Minimal lepton flavour structures lead to non-maximal 2-3 mixing

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Outline

- Lepton flavour observables: present & future data
- Lepton flavour symmetries: where do we stand
- Minimal flavour structures: what do they predict? do they correspond to a simple symmetry?
- A conspiracy for non-maximal 2-3 mixing

The importance of being massive

The discovery of neutrino oscillations amounts to

- a new energy scale, $m_\nu \sim 0.1$eV, probably associated to another one, $\Lambda = v^2/m_\nu \sim 10^{14}$GeV
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$$(m_\nu)_{ij} = c_{ij}\frac{v^2}{\Lambda}$$

The scale of flavour symmetry breaking remains unknown, and possibly out of reach

Lepton flavour parameters

$$\mathcal{L} \supset \frac{g}{\sqrt{2}} \overline{e_{L\alpha}} \gamma^\mu W_\mu^+ \nu_{L\alpha} - m_\alpha \overline{e_{L\alpha}} e_{R\alpha} - \frac{1}{2} \nu_{L\alpha} (m_\nu)_{\alpha\beta} \nu_{L\beta} + h.c.$$

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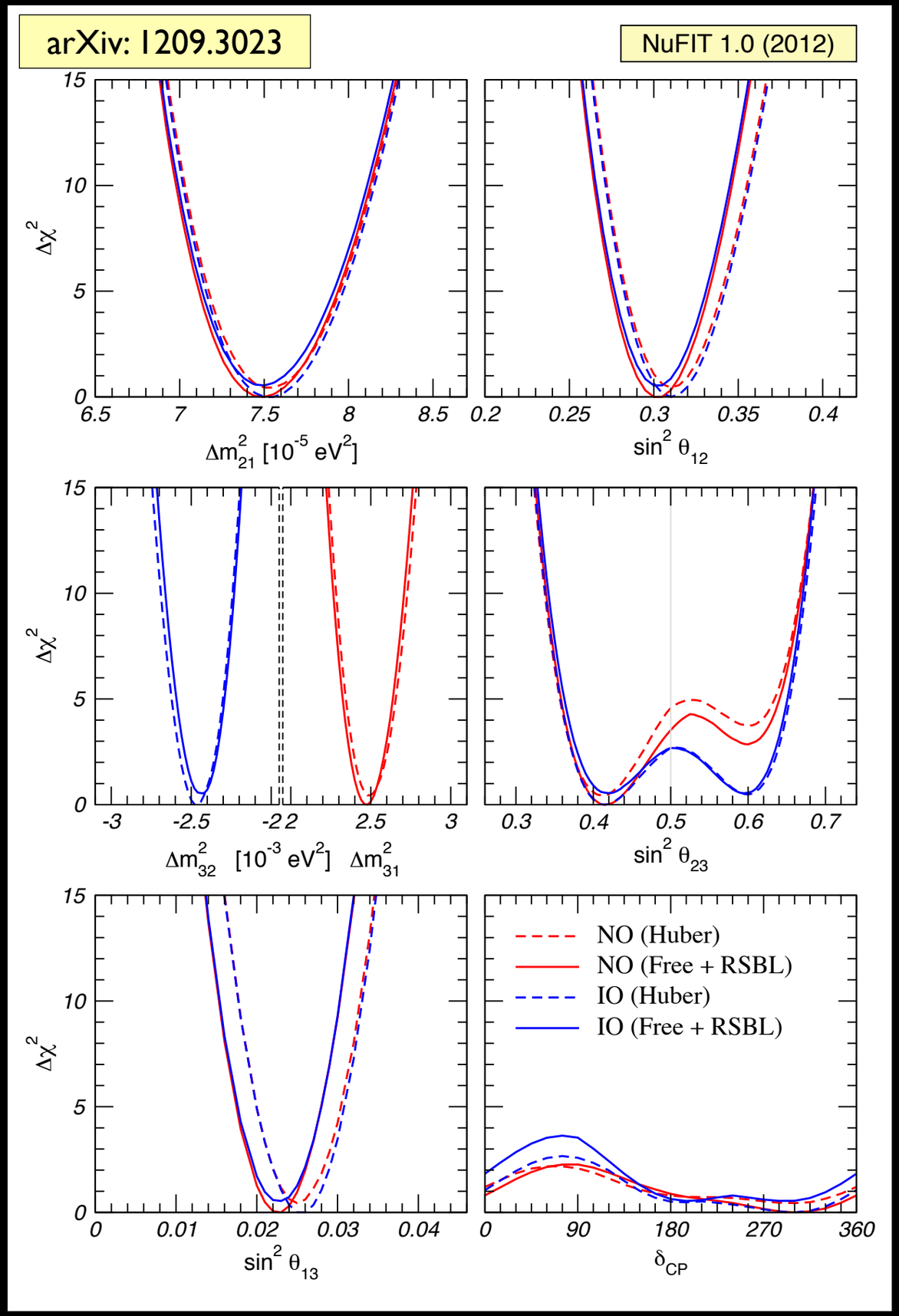
$$m_\nu = U^* \mathrm{diag}(m_1 e^{-2i\rho}, m_2, m_3 e^{-2i\sigma}) U^\dagger$$

$$U = R(\theta_{23}) R(\theta_{13}, \delta) R(\theta_{12})$$

Neutrino oscillation data

sensitivity to 3 mixing angles, 2 mass differences, 1 phase

Gonzalez-Garcia Maltoni Salvado Schwetz



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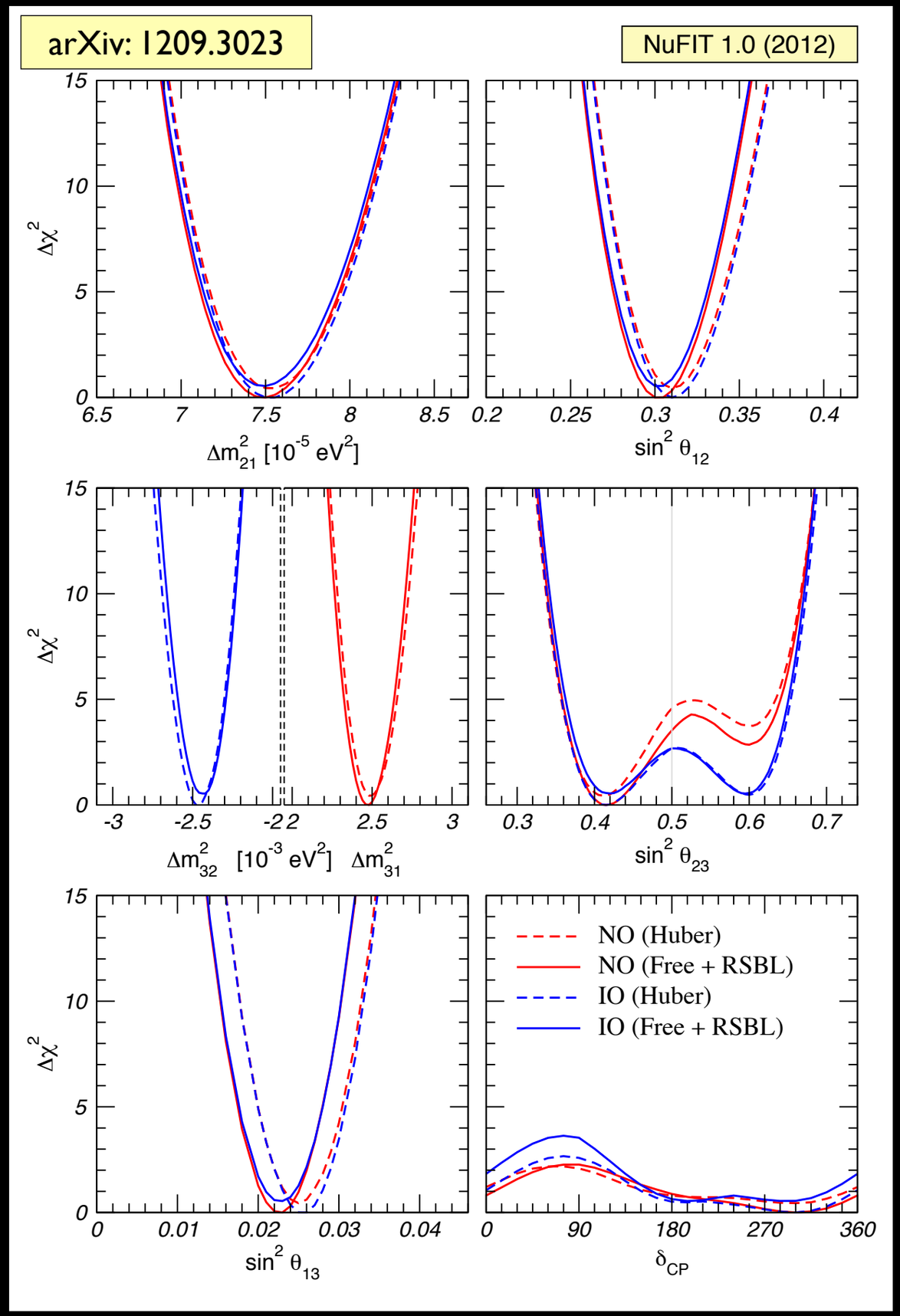

