Introduction to Hadron Collider Physics

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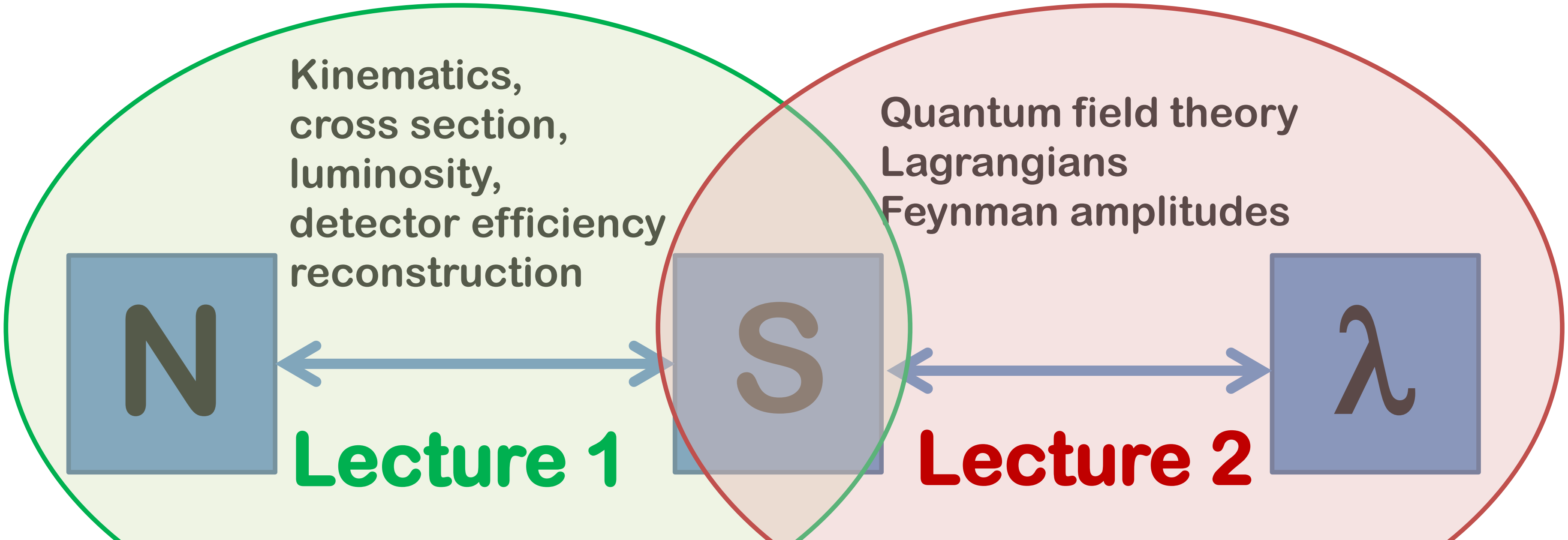
PART II

Theoretical background in HEP

General overview



Number of observed events

Number of reconstructed top quark pairs

Experimental observable

Scattering matrix

|<i|S|f>|² = probability of changing an initial state |i> to a final state |j>

Eg : |i> = pp
|f>=top pairs (+X)

Model parameters

Couplings, masses, mixing angles…

Number and nature of particles in SM

19 parameters,
12 fermions,
3 gauge interaction +SSB

Much more in SUSY…

Evolution operator

$U(t_f,t_i)$: evolution operator in interaction representation

$$|\psi(t_f)\rangle_I = U(t_f,t_i)\,|\psi(t_i)\rangle_I$$

Schrödinger equation

$$i\frac{d}{dt}U(t_f,t_i) = \mathcal{V}_I(t)U(t,t_i)$$

Where *$V_I(t)$* is the Interaction Hamiltonian

Solution :

$$U(t_f,t_i) = e^{-i\int_{t_i}^{t_f}\mathcal{V}_I(t)dt}$$

In truth, slightly more tricky because on integration bounds : needs time ordering to ensure causality…

Scattering amplitude

transition probability between

initial state |i>, at time t = -∞

and final state |f>, at time t = +∞

$$P_{i\to f} = |\langle f \mid i(+\infty)\rangle|^2 = |\langle f \mid U(-\infty, +\infty) \mid i\rangle|^2 = |S_{fi}|^2$$

S is the scattering matrix

Hamiltonian = E (kinetic) + V (potential=interaction)
Lagrangian = E – V
So interaction Hamiltonian = - interaction Lagrangian

Finally :

Important bit to know…

$$S = U(-\infty, +\infty) = e^{-i\int_{-\infty}^{+\infty} V_I(t)\,dt} = e^{i\int \boxed{\mathcal{L}_{int}}\, d^4x}$$

Theoretical background

$n \to p + e + \nu$ $m_n = 938{,}3$ MeV $m_p = 939{,}6$ MeV $\Delta m = 1{,}3$ MeV

Small objects, smaller than nucleus → Quantum mechanics

Kinetic energy ~ mass (for electron)
Massless neutrino
Electromagnetic fields

→ Special relativity
→ Field theory

Number and nature of particles vary → Second quantization

Putting everything together : new formalism

(Relativistic) Quantum Field Theory

QFT in a nutshell

Classical mechanics

Euler-Lagrange equations

Time evolution of discrete coordinate **x(t)**

Poisson brackets $\{x_i, x_j\} = \{p_i, p_j\} = 0, \quad \{x_i, p_j\} = \delta_{ij}$

Quantum physics

Schrödinger equation

Time evolution of wavefunction **ψ(x,t)**

$|\psi(x)|^2$: presence probability

Observables become operators

Canonical commutation relations

$$[\hat{x}_i, \hat{x}_j] = [\hat{p}_i, \hat{p}_j] = 0, \quad [\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}$$

Special relativity+Field theory

(Maxwell EM)

Space-time evolution of a field (continuous coordinates) **φ(x,t)**

$$\{\varphi(x,t), \varphi(x',t)\} = \{\pi(x,t), \pi(x',t)\} = 0$$

$$\{\varphi(x,t), \pi(x',t)\} = \delta(x - x')$$

Quantum field theory

Field become operators **$\hat{\varphi}$(x,t)**

Observables becomes functionals

Canonical commutation

$$[\hat{\varphi}(x,t), \hat{\varphi}(x',t)] = [\hat{\pi}(x,t), \hat{\pi}(x',t)] = 0$$

$$[\hat{\varphi}(x,t), \hat{\pi}(x',t)] = i\delta(x - x')$$

Relativistic equations

Require invariance une Lorentz Poincarré Symmetry

Boosts, Translation in space and time, Rotations

Link between spin and mathematical structure used to reprensent it

Spin 0 : scalar (number)

Klein-Gordon equation :

$$E^2 = p^2c^2 + m^2c^4 \xleftrightarrow[E \to i\hbar\frac{\partial}{\partial t},\ \vec{p} \to i\hbar\vec{\nabla}]{} \Box\psi + \frac{m^2c^2}{\hbar^2}\psi = 0$$

Or in covariant notations, natural units : $\left(m^2 + \partial_\mu \partial^\mu\right)\varphi = 0$

Spin 1 : 4-vector, Maxwell equation

Spin ½ : spinor field, Dirac equation

Field quantization

Plane wave solution for Klein gordon are of the form exp(-ikx) :

$$\partial_\mu \partial^\mu \varphi + m^2 \varphi = 0 \Rightarrow \left[(-ik_0)^2 - (i\vec{k})^2 + m^2\right] e^{-ikx} = 0$$

$$\Longrightarrow k_0 = \pm\sqrt{\vec{k}^2 + m^2} = \omega_k$$

General solution :

$$\varphi(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \frac{1}{\sqrt{2\omega_k}} \left(a_k e^{-ikx} + a_k^* e^{ikx}\right),$$

Field quantization : turns coefficients a_k and a_k^*
Into operators
Canonical commuations : $[a_i\ , a_j\ ] = [a_i^+, a_j^+] = 0 \qquad [a_i\ , a_j^+] = \delta_{ij}\mathbb{1}$

$$\hat{\varphi}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \frac{1}{\sqrt{2\omega_k}} \Big(\underbrace{\hat{a}_k e^{-ikx}}_{(1)} + \underbrace{\hat{a}_k^+ e^{ikx}}_{(2)} \Big)$$

