

Perspectives for New Physics in the Flavour Sector

G.C. Branco

CFTP/IST, U. Lisboa

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based on collaboration with

F. Botella, M. N. Rebelo, M. Nebot

and earlier work with W. Grimus and L.avoura

Some of the Open Questions in Flavour Physics.

- Why is there "Flavour"?
i.e. What is the Origin of Family replication?
- How to understand the observed spectrum
of Fermion Masses and Mixing?
Why is Leptonic Mixing Large, in
contrast to Small Quark Mixing?
- Why is there Flavour Alignment in
the Quark Sector?

• Does the Scalar Sector play a non-trivial rôle in the question of Flavour? If so, it is likely that Nature chooses a richer scalar sector. Simplest possibility: more than one Higgs doublet

Question: How to avoid Flavour changing neutral currents!

- What is the Origin of CP Violation?
Complex Yukawa couplings?
Or Spontaneously broken by the vacuum?
- How to solve the Strong CP problem?
Peccei-Quinn provide an elegant solution, but no Axions have been found !!

The various manifestations of CP violation :

- CP violation in the quark sector
- CP violation in the lepton sector ?
- CP violation needed to generate BAU

A Common Origin ?

- What about the Strong CP problem?

At present, there is solid experimental evidence for a complex V_{CKM} .

Does this imply complex Yukawa couplings?

NO!!

There are realistic models of spontaneous CP violation where the vacuum phase generates a complex V_{CKM} .

Can one have geometrical CP violation?

Generation of the Baryon Asymmetry of the Universe (BAU)

The ingredients to dynamically generate **BAU** from an initial state with zero B. A. , were formulated by Sakharov(1967)

- (i) Baryon number violation
- (ii) C and CP Violation
- (iii) Departure from thermal equilibrium

P 50

All these ingredients exist in the SM,
but it has been established that in
the SM, one cannot generate the
observed BAU :

$$\Omega_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.20 \pm .15) \times 10^{-10}$$

n_B , $n_{\bar{B}}$, n_γ number densities of baryons,
anti baryons and photons at present time.

Reasons why the SM cannot generate sufficient BAU :

(i) CP violation in the SM is too small

$$\frac{\text{tr} [H_u, H_d]^3}{T_{ew}^{12}} \approx 10^{-20}$$

(ii) Successful baryogenesis requires a strongly first order phase transition which would require a light Higgs mass

$$m_H \leq 70 \text{ GeV}$$

Weak-basis invariants are a useful to study Flavour.

Example :

$$\text{tr} [H_u, H_d]^3 \xrightarrow{\text{J. Bernabeu, G.C.B., M. Gronau}} \\ \times (m_c^2 - m_u^2) (m_t^2 - m_c^2) (m_t^2 - m_u^2) \times \\ \times (m_s^2 - m_d^2) (m_b^2 - m_s^2) (m_b^2 - m_d^2) \times$$

$\propto \det [H_u, H_d]$

for 3 generations

(C. Jarlskog)

Im Q

Q → invariant quartet of
VCKM

One may also study invariants under Higg basis transformations

G.C.B., M.N. Rebelo, Silva-Marcos ;

H. Haber, F. Gunion
S. Davidson, H. Haber

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3 Flavour Dogmas introduced
in the 70's , soon after the
creation and early success of the S.M.

Dogma 1 - Neutrinos are strictly massless !

Dogma 2 - No Z -mediated FCNC at
tree level

Dogma 3 - No Higgs mediated FCNC
at tree level.

Question : We all know the "Fate" of Dogma N°1
Will the other two Dogmas have the same
"Fate" as Dogma 1. ?

Can one violate these two dogmas in reasonable extensions of the SM? Yes!

"Reasonable" means that FCNC should be naturally suppressed, without fine-tuning.

- In the gauge sector, the Dogma can be violated through the introduction of a $Q = 1/3$ and/or $Q = 2/3$ vector-like quark.

