Probabilistic Reasoning in Frontier Physics

– inference, forecasting, decision –

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"Probability is good sense reduced to a calculus" (Laplace)

Preamble

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⇒ Probabilistic approach

An invitation to (re-)think

on foundamental aspects

of data analysis.

This first lesson:

1. *Claims of discoveries based on 'sigmas'*
   (based on a lecture to Italian teachers in Frascati, `http://www.lnf.infn.it/edu/incontri/2012/`)

2. *Basic of probabilistic inference*
   (and related topics)

Tomorrow other applications will be shown

⇒ Lorenzo Bellagamba

2011: non only Opera...

- April, CDF: absolutely unexpected excess at about 150 GeV

$$\approx 3.2\,\sigma$$

- September, Opera: neutrinos faster than light

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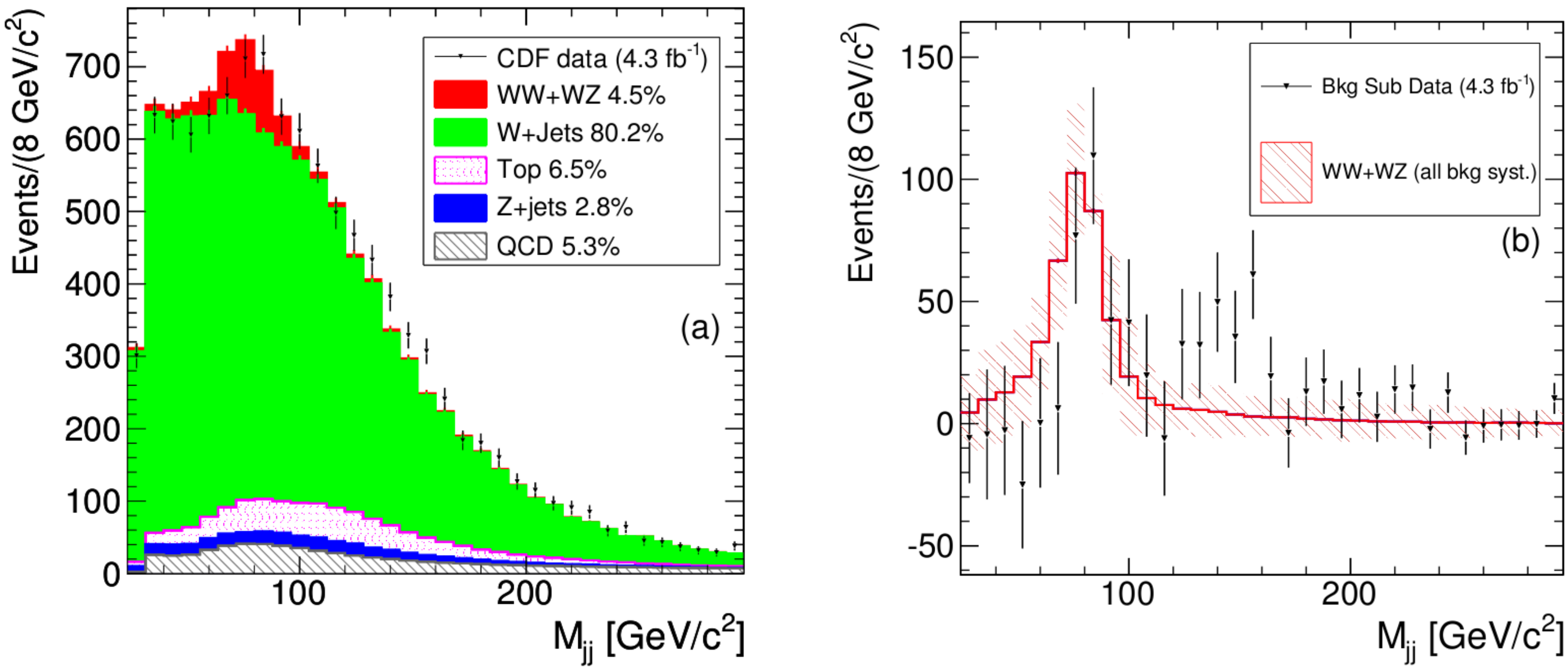
Why there was substancial scepticism towards the first two anouncements, in constrast with a cautious/pronounced optimism towards the third one?

April 2011

CDF Collaboration at the Tevatron



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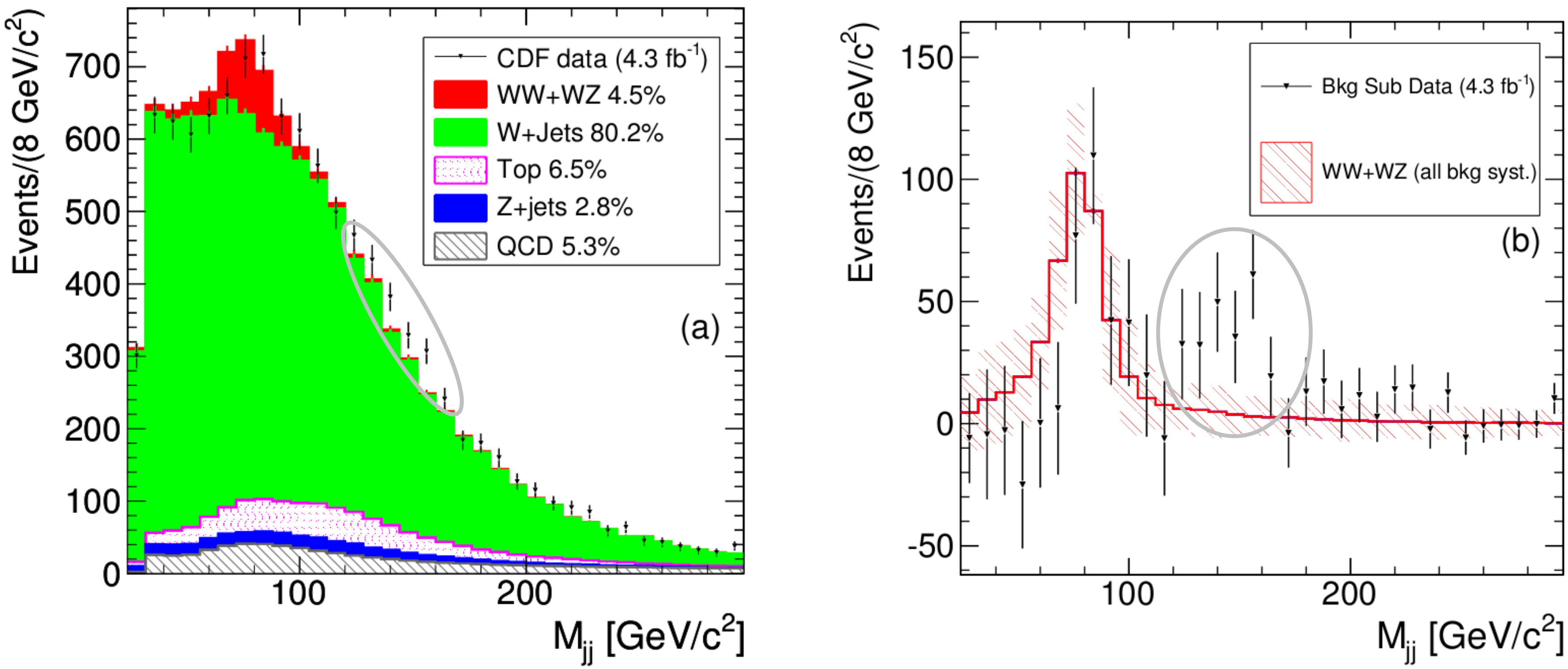

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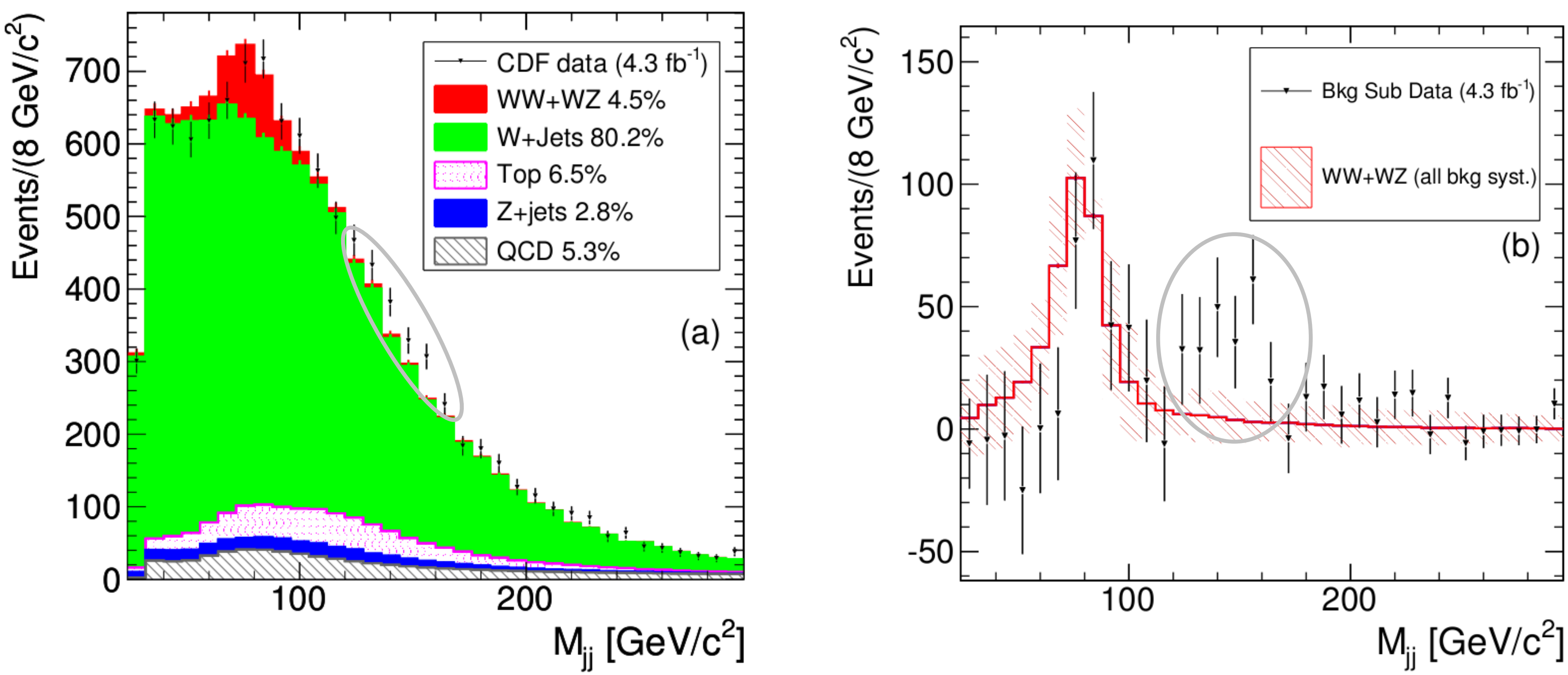

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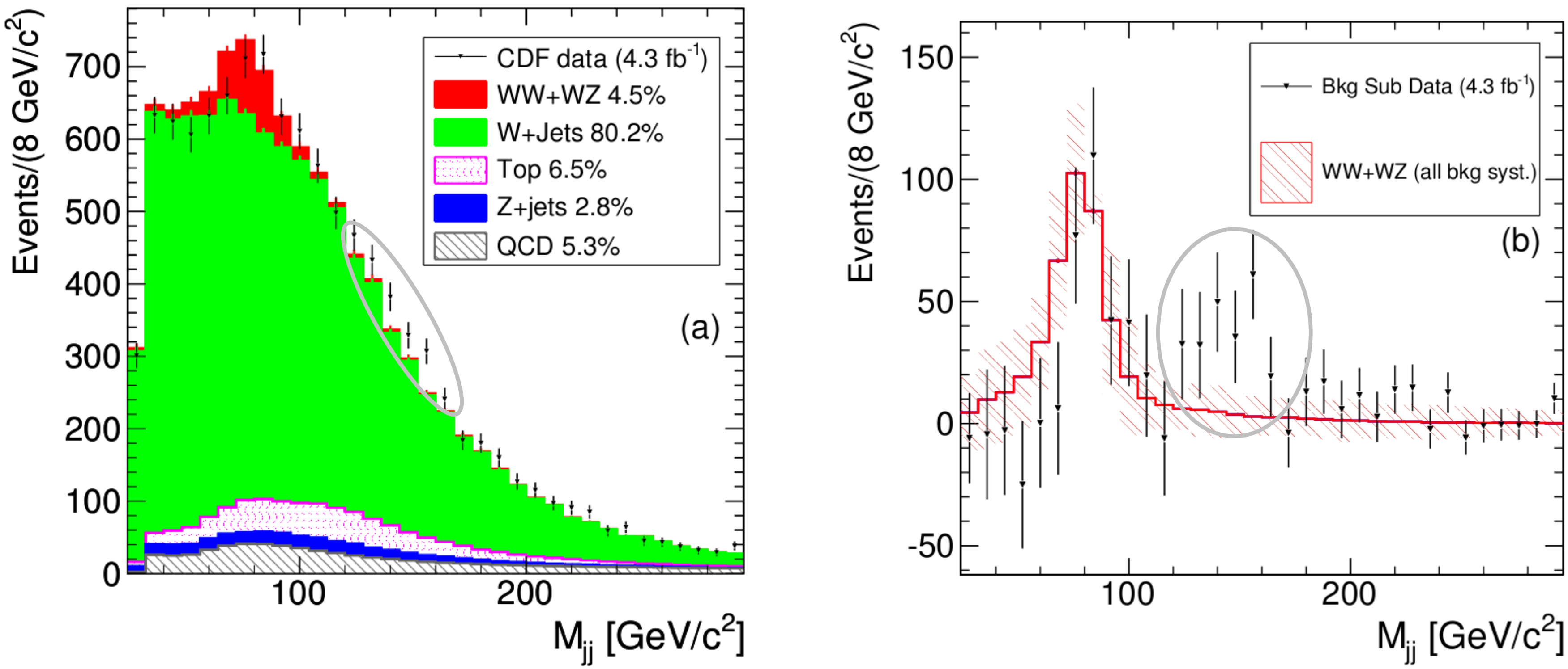

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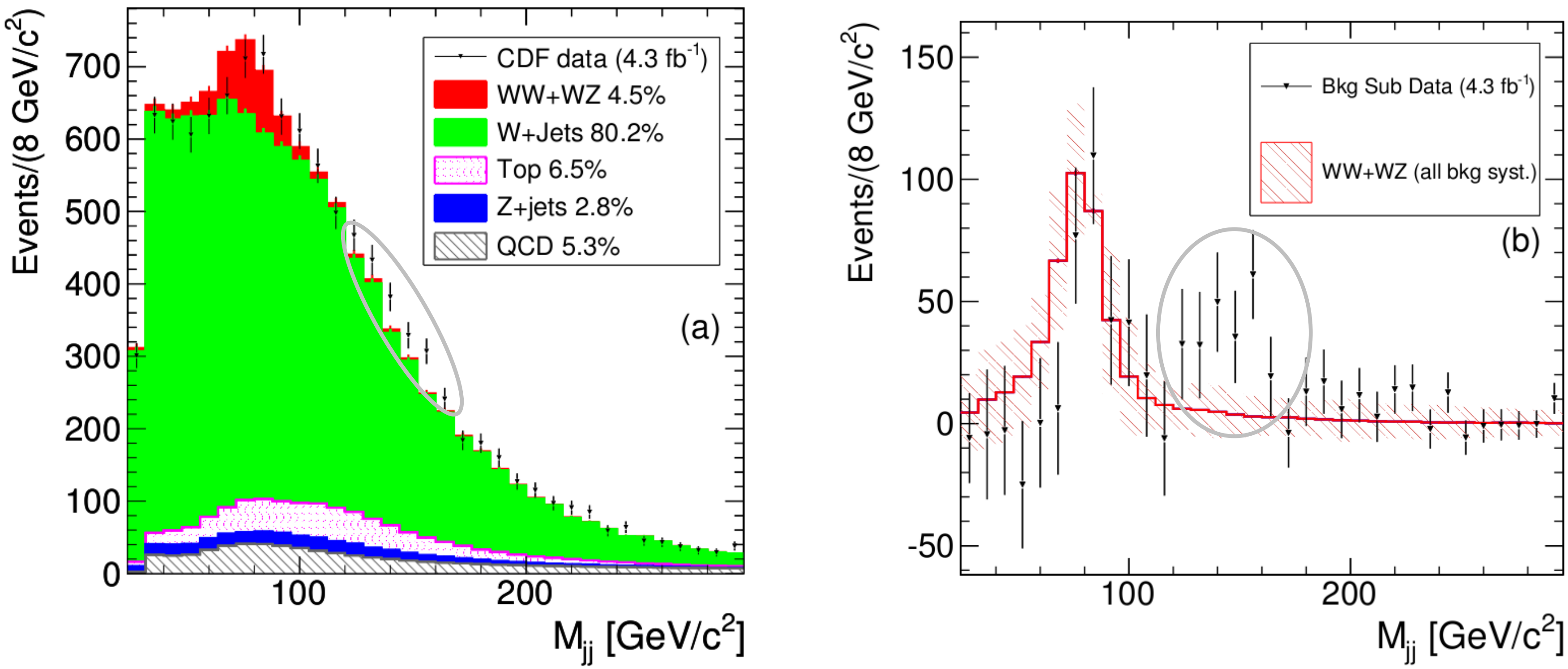
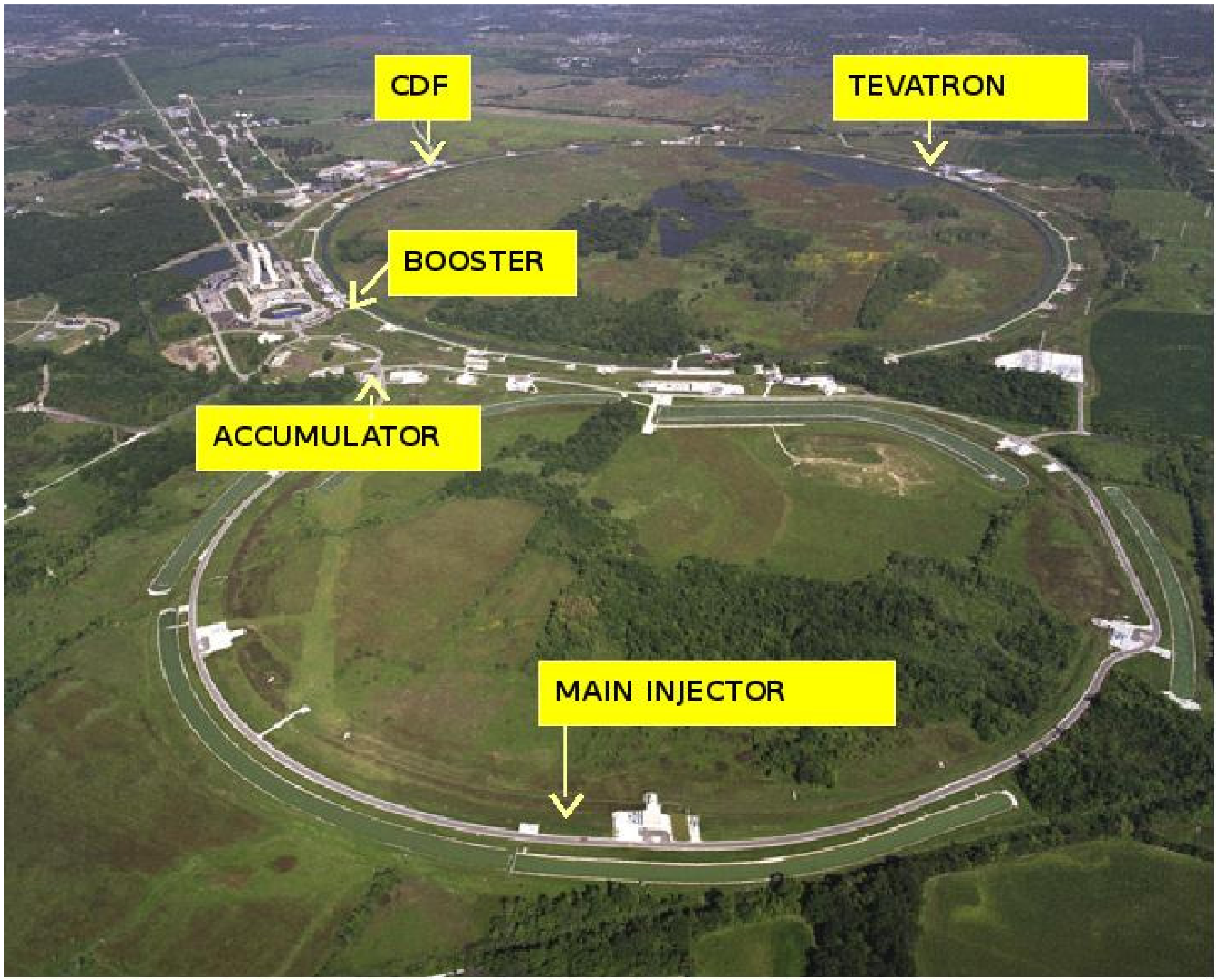


What does it mean?

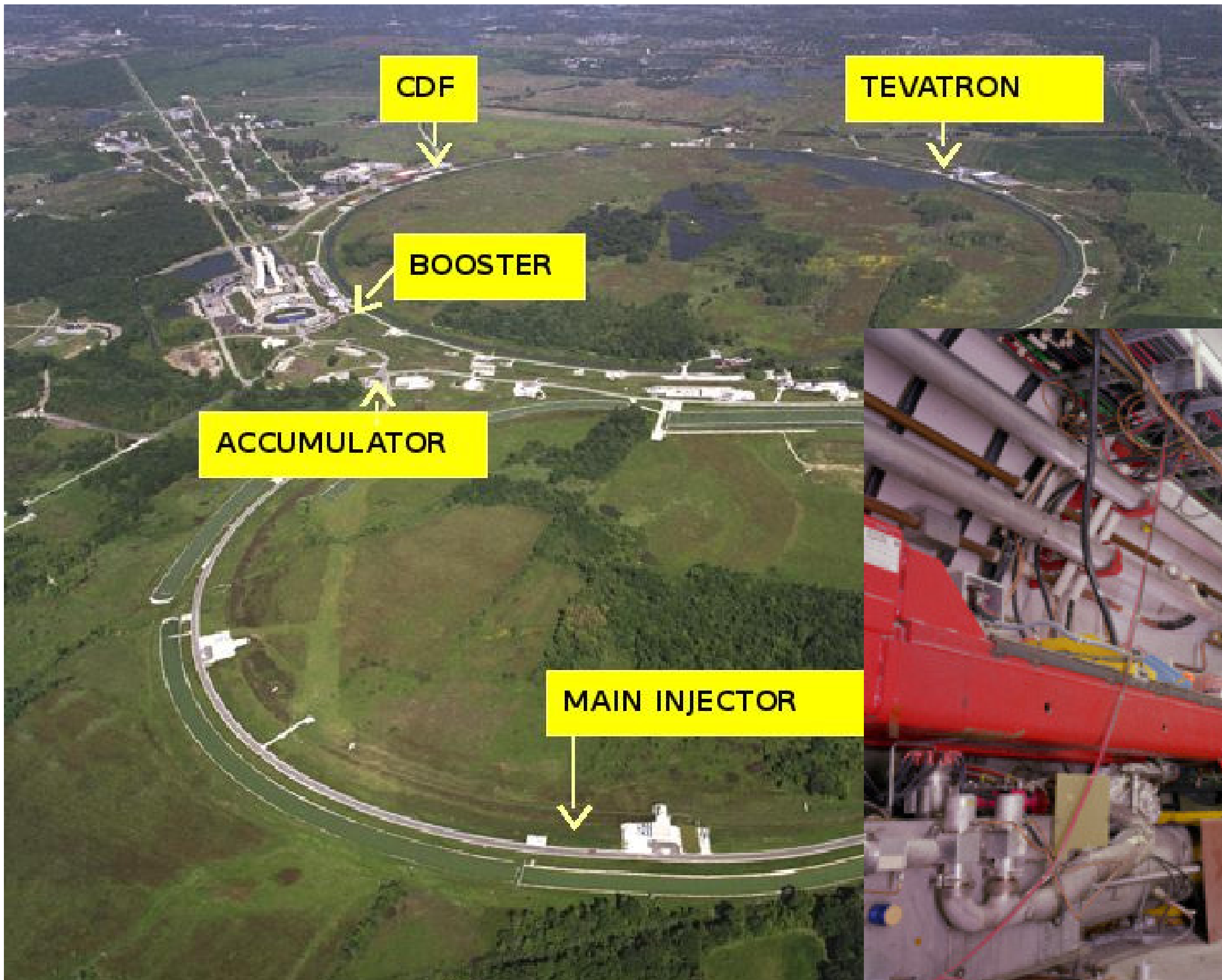
Tevatron and CDF

6.28 km, near Chicago



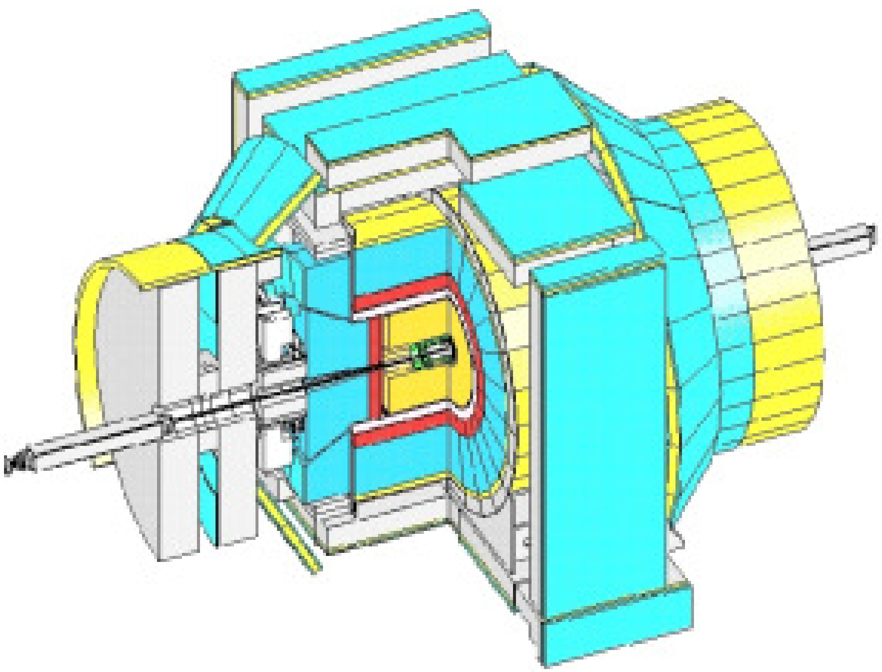
Tevatron and CDF

$p \rightarrow \cdot \leftarrow \bar{p}$ $\quad$ $[\approx 1\,\mathrm{TeV} + 1\,\mathrm{TeV}]$



Tevatron and CDF

CDF: a multipurpose ('hermetic') detector

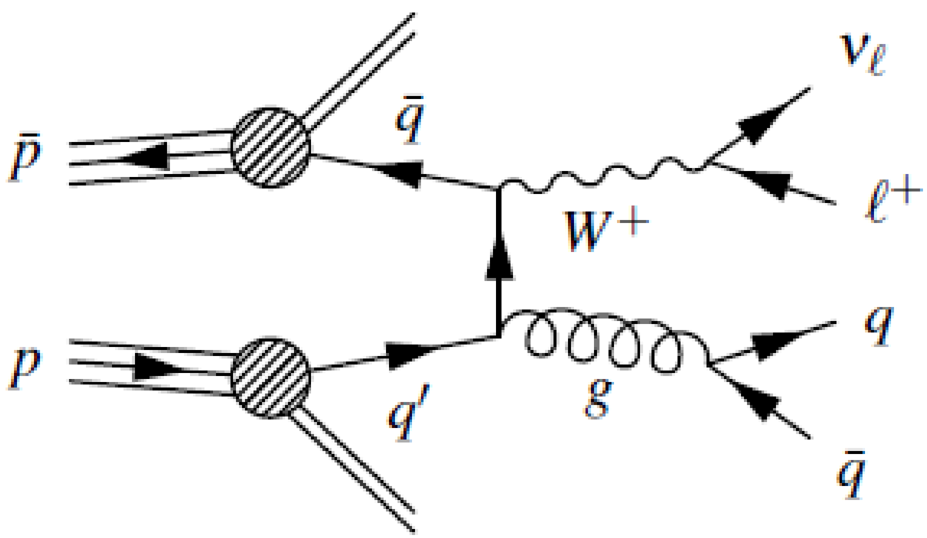


Tevatron and CDF

...a large, very sophisticated detector!