Antiparticles

Negative energy solution arise from the quadratic equation
$\rightarrow$not physical :

$$E = \omega_k : \varphi(x) \propto e^{-i(\omega_k t - \vec{k}.\vec{x})}$$
$$E = -\omega_k : \varphi(x) \propto e^{-i(-\omega_k t - \vec{k}.\vec{x})}$$

Energy is positive, absorb minus sign into the time

Particle moving towards negative time, with momentum –k

=

Antiparticle, moving towards positive time, with momentum k

$$\hat{\varphi}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \frac{1}{\sqrt{2\omega_k}} \Big(\underbrace{\hat{a}_k e^{-ikx}}_{(1)} + \underbrace{\hat{a}_k^+ e^{ikx}}_{(2)} \Big)$$

Real field φ contains :

a_k annihilation of particle with momentum k
a_k^+ creation of antiparticle with momentum k

Spinor solutions

$$\psi(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \frac{1}{\sqrt{2\omega_k}} \sum_{s=1,2}^{\text{hélicité}} \underbrace{u_{k,s}}_{spineur} \underbrace{b_{k,s}}_{op.} e^{-ikx} + \underbrace{v_{k,s}}_{spineur} \underbrace{d^+_{k,s}}_{op.} e^{ikx}$$

$$\bar{\psi}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \frac{1}{\sqrt{2\omega_k}} \sum_{s=1,2}^{\text{hélicité}} \underbrace{\bar{u}_{k,s}}_{spineur} \underbrace{b^+_{k,s}}_{op.} e^{ikx} + \underbrace{\bar{v}_{k,s}}_{spineur} \underbrace{d_{k,s}}_{op.} e^{-ikx}$$

Field Ψ contains :

- **$b_{k,s}$ annihilation of particle (e^-) with momentum k and helicity s**
- **$d_{k,s}^+$ creation of antiparticle (e^+) with momentum k and helicity s**

Field $\overline{\Psi}$ contains :

- **$b_{k,s}^+$ creation of particle (e^-) with momentum k and helicity s**
- **$b_{k,s}$ annihilation of antiparticle (e^+) with momentum k and helicity s**

Lagrangians

Free lagrangian :

create and annihilate the field : $a^+a \rightarrow \varphi^+\varphi,\ \bar{\psi}\psi,\ \partial_\mu\varphi^+\partial_\mu\varphi, \ldots$

Interaction lagrangian : electron radiates a photon

Annihilate electron
Create electron and photon

$\Big\}$ $b^+ba \rightarrow \bar{\psi}\gamma^\mu\psi A_\mu$

In general, interaction lagrangian :

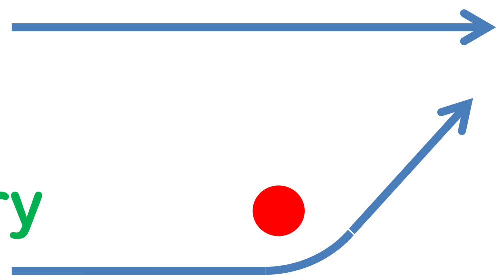
Products of fields : creation/annihilation operators
Coupling constant : interaction intensity

$$\mathcal{L} = g\Pi$$

Gauge invariance

Free propagation : straight line

Interaction : locally curved trajectory



Absorb the deformation into derivation : make the line straight again

> **Modification of the metric :** general relativity
> **Modification via internal space :** gauge theory

