced with distance!

Leads to an increased attraction between two quarks linearly with the distance between them: "colour confinement"

A comparison of QED and QCD

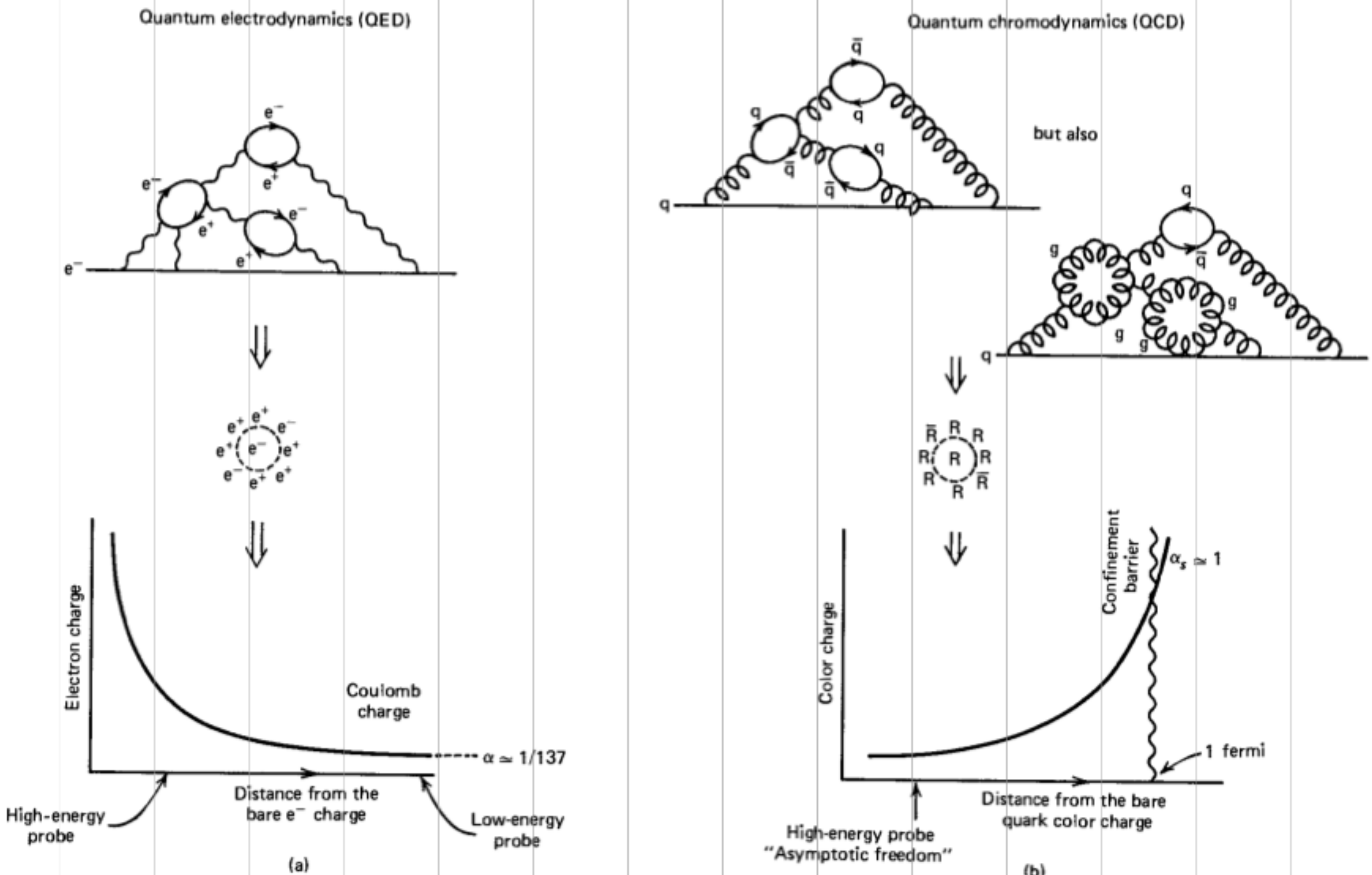


Fig. 1.5 Screening of the (a) electric and (b) color charge in quantum field theory.

Taken from Halzen and Martin, "Quarks and Leptons"

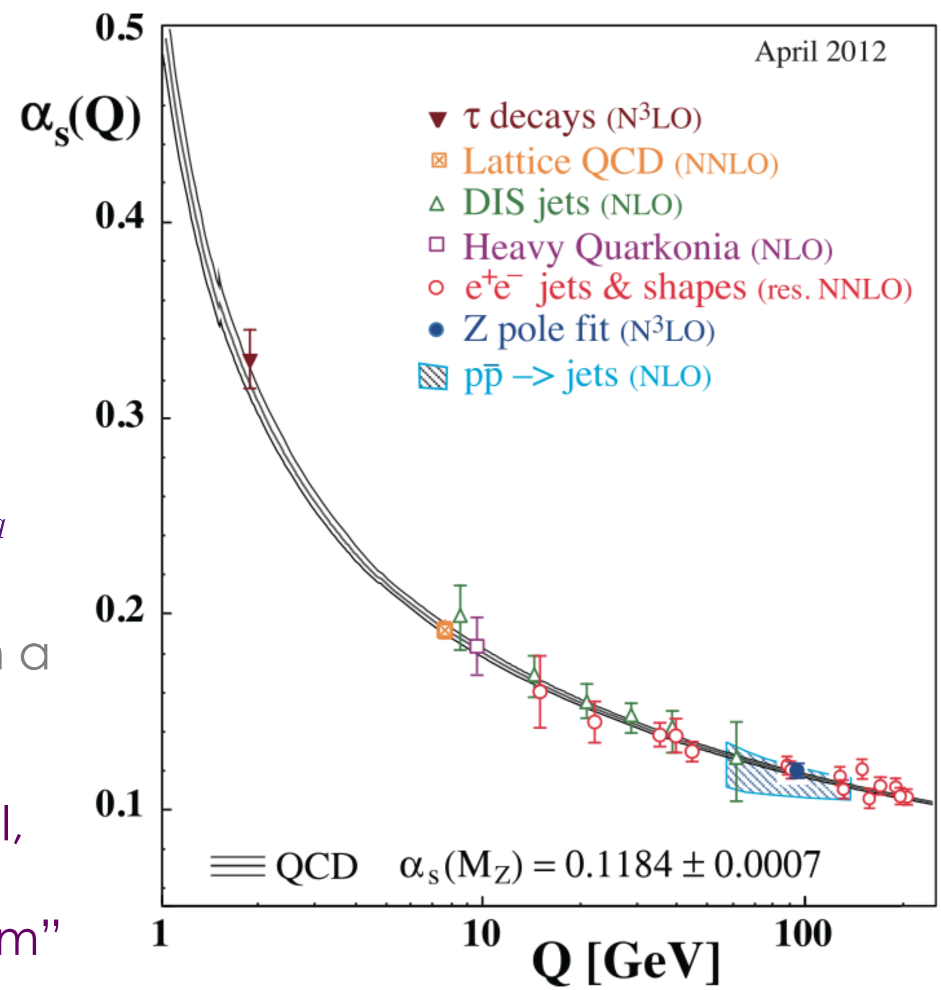
Asymptotic freedom and confinement

Charge inside colour "cloud" between two quarks experiences **smaller force at smaller distances** (or larger 4-momentum transfer, Q)

Dependence of strong [colour] coupling α_s on the scale Q is:

$$\alpha_s(Q) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)}, \qquad \beta_0 = 11 - \frac{2}{3}N_q$$

Λ is the QCD scale parameter, with a measured value of 220 MeV.

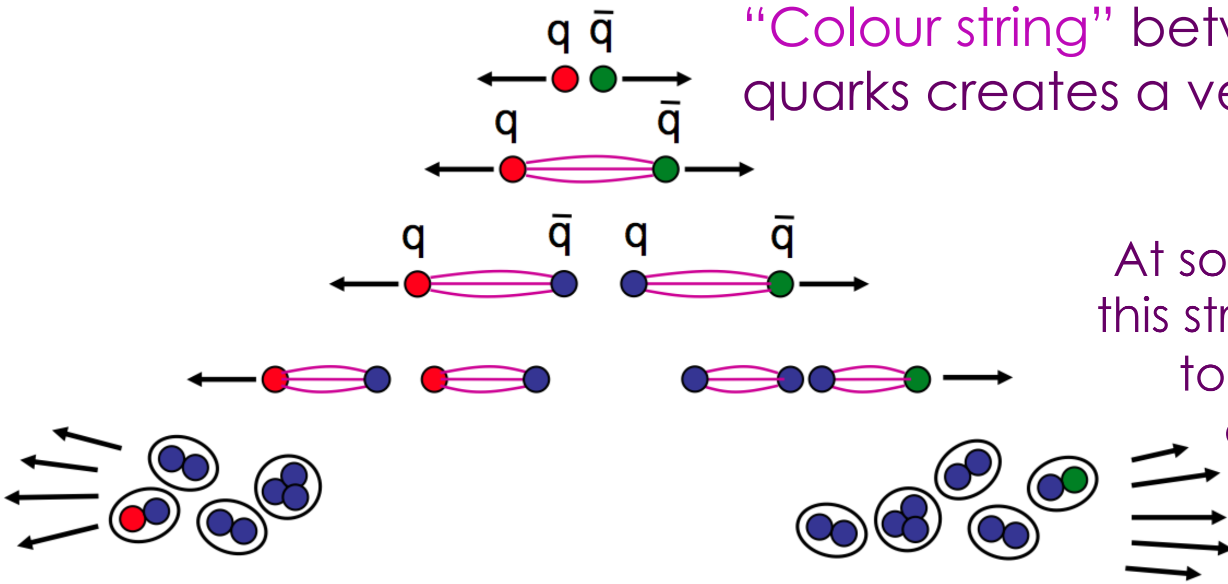


At high Q, coupling becomes small, gluons and quarks are almost free inside hadrons "asymptotic freedom"

At low Q the strong interaction becomes very strong! Hence at large distances the quarks and gluons cannot escape the hadron.

Asymptotic freedom and confinement

Qualitative picture of colour confinement with increasing distance



"Colour string" between two separating quarks creates a very strong force

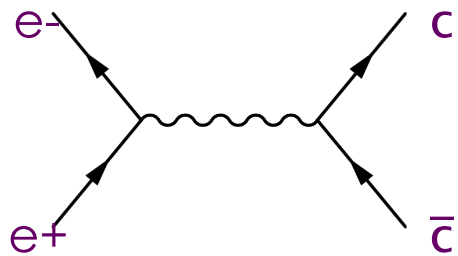
At some point the energy in this string becomes sufficient to create another quark-antiquark pair from the vacuum!

End result is that quarks (and gluons) are never isolated but instead form new hadrons following the directions of the initial quarks until relative 4-momentum is low.

Hadronisation

Colour confinement and asymptotic freedom has implications for what is observed in an experimental detector

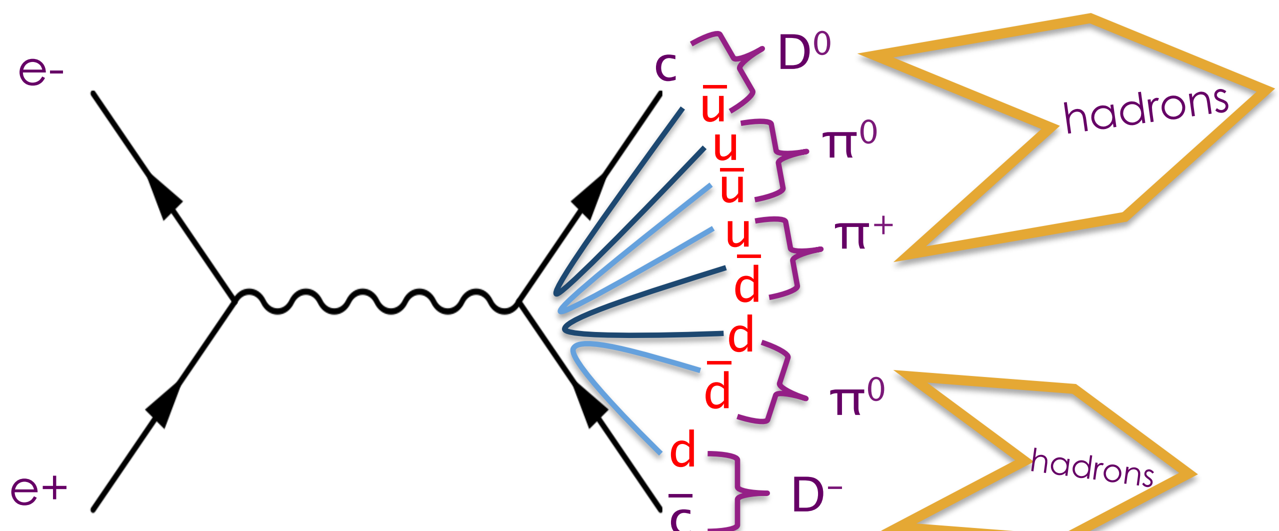
The process



is only part of the story!

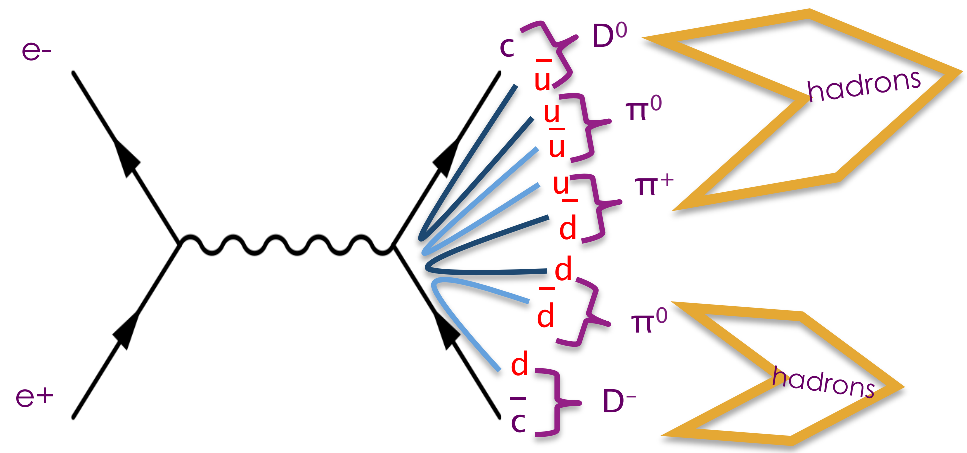
Due to colour confinement the space between the quark lines are 'understood' to be filled with many virtual gluon couplings

The 'real' picture looks something more like this:

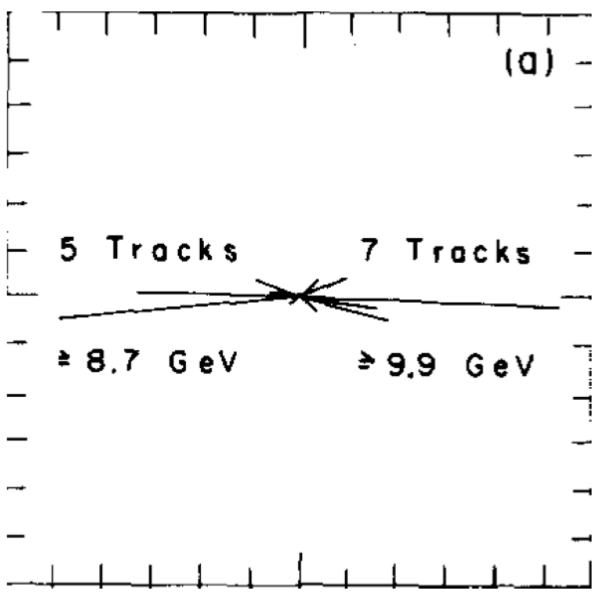


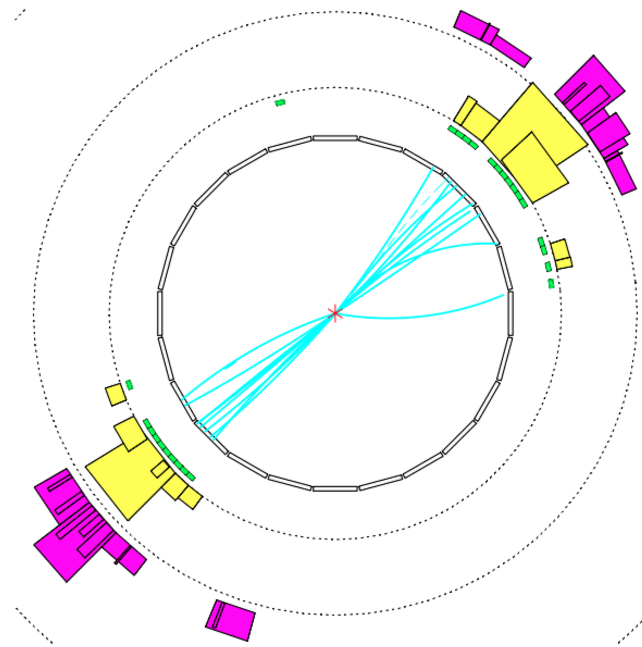
Hadronisation and observation of jets

The hadronisation process produces narrowly collimated jets of hadrons, that have properties correlated to the initial quarks (or gluons)



Below is a picture of reconstructed particle tracks from the SPEAR e+e- collider in 1975 providing first evidence for this "jet" behaviour





A later 2-jet event from the OPAL detector

Quantitative jet reconstruction in e+e-

To go further we need to clarify what we really define a jet to be!

