String Theory applied to strongly interacting systems

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Outline

Brief overview of holographic (gauge/gravity) duality

Finite temperature, holography and black holes

Transport in strongly coupled gauge theories from black hole physics

First- and second-order hydrodynamics and dual gravity

Photon/dilepton emission rates from dual gravity

Quantum liquids and holography

Other approaches

Over the last several years, holographic (gauge/gravity duality) methods were used to study strongly coupled gauge theories at finite temperature and density

These studies were motivated by the heavy-ion collision programs at RHIC and LHC (ALICE, ATLAS) and the necessity to understand hot and dense nuclear matter in the regime of intermediate coupling $\alpha_s(T_{\mathrm{RHIC}}) \sim O(1)$

As a result, we now have a better understanding of thermodynamics and especially kinetics (transport) of strongly coupled gauge theories

Of course, these calculations are done for theoretical models such as N=4 SYM and its cousins (including non-conformal theories etc).

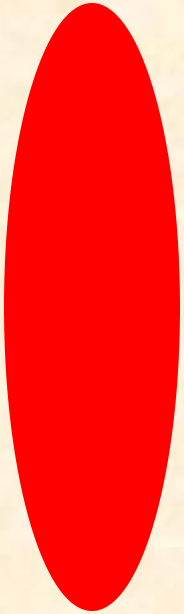
We don't know quantities such as $\frac{\eta}{s}\left(\frac{\Lambda_{\mathrm{QCD}}}{T}\right)$ for QCD

Heavy ion collision experiments at RHIC (2000-current) and LHC (2010-??) create hot and dense nuclear matter known as the "quark-gluon plasma"

(note: qualitative difference between p-p and Au-Au collisions)

Evolution of the plasma "fireball" is described by relativistic fluid dynamics (relativistic Navier-Stokes equations)

Need to know



thermodynamics (equation of state)
kinetics (first- and second-order transport coefficients)
in the regime of intermediate coupling strength:

$$\alpha_s(T_{\text{RHIC}}) \sim O(1)$$

initial conditions (initial energy density profile)

thermalization time (start of hydro evolution)

freeze-out conditions (end of hydro evolution)

Energy density vs temperature for various gauge theories

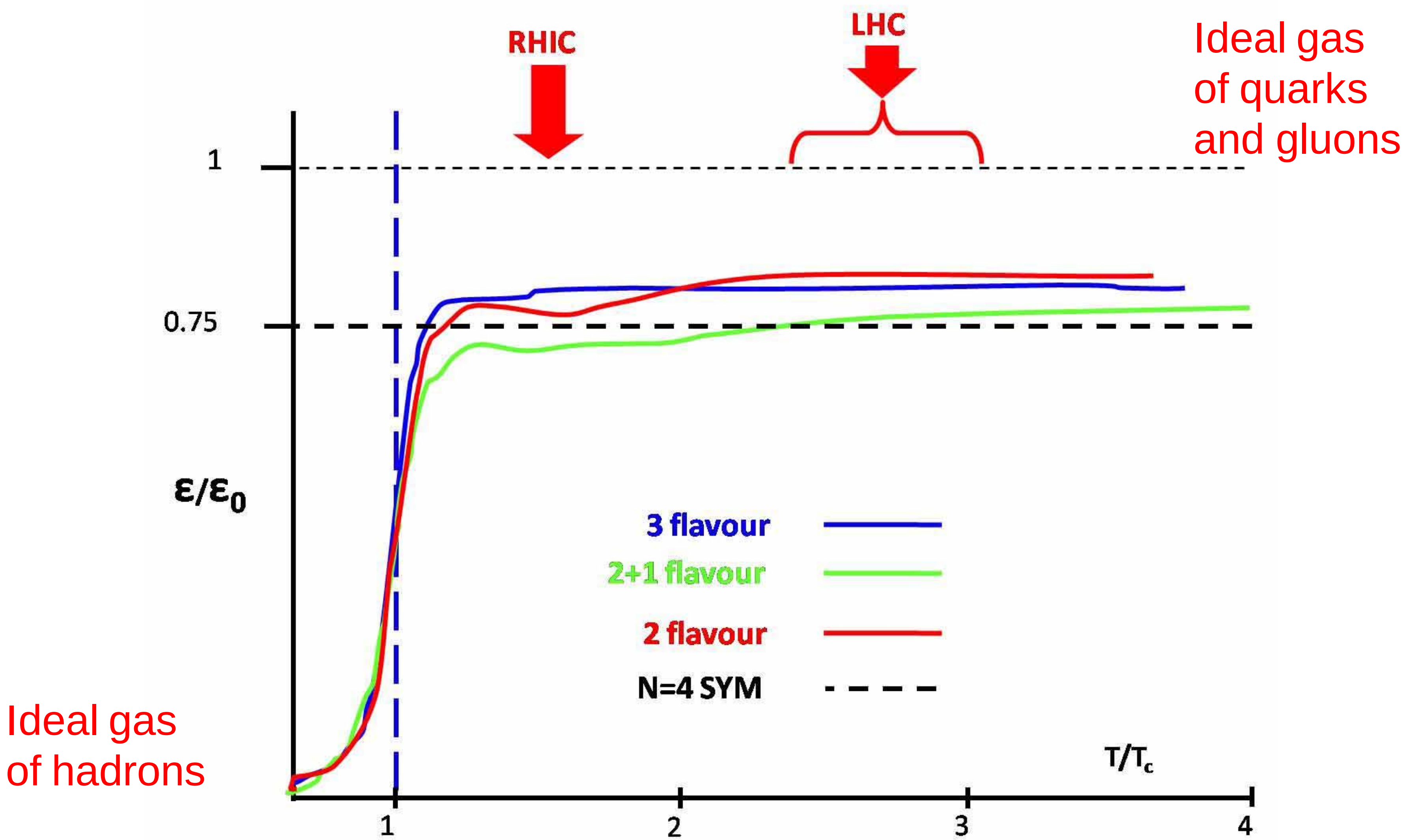
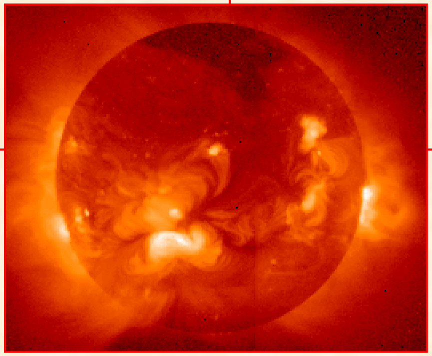


Figure: an artistic impression from Myers and Vazquez, 0804.2423 [hep-th]

Quantum field theories at finite temperature/density

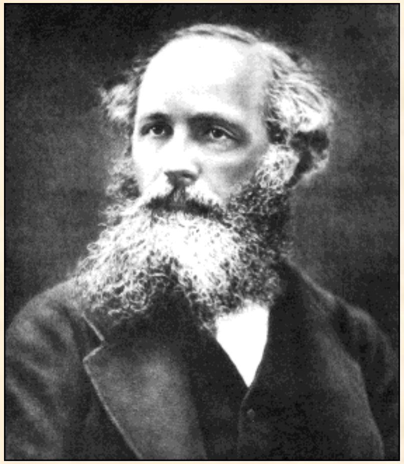


Equilibrium

entropy
equation of state
....

Near-equilibrium

transport coefficients
emission rates
....



perturbative — pQCD

non-perturbative — Lattice

perturbative — kinetic theory

non-perturbative — ????

First-order transport (kinetic) coefficients

Shear viscosity	η
Bulk viscosity	ζ
Charge diffusion constant	D_Q
Supercharge diffusion constant	D_s
Thermal conductivity	κ_T
Electrical conductivity	σ

* Expect Einstein relations such as $\frac{\sigma}{e^2 \Xi} = D_{U(1)}$ to hold

Hydrodynamics: fundamental d.o.f. = densities of conserved charges

Need to add constitutive relations!

Example: charge diffusion

Conservation law $$\partial_t j^0 + \partial_i j^i = 0$$

Constitutive relation

[Fick's law (1855)] $$j_i = -D\,\partial_i j^0 + O[(\nabla j^0)^2, \nabla^2 j^0]$$

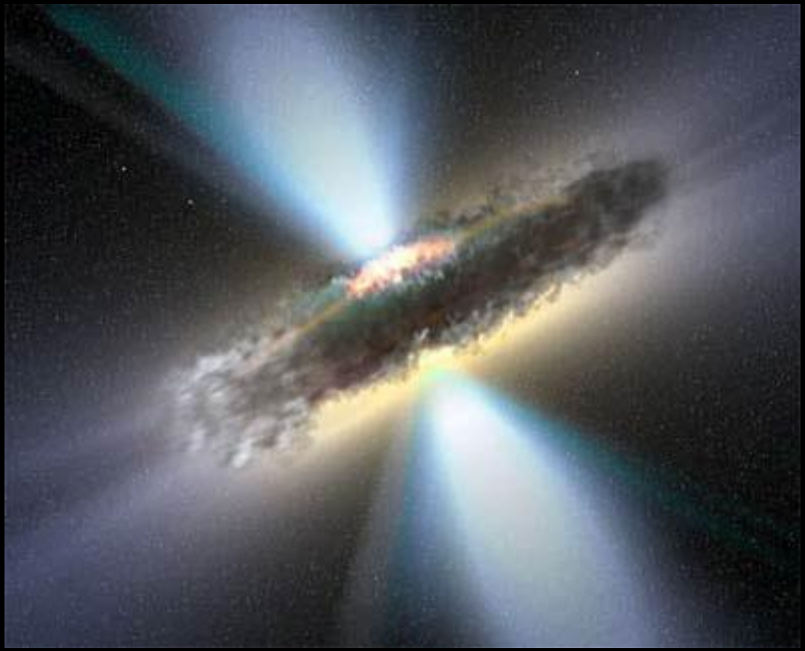
Diffusion equation $$\partial_t j^0 = D\nabla^2 j^0$$

Dispersion relation $$\omega = -i\,D\,q^2 + \cdots$$

Expansion parameters: $\omega \ll T\,, \quad q \ll T$

Hydrodynamic properties of strongly interacting hot plasmas in 4 dimensions

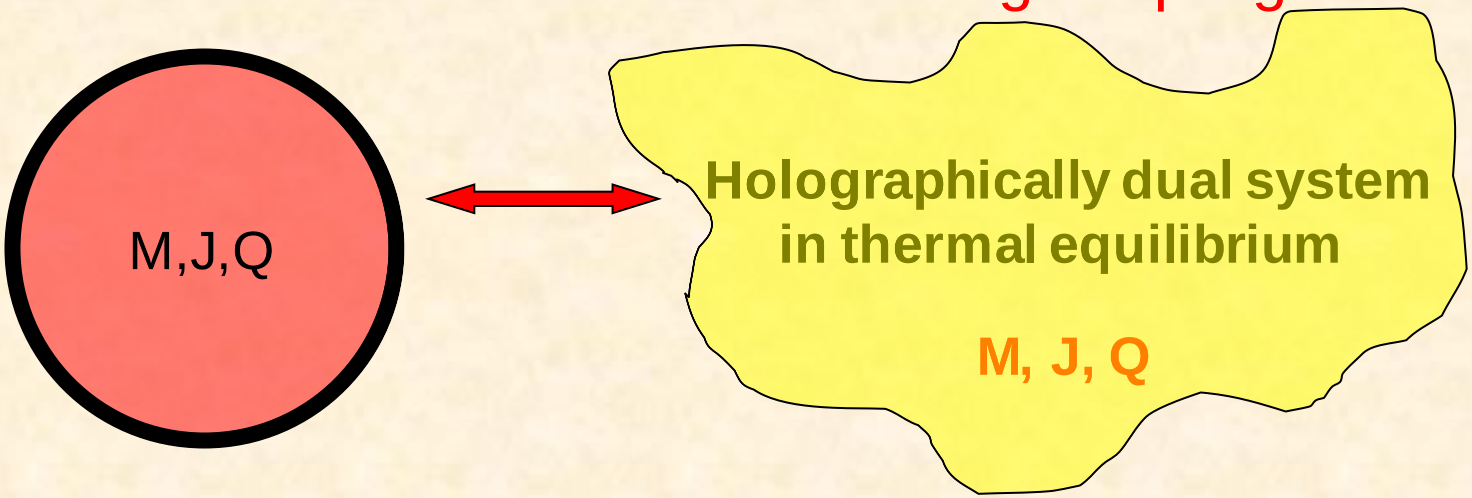
can be related (for certain models!)