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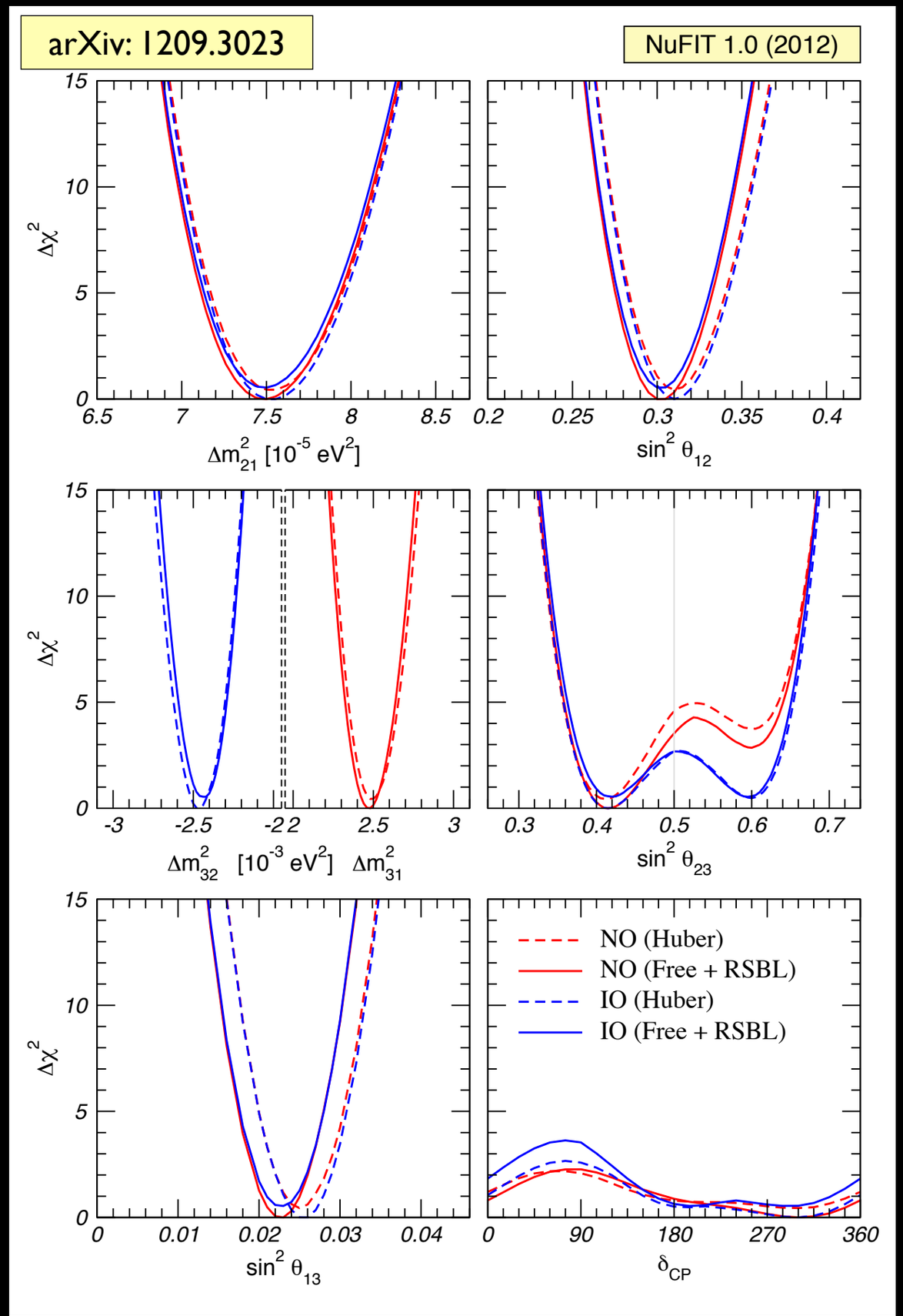
There is a 2σ preference for a non-maximal value (accelerator ν's).



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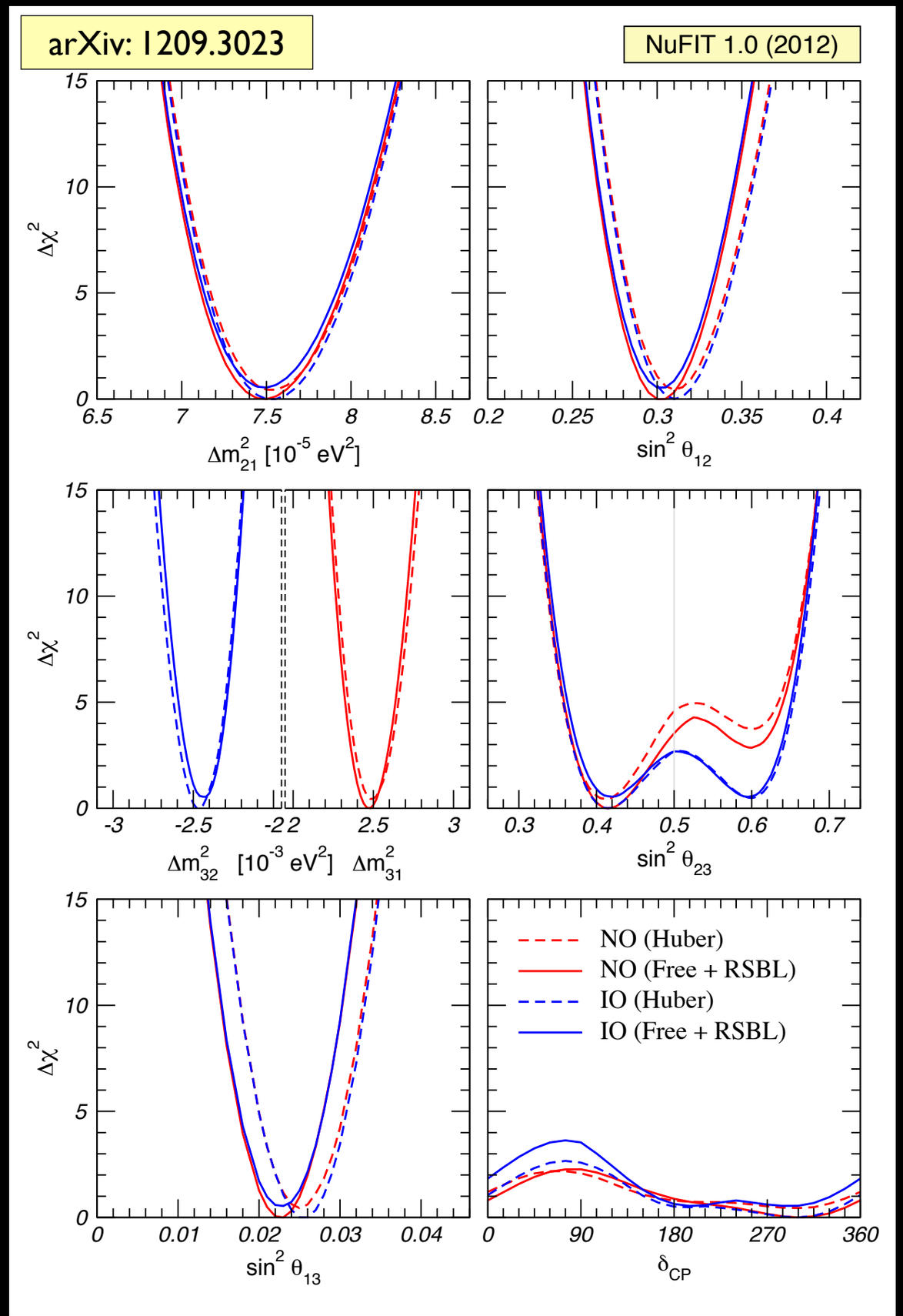
Lepton CP-violation is behind the corner ?
Some values of δ already disfavoured at 1σ