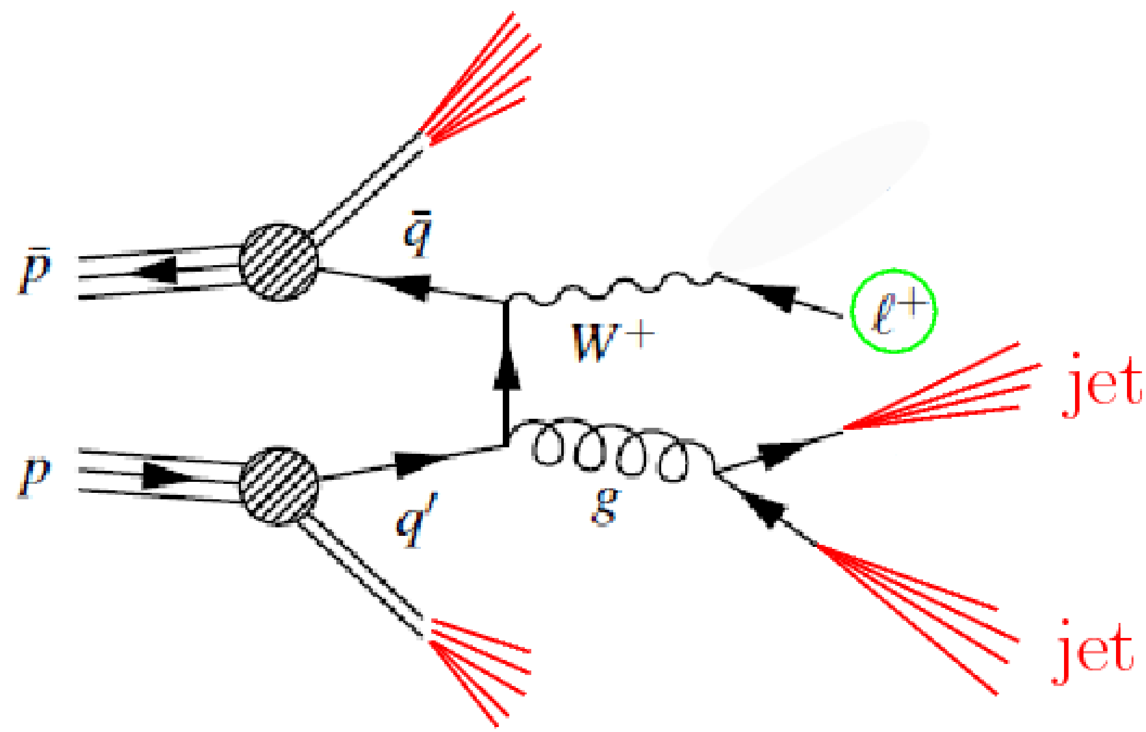
Jet-jet + W

$W + (q\overline{q})$ [+ 'remnants']



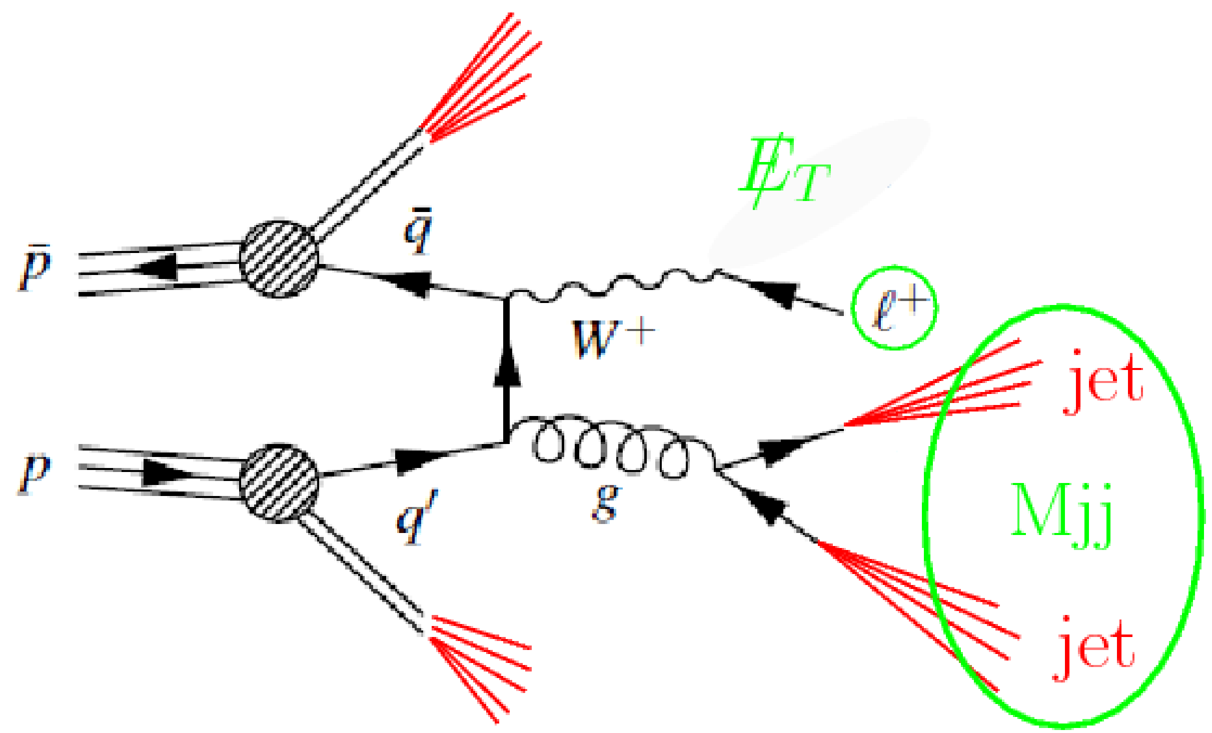
Jet-jet + W

$W + 2$jet [+ much more]



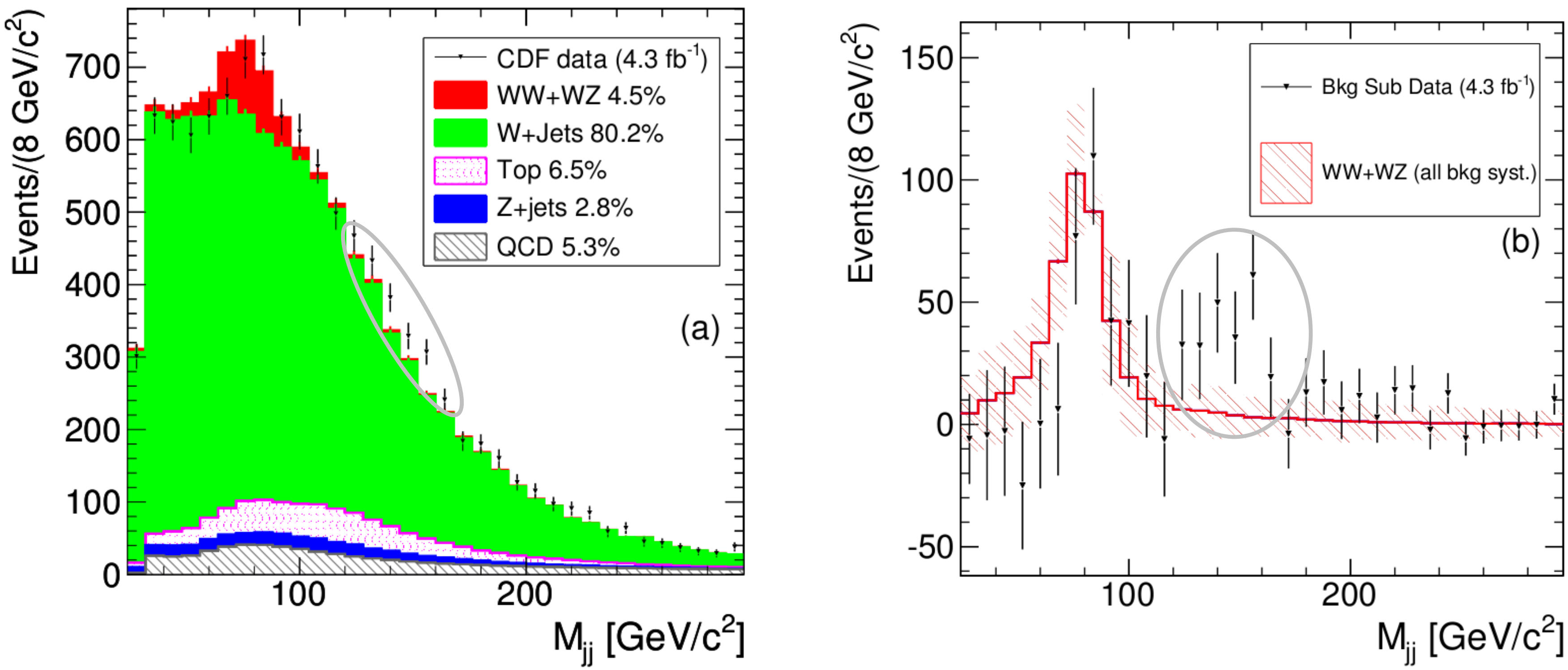
Jet-jet + W

$\Rightarrow M_{jj} + W + \ldots$



The 'bump'!

Invariant Mass Distribution of Jet Pairs Produced in Association with a W boson in $p\overline{p}$ Collisions at $\sqrt{s} = 1.96$ TeV", (CDF, 4 aprile 2011)



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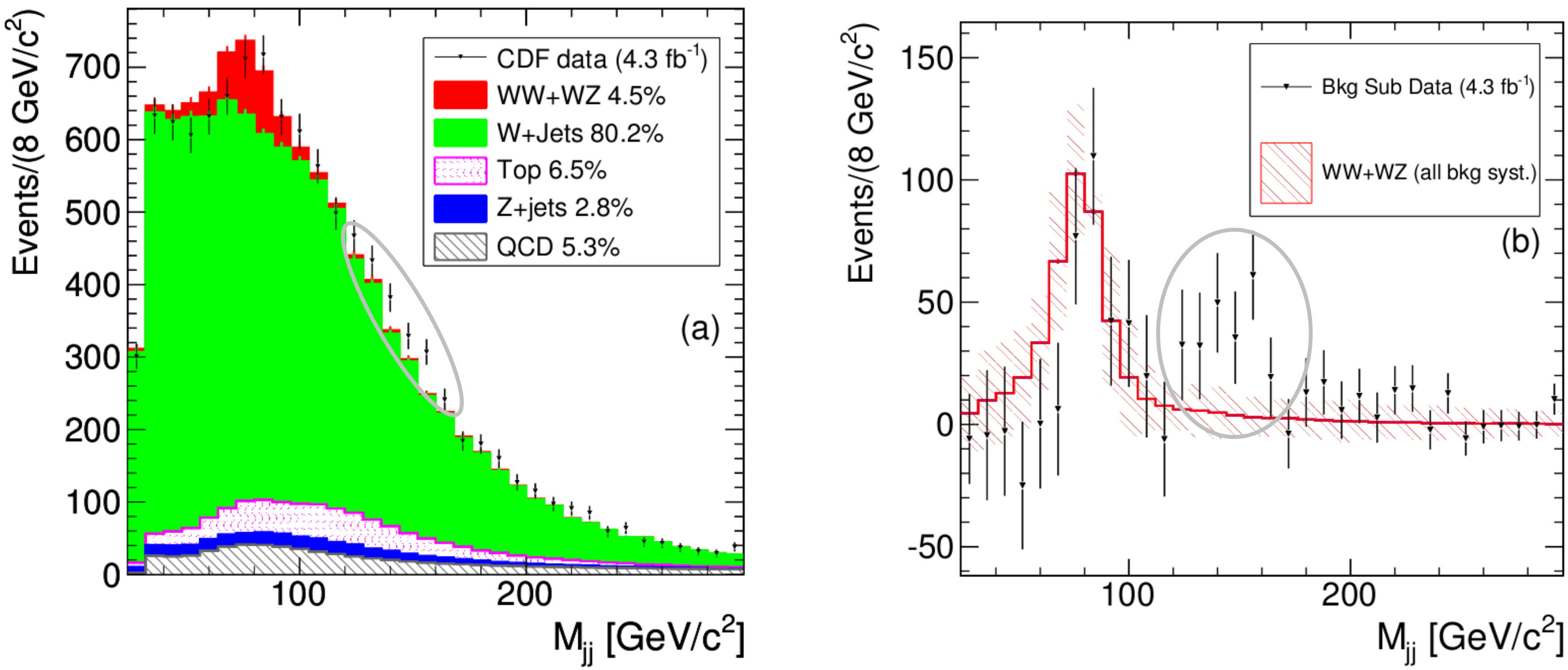

What does it mean?

Sigma and gaussian distribution

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“Functio nostra fiet. . . ”

Sigma e probability [gaussian!]

If the random number X is described by a gaussian pdf

$$
\begin{aligned}
P(-\sigma \le X \le +\sigma) &= 68.3\% \\
P(-2\,\sigma \le X \le +2\,\sigma) &= 95.4\% \\
P(-3\,\sigma \le X \le +3\,\sigma) &= 99.73\% \\
1 - P(-3\,\sigma \le X \le +3\,\sigma) &= 0.27\% \\
1 - P(-4\,\sigma \le X \le +4\,\sigma) &= 6.3 \times 10^{-5} \\
\ldots &= \ldots \\
1 - P(-6\,\sigma \le X \le +6\,\sigma) &= 2.0 \times 10^{-9}
\end{aligned}
$$

$$
\begin{aligned}
1 - P(-3.2\,\sigma \le X \le +3.2\,\sigma) &= 1.4 \times 10^{-3} \\
P(X \ge +3.17\,\sigma) &= 7.6 \times 10^{-4} \quad \surd
\end{aligned}
$$

p-value, significance and sigma

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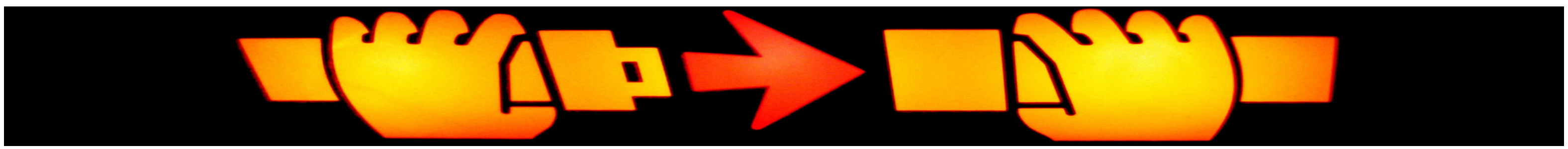

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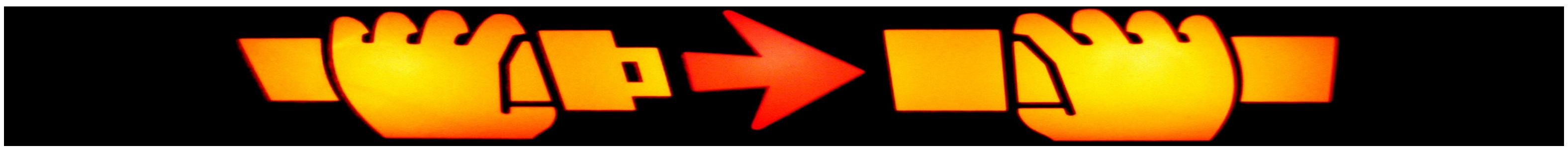

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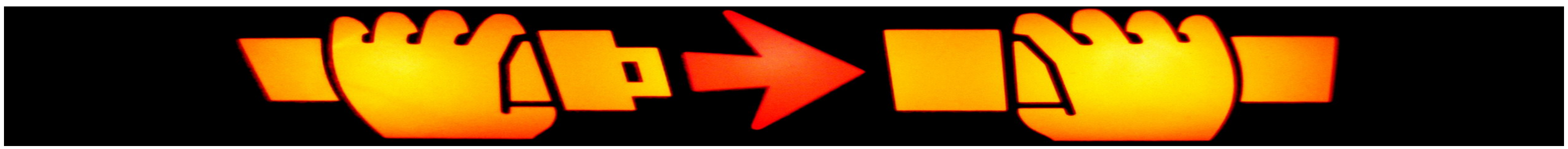

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In short,

- Is 7.6×10^{-4} a **probability**?
- **of what**?

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The New York Times, Tuesday, April 5:

“Physicists at the Fermi National Accelerator Laboratory are planning to announce Wednesday that they have found a suspicious bump in their data that could be evidence of a new elementary particle or even, some say, a new force of nature.

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Eureka!!

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[Do not ask me how 7.6×10^{-4} becomes $< 2.5 \times 10^{-3}$ (but this can be considere a minor detail. . .)]

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From my experience, journalists might make imprecisions, bad they do not invent pieces of news [. . . at least scientific ones. . . :-)]

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$1/1375 = 7.3 \times 10^{-4} \;\Rightarrow\; P(\text{No stat. fluct.}) = 99.93\%$ **!**

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> *This is a big week for particle physicists, and even they will be having many sleepless nights over the coming months trying to grasp what it all means.*
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It seems we are understanding well, besides the fact of how 99.9% becomes 99.7%. . .

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But, at the end of the post:

1. “My money is on the false alarm at the moment,...”
2. "...but I would be very happy to lose it.”
3. “And I reserve the right to change my mind rapidly as more data come in!”

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Assolutetly meaningful! (A part from the initial mismatch)

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But how <u>must</u> our convictions <u>rationally</u> change on the light of new experimental data? Is there a logical rule?

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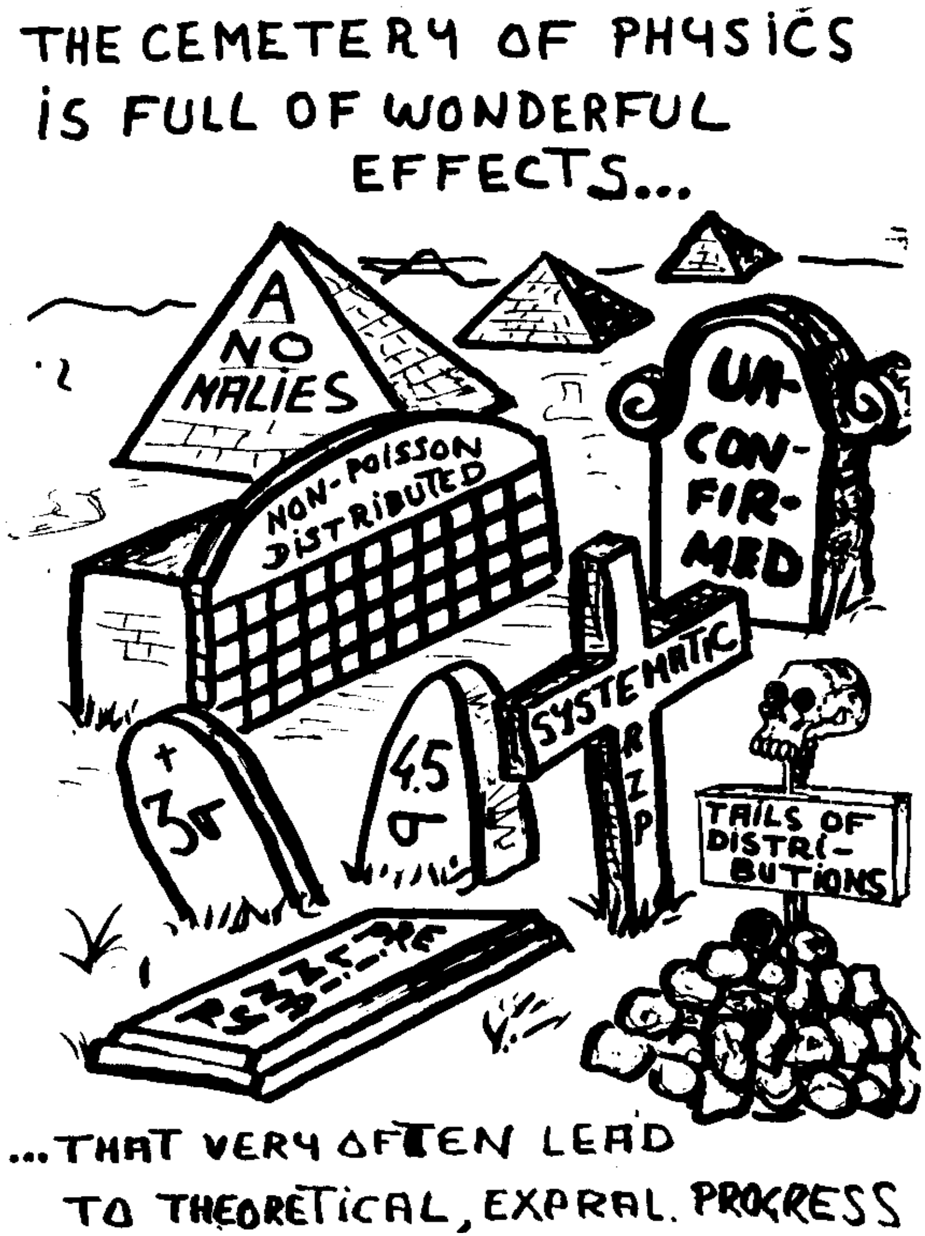
“de Rujula’s paradox”:

> *“If you disbelieve every result presented as having a 3 sigma – or ”equivalently” a 99.7% chance – of being correct. . . You will turn out to be right 99.7% of the times.”*
>
> (Alvaro de Rujula, private communication)

The cemetery of Physics



Alvaro de Rujula

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  - let's start from a 'conventional' model
    [Standard Modell, rather 'extablished theory', etc:]

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Let's review the practice and what is behind it ⇒

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It seems OK – *'obvious'*! – but it is indeed naïve for several aspects.

Proof by contradiction . . . ‘extended’. . .

Falsification rule: to what is ‘inspired’?

Proof by contradiction ... ‘extended’...

Falsification rule: to what is ‘inspired’?

Proof by contradiction of classical, deductive logic:

- Assume that a hypothesis is true;
- Derive ‘all’ logical consequence;
- If (at least) one of the consequences is known to be false, then the hypothesis is rejected.

Proof by contradiction . . . 'extended'. . .

Falsification rule: to what is 'inspired'?

Proof by contradiction of classical, deductive logic:

- Assume that a hypothesis is true;
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Popperian falsificationism

extends the reasoning to experimental sciences

Proof by contradiction . . . 'extended'. . .

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is this extension legitimate?

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  E.g. H_i being a Gaussian $f(x \,|\, \mu_i, \sigma_i)$

  $\Rightarrow$ Given any pair or parameters $\{\mu_i, \sigma_i\}$ (i.e. $\forall H_i$), <u>all</u> values of x from $-\infty$ to $+\infty$ are <u>possible</u>.

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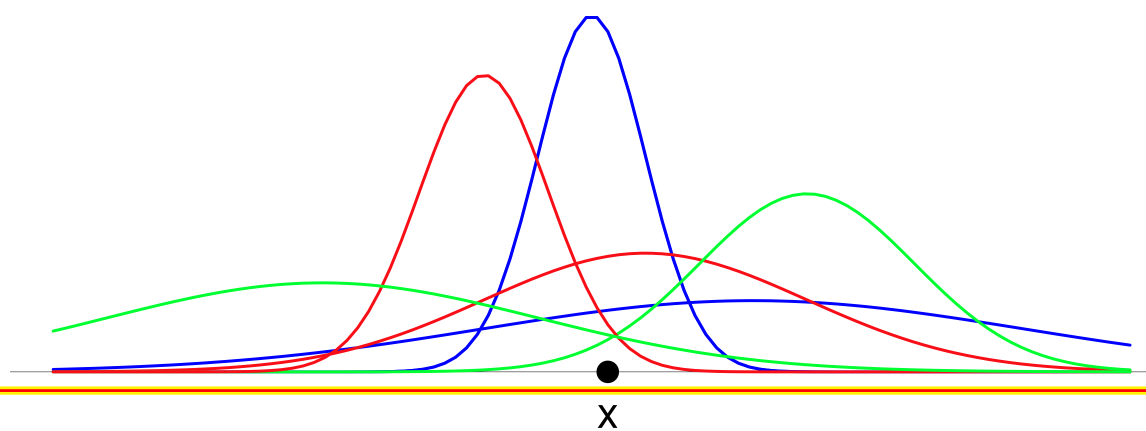
  E.g. H_i being a Gaussian $f(x \mid \mu_i, \sigma_i)$

  $\Rightarrow$ Given any pair or parameters $\{\mu_i, \sigma_i\}$ (i.e. $\forall H_i$), all values of x from $-\infty$ to $+\infty$ are possible.

  $\Rightarrow$ Having observed any value of x, none of H_i can be, strictly speaking, falsified.



Falsificationism in action. . .

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$\Rightarrow$ Practically never in the experimental sciences!

Falsificationism in action...

Obviously, this does not means that falsificationism never works, as long as no stochastic processes are involved (randomness inherent to the physical processes, or due to 'errors' in measurement). Certainly it works against itself:

- Science proceeds, in practice, rather differently:
  The natural development of Science shows that researches are carried along the directions that seem more *credibile* (and hopefully fruitful) at a given moment. A behaviour *"179 degrees or so out of phase from Popper's idea that we make progress by falsificating theories"* (Wilczek, `http://arxiv.org/abs/physics/0403115`)

Falsificationism in action. . .

Obviously, this does not means that falsificationism never works, as long as no stochastic processes are involved (randomness inherent to the physical processes, or due to ‘errors’ in measurement). Certainly it works against itself:

$\Rightarrow$ logically speaking, falsificationism has to be considered . . . falsified!

Falsificationism and statistics

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This mechanism, logically flawed, is particularly dangerous because is deeply rooted in most scientists, due to education and custom, although not supported by logic.

$\Rightarrow$ Basically responsible of all fake claims of discoveries in the past decades.

[*I am particularly worried about claims concerning our health, or the status of the planet, of which I have no control of the experimental data.*]

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B) **if** $C_i \xrightarrow[\text{small probability}]{} E$, and **we observe** E — NO

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"most likely false"

But it is behind the rational behind the statistical hypothesis tests!

Example

An Italian citizen is chosen at random and sent to take an AIDS test (test is not perfect, as it is the case in practice). *Simplified model*:

$$\begin{aligned} P(\text{Pos} \,|\, \text{HIV}) &= 100\% \\ P(\text{Pos} \,|\, \overline{\text{HIV}}) &= 0.2\% \\ P(\text{Neg} \,|\, \overline{\text{HIV}}) &= 99.8\% \end{aligned}$$

H_1='HIV' (Infected) E_1 = Positive

H_2='$\overline{\text{HIV}}$' (Not infected) E_2 = Negative

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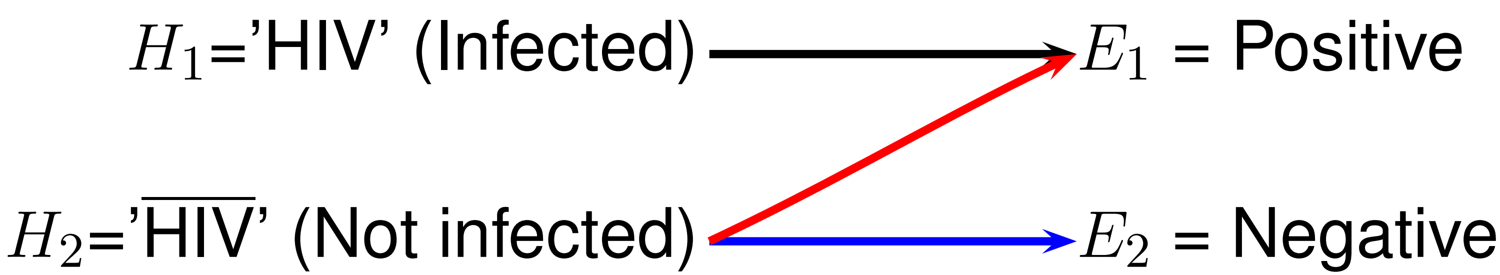

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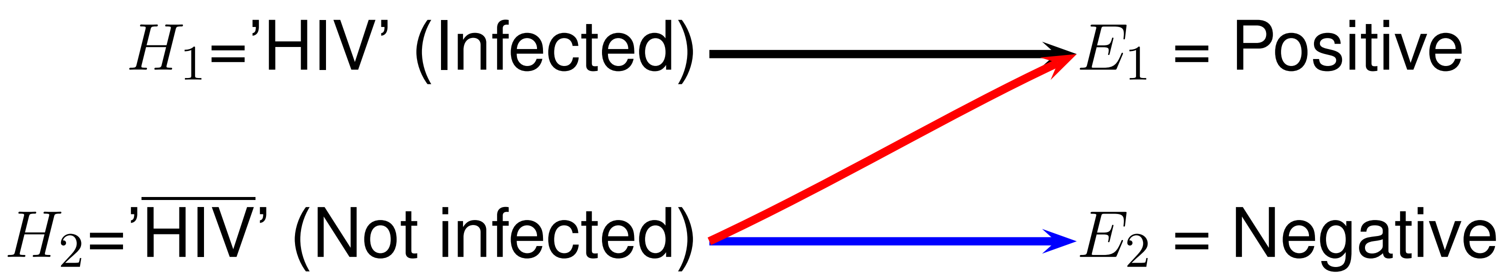

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Result: $\Rightarrow$ Positive

HIV or not HIV?

What shall we conclude?

Being $P(\text{Pos}\,|\,\overline{\text{HIV}}) = 0.2\%$ and having observed ‘Positive’, can we say

- ”It is practically impossible that the person is healthy, since it was practically impossible that an healthy person would result positive”?

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Instead, $P(\text{HIV}\,|\,\text{Pos, randomly chosen Italian}) \approx 45\%$

Think about it (a crucial information is missing!)