to fluctuations and dynamics of 5-dimensional black holes

10-dim gravity

4-dim gauge theory – large N, strong coupling



T_{Hawking} $S_{\text{Bekenstein-Hawking}}$ $\Longleftrightarrow$ T S

Gravitational+electromag fluctuations $\Longleftrightarrow$ Deviations from equilibrium

$g^{(0)}_{\mu\nu} + h_{\mu\nu}$ $A^0_\mu + a_\mu$ $\Longleftrightarrow$????

$$j_i = -D\partial_i j^0 + \cdots$$

$"\Box"\ h_{\mu\nu} = 0$ and B.C. $\Longleftrightarrow$

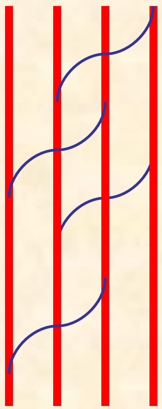
$$\partial_t j^0 + \partial_i j^i = 0$$

$$\partial_t j^0 = D\nabla^2 j^0$$

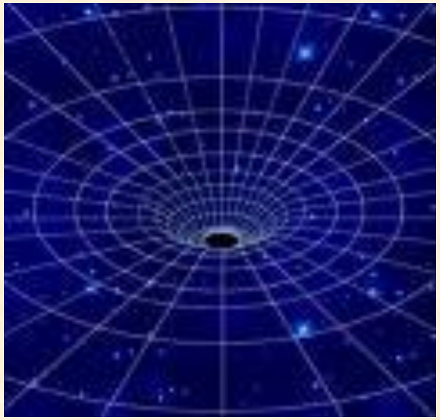
Quasinormal spectrum $\Longleftrightarrow$

$$\omega = -iDq^2 + \cdots$$

From brane dynamics to AdS/CFT correspondence



$\Longleftrightarrow$



Open strings picture:
dynamics of N_c coincident D3 branes
at low energy is described by

Closed strings picture:
dynamics of N_c coincident D3 branes
at low energy is described by

$\mathcal{N}=4$ supersymmetric
$SU(N_c)$ YM theory in 4 dim

$\Longleftrightarrow$

conjectured
exact equivalence

type IIB superstring theory
on $AdS_5 \times S^5$ backgrond

Maldacena (1997); Gubser, Klebanov, Polyakov (1998); Witten (1998)

$\mathcal{N} = 4$ supersymmetric YM theory

Gliozzi,Scherk,Olive'77
Brink,Schwarz,Scherk'77

- Field content:

$$A_\mu \qquad \Phi_I \qquad \Psi_\alpha^A \qquad \text{all in the adjoint of } SU(N)$$

$$I = 1 \dots 6 \qquad A = 1 \dots 4$$

- Action:

$$S = \frac{1}{g_{YM}^2} \int d^4x \ \mathrm{tr} \left\{ \frac{1}{2} F_{\mu\nu}^2 + (D_\mu \Phi_I)^2 - \frac{1}{2} [\Phi_I, \Phi_J]^2 + i\bar{\Psi}\Gamma^\mu D_\mu \Psi - \bar{\Psi}\Gamma^I [\Phi_I, \Psi] \right\}$$

(super)conformal field theory = coupling doesn't run

AdS/CFT correspondence

$\mathcal{N}=4$ supersymmetric $SU(N_c)$ YM theory in 4 dim $\Longleftrightarrow$ type IIB superstring theory on $AdS_5 \times S^5$ backgrond

conjectured
exact equivalence

$$Z_{\rm SYM}[J] = \langle e^{-\int J\,\mathcal{O}\,d^4x}\rangle_{\rm SYM} = Z_{\rm string}[J]$$

Generating functional for correlation functions of gauge-invariant operators $\langle \mathcal{O}\ \mathcal{O}\ \cdots \mathcal{O}\rangle$ $\Longleftrightarrow$ String partition function

In particular

$$Z_{\rm SYM}[J] = Z_{\rm string}[J] \simeq e^{-S_{\rm grav}[J]}$$

$\lambda \equiv g_{YM}^2\, N_c \gg 1$

$N_c \gg 1$

Classical gravity action serves as a generating functional for the gauge theory correlators

Holography at finite temperature and density

$$\langle \mathcal{O} \rangle = \frac{\mathrm{tr}\rho\mathcal{O}}{\mathrm{tr}\rho}$$

$$\rho = e^{-\beta H + \mu Q}$$

$$H \to T^{00} \to T^{\mu\nu} \to h_{\mu\nu}$$

$$Q \to J^0 \to J^\mu \to A_\mu$$

Nonzero expectation values of energy and charge density translate into nontrivial background values of the metric (above extremality)=horizon and electric potential = CHARGED BLACK HOLE (with flat horizon)

$$ds^2 = -F(u)\, dt^2 + G(u)\left(dx^2 + dy^2 + dz^2\right) + H(u)\, du^2$$

$T = T_H$ temperature of the dual gauge theory

$$A_0 = P(u)$$

$\mu = P(\text{boundary}) - P(\text{horizon})$ chemical potential of the dual theory

Computing transport coefficients from "first principles"

Fluctuation-dissipation theory
(Callen, Welton, Green, Kubo)

Kubo formulae allows one to calculate transport coefficients from microscopic models

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d^3x e^{i\omega t} \langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \rangle$$

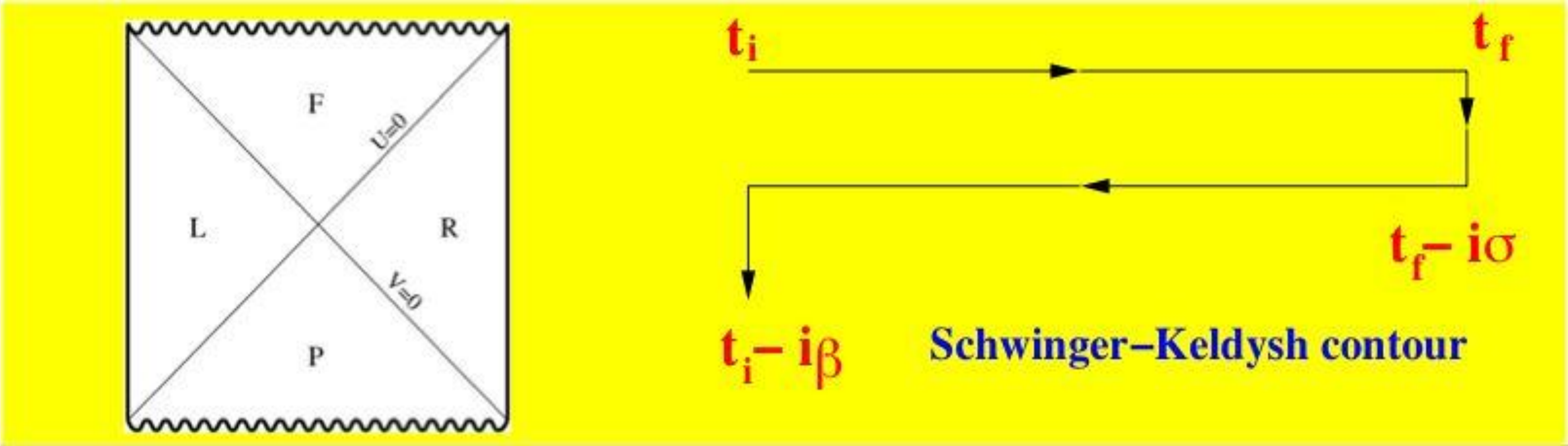
In the regime described by a gravity dual the correlator can be computed using the gauge theory/gravity duality

Computing real-time correlation functions from gravity

To extract transport coefficients and spectral functions from dual gravity, we need a recipe for computing Minkowski space correlators in AdS/CFT

The recipe of [D.T.Son & A.S., 2001] and [C.Herzog & D.T.Son, 2002] relates real-time correlators in field theory to Penrose diagram of black hole in dual gravity

Quasinormal spectrum of dual gravity = poles of the retarded correlators in 4d theory
[D.T.Son & A.S., 2001]



Computing transport coefficients from dual gravity

Assuming validity of the gauge/gravity duality,
all transport coefficients are completely determined
by the lowest frequencies
in quasinormal spectra of the dual gravitational background

(D.Son, A.S., hep-th/0205051, P.Kovtun, A.S., hep-th/0506184)

This determines kinetics in the regime of a thermal theory
where the dual gravity description is applicable

Transport coefficients and quasiparticle spectra can also be
obtained from thermal spectral functions $\chi = -2\,\mathrm{Im}\, G^R(\omega, q)$

Sound and supersymmetric sound in $4d\ \mathcal{N}=4$ SYM

In 4d CFT

$$\epsilon = 3P$$

$$\zeta = 0$$

$$\Longrightarrow$$

$$v_s = \sqrt{\frac{\partial P}{\partial \epsilon}} = \frac{1}{\sqrt{3}}$$

$$v_{SS} = \frac{P}{\epsilon} = \frac{1}{3}$$

Sound mode: $\omega = \pm \frac{q}{\sqrt{3}} - i\frac{2\,\eta}{3sT}q^2 + \cdots$

Supersound mode: $\omega = \pm \frac{q}{3} - iD_s q^2 + \cdots$

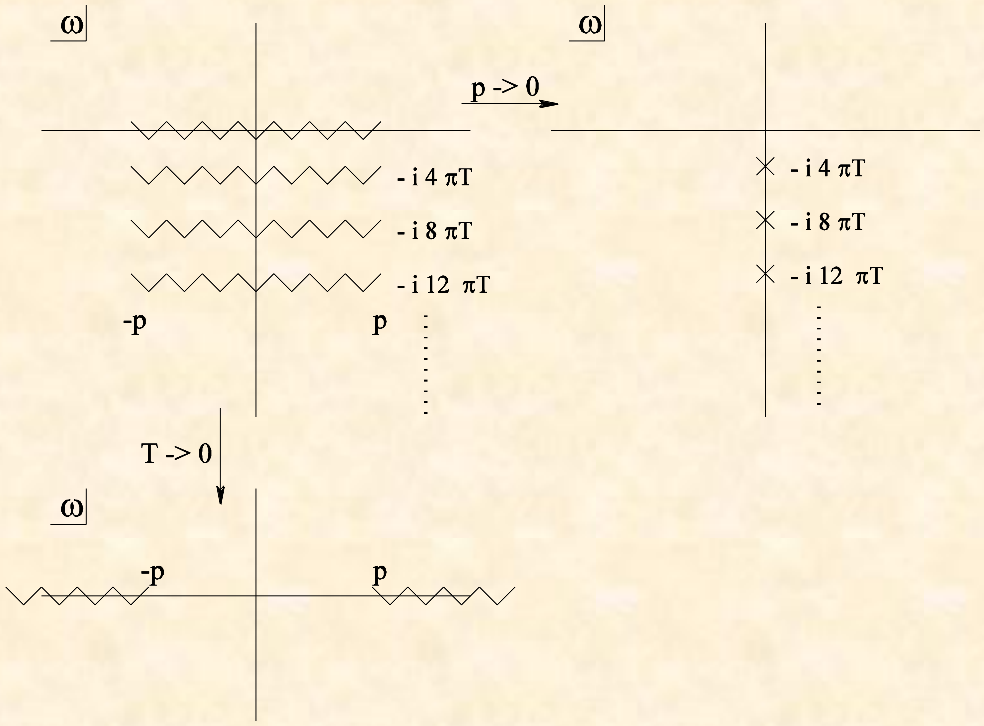
Quasinormal modes in dual gravity

Graviton: $\omega = \pm \frac{q}{\sqrt{3}} - i\frac{1}{6\pi T}q^2 + \cdots \Longrightarrow \frac{\eta}{s} = \frac{1}{4\pi}$