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- Precision in oscillation experiments means essentially:
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- Only one realistic observable is sensitive to the two Majorana-type CP-violating phases: the neutrinoless 2β-decay effective mass, $0 < m_{ee} < 0.38$ eV (90% C.L. from EXO-200, similar bound from GERDA)

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The well-known speaker replied: "It is just you who did not learn anything!"

From the neutrino mass matrix to the observables

4 precisely known parameters: $p_a = \Delta m^2_{21},\ |\Delta m^2_{31}|,\ \theta_{12},\ \theta_{13}$

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No matter how complicated the model, its predictivity depends on n only

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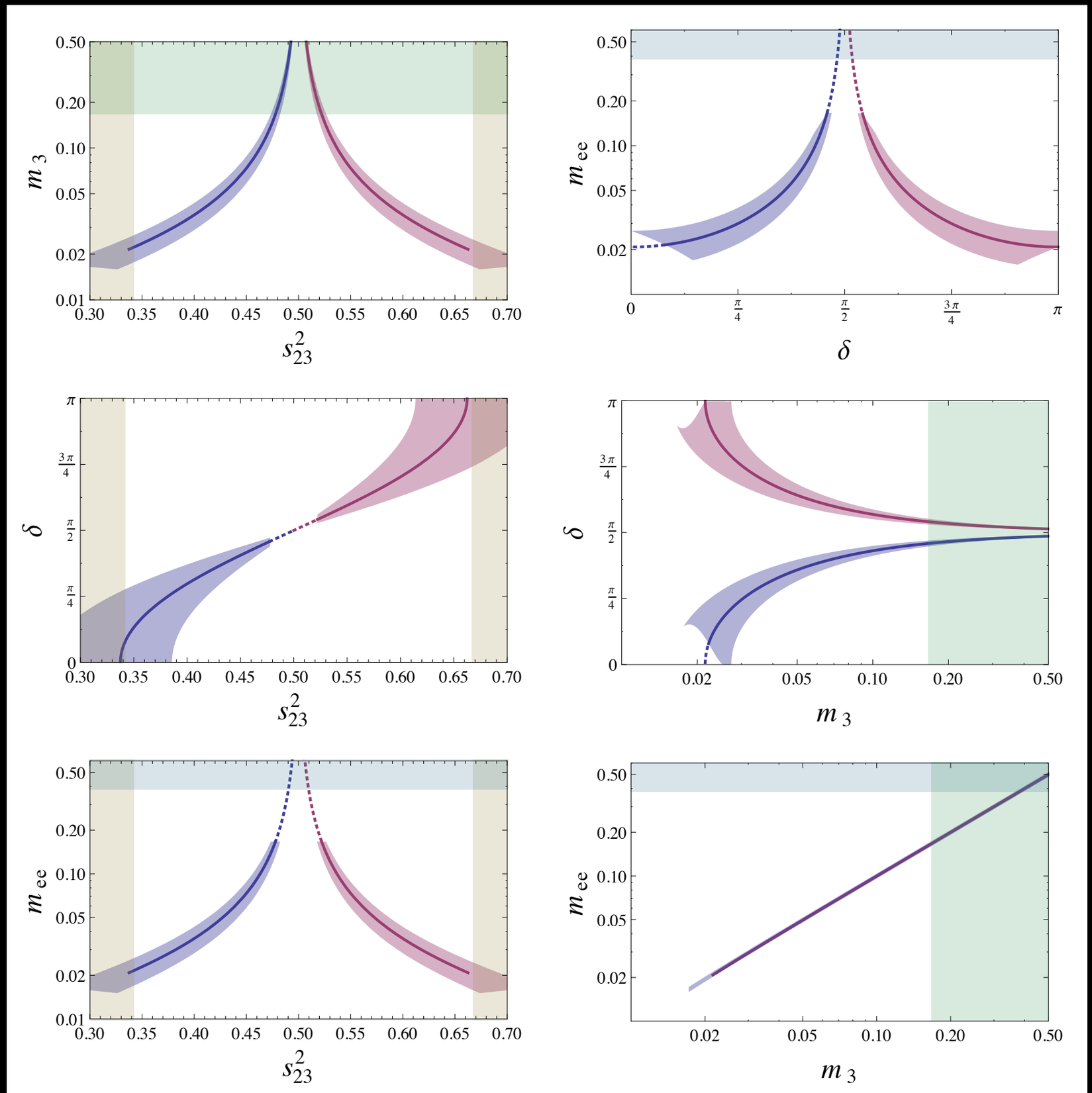