$$D_\mu = \partial_\mu - igA_\mu$$

necessitates a massless vector field

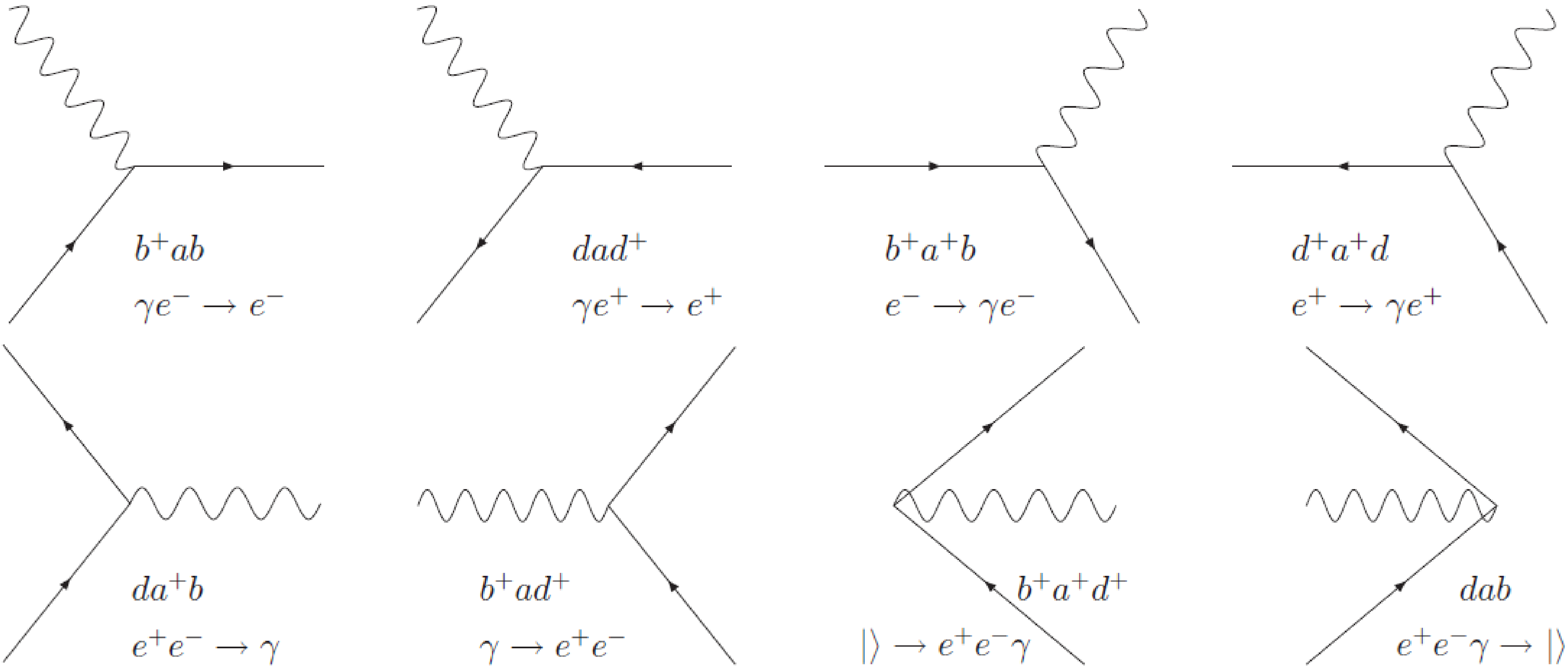
Elementary vertices

$\Psi \sim b + d^{+}$

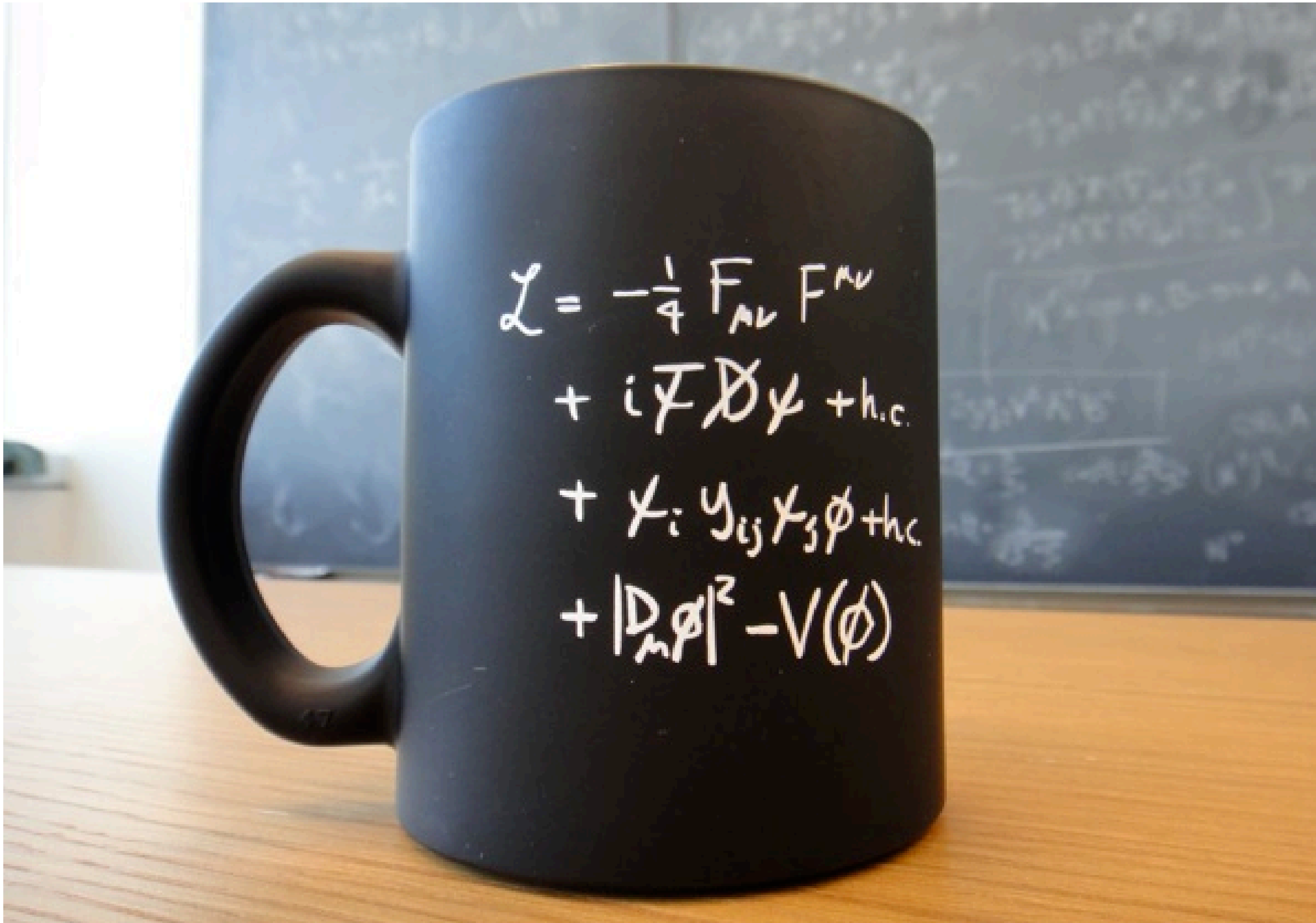
$\bar{\Psi} \sim b^{+} + d$

$A \sim a + a^{+}$

$$\mathcal{L} = g\bar{\psi}\gamma^{\mu}\psi A_{\mu}$$



SM Lagrangian (compact)



SM Lagrangian (expanded)

$$\mathcal{L}_{SM} = \sum_{\ell=e,\mu,\tau} i\bar\psi_\ell\gamma^\mu\partial_\mu\psi_\ell + \sum_{\ell'=\nu_e,\nu_\mu,\nu_\tau} i\bar\psi_{\ell'}\gamma^\mu\partial_\mu\psi_{\ell'} + \sum_i^3\sum_{q=u,c,t} i\bar\psi_{q_i}\gamma^\mu\partial_\mu\psi_{q_i} + \sum_i^3\sum_{q'=d,s,b} i\bar\psi_{q'_i}\gamma^\mu\partial_\mu\psi_{q'_i} - \frac{1}{2}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)(\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}) - \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu)$$

$$-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{1}{4}\sum_{a=1}^8(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu)(\partial^\mu G^{a\nu} - \partial^\nu G^{a\mu}) + \frac{1}{2}\partial_\mu h\partial^\mu h - \sum_{\ell=e,\mu,\tau}\frac{\lambda_\ell v}{\sqrt{2}}\bar\psi_\ell\psi_\ell - \sum_i^3\sum_{q=u,c,t}\frac{\lambda_q v}{\sqrt{2}}\bar\psi_{q_i}\psi_{q_i} - \sum_i^3\sum_{q'=d,s,b}\frac{\lambda_{q'} v}{\sqrt{2}}\bar\psi_{q'_i}\psi_{q'_i}$$

$$-\left(\frac{gv}{2}\right)^2 W^+_\mu W^{-\mu} - \frac{1}{2}\left(\frac{gv}{2\cos\theta_W}\right)^2 Z_\mu Z^\mu - \frac{1}{2}\left(-2m^2\right)^2 h^2 + \frac{g}{4\cos\theta_W}\left(\sum_{\ell=e,\mu,\tau}\bar\psi_\ell\gamma^\mu(4\sin^2\theta_W - 1 + \gamma^5)\psi_\ell Z_\mu + \sum_{\ell'=\nu_e,\nu_\mu,\nu_\tau}\bar\psi_{\ell'}\gamma^\mu(1-\gamma^5)\psi_{\ell'} Z_\mu\right)$$

$$+\frac{g}{4\cos\theta_W}\left(\sum_i^3\sum_{q=u,c,t}\bar\psi_{q_i}\gamma^\mu(1 - \frac{8}{3}\sin^2\theta_W - \gamma^5)\psi_{q_i}Z_\mu + \sum_i^3\sum_{q'=b,s,b}\bar\psi_{q'_i}\gamma^\mu(\frac{4}{3}\sin^2\theta_W - 1 + \gamma^5)\psi_{q'_i}Z_\mu\right) + \frac{g}{2\sqrt{2}}\left(\sum_{\ell=e,\mu,\tau}\bar\psi_{\nu_\ell}\gamma^\mu(1-\gamma^5)\psi_\ell W^+_\mu + \sum_{\ell=e,\mu,\tau}\bar\psi_\ell\gamma^\mu(1-\gamma^5)\psi_{\nu_\ell}W^-_\mu\right)$$

$$+\frac{g}{2\sqrt{2}}\left(\sum_i^3\sum_{\substack{q=u,c,t\\ q'=d,s,b}} V_{qq'}\bar\psi_q\gamma^\mu(1-\gamma^5)\psi_{q'_i}W^+_\mu + \sum_i^3\sum_{\substack{q=u,c,t\\ q'=d,s,b}} V^*_{qq'}\bar\psi_{q'_i}\gamma^\mu(1-\gamma^5)\psi_{q_i}W^-_\mu\right) + g_{em}\left(-\sum_{\ell=e,\mu,\tau}\bar\psi_\ell\gamma^\mu\psi_\ell A_\mu + \frac{2}{3}\sum_{q=u,c,t}\bar\psi_{q_i}\gamma^\mu\psi_{q_i}A_\mu - \frac{1}{3}\sum_{q'=d,s,b}\bar\psi_{q'_i}\gamma^\mu\psi_{q'_i}A_\mu\right)$$

$$+g_s\left(\sum_{i,j}^3\sum_a^8\sum_{q=u,c,t}\bar\psi_{q_j}\gamma^\mu\psi_{q_i}G^a_\mu T^a_{ij} + \sum_{i,j}^3\sum_a^8\sum_{q'=d,s,b}\bar\psi_{q'_j}\gamma^\mu\psi_{q'_i}G^a_\mu T^a_{ij}\right) - \sum_{\ell=e,\mu,\tau}\frac{\lambda_\ell}{\sqrt{2}}\bar\psi_\ell\psi_\ell h - \sum_i^3\sum_{q=u,c,t}\frac{\lambda_q}{\sqrt{2}}\bar\psi_{q_i}\psi_{q_i}h - \sum_i^3\sum_{q'=d,s,b}\frac{\lambda_{q'}}{\sqrt{2}}\bar\psi_{q'_i}\psi_{q'_i}h$$