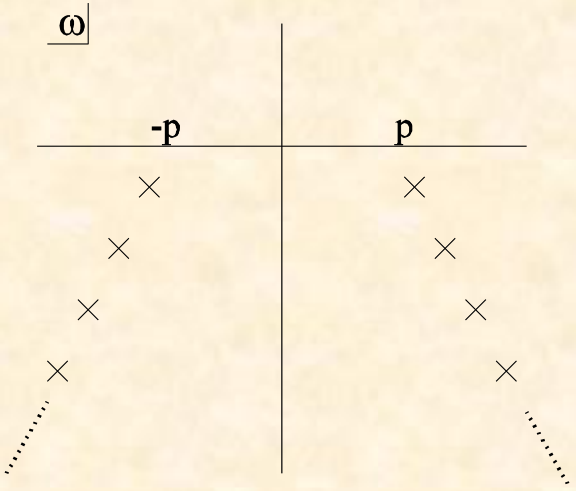
Gravitino: $\omega = \pm \frac{q}{3} - i\frac{2\sqrt{2}}{9\pi T}q^2 + \cdots \Longrightarrow D_s = \frac{2\sqrt{2}}{9\pi T}$

Analytic structure of the correlators

$g^2 N = 0$



$g^2 N = \infty$

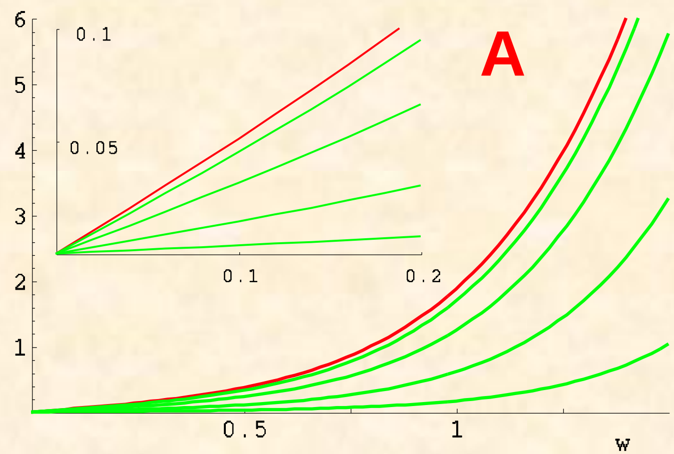


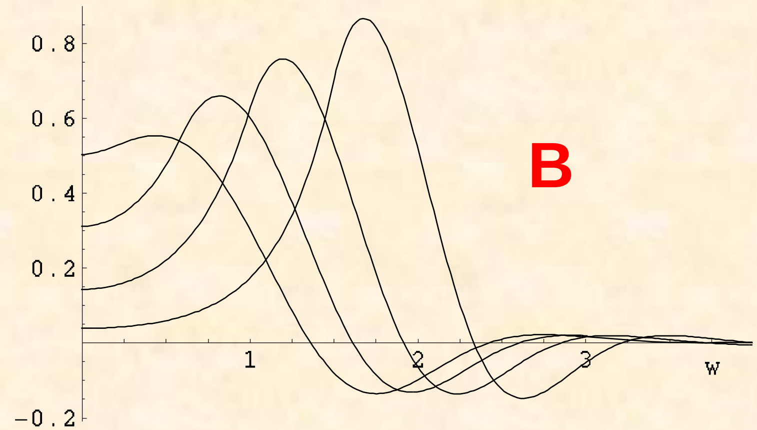
Strong coupling: A.S., hep-th/0207133

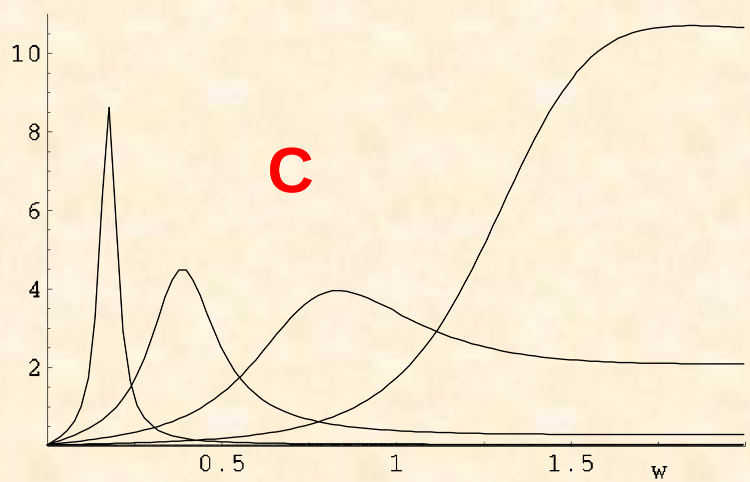
Weak coupling: S. Hartnoll and P. Kumar, hep-th/0508092

Spectral function and quasiparticles

$$\chi_{\mu\nu,\alpha\beta}(k) = \int d^4x\, e^{-ikx} \langle \left[T_{\mu\nu}(x) T_{\alpha\beta}(0) \right] \rangle = -2\, \mathrm{Im}\, G^R_{\mu\nu,\alpha\beta}(\omega, q)$$





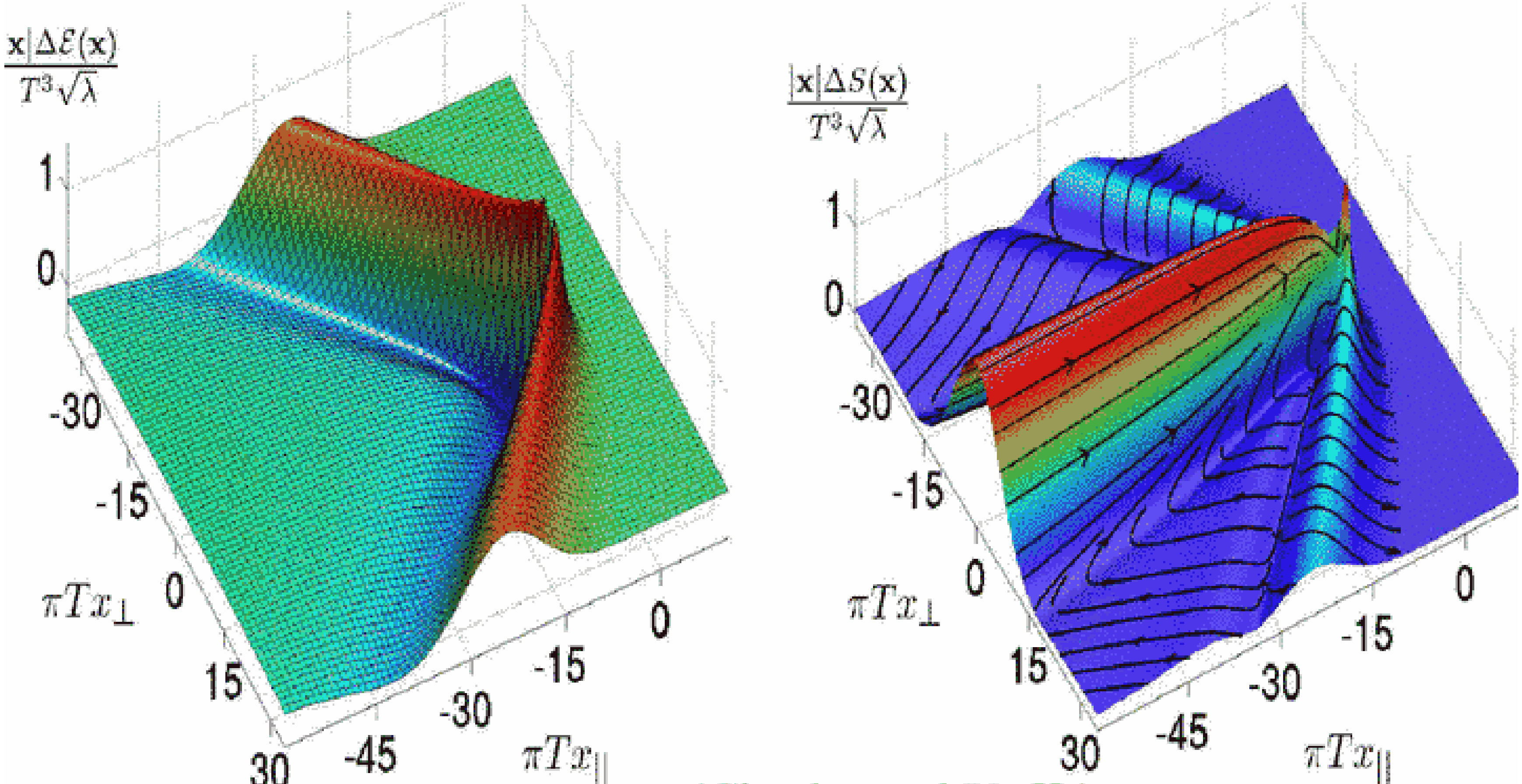


A: scalar channel

B: scalar channel - thermal part

C: sound channel

Energy and Momentum Density



(Chesler and Yaffe)

$\mathcal{N} = 4$ supersymmetric YM theory

Gliozzi,Scherk,Olive'77
Brink,Schwarz,Scherk'77

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(super)conformal field theory = coupling doesn't run

First-order transport coefficients in N = 4 SYM

in the limit $N_c \to \infty\,, \quad g^2_{YM} N_c \to \infty$

Shear viscosity $$\eta = \frac{\pi}{8} N_c^2 T^3 \left[1 + O\left(\frac{1}{(g^2 N_c)^{3/2}}, \frac{1}{N_c^2}\right)\right]$$

Bulk viscosity $$\zeta = 0$$

for non-conformal theories see Buchel et al; G.D.Moore et al Gubser et al.

