High Energy Factorization
selection of recent theory developments and prospects for phenomenology

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Selection of results obtained together with:
K. Bozek, K. Golec-Biernat, A. van Hameren, S. Jadach,
P. Kotko, D. Toton, W. Placzek, S. Sapeta, M. Slawinska, M. Skrzypek

QCD at high energies – high energy factorization

$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \sum_{a,b,c,d} \int \frac{d^2k_{1t}}{\pi}\frac{d^2k_{2t}}{\pi}\frac{1}{16\pi^2(x_1x_2S)^2}\overline{|\mathcal{M}_{ab\to cd}|}^2\delta^2(\vec{k}_{1t}+\vec{k}_{2t}-\vec{p}_{1t}-\vec{p}_{2t})$$
$$\times\, \mathcal{F}_{a/A}(x_1,k_{1t}^2,\mu^2)\,\mathcal{F}_{b/B}(x_2,k_{2t}^2,\mu^2)\,\frac{1}{1+\delta_{cd}}$$

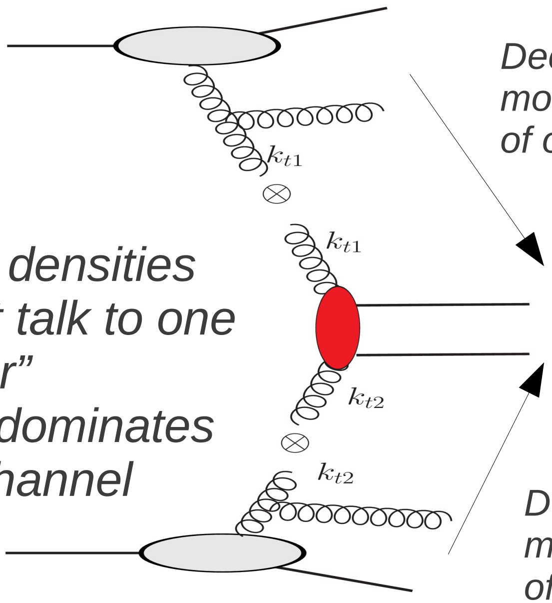
$$k_1^\mu = x_1P_1^\mu + \bar{x}_1P_2^\mu + k_{1t}^\mu \qquad k_2^\mu = x_2P_2^\mu + \bar{x}_2P_1^\mu + k_{2t}^\mu$$

$$\bar{x}_1 = \frac{k_1^2+\mathbf{k}^2}{Sx_1} \qquad \bar{x}_2 = \frac{k_2^2+\mathbf{k}^2}{Sx_2}$$

$$|\mathcal{M}_{ab\to cd}|^2 = \frac{2x_1k_1^{\mu_1}k_1^{\nu_1}}{k_1^2}\frac{2x_2k_2^{\mu_2}k_2^{\nu_2}}{k_2^2}\mathcal{M}_{ab\to cd\,\mu_1\nu_1}\mathcal{M}^*_{ab\to cd\,\mu_2\nu_2}$$

Parton densities "do not talk to one another"
Gluon dominates
In "t" channel

Decreasing longitudianl momentum fractions of off-shell partons



Decreasing longitudianl momentum fractions of off-shell partons

Gribov, Levin, Ryskin '81
Ciafaloni, Catani, Hautman '93

Originally derived for quarks in final state.
So far no tool for automated calculations

Similar and framework motivated by CGC
for pA developed by Dominguez, Marquet, Juan, Xiao' 10

High Energy Factorization - matrix elements

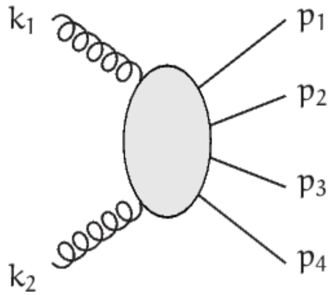
- *General theory given by Lipatov effective action. So far there is no numerical tool which generates matrix elements directly from the effective action.*

- *For collinear factorization there are: HELAC, Amagic++,AlpGen, MadGraph,...*

- *New framework which is equivalent to Lipatov effective action and which makes use of existing tools for evaluation of matrix elements*

  Van Hameren, Kotko, KK JHEP 1301 (2013) 078, Van Hameren, Kotko, KK JHEP 1212 (2012) 029 .

Kinematics of High Energy Factorization



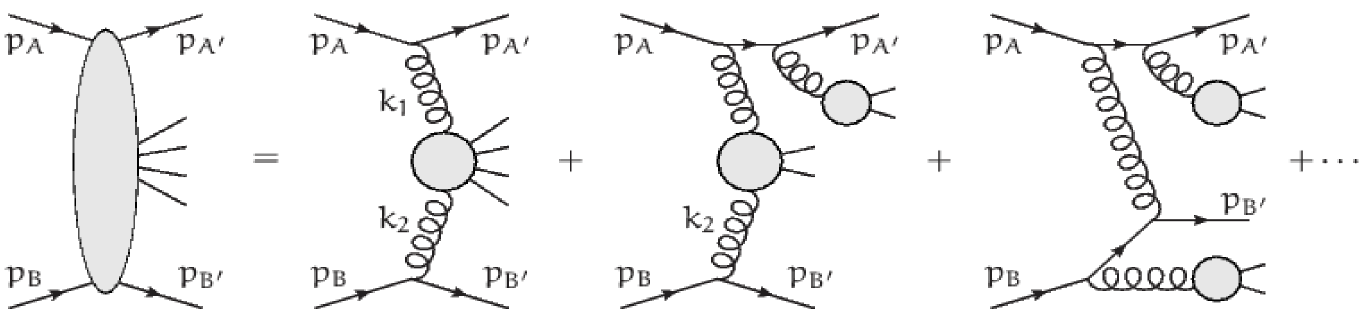
$$k_1 + k_2 = p_1 + p_2 + p_3 + p_4$$

$$k_1 = x_1 P_A + k_{\perp 1} \qquad k_2 = x_2 P_B + k_{\perp 2}$$

$$P_A \cdot k_{\perp 1} = P_A \cdot k_{\perp 2} = P_B \cdot k_{\perp 1} = P_B \cdot k_{\perp 2} = 0$$

$$P_A^2 = P_B^2 = 0$$

$$k_1^2 = k_{\perp 1}^2 \qquad k_2^2 = k_{\perp 2}^2$$



$$\ell_1 = (E, 0, 0, E) \qquad \ell_2 = (E, 0, 0, -E)$$

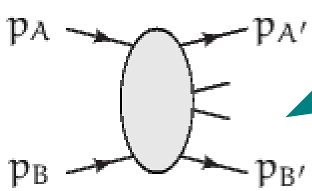
$$p_A - p_{A'} = k_1 = x_1 \ell_1 + k_{1\perp} + y_2 \ell_2 \qquad p_B - p_{B'} = k_2 = x_2 \ell_2 + k_{2\perp} + y_1 \ell_1$$