$$+\frac{g}{2\sqrt{2}}\left(\sum_i^3\sum_{\substack{q=u,c,t\\ q'=d,s,b}} V_{qq'}\bar\psi_q\gamma^\mu(1-\gamma^5)\psi_{q'_i}W^+_\mu + \sum_i^3\sum_{\substack{q=u,c,t\\ q'=d,s,b}} V^*_{qq'}\bar\psi_{q'_i}\gamma^\mu(1-\gamma^5)\psi_{q_i}W^-_\mu\right) + g_{em}\left(-\sum_{\ell=e,\mu,\tau}\bar\psi_\ell\gamma^\mu\psi_\ell A_\mu + \frac{2}{3}\sum_{q=u,c,t}\bar\psi_{q_i}\gamma^\mu\psi_{q_i}A_\mu - \frac{1}{3}\sum_{q'=d,s,b}\bar\psi_{q'_i}\gamma^\mu\psi_{q'_i}A_\mu\right)$$

$$+g_s\left(\sum_{i,j}^3\sum_a^8\sum_{q=u,c,t}\bar\psi_{q_j}\gamma^\mu\psi_{q_i}G^a_\mu T^a_{ij} + \sum_{i,j}^3\sum_a^8\sum_{q'=d,s,b}\bar\psi_{q'_j}\gamma^\mu\psi_{q'_i}G^a_\mu T^a_{ij}\right) - \sum_{\ell=e,\mu,\tau}\frac{\lambda_\ell}{\sqrt{2}}\bar\psi_\ell\psi_\ell h - \sum_i^3\sum_{q=u,c,t}\frac{\lambda_q}{\sqrt{2}}\bar\psi_{q_i}\psi_{q_i}h - \sum_i^3\sum_{q'=d,s,b}\frac{\lambda_{q'}}{\sqrt{2}}\bar\psi_{q'_i}\psi_{q'_i}h$$

$$+ig_{em}\left[\partial_\mu A_\nu W^{-\mu}W^{+\nu} + \partial_\mu W^+_\nu W^{-\nu}A^\mu + \partial_\mu W^-_\nu W^{+\mu}A^\nu - \partial_\mu A_\nu W^{-\nu}W^{+\mu} - \partial_\mu W^+_\nu W^{-\mu}A^\nu - \partial_\mu W^-_\nu W^{+\nu}A^\mu\right]$$

$$+ig\cos\theta_W\left[\partial_\mu Z_\nu W^{-\mu}W^{+\nu} + \partial_\mu W^+_\nu W^{-\nu}Z^\mu + \partial_\mu W^-_\nu W^{+\mu}Z^\nu - \partial_\mu Z_\nu W^{-\nu}W^{+\mu} - \partial_\mu W^+_\nu W^{-\mu}Z^\nu - \partial_\mu W^-_\nu W^{+\nu}Z^\mu\right] + \frac{g^2 v}{2}W^+_\mu W^{-\mu}h + \frac{g^2 v}{4\cos^2\theta_W}Z_\mu Z^\mu h - \lambda v h^3$$

$$+g_{em}^2\left[W^+_\nu W^{-\mu}A_\nu A^\mu - W^+_\mu W^{-\mu}A_\nu A^\nu\right] + g^2\cos^2\theta_W\left[W^+_\nu W^{-\mu}Z_\nu Z^\mu - W^+_\mu W^{-\mu}Z_\nu Z^\nu\right] + g^2\cos\theta_W\sin\theta_W\left[2W^+_\mu W^{-\mu}Z_\nu A^\nu - W^+_\mu W^{-\nu}A_\nu Z^\mu - W^+_\mu W^{-\nu}A^\mu Z_\nu\right]$$

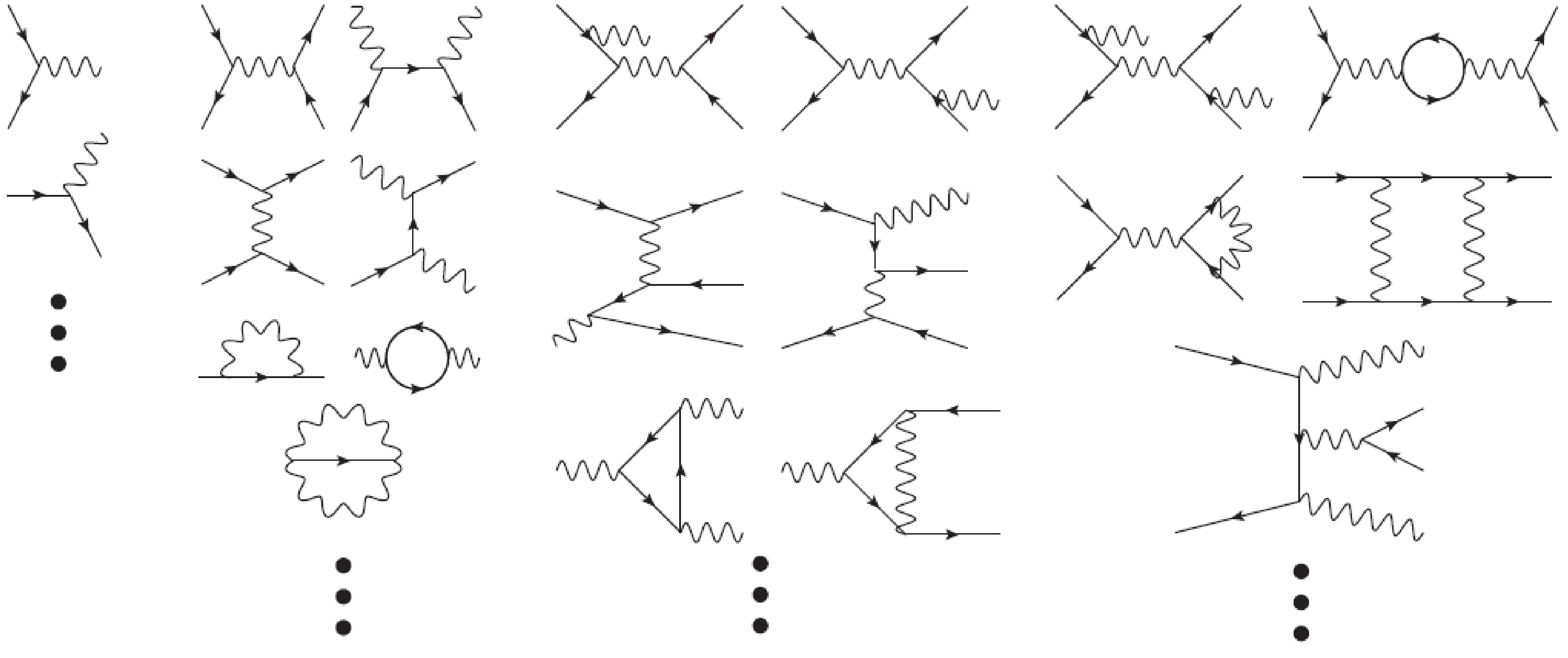
$$+\frac{g^2}{2}\left[W^-_\mu W^{-\mu}W^+_\nu W^{+\nu} - W^-_\mu W^{+\mu}W^-_\nu W^{+\nu}\right] + \frac{g^2}{4}W^+_\mu W^{-\mu}h^2 + \frac{g^2}{8\cos^2\theta_W}Z_\mu Z^\mu h^2 - \frac{\lambda}{4}h^4 - \frac{g_s}{2}\sum_{a,b,c}^8 f^{abc}(\partial_\mu G^{a\nu} - \partial_\nu G^a_\mu)G^{\mu b}G^{\nu c} - \frac{g_s^2}{4}\sum_{\substack{a,b,c\\ d,e,f}}^8 f^{abc}f^{ade}G^b_\mu G^c_\nu G^{\mu d}G^{\nu e}$$

$$g_{em} = g\sin\theta_W,\ v^2 = \frac{-m^2}{\lambda}\ (m^2<0, \lambda>0),\ m_\ell = \frac{\lambda_\ell v}{\sqrt{2}},\ m_q = \frac{3\lambda_q v}{\sqrt{2}},\ m_W = \frac{gv}{2},\ m_Z = \frac{gv}{2\cos\theta_W},\ m_h = \sqrt{-2m^2}$$