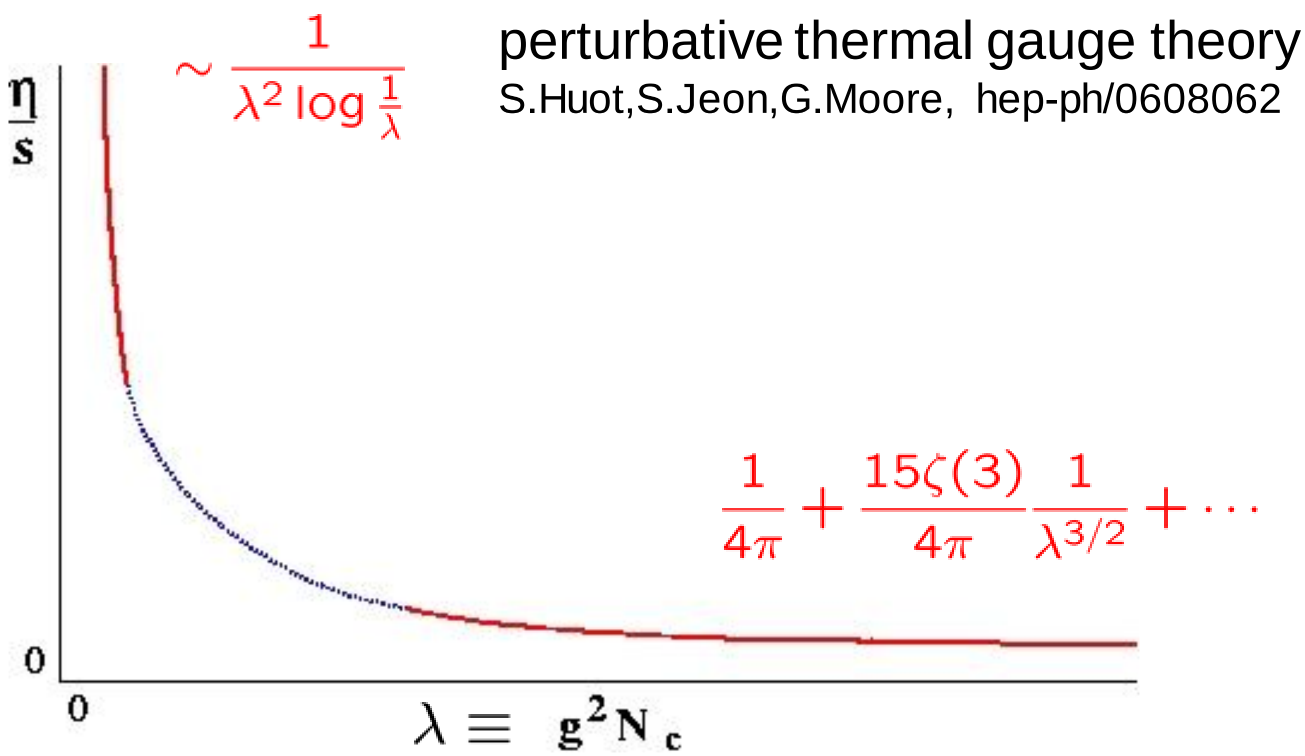
Charge diffusion constant $$D_R = \frac{1}{2\pi T} + \cdots$$

Supercharge diffusion constant $$D_s = \frac{2\sqrt{2}}{9\pi T}$$

Thermal conductivity $$\frac{\kappa_T\ \mu^2}{\eta\ T} = 8\pi^2 + \cdots$$

Electrical conductivity $$\sigma = e^2 \frac{N_c^2 T}{16\,\pi} + \cdots$$

Shear viscosity in $\mathcal{N} = 4$ SYM



Correction to $1/4\pi$: Buchel, Liu, A.S., hep-th/0406264

Buchel, 0805.2683 [hep-th]; Myers, Paulos, Sinha, 0806.2156 [hep-th]

Electrical conductivity in $\mathcal{N} = 4$ SYM

Weak coupling: $$\sigma = 1.28349 \, \frac{e^2 \, (N_c^2 - 1) \, T}{\lambda^2 \left[\ln \lambda^{-1/2} + O(1) \right]}$$

$\lambda \ll 1$

Strong coupling: $$\sigma = \frac{e^2 \, N_c^2 \, T}{16 \, \pi} + O\Big(\frac{1}{\lambda^{3/2}}\Big)$$

$\lambda \gg 1$

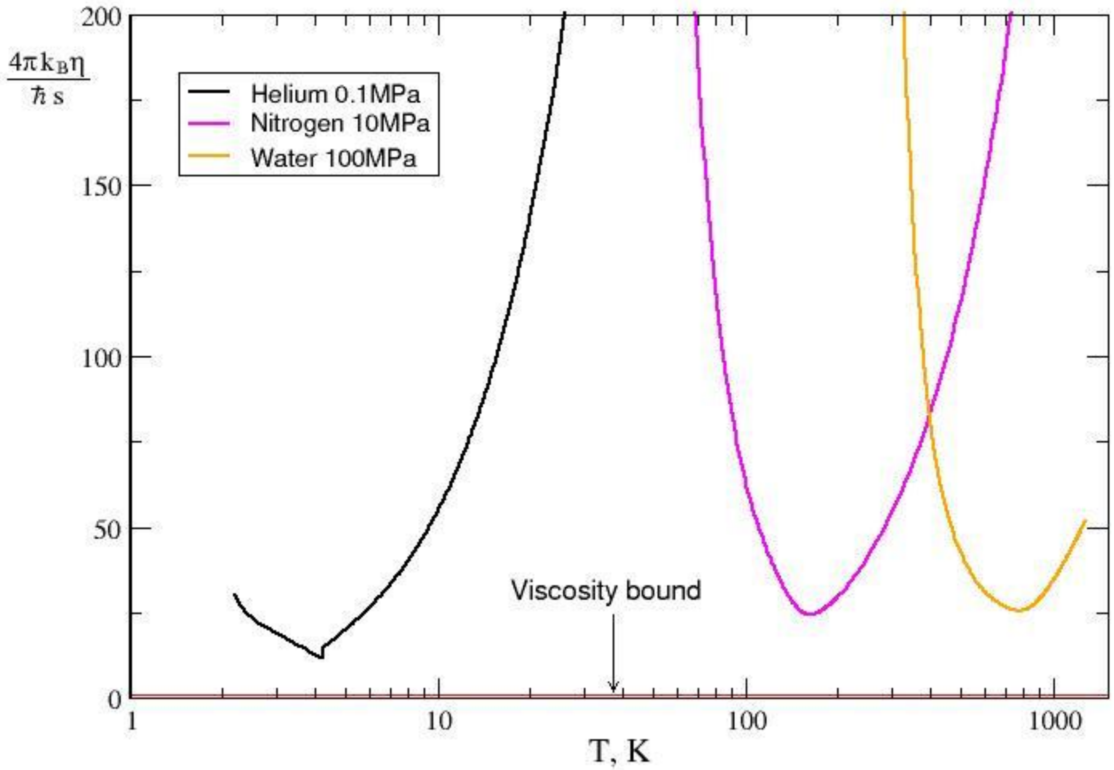
* Charge susceptibility can be computed independently: $\Xi = \frac{N_c^2 T^2}{8}$

D.T.Son, A.S., hep-th/0601157

Einstein relation holds: $$\frac{\sigma}{e^2 \Xi} = D_{U(1)} = \frac{1}{2\pi T}$$

A viscosity bound conjecture

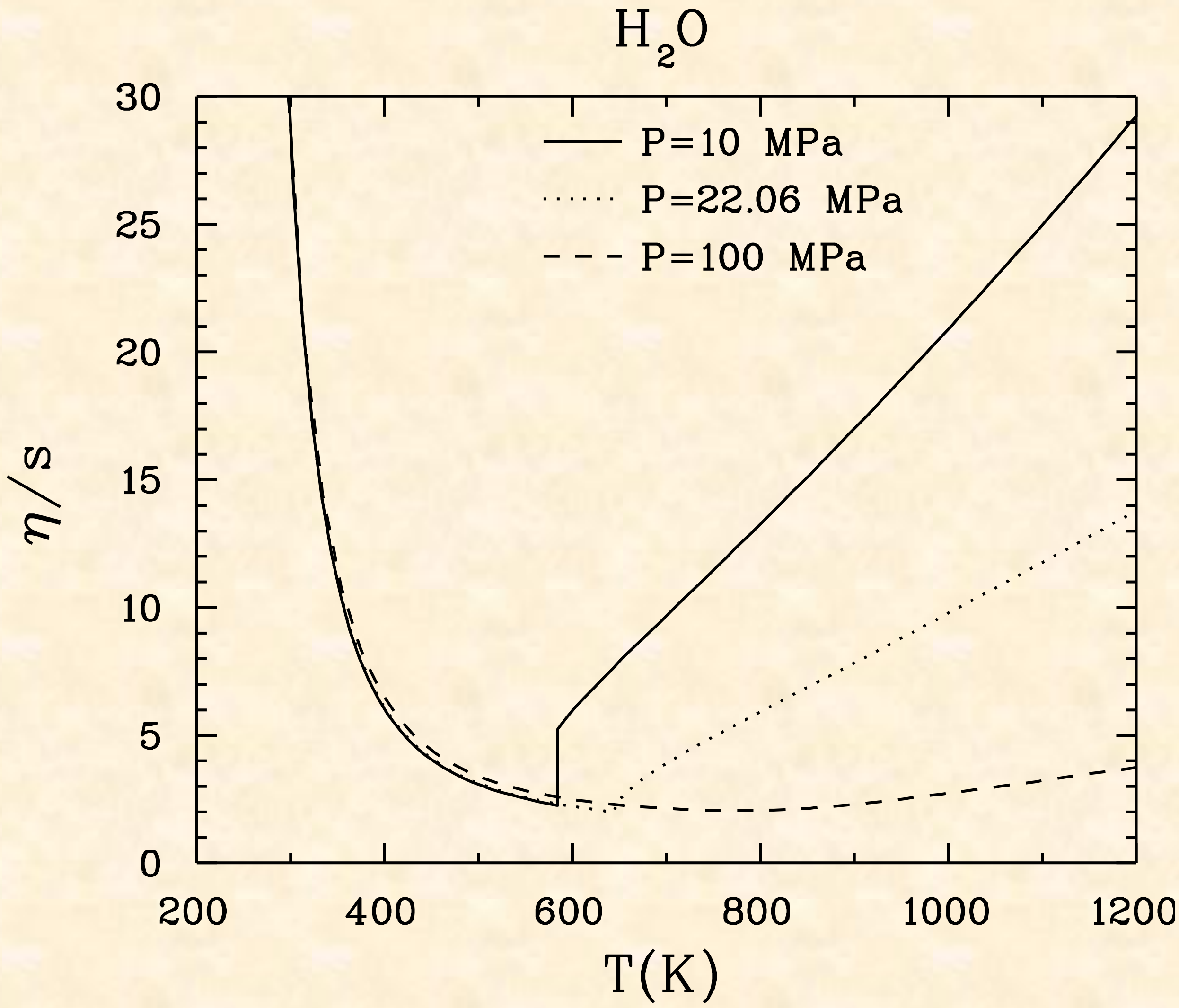
$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \approx 6.08 \cdot 10^{-13} \, K \cdot s$$