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Instead, $P(\text{HIV} \,|\, \text{Pos, randomly chosen Italian}) \approx 45\%$

$\Rightarrow$ Serious mistake! (not just 99.8% instead of 98.3%)

$P(A \mid B) \leftrightarrow P(B \mid A)$

Pay attention no to arbitrary revert conditional probabilities:

In general $P(A \mid B) \neq P(B \mid A)$

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In particular

- A cause might produce a given effect with very low probability, and nevertheless could be the most probable cause of that effect, often the only one!

‘Low probability’ events

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→ What was the probability to give exactly that number?:

$$\begin{aligned} P(X = 3.1416 \,|\, H_0) &= \int_{3.14155}^{3.14165} f_{\mathcal{G}}(x \,|\, \mu, \sigma) dx \\ &\approx f_{\mathcal{G}}(3.1416 \,|\, \mu, \sigma) \times \Delta x \\ &\approx f_{\mathcal{G}}(3.1416 \,|\, \mu, \sigma) \times 0.0001 \\ &\approx 39 \times 10^{-6} \end{aligned}$$

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BUT the greatest majority of the events of interest have very low probability (before occurring!).

For example, imagine a Gaussian random generator (H_0, with $\mu = 3, \sigma = 1$) gives us $X = 3.1416$.

$\rightarrow$ What is the probability that X comes from H_0?

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- Certainly NOT $\approx 39 \times 10^{-6}$;
- Indeed, it is exactly 1, since H_0 is the only cause which can produce that effect:

$$
\begin{aligned}
P(X = 3.1416 \,|\, H_0) &\approx 39 \times 10^{-6} \\
P(H_0 \,|\, X = 3.1416) &= 1\,.
\end{aligned}
$$

Probability of something else. . .

Besides the fact that the reasoning based only on the probability of the event given the cause is logically flawed, the ‘techical issue’ of low probability events which would lead to reject any hypothesis forces the statistician to rethink the question. . .

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$\rightarrow$ what matter is not the probability of the X, but rather the probability of X or of any other less probable number (or a number farther than X from the expected value – the story is a bit longer...):

$$P(X \geq 3.1416) \;=\; \int_{3.14155}^{+\infty} f_{\mathcal{G}}(x \,|\, \mu, \sigma)\, dx \approx 44\%$$

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$$P(X \geq 3.1416)\ [= P(X \geq x_{obs})] \ \Rightarrow \ \text{‘p-value’}$$

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- ⇒ Magically the result 'becomes' rather probable! Why, we, silly, worried about it?
- ⇒ The statisticians are happy...

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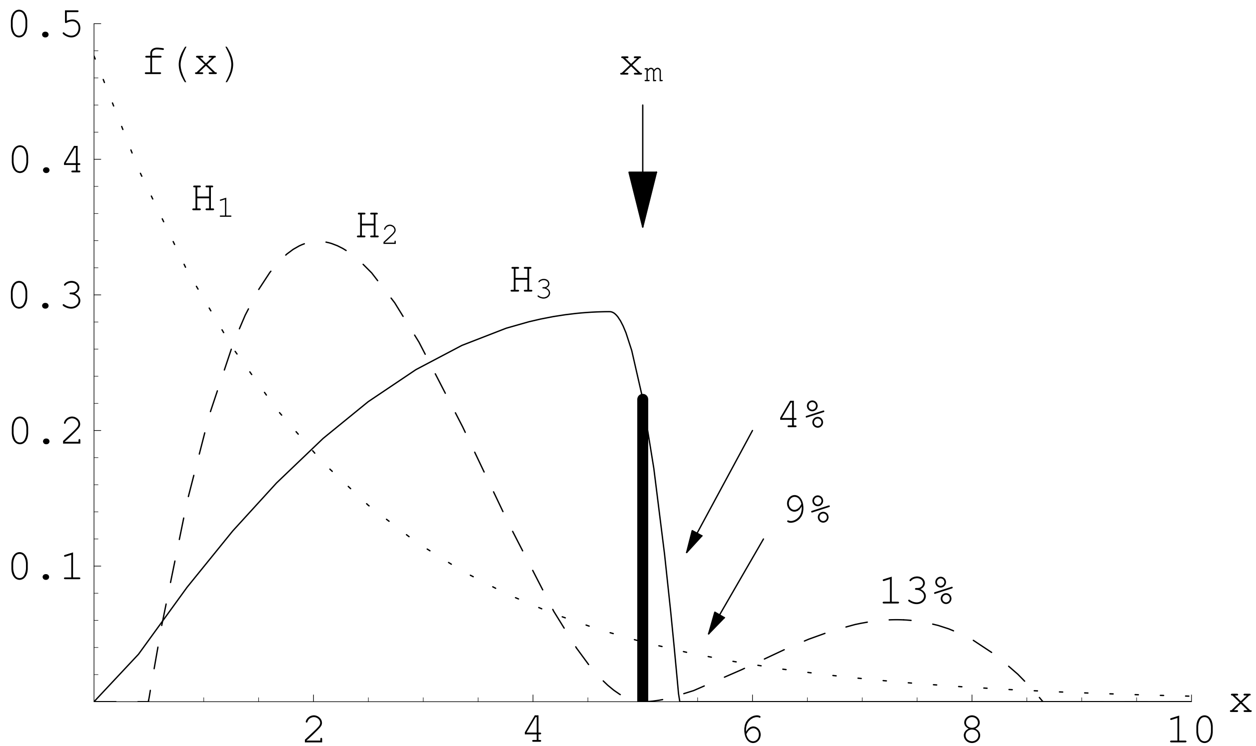
Comparing three hypotheses

Which hypothesis is favored by the experimental observation x_m?



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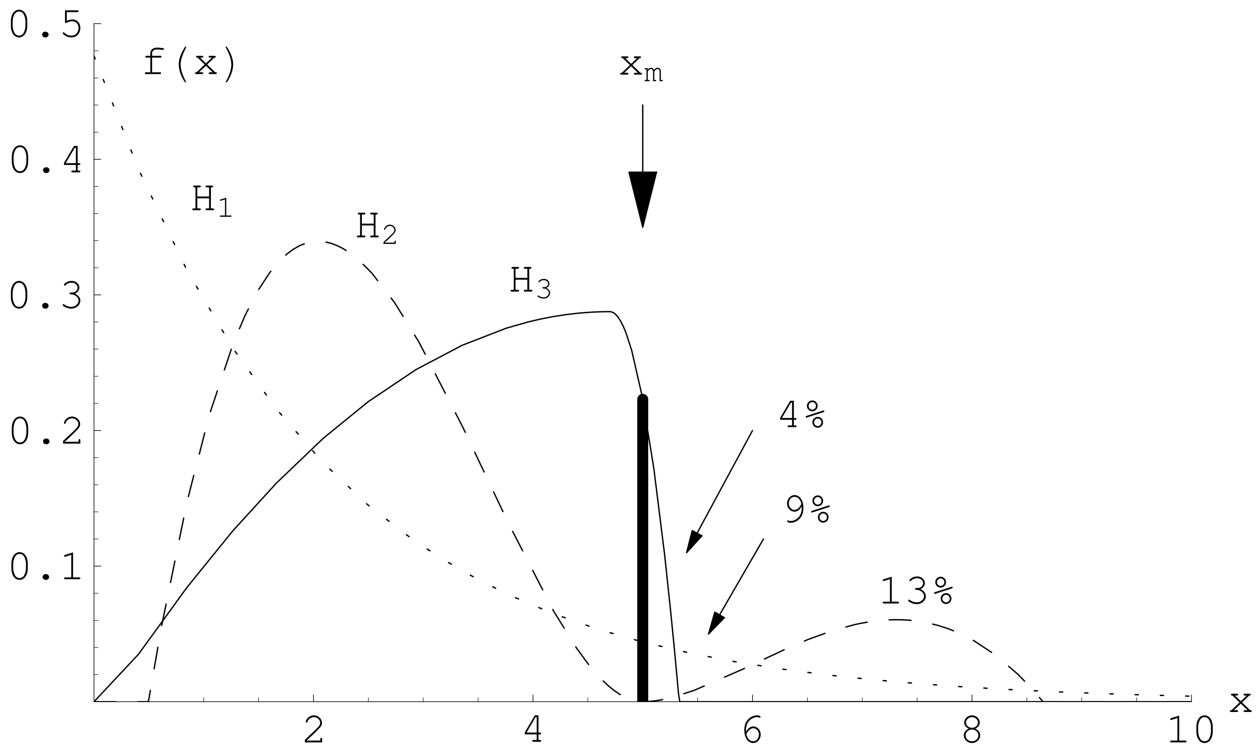

$$P(x_m \,|\, H_3) > P(x_m \,|\, H_1) > P(x_m \,|\, H_2) = 0 \quad (\,!\,)$$

Even if $P(x_m \,|\, H_i) \to 0$ (it depends on resolution)

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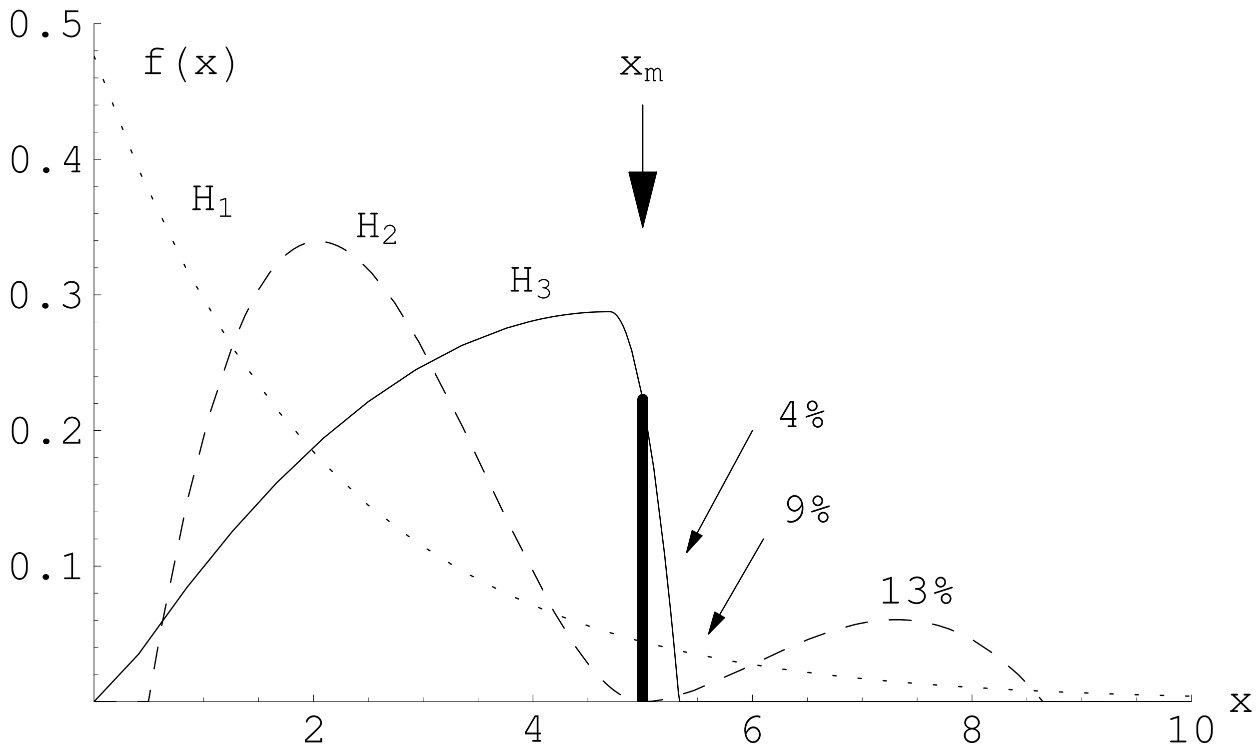

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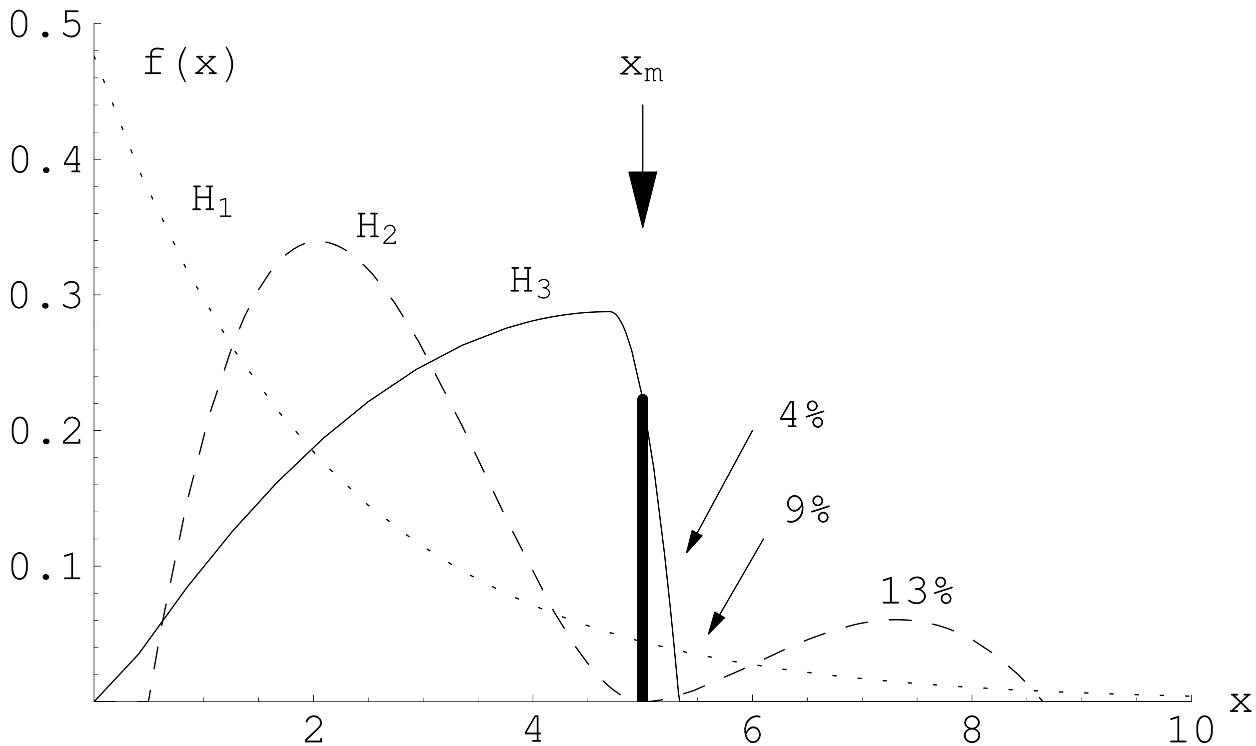

In particular, the hypothesis H_2 is (truly) falsified (impossible!), although it yields the largest 'p-value', or 'probability of the tail(s)'

An irrilevant experiment

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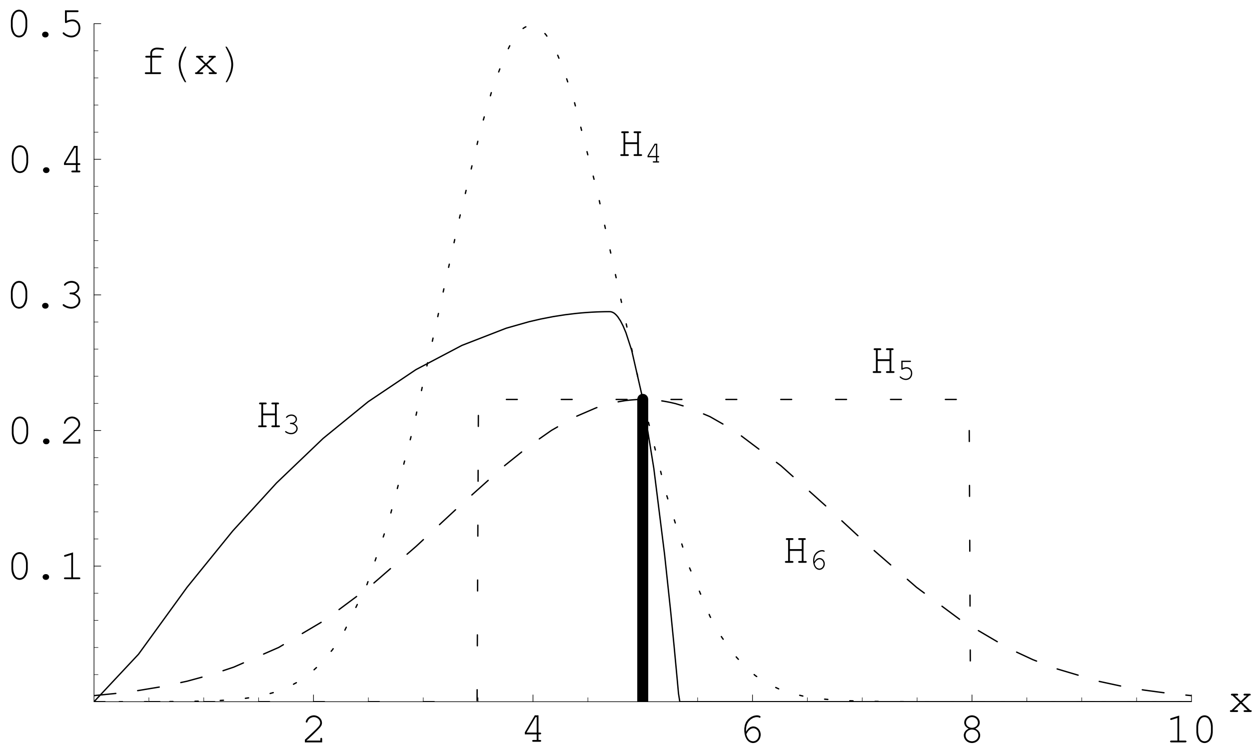

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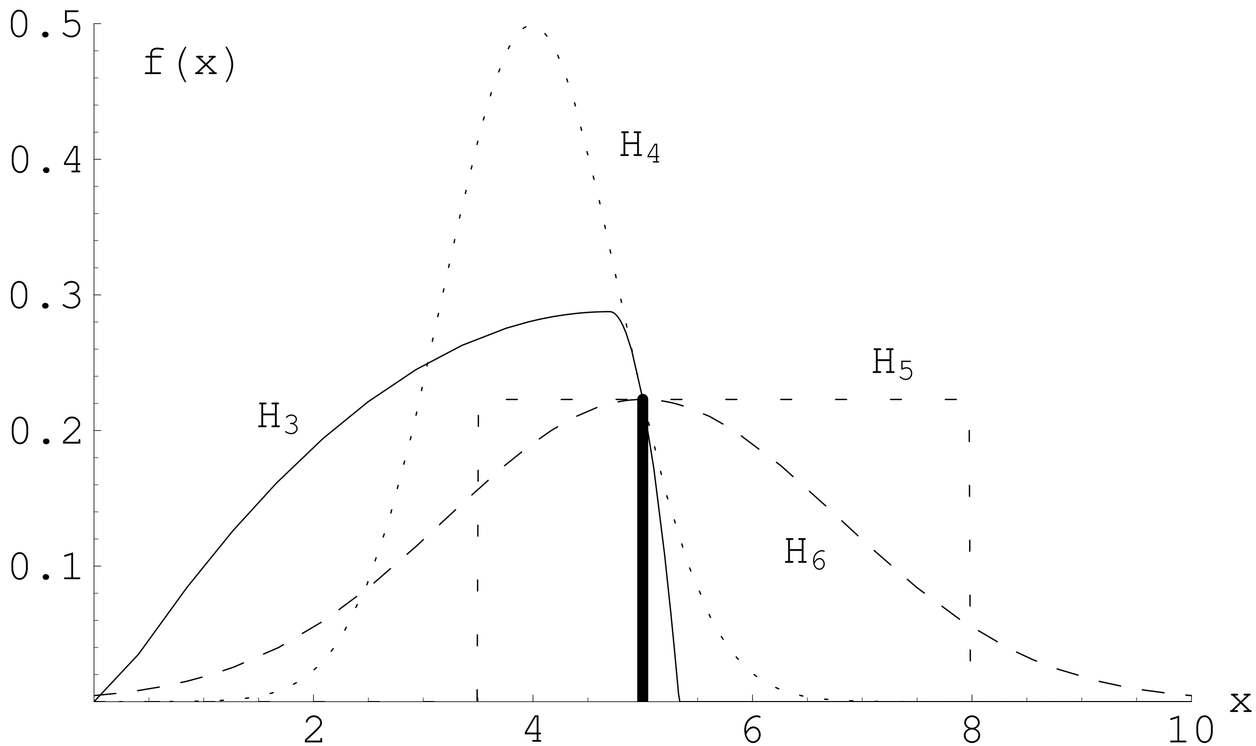

$$P(x_m \,|\, H_3) = P(x_m \,|\, H_4) = P(x_m \,|\, H_5) = P(x_m \,|\, H_6)$$

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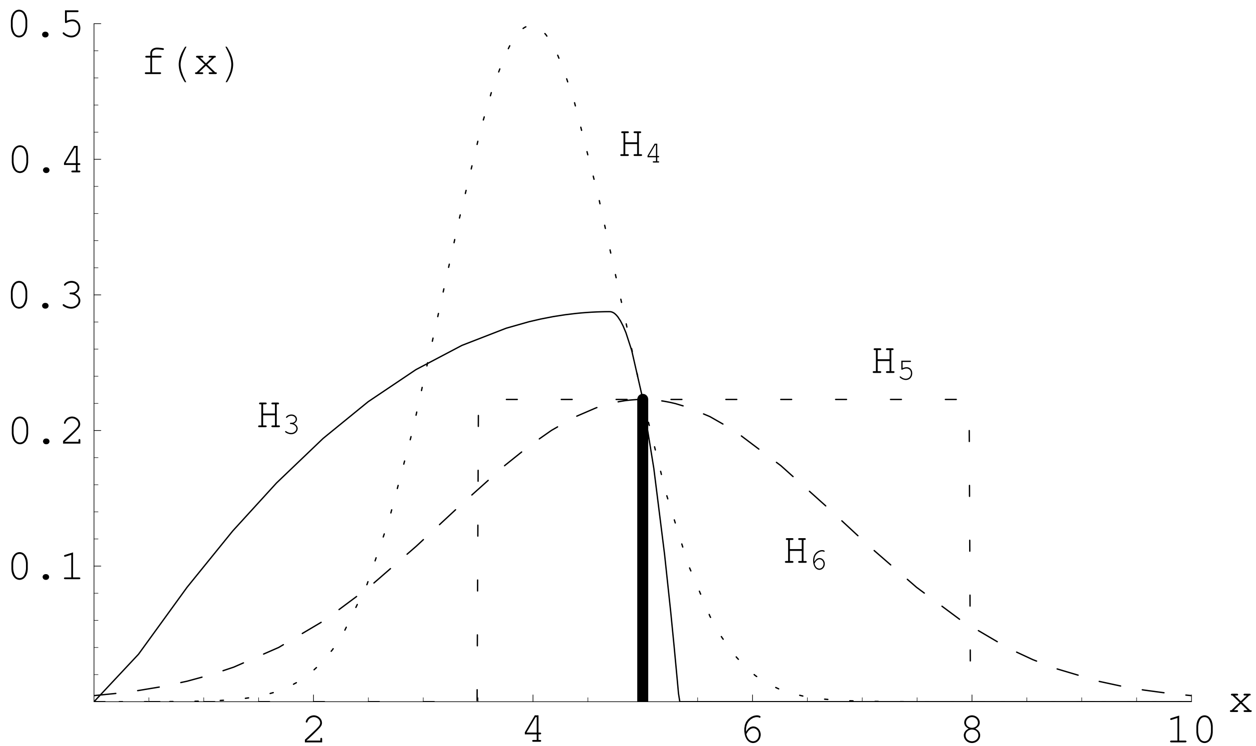

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$\Rightarrow$ *The experimental result is irrelevant!*
$\rightarrow$ we mantain our opinions about H_i

An irrelevant experiment

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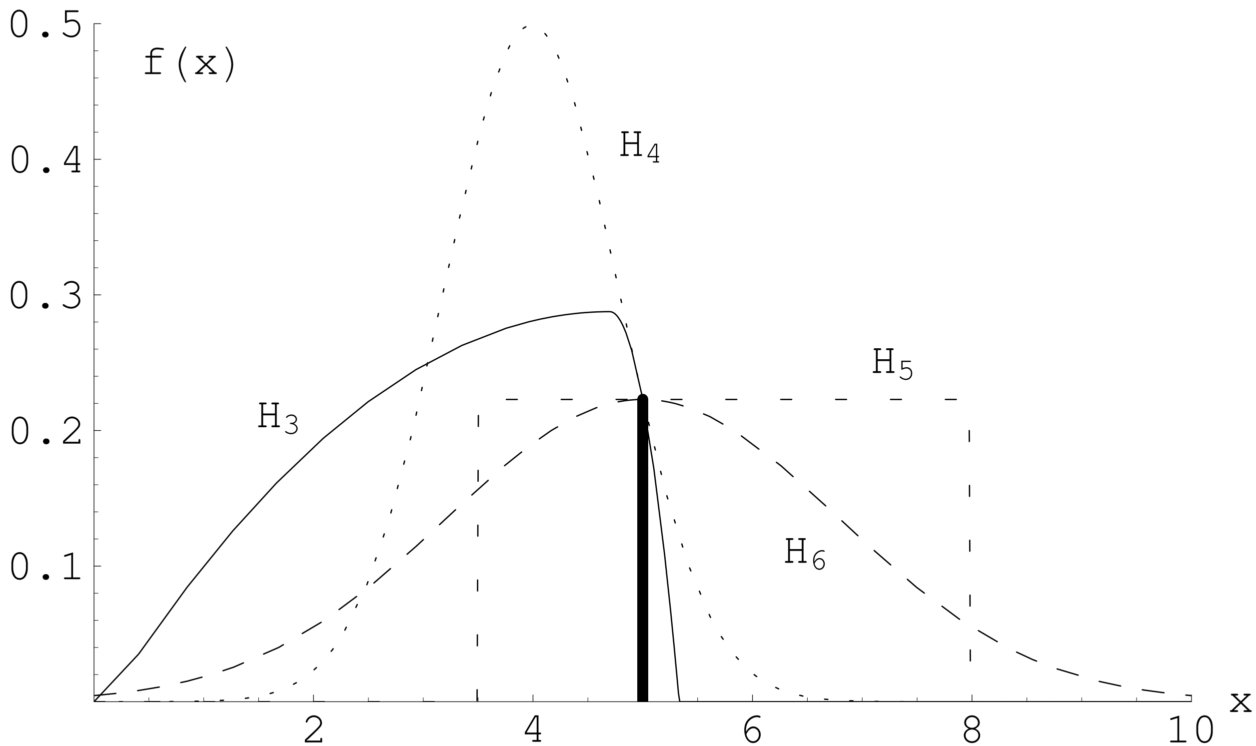

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⇒ *The experimental result is irrelevant!*

⇒ *. . . no matter what the different the p-values are!*

Which p-value?...

'p-value' = 'probability of the tail(s)'

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Of what?

Which p-value?. . .

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$\longrightarrow$ the test variable ('θ') is absolutely arbitrary:

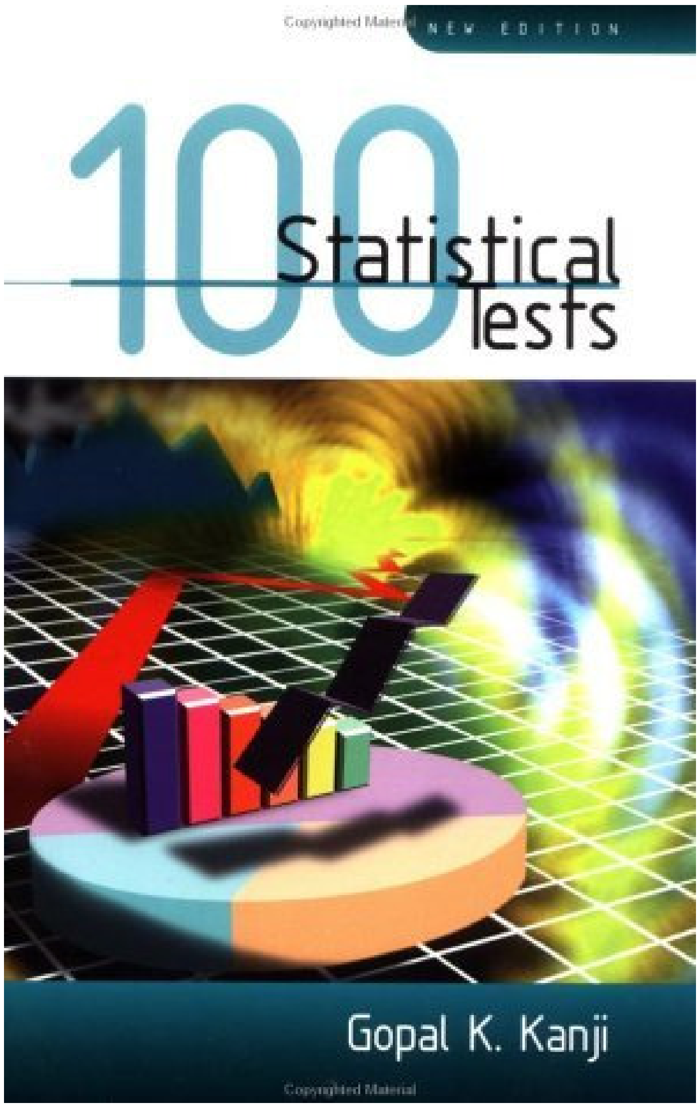
$$\theta = \theta(\mathbf{x})$$

$$\rightarrow f(\theta) \quad \text{[p.d.f]}$$

Experiment: $\rightarrow \theta_{mis} = \theta(\mathbf{x}_{mis})$

p-value $= P(\theta \geq \theta_{mis})$ ('one tail')

Which p-value?. . .