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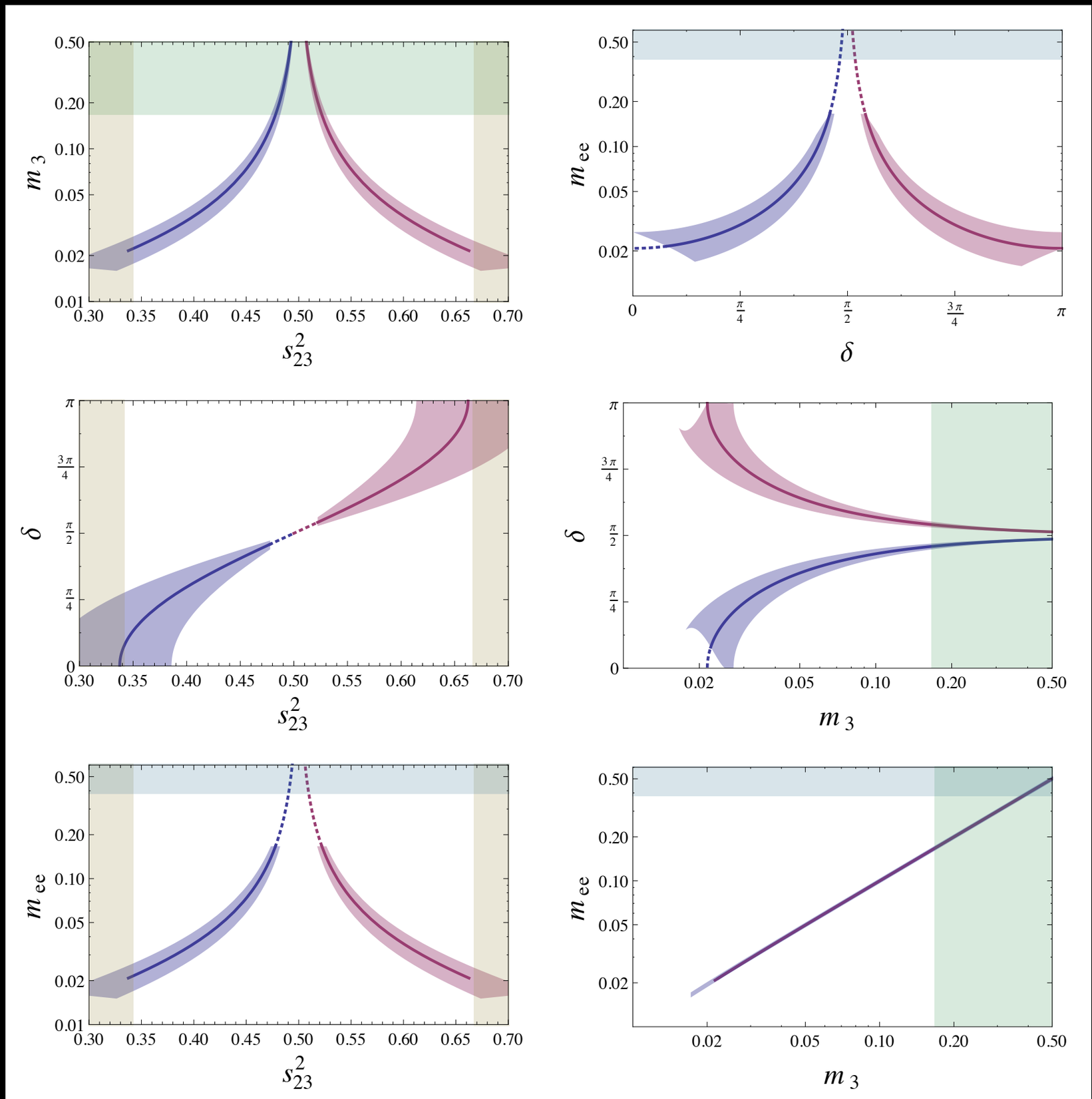

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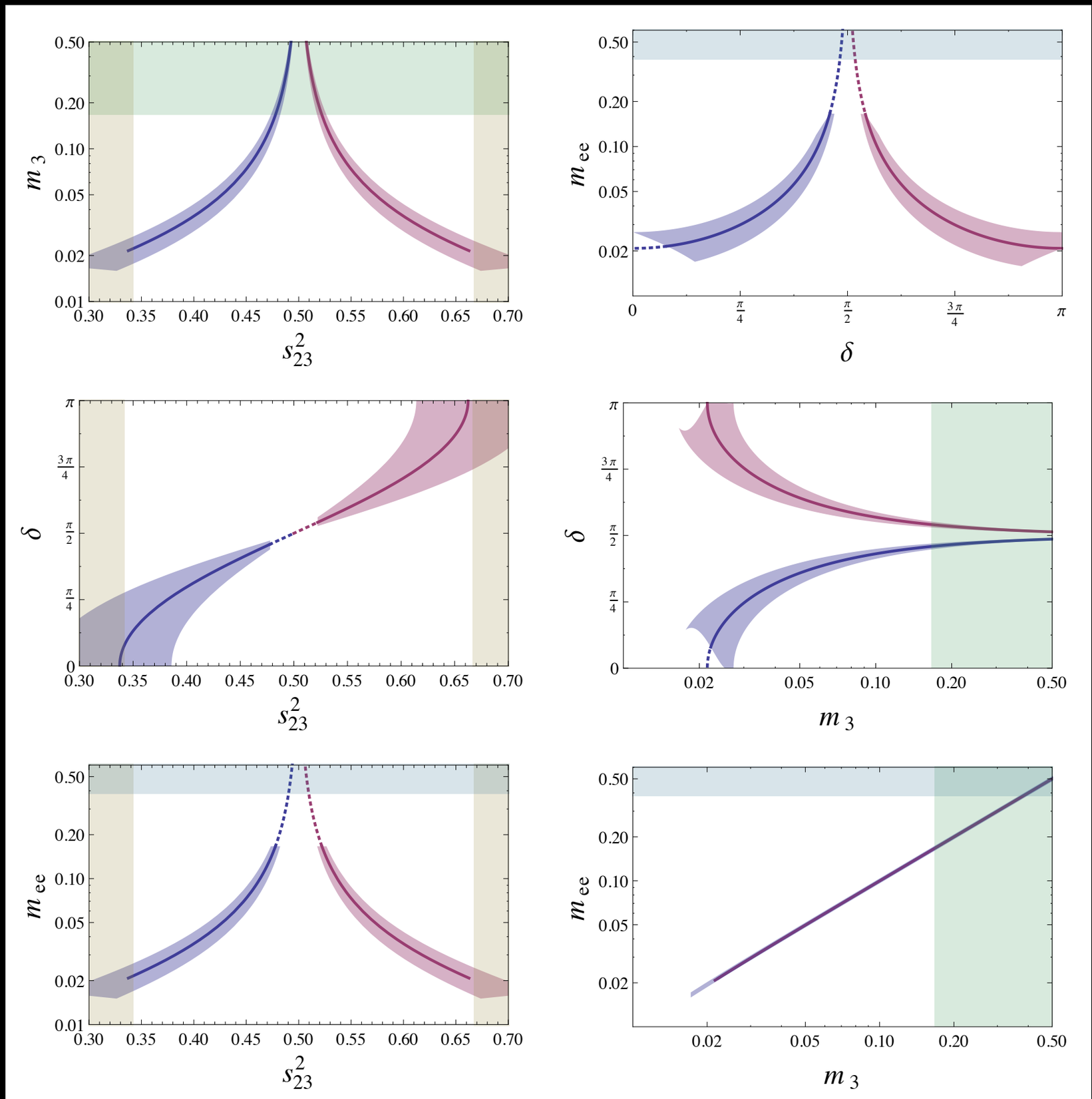

