



# Introduction to Hadron Collider Physics

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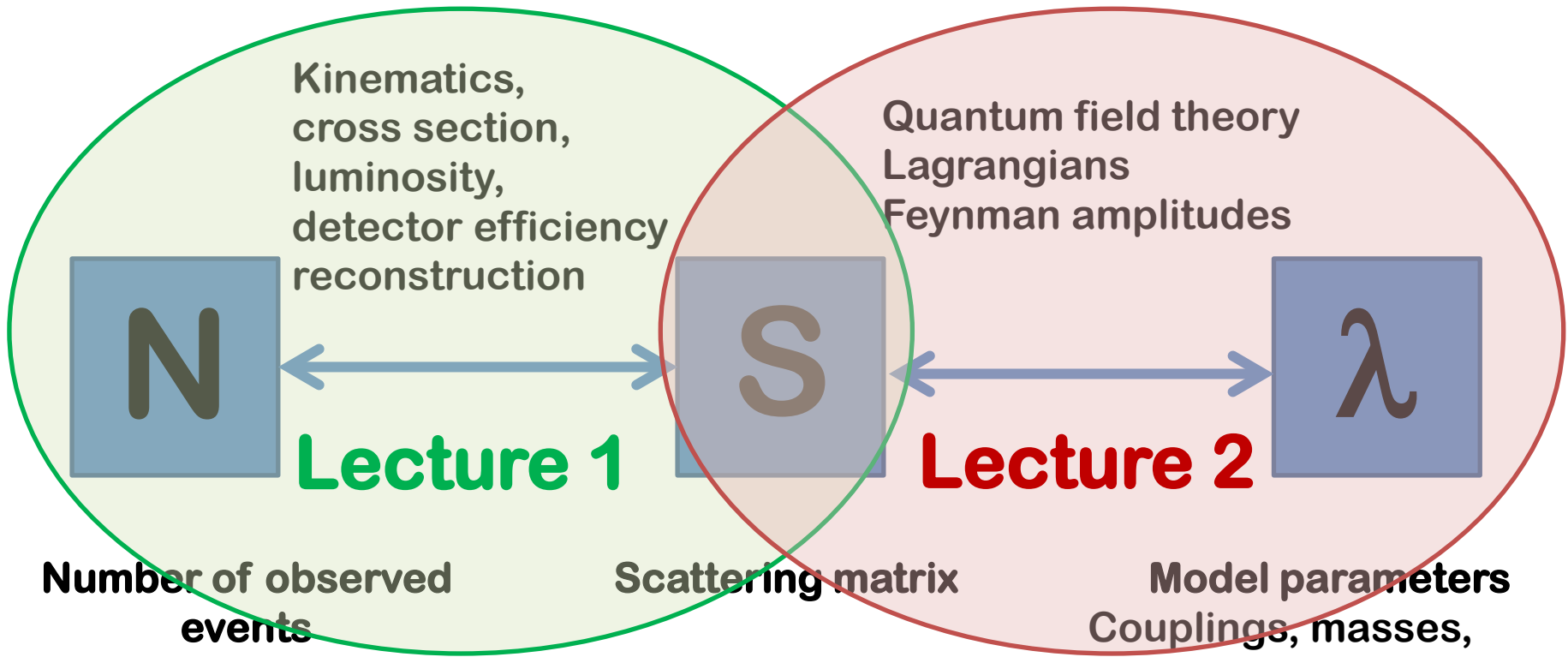
**Université Joseph Fourier/LPSC Grenoble**

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# PART II

## Theoretical background in HEP

# General overview



Kinematics,  
cross section,  
luminosity,  
detector efficiency  
reconstruction

Quantum field theory  
Lagrangians  
Feynman amplitudes

**Number of observed events**

**Scattering matrix**

**Model parameters**  
Couplings, masses,  
mixing angles...

Number of reconstructed top quark pairs

$|\langle i|S|f\rangle|^2 =$  probability of changing an initial state  $|i\rangle$  to a final state  $|j\rangle$

Number and nature of particles

Experimental observable

Eg :  $|i\rangle = pp$   
 $|f\rangle = \text{top pairs (+X)}$

in SM  
19 parameters,  
12 fermions,  
3 gauge interaction +SSB

Much more in SUSY... 3

# Evolution operator

$U(t_f, t_i)$  : evolution operator in interaction representation

$$|\psi(t_f)\rangle_I = U(t_f, t_i) |\psi(t_i)\rangle_I$$

Schrödinger equation  $i \frac{d}{dt} U(t_f, t_i) = \mathcal{V}_I(t) U(t, t_i)$

Where  $V_I(t)$  is the **Interaction Hamiltonian**

Solution :  $U(t_f, t_i) = e^{-i \int_{t_i}^{t_f} \mathcal{V}_I(t) dt}$

In truth, slightly more tricky because on integration bounds : needs time ordering to ensure causality...

# Scattering amplitude

transition probability between

initial state  $|i\rangle$ , at time  $t = -\infty$

and final state  $|f\rangle$ , at time  $t = +\infty$

$$P_{i \rightarrow f} = |\langle f | i(+\infty) \rangle|^2 = |\langle f | U(-\infty, +\infty) | i \rangle|^2 = |S_{fi}|^2$$

**S is the scattering matrix**

Hamiltonian = E (kinetic) + V (potential=interaction)

Lagrangian = E - V

So **interaction Hamiltonian = - interaction Lagrangian**

**Finally :**

**Important bit to know...**

$$S = U(-\infty, +\infty) = e^{-i \int_{-\infty}^{+\infty} \mathcal{V}_I(t) dt} = e^{i \int \mathcal{L}_{int} d^4x}$$

# Theoretical background

$$n \rightarrow p + e + \nu \quad m_n = 938,3 \text{ MeV} \quad m_p = 939,6 \text{ MeV} \quad \Delta m = 1,3 \text{ MeV}$$

Small objects, smaller than nucleus → Quantum mechanics

Kinetic energy  $\sim$  mass (for electron) → Special relativity  
Massless neutrino → Field theory  
Electromagnetic fields

Number and nature of particles vary → Second quantization

Putting everything together : new formalism

**(Relativistic) Quantum Field Theory**

# QFT in a nutshell

## Classical mechanics

Euler-Lagrange equations

Time evolution of discrete coordinate  $\mathbf{x}(t)$

Poisson brackets  $\{x_i, x_j\} = \{p_i, p_j\} = 0, \quad \{x_i, p_j\} = \delta_{ij}$

## Quantum physics

Schrödinger equation

Time evolution of wavefunction  $\psi(\mathbf{x}, t)$

$|\psi(\mathbf{x})|^2$  : presence probability

Observables become operators

Canonical commutation relations

$$[\hat{x}_i, \hat{x}_j] = [\hat{p}_i, \hat{p}_j] = 0, \quad [\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}$$

## Special relativity+Field theory (Maxwell EM)

Space-time evolution of a field  
(continuous coordinates)  $\varphi(\mathbf{x}, t)$

$$\{\varphi(x, t), \varphi(x', t)\} = \{\pi(x, t), \pi(x', t)\} = 0$$

$$\{\varphi(x, t), \pi(x', t)\} = \delta(x - x')$$

## Quantum field theory

Field become operators  $\hat{\varphi}(\mathbf{x}, t)$

Observables becomes functionals

Canonical commutation  $[\hat{\varphi}(x, t), \hat{\varphi}(x', t)] = [\hat{\pi}(x, t), \hat{\pi}(x', t)] = 0$

$$[\hat{\varphi}(x, t), \hat{\pi}(x', t)] = i\delta(x - x')$$

# Relativistic equations

Require invariance une Lorentz Poincaré Symmetry

**Boosts, Translation in space and time, Rotations**

Link between spin and mathematical structure used to represent it

**Spin 0** : scalar (number)

Klein-Gordon equation :

$$E^2 = p^2 c^2 + m^2 c^4 \xrightarrow{E \rightarrow i\hbar \frac{\partial}{\partial t}, \vec{p} \rightarrow i\hbar \vec{\nabla}} \square \psi + \frac{m^2 c^2}{\hbar^2} \psi = 0$$

**Or in covariant notations, natural units** :  $(m^2 + \partial_\mu \partial^\mu) \varphi = 0$

**Spin 1** : 4-vector, Maxwell equation

**Spin 1/2** : spinor field, Dirac equation



# Field quantization

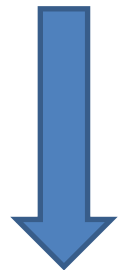
Plane wave solution for Klein gordon are of the form  $\exp(-ikx)$  :

$$\partial_\mu \partial^\mu \varphi + m^2 \varphi = 0 \Rightarrow \left[ (-ik_0)^2 - (i\vec{k})^2 + m^2 \right] e^{-ikx} = 0$$

$$\Rightarrow k_0 = \pm \sqrt{\vec{k}^2 + m^2} = \omega_k$$

General solution :

$$\varphi(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \frac{1}{\sqrt{2\omega_k}} \left( a_k e^{-ikx} + a_k^* e^{ikx} \right),$$