Verbally, "*a cluster of particles (or tracks/energy deposits) or energy flow in a restricted spatial region*"

Jets are our connection between quarks and gluons of QCD and signals measured in detectors.

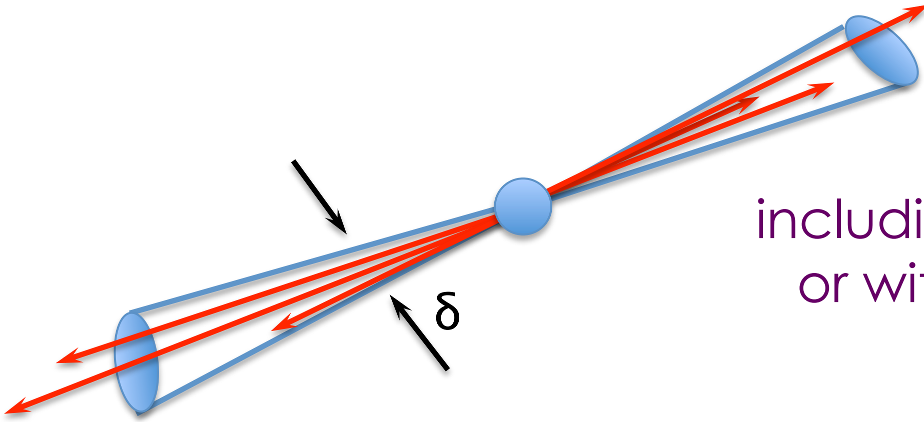
Need a clear algorithmic definition if comparison between theory and data is to be made!

At the most basic level the jet definition needs to:

1. **Be able to be applied to both data and theory predictions**
2. **Provide a close relationship between partons and jets**
3. **Have no ambiguities in the definition**

Early jet definitions in e+e-

Example of an early jet definition (Sterman & Weinberg 1978):



Define a dijet event by including anything below energy ε or within angle δ into dijet system

Problematic prescription:

- **Where do we place the cones?**
- **What happens if the cones overlap?**
- **How do we generalise the algorithm to other collision types?**

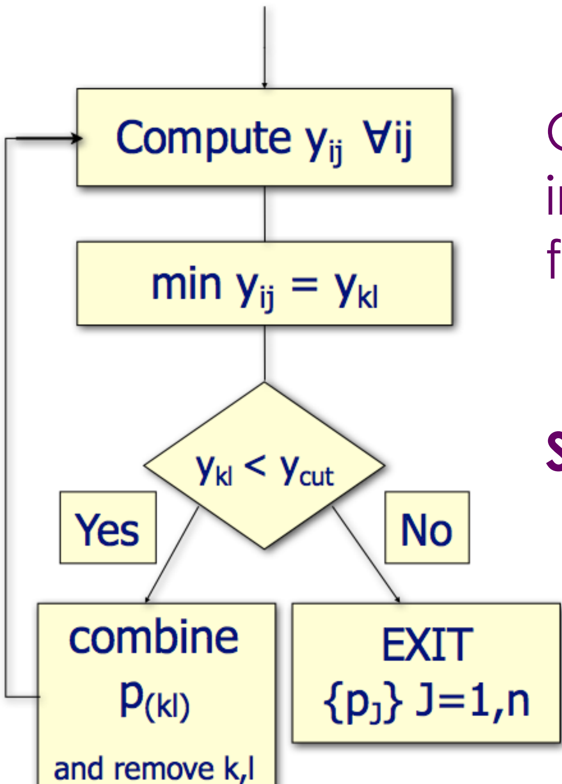
Need for well-defined scheme led to JADE recombination algorithmic prescription…

JADE e+e- jet algorithm

The JADE jet recombination algorithm first defines a metric y_{ij} as a measure of distance in momentum space:

$$y_{ij} = \frac{2E_i E_j (1 - \cos\theta_{ij})}{E_{CM}^2} \approx \frac{m_{ij}^2}{E_{CM}^2}$$

a resolution criterion y_{cut}, and a procedure for recombination:



Closest pairing if below y_{cut} are combined into one "particle" and constituents removed from consideration until only n jets remain.

Some strengths:

- All particles assigned a jet unambiguously
- Algorithm is "infrared safe"
- Algorithm is "collinear-safe"

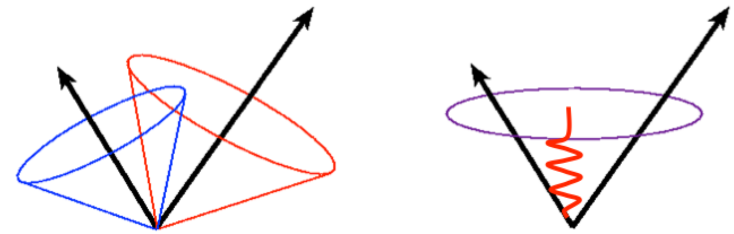
What is meant for a jet algorithm to be "infrared safe" and "collinear safe"?

Infrared-safe:

The addition of a further soft particle to the event should not change the configuration of the jets.

(That is, two jets defined by the algorithm should not get redefined as a single more energetic jet be a soft particle in between the previously defined jets)

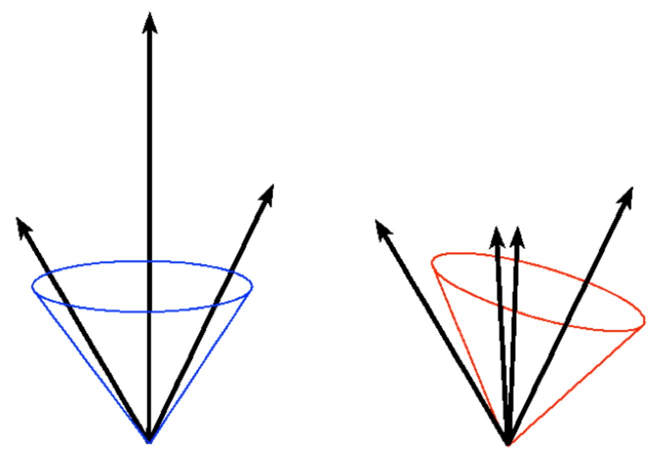
[In JADE: $j_{ij} \rightarrow 0$ as $E_i \rightarrow 0$ or $E_j \rightarrow 0$] ✔



Collinear-safe:

The jet configuration should not change by the replacement of a single resolved particle by two collinear particles

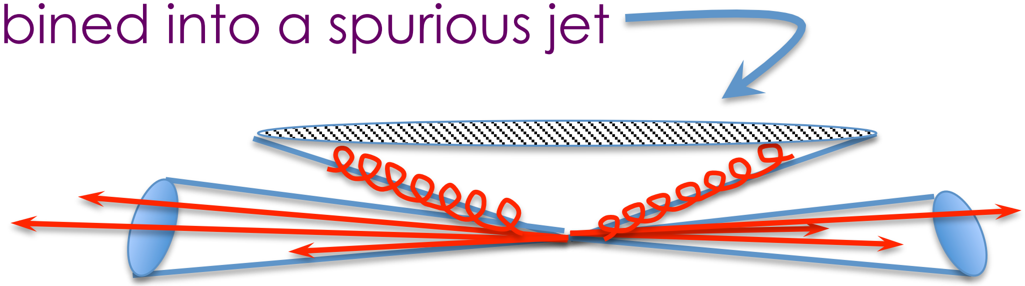
[In JADE: $y_{ij} \rightarrow 0$ for $\theta_{ij} \rightarrow 0$] ✔



A weakness of the JADE algorithm

The JADE jet recombination algorithm has a specific weakness:

- in QCD soft gluons are copiously radiated
- soft gluons spatially well-separated can nonetheless be combined into a spurious jet



Behaviour arises from peculiarities of JADE distance metric:
$y_{ij} \propto 2E_iE_j(1-\cos\theta_{ij})$ two soft (E~0) gluons can be very "close"

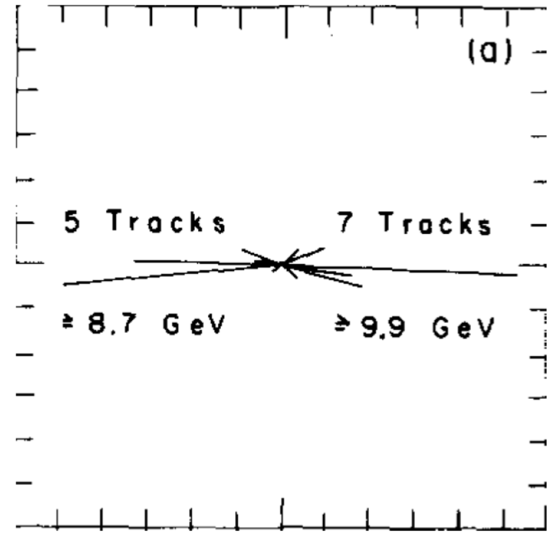
Improvement called the k_T(Durham) algorithm solves the problem, attaching soft collinear radiation to the correct jet redefining metric:

$$y_{ij} = \frac{2\min(E_i, E_j)(1-\cos\theta_{ij})}{E_{CM}^2} \rightarrow \frac{k_T^2}{E_{CM}^2} \text{ as } \theta_{ij} \rightarrow 0$$

where k_T is the minimum relative momentum of i and j

Observation of jets

Study of the kinematics of these events supports their "jet"-like nature

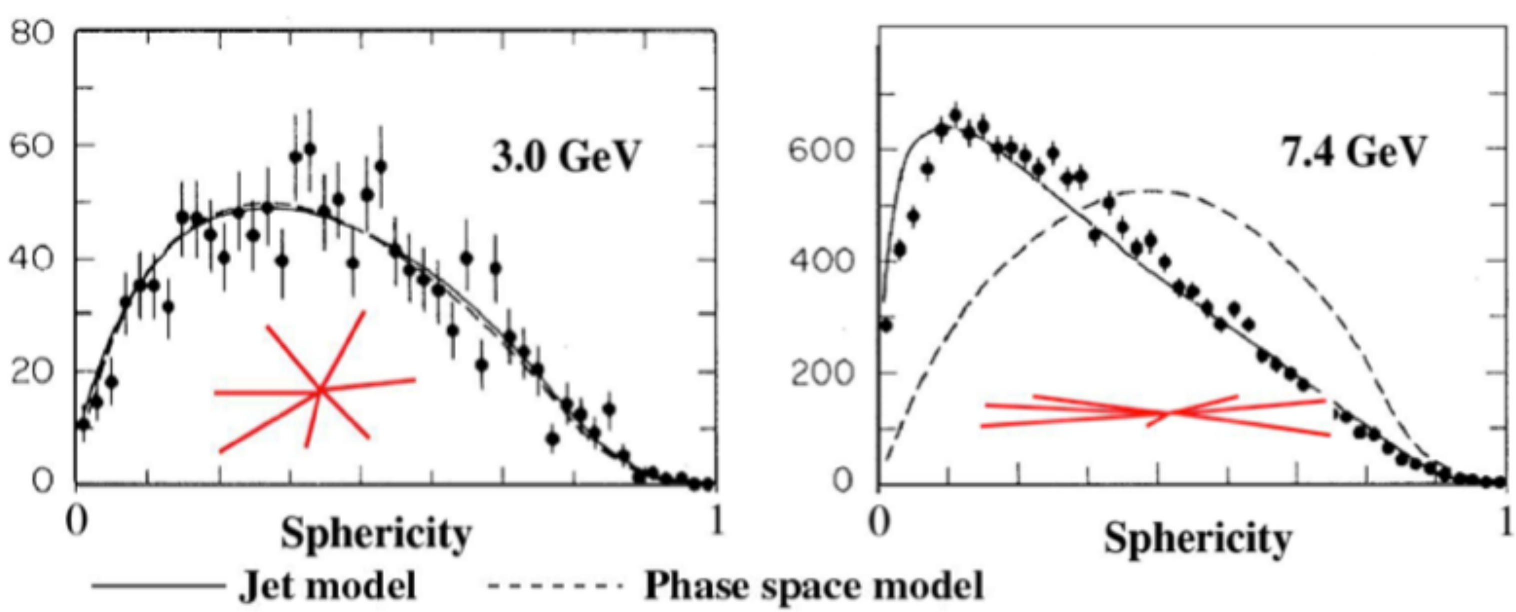


Observed "sphericity", S, distribution:

$$S = \frac{3(\sum_i p_{T,i}^2)_{\min}}{2(\sum_i p_i^2)}$$

S~0 'jet-like'
S~1 isotropic

Predicted to peak toward lower **S** as energy increased, as observed



Confirmation of spin ½ quark jets

Study of the angular distribution of two-jet events provides access to the properties of the initial quarks

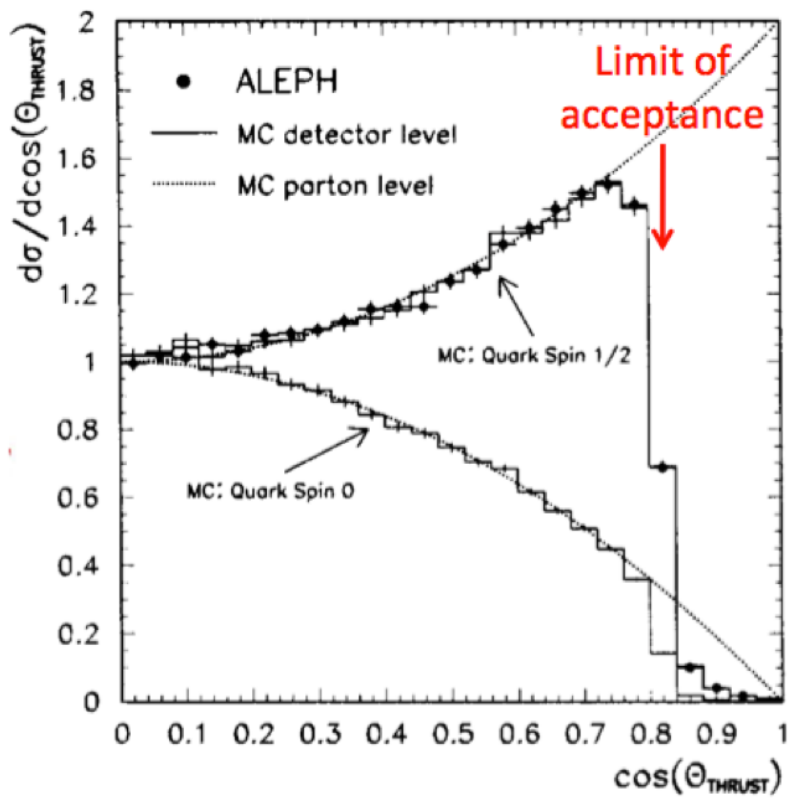
Determine angular distribution of the thrust axis, T, the axis which maximises the transverse and longitudinal momentum of particles in the event

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$

$$\frac{d\sigma}{d\cos\Theta_{TH}} \propto 1 + \alpha\cos^2\Theta_{TH}$$

$$\text{Spin 1/2 quarks} \implies \alpha = +1$$

$$\text{Spin 0 quarks} \implies \alpha = -1$$

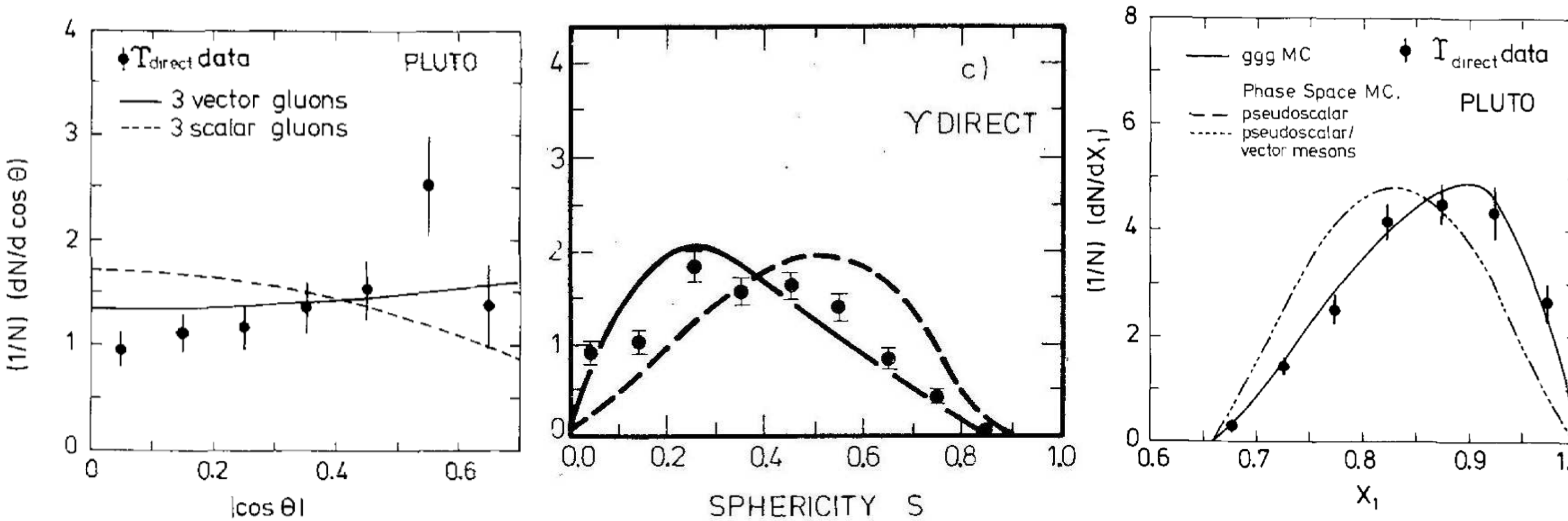


The Upsilon and vector gluons

The Upsilon meson (a bound state of a b and anti-b quark) first observed in di-muon decay mode in 1979.