Needed to keep quarks on shell. Usually neglected

Towards automation of High Energy Factorization

Van Hameren, Kotko, KK
JHEP 1301 (2013) 078

Let us consider $\mathcal{A}(g^* g^* \to X)$



Must be gauge invariant

Introduce complex p_A, p_B, $p_{A'}$ $p_{B'}$

$$p_A = (\Lambda + x_1)\ell_1 + \kappa_{13}\ell_3$$
$$p_{A'} = \Lambda\ell_1 - \kappa_{14}\ell_4$$
$$p_A - p_{A'} = x_1\ell_1 + k_{1\perp}$$

$$p_B = (\Lambda + x_2)\ell_2 + \kappa_{24}\ell_4$$
$$p_{B'} = \Lambda\ell_2 - \kappa_{23}\ell_3$$
$$p_B - p_{B'} = x_2\ell_2 + k_{2\perp}$$

$$\ell_3^\mu = \tfrac{1}{2}\langle \ell_2 - | \gamma^\mu | \ell_1 - \rangle \qquad \ell_4^\mu = \tfrac{1}{2}\langle \ell_1 - | \gamma^\mu | \ell_2 - \rangle$$

$$p_A^\mu = (\Lambda + x_1)\ell_1^\mu - \frac{\ell_4 \cdot k_{1\perp}}{\ell_1 \cdot \ell_2}\ell_3^\mu \qquad p_{A'}^\mu = \Lambda\ell_1^\mu + \frac{\ell_3 \cdot k_{1\perp}}{\ell_1 \cdot \ell_2}\ell_4^\mu$$

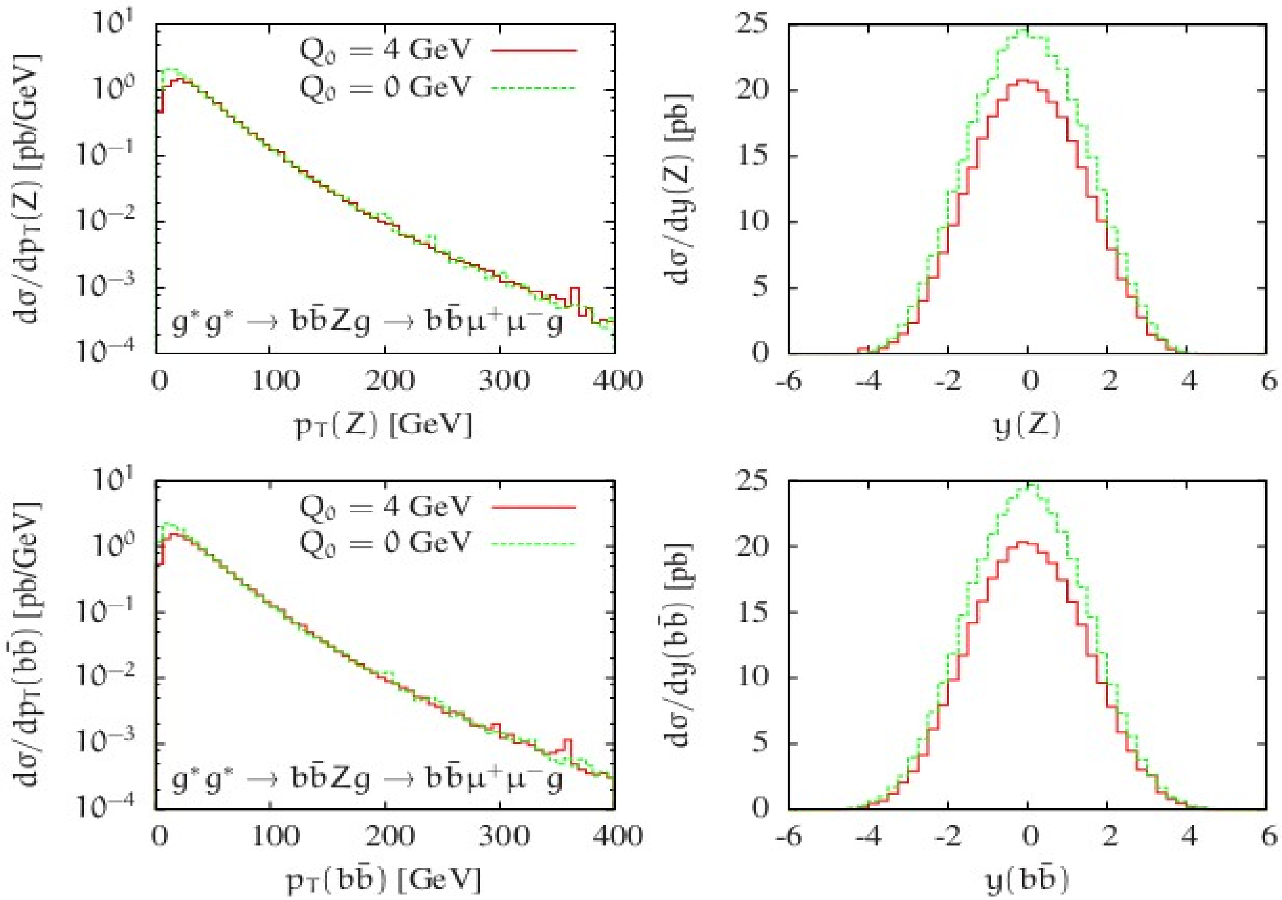
$$p_B^\mu = (\Lambda + x_2)\ell_2^\mu - \frac{\ell_3 \cdot k_{2\perp}}{\ell_1 \cdot \ell_2}\ell_4^\mu \qquad p_{B'}^\mu = \Lambda\ell_2^\mu + \frac{\ell_4 \cdot k_{2\perp}}{\ell_1 \cdot \ell_2}\ell_3^\mu$$