**Field quantization** : turns coefficients  $a_k$  and  $a_k^*$

Into operators

Canonical commutations :  $[a_i, a_j] = [a_i^+, a_j^+] = 0$        $[a_i, a_j^+] = \delta_{ij} \mathbb{1}$

$$\hat{\varphi}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \frac{1}{\sqrt{2\omega_k}} \left( \underbrace{\hat{a}_k e^{-ikx}}_{(1)} + \underbrace{\hat{a}_k^+ e^{ikx}}_{(2)} \right)$$

# Antiparticles

Negative energy solution arise from the quadratic equation

→not physical :

$$E = \omega_k : \varphi(x) \propto e^{-i(\omega_k t - \vec{k} \cdot \vec{x})}$$

$$E = -\omega_k : \varphi(x) \propto e^{-i(-\omega_k t - \vec{k} \cdot \vec{x})}$$

Energy is positive, absorb minus sign into the time

Particle moving towards negative time, with momentum  $-\mathbf{k}$

=

Antiparticle, moving towards positive time, with momentum  $\mathbf{k}$

$$\hat{\varphi}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \frac{1}{\sqrt{2\omega_k}} \left( \underbrace{\hat{a}_k e^{-ikx}}_{(1)} + \underbrace{\hat{a}_k^+ e^{ikx}}_{(2)} \right)$$

**Real field  $\varphi$  contains :**

$a_k$  annihilation of particle with momentum  $\mathbf{k}$

$a_k^+$  creation of antiparticle with momentum  $\mathbf{k}$

# Spinor solutions

$$\psi(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \frac{1}{\sqrt{2\omega_k}} \sum_{s=1,2}^{\text{hélicité}} \underbrace{u_{k,s}}_{\text{spineur}} \underbrace{b_{k,s}}_{\text{op.}} e^{-ikx} + \underbrace{v_{k,s}}_{\text{spineur}} \underbrace{d_{k,s}^+}_{\text{op.}} e^{ikx}$$

$$\bar{\psi}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \frac{1}{\sqrt{2\omega_k}} \sum_{s=1,2}^{\text{hélicité}} \underbrace{\bar{u}_{k,s}}_{\text{spineur}} \underbrace{b_{k,s}^+}_{\text{op.}} e^{ikx} + \underbrace{\bar{v}_{k,s}}_{\text{spineur}} \underbrace{d_{k,s}}_{\text{op.}} e^{-ikx}$$

**Field  $\Psi$  contains :**

$b_{k,s}$  **annihilation of particle** ( $e^-$ ) with momentum  $k$  and helicity  $s$

$d_{k,s}^+$  **creation of antiparticle** ( $e^+$ ) with momentum  $k$  and helicity  $s$

**Field  $\bar{\Psi}$  contains :**

$b_{k,s}^+$  **creation of particle** ( $e^-$ ) with momentum  $k$  and helicity  $s$

$b_{k,s}$  **annihilation of antiparticle** ( $e^+$ ) with momentum  $k$  and helicity  $s$

# Lagrangians

**Free lagrangian :**

**create and annihilate the field :  $a^\dagger a \rightarrow \varphi^\dagger \varphi, \bar{\psi} \psi, \partial_\mu \varphi^\dagger \partial_\mu \varphi, \dots$**

**Interaction lagrangian : electron radiates a photon**

**Annihilate electron  
Create electron and photon** }  **$b^\dagger b a \rightarrow \bar{\psi} \gamma^\mu \psi A_\mu$**

**In general, interaction lagrangian :**

**Products of fields : creation/annihilation operators**

**Coupling constant : interaction intensity**

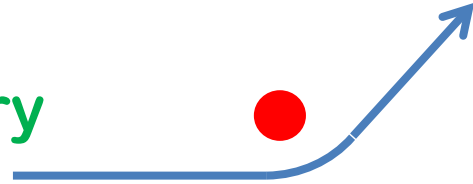
$$\mathcal{L} = g\Pi$$

# Gauge invariance

Free propagation : **straight line**



Interaction : **locally curved trajectory**



Absorb the deformation into derivation : make the line straight again

Modification of the metric : **general relativity**

Modification via internal space : **gauge theory**

$$D_{\mu} = \partial_{\mu} - igA_{\mu}$$

**necessitates a massless vector field**

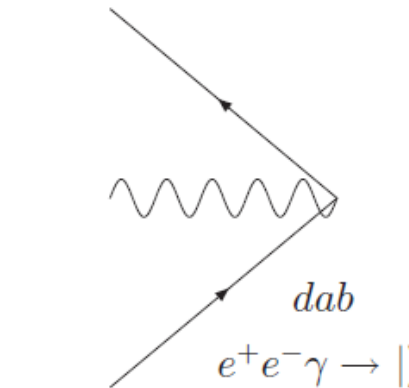
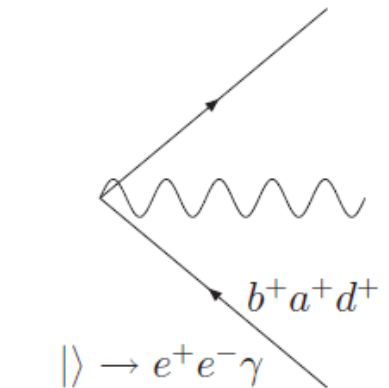
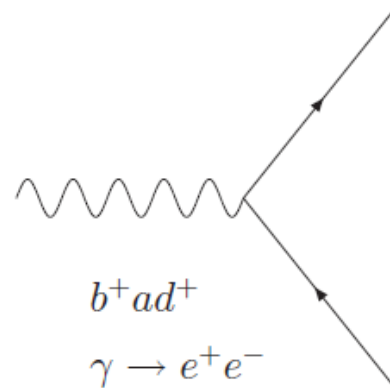
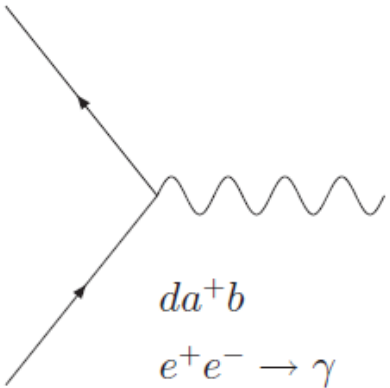
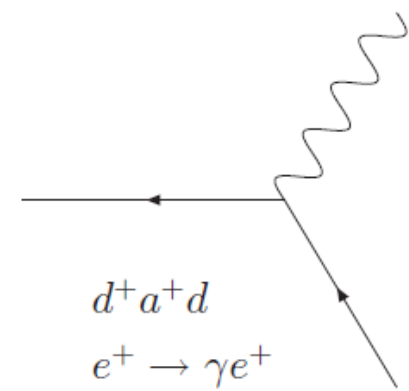
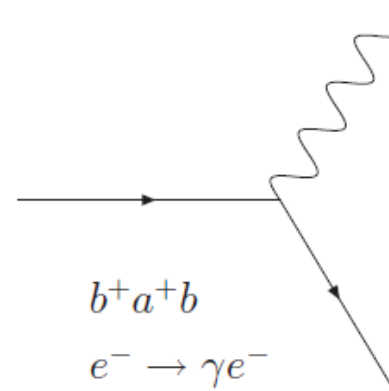
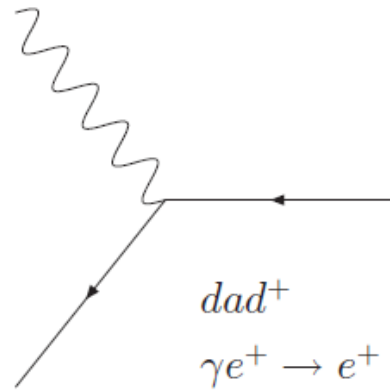
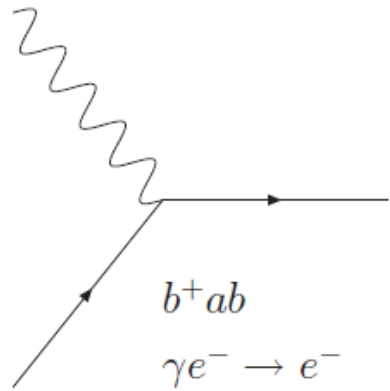
# Elementary vertices