Perturbative development

$$S = e^{i\int \mathcal{L}_{int} d^4x}$$

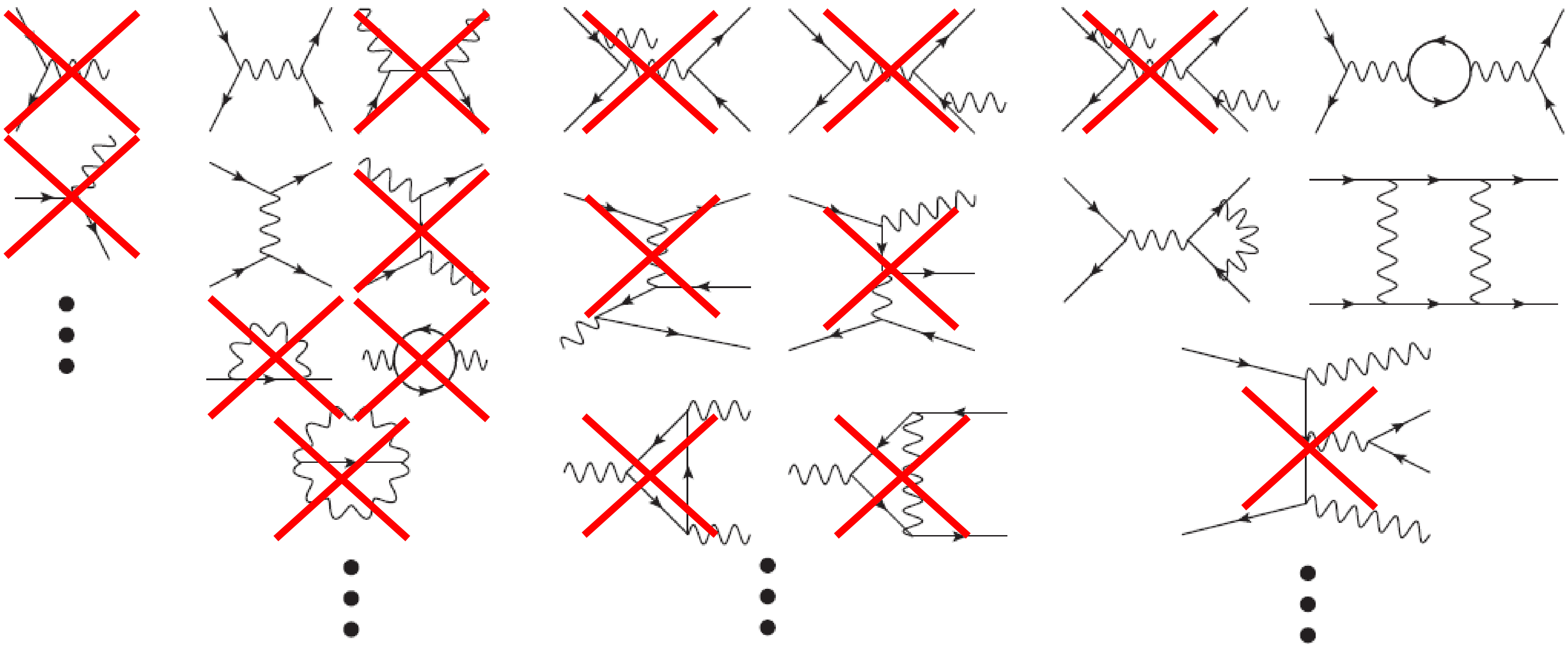
$$i\sqrt{\alpha}\int \mathcal{L}dx + \frac{(i\sqrt{\alpha})^2}{2}\iint \mathcal{L}\mathcal{L}'dxdx' + \frac{(i\sqrt{\alpha})^3}{3!}\iiint \mathcal{L}\mathcal{L}'\mathcal{L}''dxdx'dx'' + \frac{(i\sqrt{\alpha})^4}{4!}\iiiint \mathcal{L}\mathcal{L}'\mathcal{L}''\mathcal{L}'''dxdx'dx''dx''' + \ldots$$



Perturbative development

$f\bar{f} \rightarrow f\bar{f}$

$$i\sqrt{\alpha}\int \mathcal{L}dx + \frac{(i\sqrt{\alpha})^2}{2}\iint \mathcal{L}\mathcal{L}'dxdx' + \frac{(i\sqrt{\alpha})^3}{3!}\iiint \mathcal{L}\mathcal{L}'\mathcal{L}''dxdx'dx'' + \frac{(i\sqrt{\alpha})^4}{4!}\iiiint \mathcal{L}\mathcal{L}'\mathcal{L}''\mathcal{L}'''dxdx'dx''dx''' + \ldots$$

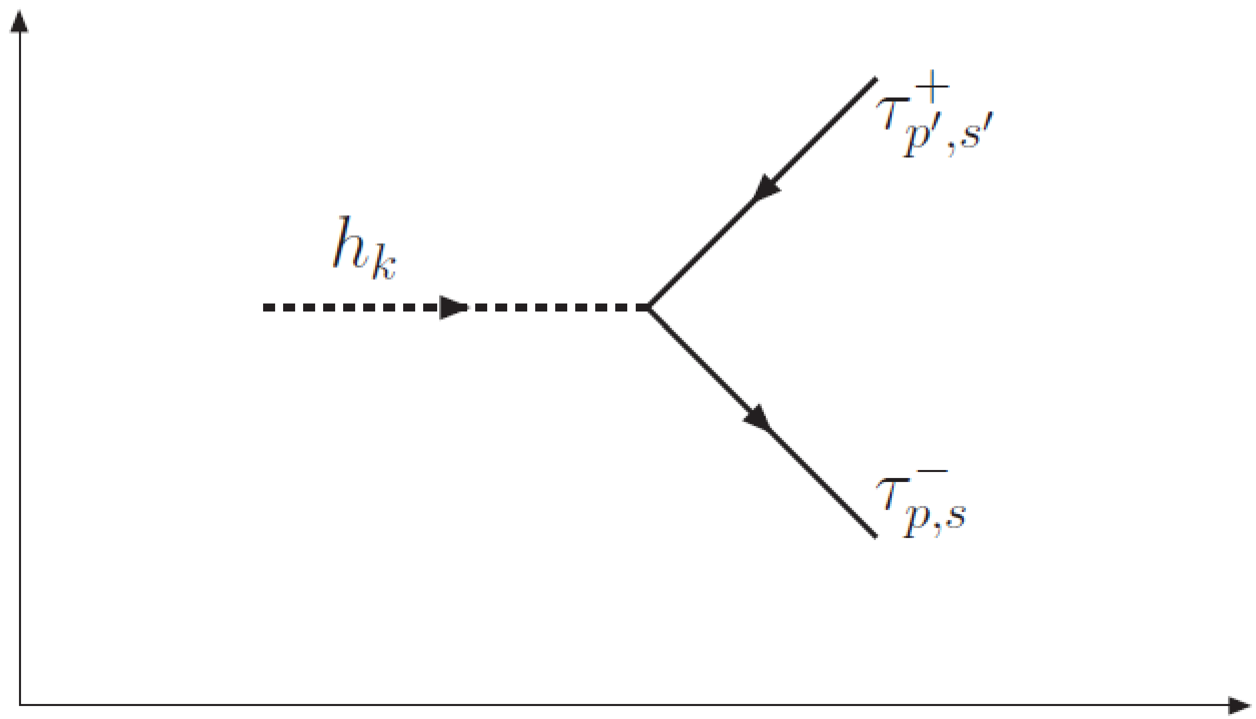


S_{fi} = <f|S|i> : select diagrams that connect |i>and |f> .