$$p_A^\mu - p_{A'}^\mu = x_1\ell_1^\mu + k_{1\perp}^\mu \qquad p_B^\mu - p_{B'}^\mu = x_2\ell_2^\mu + k_{2\perp}^\mu$$

$$p_A^2 = p_{A'}^2 = p_B^2 = p_{B'}^2 = 0$$

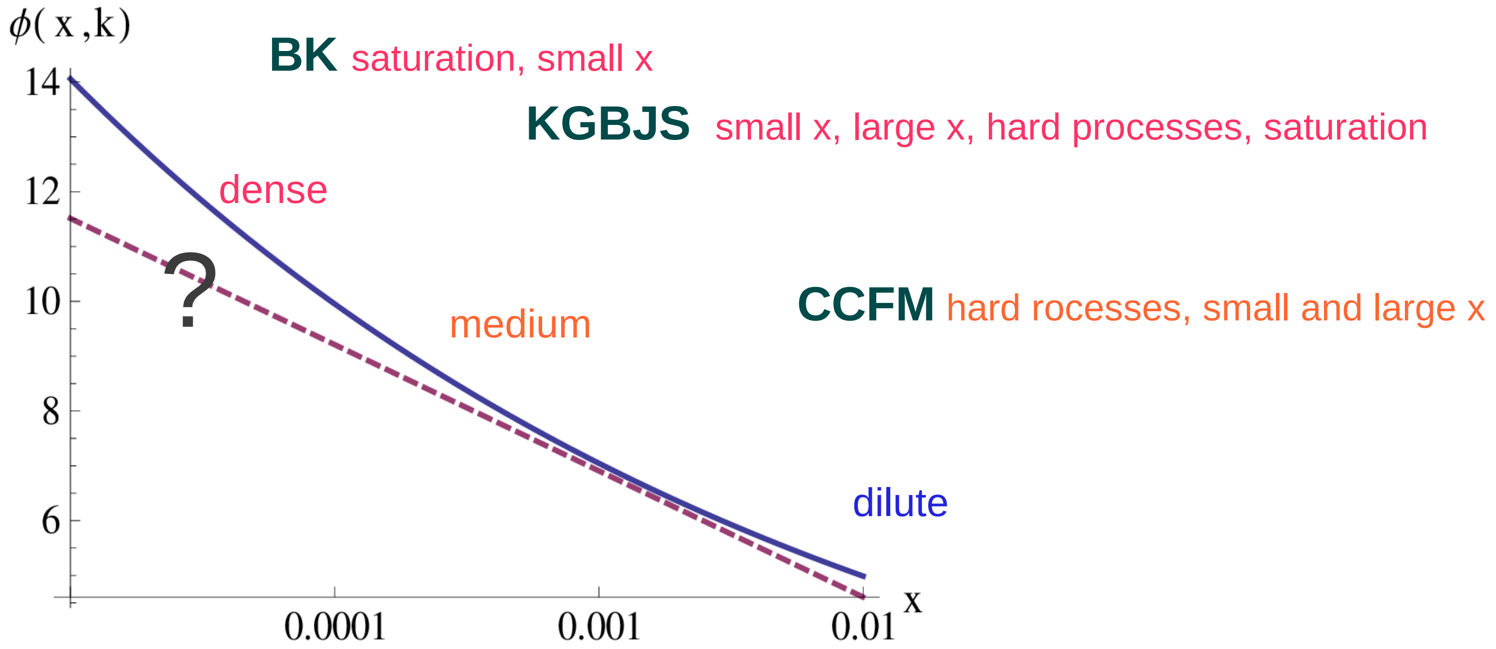
In agreement with Lipatov action

Preliminary application of the formalism



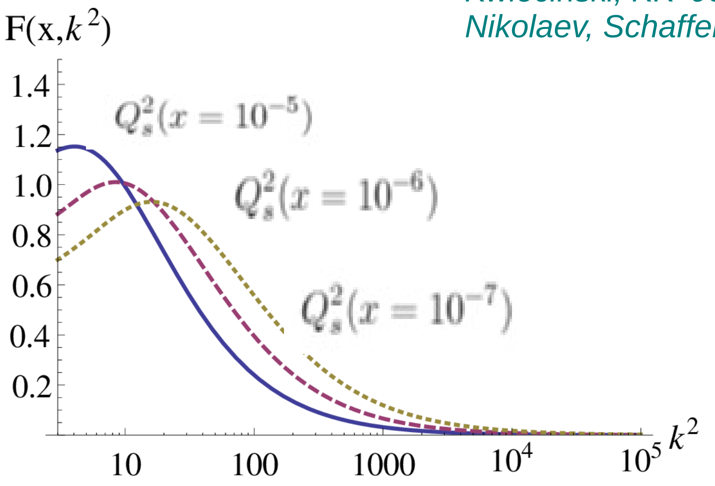
Realistic gluon densities are going to be used. So far just simple model.

Forward physics as the way to constrain gluon both at large and small pt



High energy factorizable gluon density with saturation

Kwiecinski, KK '03
Nikolaev, Schaffer '04



Gluon momentum density

$$\mathcal{F}(x,k^2)=\mathcal{F}_0(x,k^2)+\overline{\alpha}_s\int_{x/x_0}^1\frac{dz}{z}\int_0^\infty\frac{dl^2}{l^2}\left[\frac{l^2\mathcal{F}(x/z,l^2)-k^2\mathcal{F}(x/z,k^2)}{|k^2-l^2|}+\frac{k^2\mathcal{F}(x/z,k^2)}{\sqrt{(4l^4+k^4)}}\right]$$
$$-\frac{2\alpha_s^2\pi}{N_cR^2}\int_{x/x_0}^1\frac{dz}{z}\left\{\left[\int_{k^2}^\infty\frac{dl^2}{l^2}\mathcal{F}(x/z,l^2)\right]^2+\mathcal{F}(x/z,k^2)\int_{k^2}^\infty\frac{dl^2}{l^2}\ln\left(\frac{l^2}{k^2}\right)\mathcal{F}(x/z,l^2)\right\}$$

target's radius

Gluon number density

$$\Phi_b(x,k^2)=\Phi_{0\,b}(x,k^2)+\overline{\alpha}_s\int_{x/x_0}^1\frac{dz}{z}\int_0^\infty\frac{dl^2}{l^2}\left[\frac{l^2\Phi_b(x/z,l^2)-k^2\Phi_b(x/z,k^2)}{|k^2-l^2|}+\frac{k^2\Phi_b(x/z,k^2)}{\sqrt{(4l^4+k^4)}}\right]-\overline{\alpha}_s\int_{x/x_0}^1\frac{dz}{z}\Phi_b^2(x/z,k^2)$$