$$\Psi \sim \mathbf{b} + \mathbf{d}^+$$

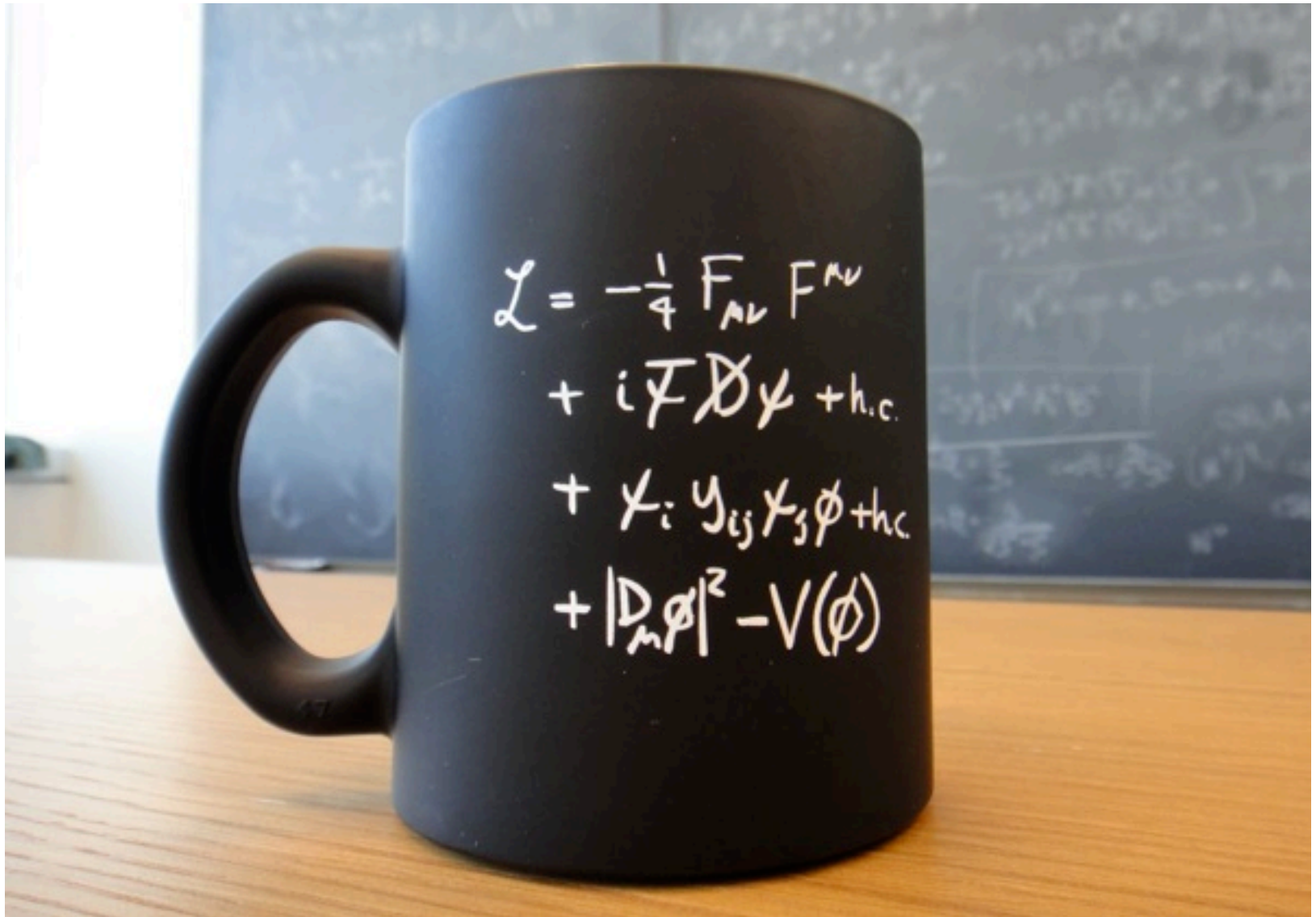
$$\bar{\Psi} \sim \mathbf{b}^+ + \mathbf{d}$$

$$\mathbf{A} \sim \mathbf{a} + \mathbf{a}^+$$

$$\mathcal{L} = g \bar{\psi} \gamma^\mu \psi A_\mu$$



# SM Lagrangian (compact)



# SM Lagrangian (expanded)

$$\begin{aligned}
\mathcal{L}_{SM} = & \sum_{\ell=e,\mu,\tau} i\bar{\psi}_\ell \gamma^\mu \partial_\mu \psi_\ell + \sum_{\ell'=\nu_e,\nu_\mu,\nu_\tau} i\bar{\psi}_{\ell'} \gamma^\mu \partial_\mu \psi_{\ell'} + \sum_i^3 \sum_{a=u,c,t} i\bar{\psi}_{q_i} \gamma^\mu \partial_\mu \psi_{q_i} + \sum_i^3 \sum_{a'=d,s,b} i\bar{\psi}_{q'_i} \gamma^\mu \partial_\mu \psi_{q'_i} - \frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}) - \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) \\
& - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{1}{4} \sum_{a=1}^8 (\partial_\mu G_\nu^a - \partial_\nu G_\mu^a)(\partial^\mu G^{a\nu} - \partial^\nu G^{a\mu}) + \frac{1}{2} \partial_\mu h \partial^\mu h - \sum_{\ell=e,\mu,\tau} \frac{\lambda_\ell v}{\sqrt{2}} \bar{\psi}_\ell \psi_\ell - \sum_i^3 \sum_{q=u,c,t} \frac{\lambda_q v}{\sqrt{2}} \bar{\psi}_{q_i} \psi_{q_i} - \sum_i^3 \sum_{q'=d,s,b} \frac{\lambda_{q'} v}{\sqrt{2}} \bar{\psi}_{q'_i} \psi_{q'_i} \\
& - \left(\frac{gv}{2}\right)^2 W_\mu^+ W^{-\mu} - \frac{1}{2} \left(\frac{gv}{2 \cos \theta_W}\right)^2 Z_\mu Z^\mu - \frac{1}{2} (-2m^2)^2 h^2 + \frac{g}{4 \cos \theta_W} \left( \sum_{\ell=e,\mu,\tau} \bar{\psi}_\ell \gamma^\mu (4 \sin^2 \theta_W - 1 + \gamma^5) \psi_\ell Z_\mu + \sum_{\ell'=\nu_e,\nu_\mu,\nu_\tau} \bar{\psi}_{\ell'} \gamma^\mu (1 - \gamma^5) \psi_{\ell'} Z_\mu \right) \\
& + \frac{g}{4 \cos \theta_W} \left( \sum_i^3 \sum_{q=u,c,t} \bar{\psi}_{q_i} \gamma^\mu (1 - \frac{8}{3} \sin^2 \theta_W - \gamma^5) \psi_{q_i} Z_\mu + \sum_i^3 \sum_{q'=d,s,b} \bar{\psi}_{q'_i} \gamma^\mu (\frac{4}{3} \sin^2 \theta_W - 1 + \gamma^5) \psi_{q'_i} Z_\mu \right) + \frac{g}{2\sqrt{2}} \left( \sum_{\ell=e,\mu,\tau} \bar{\psi}_{\nu_\ell} \gamma^\mu (1 - \gamma^5) \psi_\ell W_\mu^+ + \sum_{\ell=e,\mu,\tau} \bar{\psi}_\ell \gamma^\mu (1 - \gamma^5) \psi_{\nu_\ell} W_\mu^- \right) \\
& + \frac{g}{2\sqrt{2}} \left( \sum_i^3 \sum_{\substack{q=u,c,t \\ q'=d,s,b}} V_{qq'} \bar{\psi}_q \gamma^\mu (1 - \gamma^5) \psi_{q'} W_\mu^+ + \sum_i^3 \sum_{\substack{q=u,c,t \\ q'=d,s,b}} V_{qq'}^* \bar{\psi}_{q'} \gamma^\mu (1 - \gamma^5) \psi_q W_\mu^- \right) + g_{em} \left( - \sum_{\ell=e,\mu,\tau} \bar{\psi}_\ell \gamma^\mu \psi_\ell A_\mu + \frac{2}{3} \sum_{q=u,c,t} \bar{\psi}_{q_i} \gamma^\mu \psi_{q_i} A_\mu - \frac{1}{3} \sum_{q'=d,s,b} \bar{\psi}_{q'_i} \gamma^\mu \psi_{q'_i} A_\mu \right) \\
& + g_s \left( \sum_{i,j}^3 \sum_a^8 \sum_{q=u,c,t} \bar{\psi}_{q_j} \gamma^\mu \psi_{q_i} G_\mu^a T_{ij}^a + \sum_{i,j}^3 \sum_a^8 \sum_{q'=d,s,b} \bar{\psi}_{q'_j} \gamma^\mu \psi_{q'_i} G_\mu^a T_{ij}^a \right) - \sum_{\ell=e,\mu,\tau} \frac{\lambda_\ell}{\sqrt{2}} \bar{\psi}_\ell \psi_\ell h - \sum_i^3 \sum_{q=u,c,t} \frac{\lambda_q}{\sqrt{2}} \bar{\psi}_{q_i} \psi_{q_i} h - \sum_i^3 \sum_{q'=d,s,b} \frac{\lambda_{q'}}{\sqrt{2}} \bar{\psi}_{q'_i} \psi_{q'_i} h \\
& + \frac{g}{2\sqrt{2}} \left( \sum_i^3 \sum_{\substack{q=u,c,t \\ q'=d,s,b}} V_{qq'} \bar{\psi}_q \gamma^\mu (1 - \gamma^5) \psi_{q'} W_\mu^+ + \sum_i^3 \sum_{\substack{q=u,c,t \\ q'=d,s,b}} V_{qq'}^* \bar{\psi}_{q'} \gamma^\mu (1 - \gamma^5) \psi_q W_\mu^- \right) + g_{em} \left( - \sum_{\ell=e,\mu,\tau} \bar{\psi}_\ell \gamma^\mu \psi_\ell A_\mu + \frac{2}{3} \sum_{q=u,c,t} \bar{\psi}_{q_i} \gamma^\mu \psi_{q_i} A_\mu - \frac{1}{3} \sum_{q'=d,s,b} \bar{\psi}_{q'_i} \gamma^\mu \psi_{q'_i} A_\mu \right) \\
& + g_s \left( \sum_{i,j}^3 \sum_a^8 \sum_{q=u,c,t} \bar{\psi}_{q_j} \gamma^\mu \psi_{q_i} G_\mu^a T_{ij}^a + \sum_{i,j}^3 \sum_a^8 \sum_{q'=d,s,b} \bar{\psi}_{q'_j} \gamma^\mu \psi_{q'_i} G_\mu^a T_{ij}^a \right) - \sum_{\ell=e,\mu,\tau} \frac{\lambda_\ell}{\sqrt{2}} \bar{\psi}_\ell \psi_\ell h - \sum_i^3 \sum_{q=u,c,t} \frac{\lambda_q}{\sqrt{2}} \bar{\psi}_{q_i} \psi_{q_i} h - \sum_i^3 \sum_{q'=d,s,b} \frac{\lambda_{q'}}{\sqrt{2}} \bar{\psi}_{q'_i} \psi_{q'_i} h \\
& + ig_{em} [\partial_\mu A_\nu W^{-\mu} W^{+\nu} + \partial_\mu W_\nu^+ W^{-\nu} A^\mu + \partial_\mu W_\nu^- W^{+\mu} A^\nu - \partial_\mu A_\nu W^{-\nu} W^{+\mu} - \partial_\mu W_\nu^+ W^{-\mu} A^\nu - \partial_\mu W_\nu^- W^{+\mu} A^\nu] \\
& + ig \cos \theta_W [\partial_\mu Z_\nu W^{-\mu} W^{+\nu} + \partial_\mu W_\nu^+ W^{-\nu} Z^\mu + \partial_\mu W_\nu^- W^{+\mu} Z^\nu - \partial_\mu Z_\nu W^{-\nu} W^{+\mu} - \partial_\mu W_\nu^+ W^{-\mu} Z^\nu - \partial_\mu W_\nu^- W^{+\mu} Z^\nu] + \frac{g^2 v}{2} W_\mu^+ W^{-\mu} h + \frac{g^2 v}{4 \cos^2 \theta_W} Z_\mu Z^\mu h - \lambda v h^3 \\
& + g_{em}^2 [W_\nu^+ W^{-\mu} A_\nu A^\mu - W_\mu^+ W^{-\mu} A_\nu A^\nu] + g^2 \cos^2 \theta_W [W_\nu^+ W^{-\mu} Z_\nu Z^\mu - W_\mu^+ W^{-\mu} Z_\nu Z^\nu] + g^2 \cos \theta_W \sin \theta_W [2W_\mu^+ W^{-\mu} Z_\nu A^\nu - W_\mu^+ W^{-\nu} A_\nu Z^\mu - W_\mu^+ W^{-\nu} A^\mu Z_\nu] \\
& + \frac{g^2}{2} [W_\mu^- W^{-\mu} W_\nu^+ W^{+\nu} - W_\mu^- W^{+\mu} W_\nu^- W^{-\nu}] + \frac{g^2}{4} W_\mu^+ W^{-\mu} h^2 + \frac{g^2}{8 \cos^2 \theta_W} Z_\mu Z^\mu h^2 - \frac{\lambda}{4} h^4 - \frac{g_s}{2} \sum_{a,b,c}^8 f^{abc} (\partial_\mu G^{a\nu} - \partial_\nu G_\mu^a) G^{\mu b} G^{\nu c} - \frac{g_s^2}{4} \sum_{a,b,c,d,e,f}^8 f^{abc} f^{ade} G_\mu^b G_\nu^c G^{\mu d} G^{\nu e}
\end{aligned}$$