This state was predicted to mainly decay into three gluons in QCD

Data from this decay were studied and the angle between thrust axis and beam axis, sphericity and the scaled fractional momentum were compared to scalar and vector 3-gluon models.

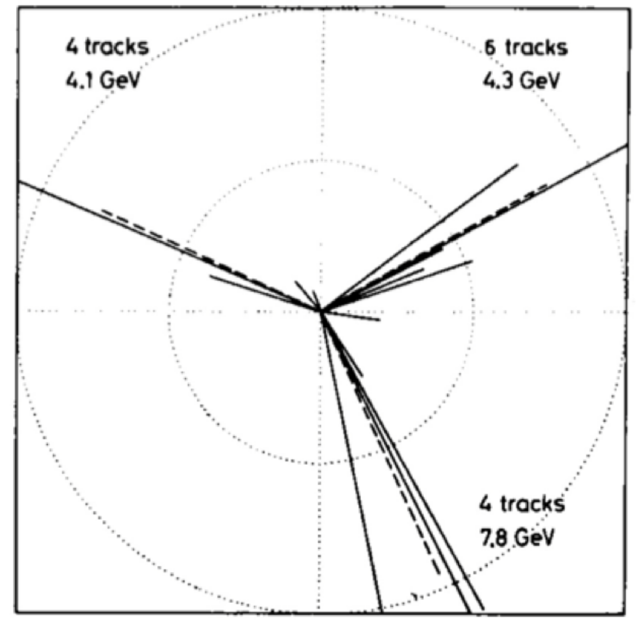


Measurements support the 3-vector gluon interpretation of QCD.

3-jet events and evidence for the gluon

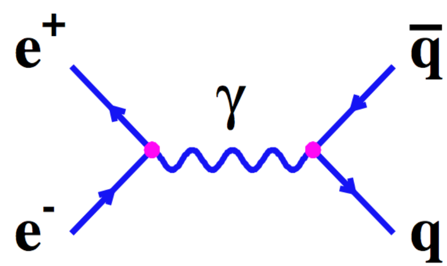
First unambiguous direct evidence for the physical existence of the gluon came from PETRA, where three jet events were first observed.

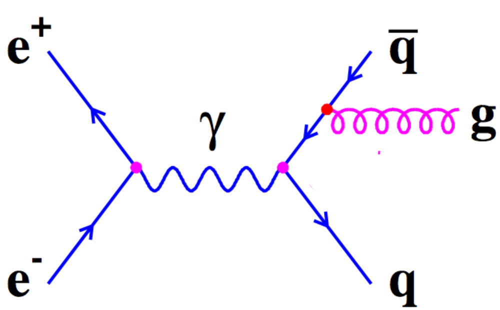
TASSO



As quark-antiquark pairs formed together, emission of an odd-number of jets had to come from gluon radiated off one of the quarks.

As well as the diagram:

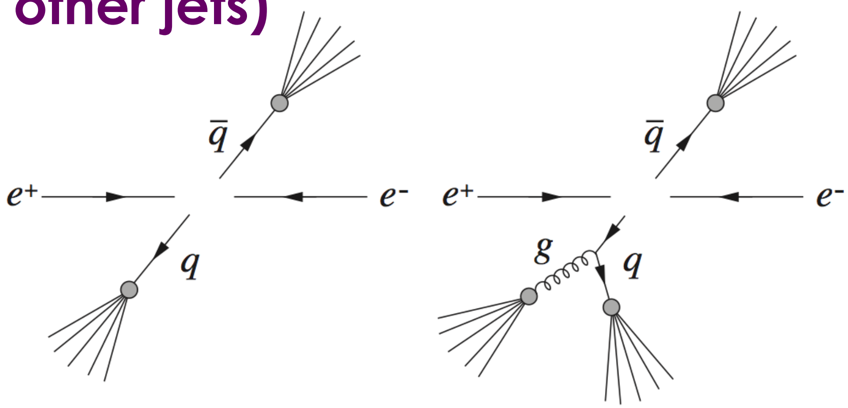


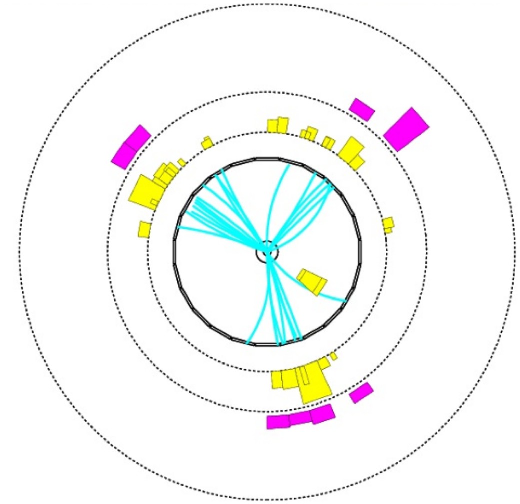


was also possible (suppressed as #three-jet events / #two jet events ~ α_s=0.1)

3-jet events and evidence for the gluon

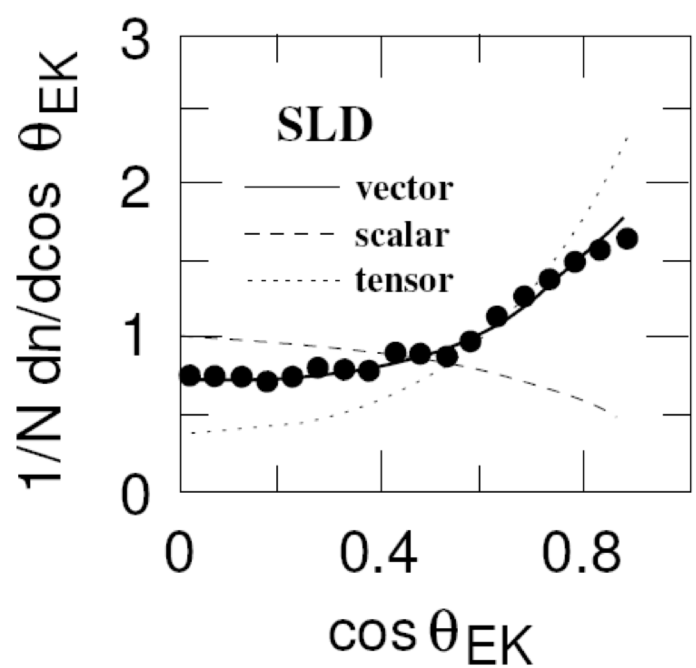
Hard gluon radiation leads to three-jet events (where the gluon is not collinear with other jets)





The probability to radiate a soft gluon is larger than to radiate off a hard gluon.

Determined at LEP that if the three jets are ordered by energy, the **gluon jet** should be the **third** jet **75%** of the time

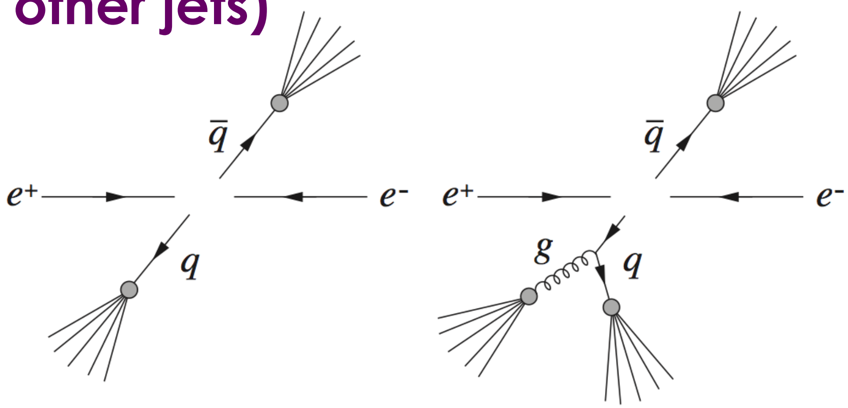


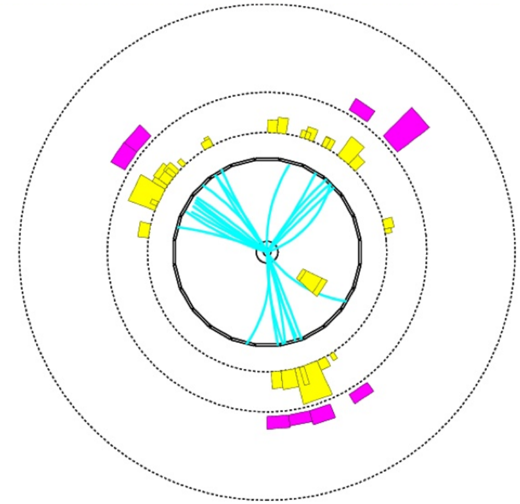
Angle between axis of jets (2,3) relative to jet 1 in centre-of-mass frame of dijet system sensitive to gluon spin

– data clearly in favour of spin-1 of QCD

3-jet events and evidence for the gluon

Hard gluon radiation leads to three-jet events (where the gluon is not collinear with other jets)



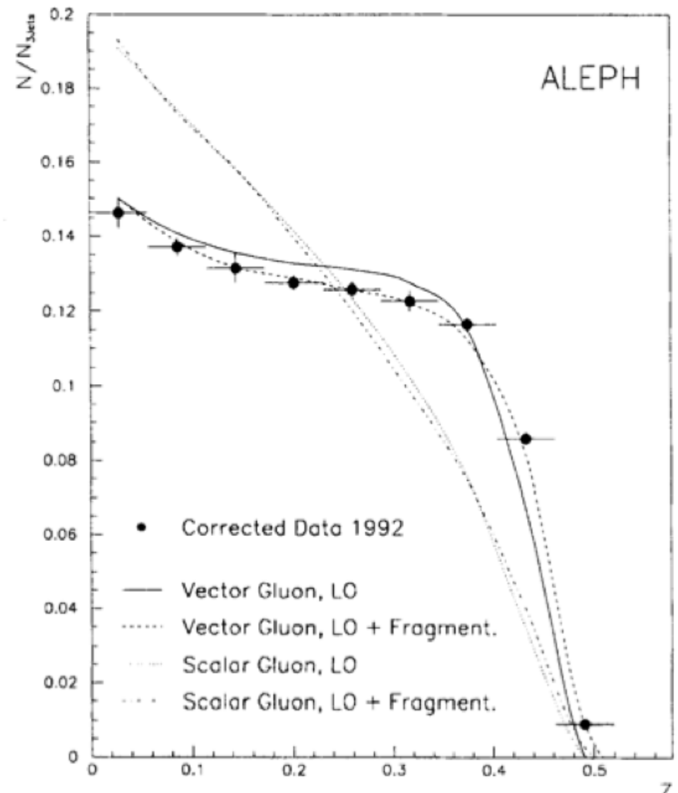


The probability to radiate a soft gluon is larger than to radiate off a hard gluon.

Determined at LEP that if the three jets are ordered by energy, the **gluon jet** should be the **third** jet **75%** of the time

The distribution of energy difference between the 2nd and 3rd jets is distinctly different for vector and scalar gluon hypotheses

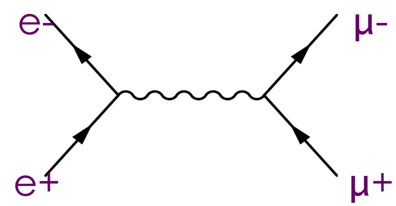
– data clearly in favour of spin-1 of QCD

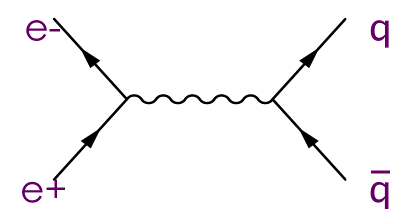


The R ratio: a correction for QCD

Earlier we discussed the ratio

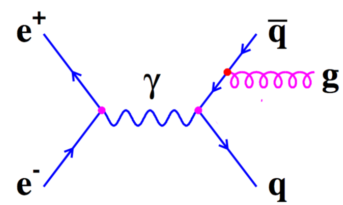
$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum_q e_q^2$$





using the similarity between $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to q\bar{q}$ scattering

The presence of the diagram



makes things a little more complicated!

No closed form for full QCD calculations, rely on perturbative techniques, expanding in powers of α_s.