Balitsky 96, Kovchegov 99

$$\Phi_b(x,k^2)=\Phi(x,k^2)S(b)\qquad\qquad \int d^2\mathbf{b}\,S(b)=1,\quad\int d^2b\,S^2(b)=\frac{1}{\pi R^2}$$

$$\Phi(x,k^2)=\Phi_0(x,k^2)+\overline{\alpha}_s\int_{x/x_0}^1\frac{dz}{z}\int_0^\infty\frac{dl^2}{l^2}\left[\frac{l^2\Phi(x/z,l^2)-k^2\Phi(x/z,k^2)}{|k^2-l^2|}+\frac{k^2\Phi(x/z,k^2)}{\sqrt{(4l^4+k^4)}}\right]-\frac{\overline{\alpha}_s}{\pi R^2}\int_{x/x_0}^1\frac{dz}{z}\Phi^2(x/z,k^2)$$

Interesting properties
travelling wave solution
Munier, Peschanski '03

inversion

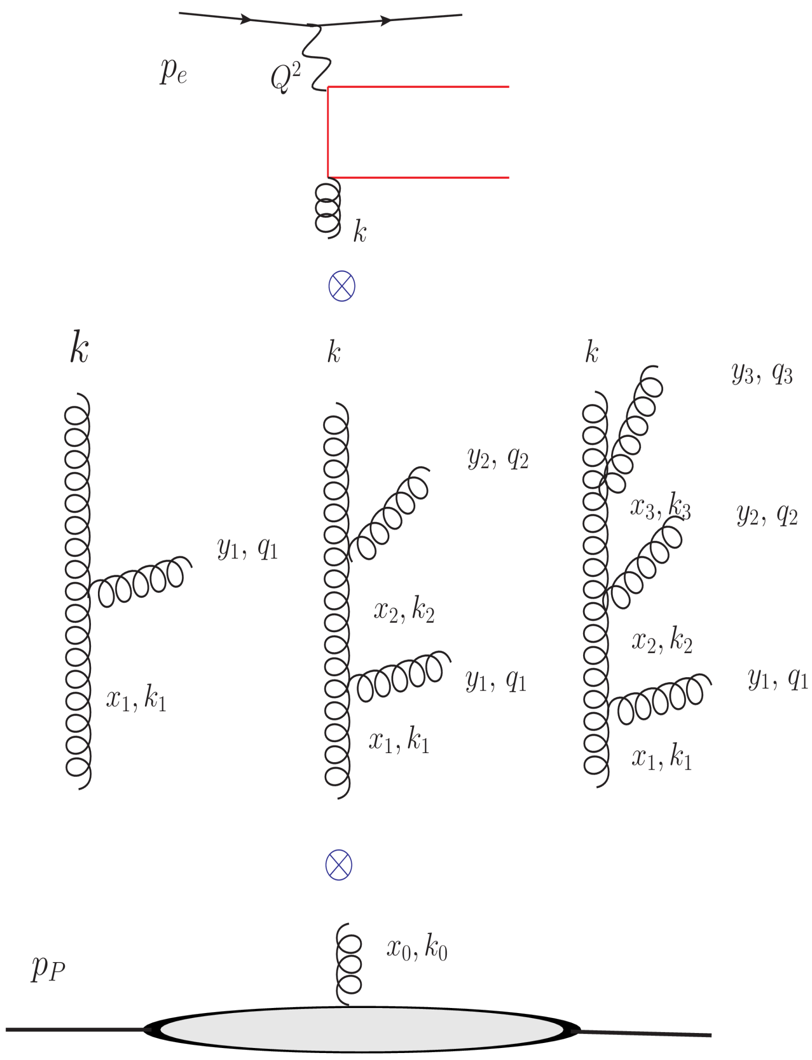
$$\mathcal{F}(x,k^2)=\frac{N_c}{4\alpha_s\pi^2}k^2\nabla_k^2\Phi(x,k^2)\quad\longrightarrow\quad\Phi(x,k^2)=\frac{\alpha_s\pi^2}{N_c}\int_{k^2}^\infty\frac{dl^2}{l^2}\ln\frac{l^2}{k^2}\mathcal{F}(x,l^2)$$

possible due to color transparency of the dipole amplitude

CCFM evolution equation - evolution with observer

Catani,Ciafaloni,Fiorani Marchesin '88

Recent review: Avsar, Iancu '09



In x → 1 region where emitted gluons are soft the dominant contribution to the amplitude comes from the angular ordered region.

$$\bar{\xi} > \xi_i > \xi_{i-1} > \ldots > \xi_1 > \xi_0$$

The same structure for x → 0 although the softest emitted gluons are harder than internal.

$$q_i = \alpha_i\, p_P + \beta_i\, p_e + q_{t\,i} \qquad s = (p_P + p_e)^2$$

$$\eta_i = \frac{1}{2}\ln(\xi_i) \equiv \frac{1}{2}\ln\left(\frac{\beta_i}{\alpha_i}\right) = \ln\left(\frac{|\mathbf{q}_i|}{\sqrt{s}\,\alpha_i}\right) \qquad \tan\frac{\theta_i}{2} = \frac{|\mathbf{q}_i|}{\sqrt{s}\,\alpha_i}$$