Feynman diagrams

scalar boson — fermion (spinor) — photon, W, Z — gluon

gauge bosons (vectors)



Feynman diagrams

Transition probability

S_{fi} = Phase space x Feynman amplitude

Kinematics (E/P conservation) — **Dynamics (couplings)**

Space-time diagrams ⇔ Feynman amplitude

2→2 process :

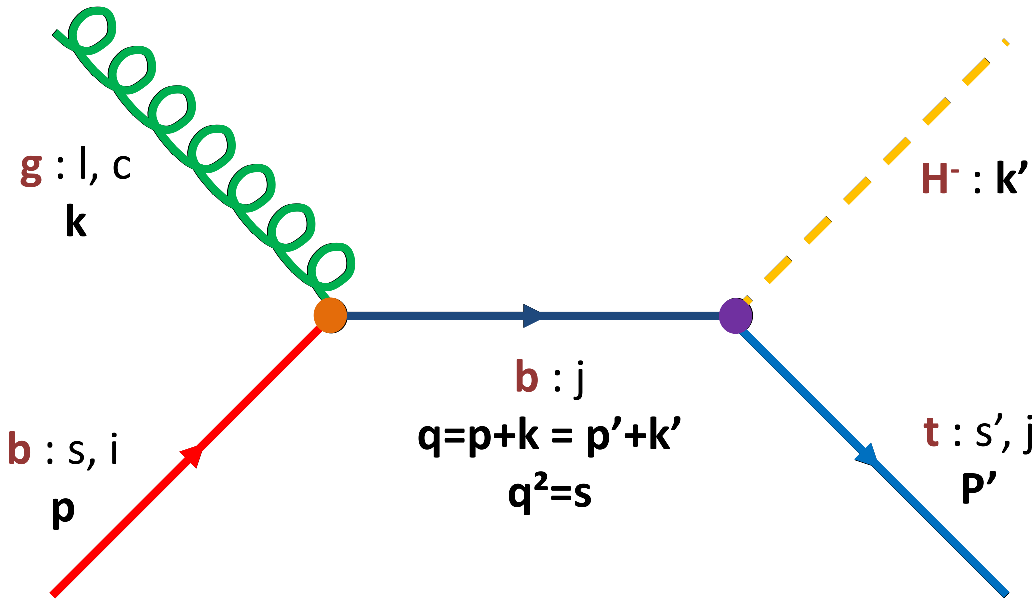
$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{p_f}{p_i}|\mathcal{M}_{fi}|^2$$

Each vertex is proportionnal to the coupling x charge

Each propagator (internal line) is proportionnal to $1/(m^2-p^2)$

Feynman rules

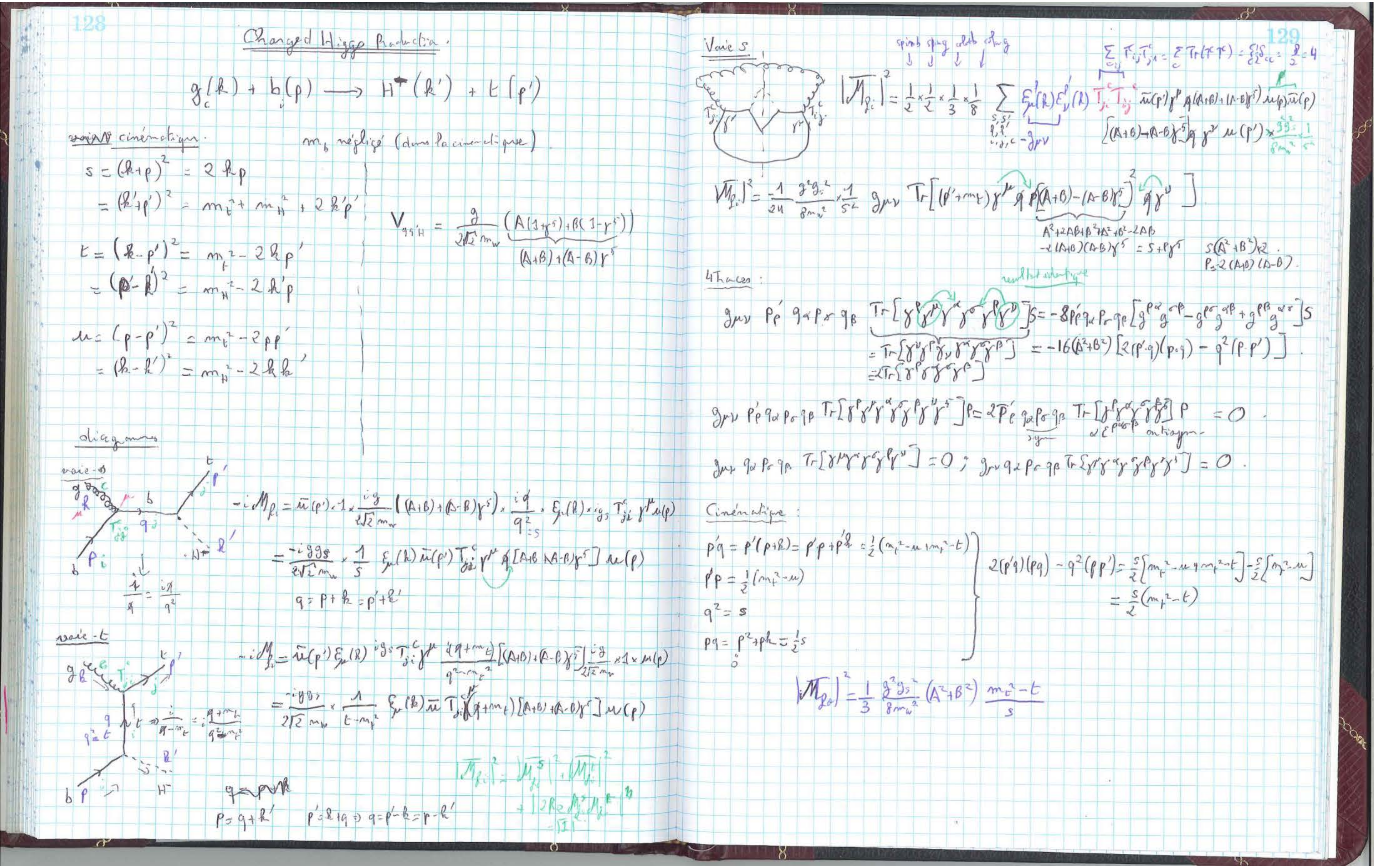
Each line and each vertex correspond to a multiplicative factor



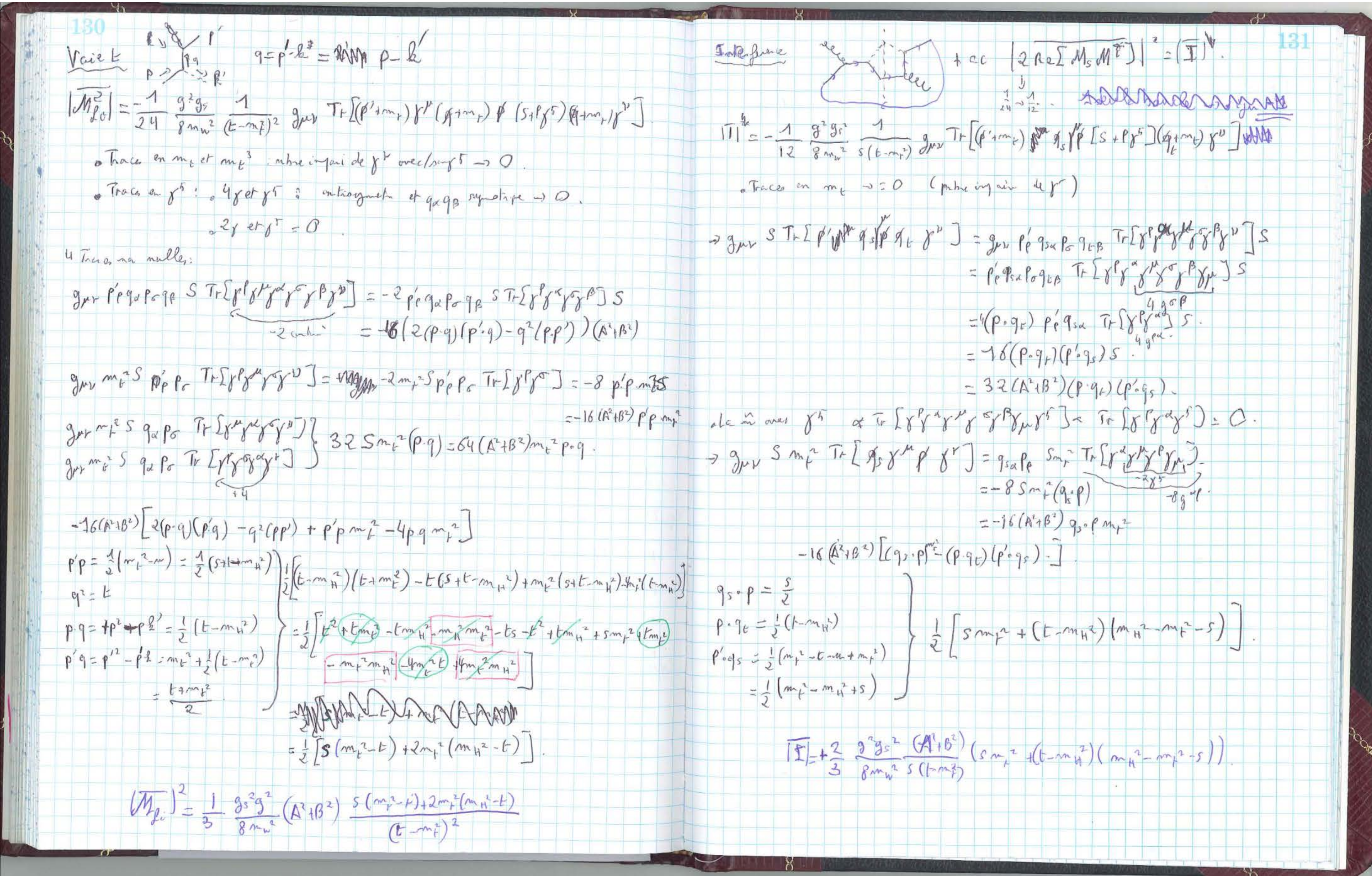
$$-i\mathcal{M}_{fi} = \bar{u}_{s'}(p') \times 1 \times \frac{ig}{2\sqrt{2}m_W}\left[(A+B)+(A-B)\,\gamma^5\right] \times i\frac{\not{q}}{s} \times ig_s T^c_{ji}\gamma^\mu \times \varepsilon^\mu_l(k) \times u_s(p)$$