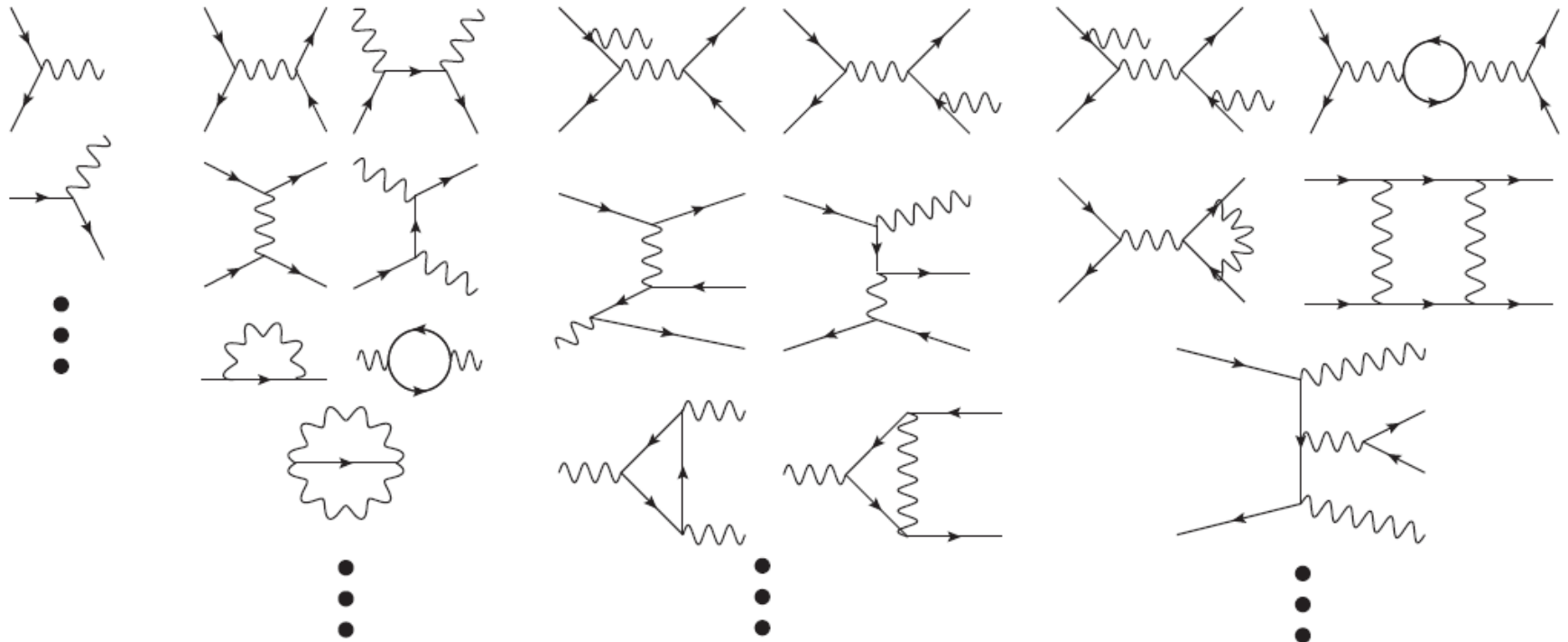
$$g_{em} = g \sin \theta_W, \quad v^2 = \frac{-m^2}{\lambda} \quad (m^2 < 0, \lambda > 0), \quad m_\ell = \frac{\lambda_\ell v}{\sqrt{2}}, \quad m_q = \frac{3\lambda_q v}{\sqrt{2}}, \quad m_W = \frac{gv}{2}, \quad m_Z = \frac{gv}{2 \cos \theta_W}, \quad m_h = \sqrt{-2m^2}$$