Naturally small violations of 3×3 unitarity of V^{CKM}

Z -mediated, Naturally suppressed FCNC at tree level

~~Yukawa interactions in the General two Higgs doublet Model~~

$$-\mathcal{L}_Y = \bar{Q}_L^0 \Gamma_1 \phi_1 d_R^0 + \bar{Q}_L^0 \Gamma_2 \phi_2 d_R^0 + \bar{Q}^0 \Delta_1 \tilde{\phi}_1 u_R^0 + \bar{Q}_L^0 \Delta_2 \tilde{\phi}_2 u_R^0 + h.c.$$

Quark mass matrices:

$$M_d = \frac{1}{\sqrt{2}} (v_1 \Gamma_1 + v_2 e^{i\alpha} \Gamma_2) ; M_u = \frac{1}{\sqrt{2}} (v_1 \Delta_1 + v_2 e^{-i\alpha} \Delta_2)$$

Diagonalized by:

$$(U_d)_L^+ M_d (U_d)_R = D_d = \text{diag}(m_d, m_s, m_b)$$

$$(U_u)_L^+ M_u U_{uR} = D_d = \text{diag}(m_u, m_c, m_t)$$

Expanding around the vacuum:

$$\langle \phi_j^0 \rangle = e^{i\alpha_j} \frac{1}{\sqrt{2}} (v_j + \rho_j + i\eta_j) \quad j=1,2$$

It is useful to introduce new fields:

$$\begin{bmatrix} H^0 \\ R \end{bmatrix} = O \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix}; \begin{bmatrix} G^0 \\ I \end{bmatrix} = O \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}; \begin{bmatrix} G^+ \\ H^+ \end{bmatrix} = O \begin{bmatrix} \phi_1^+ \\ \phi_2^+ \end{bmatrix}$$

$$O = \frac{1}{V} \begin{bmatrix} v_1 & v_2 \\ v_2 & -v_1 \end{bmatrix}; V = \sqrt{v_1^2 + v_2^2} \approx 246 \text{ GeV}$$

$H^0 \rightarrow$ has couplings to quarks proportional to mass matrices

$G^0 \rightarrow$ neutral pseudo Goldstone boson

$H^\pm \rightarrow$ charged pseudo-Goldstone bosons

Neutral and charged Higgs interactions¹⁴ in the quark sector

$$\begin{aligned}
 -\mathcal{L}_Y = & \bar{d}_L^\circ \frac{1}{v} (M_d H^\circ + N_d^\circ R + i N_d^\circ I) d_R^\circ + \\
 & + \bar{u}_L^\circ \frac{1}{v} (M_u H^\circ + N_u^\circ R + i N_u^\circ I) u_R^\circ + \\
 & + \frac{\sqrt{2}}{v} H^+ (\bar{u}_L^\circ N_d^\circ d_R^\circ - \bar{u}_R^\circ N_u^\circ d_L^\circ) + h.c.
 \end{aligned}$$

$$N_d^\circ = \frac{1}{\sqrt{2}} (v_2 \Gamma_1 - v_1 e^{i\alpha} \Gamma_2); \quad N_u^\circ = \frac{1}{\sqrt{2}} (v_2^{\Delta_1} v_1 \bar{e}^{-i\alpha} \Gamma_2).$$

In the quark mass eigenstate basis N_d, N_u are not flavour diagonal.

Physical neutral Higgs are combinations of H°, R, I

Yukawa Couplings in terms of quark mass eigenstates :

$$\begin{aligned}
 \mathcal{L}_Y = & \frac{\sqrt{2} H^+}{v} \bar{u} \left[-V N_d \gamma_R + N_u^+ V \gamma_L \right] d + \text{h.c.} - \\
 & - \frac{H^0}{v} \left[\bar{u} D_u u + \bar{d} D_d d \right] - \\
 & - \frac{R}{v} \left[\bar{u} (N_u \gamma_R + N_u^+ \gamma_L) u + \bar{d} (N_d \gamma_R + N_d^+ \gamma_L) d \right] \\
 & + \frac{T}{v} \left[\bar{u} (N_u \gamma_R - N_u^+ \gamma_L) u - \bar{d} (N_d \gamma_R - N_d^+ \gamma_L) d \right]
 \end{aligned}$$

$$\gamma_L = \frac{1}{2}(1 - \gamma_5) \quad ; \quad \gamma_R = \frac{1}{2}(1 + \gamma_5) ; \quad V \text{ is the CKM matrix}$$