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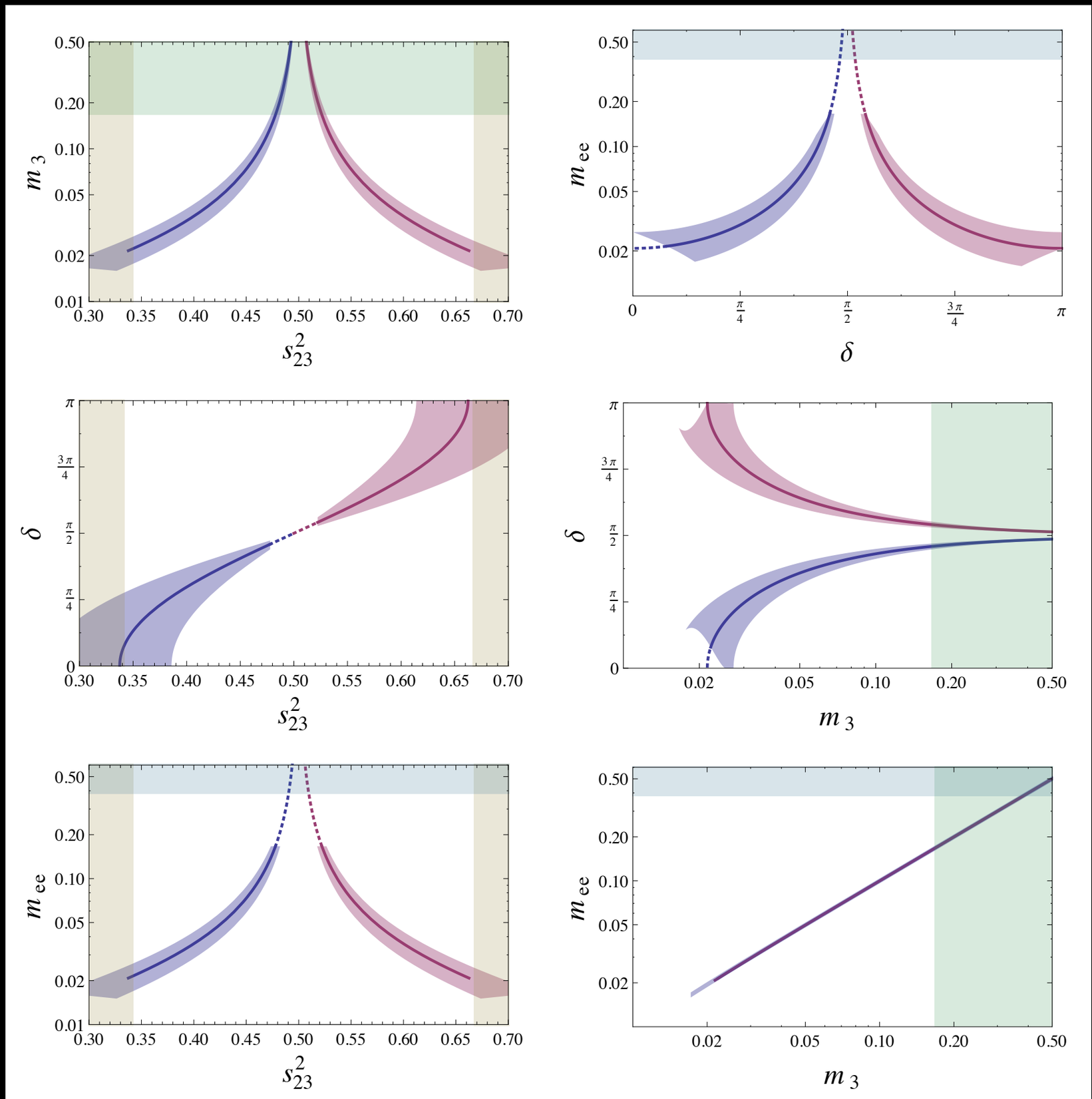

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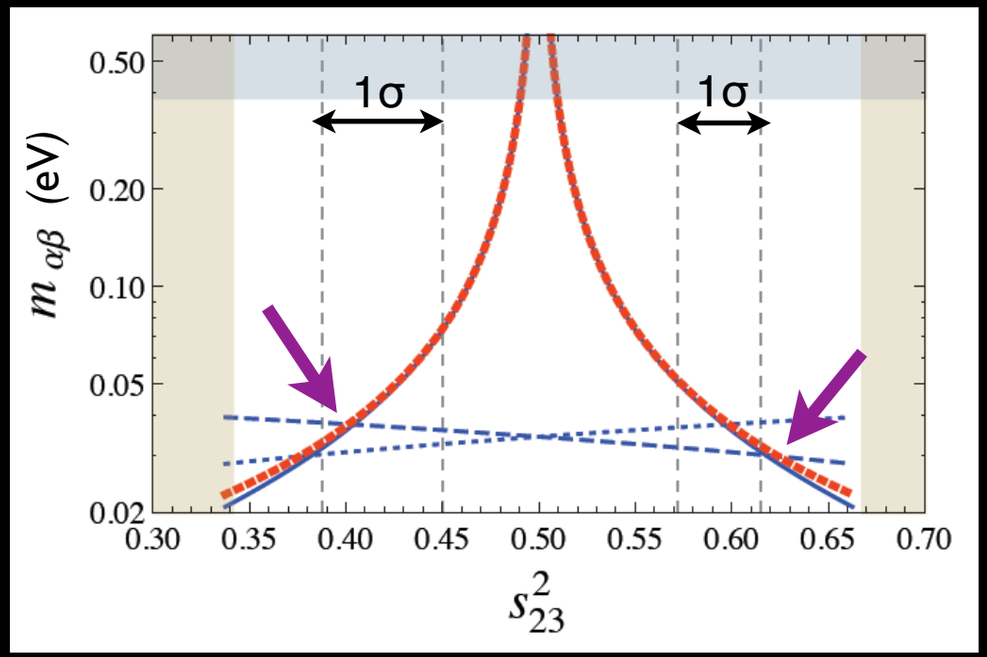

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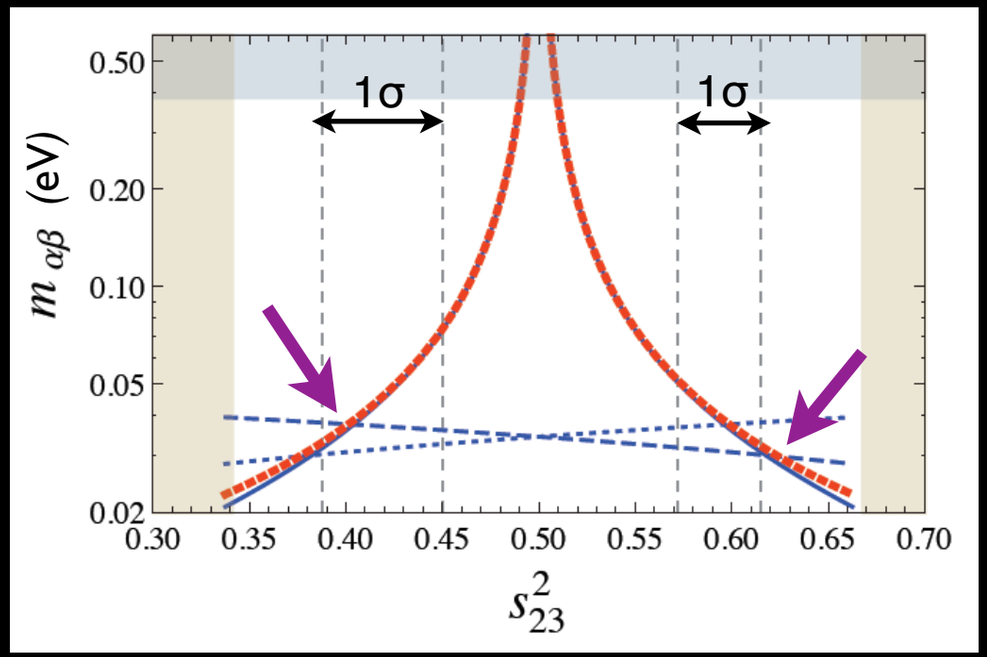