# Perturbative development

$$S = e^{i \int \mathcal{L}_{int} d^4x}$$

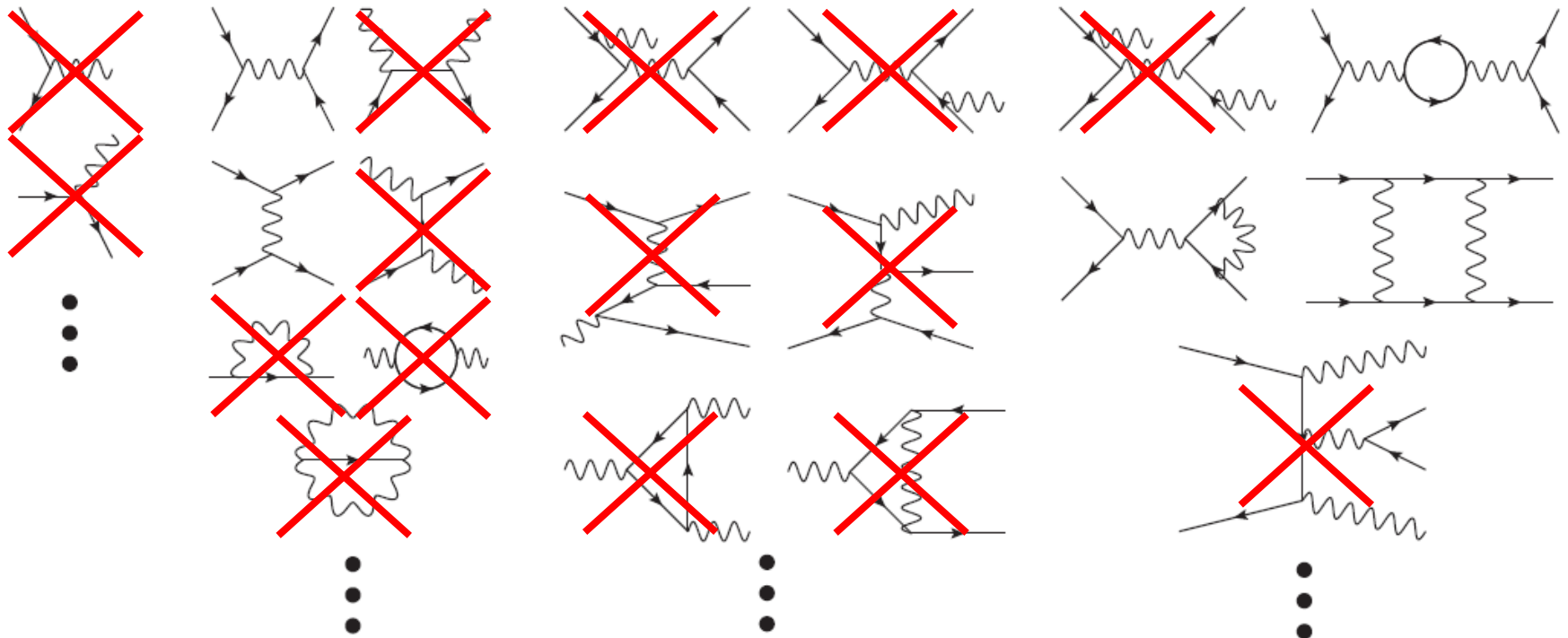
$$i\sqrt{\alpha} \int \mathcal{L} dx + \frac{(i\sqrt{\alpha})^2}{2} \iint \mathcal{L}\mathcal{L}' dx dx' + \frac{(i\sqrt{\alpha})^3}{3!} \iiint \mathcal{L}\mathcal{L}'\mathcal{L}'' dx dx' dx'' + \frac{(i\sqrt{\alpha})^4}{4!} \iiiii \mathcal{L}\mathcal{L}'\mathcal{L}''\mathcal{L}''' dx dx' dx'' dx''' + \dots$$



# Perturbative development


$$\mathbf{f\bar{f}} \rightarrow \mathbf{f\bar{f}}$$


$$i\sqrt{\alpha} \int \mathcal{L} dx + \frac{(i\sqrt{\alpha})^2}{2} \iint \mathcal{L}\mathcal{L}' dx dx' + \frac{(i\sqrt{\alpha})^3}{3!} \iiint \mathcal{L}\mathcal{L}'\mathcal{L}'' dx dx' dx'' + \frac{(i\sqrt{\alpha})^4}{4!} \iiiii \mathcal{L}\mathcal{L}'\mathcal{L}''\mathcal{L}''' dx dx' dx'' dx''' + \dots$$





$S_{fi} = \langle f | S | i \rangle$  : select diagrams that connect  $|i\rangle$  and  $|f\rangle$  .

# Feynman diagrams

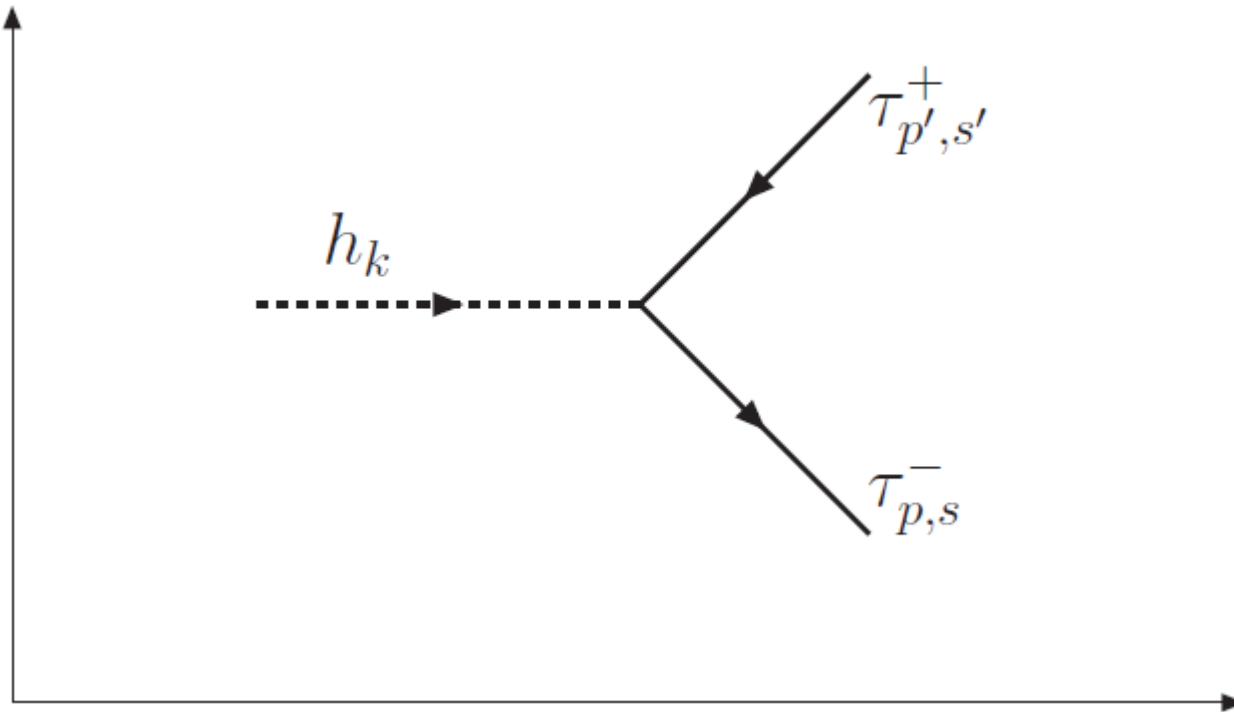
  
scalar boson

  
fermion (spinor)

  
photon, W, Z

  
gluon

gauge bosons (vectors)



# Feynman diagrams

## Transition probability

$$S_{fi} = \text{Phase space} \times \text{Feynman amplitude}$$



## Space-time diagrams $\Leftrightarrow$ Feynman amplitude

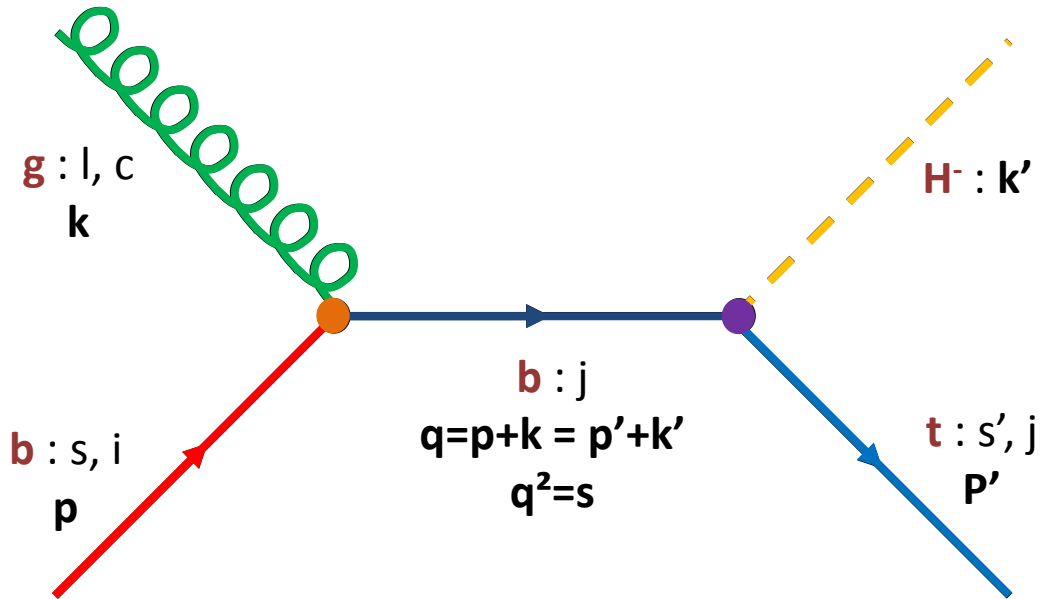
$$2 \rightarrow 2 \text{ process : } \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{p_f}{p_i} |\mathcal{M}_{fi}|^2$$

Each **vertex** is proportionnal to the **coupling x charge**

Each **propagator** (internal line) is proportionnal to  **$1/(m^2-p^2)$**

# Feynman rules

Each line and each vertex correspond to a multiplicative factor



$$-i\mathcal{M}_{fi} = \bar{u}_{s'}(\mathbf{p}') \times \mathbf{1} \times \frac{ig}{2\sqrt{2}m_W} [(A+B) + (A-B)\gamma^5] \times i\frac{\not{q}}{s} \times ig_s T_{ji}^c \gamma^\mu \times \epsilon_l^\mu(k) \times u_s(p)$$

$$-i\mathcal{M}_{fi} = \frac{-gg_s}{2\sqrt{2}m_W s} T_{ji}^c \epsilon_l^\mu(k) \bar{u}_{s'}(\mathbf{p}') [(A+B) + (A-B)\gamma^5] \not{q} \gamma^\mu u_s(p)$$