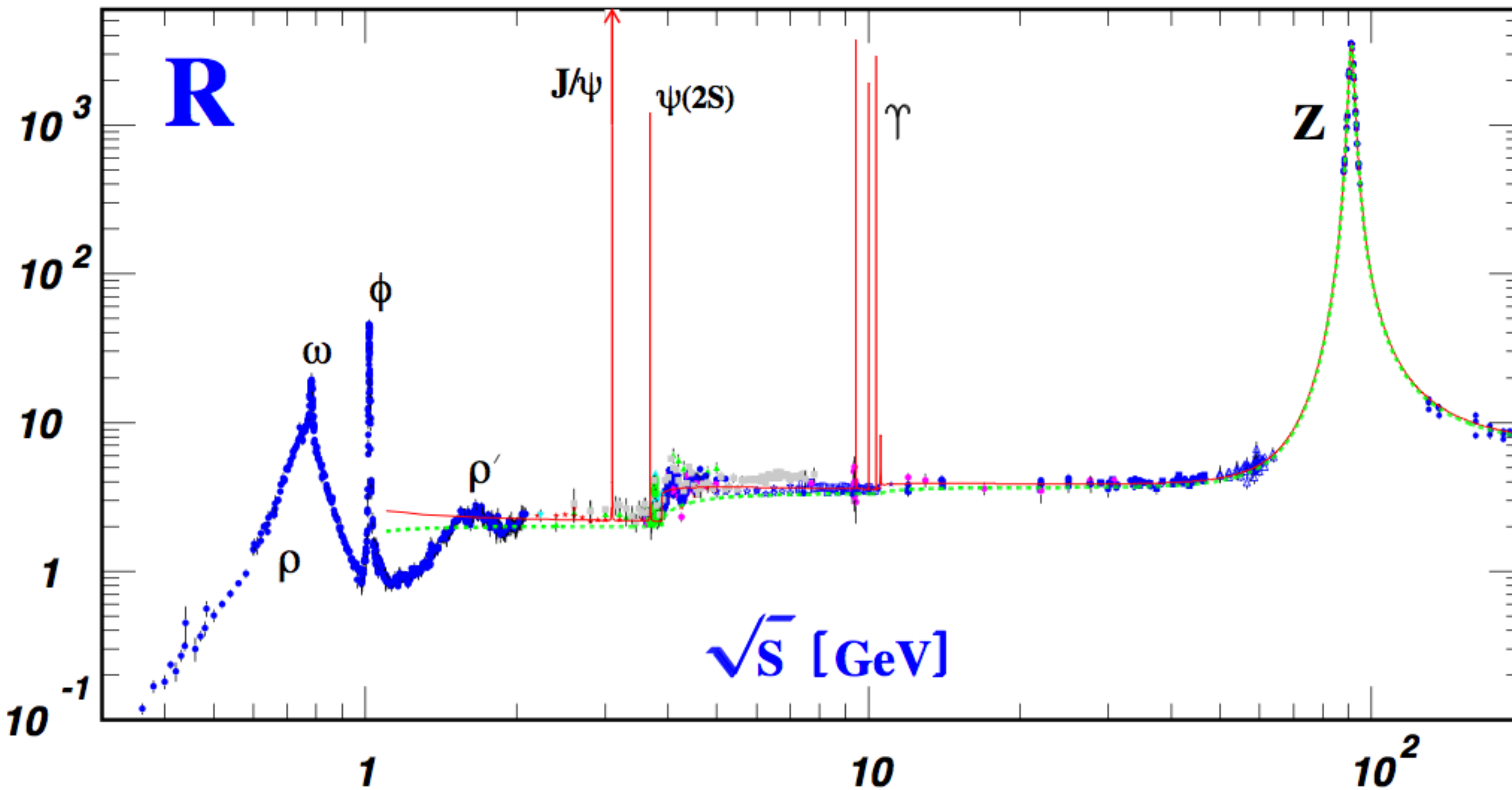
The above is the *pure EW prediction*. By accounting for the contribution of single gluon radiation (one power of α_s) the expression becomes:

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum_q e_q^2 \left(1 + \frac{\alpha_s(Q^2)}{\pi}\right)$$

QCD correction to R

Leading order correction for QCD shown in red on plot below

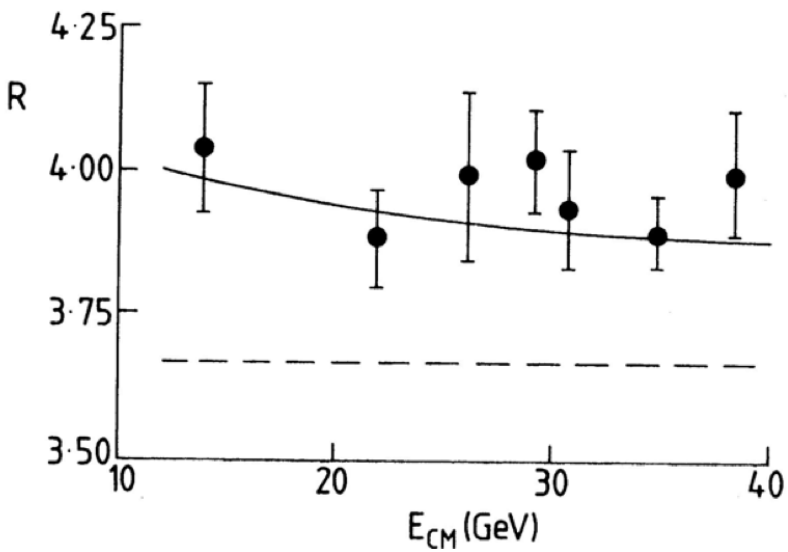
$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum_q e_q^2 \left(1 + \frac{\alpha_s(Q^2)}{\pi}\right)$$



Determination of strong coupling

Strong coupling can be measured at various scales directly with:

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum_q e_q^2 \left(1 + \frac{\alpha_s(Q^2)}{\pi}\right)$$

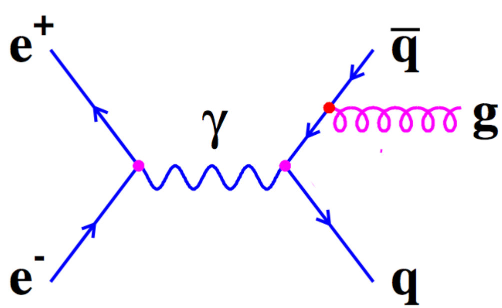


Very clean measurement, if somewhat imprecise:

$$\sum_q Q_q^2 = 3\frac{2}{3}$$

$$\Longrightarrow \left(1 + \frac{\alpha_s(E_{CM}^2)}{\pi}\right) \approx \frac{3.9}{3.67}$$

$$\Longrightarrow \alpha_s\left(E_{CM}^2 = 25^2\right) \approx 0.20$$



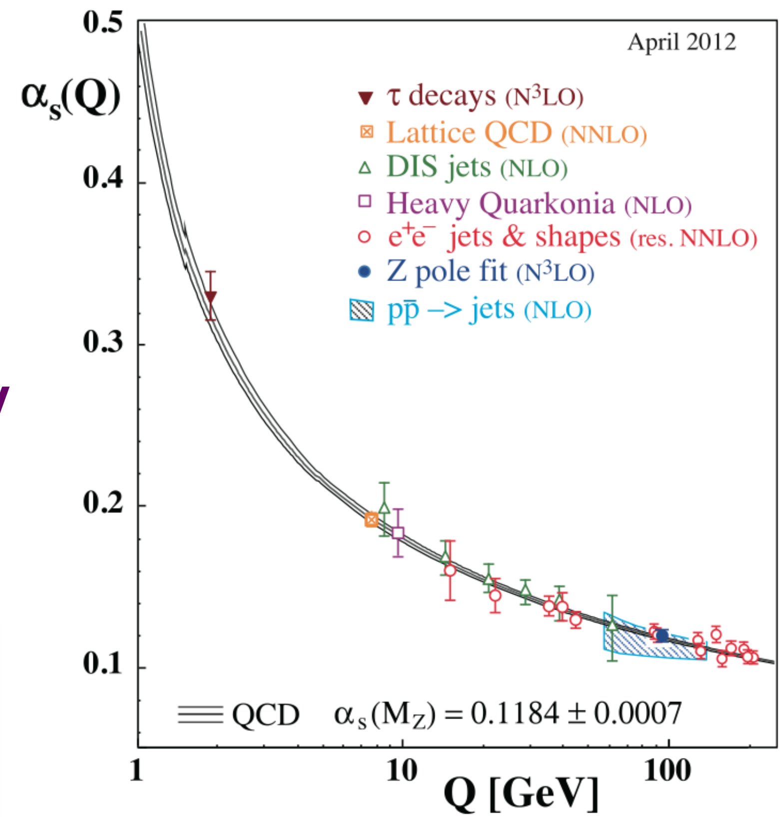
Also measure relative 3-to-2-jet rate

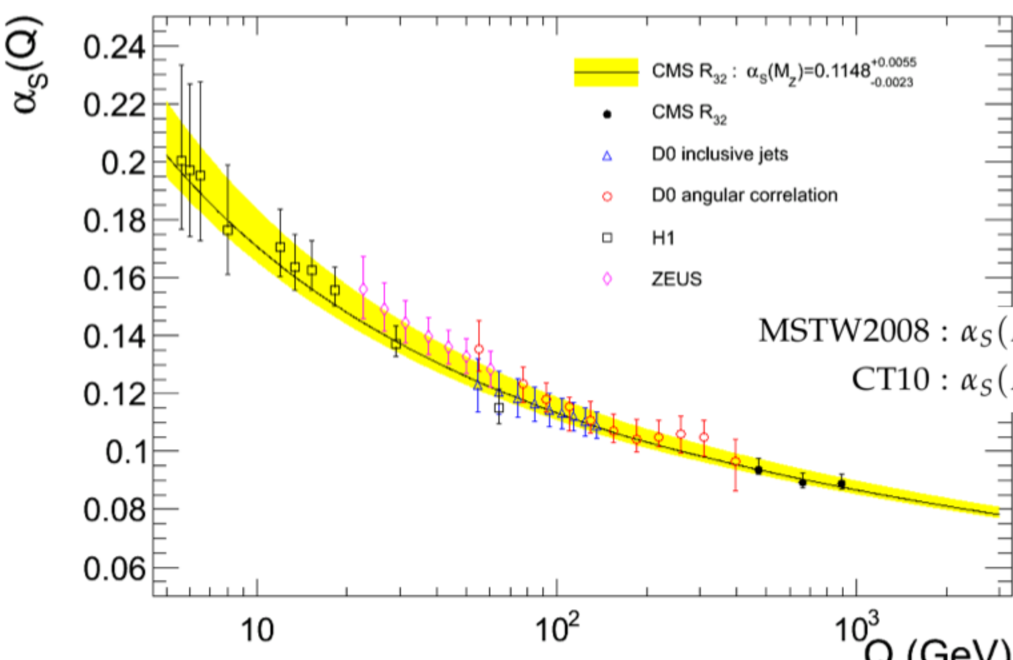
$$\frac{\sigma(3-jet)}{\sigma(2-jet)} = \frac{\sigma(q\bar{q}g)}{\sigma(q\bar{q})} \propto \alpha_s$$

Running of the strong coupling

Now have measurements from wide variety of sources

Together testing running of strong coupling spanning scales from 1 GeV to 1 TeV…

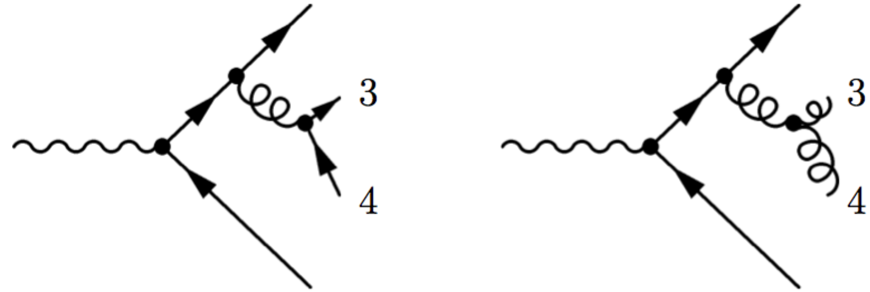




$$\text{MSTW2008}: \alpha_S(M_Z) = 0.1135 \pm 0.0096(exp.)$$
$$\text{CT10}: \alpha_S(M_Z) = 0.1130 \pm 0.0080(exp.)$$

4-jet events: gluon self-interactions

Four-jet events sensitive to proposed gluon self-interaction vertex

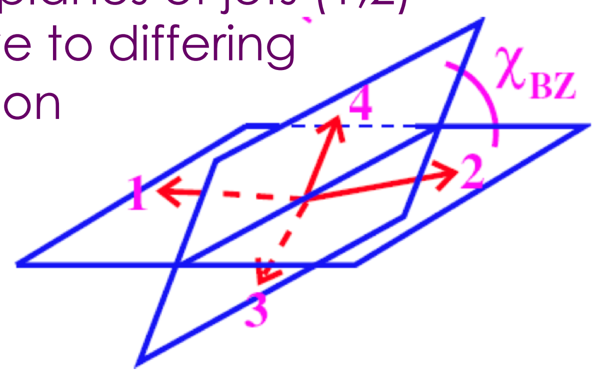


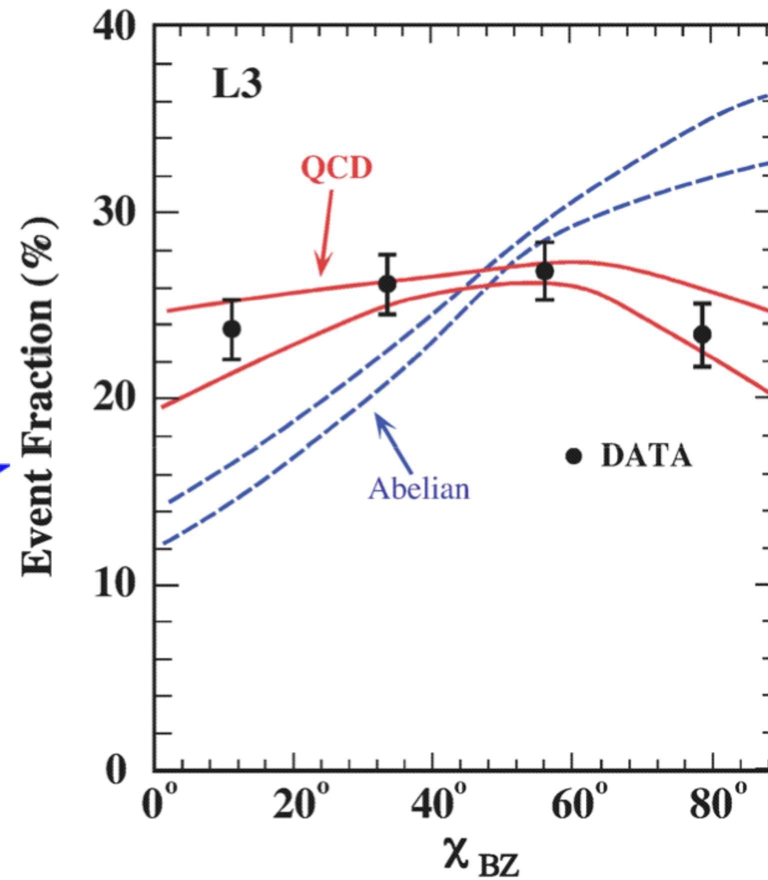
QCD an SU(3) **non-abelian** field theory (compare QED U(1) abelian).

$q\bar{q}g$ vertex consists of two spin-½ quarks and a spin-1 gluon

ggg vertex consists of three spin-1 gluons

Angle between planes of jets (1,2) and (3,4) sensitive to differing angular distribution





Data confirms non-abelian SU(3) structure of QCD!

QCD colour factors

Coming back to colour: SU(3) QCD theory predicts 8 (9) gluons

Eight colour octet combinations:

$$R\bar{G}, R\bar{B}, G\bar{R}, G\bar{B}, B\bar{R}, B\bar{G}, \sqrt{\frac{1}{2}}(R\bar{R} - G\bar{G}), \sqrt{\frac{1}{6}}(R\bar{R} + G\bar{G} - 2B\bar{B})$$

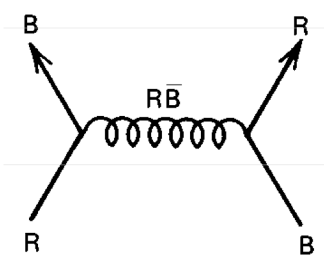
and one singlet combination (non-interacting):

$$\sqrt{\frac{1}{3}}(R\bar{R} + G\bar{G} + B\bar{B})$$

In QED, strength between two quarks: $e^{q1}e^{q2}\alpha$

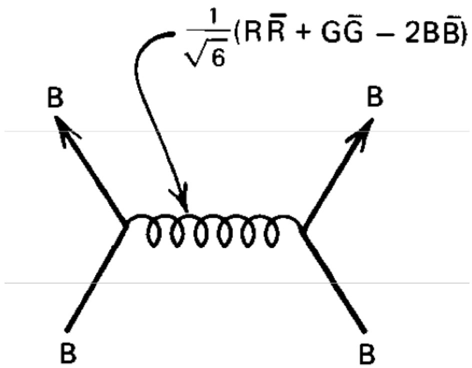
In QCD, strength of single gluon exchange: $\frac{1}{2}c_1c_2\alpha_s$

where c_1 and c_2 are "*colour coefficients*" of associated vertices



$$\text{Call } C_F = \frac{1}{2}|c_1c_2| \text{ the 'colour factor'}$$

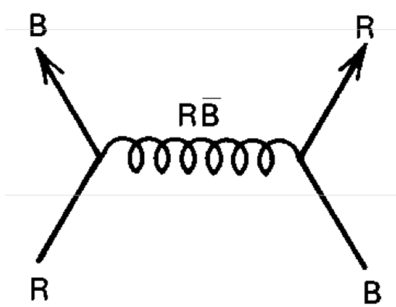
What is the colour factor predicted by QCD?