$$\bar{\xi} = p^2/(x^2 s)$$

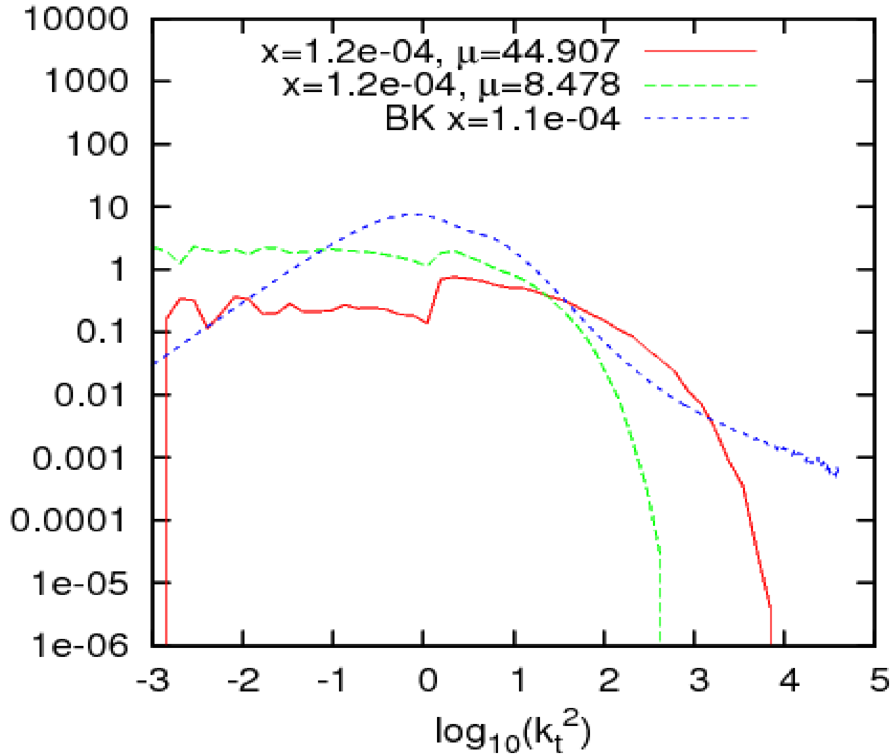
$$z_i = x_i/x_{i-1}$$

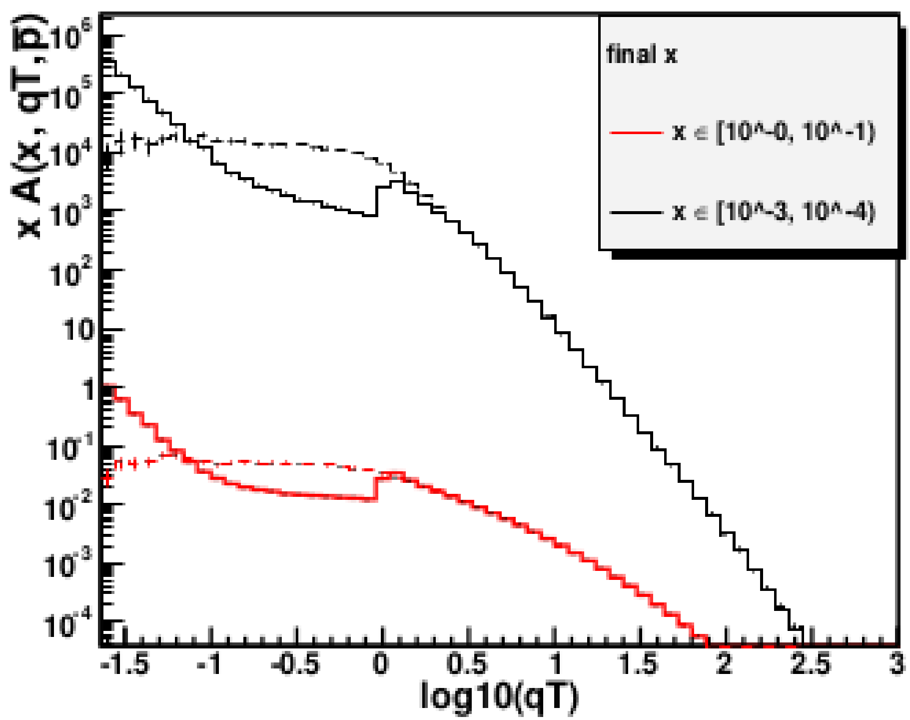
$$dP_i^g = \frac{\alpha_S}{2\pi} dz_i \frac{d^2 q_i}{q_i^2} P_{gg}(z_i)\theta(q_i - z_{i-1}q_{i-1})(1 - z_i)$$

lemented in CASCADE Monte Carlo H. Jung 02

v program CohRad is developed by M. Slawinska, S. Jadach, KK

Forward physics as the way to constrain gluon both at large and small pt





Slawinska, Jadach, KK,Arxiv:1302.0293

- *Too flat behaviour of at large k_t in BK*
- *No universality of CCFM distribution at small k_t*
- *Lack of saturation in CCFM at at small k_t*

Needed framework which unifies both correct behaviors

BK equation in the resummed exclusive form for gluon number density

$$\Phi(x,k^2) = \Phi_0(x,k^2) + \overline{\alpha}_s \int_{x/x_0}^{1} \frac{dz}{z} \int_0^\infty \frac{dl^2}{l^2} \left[\frac{l^2 \Phi(x/z,l^2) - k^2 \Phi(x/z,k^2)}{|k^2 - l^2|} + \frac{k^2 \Phi(x/z,k^2)}{\sqrt{(4l^4 + k^4)}} \right] - \frac{\overline{\alpha}_s}{\pi R^2} \int_{x/x_0}^{1} \frac{dz}{z} \Phi^2(x/z,k^2)$$