Minimum of $\frac{\eta}{s}$ in units of $\frac{\hbar}{4\pi k_B}$

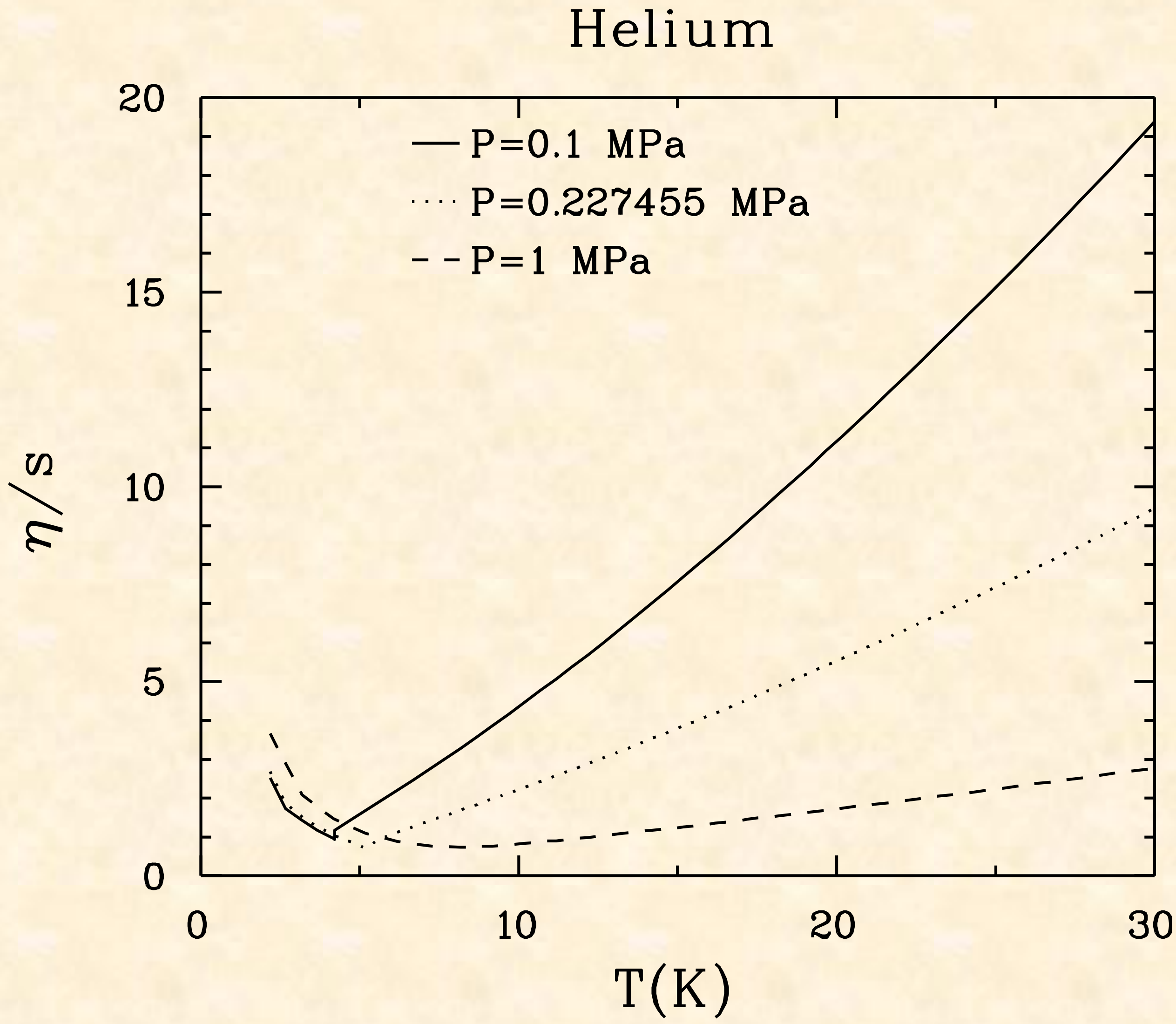
Xe	84
Kr	57
CO_2	32
H_2O	25
C_2H_5OH	22
Ne	17
He	8.8

P.Kovtun, D.Son, A.S., hep-th/0309213, hep-th/0405231



$(\eta/s)_{\mathrm{min}} \sim 25$ in units of $\dfrac{\hbar}{4\pi k_B}$

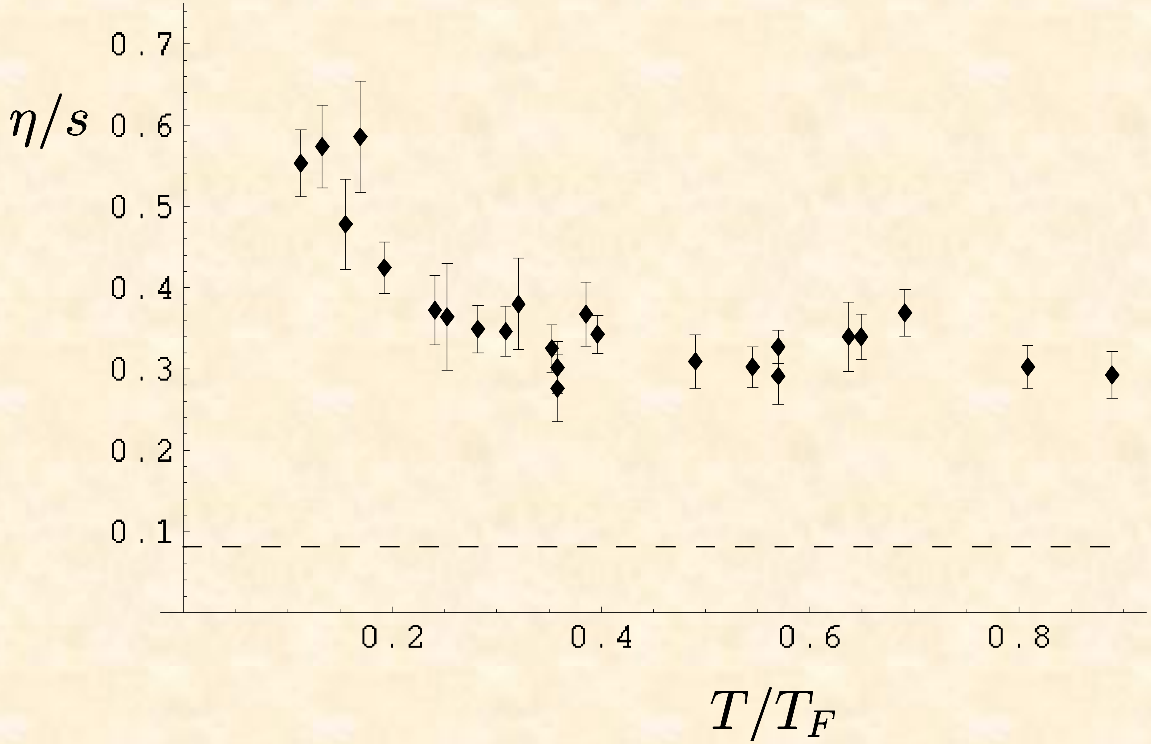
Chernai, Kapusta, McLerran, nucl-th/0604032



$(\eta/s)_{\mathrm{min}} \sim 8.8$ in units of $\dfrac{\hbar}{4\pi k_B}$

Chernai, Kapusta, McLerran, nucl-th/0604032

Viscosity-entropy ratio of a trapped Fermi gas

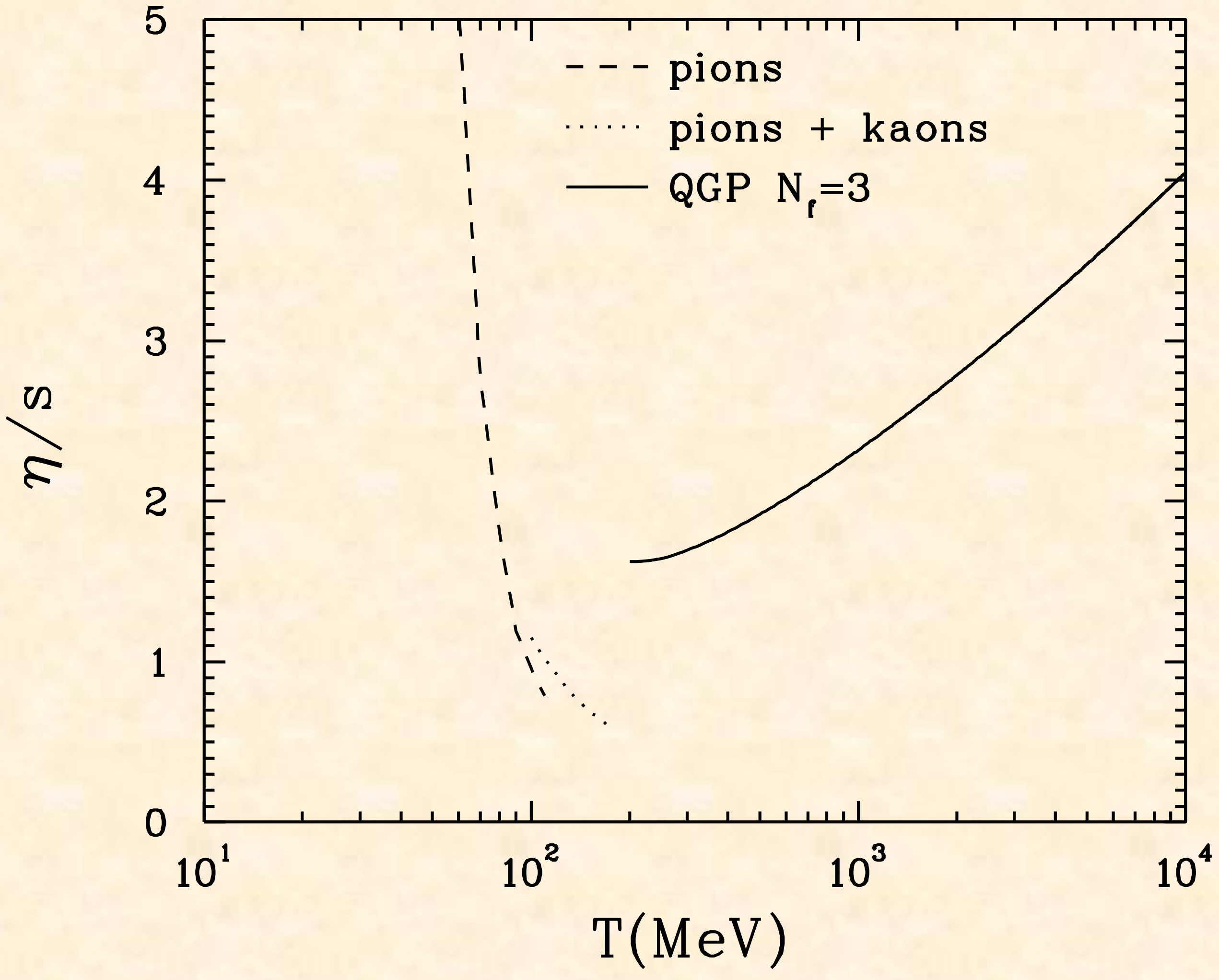


$\eta/s \sim 4.2$ in units of $\frac{\hbar}{4\pi k_B}$

T.Schafer, cond-mat/0701251

(based on experimental results by Duke U. group, J.E.Thomas et al., 2005-06)