Consider colour factor between two quarks of same colour (**B**)

Of all 8 quarks, only one contributes in exchange, and only $B\bar{B}$

$$\text{Here } C_F = \frac{1}{2}|(-2\frac{1}{\sqrt{6}})(-2\frac{1}{\sqrt{6}})| = \frac{1}{3}$$



Another example, for **R** and **B** quarks:

This time, only $R\bar{B}$ contributes

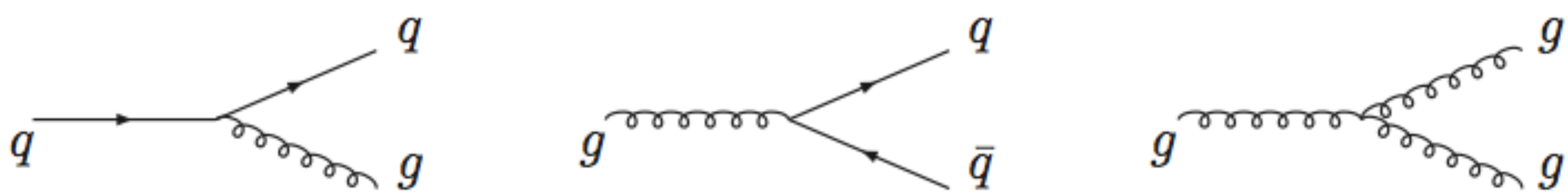
$$C_F = \frac{1}{2}|1 \cdot 1| = \frac{1}{2}$$

Quark and gluon jet differences?

From the Colour Factor calculation, QCD group structure SU(3) predicts total relative probabilities for the three transitions:

1. gluon radiation ($q \to gq$),
2. gluon splitting ($g \to q\bar{q}$),
3. triple gluon vertex ($g \to gg$),

to be **C_F=4/3, T_F=1/2, C_A=3 – "QCD colour factors"**.

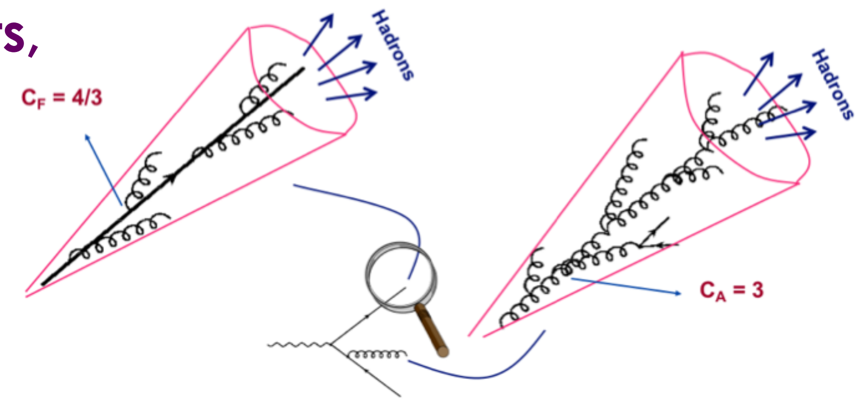


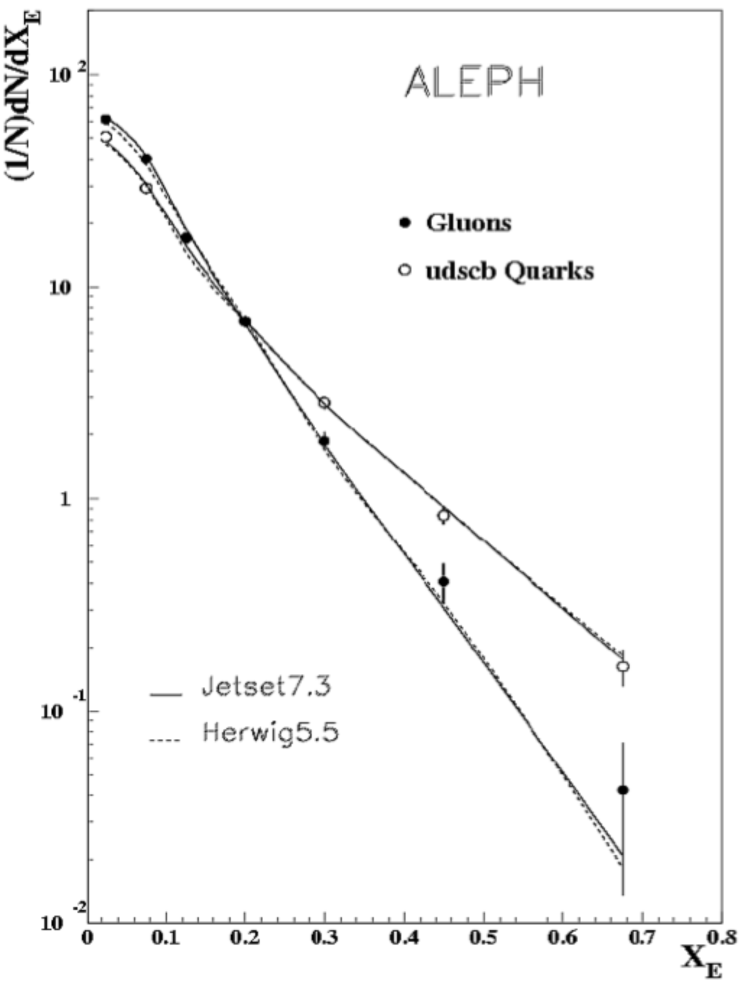
Can therefore expect differences from quark and gluon jets:

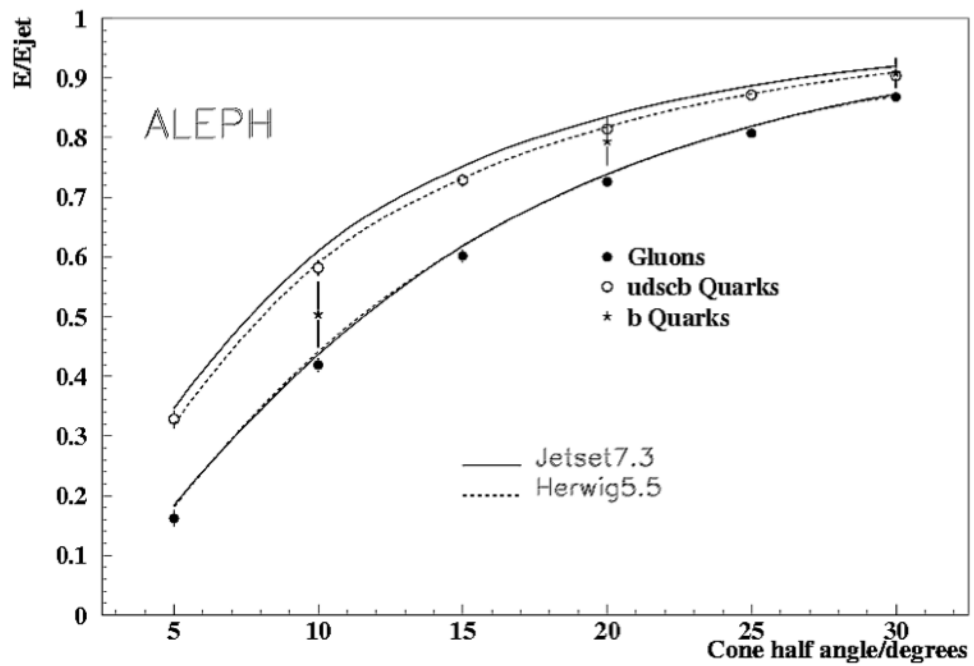
1. Larger particle multiplicity in gluon jets from C_A/C_F
2. Softening of momentum distributions in particles from gluon jet

Quark and gluon jet differences?

Defining cone around observed jets, can study properties to separate gluon/quark jets

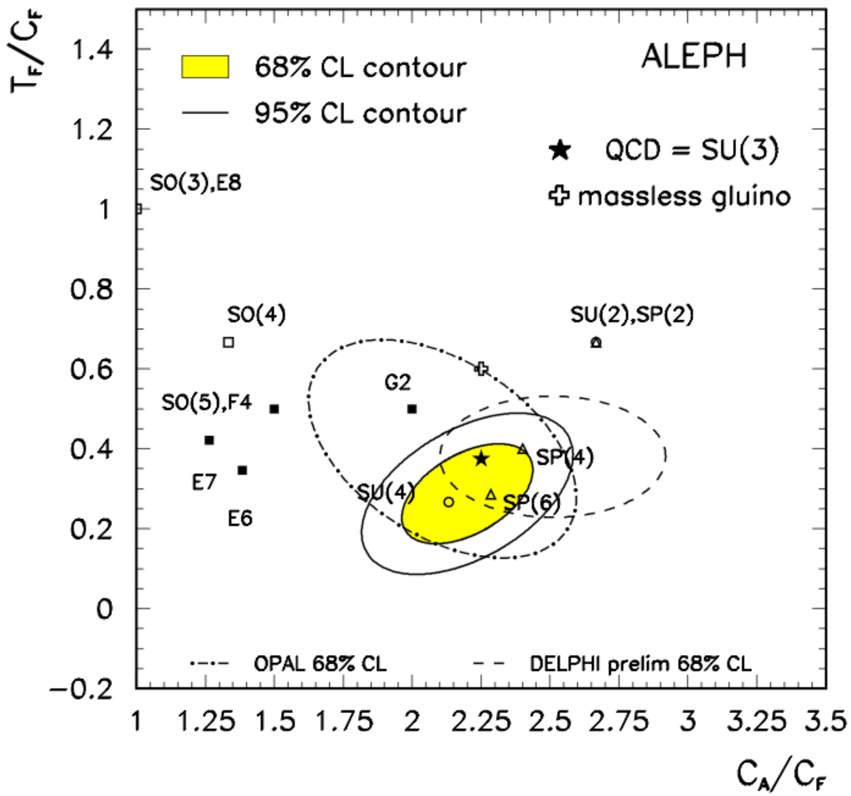


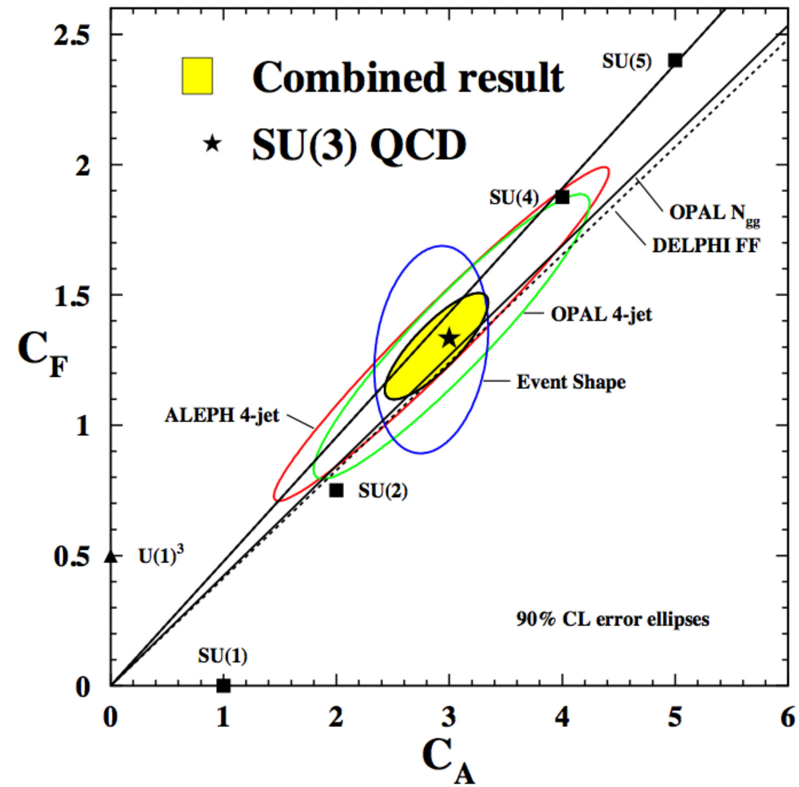




Measurement of QCD colour factors

Simultaneous measurement of C_A/C_F and T_F/C_F in e+e- collisions possible through study of angular correlations in four jet events and C_F and C_A through event shape variables





Best combination measurement gives C_A=2.89±0.21 [QCD SU(3) = 3] and C_F=1.30±0.09 [QCD SU(3)=1.33]

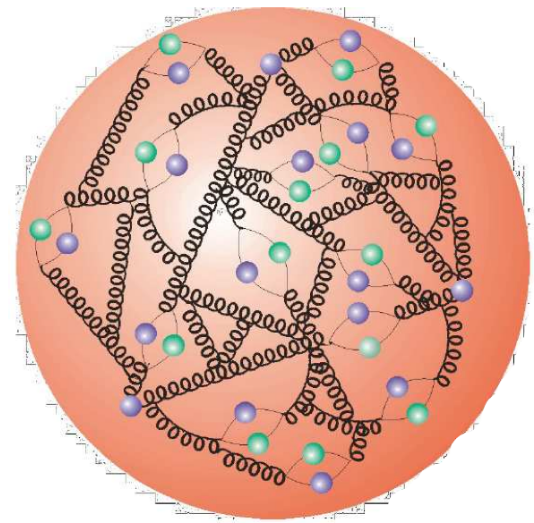
Proton structure: confinement implications

Confinement shown to restrict quarks to baryons, along with colour field of soft gluons: implications for proton structure

Three constituent quarks of proton are constantly interacting, emitting and reabsorbing gluons, that themselves emit more gluon/quark pairs.

Prediction of proton structure from QCD is complex!

Proton = 3 valence quarks + many soft gluons and quark-antiquark pairs



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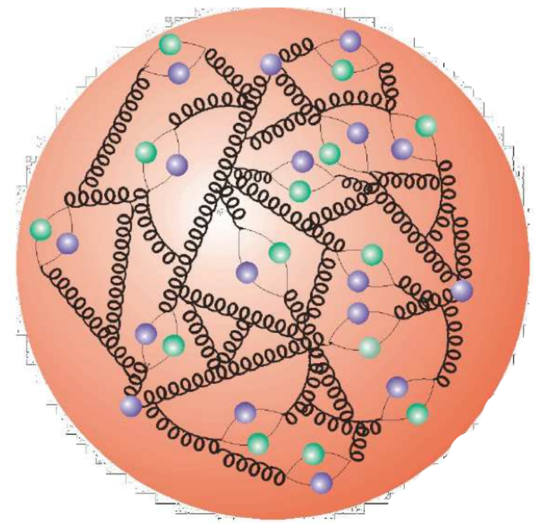


Prediction of proton structure from QCD is complex!

Proton = 3 valence quarks + many soft gluons and quark-antiquark pairs

Valence quarks carry quantum numbers of proton (isospin, strangeness etc.), but gluons and quark-antiquark "sea" can carry momentum/energy/spin

Need a camera with very fast shutter in order to take snapshot of the sea!

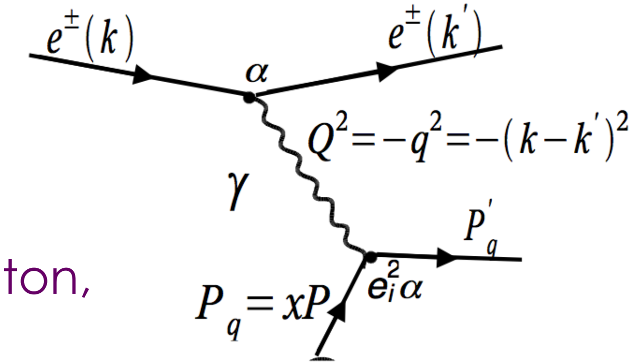
A proton with high energies in the lab frame will have **proper time slowed**. If probed with high energy electron, such 'snapshots' can be revealed…

Electron-proton scattering

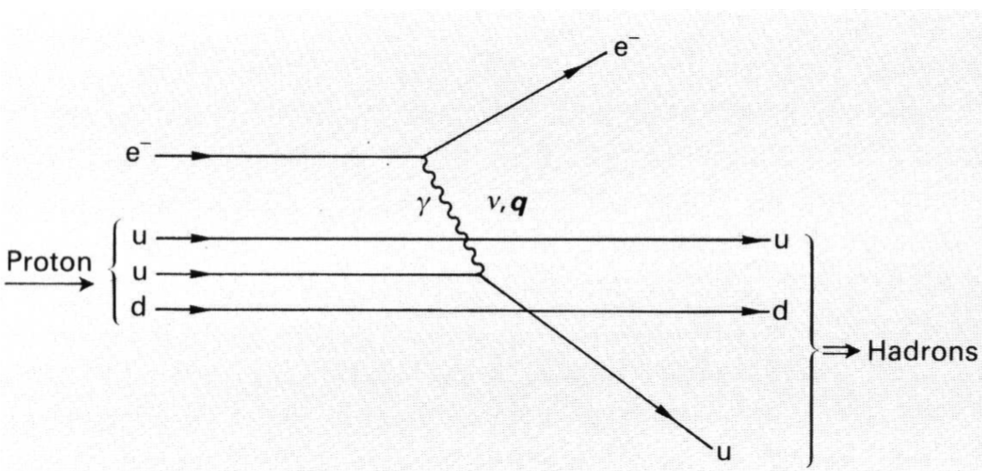
At low energies electrons scatter elastically off protons:

$$e^- + p \rightarrow e^- + p$$

Electron does not probe structure of the proton, and cross-section similar to eμ scattering



At higher energies, resolving power of virtual photon improves, scatters off individual constituents rather than proton as a whole.