$$-i\mathcal{M}_{fi} = \frac{-gg_s}{2\sqrt{2}m_W s} T^c_{ji}\varepsilon^\mu_l(k)\bar{u}_{s'}(p')\left[(A+B)+(A-B)\gamma^5\right]\not{q}\gamma^\mu u_s(p)$$

LO Feynman calculation



LO Feynman calculation (2)



LO Feynman calculation (end)

132

$$\overline{|\mathcal{M}_{fi}|^2} = \frac{1}{3} g_s^2 \frac{G_F}{\sqrt{2}} (A^2+B^2) \left[\frac{m_t^2 - t}{s} + \frac{s(m_t^2 - t) + 2m_t^2(m_H^2 - t)}{(t-m_t^2)^2} + 2\frac{s m_t^2 + (t-m_H^2)(m_H^2 - m_t^2 - s)}{s(t-m_t^2)} \right]$$

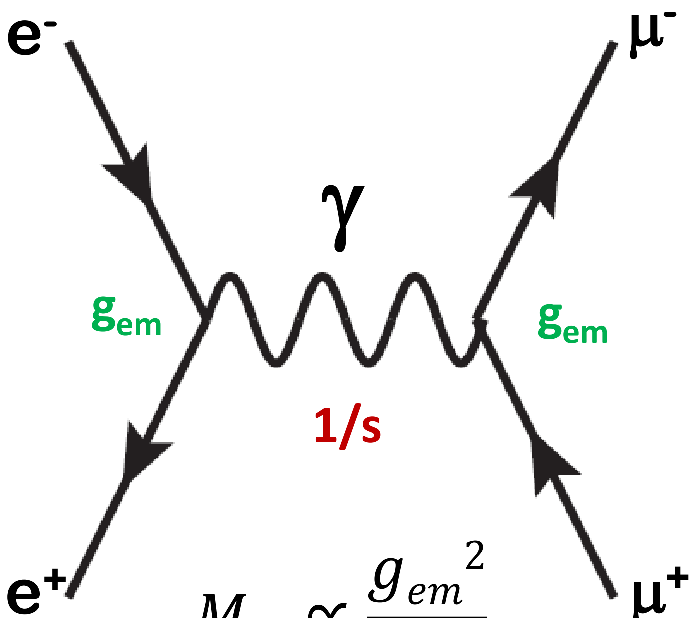
$$d\sigma = \frac{1}{F} \overline{|\mathcal{M}_{fi}|^2}\, dPS^{(2)} = \frac{1}{16\pi} \frac{1}{\underbrace{\lambda(s,0,0)}_{=s}} |\mathcal{M}_{fi}|^2\, dt$$

$$\frac{d\sigma}{dt} = \frac{1}{12} \alpha_s \frac{G_F}{\sqrt{2}} \left[A^2+B^2\right] \frac{M}{s}$$

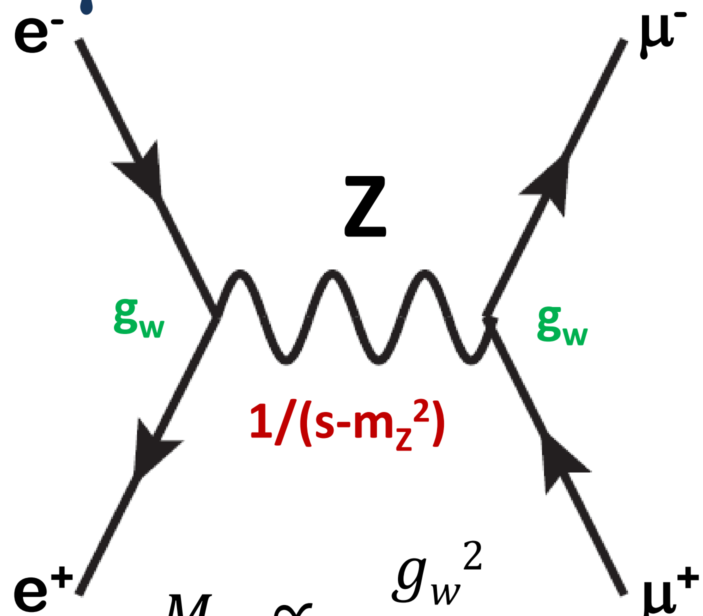
Section efficace Partonique

$s \to \hat{s}$, $t \to \hat{t}$.

$e^+e^- \to \mu^+\mu^-$



$$M_{fi} \propto \frac{g_{em}{}^2}{s}$$



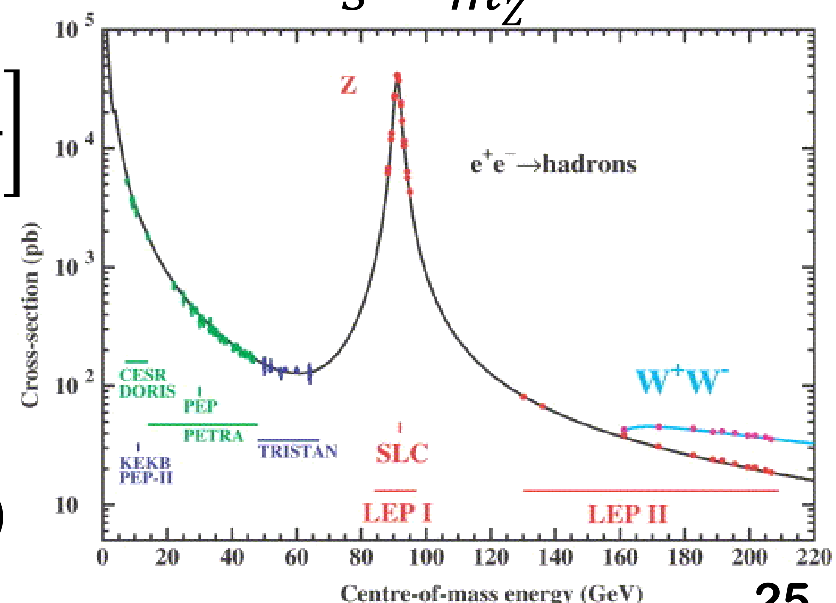
$$M_{fi} \propto \frac{g_w{}^2}{s - m_Z{}^2}$$

$$\sigma \propto s\left[\frac{g_{em}{}^4}{s^2} + \frac{2g_{em}{}^2 g_w{}^2}{s(s - m_Z{}^2)} + \frac{g_w{}^4}{(s - m_Z{}^2)^2}\right]$$