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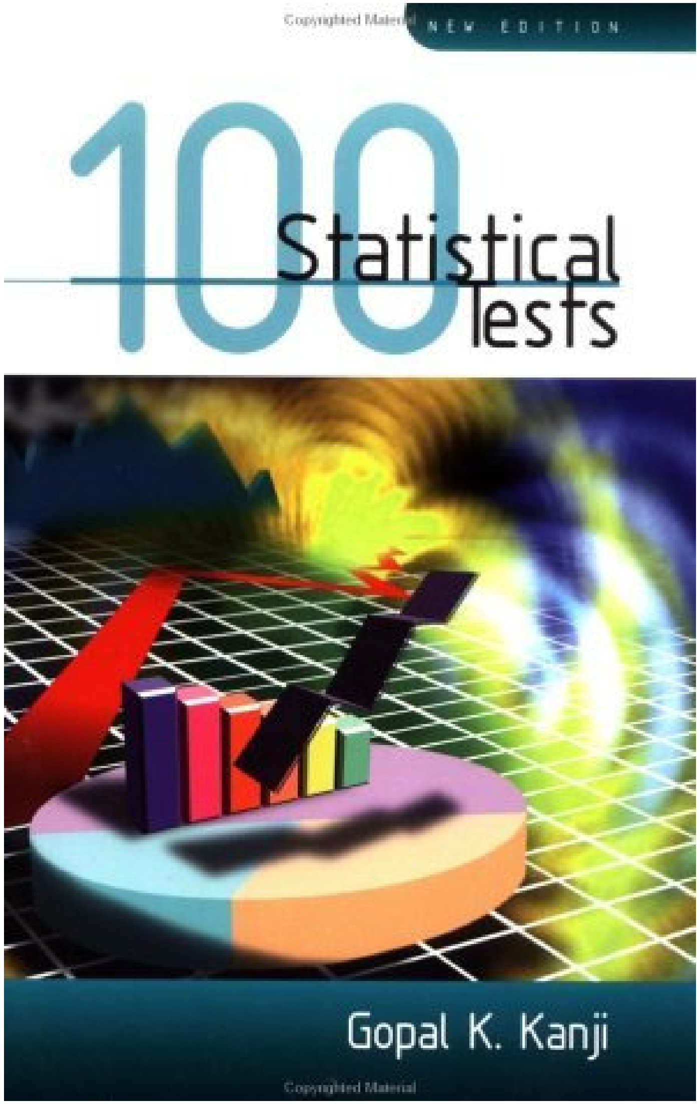

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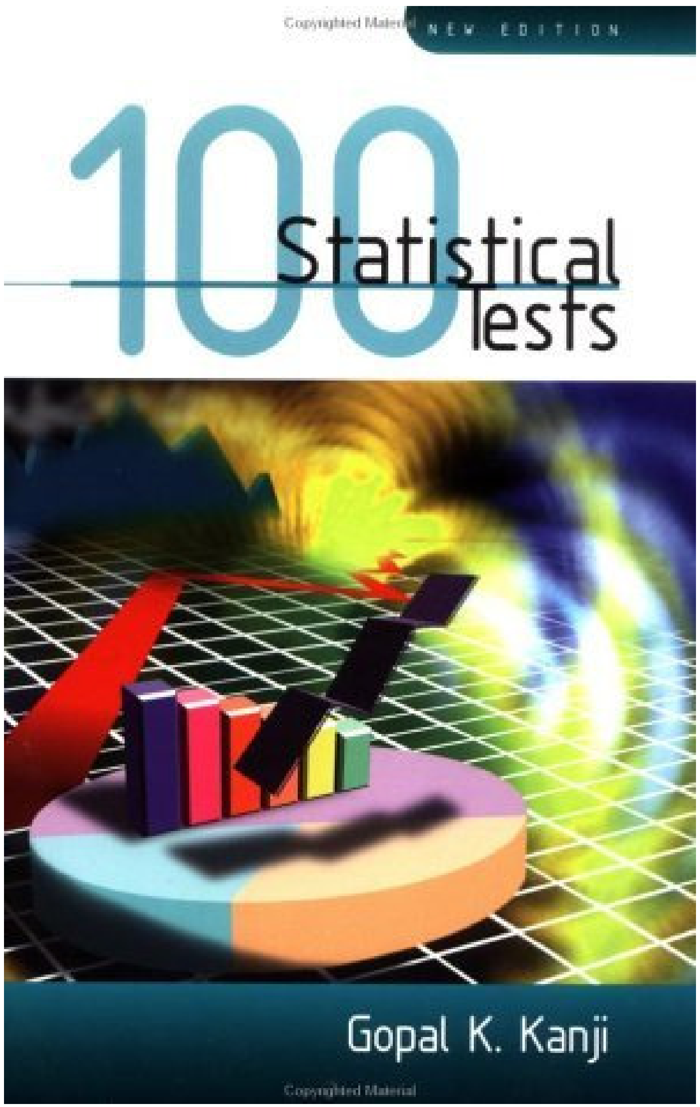

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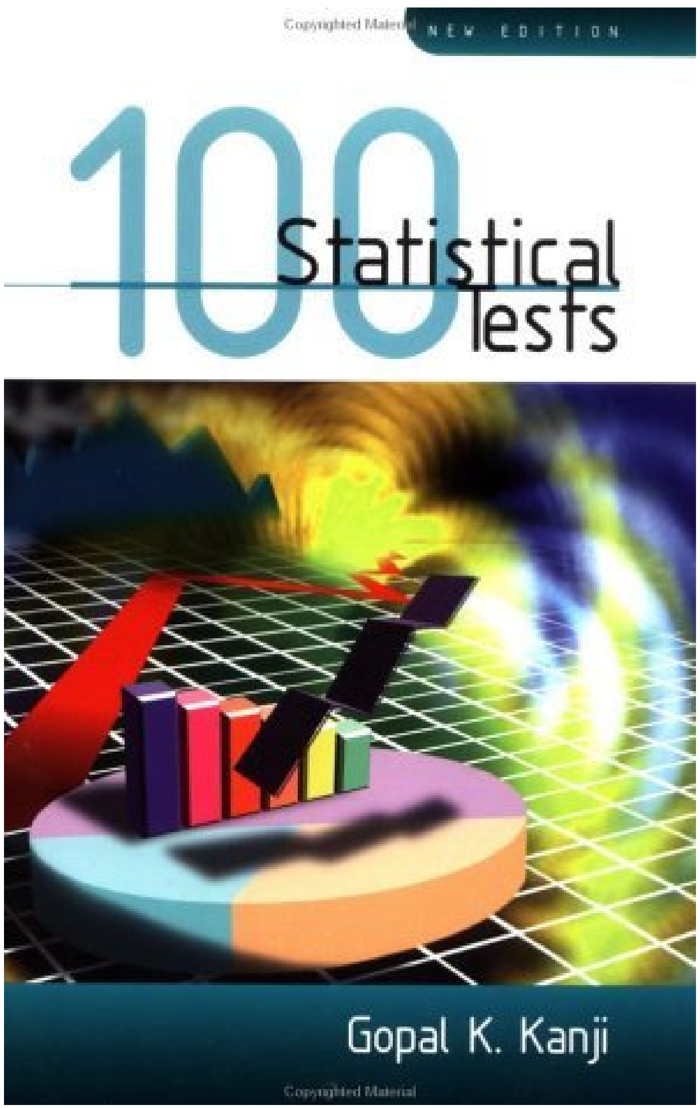

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⇒ practitioner chose the one that provide the result they like better:
→ *like if you go around until "someone agrees with you"*

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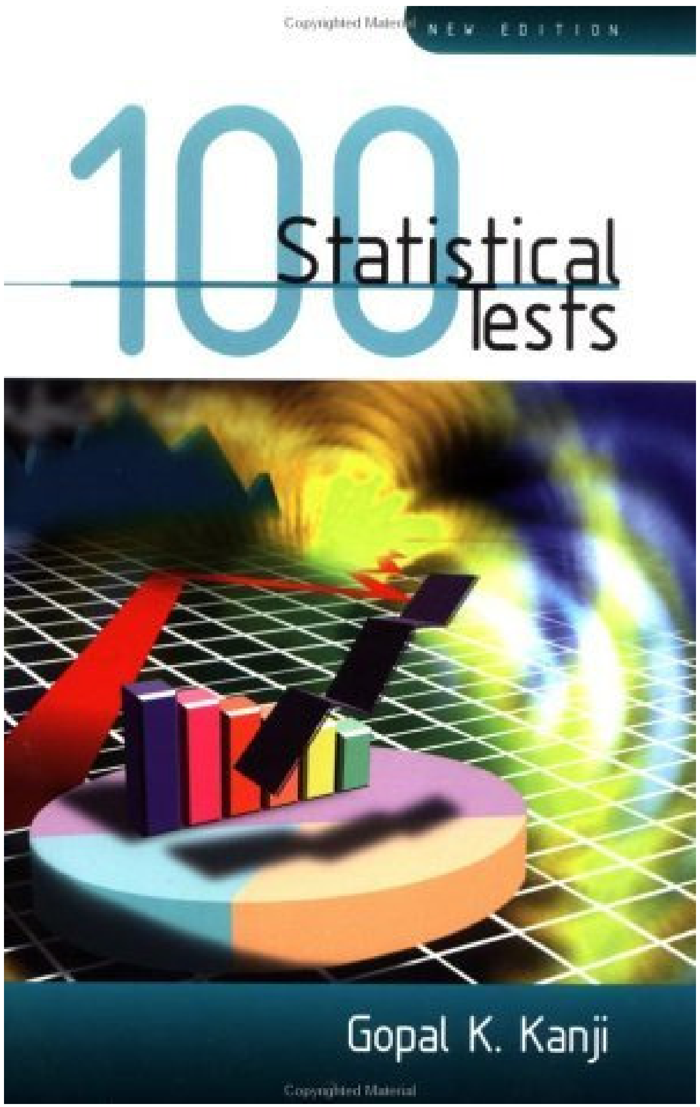

- far from exhaustive list,
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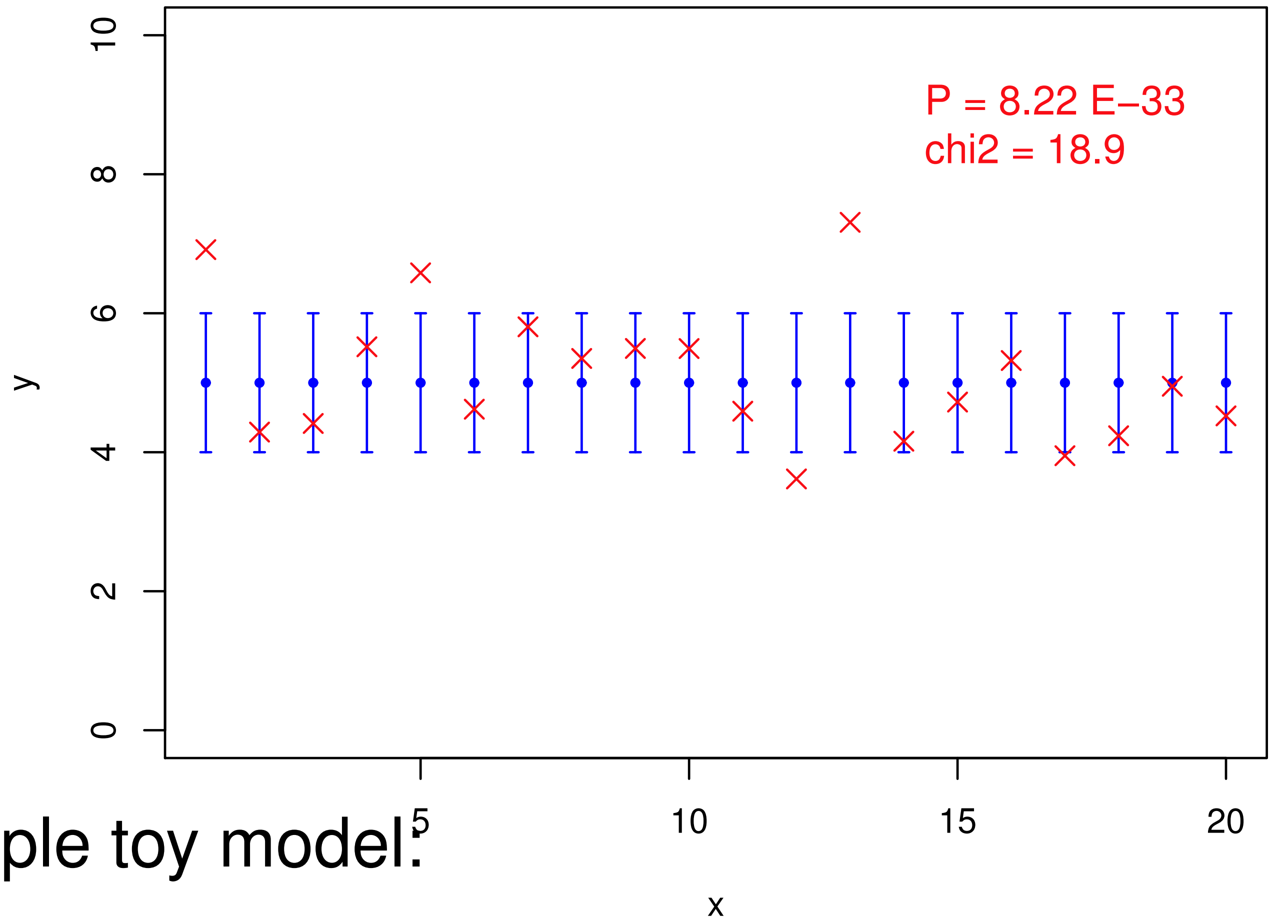
$\Rightarrow$ practitioner chose the one that provide the result they like better:
$\rightarrow$ *like if you go around until "someone agrees with you"*

- personal **'golden rule'**:
"the more exotic is the name of the test, the less I believe the result", because I'm pretty shure that several 'normal' tests have been descarded in the meanwhile...

χ^2... the mother of all p-values

Theory Vs experiment *(bars: expectation uncertainty)*:



Very simple toy model:

- True value of y: 5, independently of x (a.u.);
- Gaussian instrumental error with $\sigma = 1$.

Probability of the data sample

$P = 8.22 \times 10^{-33}$ is the probability of the 'configuration' of experimental points:

- obtained multiplying the probability of each point (independent measurements):

$$P = \prod_i P_i$$

  where
$$P_i = \int_{y_{m_i}-\Delta y/2}^{y_{m_i}+\Delta y/2} f(y)dy$$

- as seen, P_i depends on the 'resoluzion' Δy (instrumental 'discretization'):

$\rightarrow$ we use $\Delta y = \frac{1}{10}\,\sigma$

'Distance' Experiment-theory: χ^2

The costruction of the χ^2 is very popular (usually in first lab. courses – 'Fisichetta'):

$$\chi^2 = \sum_i \left(\frac{y_{m_i} - y_{th_i}}{\sigma_i}\right)^2$$

$$\rightarrow \sum_i \left(\frac{y_{m_i} - y_0}{\sigma}\right)^2$$

$$\begin{array}{rcll} \chi^2 & \sim & \Gamma(\nu/2, 1/2) & [\rightarrow \nu = 20] \\ \mathrm{E}[\chi^2] & = & \nu & [\rightarrow 20] \\ \mathrm{Var}[\chi^2] & = & 2\nu & [\rightarrow 40] \\ \mathrm{Std}[\chi^2] & = & \sqrt{2\nu} & [\rightarrow 6.3] \\ \Rightarrow & & \boxed{\chi^2 = 20 \pm 6} & \end{array}$$

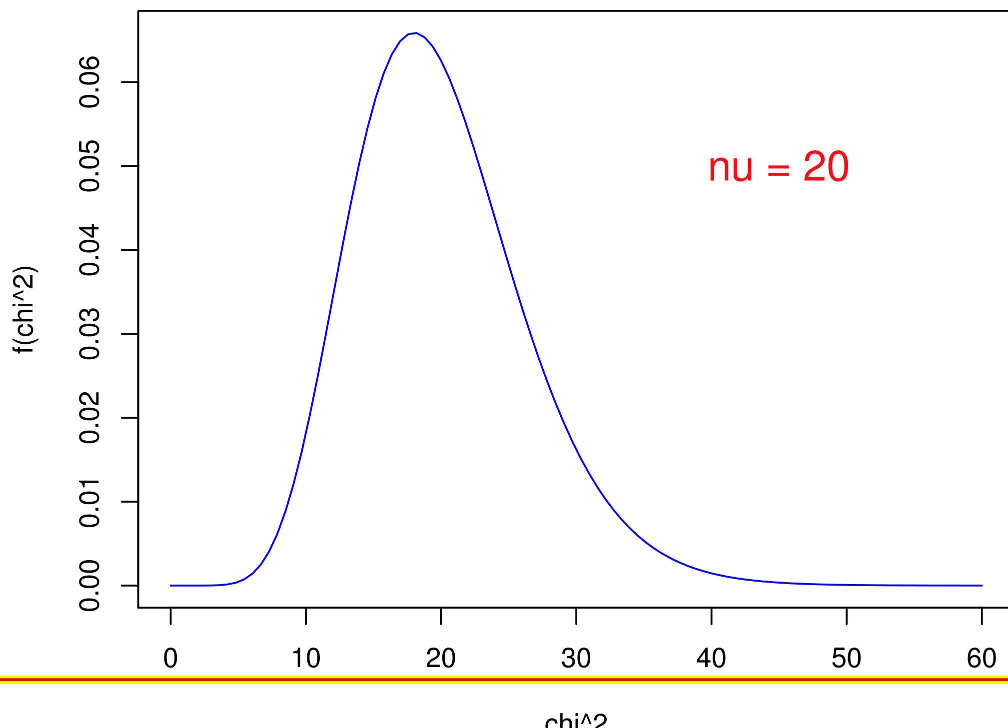
Our expectations about χ^2

$$\mathsf{E}[\chi^2] \;=\; \nu \qquad [\to 20]$$

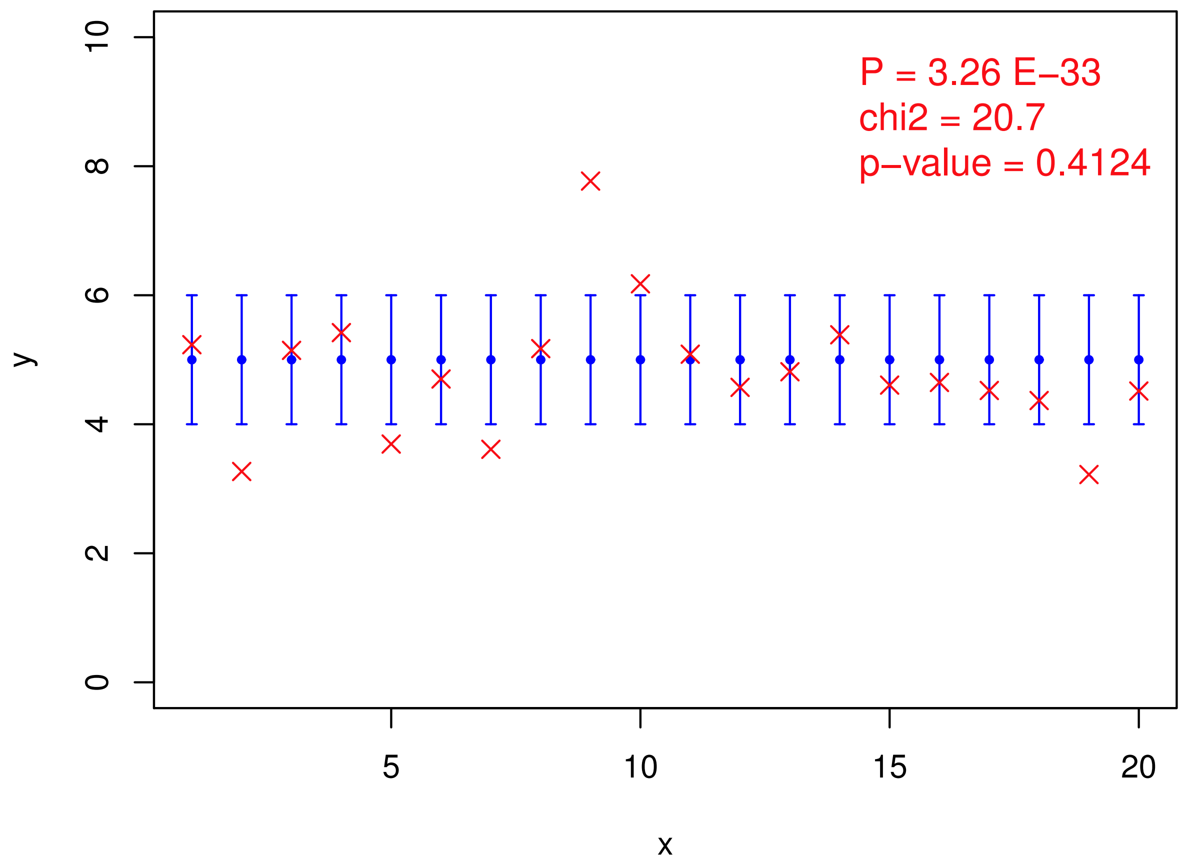
$$\mathsf{Std}[\chi^2] \;=\; \sqrt{2\nu} \qquad [\to 6.3]$$

$$\Rightarrow \quad \boxed{\chi^2 = 20 \pm 6}$$

[mode: 18]



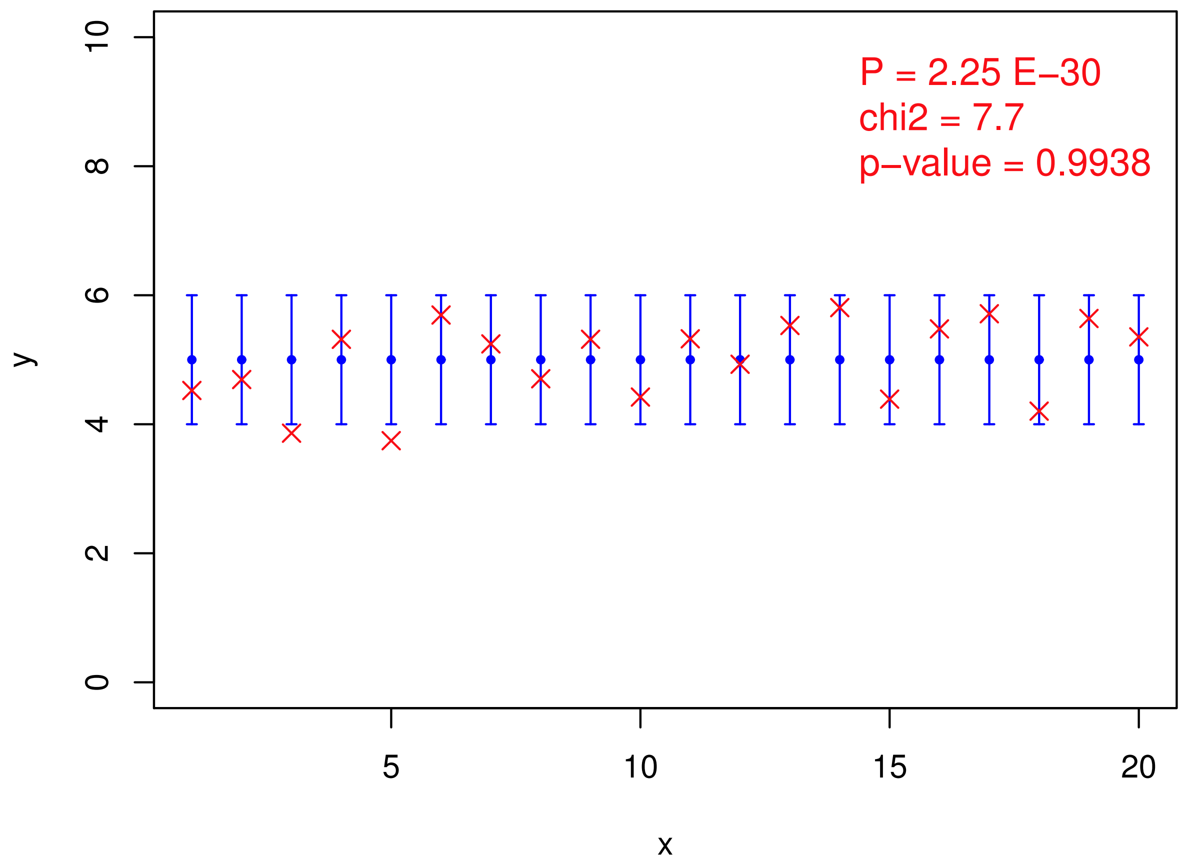
Some examples



In the average.

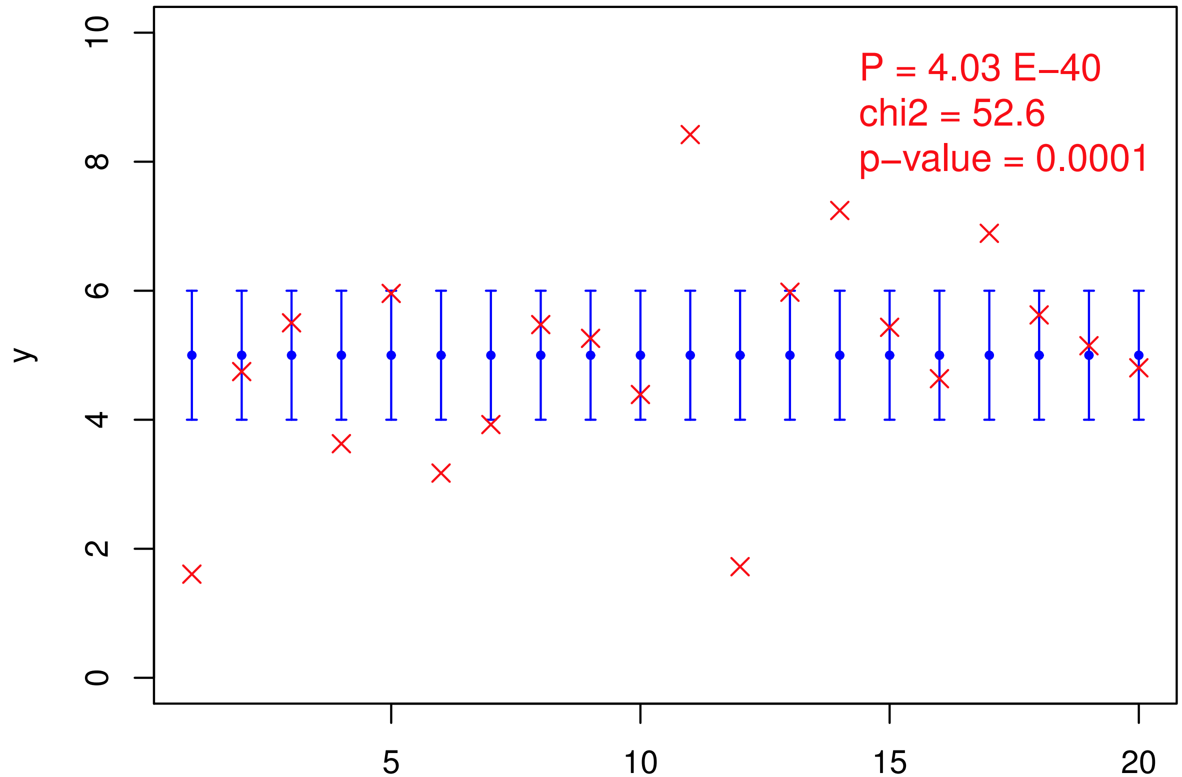
(but someone could see the points forming a 'constellation'. . .)

Some examples



Too good?