Scattered quark tends to gain a lot of energy/momentum

Leads to break up of proton and hadronisation

No more elastic, but Deep Inelastic Scattering:

$$e^- + p \rightarrow e^- + X$$

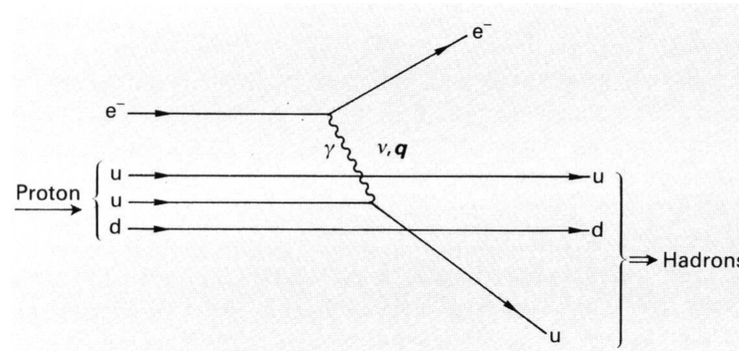
Deep Inelastic Scattering (ep)

Deep Inelastic Scattering kinematics:

$$e^-(p_1) + p(p_2) \to e^-(p_3) + X(p_4)$$

$$q = (v, \vec{q}) \equiv p_1 - p_3$$

$$-q^2 = \vec{q}^2 - v^2 \equiv Q^2 > 0$$



In Deep Inelastic Scattering, photon is **deeply** virtual, with large $Q^2 \geq 10\ \mathrm{GeV}$
Quantity $1/\sqrt{Q^2}$ is measure of spatial resolution, so at large Q^2 we get information on the deep structure of the proton (*destroying it in the process*).

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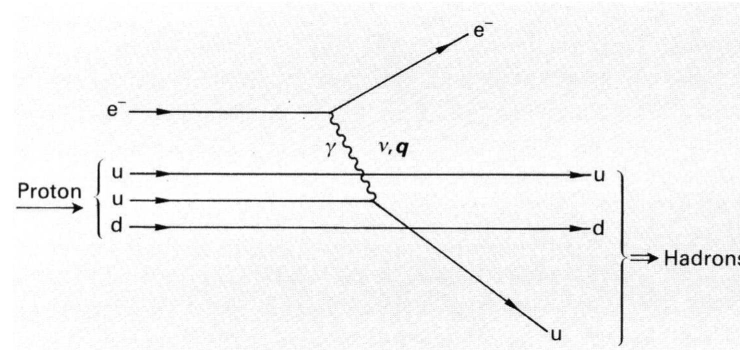

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If quark hit by photon initially carrying fraction x of proton 4-momentum $k_1 = xp_2$ then $k_2 = k_1 + q = xp_2 + q$. In QCD quark ~free in proton so electron-quark scattering is elastic and $k_1^2 = k_2^2$:

$$k_2^2 = (k_1 + q)^2 = k_1^2 + q^2 + 2x(p_2 q) \implies x = -\frac{q^2}{2(p_2 q)} = \frac{Q^2}{2M_p v}$$

where x is known as "Bjorken's scaling variable" or just "Bjorken x"

Structure functions

Cross-section for ep DIS looks like:

$$\frac{d^2\sigma}{d\cos\theta dx} = \frac{\pi\alpha^2(p_1 + xp_2)}{q^4}\left(1 + \cos^4\frac{\theta}{2}\right) F_2^{ep}(x, Q^2)$$

Equivalent to the scattering of electrons of momentum p_1 on a set of point-like particles of momenta xp_2, times the probability of finding such a particle.

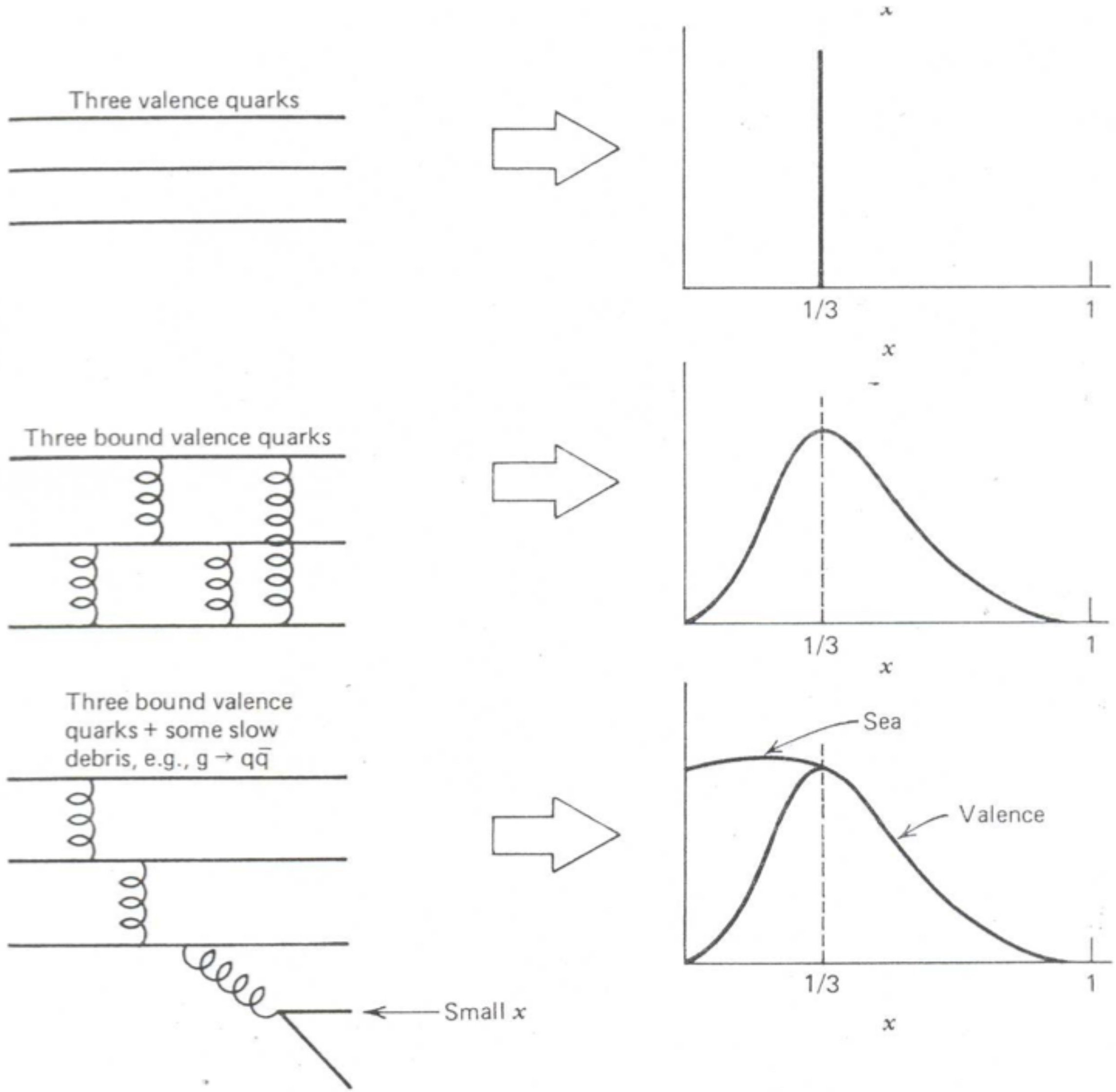
F_2^{ep} called the "structure function" of the proton, related to distributions of quarks and antiquarks in the proton:

$$F_2^{ep} = \frac{4}{9}x[u(x) + \bar{u}(x)] + \frac{1}{9}x[d(x) + \bar{d}(x)] + \frac{1}{9}x[s(x) + \bar{s}(x)]$$

Here the q(x) describe probabilities of finding quark q in proton, carrying fraction x of proton momentum [neglect c, b, t quarks here as heavy]

Proton momentum distribution models

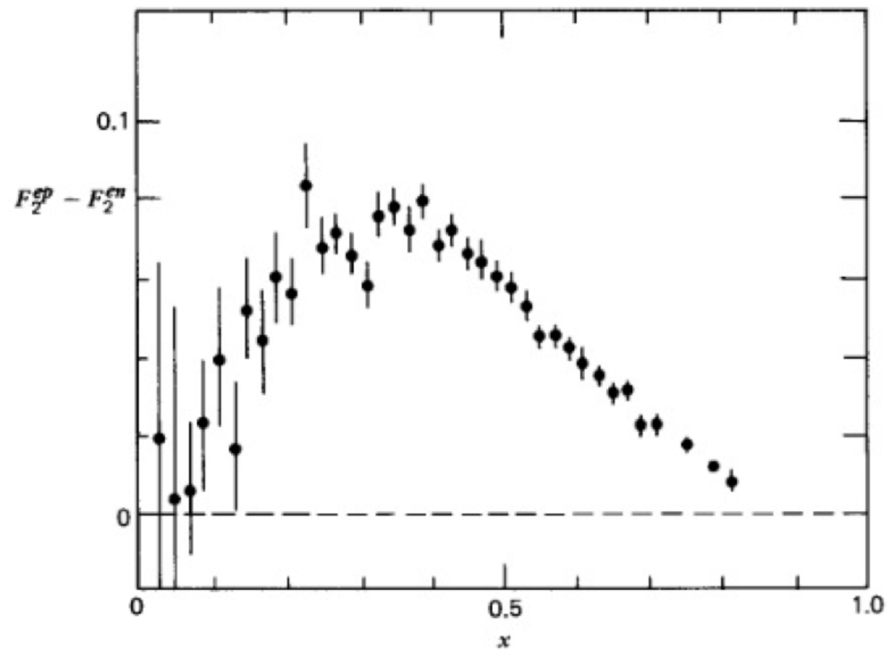
Different models of the proton have distinctive predictions for the momentum distribution…



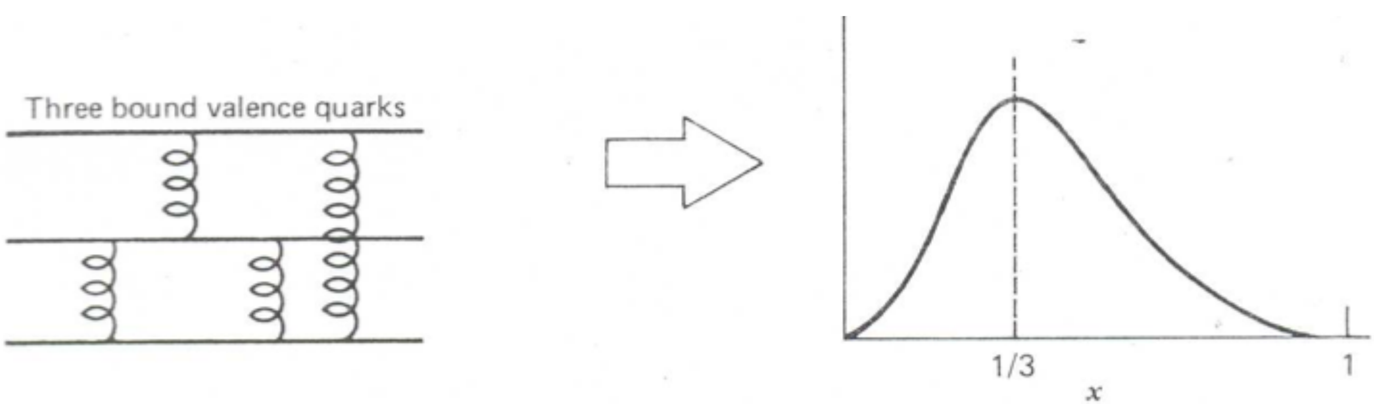
Valence quark momentum distribution

Can isolate and study valence quark momentum distribution through study of difference of proton and neutron structure functions

$$\frac{1}{x}[F_2^{ep}(x) - F_2^{en}(x)]$$
$$= \frac{1}{3}[u_v(x) - d_v(x)]$$



Experimental results really do look like the three bound valence quark expectation (<x>~1/3)



Quark distribution in the proton

Determining the quark distribution in the proton requires additional information:

- Due to isospin symmetry, u-quarks in proton have same distribution as d-quarks in neutron.
- DIS en-scattering can be observed with Deuterium target.
- Neutrino beam fixed target DIS (charged weak interactions) (Anti-)Neutrino sees mainly d and anti-u (u and anti-d)

These inputs together with proton DIS results are sufficient to determine quark and antiquark distribution functions independently

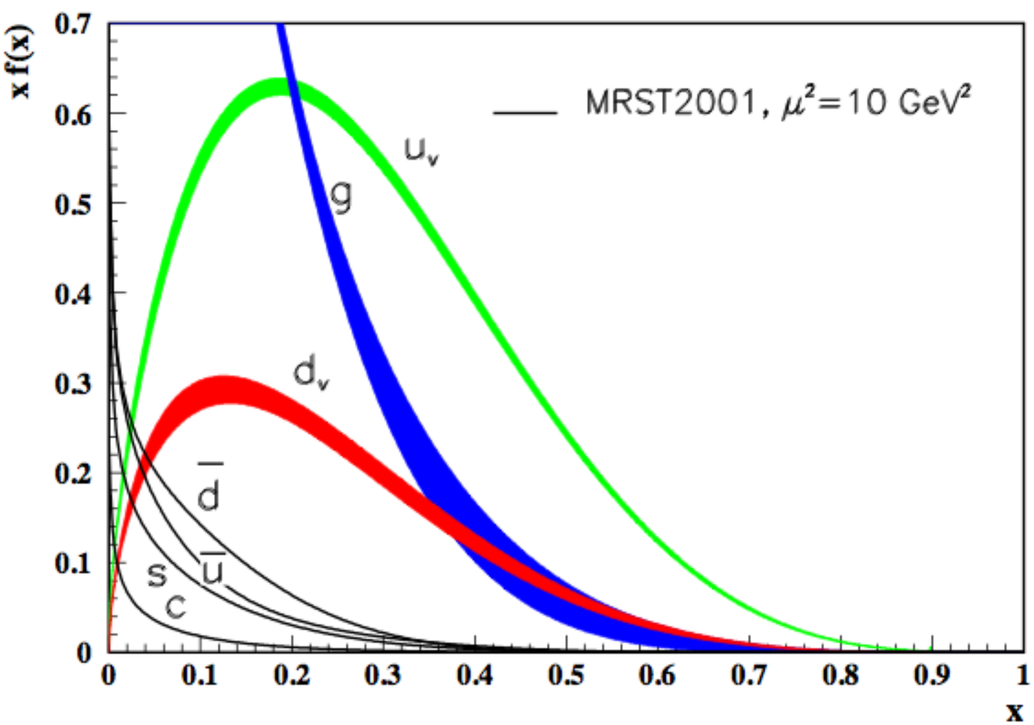
Gluon distribution from "scaling violation" information (see later)

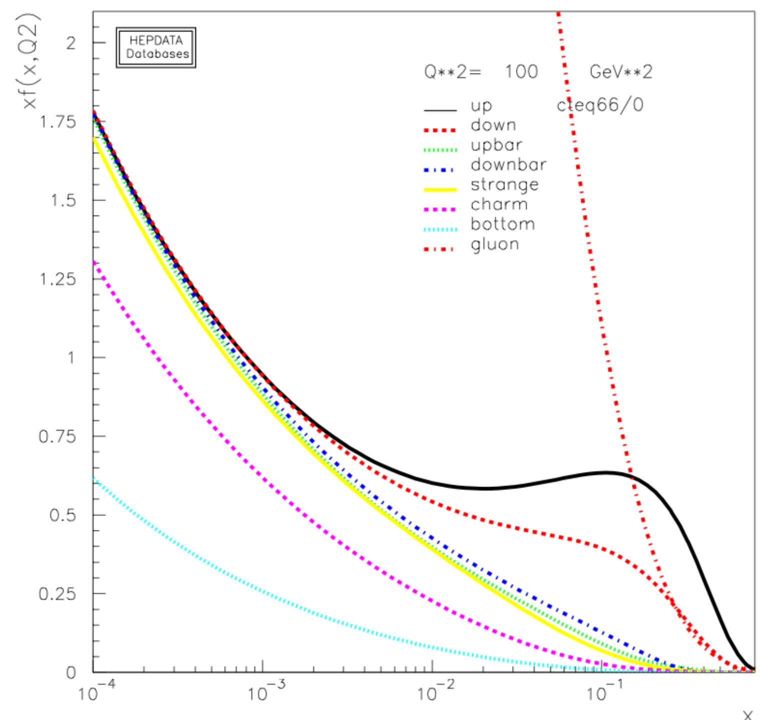
Quark distribution in the proton

Valence u and d dominate at high x, only account for ~30% momentum

Gluons take ~50% of total momentum, remaining 20% in quark sea!