Flavour changing neutral currents are controlled by :

$$N_d = \frac{1}{\sqrt{2}} U_{dL}^\dagger (v_2 \Gamma_1 - v_1 e^{i\alpha} \Gamma_2) U_{dR}$$

$$N_u = \frac{1}{\sqrt{2}} U_{uL}^\dagger (v_2 \Delta_1 - v_1 \bar{e}^{-i\alpha} \Delta_2) U_{uR}$$

For generic two Higgs doublet models
 N_u, N_d are non-diagonal, arbitrary
matrices.



too large Higgs mediated Flavour changing
neutral currents, unless a suppression mechanism
is introduced

2)

G.C.B, Grimus, W and Lavoura, L (BGL) have shown that it is possible to find a symmetry which when imposed on a 2-Higgs doublet extension of the SM, leads to a structure of the Yukawa couplings such that there are FCNC at tree level, with strength completely controlled by V_{CKM} , i.e., N_d , N_u only depend on V_{CKM} and on $\frac{v_1}{v_2}$.

Possible choice of the symmetry S :

$$Q_L^{\circ} \rightarrow \exp(i\alpha) Q_L^{\circ} \quad ; \quad U_R^{\circ} \rightarrow \exp(i2\alpha) U_R^{\circ}$$

$$\phi_2 \rightarrow \exp(i\alpha) \phi_2, \text{ where } \alpha \neq 0, \pi$$

all other fields transform trivially under S .
 This leads to the Yukawa couplings:

$$\Gamma_1 = \begin{bmatrix} * & * & * \\ * & * & * \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ * & * & * \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad \Delta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & * \end{bmatrix}$$

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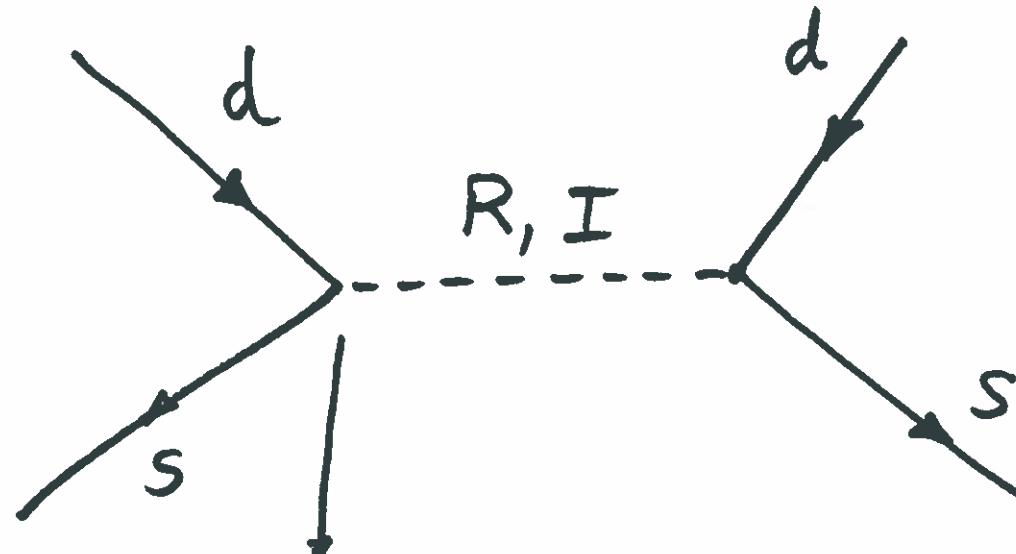
These Yukawa structures lead to :

$$(N_d)_{ij} = \frac{v_2}{v_1} (D_d)_{ij} - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) [V_{CKM}]_{3j} \left[V_{CKM}^* \right]_{3i} (D_d)_{jj}$$

$$(N_u) = -\frac{v_1}{v_2} \text{diag.}(0, 0, m_t) + \frac{v_2}{v_1} \text{diag.}(m_u, m_c, 0)$$