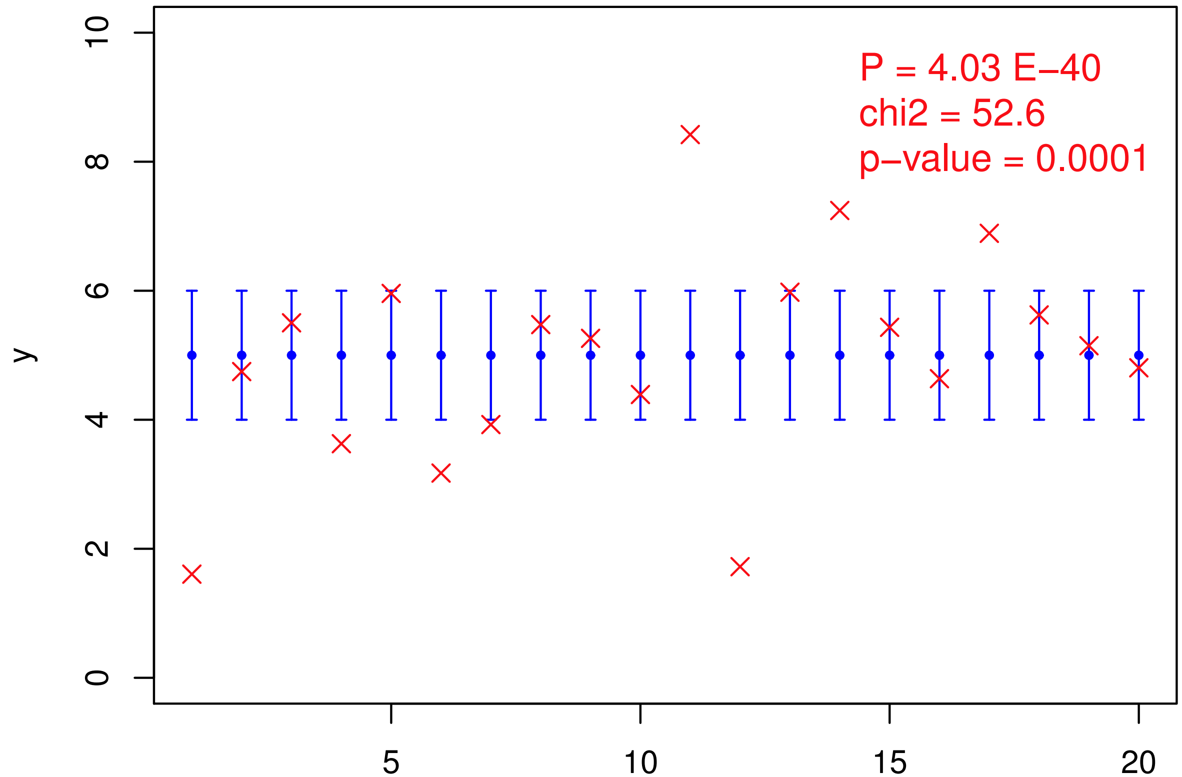
Some examples



$\chi^2 = 52.6$, with a p-value $= 0.93 \times 10^{-4}$

At limit?

Some examples

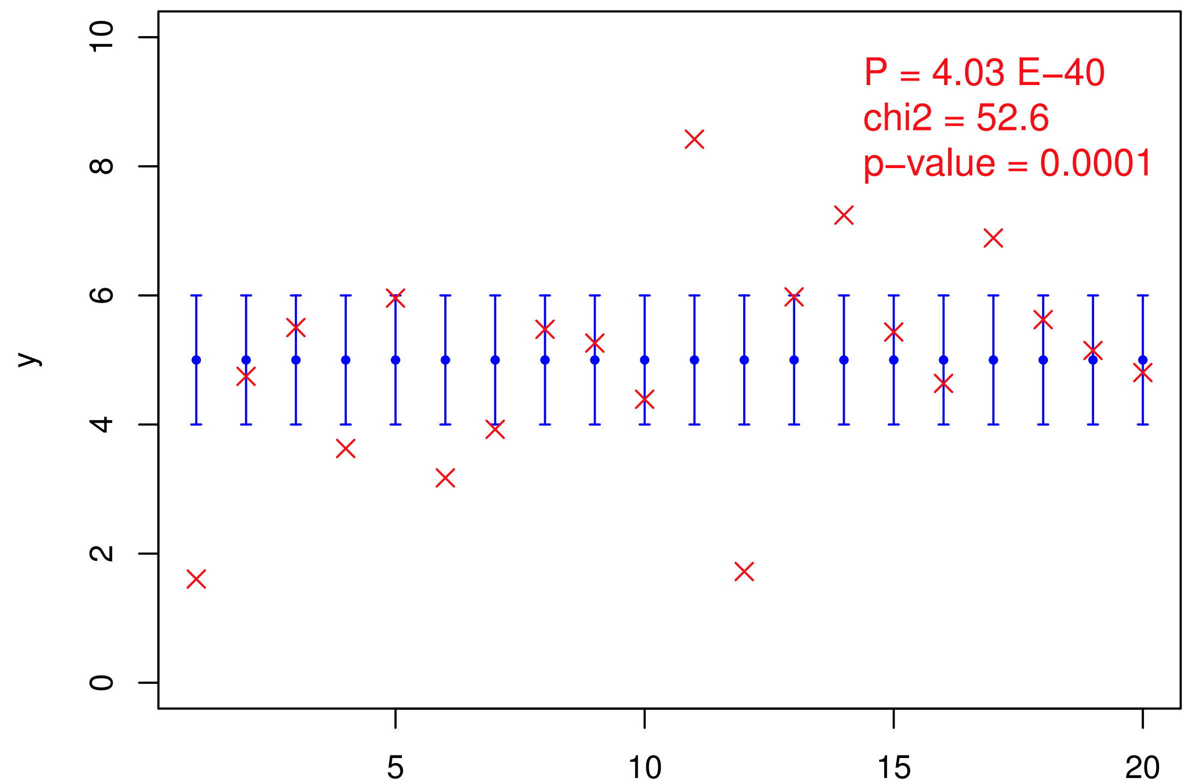


$\chi^2 = 52.6$, with a p-value $= 0.93 \times 10^{-4}$

At limit? Just come out at the first time (October 9, 13:01)

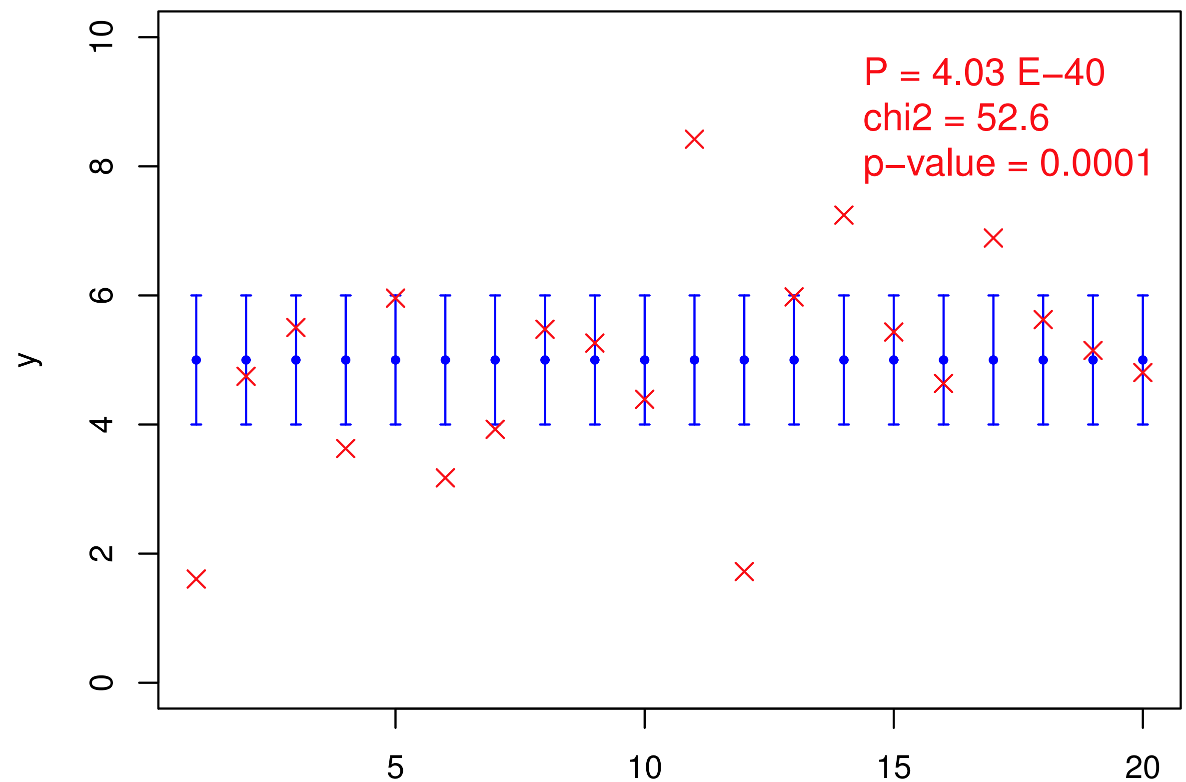
```
while(chi2.ym() < 38) source("chi2_1.R")
```

Some examples



Note: χ^2_{mis} 52.6 is 5.1σ from its expectation [$\frac{52.6-20}{\sqrt{40}} = 5.1$]

Some examples



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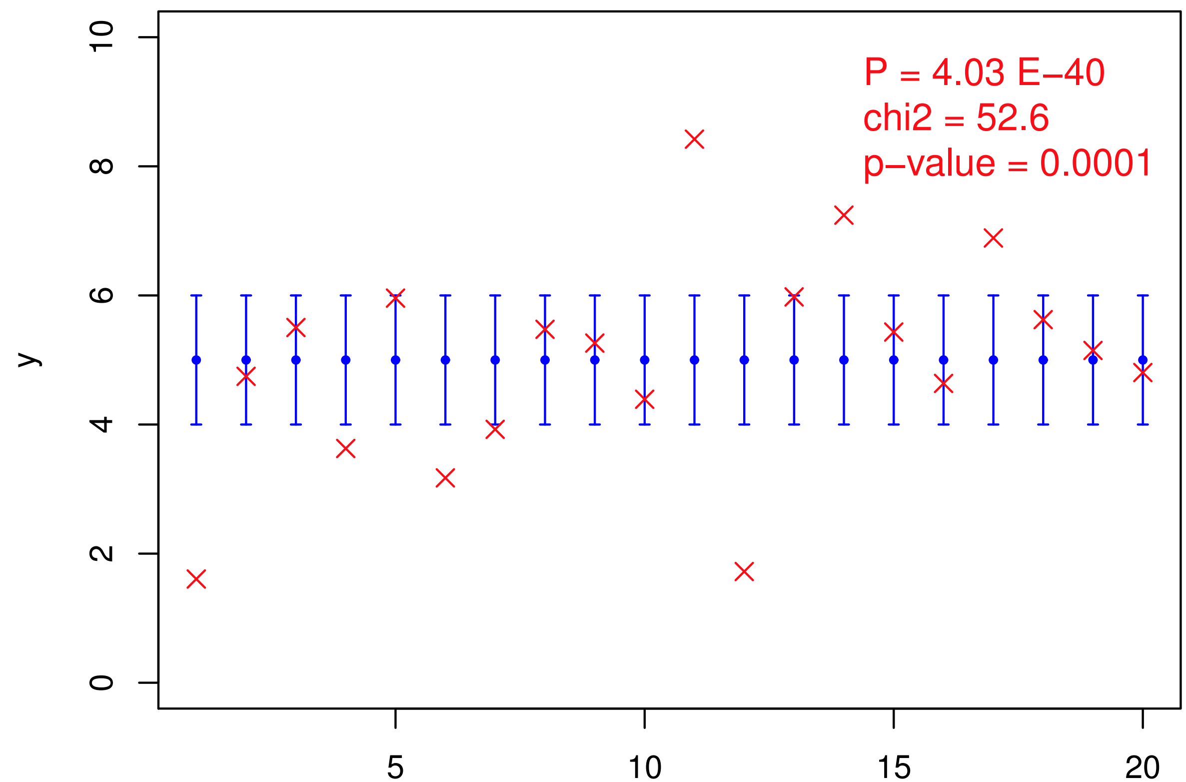

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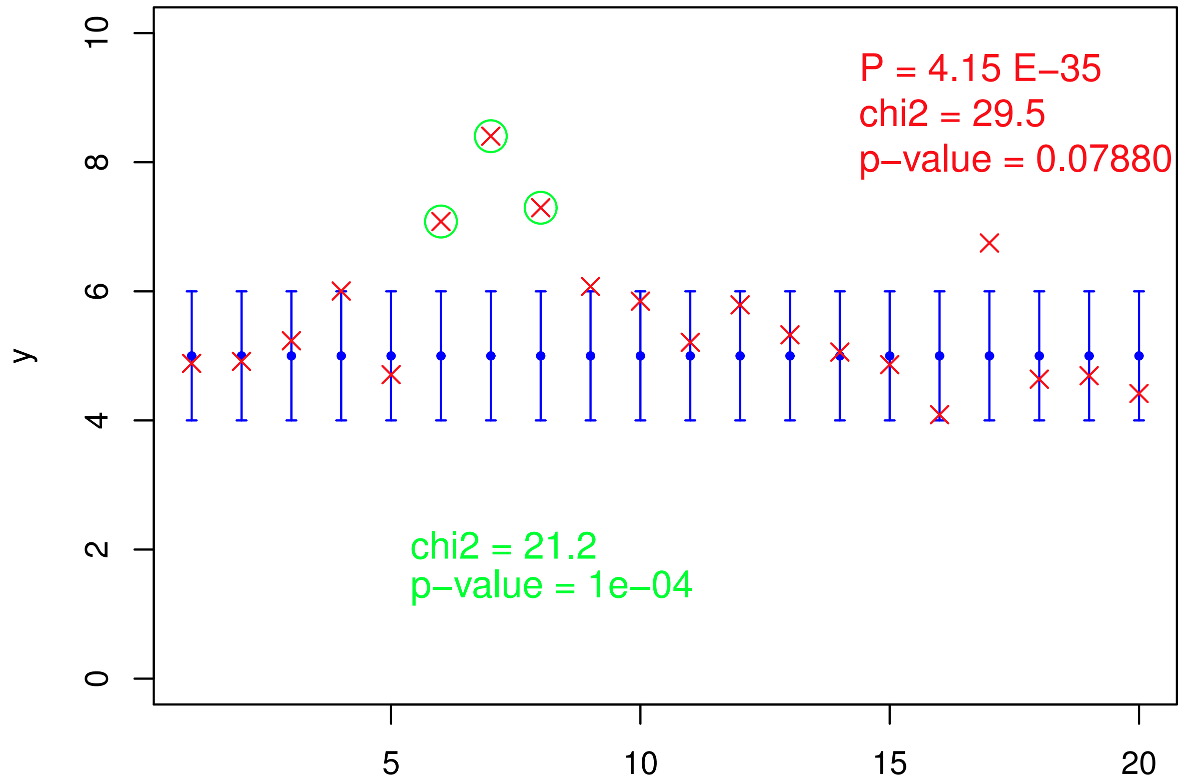
(as if there were already not enough confusion. . .)

The art of χ^2

Sometimes the χ^2 test does not give “the wished result”



Then it is calculated in the ‘suspicious region’

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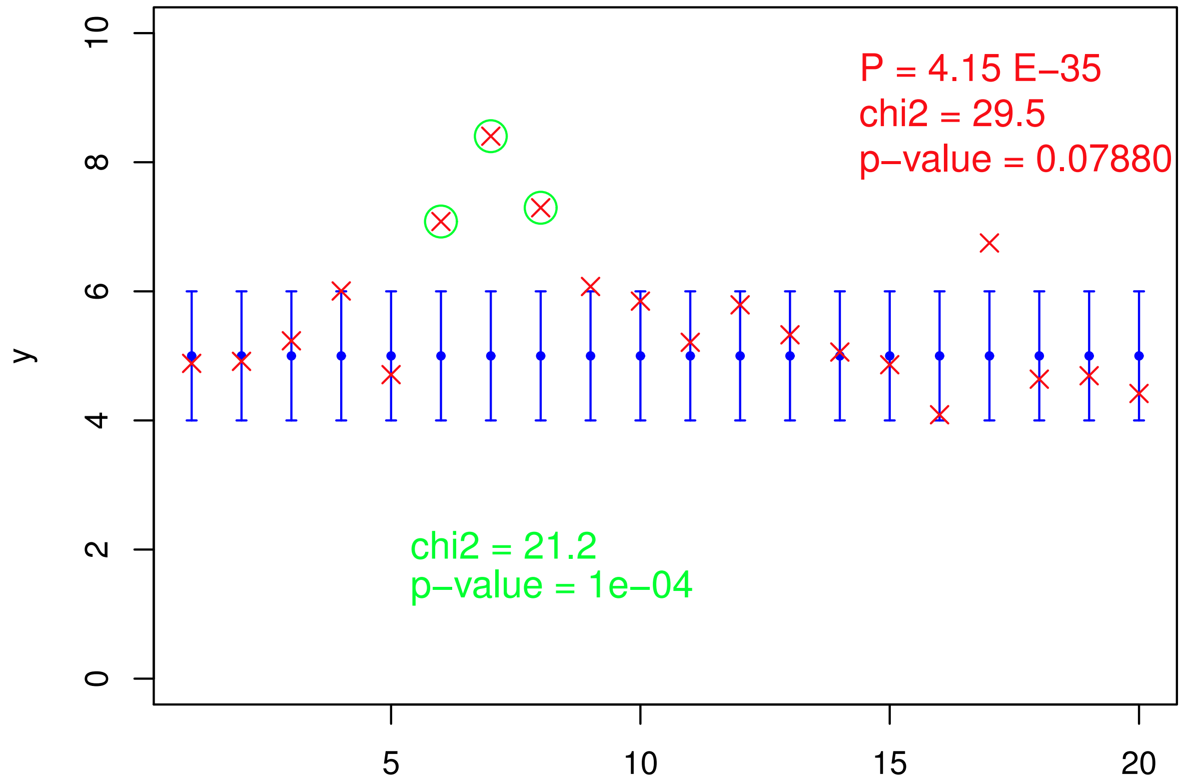

Then it is calculated in the 'suspicious region'

$\Rightarrow$ If we add the two side points, χ^2 becomes 22.2.

$\Rightarrow$ But with 5 points we had got a p-value of 5×10^{-4}

p-value: what they are

p-value:

- Probability of the tail(s) of a ‘test variable’ (a “statistic”):

$$P(\theta \geq \theta_{mis}) = \int_{\theta_{mis}}^{\infty} f(\theta \,|\, H_0)\, d\theta$$

$$P[(\theta \geq \theta_{mis}) \cap (\theta \leq (\theta^c)_{mis})] = 1 - \int_{(\theta^c)_{mis}}^{\theta_{mis}} f(\theta \,|\, H_0)\, d\theta$$

- θ is an arbitrary function of the data.
- …and often of a subsample of the data.
- $f(\theta \,|\, H_0)$ is obtained ‘somehow’, analitically, numerically, or by Monte Carlo methods.

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- What we wanted:
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$\Rightarrow$ BUT the p-value do not provide this:

$$P(\theta \geq \theta_{mis} \,|\, H_0) \not\Leftrightarrow P(H_0 \,|\, \theta_{mis})$$

$\Rightarrow$ Although they are erroneously confused with this!

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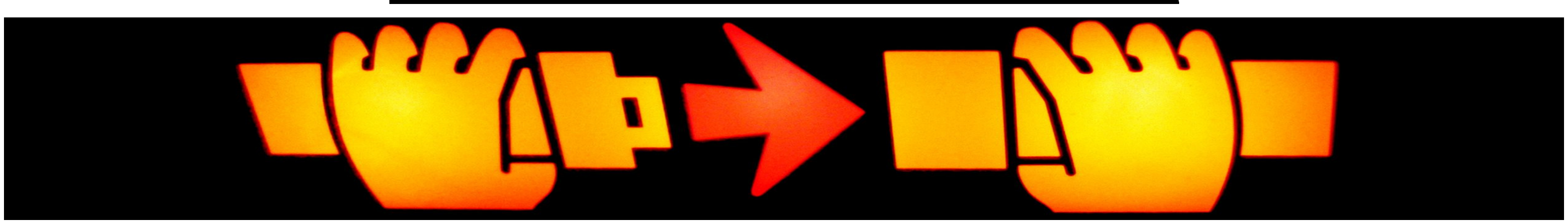
Tight seat belts!



Misunderstandings p-values

http://en.wikipedia.org/wiki/P-value#Misunderstandings

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3. The p-value is not the probability of falsely rejecting the null hypothesis.

   ...

7. ...

The 5 sigma Higgs!

July 2012

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http://www.roma1.infn.it/~dagos/badmath/#added

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  - 'Mismatch' between our natural way of thinking and the statistics theory:
  - $P(H_0 \,|\, \text{data}) \longleftrightarrow P(\theta \geq \theta_{mis} \,|\, H_0)$

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  But weren't already Gaussians, χ^2, σ's, etc.?
- The 'classical' framework of hypothesis tests misses – because explicitly forbitten! – the foundamental thing we need in our game:
- It is enough get rid of '900 statisticians (the 'frequentists') and reload 'serious guys',
  $\rightarrow$ restart from Laplace, together with Gauss, Bayes, etc.,

Beliefs and bets

Recover the natural concept of probability

- “how much I am confident in something”
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"The usual touchstone, whether that which someone asserts is merely his persuasion – or at least his subjective conviction, that is, his firm belief – is betting. It often happens that someone propounds his views with such positive and uncompromising assurance that he seems to have entirely set aside all thought of possible error. A bet disconcerts him. Sometimes it turns out that he has a conviction which can be estimated at a value of one ducat, but not of ten. For he is very willing to venture one ducat, but when it is a question of ten he becomes aware, as he had not previously been, that it may very well be that he is in error." (Kant)

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- "I am rationally ready to change my opinion"
- "...but more unlikelly hypotheses initially were, the stronger evidence is needed, possible provided (independently) by several persons I trust"

Laplace's "Bayes Theorem"

"The greater the probability of an observed event given any one of a number of causes to which that event may be attributed, the greater the likelihood of that cause {given that event}.

$$P(C_i \,|\, E) \propto P(E \,|\, C_i)$$

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"The greater the probability of an observed event given any one of a number of causes to which that event may be attributed, the greater the likelihood of that cause {given that event}. The probability of the existence of any one of these causes {given the event} is thus a fraction whose numerator is the probability of the event given the cause, and whose denominator is the sum of similar probabilities, summed over all causes.

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$$P(C_i\,|\,E) = \frac{P(E\,|\,C_i)\,P(C_i)}{\sum_j P(E\,|\,C_j)\,P(C_j)}$$

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$$P(C_i \,|\, E) \;=\; \frac{P(E \,|\, C_i)\, P(C_i)}{\sum_j P(E \,|\, C_j)\, P(C_j)}$$

"This is the fundamental principle (*) of that branch of the analysis of chance that consists of reasoning *a posteriori* from events to causes"

(*) In his "Philosophical essay" Laplace calls 'principles' the 'fondamental rules'.

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Note: denominator is just a normalization factor.

$$\Rightarrow \qquad P(C_i \,|\, E) \;\propto\; P(E \,|\, C_i)\, P(C_i)$$

Most convenient way to remember Bayes theorem

Laplace's teaching

$$\frac{P(H_0\,|\,\text{data})}{P(H_1\,|\,\text{data})} = \frac{P(\text{dati}\,|\,H_0)}{P(\text{dati}\,|\,H_1)} \times \frac{P(H_0)}{P(H_1)}$$

- We should possible use the data, rather then the test variables 'θ' (χ^2 etc);
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- If $P(\text{data}\,|\,H_i) = 0$, it follows $P(H_i\,|\,\text{data}) = 0$:
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- There is no conceptual problem with the fact that $P(\text{dati}\,|\,H_1) \to 0$ (e.g. 10^{-37}), provided the ratio $P(\text{dati}\,|\,H_0)/P(\text{dati}\,|\,H_1)$ is not undefined.

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Someone would object that p-values and, in general, ‘hypothesis tests’ *usually* do work!

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- Certainly! I agree!
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- But now we are also able to explain the reason.

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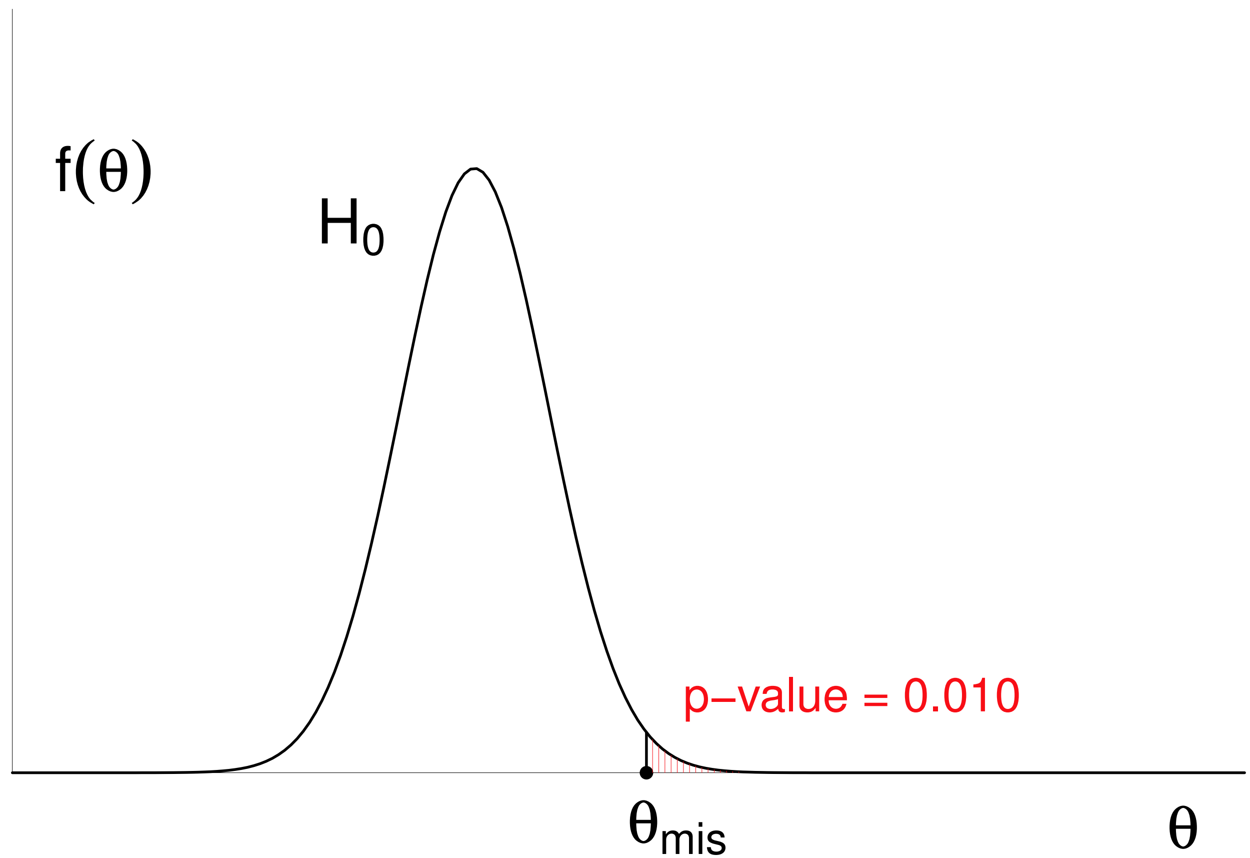

Why should the observation of θ_{mis} should diminish our confidence on H_0?

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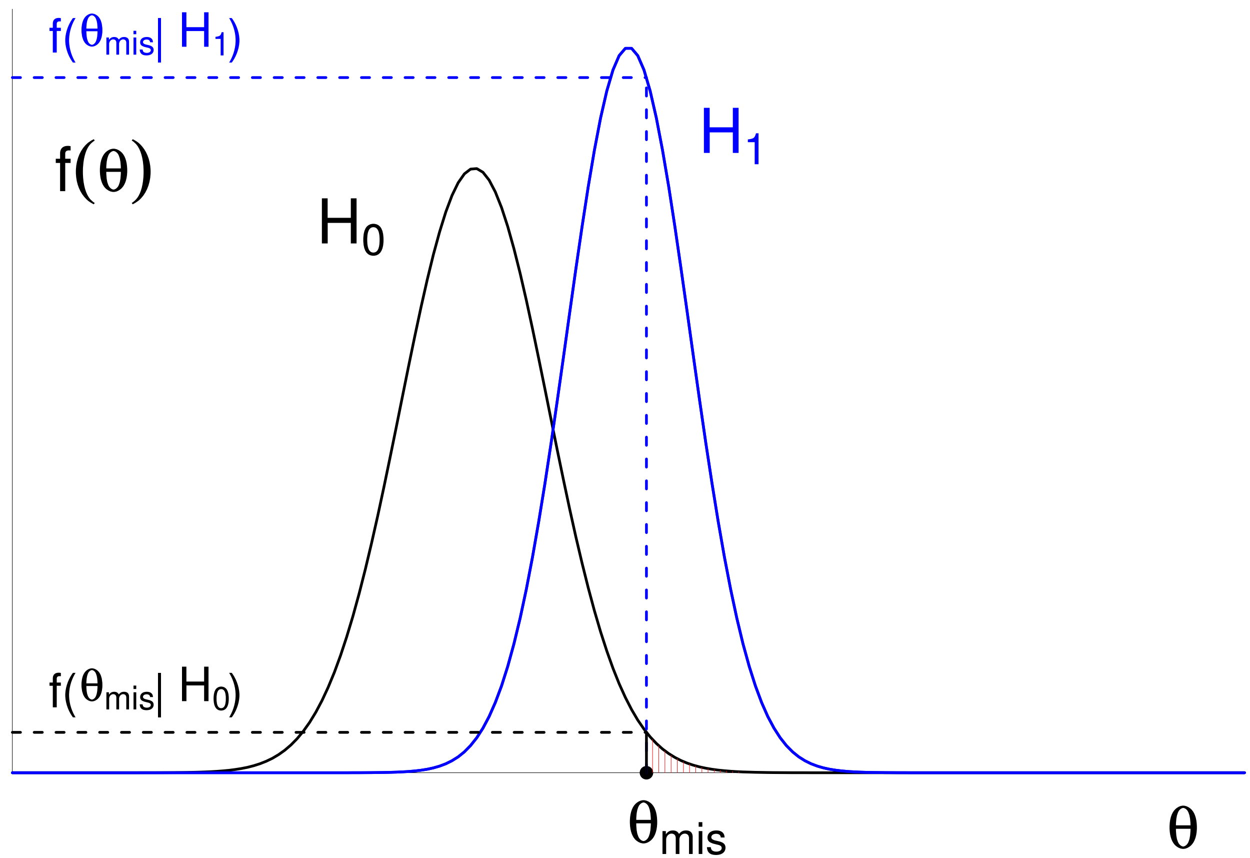

Because *often* we give *some chance* to a possible alternative hypothesis H_1, even if we are not able to exactly formulate it.