# LO Feynman calculation

## Charged Higgs Production

$$g_c(k) + b_i(p) \rightarrow H^+(k') + t(p')$$

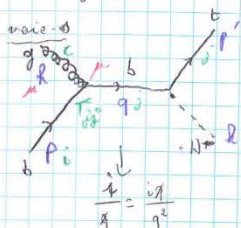
cinématique

$m_b$  néglige (dans la cinématique)

$$\begin{aligned} s &= (k+p)^2 = 2kp \\ &= (k+p')^2 = m_t^2 + m_H^2 + 2kp' \\ t &= (k-p')^2 = m_t^2 - 2kp' \\ &= (p-k')^2 = m_H^2 - 2kp' \\ u &= (p-p')^2 = m_t^2 - 2pp' \\ &= (k-k')^2 = m_H^2 - 2kk' \end{aligned}$$

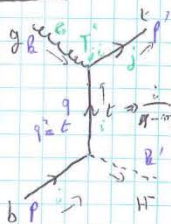
$$V_{g_i H} = \frac{g}{2\sqrt{2}m_W} \frac{(A(1+y^2) + B(1-y^2))}{(A+B)(A-B)y^2}$$

diagrammes



$$\begin{aligned} -iM_{p_i} &= \bar{u}(p') \cdot 1 \times \frac{ig}{2\sqrt{2}m_W} (A+B)(y^2) \cdot \frac{ig}{q^2-s} \cdot \xi_\mu(k) \cdot ig_s T_{3c}^c \gamma^\mu u(p) \\ &= \frac{-ig g_s}{2\sqrt{2}m_W} \cdot \frac{1}{s} \cdot \xi_\mu(k) \bar{u}(p') T_{3c}^c \gamma^\mu [A+B(A-B)y^2] u(p) \\ q &= p+k = p'+k' \end{aligned}$$

voie-t

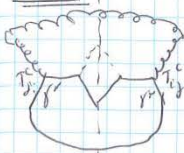


$$\begin{aligned} -iM_{t_i} &= \bar{u}(p') \xi_\mu(k) \cdot ig_s T_{3c}^c \gamma^\mu \frac{1}{q^2-m_t^2} [A+B(A-B)y^2] \cdot \frac{ig}{2\sqrt{2}m_W} \cdot 1 \times u(p) \\ &= \frac{-ig g_s}{2\sqrt{2}m_W} \cdot \frac{1}{t-m_t^2} \cdot \xi_\mu(k) \bar{u}(p') T_{3c}^c \gamma^\mu [A+B(A-B)y^2] u(p) \end{aligned}$$

$$q = p+k \quad p' = k+q \Rightarrow q = p'-k = p-k'$$

$$M_{p_i}^2 = M_{t_i}^2 + M_{s_i}^2 + 2 \text{Re}(M_{p_i} M_{t_i}^*)$$

Voie s



splitting of gluon into top and anti-top

$$|M_{p_i}|^2 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{8} \sum_{S, S', C, C'} \xi_\mu(k) \xi_\nu(k) T_{3c}^c T_{3c}^{c'} \bar{u}(p') \gamma^\mu [A+B(A-B)y^2] u(p) \times \dots$$

$$|M_{p_i}|^2 = \frac{1}{24} \frac{g^2 g_s^2}{8m_W^2} \frac{1}{s^2} g_{\mu\nu} \text{Tr} [(\not{p} + m_t) \gamma^\mu \not{q} [A+B(A-B)y^2] \not{q} \gamma^\nu]$$

$A^2 + 2AB + B^2 = (A+B)^2$   
 $-2(A-B)(A-B)y^2 = 5 + 4y^2$   
 $s(A^2 + B^2) = 2$   
 $p = 2(A-B)(A-B)y^2$

4 Traces

$$g_{\mu\nu} p'_\alpha q_\beta p_\gamma q_\delta \text{Tr} [\gamma^\mu \not{p}' \gamma^\alpha \not{q} \gamma^\nu \not{p} \gamma^\beta \not{q} \gamma^\delta \not{p}] = -8 p'_\alpha p_\beta p_\gamma q_\delta [g^{\mu\alpha} g^{\nu\beta} - g^{\mu\beta} g^{\nu\alpha} + g^{\mu\gamma} g^{\nu\delta} - g^{\mu\delta} g^{\nu\gamma}]$$

$$= \text{Tr} [\gamma^\mu \not{p}' \gamma^\alpha \not{q} \gamma^\nu \not{p} \gamma^\beta \not{q} \gamma^\delta \not{p}] = -16(A^2 + B^2) [2(p' \cdot q)(p \cdot q) - q^2(p \cdot p')]$$

$$g_{\mu\nu} p'_\alpha p_\beta p_\gamma q_\delta \text{Tr} [\gamma^\mu \not{p}' \gamma^\alpha \not{q} \gamma^\nu \not{p} \gamma^\beta \not{q} \gamma^\delta \not{p}] = 2 p'_\alpha p_\beta p_\gamma q_\delta \text{Tr} [\gamma^\mu \not{p}' \gamma^\alpha \not{q} \gamma^\nu \not{p} \gamma^\beta \not{q} \gamma^\delta \not{p}] = 0$$

$$g_{\mu\nu} q_\alpha p_\beta q_\gamma \text{Tr} [\gamma^\mu \not{q} \gamma^\alpha \not{p} \gamma^\nu \not{q} \gamma^\beta \not{p} \gamma^\gamma \not{q}] = 0 ; g_{\mu\nu} q_\alpha p_\beta q_\gamma \text{Tr} [\gamma^\mu \not{q} \gamma^\alpha \not{p} \gamma^\nu \not{q} \gamma^\beta \not{p} \gamma^\gamma \not{q}] = 0$$

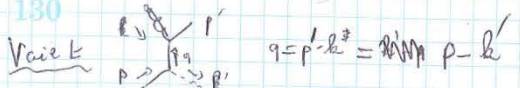
Cinématique

$$\begin{aligned} p'q &= p'(p+k) = p'p + p'k = \frac{1}{2}(m_t^2 - u + m_t^2 - t) \\ p'p &= \frac{1}{2}(m_t^2 - u) \\ q^2 &= s \\ pq &= p^2 + pk = \frac{1}{2}s \end{aligned}$$

$$\left. \begin{aligned} 2(p'q)(pq) - q^2(p'p) &= \frac{s}{2} [m_t^2 - u + m_t^2 - t] - \frac{s}{2} [m_t^2 - u] \\ &= \frac{s}{2} (m_t^2 - t) \end{aligned} \right\}$$

$$|M_{p_i}|^2 = \frac{1}{3} \frac{g^2 g_s^2}{8m_W^2} (A^2 + B^2) \frac{m_t^2 - t}{s}$$

# LO Feynman calculation (2)



Trace  $M_{fi}^2 = -\frac{1}{24} \frac{g^2 g_s^2}{8m_W^2 (t-m_f^2)^2} g_{\mu\nu} \text{Tr}[(\not{p} + m_f) \gamma^\mu (\not{p} + m_f) \not{S} (\not{p} + m_f) \gamma^\nu (\not{p} + m_f)]$

- Trace en  $m_f$  et  $m_f^3$  : nombre impair de  $\gamma^\mu$  avec  $\not{p}$  ou  $\not{S}$   $\rightarrow 0$ .
- Trace en  $\gamma^5$  : 4  $\gamma$  et  $\gamma^5$  : orthogonaux et  $q_\mu q_\nu$  symétrique  $\rightarrow 0$ .
- $2\gamma$  et  $\gamma^T = 0$ .

4 Traces non nulles:

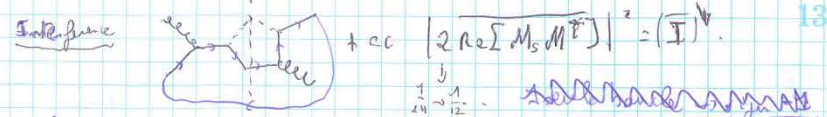
$g_{\mu\nu} \not{p} \not{q} \not{p} \not{q} S \text{Tr}[\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^\lambda \gamma^\lambda \gamma^\rho \gamma^\sigma] = -2 \not{p} \not{q} \not{p} \not{q} S \text{Tr}[\gamma^\rho \gamma^\sigma \gamma^\rho \gamma^\sigma] S$   
 $= -16 (2(p \cdot q)(p' \cdot q) - q^2(p \cdot p')) (A^2 + B^2)$

$g_{\mu\nu} m_f^2 S \not{p} \not{p} \not{p} \text{Tr}[\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\rho] = -8 \not{p} \not{p} m_f^2 S$   
 $= -16 (A^2 + B^2) \not{p} \not{p} m_f^2$

$g_{\mu\nu} m_f^2 S q_\alpha p_\sigma \text{Tr}[\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^\alpha \gamma^\beta \gamma^\rho \gamma^\sigma] = 32 S m_f^2 (p \cdot q) = 64 (A^2 + B^2) m_f^2 p \cdot q$   
 $g_{\mu\nu} m_f^2 S q_\alpha p_\sigma \text{Tr}[\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^\alpha \gamma^\beta \gamma^\rho \gamma^\sigma] = 32 S m_f^2 (p \cdot q) = 64 (A^2 + B^2) m_f^2 p \cdot q$