In this particular BGL model there are FCNC only in the down sector, but there is a strong, natural suppression of the most "dangerous" processes

$K^0 - \bar{K}^0$ mixing



λ^{10} suppression!!

$$V_{td} V_{ts}^* \sim \lambda^5$$

There are 6 different BGL models
in the quark sector

With so many Open Questions, it
is very likely that there is
New Physics beyond the LSM.

The fundamental question :

Where to search for New Physics?

Energy Frontier? Answer:

Intensity Frontier? Both !!

3×3 Unitarity of $\sqrt{V_{CKM}}$ ²²

A crucial ingredient in testing
the flavour sector of the SM.

How to test 3×3 unitarity of
 V_{CKM} ? Can one have a sensible
theory where 3×3 unitarity is violated?

Answer : Yes!

Consider an extension of the SM, where the quark mixing matrix, is of arbitrary size :

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} & \dots \\ V_{cd} & V_{cs} & V_{cb} & \dots \\ V_{td} & V_{ts} & V_{tb} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

If we do not impose 3×3 unitarity how many physical parameters are there in the 3×3 sector of V ?

Counting of Parameters :

Moduli : 9

Invariant phases : $9 - 5 = 4$

↓
removed by rephasing
of standard quark
phases

So altogether one has 13 independent physical quantities

Choice of rephasing invariant phases

$$\beta \equiv \arg (-V_{cd} V_{tb} V_{cb}^* V_{td}^*)$$

$$\gamma \equiv \arg (-V_{ud} V_{cb} V_{ub}^* V_{cd}^*)$$

$$\chi \equiv \beta_s \equiv \arg (-V_{cb} V_{ts} V_{cs}^* V_{tb}^*)$$

$$\chi' = \arg (-V_{us} V_{cd} V_{ud}^* V_{cs}^*)$$

Note that $\alpha \equiv \arg (-V_{ub} V_{td} V_{ud}^* V_{tb}^*)$
is not independent of β, γ .

$$\alpha + \beta + \gamma = \pi \text{ by definition!}$$

3×3 unitarity of V_{CKM}
 implies exact relations among
 these 13 physical quantities :

Examples :

(db) →

$$\frac{|V_{ub}|}{|V_{td}|} = \frac{\sin\beta}{\sin\delta} \frac{|V_{tb}|}{|V_{ud}|}$$

$$|V_{ub}| = \frac{|V_{cd}| |V_{cb}|}{|V_{ud}|} \frac{\sin\beta}{\sin(\delta+\beta)}$$

Relations involving the "small" rephasing invariant phases β_s and χ' :

$(ct) \rightarrow$

$$\sin \beta_s = \frac{|V_{td}|}{|V_{ts}|} \frac{|V_{cd}|}{|V_{cs}|} \sin \beta$$

$u_c \rightarrow$

$$\sin \chi' = \frac{|V_{ub}|}{|V_{us}|} \frac{|V_{cb}|}{|V_{cs}|} \sin \delta$$

Since present and future B factories will provide increasingly more precise values of moduli and invariant phases, it is crucial to have Exact Relations

Suppose that New Physics respects 3×3 unitarity of V_{CKM} but gives new contributions to $B_d - \overline{B}_d$ and $B_s - \overline{B}_s$ mixings, parameterized by :