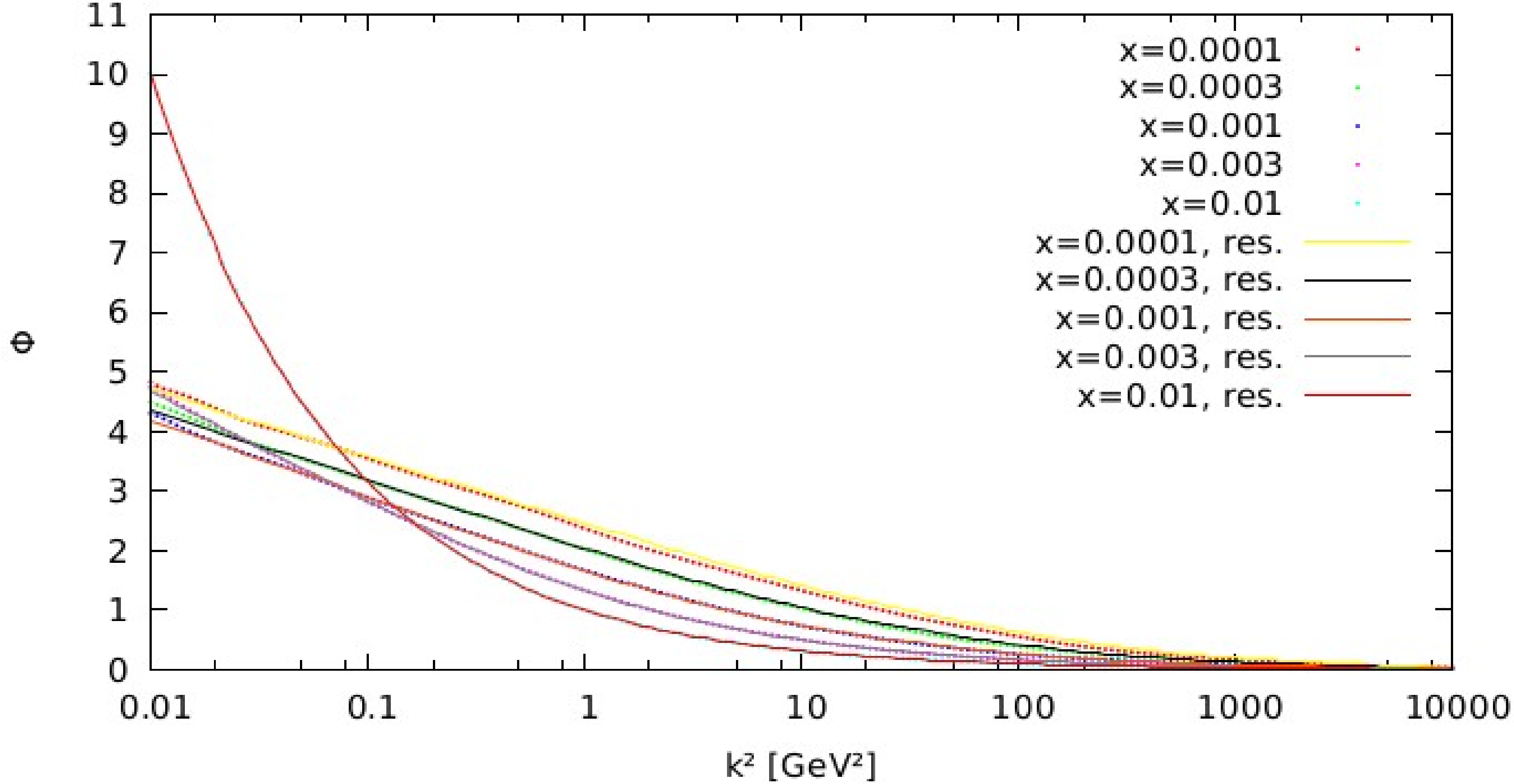
KK , JHEP 1202 (2012) 117

$$\begin{aligned}\Phi(x,k^2) &= \tilde{\Phi}^0(x,k^2) \\ &+ \overline{\alpha}_s \int_x^1 dz \int \frac{d^2\mathbf{q}}{\pi q^2} \theta(q^2 - \mu^2) \frac{\Delta_R(z,k,\mu)}{z} \left[\Phi(\frac{x}{z}, |\mathbf{k}+\mathbf{q}|^2) - q^2 \delta(q^2 - k^2)\, \Phi^2(\frac{x}{z}, q^2) \right]\end{aligned}$$

$$\Delta_R(z,k,\mu) \equiv \exp\left(-\overline{\alpha}_s \ln \tfrac{1}{z} \ln \tfrac{k^2}{\mu^2} \right)$$

- *The same resumed piece for linear and nonlinear*
- *Initial distribution also gets multiplied by the Regge form factor*
- *New scale introduced to equation. One has to check dependence of the solution on it*
- *Suggestive form to promote the CCFM equation to nonlinear equation which is more suitable for description of final states*

Gluon number density before and after resummation



calculations done
by postdoc *D. Toton*
and master student *K. Bozek*

Very good agreement.
Checked also stability varying the cutoff scale which should go to zero

Extension of CCFM to non linear equation for gluon number density

- The second argument should be kt motivated by analogy to BK
- The third argument should reflect locally the angular ordering

$$\Phi(x,k^2) = \tilde{\Phi}^0(x,k^2) + \overline{\alpha}_s \int_x^1 dz \int \frac{d^2\mathbf{q}}{\pi q^2}\,\theta(q^2-\mu^2)\frac{\Delta_R(z,k,\mu)}{z}\left[\Phi(\frac{x}{z},|\mathbf{k}+\mathbf{q}|^2) - q^2\delta(q^2-k^2)\,\Phi^2(\frac{x}{z},q^2)\right] \quad (1$$

$$\mathcal{E}(x,k^2,p) = \mathcal{E}_0(x,k^2,p) + \bar{\alpha}_s \int_x^1 dz \int \frac{d^2\bar{\mathbf{q}}}{\pi \bar{q}^2}\,\theta(p-z\bar{q})\Delta_s(p,z\bar{q})\left(\frac{\Delta_{ns}(z,k,q)}{z} + \frac{1}{1-z}\right)\left[\mathcal{E}\left(\frac{x}{z},k'^2,\bar{q}\right) - \bar{q}^2\delta(\bar{q}^2-k^2)\,\mathcal{E}^2(\frac{x}{z},\bar{q}^2,\bar{q})\right].$$

Extension of CCFM to nonlinear equation for gluon momentum density

The same procedure of resummation can be applied to the high energy factorizable gluon density. The stucture of nonlinearity does not introduce complications:

KK JHEP 1212 (2012) 033

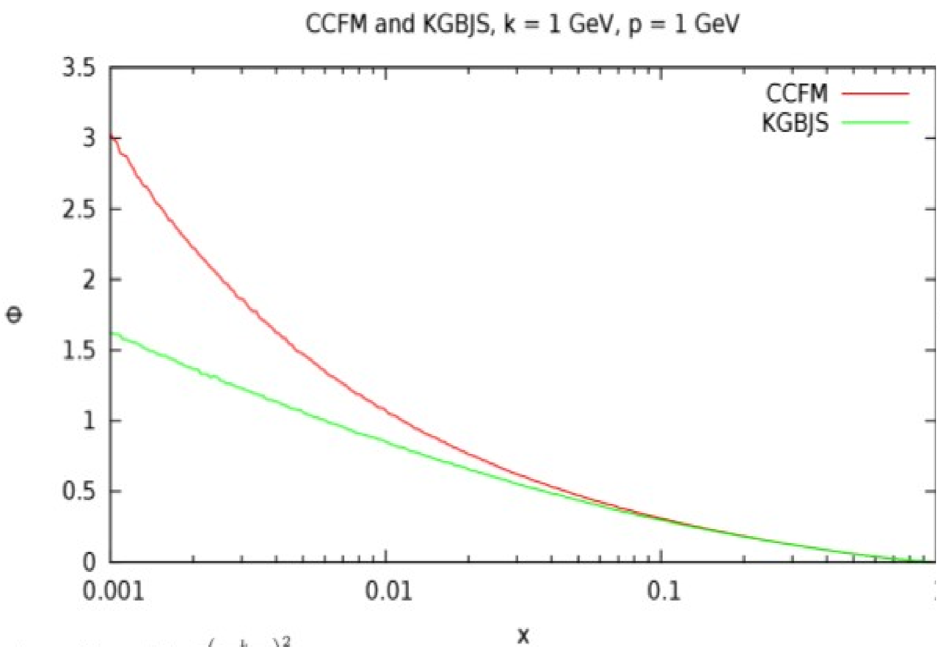
$$\mathcal{F}(x,k^2) = \tilde{\mathcal{F}}_0(x,k^2) + \overline{\alpha}_s \int_{x/x_0}^{1} \frac{dz}{z} \Delta_R(z,k,\mu) \Bigg\{ \int \frac{d^2\mathbf{q}}{\pi q^2} \theta(q^2-\mu^2) \mathcal{F}(\frac{x}{z}, |\mathbf{k}+\mathbf{q}|^2) - \frac{\pi \alpha_s^2}{4N_c R^2} k^2 \nabla_k^2 \left[\int_{k^2}^{\infty} \frac{dl^2}{l^2} \ln \frac{l^2}{k^2} \mathcal{F}(x,l^2) \right]^2 \Bigg\}$$

Include coherence

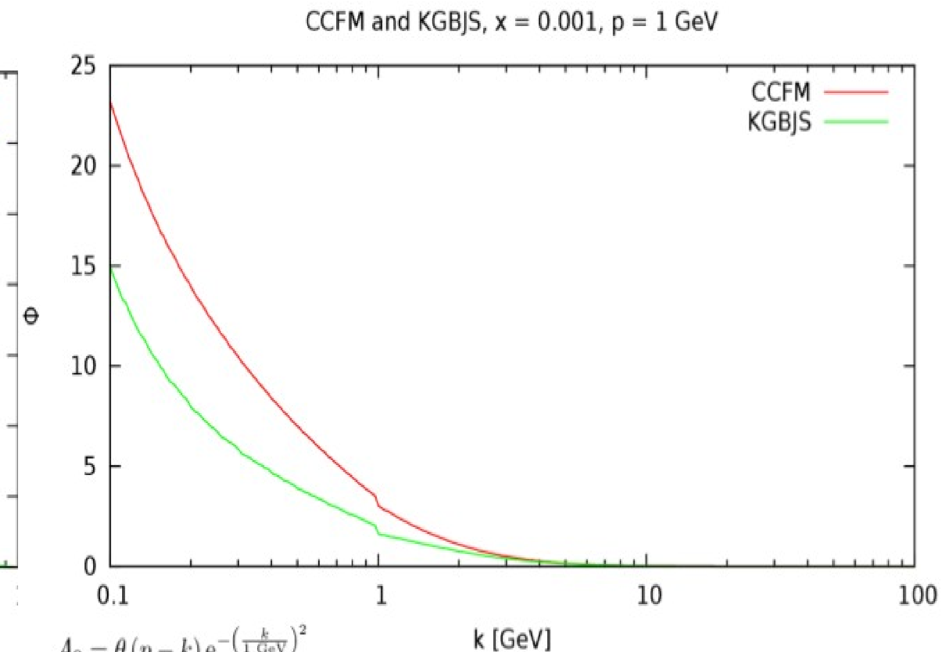
$$\mathcal{F}(x,k^2,p) = \tilde{\mathcal{F}}_0(x,k^2,p) + \overline{\alpha}_s \int \frac{d^2\mathbf{q}}{\pi q^2} \int_{x/x_0}^{1} \frac{dz}{z} \theta(p-qz) \Delta_{ns}(z,k,q) \Bigg\{ \mathcal{F}(\frac{x}{z}, |\mathbf{k}+\mathbf{q}|^2, q) - \frac{\pi \alpha_s^2}{4N_c R^2} q^2 \delta(q^2-k^2) \nabla_q^2 \left[\int_{q^2}^{\infty} \frac{dl^2}{l^2} \ln \frac{l^2}{k^2} \mathcal{F}(x/z, l^2, l) \right]^2 \Bigg\}$$

Properties of the solution of new equation for gluon number density

To be released soon



$\mathcal{A}_0 = \theta(p-k)\, e^{-\left(\frac{k}{1\,\mathrm{GeV}}\right)^2}$



$\mathcal{A}_0 = \theta(p-k)\, e^{-\left(\frac{k}{1\,\mathrm{GeV}}\right)^2}$

calculations done by postdoc *D. Toton*

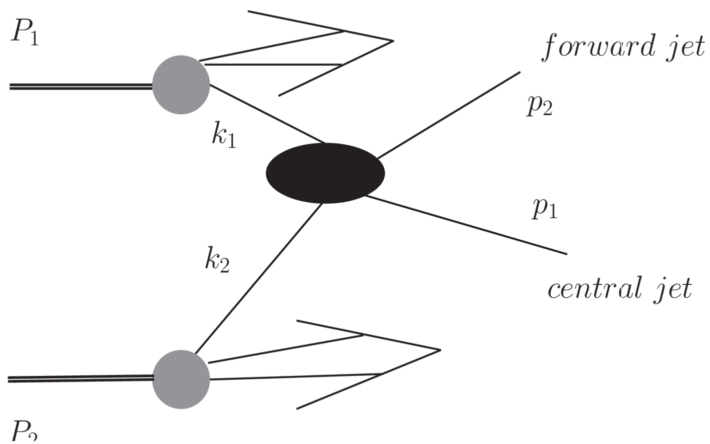
Solution also obtained in *Deak arxiv:1209.6092*