$-16(A^2+B^2) [2(p \cdot q)(p' \cdot q) - q^2(p \cdot p') + \not{p} \not{p} m_f^2 - 4p \cdot q m_f^2]$   
 $p \cdot p' = \frac{1}{2}(m_f^2 - m_f^2) = \frac{1}{2}(s + t - m_H^2)$   
 $q^2 = t$   
 $p \cdot q = p^0 q^0 - \vec{p} \cdot \vec{q} = \frac{1}{2}(t - m_H^2)$   
 $p' \cdot q = p'^0 q^0 - \vec{p}' \cdot \vec{q} = m_f^2 + \frac{1}{2}(t - m_f^2) = \frac{t + m_f^2}{2}$   
 $\left[ \frac{1}{2} \left( (t - m_H^2)(t + m_f^2) - t(s + t - m_H^2) + m_f^2(s + t - m_H^2) - 4m_f^2 t \right) \right]$   
 $= \frac{1}{2} [s(m_f^2 - t) + 2m_f^2(m_H^2 - t)]$

$(M_{fi}^2)^2 = \frac{1}{3} \frac{g_s^2 g^2}{8m_W^2} (A^2 + B^2) \frac{s(m_f^2 - t) + 2m_f^2(m_H^2 - t)}{(t - m_f^2)^2}$



Trace  $M_{fi}^2 = -\frac{1}{12} \frac{g^2 g_s^2}{8m_W^2 s(t-m_f^2)} g_{\mu\nu} \text{Tr}[(\not{p} + m_f) \gamma^\mu (\not{p} + m_f) \not{S} (\not{p} + m_f) \gamma^\nu (\not{p} + m_f)]$

- Trace en  $m_f$   $\rightarrow 0$  (nombre impair de  $\gamma$ )

$\rightarrow g_{\mu\nu} S \text{Tr}[\not{p} \not{p} \not{q} \not{p} \not{q} \not{S}] = g_{\mu\nu} \not{p} \not{p} \not{q} \not{p} \not{q} \not{S} \text{Tr}[\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^\lambda \gamma^\lambda \gamma^\rho \gamma^\sigma] S$   
 $= \not{p} \not{p} \not{q} \not{p} \not{q} \not{S} \text{Tr}[\gamma^\rho \gamma^\sigma \gamma^\rho \gamma^\sigma] S$   
 $= 4(p \cdot q) \not{p} \not{p} \not{q} \not{S} \text{Tr}[\gamma^\rho \gamma^\sigma \gamma^\rho \gamma^\sigma] S$   
 $= 16(p \cdot q) \not{p} \not{p} \not{q} \not{S} S$   
 $= 32(A^2 + B^2)(p \cdot q)(p' \cdot q_s)$

etc.  $\tilde{m}$  mes  $\gamma^5 \propto \text{Tr}[\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^\lambda \gamma^\lambda \gamma^\rho \gamma^\sigma \gamma^5] = \text{Tr}[\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^5] = 0$

$\rightarrow g_{\mu\nu} S m_f^2 \text{Tr}[\not{q} \not{S} \not{p} \not{q} \not{S}] = q_\alpha p_\beta S m_f^2 \text{Tr}[\gamma^\alpha \gamma^\beta \gamma^\mu \gamma^\nu \gamma^\alpha \gamma^\beta] S$   
 $= -8 S m_f^2 (q \cdot p) = -16(A^2 + B^2) q_\alpha p_\beta m_f^2$

$-16(A^2 + B^2) [(q \cdot p) \not{p} \not{p} - (p \cdot q)(p' \cdot q_s)]$

$q \cdot p = \frac{s}{2}$   
 $p \cdot q = \frac{1}{2}(t - m_H^2)$   
 $p' \cdot q_s = \frac{1}{2}(m_f^2 - t - m_f^2) = \frac{1}{2}(m_f^2 - m_H^2 + s)$   
 $\left. \begin{matrix} q \cdot p = \frac{s}{2} \\ p \cdot q = \frac{1}{2}(t - m_H^2) \\ p' \cdot q_s = \frac{1}{2}(m_f^2 - t - m_f^2) \end{matrix} \right\} \frac{1}{2} [s m_f^2 + (t - m_H^2)(m_H^2 - m_f^2 - s)]$

$(I) = \frac{2}{3} \frac{g_s^2 g^2}{8m_W^2 s(t-m_f^2)} (A^2 + B^2) (s m_f^2 + (t - m_H^2)(m_H^2 - m_f^2 - s))$

# LO Feynman calculation (end)

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$$|\overline{\mathcal{M}}_{fi}| = \frac{1}{3} g_s^2 \frac{G_F}{\sqrt{2}} (A^2 + B^2) \left[ \frac{m_t^2 - t}{s} + \frac{s(m_t^2 - t) + 2m_t^2(m_H^2 - t)}{(t - m_t^2)^2} + 2 \frac{sm_t^2 + (t - m_H^2)(m_H^2 - m_t^2 - s)}{s(t - m_t^2)} \right]$$

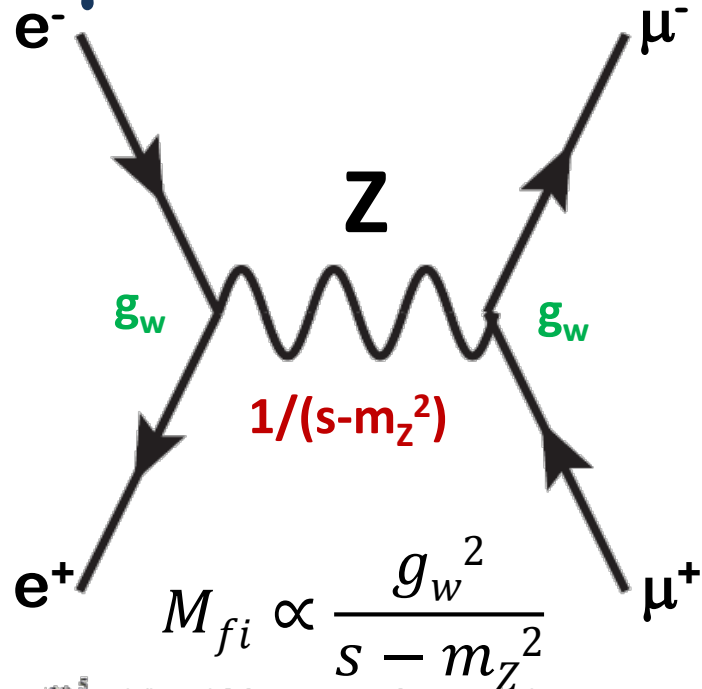
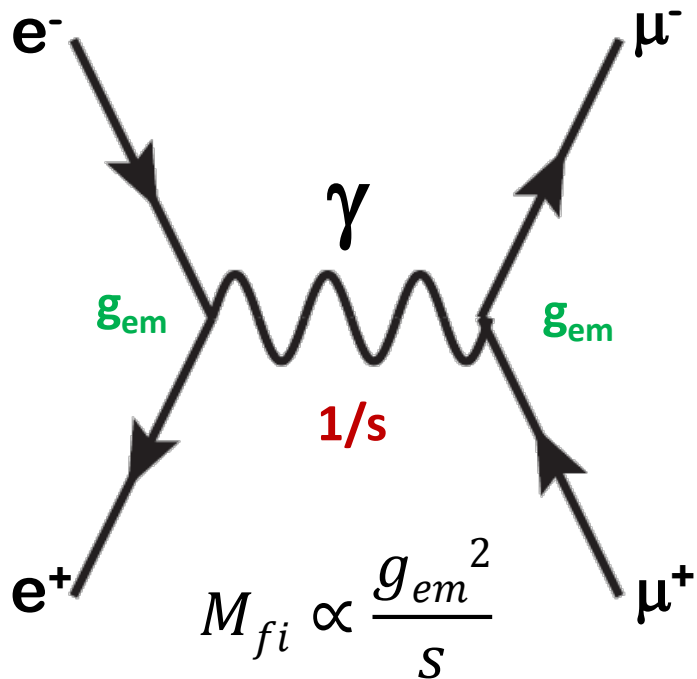
$$d\sigma = \frac{1}{F} |\overline{\mathcal{M}}_{fi}|^2 dPS^{(2)} = \frac{1}{16\pi} \frac{1}{\lambda(s, 0, 0)} |\overline{\mathcal{M}}_{fi}|^2 dt$$

$$\frac{d\sigma}{dt} = \frac{1}{12} \alpha_s \frac{G_F}{\sqrt{2}} (A^2 + B^2) \frac{M}{s}$$