Example : SUSY

$$M_{12}^{(q)} = (M_{12}^{(q)})^{SM} r_q^2 e^{-z_i \phi_q}$$

$$q = d, s$$

$$\text{In the SM, } r_d = r_s = 1; \phi_d = \phi_s = 0$$

In the presence of New Physics,
one does not measure β, β_s , but

$$\bar{\beta} = \beta + \phi_d$$

$$\bar{\beta}_s = \beta_s + \phi_s$$

On the other hand, γ is not affected
by the presence of New Physics in $B_d - \bar{B}_d$
and $B_s - \bar{B}_s$ mixings. The fact that
experiment showed that $\gamma \neq 0, \pi$
it is a clear evidence that V_{CKM} is complex

Extraction of ϕ_d , ϕ_s from
observables:

$$\tan(\phi_d) = \frac{R_u \sin(\delta + \bar{\beta}) - \sin(\bar{\beta})}{\cos(\bar{\beta}) - R_u \cos(\delta + \bar{\beta})}$$

$$R_u = \frac{|V_{ud}| |V_{ub}|}{|V_{cd}| |V_{cb}|}$$

$$\tan(\phi_s) = \frac{\sin \bar{\beta}_s - C \sin(\delta - \bar{\beta}_s)}{C \cos(\delta - \bar{\beta}_s) + \cos(\bar{\beta}_s)}$$

$$C = \frac{|V_{us}| |V_{ub}|}{|V_{cs}| |V_{cb}|}$$

At present, there is no clear deviation from the Standard Model.

But New Physics is allowed at 10% level. For a recent reference, see Jérôme Charles et al, arXiv:1309.2293 (September 2013)

Belle II and LHCb have the potential to discover New Physics.

If New Physics is discovered

how to distinguish between different
models of N.P.?

Possible Answer: Study correlations
among various observables!

A. Buras and Jennifer Gиррбах

arXIV 1306.3775
(2013)

F. Botella, G.C.B., M. Nebot

J. Phys. Conf. Series 447
(2013)

Can one have a model where
gauge mediated FCNC exist at
tree level, but are naturally suppressed?

Answer : Yes!!

A Minimal Model

Consider an extension of the SM,
where the following new fields are introduced:

- A vectorial quark D° , with both D_L° and D_R° are $SU(2)_L$ singlets with charge $Q = -1/3$ (or $Q = 2/3$)
- 3 right-handed neutrinos ν_R° ,
- A neutral complex singlet S

- Since we want to have Spontaneous CP violation, we impose CP invariance at the Lagrangian level: All couplings real.
- Introduce a Z_4 symmetry on the Lagrangian, under which :

$$\chi_e^\circ \rightarrow i \chi_l^\circ ; e_{Rj}^\circ \rightarrow i e_{Rj}^\circ ; \nu_{Rj}^\circ \rightarrow i \nu_{Rj}^\circ$$

$$D^\circ \rightarrow -D^\circ ; S \rightarrow -S$$

The Z_4 symmetry is crucial to obtain a solution of the Strong CP problem and Leptogenesis

Scalar Potential

The Scalar potential contains various terms which do not have phase dependence, but there are terms with phase dependence.

$$V_{\text{phase}} = [\mu^2 + \lambda_1 S^* S + \lambda_2 \phi^+ \phi] (S^2 + S^{*2}) + \\ + \lambda_3 (S^4 + S^{*4})$$

There is a range of the parameters of the Higgs potential, where the minimum is at:

$$\langle \phi^+ \rangle = \frac{v}{\sqrt{2}} \quad ; \quad \langle S \rangle = \frac{V}{\sqrt{2}} e^{i\theta}$$

Most general $SU(2)_L \times U(1) \times SU(3)_C \times Z_4$ invariant Yukawa couplings in the quark sector :

$$\mathcal{L}_Y = -(\bar{u}^0 \bar{d}^0)_L i [g_{ij} \phi d_R^0 R_j + h_{ij} \tilde{\phi} u_R^0 R_j] - \bar{M} (\bar{D}_L^0 D_R^0) \\ - (f_i S + f' S^*) \bar{D}^0 d_R^0 R_i + h.c.$$