QCD



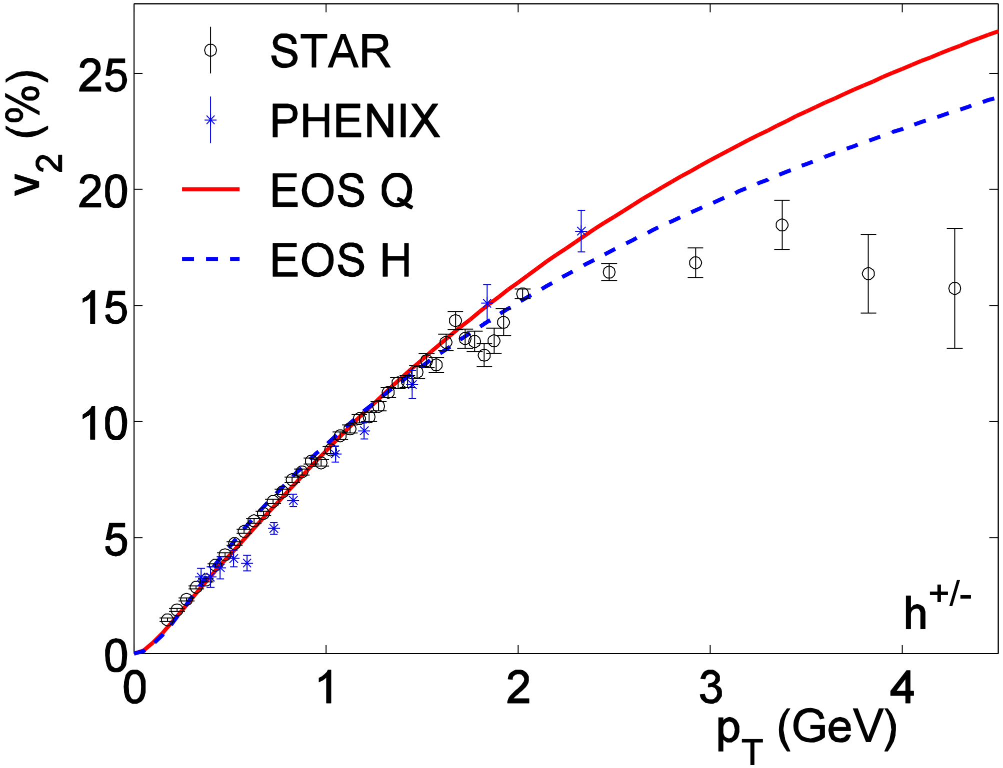
Chernai, Kapusta, McLerran, nucl-th/0604032

Viscosity "measurements" at RHIC

Viscosity is ONE of the parameters used in the hydro models describing the azimuthal anisotropy of particle distribution

$$\frac{d^2N^i}{dp_T d\phi} = N_0^i \left[1 + 2v_2^i(p_T)\cos 2\phi + \cdots\right]$$

$v_2^i(p_T)$ -elliptic flow for particle species "i"



Elliptic flow reproduced for

$0 < \eta/s \leq 0.3$

e.g. Baier, Romatschke, nucl-th/0610108

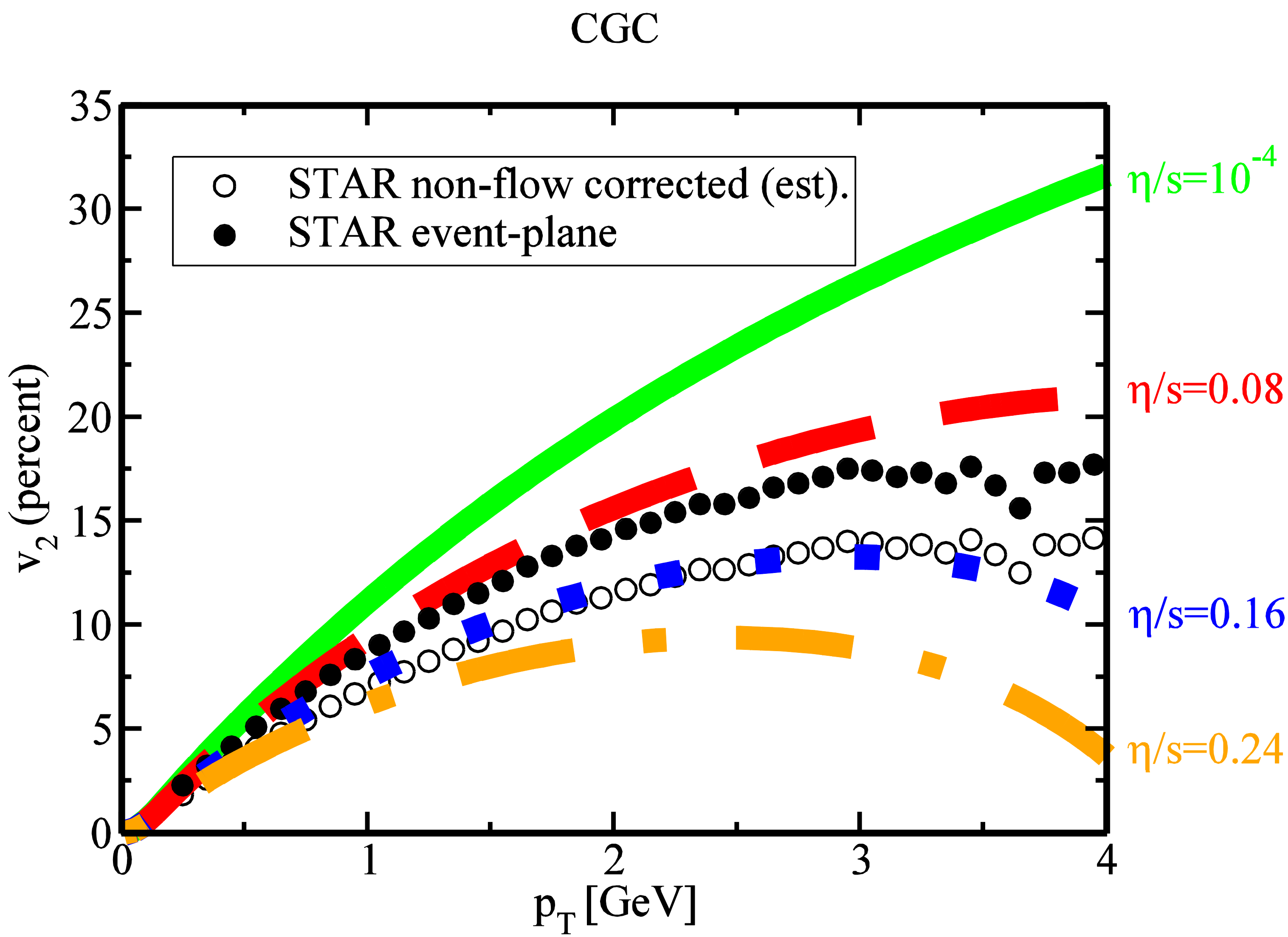
Perturbative QCD:

$\eta/s\,(T_{\text{RHIC}}) \approx 1.6 \sim 1.8$

Chernai, Kapusta, McLerran, nucl-th/0604032

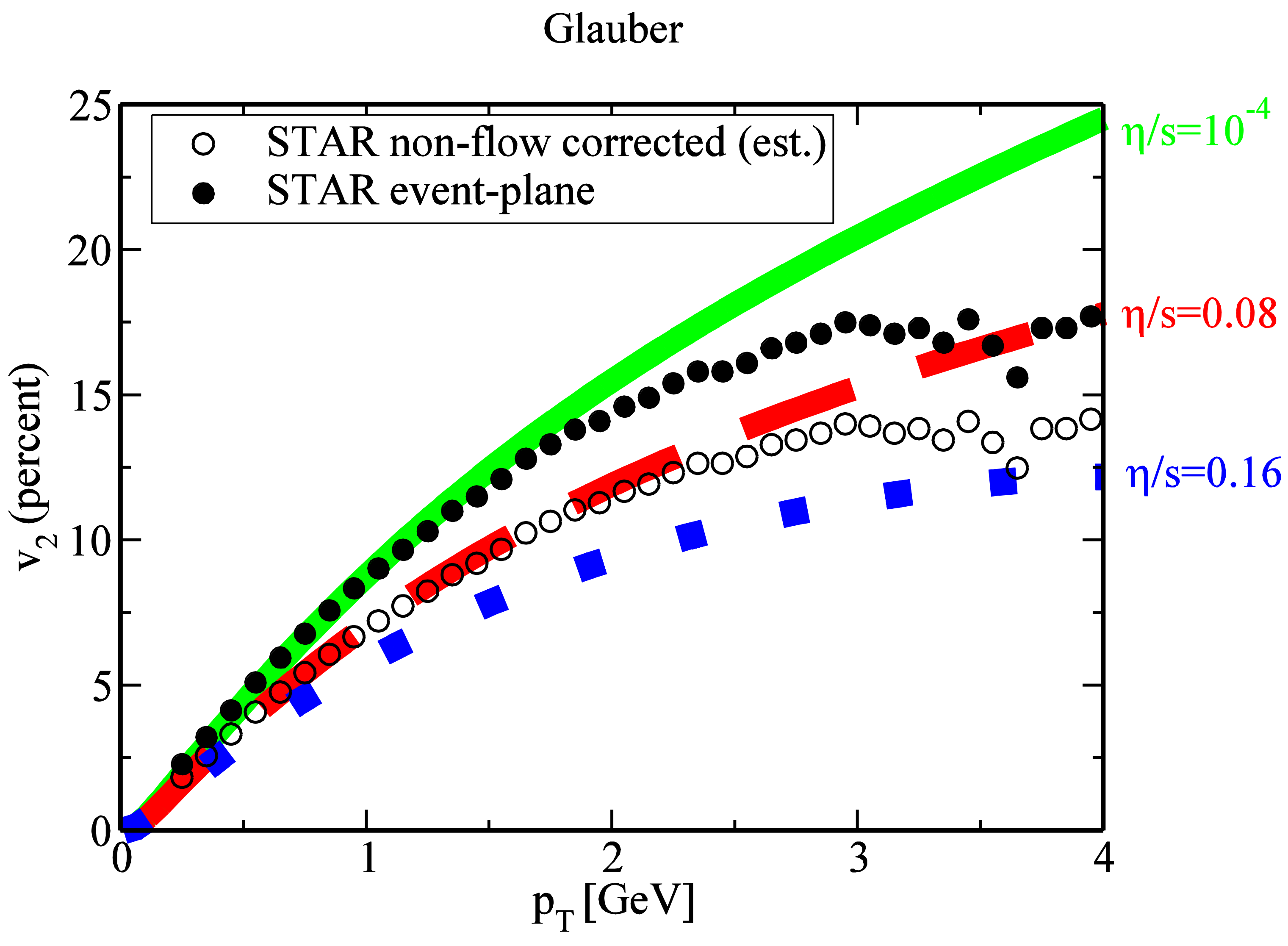
SYM: $\eta/s \approx 0.09 \sim 0.28$

Elliptic flow with color glass condensate initial conditions



Luzum and Romatschke, 0804.4015 [nuc-th]

Elliptic flow with Glauber initial conditions



Luzum and Romatschke, 0804.4015 [nuc-th]