High energy prescription and forward-central di-jets including saturation

$$\frac{d\sigma}{dy_1 dy_2 dp_{1t} dp_{2t} d\Delta\phi} = \sum_{a,c,d} \frac{p_{t1} p_{t2}}{8\pi^2 (x_1 x_2 S)^2} |\mathcal{M}_{ag\to cd}|^2 x_1 f_{a/A}(x_1,\mu^2)\,\mathcal{F}_{g/B}(x_2,k^2) \frac{1}{1+\delta_{cd}}$$

Deak,Jung, Hautmann,KK
JHEP 0909:121,2009

$S = 2P_1 \cdot P_2$



- Knowing well parton densities at largr x one can get information about low x physics
- Framework goes recently under name "hybride framework"

$$x_1 = \frac{1}{\sqrt{S}}\left(p_{t1}e^{y_1} + p_{t2}e^{y_2}\right) \xrightarrow{y_1\sim 0,\, y_2 \gg 0} \sim 1$$

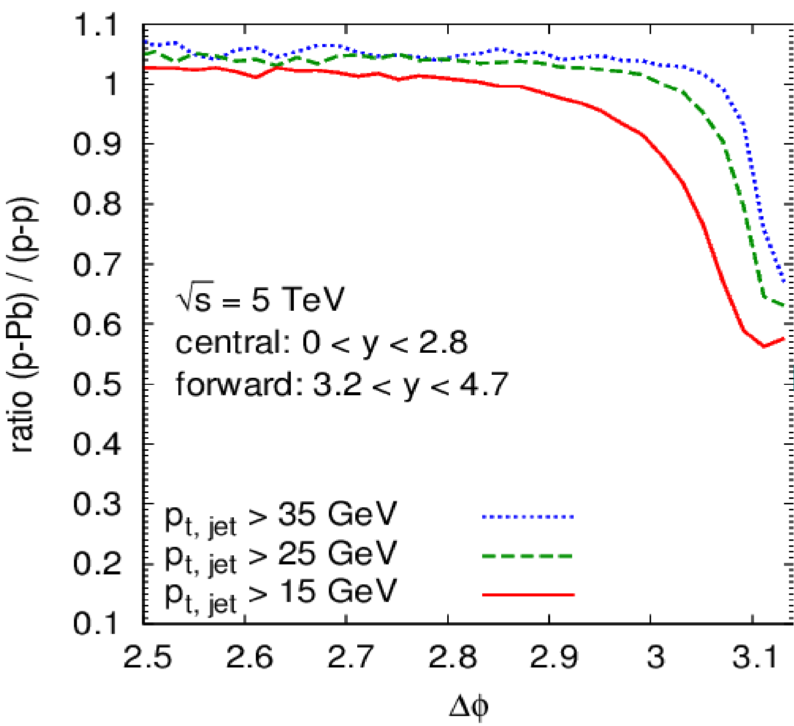
$$x_2 = \frac{1}{\sqrt{S}}\left(p_{t1}e^{-y_1} + p_{t2}e^{-y_2}\right) \qquad \ll 1$$

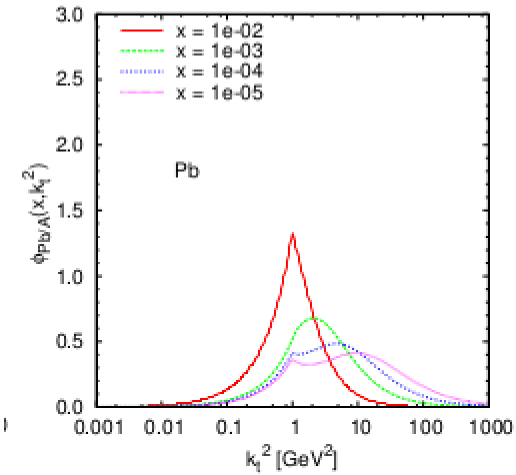
$$k_1^\mu = x_1 P_1^\mu$$

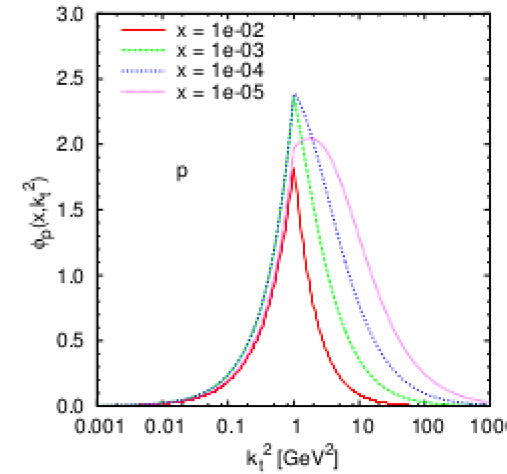
$$k_2^\mu = x_2 P_2^\mu + k_t^\mu$$

Signatures of saturation in p-p and p-Pb

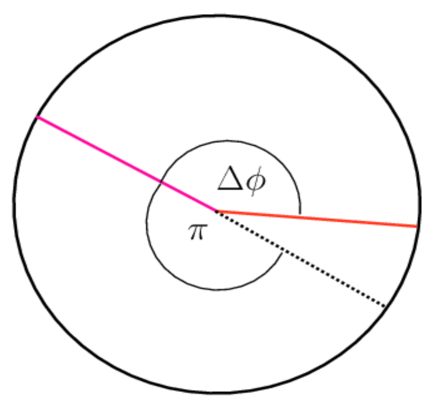
S.Sapeta. KK
Phys.Rev. D86 (2012) 094043







Studied also context of RHIC
Albacete, Marquet '10
Phys.Rev.Lett. 105 (2010) 162301



Reflects~ k^2
behavior of gluon at small k
but possible corrections from
generalized high energy factorization

Prospects for the future

- *Provide tool for simulation processes in High Energy Factorisation with the functionality like e.g. HELAC-PHEGAS*
  more than 10 particles in the final state, tree level

- *Provide tool based on High Energy Factorisation to calculate just matix elements which could be used also by external users e.g. CASCADE Monte Carlo*

- *Parton densities dependent on x, kt, hard scale with saturation*

- *Monte Carlo with saturation included*