Quark mass matrix for down-type quarks :

$$(\bar{d}_{1L}^0 \bar{d}_{2L}^0 \bar{d}_{3L}^0 \bar{D}_L^0) \begin{bmatrix} 3 \times 3, \text{ real} : & & & \\ \bar{m}_d & \vdots & 0 & \\ \hline M_1 & M_2 & M_3 & \bar{M} \end{bmatrix} \begin{bmatrix} d_{1R}^0 \\ d_{2R}^0 \\ d_{3R}^0 \\ D_R^0 \end{bmatrix}$$

"zero"
due to
 Z_4
symmetry

$$M_j = f_j V e^{i\theta} + f'_j V e^{-i\theta}$$

$$\mathcal{M} = \begin{pmatrix} m_d & ; & 0 \\ ; & ; & ; \\ \bar{M}, \bar{M}_2, \bar{M}_3, & \bar{M} & ; \end{pmatrix};$$

$$U_L^+ (\mathcal{M} \mathcal{M}^+) U_L = \text{diag.}(d^2, D^2)$$

$U_L = \begin{bmatrix} K & R \\ S & T \end{bmatrix}$; One can easily derive:

$$K^{-1} \left[d^2 \begin{bmatrix} T & M^T M & d^2 \\ d^2 & T & M^T \\ M M^T & M^2 & T \end{bmatrix} \right] K = d^2$$

$\xrightarrow{\text{Complex}}$

A remarkable feature of the Model :

The phase θ arising from $\langle S \rangle$, generates a non-trivial CKM phase, provided $|M_j|$ and \bar{M} are of the same order of magnitude. (This is "natural")

$$K^{-1} m_{\text{eff}}^+ m_{\text{eff}}^- K = \text{diag. } (m_d^2, m_s^2, m_b^2)$$

$$m_{\text{eff}}^+ m_{\text{eff}}^- = m_d m_d^\dagger - \frac{m_d M^\dagger M m_d^\dagger}{M M^\dagger + \bar{M}^2}$$

$$M_j = (f_j V e^{i\theta} + f_j' V \bar{e}^{-i\theta})$$

Naturally small
deviations of 3×3 unitarity

Naturally Small
Flavour-Changing
Neutral Currents

For definiteness, consider the case of one isosinglet $Q = -\frac{1}{3}$
 3×3 CKM quark

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} (\bar{u} \bar{c} \bar{t})_L \gamma^\mu \begin{bmatrix} \overset{\uparrow}{K} R \\ d s b D \end{bmatrix} W_\mu^+$$

$$\begin{aligned} \mathcal{L}_Z = & -\frac{g}{2 \cos \theta_W} \left\{ (\bar{u} \bar{c} \bar{t})_L \gamma^\mu \begin{bmatrix} u \\ c \\ t \end{bmatrix} - [\bar{d} \bar{s} \bar{b} \bar{D}] \begin{bmatrix} K^+ K^+ R^+ R^+ \\ R^+ K^+ R^+ R^+ \end{bmatrix} \gamma^\mu \begin{bmatrix} d \\ s \\ b \\ D \end{bmatrix} \right. \\ & \left. - \sin^2 \theta_W J_{dm}^\mu \right\} Z_\mu \end{aligned}$$

Why deviations of 3×3 unitarity are naturally small :

$$U_L^\dagger M M^+ U_L = \text{diag.}(m_d^2, m_s^2, m_b^2, M_D^2)$$

$$U_L = \begin{bmatrix} K & R \\ S & T \end{bmatrix} ; \quad K^+ K + S^+ S = 1$$

but $S \approx -\frac{M m_d^2 K}{M^2} \rightarrow O(m/M) ;$

$K^+ K = 1 - O(m^2/M^2)$. Note that there is nothing strange about violations of 3×3 unitarity.

The PMNS matrix is not unitary in the framework of seesaw mechanism, type I.