Viscosity/entropy ratio in QCD

Theories with gravity duals in the regime where the dual gravity description is valid

$$\frac{\eta}{s} = \frac{1}{4\pi} \approx 0.08$$

(universal limit)

Kovtun, Son & A.S; Buchel; Buchel & Liu, A.S

QCD: RHIC elliptic flow analysis suggests

$$0 < \frac{\eta}{s} < 0.2$$

QCD: (Indirect) LQCD simulations

H.Meyer, 0805.4567 [hep-th]

$$0.08 < \frac{\eta}{s} < 0.16$$

$$1.2\, T_c < T < 1.7\, T_c$$

Trapped strongly correlated cold alkali atoms

T.Schafer, 0808.0734 [nucl-th]

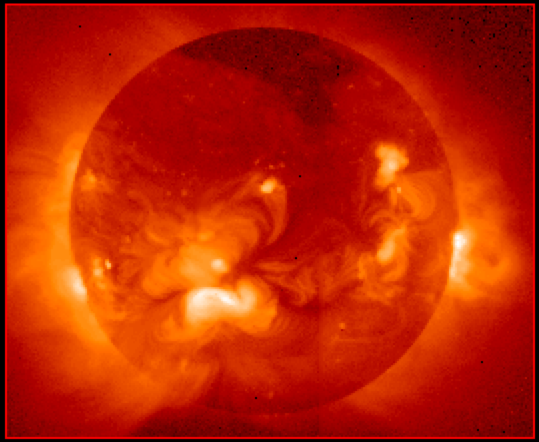
$$\left(\frac{\eta}{s}\right)_{\text{min}} \approx 0.5$$

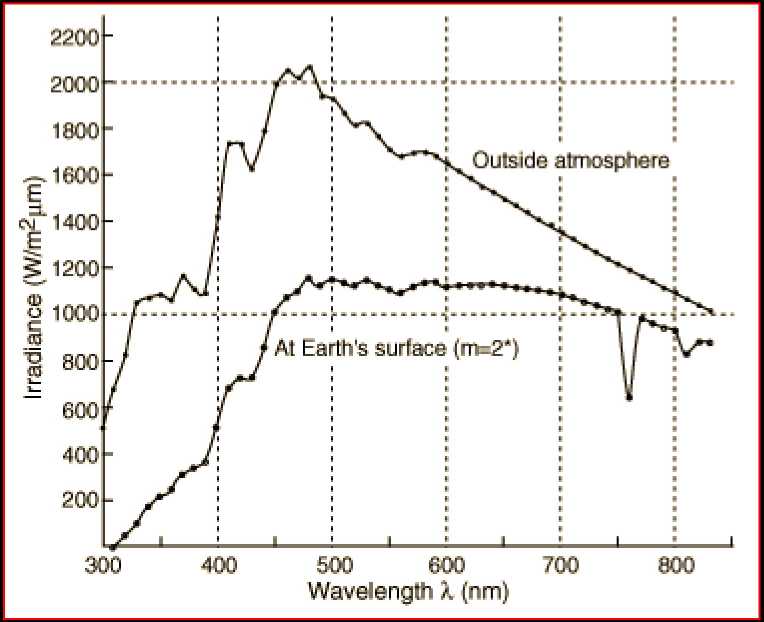
Liquid Helium-3

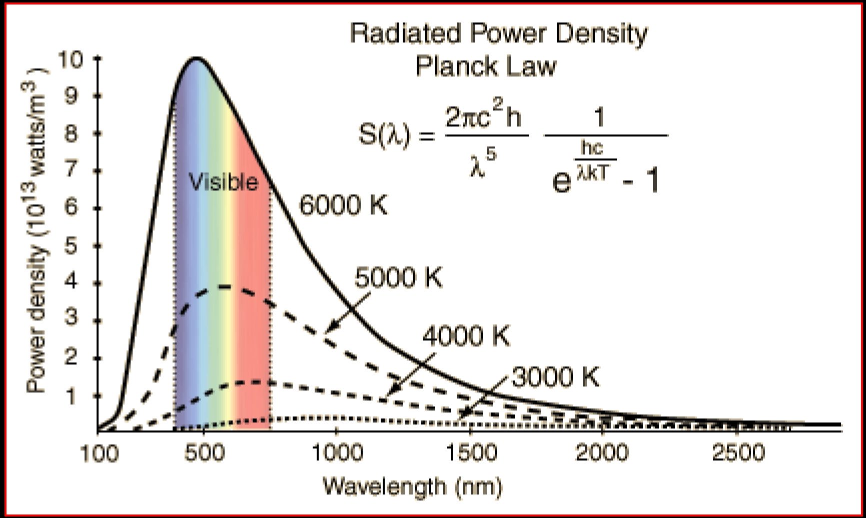
$$\left(\frac{\eta}{s}\right)_{\text{min}} \approx 0.7$$

Photon and dilepton emission from supersymmetric Yang-Mills plasma

S. Caron-Huot, P. Kovtun, G. Moore, A.S., L.G. Yaffe, hep-th/0607237







Photon emission from SYM plasma

Photons interacting with matter: $e\, J_\mu^{\mathrm{EM}}\, A^\mu$

To leading order in e $d\Gamma_\gamma = \frac{d^3k}{(2\pi)^3} \frac{e^2}{2|k|} \eta^{\mu\nu} C^<_{\mu\nu}(k^0 = |k|)$

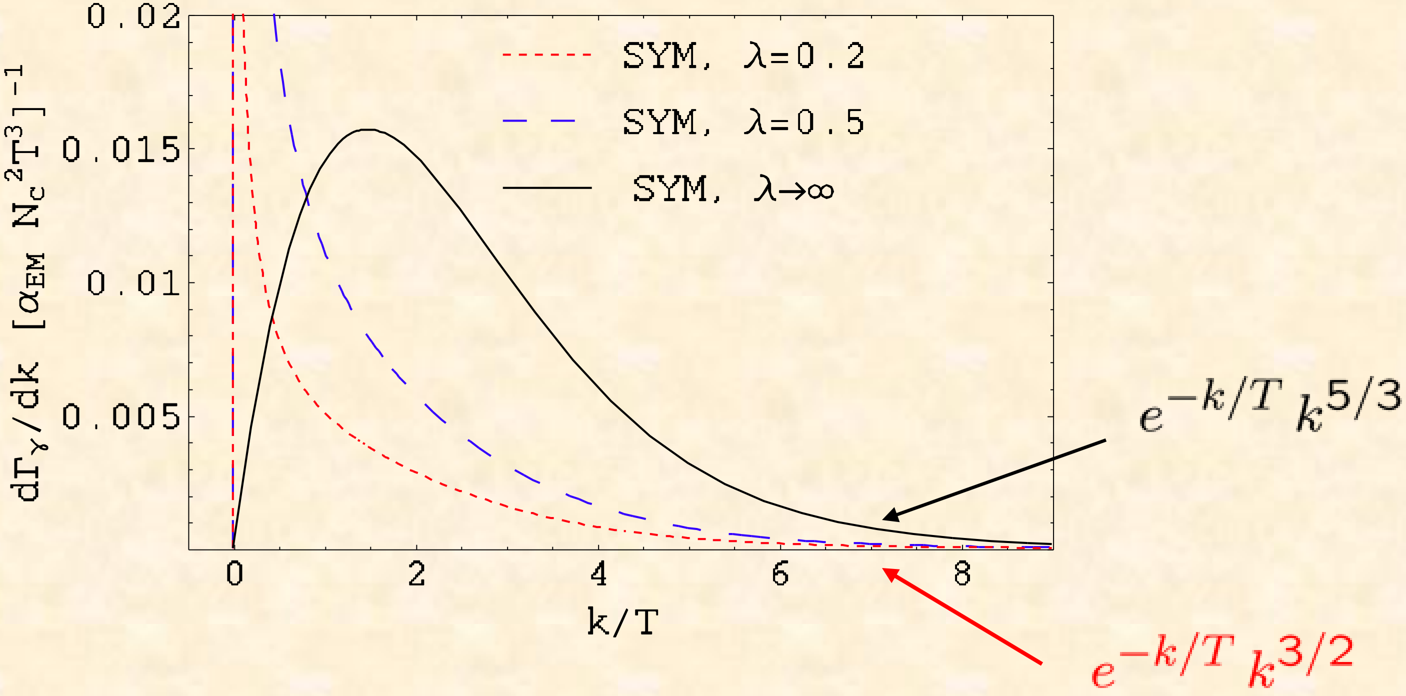
$$C^<_{\mu\nu} = \int d^4X e^{-iKX} \langle J_\mu^{\mathrm{EM}}(0) J_\nu^{\mathrm{EM}}(X) \rangle$$

Mimic J_μ^{EM} by gauging global R-symmetry $U(1) \subset SU(4)$

$$\mathcal{L} = \mathcal{L}_{\mathcal{N}=4\,\mathrm{SYM}} + e\, J_\mu^3\, A^\mu - \frac{1}{4} F_{\mu\nu}^2$$

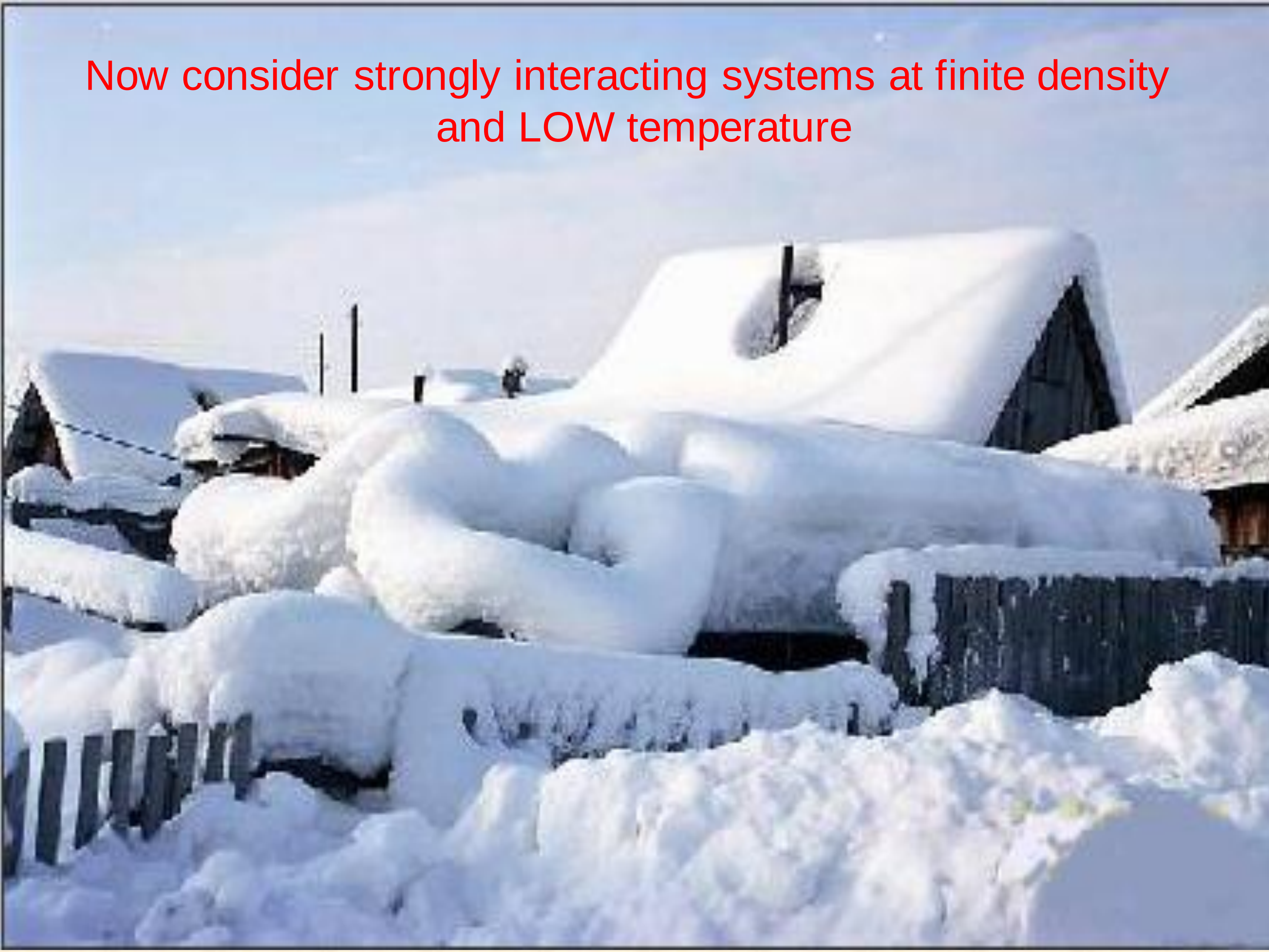
Need only to compute correlators of the R-currents J_μ^3