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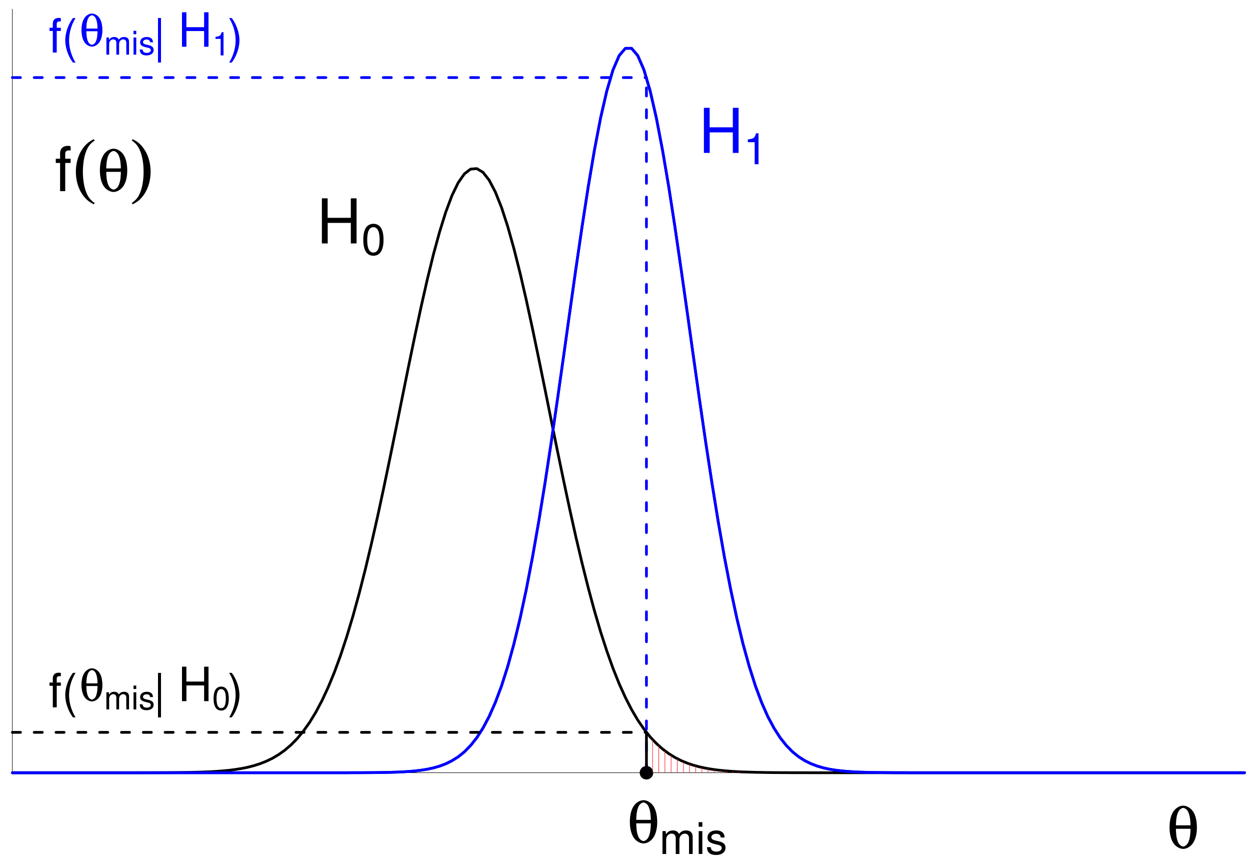

Indeed, what really matters is not the area to the right of θ_{mis}. What matters is the ratio of $f(\theta_{mis}\,|\,H_1)$ to $f(\theta_{mis}\,|\,H_0)$!
$\Rightarrow$ to a 'small' area it corresponds a 'small' $f(\theta_{mis}\,|\,H_0)$.

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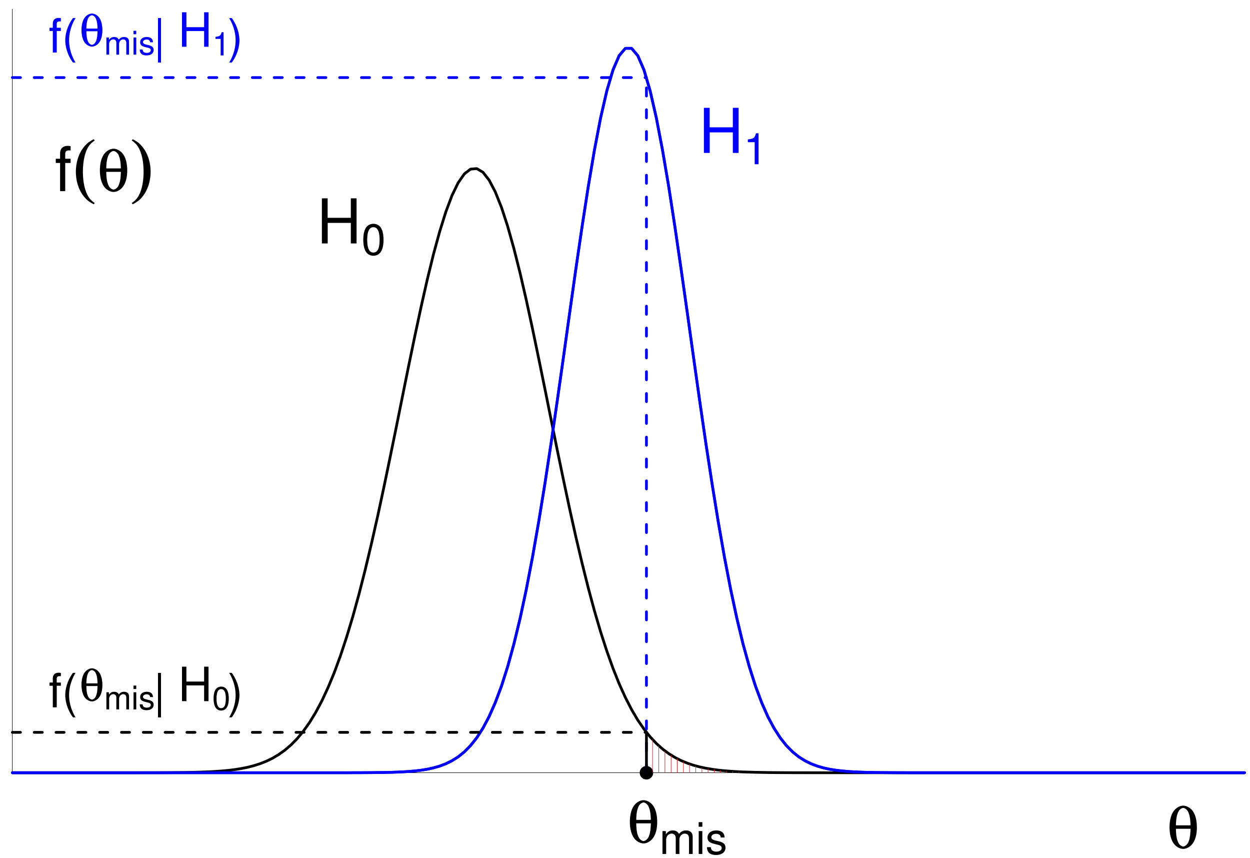

But is the alternative hypothesis H_1 is unconcievable, or hardly believable, the 'smalleness' of the area is irrelevant

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Don’t get confused by sigma’s and ’strange significances’ that do not tell you how how much to believe in the claim.

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$\rightarrow$ The excess is surely a particle only if it is the Higgs!

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It is a question of Physics not (only) of statistics:

- success of standard model;
- radiative corrections
  (the diagrams entering R.C. are essentially the same the produce the Higgs in the final state!)

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We have often listened in the past year the following statement:

> “We have discovered at CERN a new particle.
> We have to understand if it is the Higgs boson”

This statement implies that our confidence that the ≈ 126 GeV ‘excess’ is a new particle is due from the 5 sigmas alone.

It is a question of Physics not (only) of statistics:

- success of standard model;
- radiative corrections
  (the diagrams entering R.C. are essentially the same the produce the Higgs in the final state!)
- Physics is something SERIOUS! (not a statistician’s toy)

Conclusions of Part 1

Philip Ball (Guardian, 23 dicembre 2011)
(http://www.guardian.co.uk/commentisfree/2011/de

"So D'Agostini recommends that, instead of heeding impressive-sounding statistics, we should ask what scientists actually believe. Better, we should find out if they had put money on it – and how much. After all, that is a tactic endorsed by none other than Kant."

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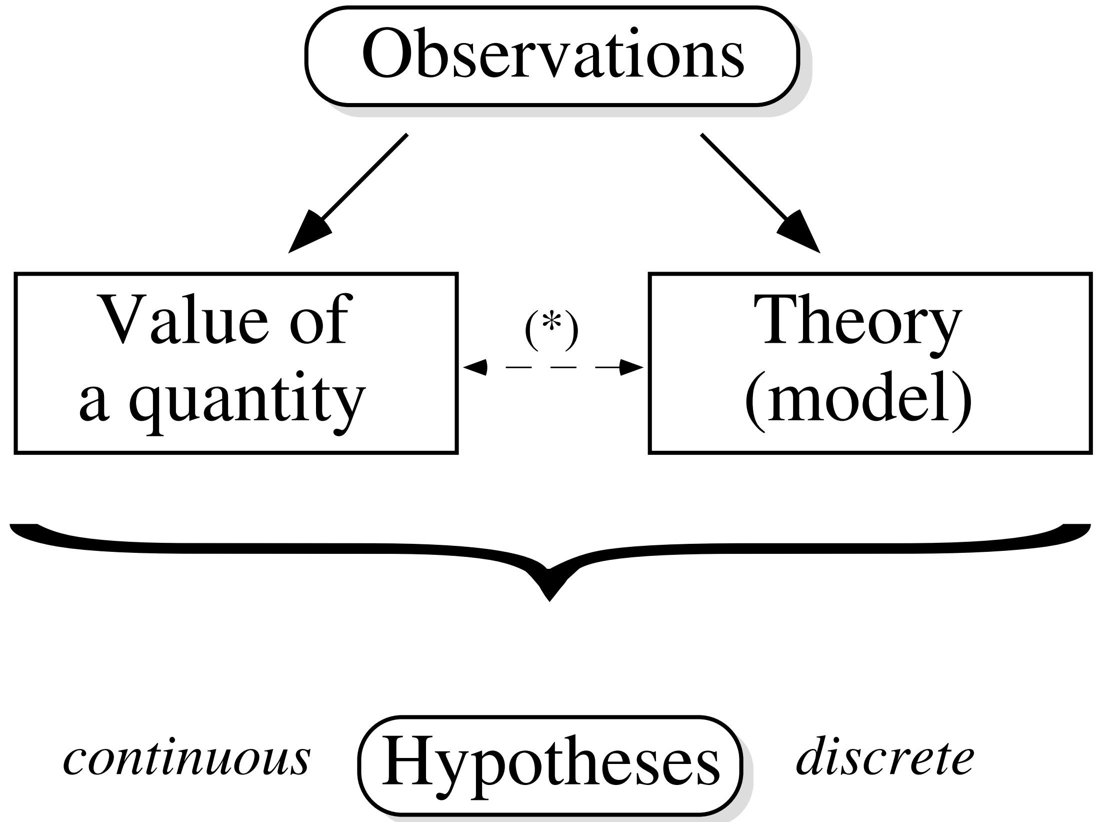
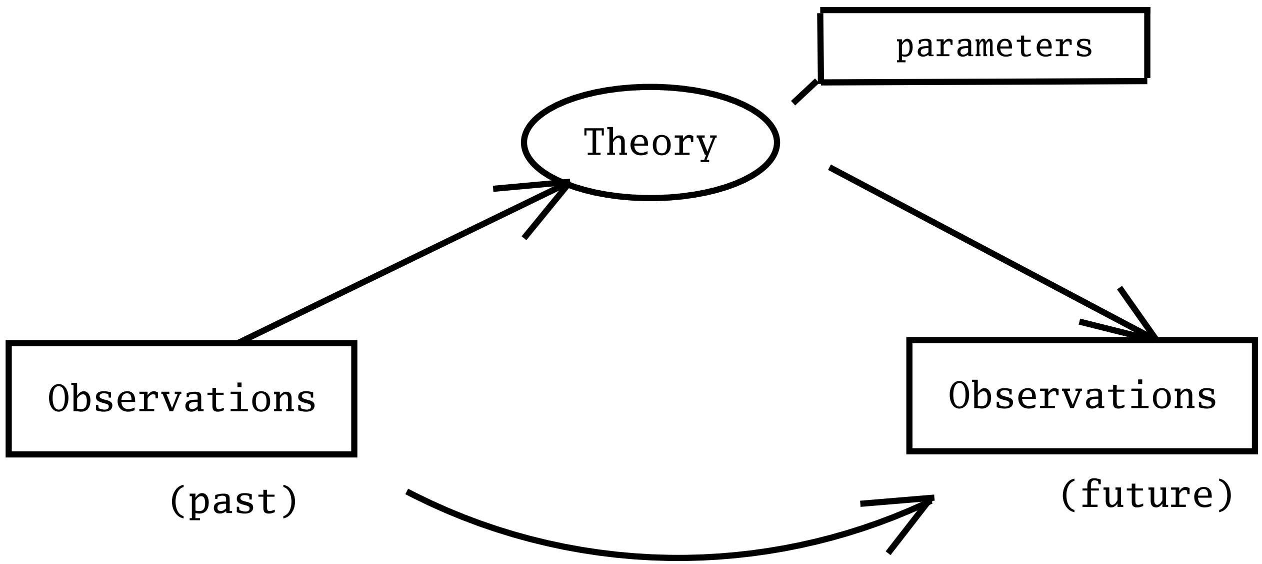
$\Longrightarrow$ *He has finally won both bets!*

Physics



(*) A quantity might be meaningful only within a theory/model

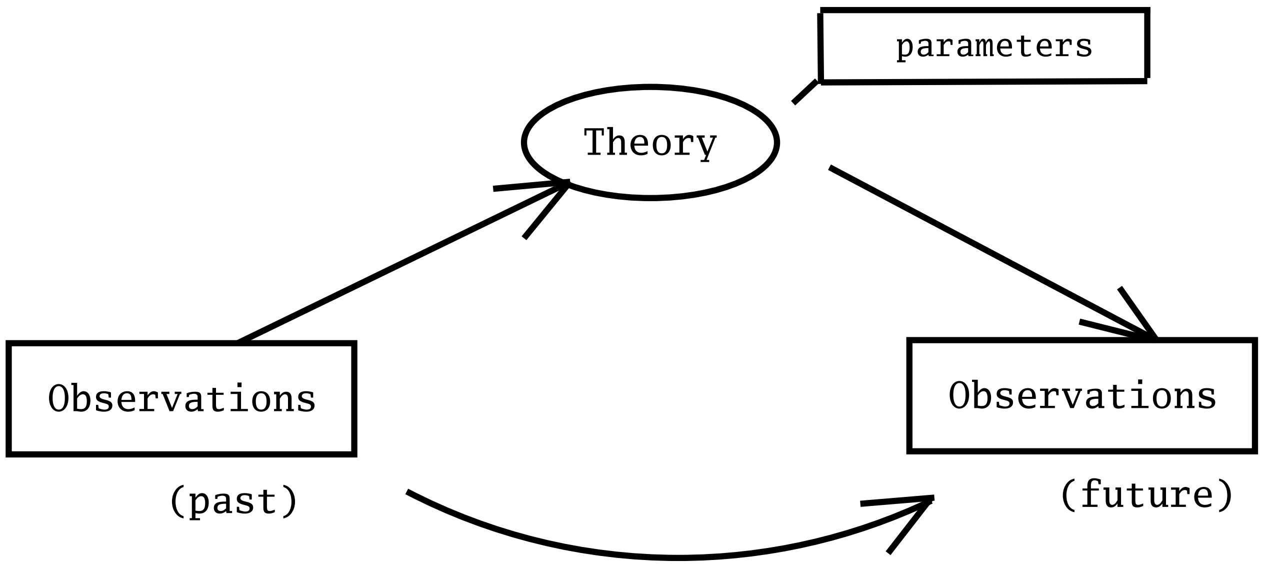
From past to future



Task of physicists:

- Describe/understand the physical world
  $\Rightarrow$ inference of laws and their parameters
- Predict observations
  $\Rightarrow$ forecasting

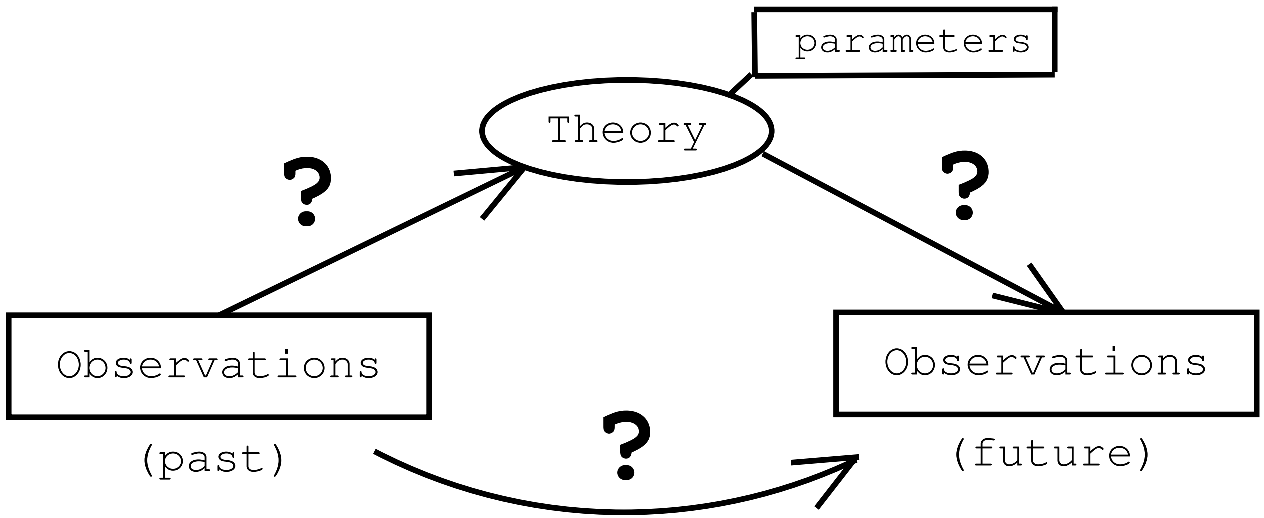
From past to future



⇒ Uncertainty:

1. Given the past observations, in general we are not sure about the theory parameters (and/or the theory itself)
2. Even if we were sure about theory and parameters, there could be internal (e.g. Q.M.) or external effects (initial/boundary conditions, 'errors', etc) that make the forecasting uncertain.

Deep source of uncertainty



Uncertainty:

Theory	— ? ⟶	Future observations
Past observations	— ? ⟶	Theory
Theory	— ? ⟶	Future observations

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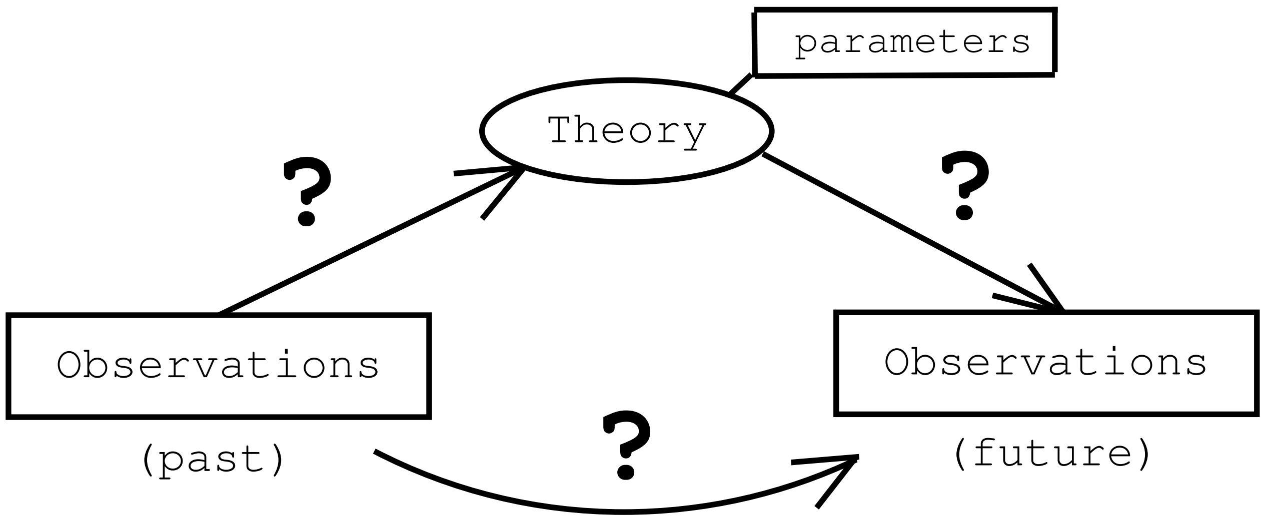

Uncertainty:

Theory	— ? ⟶	Future observations
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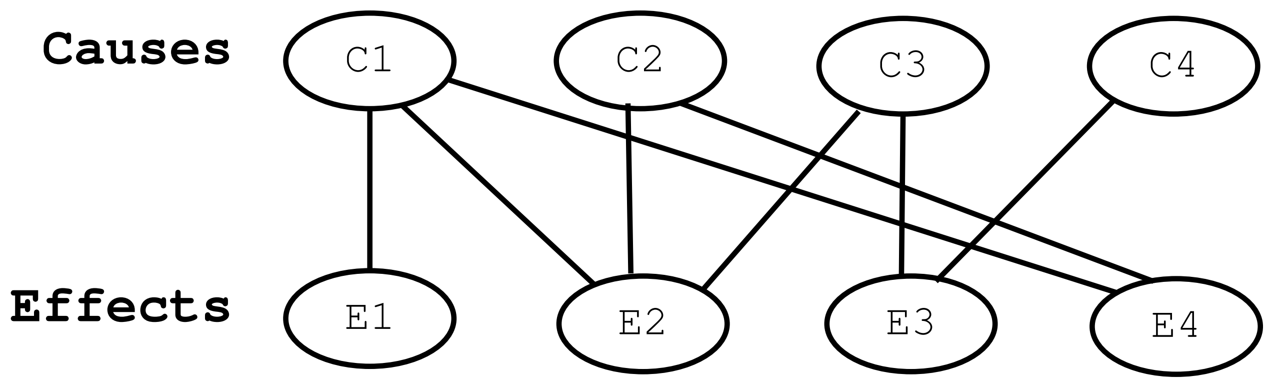
$\Longrightarrow$ **Uncertainty about causal connections**

CAUSE $\Longleftrightarrow$ **EFFECT**

Causes $\rightarrow$ effects

The same *apparent* cause might produce several,different effects



Given an observed effect, we are not sure about the exact cause that has produced it.

Causes $\to$ effects

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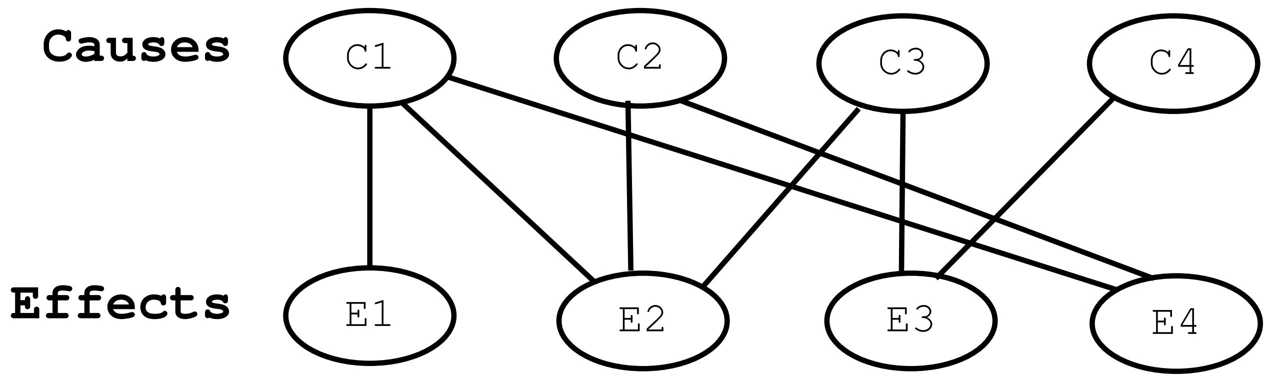

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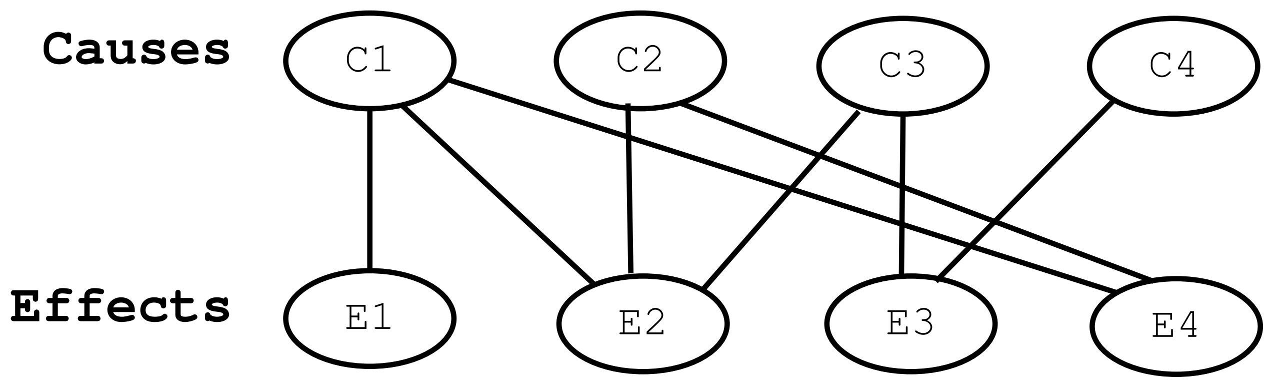

Given an observed effect, we are not sure about the exact cause that has produced it.

$$\mathbf{E_2} \Rightarrow \{C_1,\, C_2,\, C_3\}?$$

The “essential problem” of the Sciences

“Now, these problems are classified as *probability of causes*, and are most interesting of all their scientific applications. I play at *écarté* with a gentleman whom I know to be perfectly honest. What is the chance that he turns up the king? It is 1/8. This is a problem of the probability of effects.

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I play with a gentleman whom I do not know. He has dealt ten times, and he has turned the king up six times. What is the chance that he is a sharper? This is a problem in the probability of causes. It may be said that it is the essential problem of the experimental method.”