Determined "parton density functions":





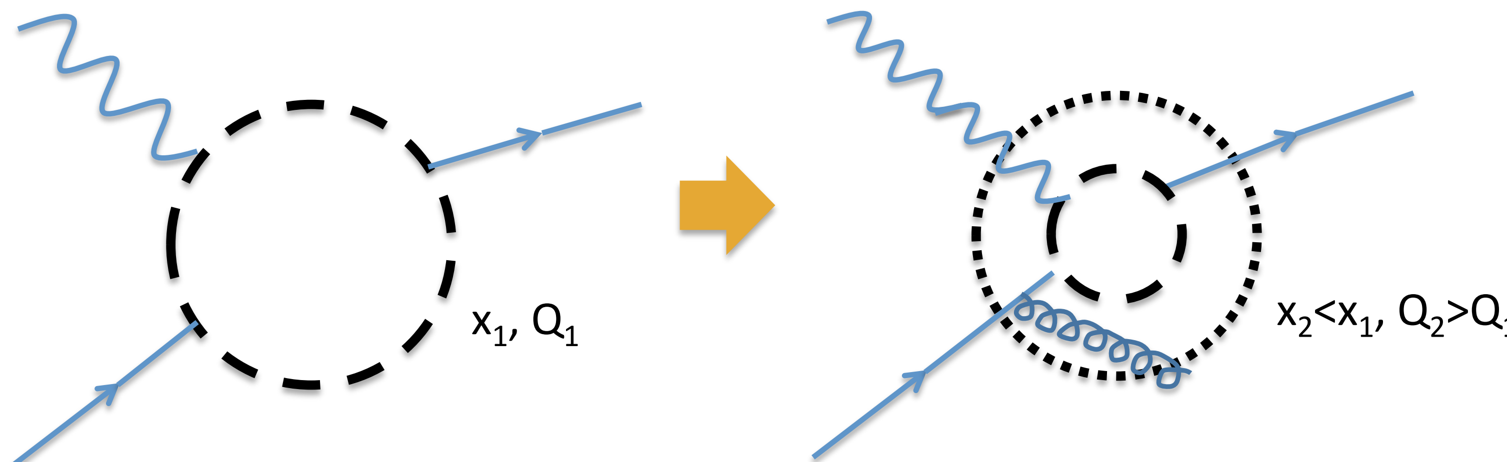
Scaling violations

So far only considered dependence of structure functions on x.

Known as **scale invariance**, but is not the case if the partons in the proton are free

When Q^2 is high, strong coupling is small (asymptotic freedom) so at high Q^2 are almost free but not quite – surrounded by parton cloud

Structure function F_2 must therefore have a Q^2 "resolution" as well as x dependence: **scaling violation**



Generally at larger Q^2, QCD interaction would tend to lead to softer distribution function

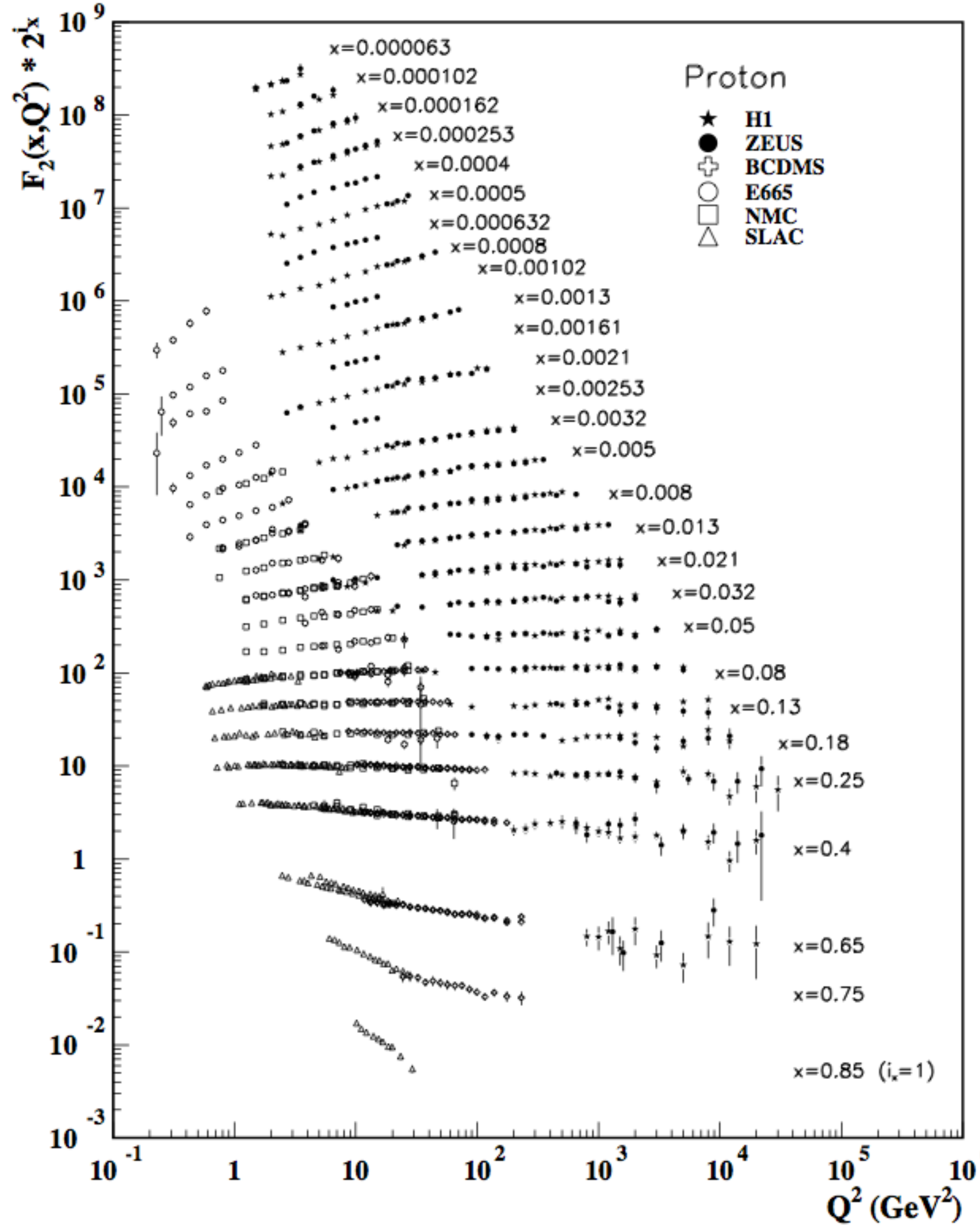
Experimental evidence for scaling violation

Figure shows example of $F_2(x,Q^2)$ measurements at various x values vs. Q^2

If scaling were exact, all curves horizontal straight lines.

However, we see that at small x F_2 increases with Q^2, while at large x F_2 decreases with Q^2 in line with prediction of QCD.

Resolve increasing numbers of soft partons with increasing Q^2 and high x momentum fraction is depleted



Scaling violation and gluon density

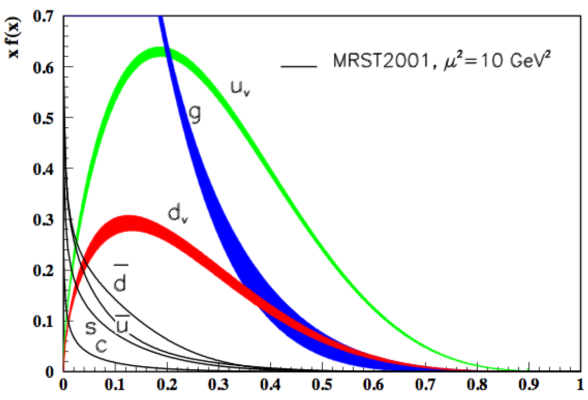
Low x behaviour of scaling violation is useful for measuring gluon density.

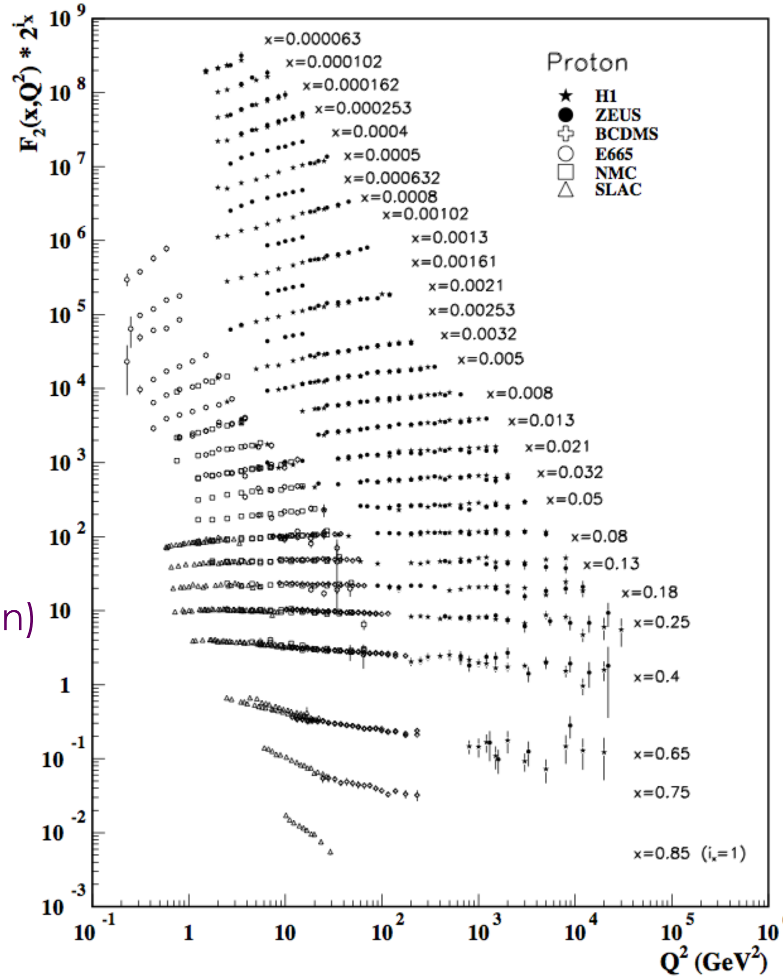
At small x, gluon splitting dominates and:

$$xg(x,Q^2) \approx \frac{27\pi}{10\alpha_s(Q^2)} \frac{dF_2(x,Q^2)}{d\ln Q^2}$$

Scaling violation at low x is a proxy for measurement of gluon density!

(Large uncertainties – other data also used to constrain)





DGLAP evolution

QCD cannot predict exact shape of F_2 but can predict how PDFs evolve in (x, Q^2) given a starting $F_2(x,Q^2)$ from measurement!

Scaling violations can be tested at each value of x at particular Q^2

'DGLAP' equation quantifies how to evolve scaling violations from particular scale to another:

$$\frac{d}{d\log Q^2} q(x, Q^2) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} q(y, Q^2) P_{qq}\left(\frac{x}{y}\right) + \mathcal{O}(\alpha_s^2)$$

At LO, can interpret as **quark with momentum fraction x** could have come from a **parent quark** with larger **momentum y**, which has **radiated a gluon** with fraction x/y momentum.

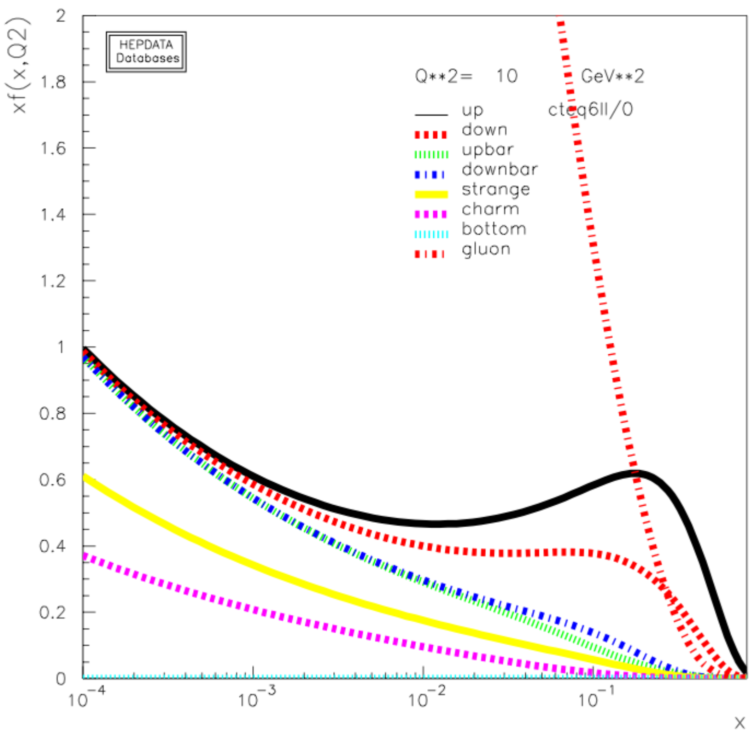
Probability of this occurring proportional to $\alpha_s P_{qq}(x/y)$

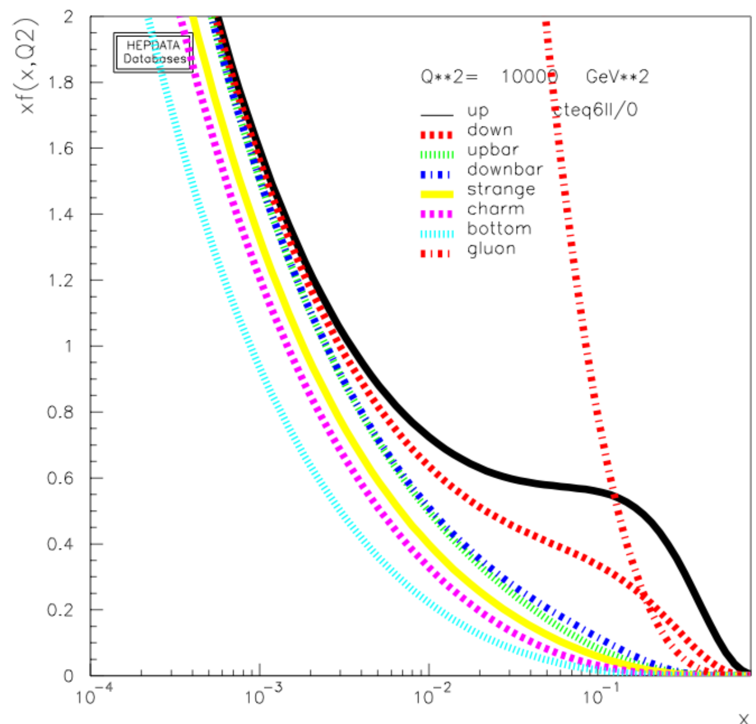
DGLAP evolution

QCD cannot predict exact shape of F_2 but can predict how PDFs evolve in (x, Q^2) given a starting $F_2(x,Q^2)$ from measurement!