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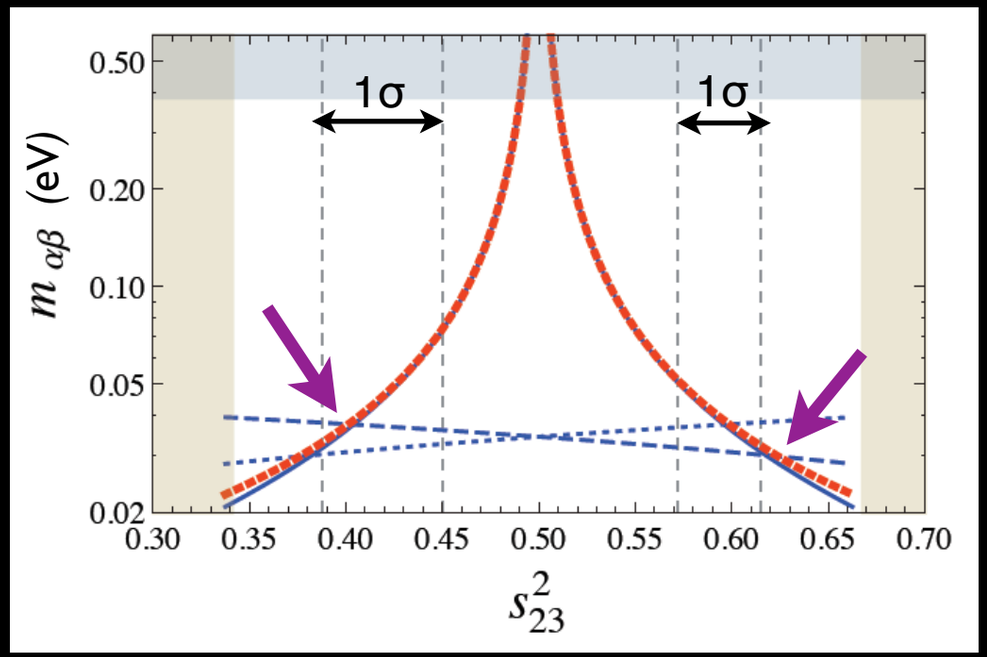

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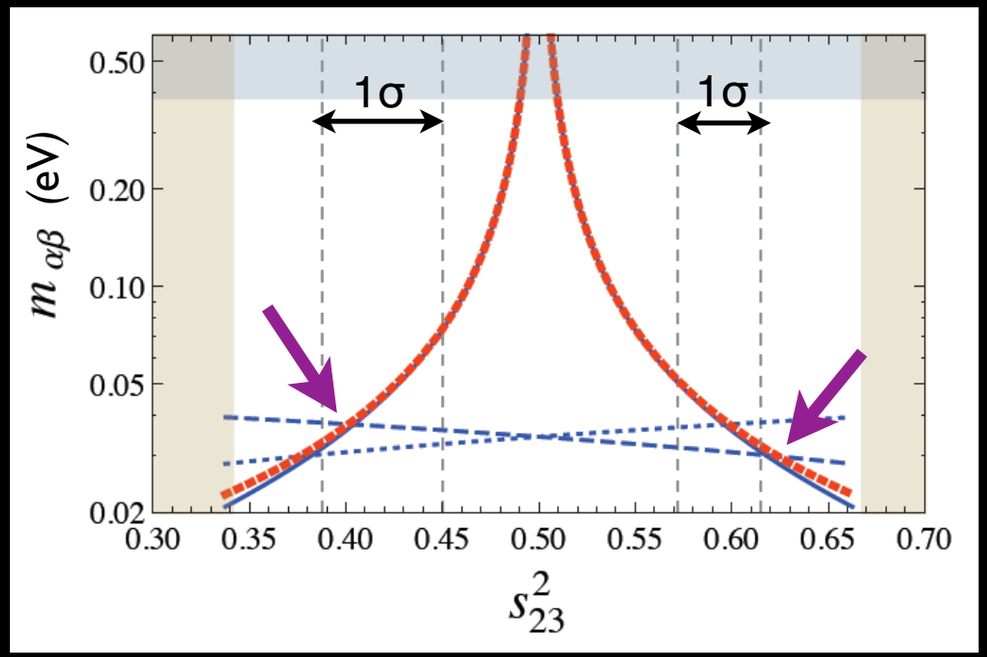

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$$\begin{cases} \sin^2\theta_{23} = 0.40^{+0.02}_{-0.01} \\ \cos\delta = 0.59^{+0.12}_{-0.14} \\ m_{\text{light}} = m_3 = 0.037^{+0.001}_{-0.002}\,\text{eV} \\ m_{ee} = 0.036^{+0.002}_{-0.001}\,\text{eV} \end{cases} \qquad \begin{cases} \sin^2\theta_{23} = 0.62^{+0.03}_{-0.02} \\ \cos\delta = -0.75^{+0.15}_{-0.12} \\ m_{\text{light}} = m_3 = 0.0289^{+0.0002}_{-0.0001}\,\text{eV} \\ m_{ee} = 0.0284^{+0.0000}_{-0.0001}\,\text{eV} \end{cases}$$

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All this can be realized with the order-12 group $Q_6 \equiv D'_3$ that has representations 2_1, 2_2, 1, 1', 1'', 1'''

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Is a minimal model convincing ?

let us try to find a pretty one

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- To enforce this result a few technicalities are needed.
  A different, economical implementation of the A_4 symmetry can lead to a better agreement with data

Off-diagonal ν masses

As in the radiative model by Zee (PLB93,389,1980) and Wolfenstein (NPB175,93,1980)

$$M_\nu^{\rm off} = \begin{pmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{pmatrix}$$

This is perfectly A_4-symmetric, if leptons (l_e,l_μ,l_τ) and flavons (φ_e,φ_μ,φ_τ) form A_4-triplets:
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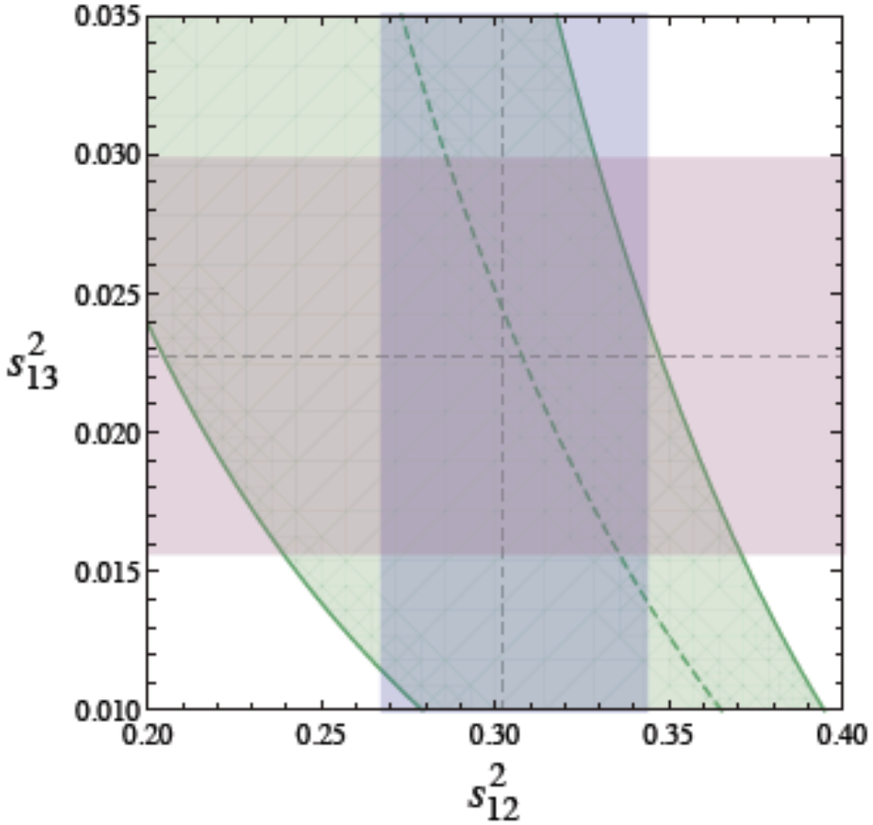
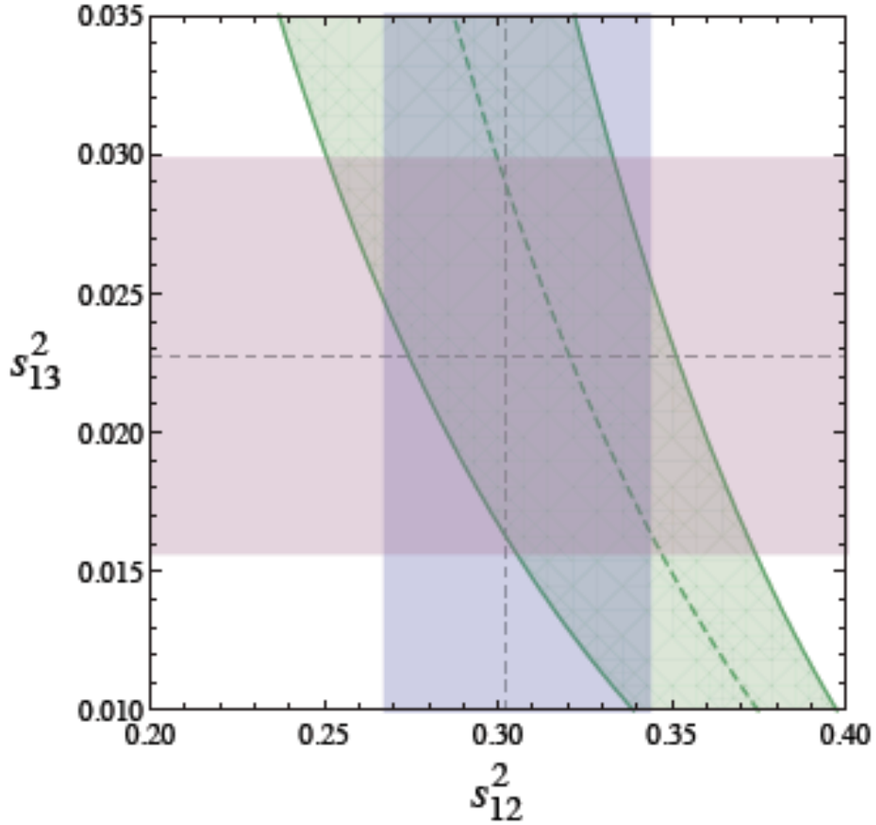
$\theta_{23} < \pi/4$ $\qquad$ $\theta_{23} > \pi/4$