(H. Poincaré – *Science and Hypothesis*)

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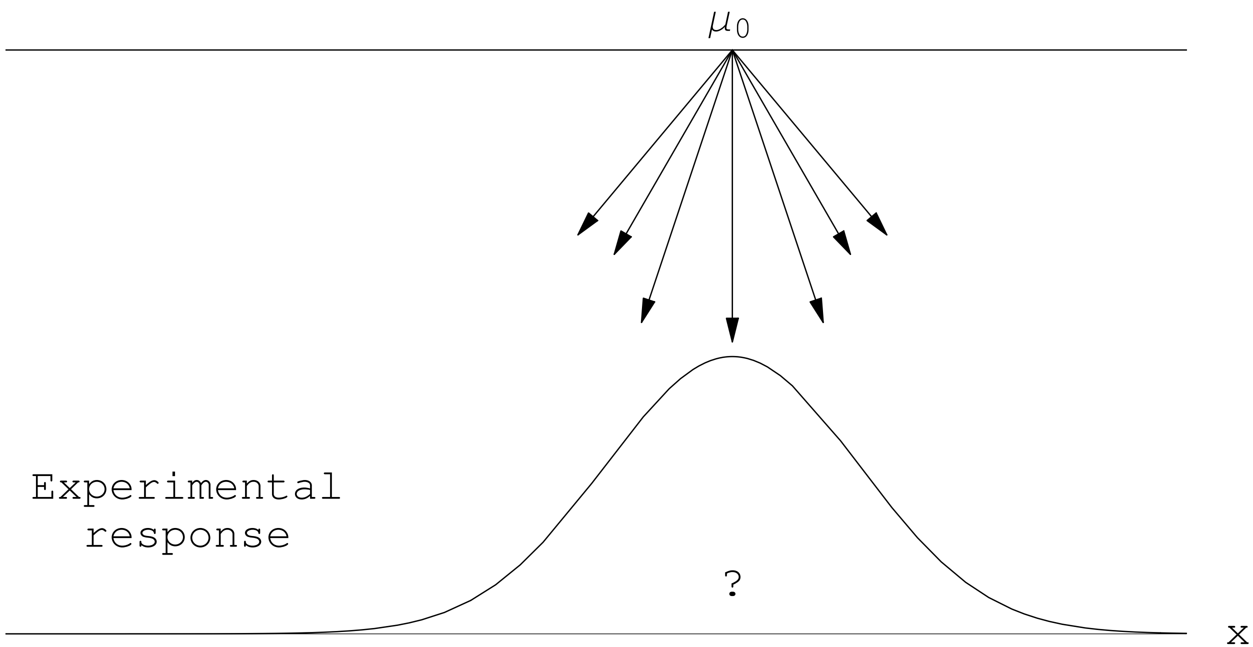
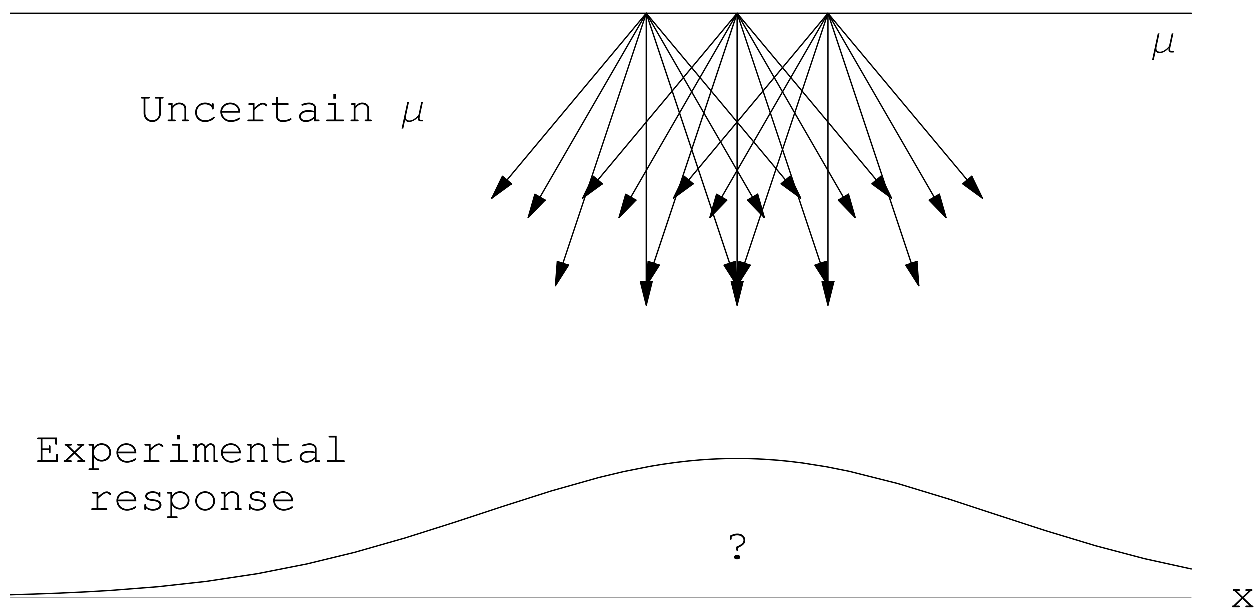
Why physics students are not taught how to tackle this kind of problems?

From ‘true value’ to observations



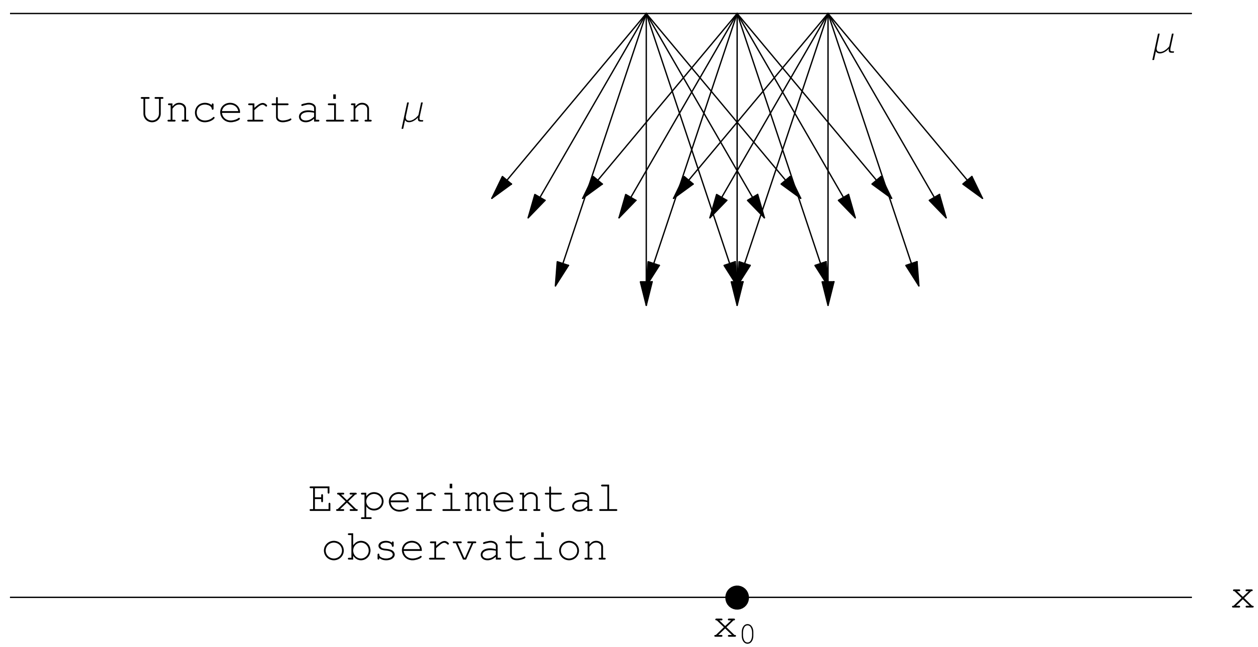
Given μ (exactly known) we are uncertain about x

From ‘true value’ to observations



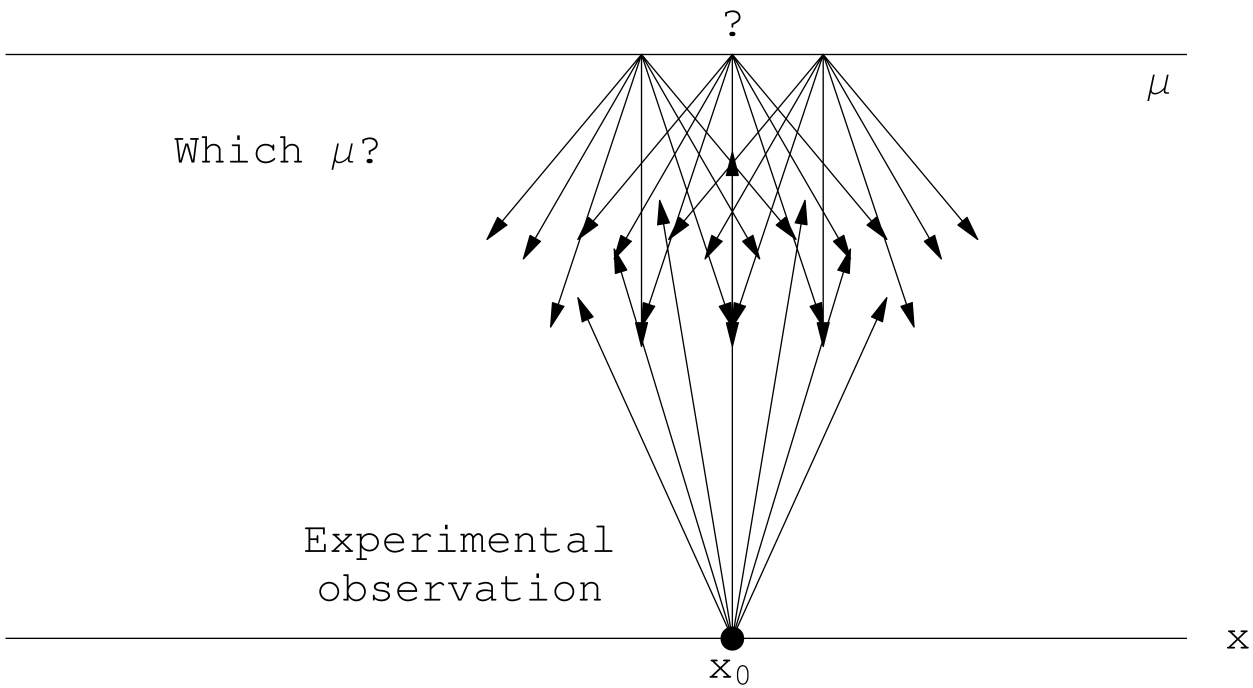
Uncertainty about μ makes us more uncertain about x

. . . and back: Inferring a true value



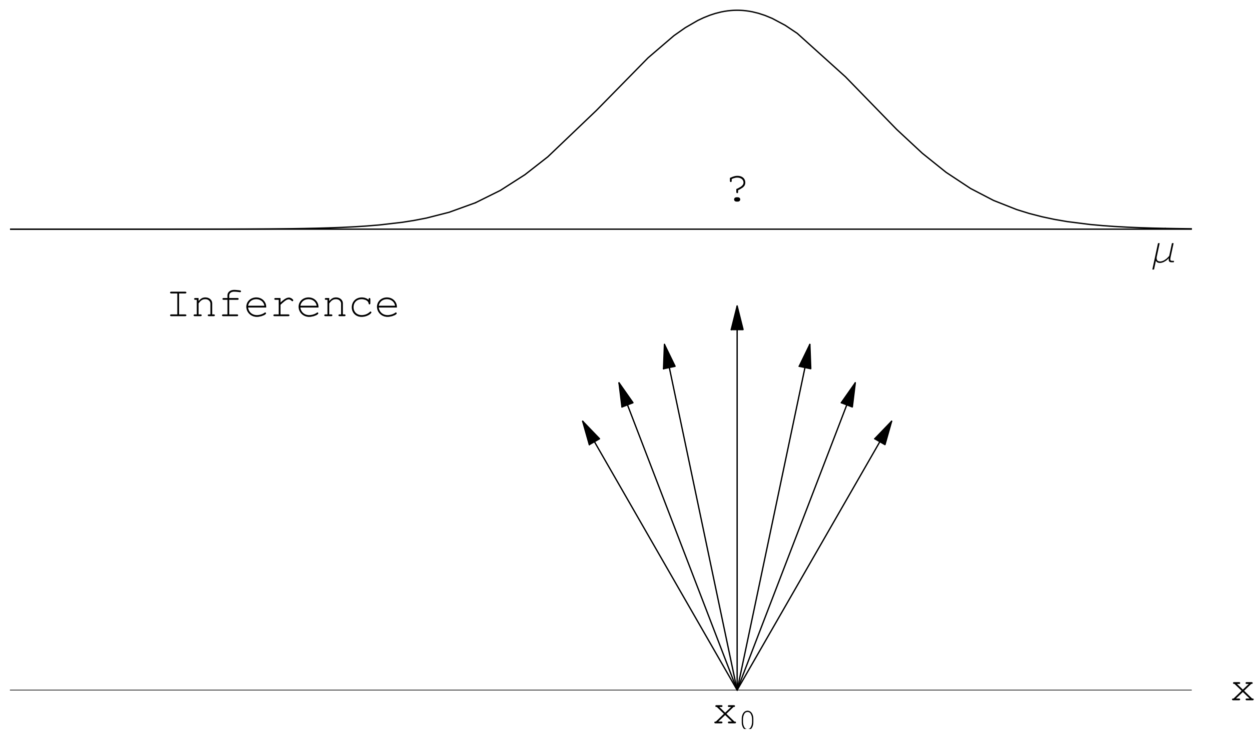
The observed data is certain: $\rightarrow$ 'true value' uncertain.

. . . and back: Inferring a true value



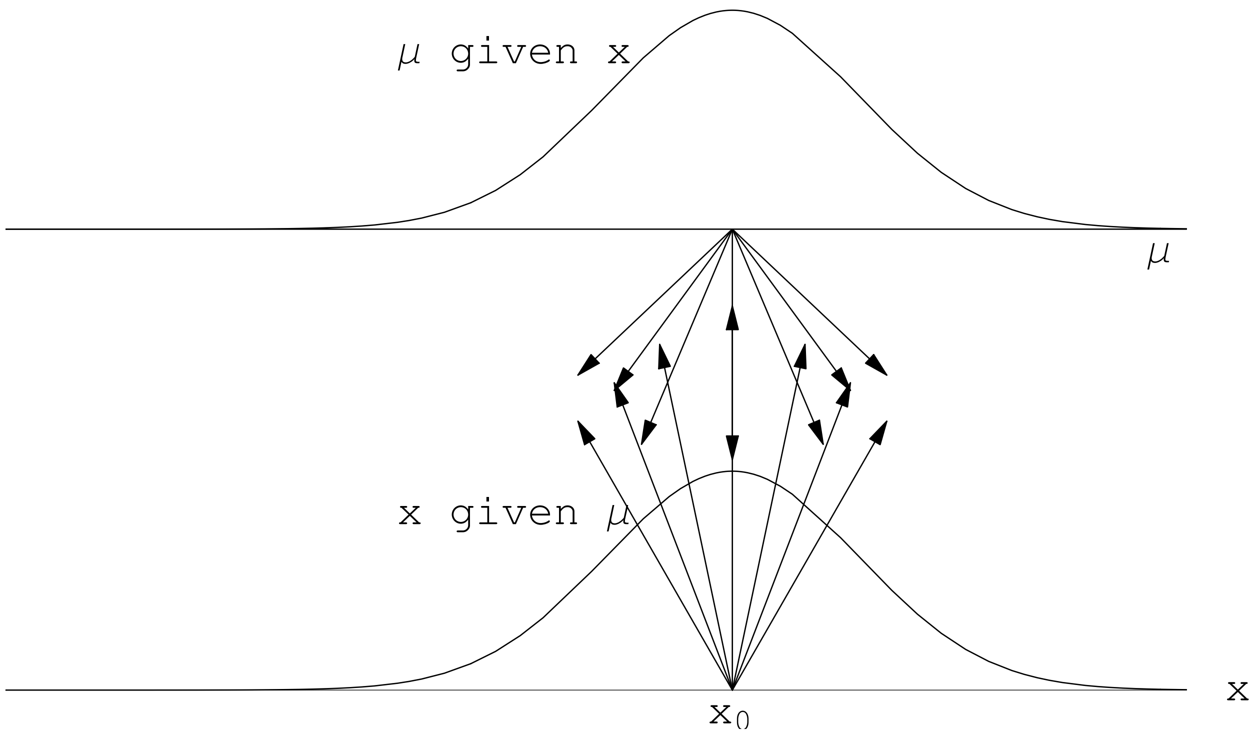
Where does the observed value of x comes from?

. . . and back: Inferring a true value



We are now uncertain about μ, given x.

...and back: Inferring a true value



Note the symmetry in reasoning.

A very simple experiment

Let's make an experiment

A very simple experiment

Let's make an experiment

- Here
- Now

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For simplicity

- μ can assume only six possibilities:

$$\mathbf{0, 1, \ldots, 5}$$

- x is binary:

$$\mathbf{0, 1}$$

$$[\,(1, 2);\ \text{Black/White};\ \text{Yes/Not};\ \ldots\,]$$

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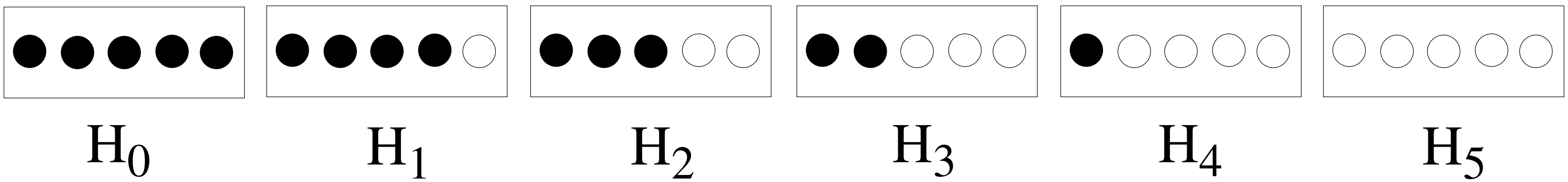
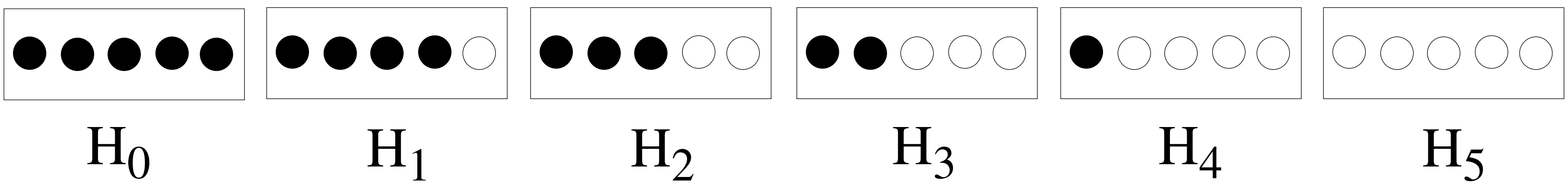
$\Rightarrow$ Later we shall make μ continous.

Which box? Which ball?



Let us take randomly one of the boxes.

Which box? Which ball?



Let us take randomly one of the boxes.

We are in a state of uncertainty concerning several *events*, the most important of which correspond to the following questions:

(a) Which box have we chosen, $H_0, H_1, \ldots, H_5$?

(b) If we extract randomly a ball from the chosen box, will we observe a white ($E_W \equiv E_1$) or black ($E_B \equiv E_2$) ball?

Our certainties:

$$\begin{aligned} \cup_{j=0}^{5} H_j &= \Omega \\ \cup_{i=1}^{2} E_i &= \Omega\,. \end{aligned}$$

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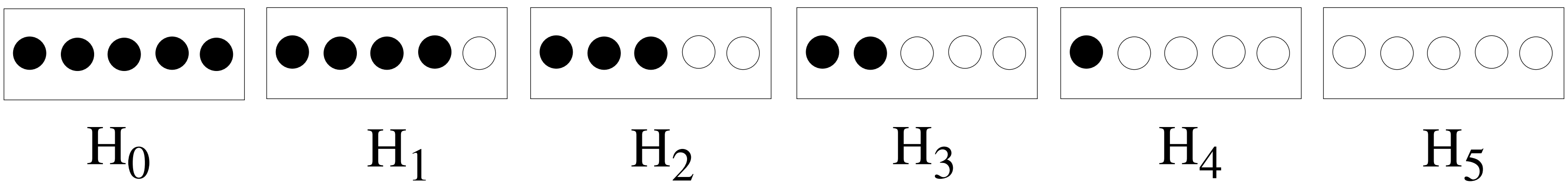

Let us take randomly one of the boxes.

- What happens after we have extracted one ball and looked its color?
  - Intuitively feel *how to* roughly *change* our opinion about
    - the possible cause
    - a future observation

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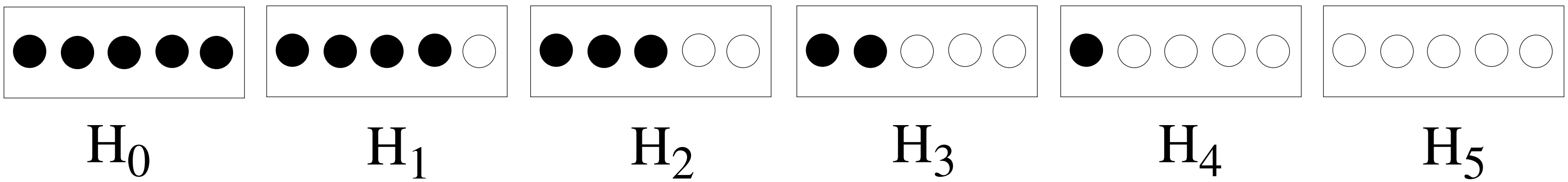

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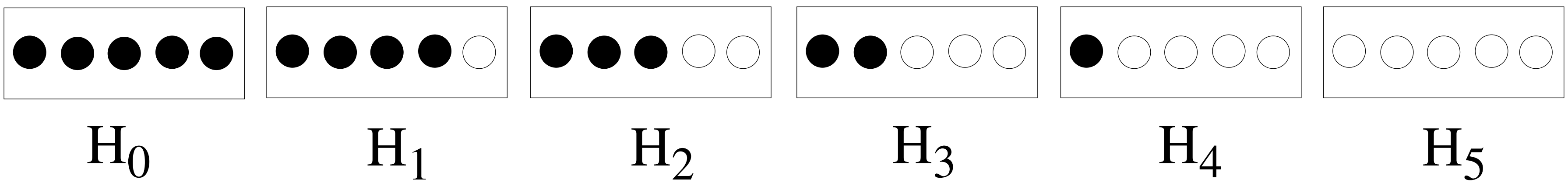

Let us take randomly one of the boxes.

- What happens after we have extracted one ball and looked its color?
  - Intuitively feel *how to* roughly *change* our opinion about
    - the possible cause
    - a future observation
  - Can we do it *quantitatively*, in an ‘objective way’?
- And after a sequence of extractions?

The toy inferential experiment

The aim of the experiment will be to guess the content of the box without looking inside it, only extracting a ball, record its color and reintroducing in the box

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This toy experiment is conceptually very close to what we do in Physics

⇒ try to guess what we cannot see (the electron mass, a branching ratio, etc)

…from what we can see (somehow) with our senses.

The rule of the game is that we are not allowed to watch inside the box! (As we cannot open an electron and read its properties, unlike we read the MAC address of a PC interface.)

Where is probability?

We all agree that the experimental results change

- the probabilities of the box compositions;
- the probabilities of a future outcomes,

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Where is the probability?

Certainly not *in* the box!

Subjective nature of probability

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Probability depends on the status of information of the *subject* who evaluates it.

Probability is always conditional probability

"Thus whenever we speak loosely of 'the probability of an event', it is always to be understood: probability with regard to a certain given state of knowledge"

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$$P(E) \longrightarrow P(E \,|\, I_s)$$

where I_s is the information available to *subject* s.

What are we talking about?

"Given the state of our knowledge about everything that could possible have any bearing on the coming true. . .

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$\Rightarrow$ How much we believe something

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$\rightarrow$ 'Degree of belief' $\leftarrow$

Beliefs and ‘coherent’ bets

Remarks:

- Subjective does not mean arbitrary!

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  “The usual touchstone, whether that which someone asserts is merely his persuasion – or at least his subjective conviction, that is, his firm belief – is betting. It often happens that someone propounds his views with such positive and uncompromising assurance that he seems to have entirely set aside all thought of possible error. A bet disconcerts him. Sometimes it turns out that he has a conviction which can be estimated at a value of one ducat, but not of ten. For he is very willing to venture one ducat, but when it is a question of ten he becomes aware, as he had not previously been, that it may very well be that he is in error.” (Kant)

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11/07 20:30							
VOJVODINA - HIBERNIANS	1,05	10,00	25,00	3,10	1,30	2,55	1,42
GLENTORAN - KR REYKJAV	4,75	3,50	1,65	1,90	1,75	1,75	1,90
HONV BUDAP. - CELIK NIKS.	1,15	7,00	12,00	2,80	1,35	2,00	1,70
GERMANIA - OLANDA	1,15	6,50	13,00	2,50	1,45	2,20	1,57
11/07 20:45							
S PATRICKS - ZALGIRIS	1,90	3,40	3,50	1,75	1,90	1,73	1,95
11/07 21:00							
LIBERTAS - SARAJEVO	22,00	8,00	1,08	3,20	1,28	2,25	1,55
11/07 22:00							
STJARNAN - HAFNARFJOR	2,65	3,40	2,35	2,15	1,60	1,50	2,35

1 X 2

Beliefs and ‘coherent’ bets

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$$\rightarrow P(3477 \leq M_{Sun}/M_{Sat} \leq 3547 \,|\, I(\text{Laplace})) = 99.99\%$$

‘C.L.’ Vs Degree of Confidence

Is a ‘conventional’ 95% C.L. lower/upper bound a 19 to 1 bet?

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For more on the subject:

http://arxiv.org/abs/1112.3620

http://www.roma1.infn.it/~dagos/badmath/#added

Mathematics of beliefs

The good news:

The basic laws of degrees of belief are the same we get from the inventory of favorable and possible cases, or from events occurred in the past.

[Details skipped. . .]

Basic rules of probability

1. $0 \leq P(A \,|\, I) \leq 1$
2. $P(\Omega \,|\, I) = 1$
3. $P(A \cup B \,|\, I) = P(A \,|\, I) + P(B \,|\, I) \quad$ [if $P(A \cap B \,|\, I) = \emptyset$]

4. $P(A \cap B \,|\, I) = P(A \,|\, B, I) \cdot P(B \,|\, I) = P(B \,|\, A, I) \cdot P(A \,|\, I)$

Remember that probability is always conditional probability!

I is the background condition (related to information 'I'_s')

$\rightarrow$ usually implicit (we only care on 're-conditioning')

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Note: 4. does not define conditional probability.
(Probability is always conditional probability!)

Mathematics of beliefs

An even <u>better news</u>:

The fourth basic rule
can be fully exploided!