Photoproduction rate in SYM



(Normalized) photon production rate in SYM for various values of ‘t Hooft coupling

$$\frac{d\Gamma_\gamma}{dk\,\alpha_{em} N_c^2 T^3} = n_B(k)\left(\frac{k}{4\pi T}\right)^2 \left|{}_2F_1\left(1-\frac{(1+i)k}{4\pi T}, 1+\frac{(1-i)k}{4\pi T}; 1-\frac{ik}{2\pi T}; -1\right)\right|^{-2}$$


Now consider strongly interacting systems at finite density
and LOW temperature

Probing quantum liquids with holography

Quantum liquid in p+1 dim	Low-energy elementary excitations	Specific heat at low T
Quantum Bose liquid	phonons	$\sim T^p$
Quantum Fermi liquid (Landau FLT)	fermionic quasiparticles + bosonic branch (zero sound)	$\sim T$

Departures from normal Fermi liquid occur in

- 3+1 and 2+1 –dimensional systems with strongly correlated electrons
- In 1+1 –dimensional systems for any strength of interaction (Luttinger liquid)

One can apply holography to study strongly coupled Fermi systems at low T

The simplest candidate with a known holographic description is

$SU(N_c)$ $\mathcal{N}=4$ SYM coupled to N_f $\mathcal{N}=2$ fundamental hypermultiplets

at finite temperature T and nonzero chemical potential associated with the "baryon number" density of the charge $U(1)_B \subset U(N_f)$

There are two dimensionless parameters: $\frac{n_q^{1/3}}{T}$ $\frac{M}{T}$

n_q is the baryon number density

M is the hypermultiplet mass

The holographic dual description in the limit $N_c \gg 1\,,\; g_{YM}^2 N_c \gg 1\,,\; N_f$ finite is given by the D3/D7 system, with D3 branes replaced by the AdS-Schwarzschild geometry and D7 branes embedded in it as probes.

Karch & Katz, hep-th/0205236

AdS-Schwarzschild black hole (brane) background

$$ds^2 = \frac{r^2}{R^2}\left[-\left(1-\frac{r_H^4}{r^4}\right)dt^2 + d\vec{x}^2\right] + \left(1-\frac{r_H^4}{r^4}\right)^{-1}\frac{R^2}{r^2}dr^2$$

D7 probe branes

$$S_{DBI} = -N_f T_{D7}\int d^8\xi \sqrt{-\det(g_{ab}+2\pi\alpha' F_{ab})}$$

The worldvolume U(1) field A_μ couples to the flavor current J^μ at the boundary

Nontrivial background value of A_0 corresponds to nontrivial expectation value of J^0

We would like to compute

- the specific heat at low $(T n_q^{-1/3} \ll 1)$ temperature
- the charge density correlator $G^R \sim \langle J^0(k)\, J^0(-k)\rangle$

★ The specific heat (in p+1 dimensions):

$$c_V = \mathcal{N}_q\, p \left(\frac{4\pi}{p+1}\right)^{2p+1} \frac{T^{2p}}{n_q} \left[1 + O(T n_q^{-\frac{1}{p}})\right]$$

(note the difference with Fermi $c_V \sim T$ and Bose $c_V \sim T^p$ systems)

★ The (retarded) charge density correlator $G^R \sim \langle J^0(k)\, J^0(-k)\rangle$
has a pole corresponding to a propagating mode (zero sound)
- even at zero temperature

$$\omega = \pm\frac{q}{\sqrt{p}} - \frac{i\,\Gamma(\frac{1}{2})\, q^2}{n_q^{\frac{1}{p}}\Gamma(\frac{1}{2}-\frac{1}{2p})\Gamma(\frac{1}{2p})} + O(q^3)$$

(note that this is NOT a superfluid phonon whose attenuation scales as q^{p+1})

New type of quantum liquid?

Other avenues of (related) research

Bulk viscosity for non-conformal theories (Buchel, Gubser,…)

Non-relativistic gravity duals (Son, McGreevy,…)

Gravity duals of theories with SSB (Kovtun, Herzog,…)

Bulk from the boundary (Janik,…)

Navier-Stokes equations and their generalization from gravity (Minwalla,…)

Quarks moving through plasma (Chesler, Yaffe, Gubser,…)

Epilogue

- On the level of theoretical models, there exists a connection between near-equilibrium regime of certain strongly coupled thermal field theories and fluctuations of black holes

- This connection allows us to compute transport coefficients for these theories

- At the moment, this method is the only theoretical tool available to study the near-equilibrium regime of strongly coupled thermal field theories

- The result for the shear viscosity turns out to be universal for all such theories in the limit of infinitely strong coupling

- Influences other fields (heavy ion physics, condmat)

THANK YOU

A hand-waving argument

$$\eta \sim \rho v l \sim \rho v^2 \tau \sim n m v^2 \tau \sim n \epsilon \tau$$

$$s \sim n$$

Thus $$\frac{\eta}{s} \sim \epsilon\tau \geq \hbar$$

Gravity duals fix the coefficient: $$\frac{\eta}{s} \geq \hbar/4\pi$$

Outlook

- Gravity dual description of thermalization ?

- Gravity duals of theories with fundamental fermions:

  - phase transitions
  - heavy quark bound states in plasma
  - transport properties

- Finite ‘t Hooft coupling corrections to photon emission spectrum

- Understanding 1/N corrections

- Phonino

Energy density vs temperature for various gauge theories

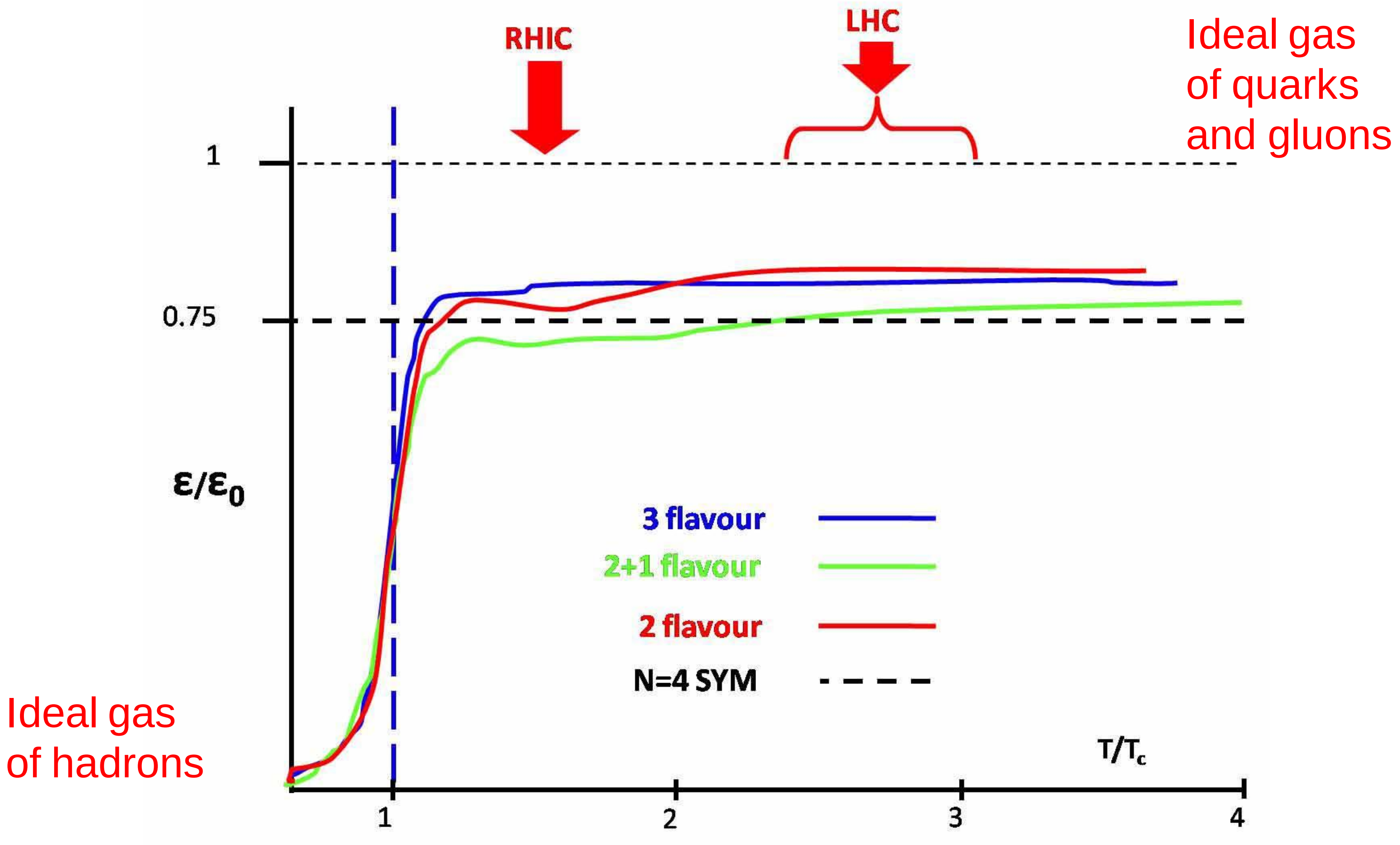


Figure: an artistic impression from Myers and Vazquez, 0804.2423 [hep-th]

Gauge-string duality can explore and qualitatively explain (with model-dependent limitations!) QGP-related phenomena such as

Rapid thermalization

Large elliptic flow

Jet quenching

Photon/dilepton emission rates

The bulk and the boundary in AdS/CFT correspondence

$$ds^2 = \frac{\eta_{\mu\nu}\, dx^\mu\, dx^\nu + dz^2}{z^2}$$

UV/IR: the AdS metric is invariant under $z \to \Lambda\, z \quad x \to \Lambda\, x$

z plays a role of inverse energy scale in 4D theory