3σ range for θ_{12}
and
3σ range for θ_{13}
superimpose perfectly with
1σ range for θ_{23}

Off-diagonal + the identity

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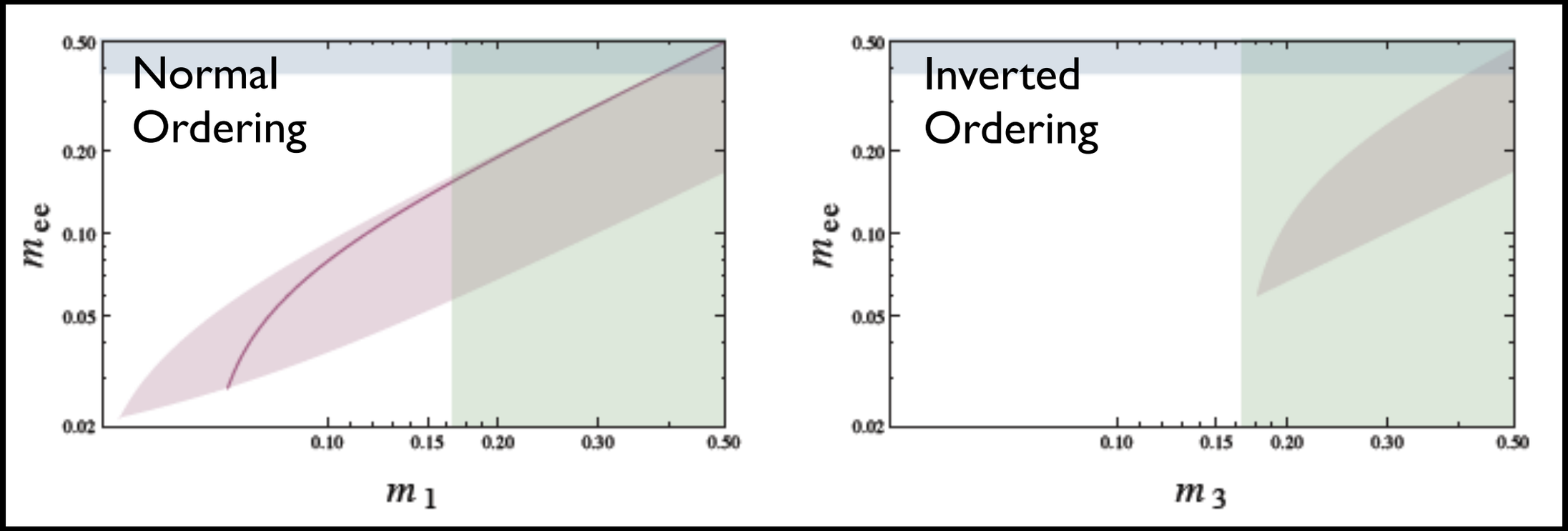

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- What is the mechanism inducing the off-diagonal & the identity terms? Radiative, seesaw, both? At what energy scale(s)?

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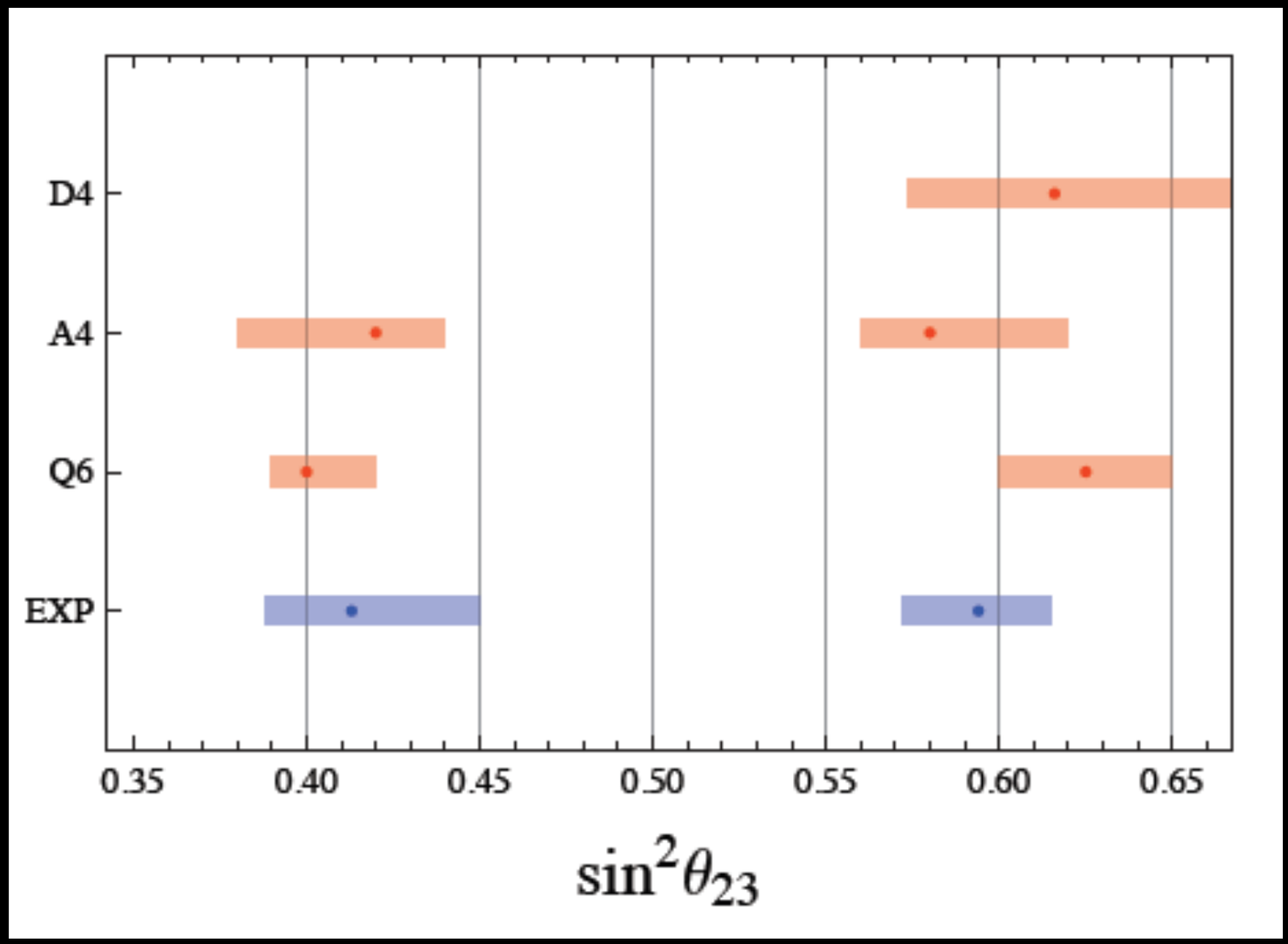