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(Liberated by a curious ideology that forbits its use)

A simple, powerful formula

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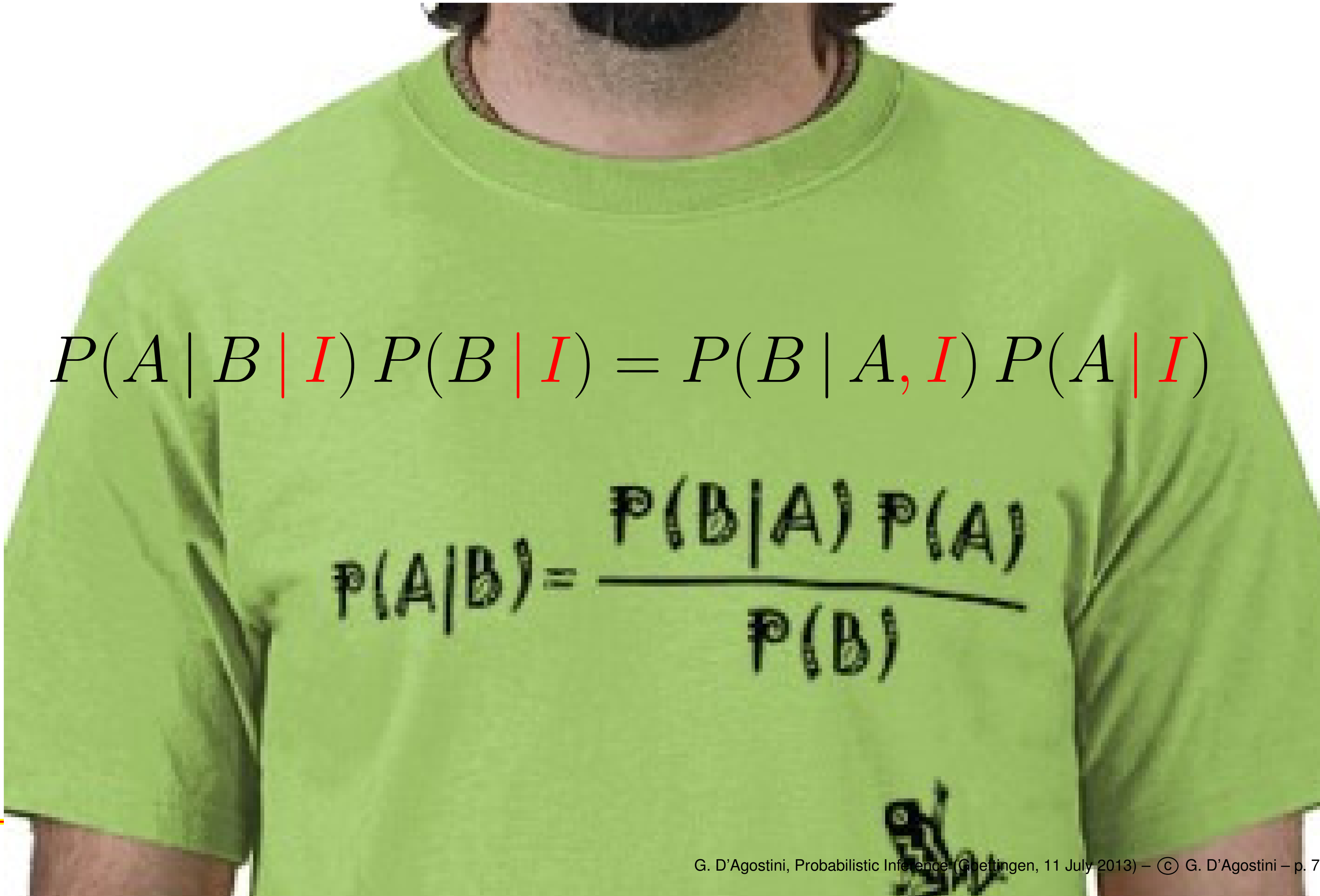
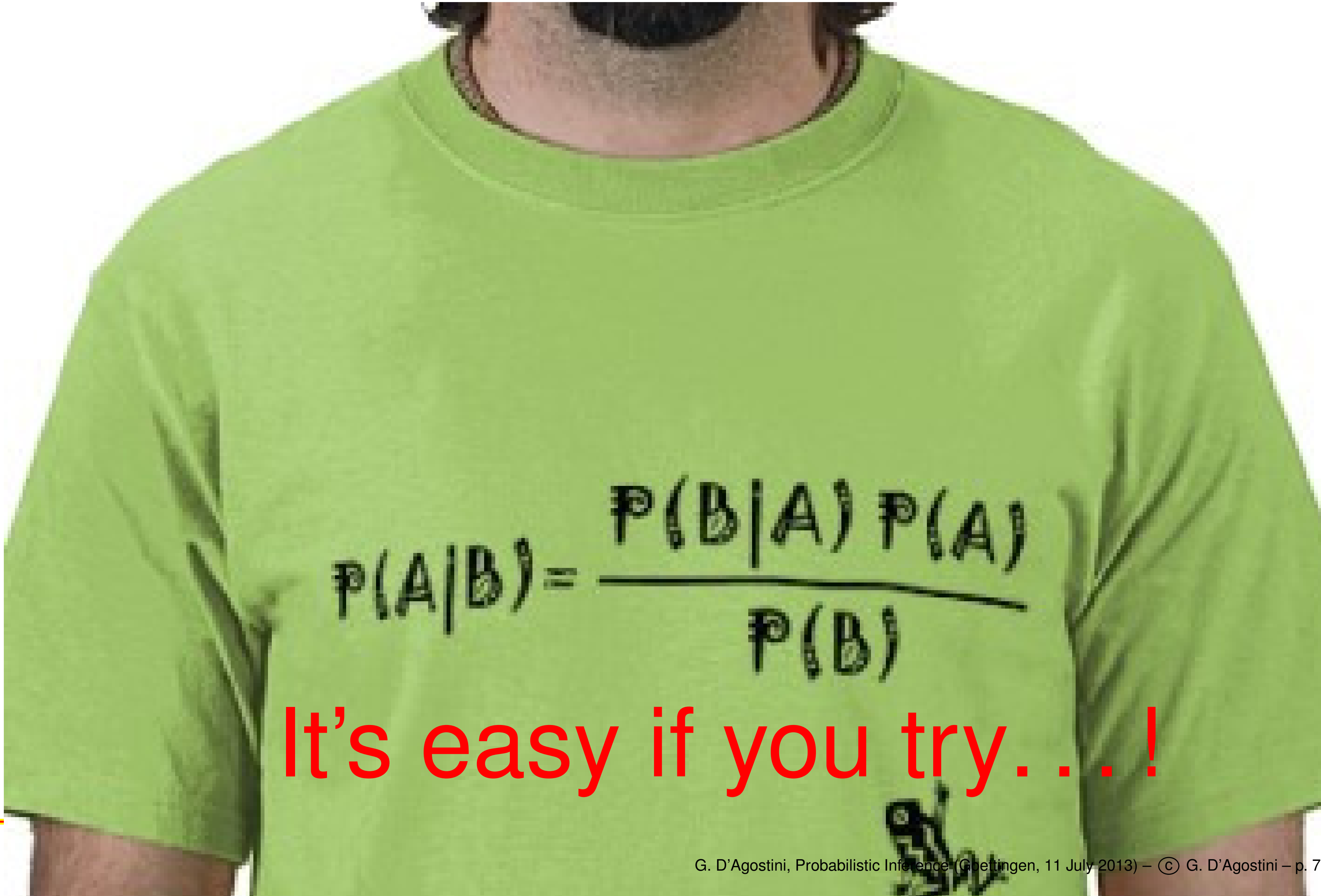


$$P(A\,|\,B\,|\,I)\,P(B\,|\,I) = P(B\,|\,A, I)\,P(A\,|\,I)$$

A simple, powerful formula

Take the courage to use it!

A simple, powerful formula



It’s easy if you try. . . !

Telling it with Gauss' words

A quote from the Princeps Mathematicorum (Prince of Mathematicians) is a must in this town and in this place.

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$$P(C_i \,|\, \text{data}) = \frac{P(\text{data} \,|\, C_i)}{P(\text{data})}\, P_0(C_i)$$

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"post illa observationes" *"ante illa observationes"*

(Gauss)

Bayes formulae

The essence is all contained in the fourth basic rule of probability theory:

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The essence is all contained in the fourth basic rule of probability theory:

$$\frac{P(C_i \,|\, E, I)}{P(C_i \,|\, I)} = \frac{P(E \,|\, C_i, I)}{P(E \,|\, I)}$$

Bayes formulae

The essence is all contained in the fourth basic rule of probability theory:

$$\frac{P(C_i \,|\, E, I)}{P(C_i \,|\, I)} = \frac{P(E \,|\, C_i, I)}{P(E \,|\, I)}$$

$$P(C_i \,|\, E, I) = \frac{P(E \,|\, C_j, I)}{P(E \,|\, I)} P(C_i \,|\, I)$$

Bayes formulae

The essence is all contained in the fourth basic rule of probability theory:

$$\frac{P(C_i \,|\, E, I)}{P(C_i \,|\, I)} = \frac{P(E \,|\, C_i, I)}{P(E \,|\, I)}$$

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or even (my preferred form to grasp its meaning):

$$\frac{P(C_i \,|\, E \,|\, I)}{P(C_j \,|\, E \,|\, I)} = \frac{P(E \,|\, C_i \,|\, I)}{P(E \,|\, C_j \,|\, I)} \cdot \frac{P(C_i \,|\, I)}{P(C_j \,|\, I)}$$

Bayesian parametric inference

If we want to infer a continuous parameter p from a set of data

$\rightarrow$ straghtforwad extension to probability density functions (pdf)

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$$f(p \,|\, \text{data}, I) \;\propto\; f(\text{data} \,|\, p, I) \cdot f(p \,|\, I)$$

Bayesian parametric inference

If we want to infer a continuous parameter *p* from a set of data

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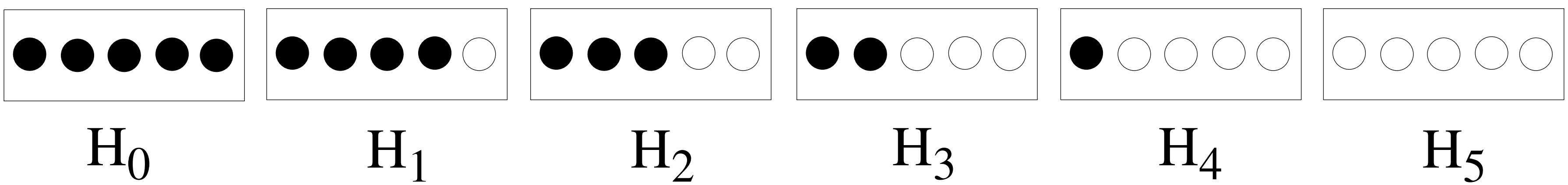
⇒ Several examples tomorrow by Lorenzo

Application to the six box problem



Remind:

- $E_1 =$ White
- $E_2 =$ Black

Collecting the pieces of information we need

Our tool:

$$P(H_j \,|\, E_i, I) = \frac{P(E_i \,|\, H_j, I)}{P(E_i \,|\, I)}\, P(H_j \,|\, I)$$

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Our prior belief about H_j

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Probability of E_i under a well defined hypothesis H_j
It corresponds to the 'response of the apparatus in measurements.
$\rightarrow$ likelihood (traditional, rather confusing name!)

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$\rightarrow$ How much we are confident that E_i will occur.

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Probability of E_i taking account all possible H_j
$\rightarrow$ How much we are confident that E_i will occur.
We can rewrite it as
$P(E_i \,|\, I) = \sum_j P(E_i \,|\, H_j,\, I) \cdot P(H_j \,|\, I)$

We are ready

Now that we have set up our formalism, let's play a little

- analyse real data
- some simulations

Then

- $H_j \longleftrightarrow j \longleftrightarrow p_j$
- extending p to a continuum:

  $\Rightarrow$ Bayes' billiard

  (prototype for all questions related to efficiencies, branching ratios)
- On the meaning of p

Which box? Which ball?

Inferential/forecasting history:

1. $k = 0$
   $P_0(H_j) = P(H_j \,|\, I_0)$ (priors)
2. `begin loop`:
   $k = k + 1$
   $\Rightarrow E^{(k)}$ (k-th extraction)
3. $P_k(H_j \,|\, I_k) \propto P(E^{(k)} \,|\, H_j) \times P_{k-1}(H_j \,|\, I_k)$

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Let's play!

Bayes' billiard

This is the original problem in the theory of chances solved by Thomas Bayes in late '700:

- imagine you roll a ball at random on a billiard;
- you mark the relative position of the ball along the billiard's length (l/L) and remove the ball
- then you roll at random other balls
  - write down if it stopped left or right of the first ball;
  - remove it and go on with n balls.
- Somebody has to guess the position of the first ball knowing only how mane balls stopped left and how many stoppe right

Bayes' billiard and Bernoulli trials

It is easy to recongnize the analogy:

- Left/Right $\rightarrow$ Success/Failure
- if Left $\leftrightarrow$ Success:
  - $l/L \leftrightarrow p$ of binomial (Bernoulli trials)

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Imagine a sequence $\{S, S, F, S, \ldots\}$ [f_0 is uniform]:

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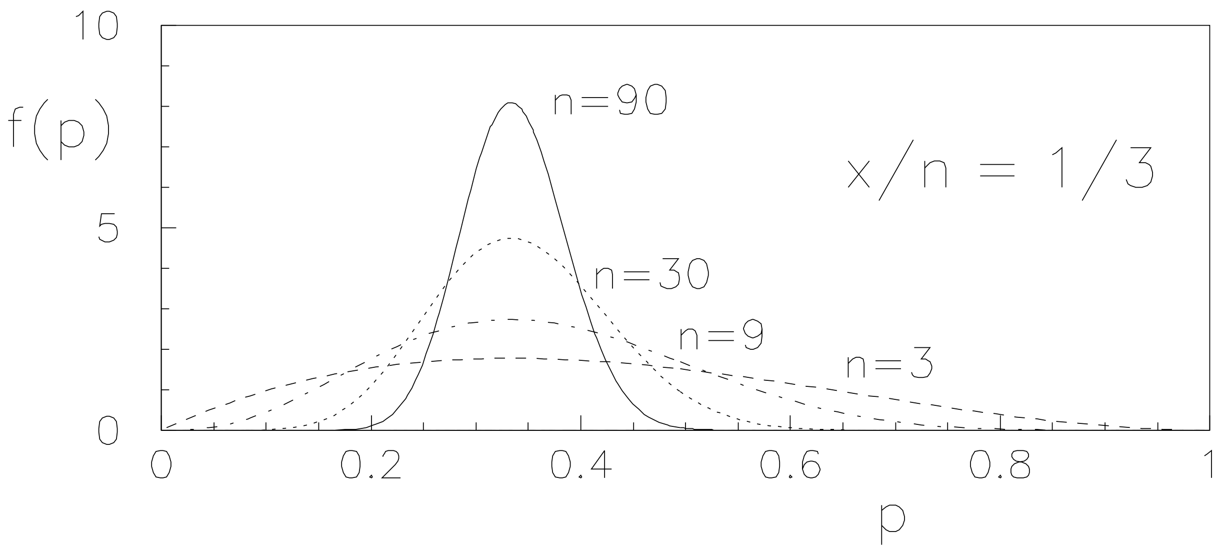
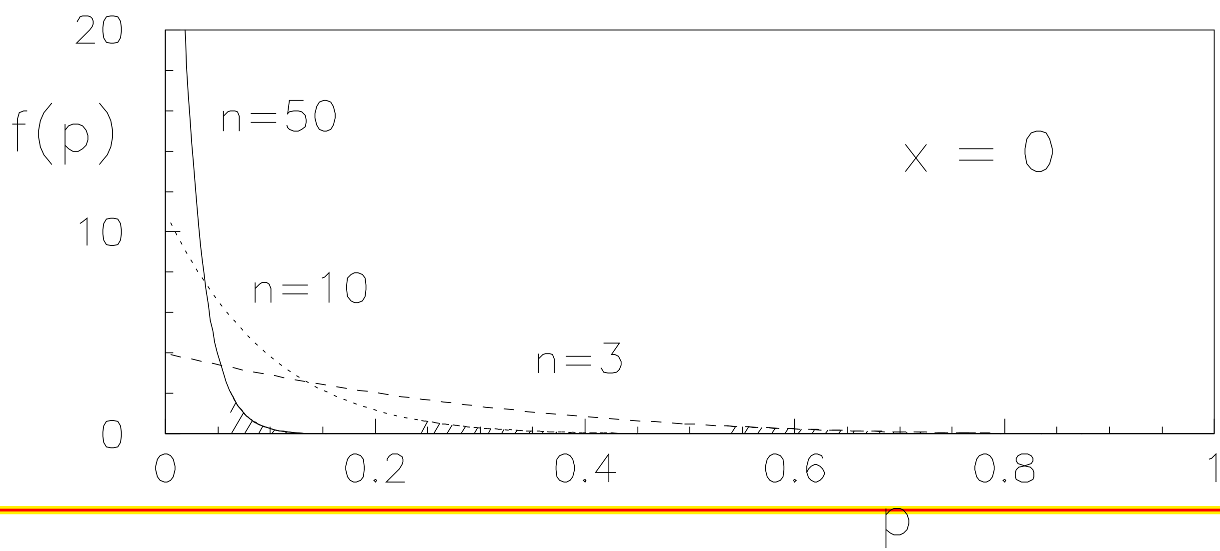
f(p\,|\,x,n) &\propto p^x(1-p)^{(n-x)} \qquad\qquad [x = \#S]
\end{aligned}
$$

Inferring the Binomial p

$$f(p\,|\,x,n,\mathcal{B}) = \frac{(n+1)!}{x!\,(n-x)!}\,p^x\,(1-p)^{n-x}\,,$$





Inferring the Binomial p

$$f(p\,|\,x,n,\mathcal{B}) = \frac{(n+1)!}{x!\,(n-x)!}\,p^x\,(1-p)^{n-x}\,,$$

$$\mathsf{E}(p) = \frac{x+1}{n+2}$$

Laplace's rule of successions

$$\mathsf{Var}(p) = \frac{(x+1)(n-x+1)}{(n+3)(n+2)^2}$$

$$= \mathsf{E}(p)\,\left(1-\mathsf{E}(p)\right)\,\frac{1}{n+3}.$$

Interpretation of E(p)

Think at any future event $E_{i>n}$
$\Rightarrow$ if we were sure of p, then our confidence on $E_{i>n}$ will be exactly p, i.e.

$$P(E_i \,|\, p) = p\,.$$

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$$\begin{aligned}
P(E_{i>n}\,|\,x,n,\mathcal{B}) &= \int_0^1 P(E_i\,|\,p)\,f(p\,|\,x,n,\mathcal{B})\,\mathrm{d}p \\
&= \int_0^1 p\,f(p\,|\,x,n,\mathcal{B})\,\mathrm{d}p \\
&= \mathrm{E}(p) \\
&= \frac{x+1}{n+2} \quad \text{(for uniform prior)}\,.
\end{aligned}$$

From frequencies to probabilities

$$E(p) = \frac{x+1}{n+2}$$

Laplace's rule of successions

$$\mathrm{Var}(p) = E(p)\,(1 - E(p))\,\frac{1}{n+3}.$$

For 'large' n, x and $n-x$: asymptotic behaviors of $f(p)$:

$$E(p) \approx p_m = \frac{x}{n} \quad \text{[with } p_m \text{ mode of } f(p)]$$

$$\sigma_p \approx \sqrt{\frac{p_m\,(1-p_m)}{n}} \xrightarrow[n\to\infty]{} 0$$

$$p \sim \mathcal{N}(p_m, \sigma_p)\,.$$

Under these conditions the frequentistic "definition" (evaluation rule!) of probability (x/n) is recovered.

Special case with $x = 0$

$$
\begin{aligned}
f(p\,|\,0, n, \mathcal{B}) &= (n+1)\,(1-p)^n \\
F(p\,|\,0, n, \mathcal{B}) &= 1-(1-p)^{n+1} \\
p_m &= 0 \\
\mathrm{E}(p) &= \frac{1}{n+2} \longrightarrow \frac{1}{n} \\
\sigma(p) &= \sqrt{\frac{(n+1)}{(n+3)(n+2)^2}} \longrightarrow \frac{1}{n}
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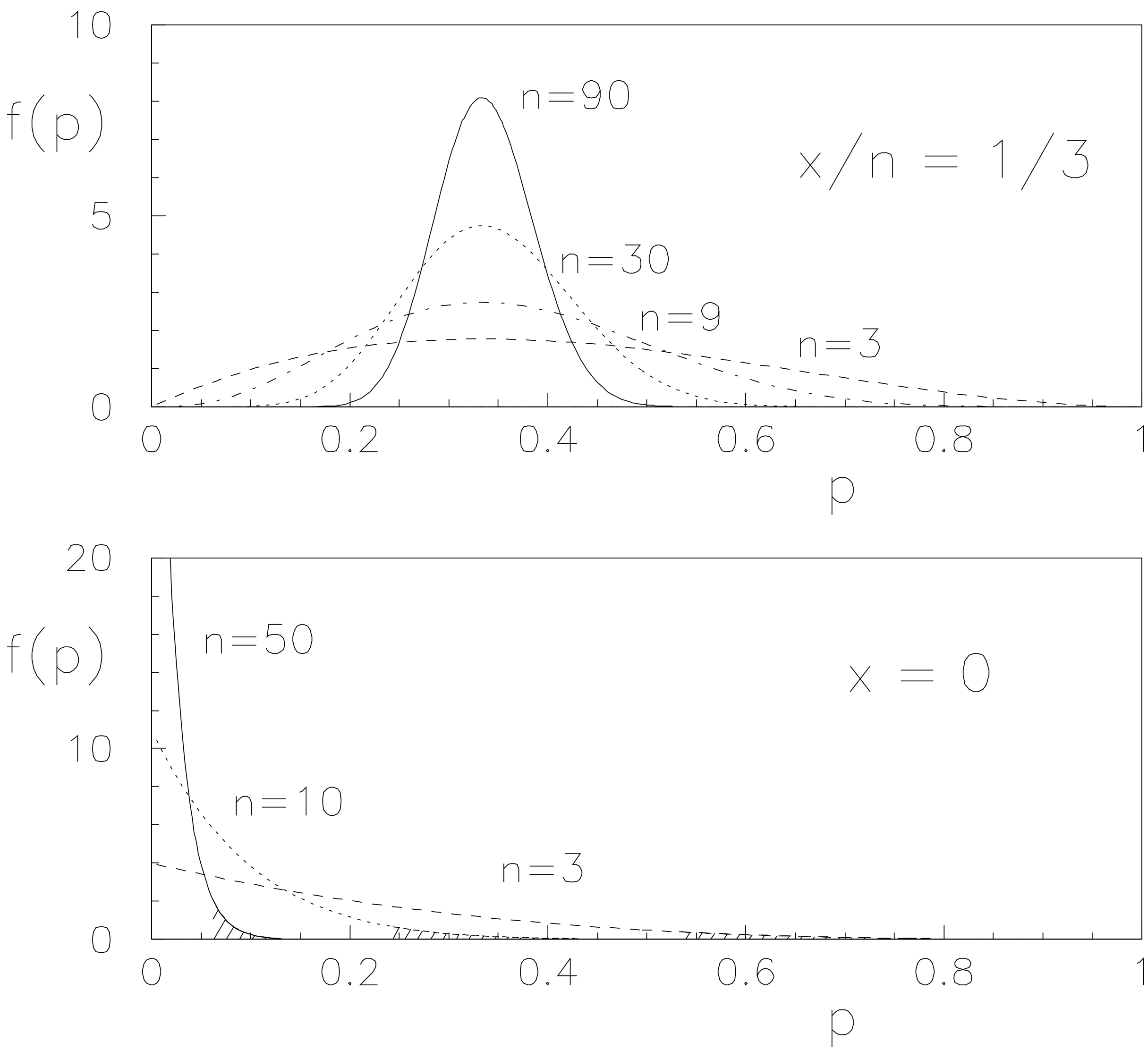
$$

$$
\begin{aligned}
P(p \le p_u \,|\, 0, n, \mathcal{B}) &= 95\% \\
&\Rightarrow \quad p_u = 1 - \sqrt[n+1]{0.05}\ :
\end{aligned}
$$

Probabilistic upper bound

Special case with $x = 0$



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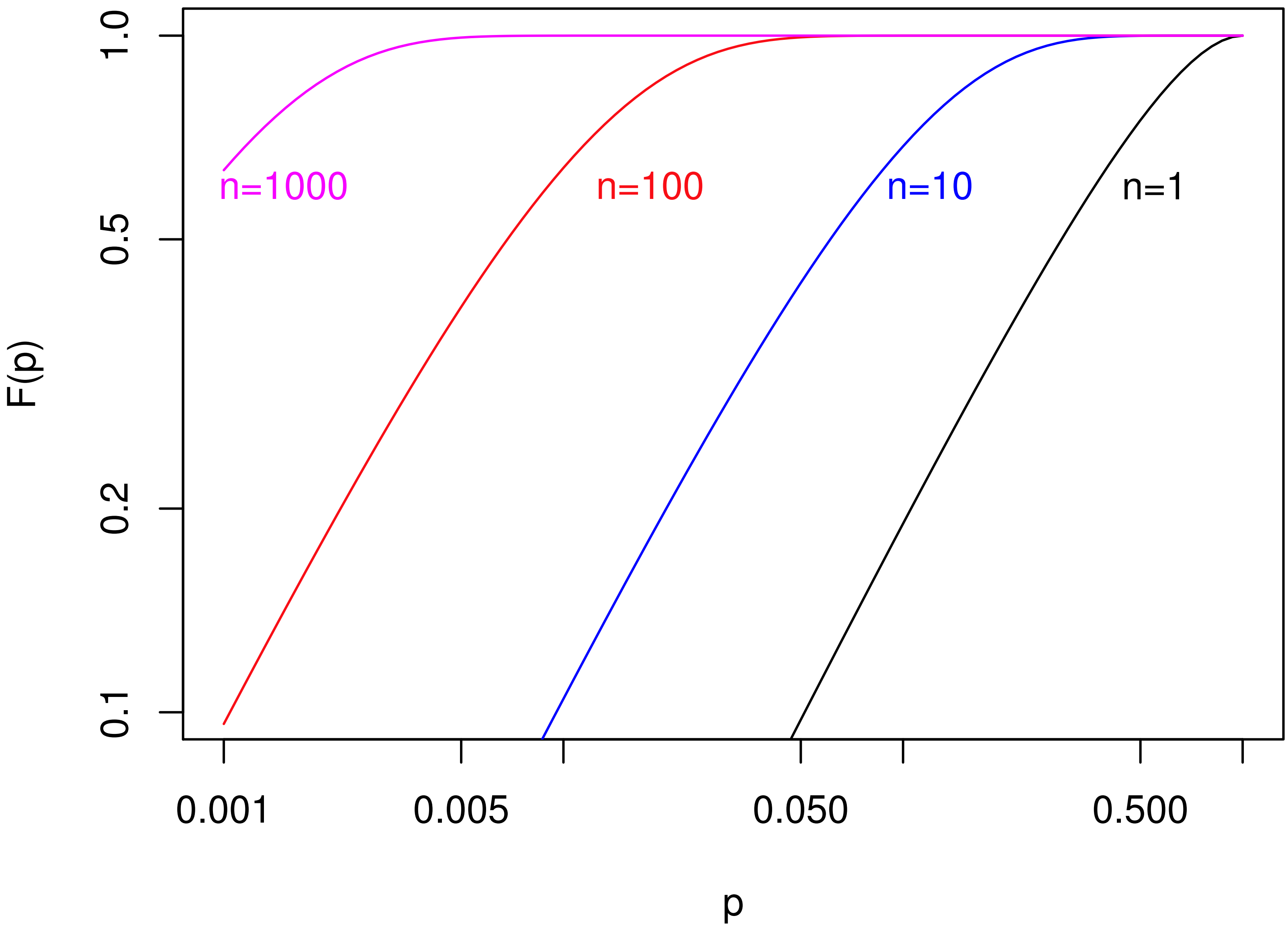

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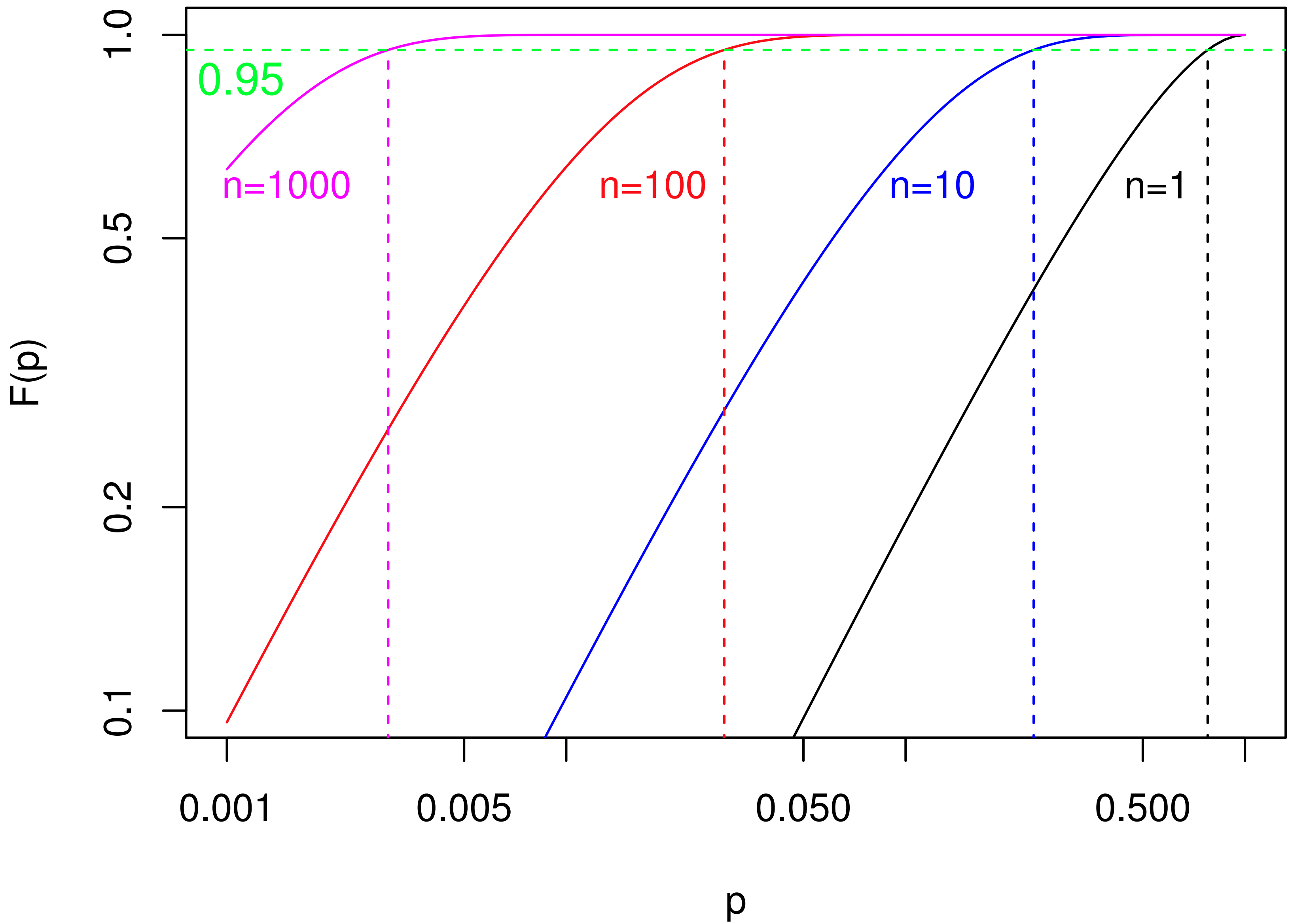

Special case with $x = 0$

For the case $x = n$

(like ‘observing’ a 100% efficiency):

$\rightarrow$ just reason on the complementary parameter

$$q = 1 - p$$

Continuing the game

We have seen ho to tackle with a single idea problems that are treated difefrently in 'standard statistics':

- comparing hypotheses
- parametric inference

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- etc.

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We have no longer excuses!!

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$\Rightarrow$ some ‘appetizers’ will be provided tomorrow by Lorenzo

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  (Diffidate chi vi promette di far germogliar zecchini nel Campo dei Miracoli! – Pinocchio docet)

Conclusions

- The probabilistic framework basically set up by Laplace in his monumental work is healthy and grows up well (browse e.g. Amazon.com)

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- But it is now possible thank to progresses in applied mathematics and computation.

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- Its consistent application in small-complex problems was prohibitive many years ago.
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Conclusions

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- But it is now possible thank to progresses in applied mathematics and computation.
- It makes little sense to stick to old 'ah hoc' methods that had their *raison d'être* in the computational barrier.
- Mistrust all results that sound as 'confidence', 'probability' etc about physics quantities, if they are obtained by methods that do not contemplate 'beliefs'.