A possible solution to the
Strong CP problem :

$$\bar{\Theta} = \Theta_{QCD} + \Theta_{QFD} ;$$

$$\mathcal{L}_\Theta = \Theta_{QCD} \frac{g_s^2}{32\pi^2} F \tilde{F} ; \quad \Theta_{QFD} = \arg \det(M_u M_d)$$

\downarrow
 $= 0$ due to

CP invariance
of the Lagrangian

\downarrow
 $= 0$ due to the

Z_4 symmetry

Leptonic Sector and Leptogenesis

The Z_4 symmetry forbids the inclusion in the Lagrangian of terms of the type :

$$\nu_R^{\circ T} C M \nu_R^\circ \rightarrow \begin{array}{l} \text{not invariant} \\ \text{under } Z_4 \end{array}$$

But allows the following couplings:

$$f_\nu \nu_R^T C S \nu_R^\circ + f'_\nu \nu_R^T C S^* \nu_R^\circ$$

After S acquires a vev: $\langle S \rangle = V e^{i\alpha}$

a mass term is generated:

$$M = V(f_\nu^+ \cos \alpha + i f_\nu^- \sin \alpha);$$

$$f_\pm' = f_\nu^+ \pm f_\nu^-$$

6×6 neutrino mass matrix:

$$\mathcal{M} = \begin{bmatrix} 0 & m_D \\ m_D^T & M \end{bmatrix}$$

real

Complex

Leptogenesis is viable

Conclusions

- We are entering the Higgs Era in Particle Physics.
- A crucial question is : Will Higgs particle(s) play an important rôle in Flavour?
- My guess: Yes, it will not be just Vanilla Higgs.
- It may take some time until Higgs Flavour be tasted experimentally. We have to be patient and remember Neutrinos
- We can safely violate the two Flavour Dogmas.

My Hope :

New Physics will be discovered in the Flavour Sector

in the Next round of Experiments

For definiteness, let us consider an extension of the SM where n $SU(2) \times U(1)$ scalar doublets are introduced. In order to include the possible existence of "family" symmetries of the Lagrangian under which the scalar doublets transform non-trivially, one has to consider the most general CP transformation which leaves invariant the kinetic energy terms of the scalar doublets :

$$\text{CP } \phi_i (\text{CP})^+ = \sum_{j=1}^n U_{ij} \phi_j^*$$

7 Let us assume that the vacuum is CP invariant,
meaning that :

$$CP |0\rangle = |0\rangle$$

One can then derive , the following relation :

$$\sum_{j=1}^n U_{ij} \langle 0 | \Phi_j | 0 \rangle^* = \langle 0 | \phi_i | 0 \rangle$$

If the vacuum is such that none of the symmetries allowed by the Lagrangian satisfy the above equation, then this means that the vacuum is not CP invariant and we say that CP is spontaneously broken.

There are various examples of multi-Higgs models with "family" symmetries, where in a certain region of parameter space, the minimum of the scalar potential corresponds to fixed values of the vacuum phases, which do not depend on the specific values of the parameters of the potential.

$$\mathbb{Z}_2 \text{ symmetry} \rightarrow \langle \phi_1^0 \rangle = v, \quad \langle \phi_2^0 \rangle = v e^{i\pi/2}$$

$$S_3 \text{ symmetry} \rightarrow \langle \phi_1^0 \rangle = v, \quad \langle \phi_2^0 \rangle = v e^{i2\pi/3}, \quad \phi_3 = v e^{i4\pi/3}$$

It has been shown by G.B, Girard, J.M., Grimbs, W
that, contrary to naïve expectation
all these vacua are CP conserving.

However, an example was found, based
on the group $\Delta(27)$ leading to genuine
CP and T geometrical valuation.

Recently, interesting other examples have
been found :

I. M. Varzilas, D. E. Costa, P. Lester Phys. Lett (2012)

I. P. Ivanov, L. Lavanya, Eur. Phys. (2013)

Realistic model ?