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MF & Albert Villanova del Moral
1303.5284 - summary plot

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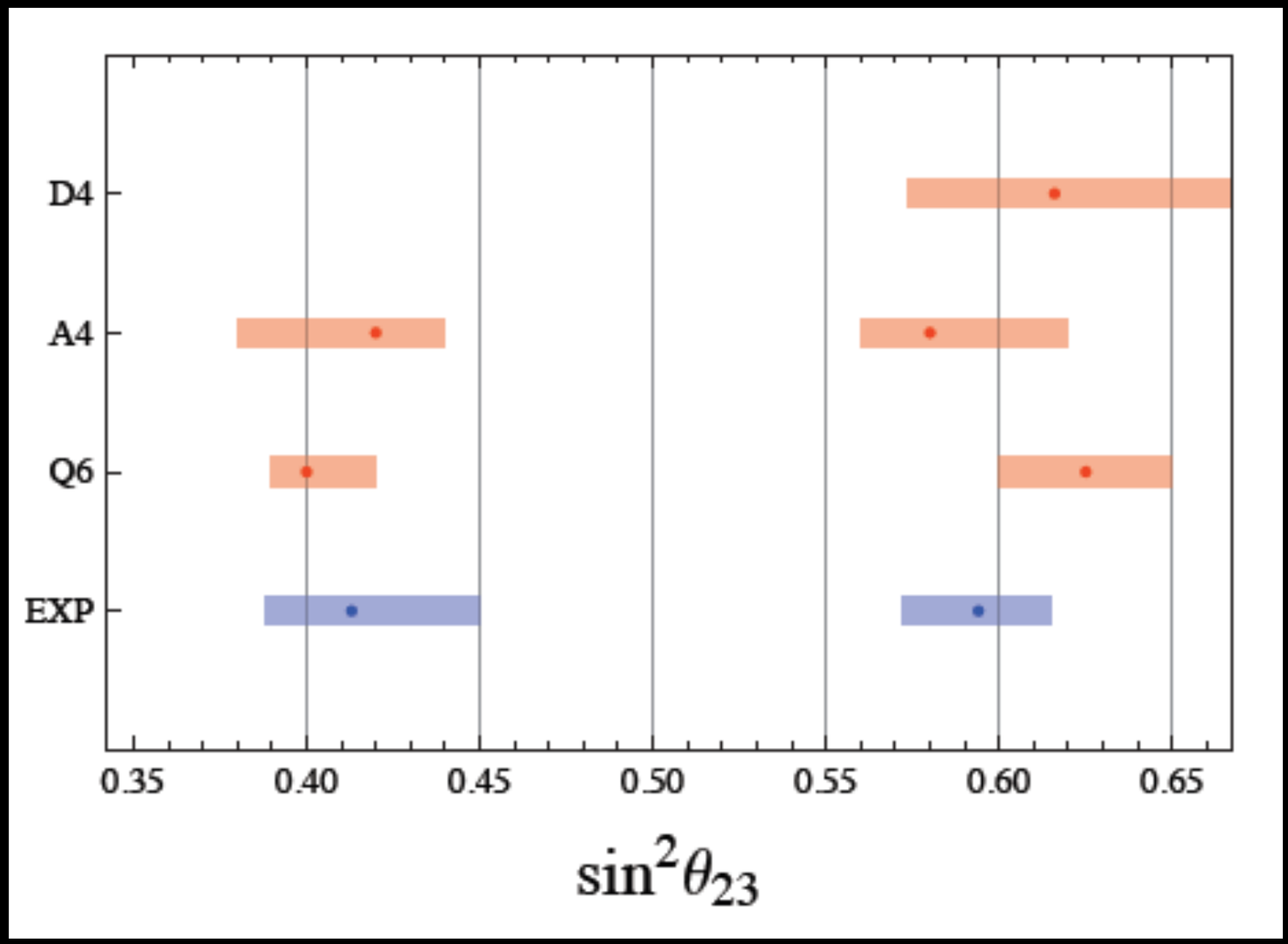
top-down 1+2 model

off-diagonal + identity model

bottom-up 1+2 model

global 3ν fit at 1σ



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- Simplicity should be searched for in the structure of the mass matrices, not in the value of the observables
- Minimal models can sharply predict future observables
- A particularly neat possibility: M_ν = off-diagonal + identity
- In general, with the least possible assumptions, we found a preference for a deviation from maximal θ_{23} of the presently preferred size

A top-down model

For more general models of this type, see Hernandez & Smirnov 2012

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- Do not postulate a given form for M_ν and M_e, rather build a flavour model that (i) is minimal according to a few theoretical criteria (ii) is viable

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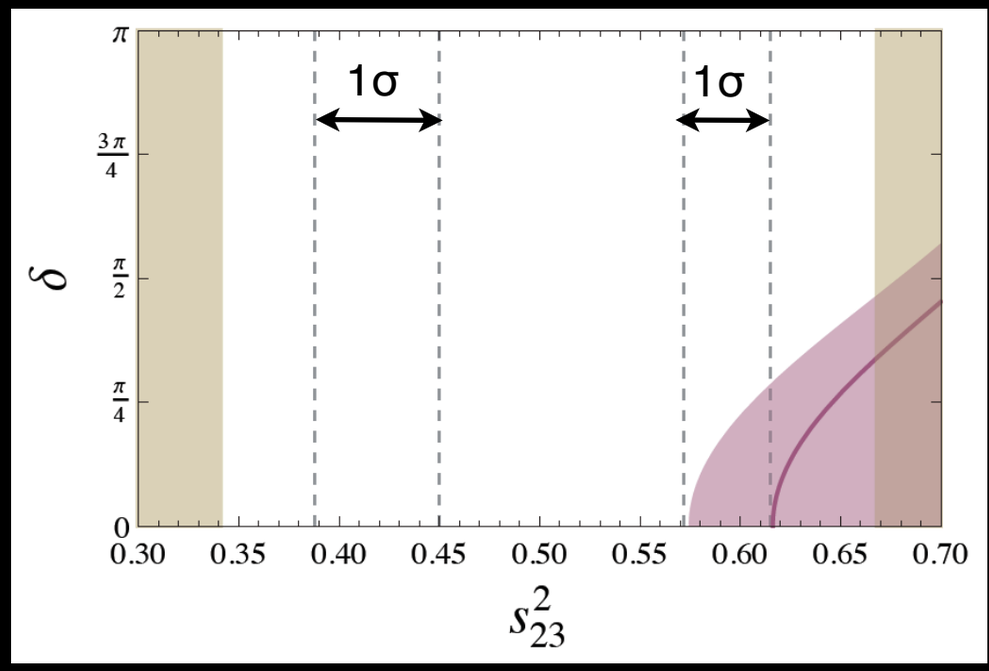
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