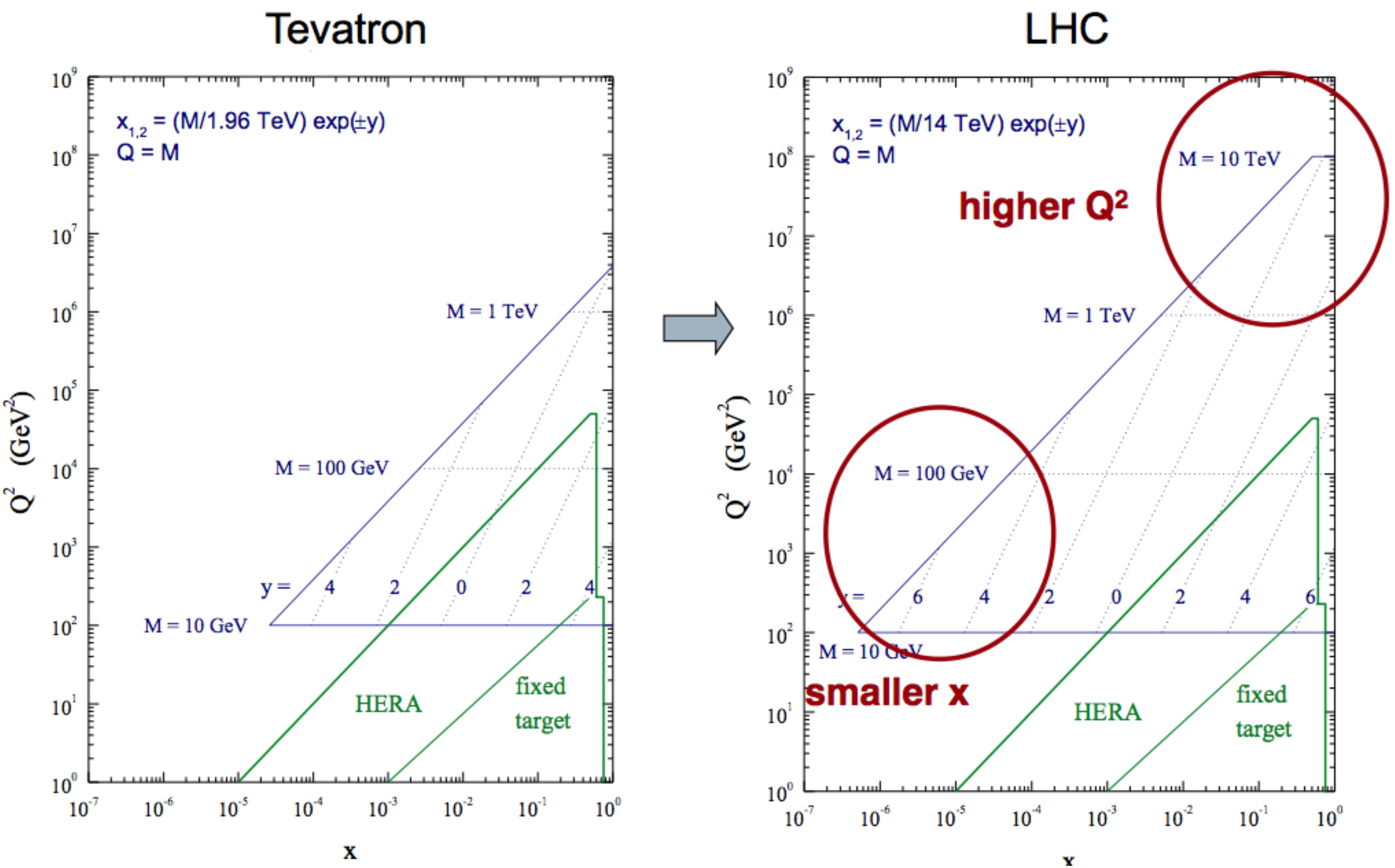
DGLAP evolution tested and scaling works well!

$$\frac{d}{d\log Q^2} q(x,Q^2) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} q(y,Q^2) P_{qq}\left(\frac{x}{y}\right) + \mathcal{O}(\alpha_s^2)$$





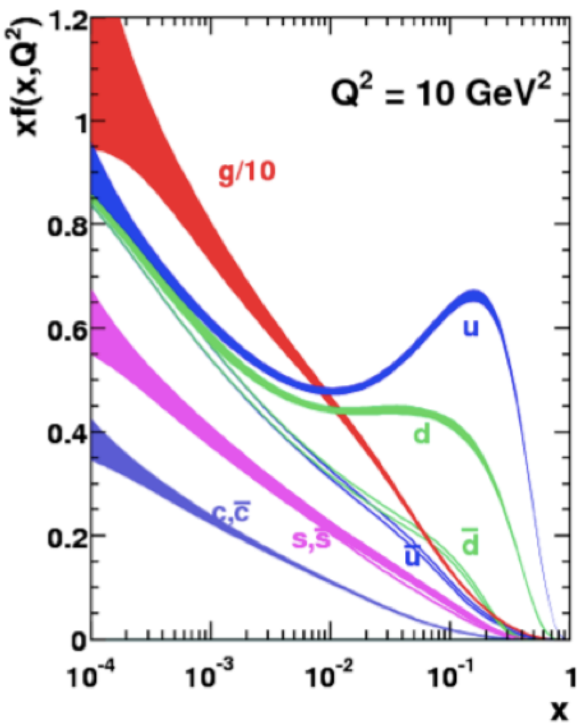
Density function evolution

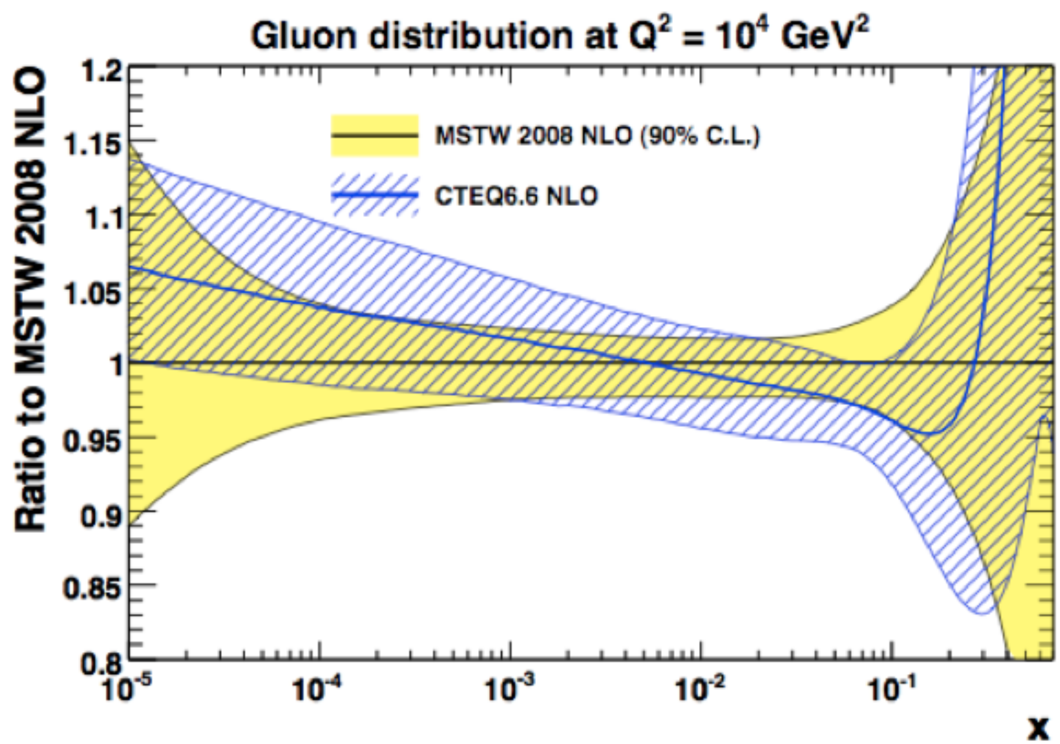


Density function fits to data

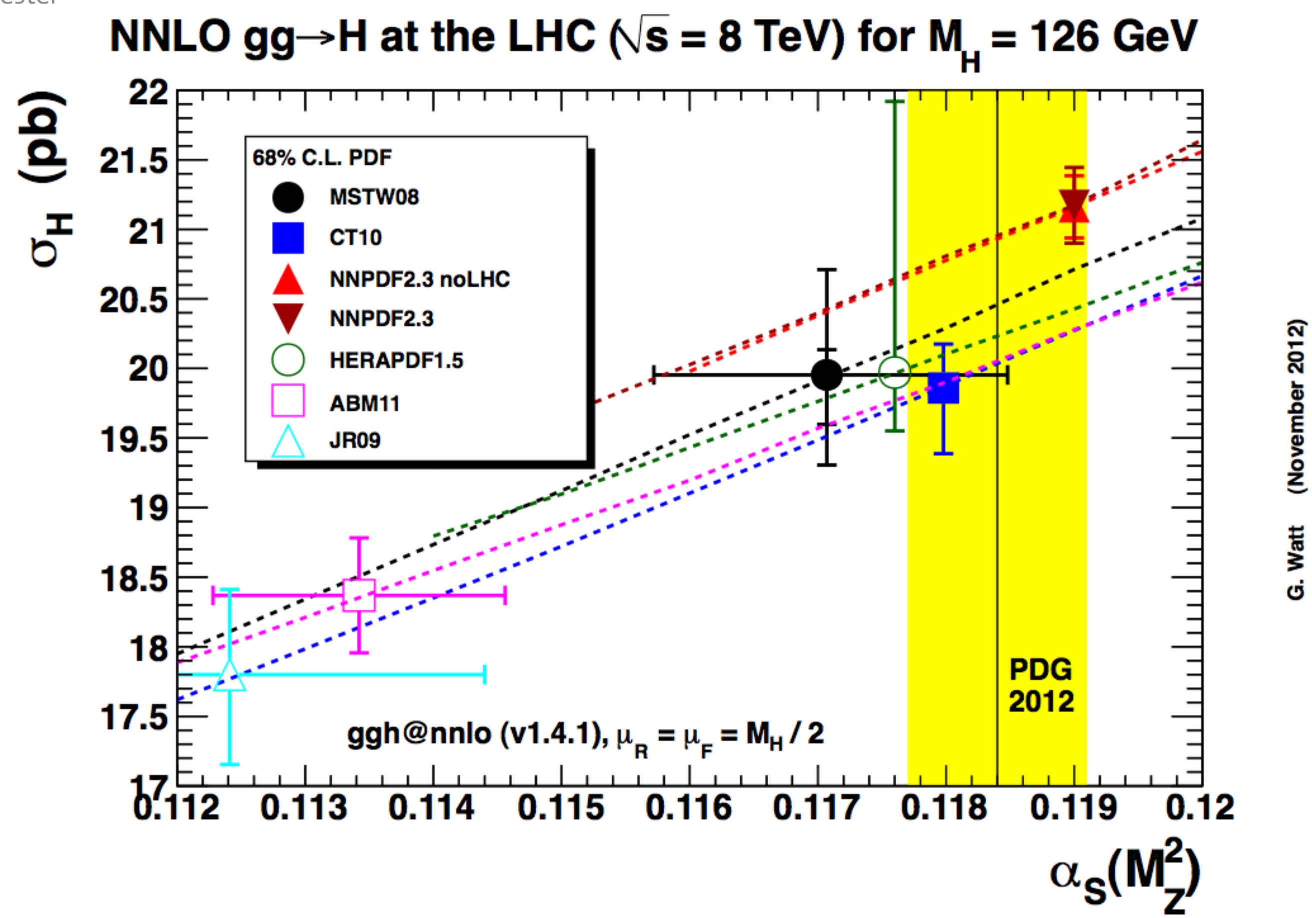
Density functions and their evolution well-established theoretically

Reality a little trickier… need to fit datasets for initial evolution.
What data to use? Correlations? Room for interpretation…





Density functions and precision studies



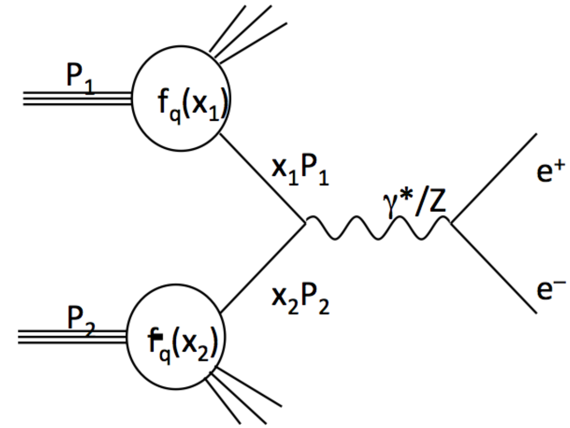
Precise quark and gluon densities needed across whole x range from ep machines to predict new signals and model old backgrounds at the LHC!

Data fitted to extract densities and evolve a bit of an art… some uncertainty

Factorisation theorem

The link with hadron-hadron collisions!

Want to look at proton-proton collisions?
Two protons, two PDFs!



Important theorem: 'factorisation theorem'

$$\sigma_{AB\to\phi+X} = \sum_{ab}\int_0^1 dx_1 dx_2 f_{a/A}(x_1,Q^2) f_{b/B}(x_2,Q^2)\hat{\sigma}_{ab\to\phi}\left(\hat{s},\alpha_s(Q^2)\right)$$

Collins & Soper developed (1987) developed framework to show DIS structure functions can be used in hadron-hadron scattering: **universality**

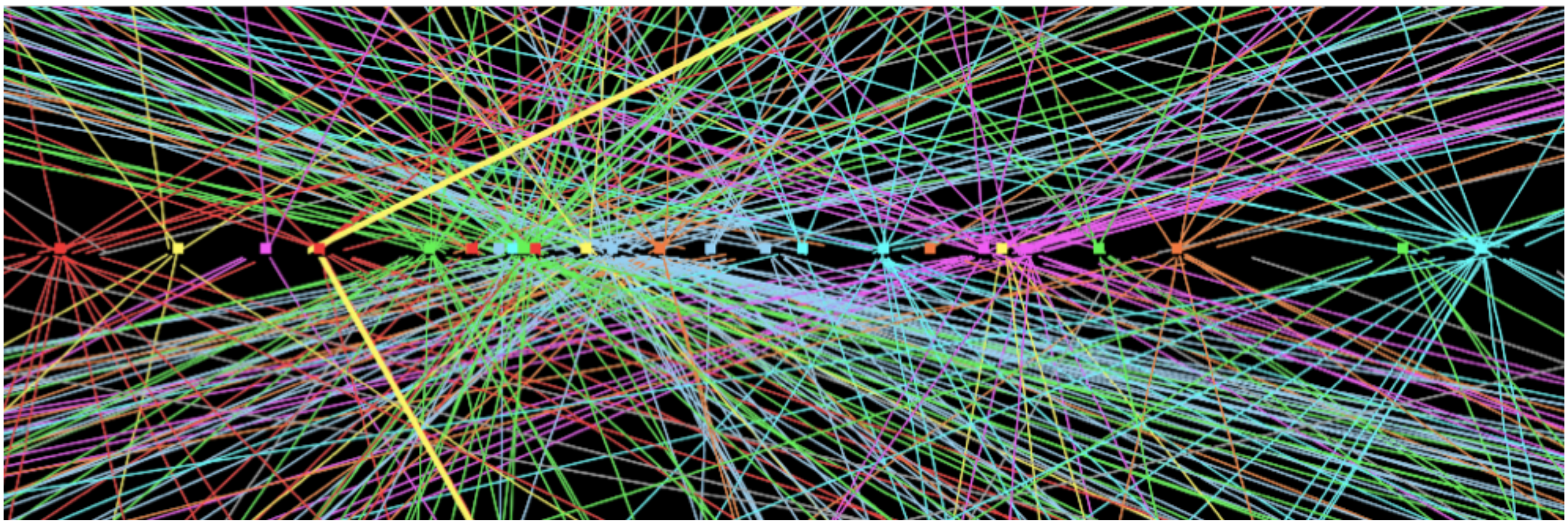
Factorised hadron-hadron process into non-perturbative part (PDFs) and parton-level scattering amplitude [calculable]

Factorisation hypothesis has been questioned – does it break down at some point?...

Next steps...

Now we know in principle how to understand hadron-hadron collisions, given experience and expertise from e+e- and ep(n)...

Plenty of new challenges await us, in jet definitions, reconstruction and calibration, underlying event, pile-up...



Hadron-hadron collision environment can get pretty messy!
(Real Z boson production event in ATLAS 2012 data)

Summary of today

- What is QCD, and what does it predict?
- What is colour – experimental verification?
- Jets and algorithmic definitions
- Reality of gluons and quarks
- Precision predictions in e+e-
- Electron-proton scattering
- Substructure of the proton and evolution with scale
- Implications for hadron-hadron scattering and the LHC…

Next time:

- Formalism of hadron-hadron collision calculations
- Jet algorithms at hadron colliders
- Underlying event
- Multiple parton interactions
- Pile-up
- Jet measurements at hadron colliders,
  - Precision tests and input to QCD theory, searches for new phenomena