Section efficace Partonique  
 $s \rightarrow \hat{s}, t \rightarrow \hat{t}$



$$e^+e^- \rightarrow \mu^+\mu^-$$

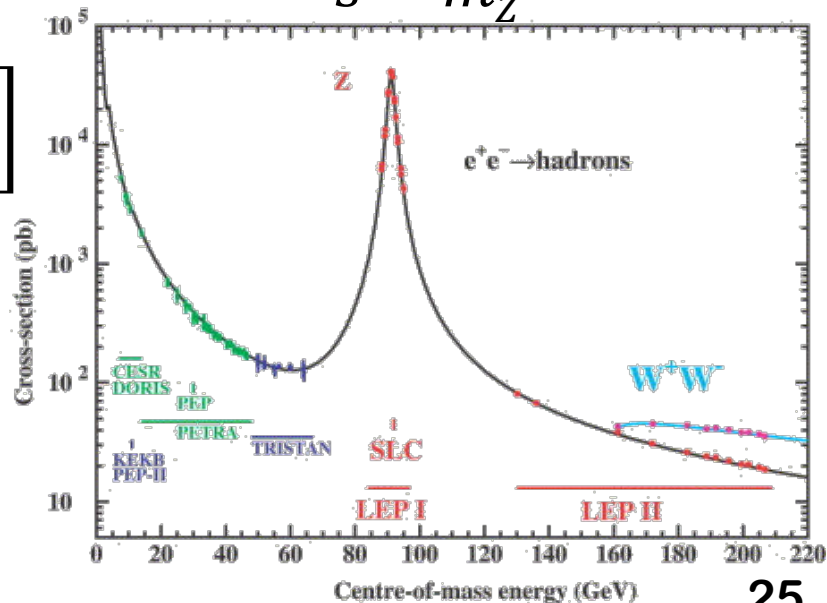


$$\sigma \propto s \left[ \frac{g_{em}^4}{s^2} + \frac{2g_{em}^2 g_w^2}{s(s-m_Z^2)} + \frac{g_w^4}{(s-m_Z^2)^2} \right]$$

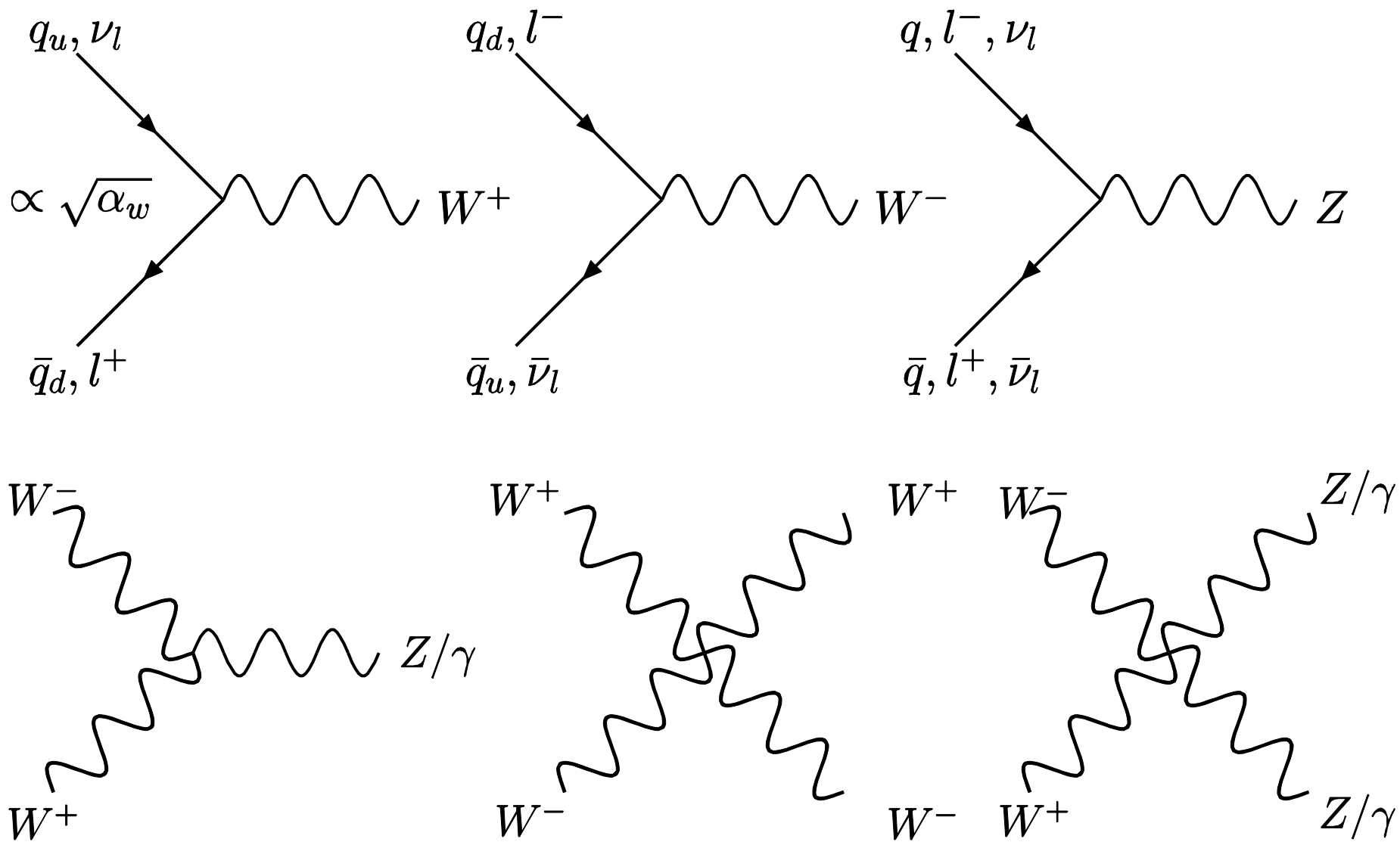
$$s \ll m_Z \quad \sigma \propto g_{em}^4/s \propto \alpha_{em}^2/s$$

$$s = m_Z \quad \text{resonant effect at Z peak}$$

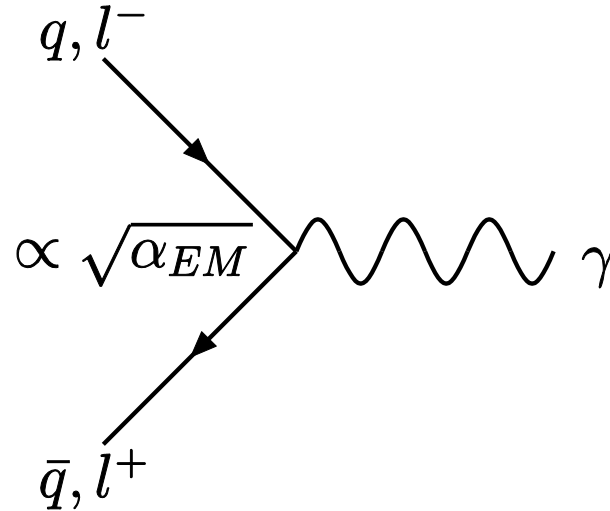
$$s \gg m_Z \quad \sigma \propto 4g_{em}^4/s \quad (\text{assume } g_{em} \sim g_w)$$



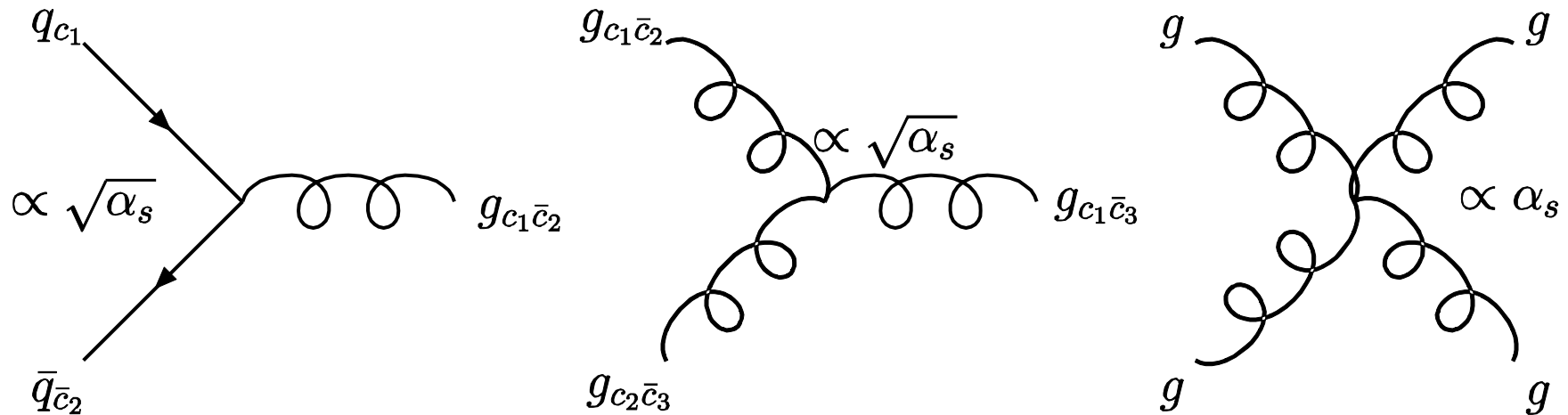
# SM vertices : Weak interaction



# SM vertices : EM interaction



# strong interaction



# SM vertices : Higgs sector