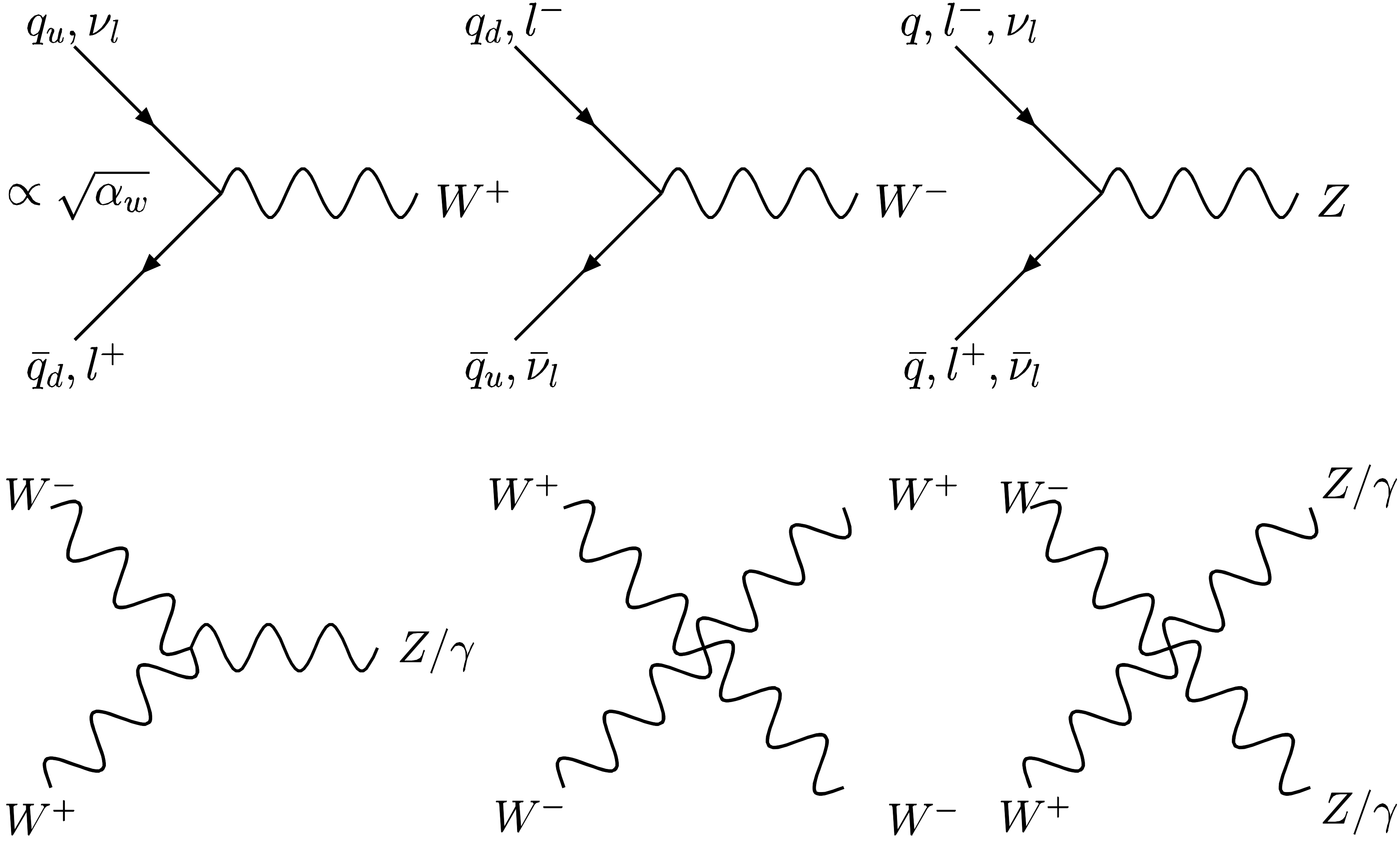
$s \ll m_Z$ $\sigma \propto$ **$g_{em}{}^4/s \propto \alpha_{em}{}^2/s$**

$s = m_Z$ resonnant effect at Z peak

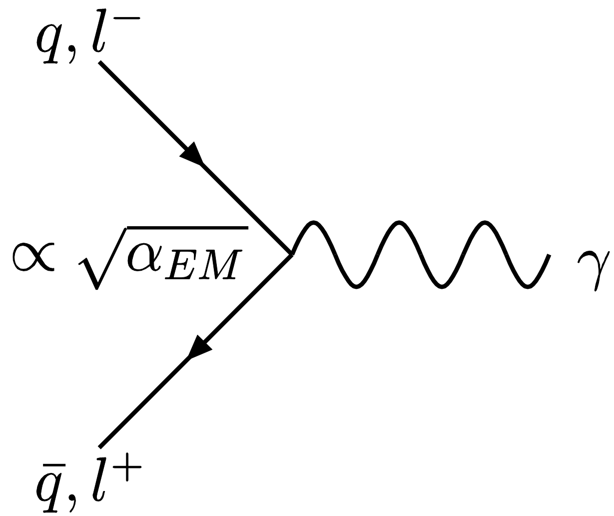
$s \gg m_Z$ $\sigma \propto$ **$4g_{em}{}^4/s$ (assume $g_{em} \sim g_w$)**



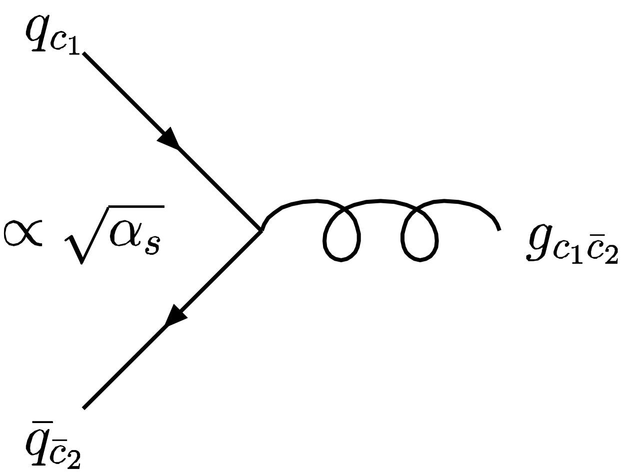
SM vertices : Weak interaction

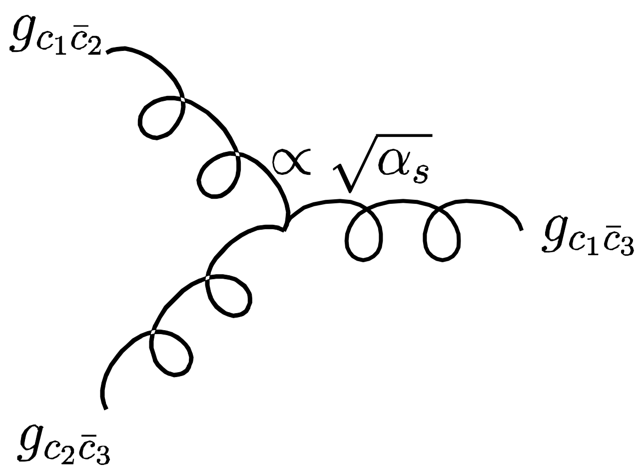


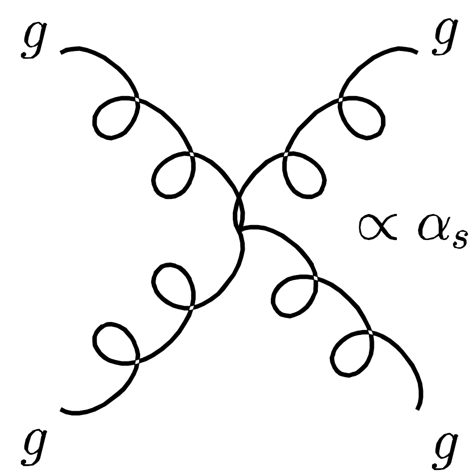
SM vertices : EM interaction



strong interaction







SM vertices : Higgs sector