Theoretical Limits on the (Non-)Existence of Lorentz Frame Near Black Hole Singularity *

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* based on my work with **Doug Moore**

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OUTLINE OF THE TALK

Prelude

- Quantum Gravity
- "Desert"
- Firewall
- Dark Energy

FORESTING THE DESERT!

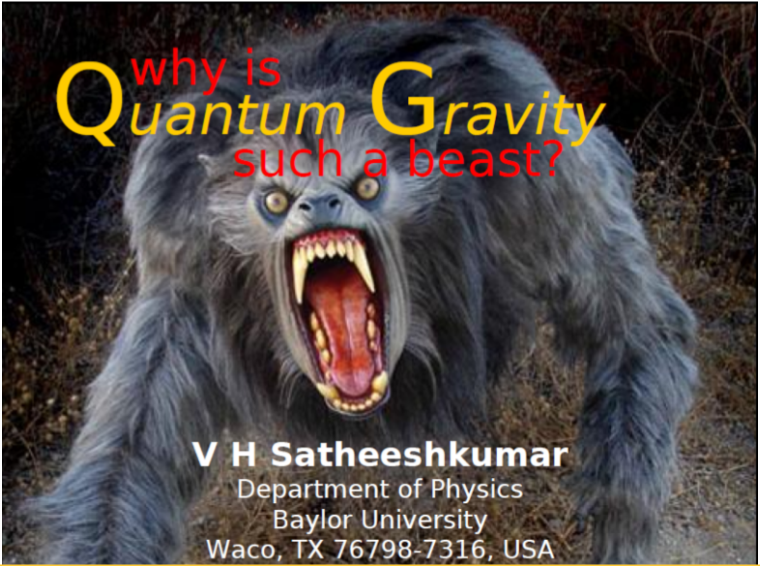
It seems very unlikely that the usual incremental increase of knowledge from a combination of theory and experiment will ever get us where we want to go, that is, to the Planck scale. Under this circumstance our best hope is an examination of fundamental principles, paradoxes and contradictions, and the study of gedanken experiments.

Gravity at High Energies

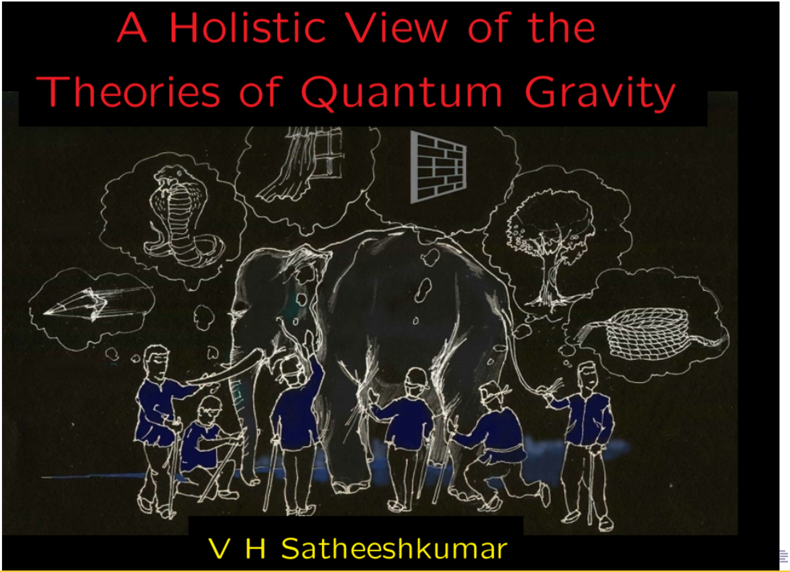
All tests of general relativity in the weak gravity field limit, like in our solar system, fall directly along the lines of what Einstein predicted. I wouldn't brag about it! But there is another regime – which has yet to be tested, and which is the hardest to test – that represents the strong gravitational field limit. And according to Einstein, gravity is strongest near black holes. This makes black holes the ultimate experimental testing grounds for Einstein's theory of general relativity.

The need to continue testing Einstein's theory of gravity may seem superfluous

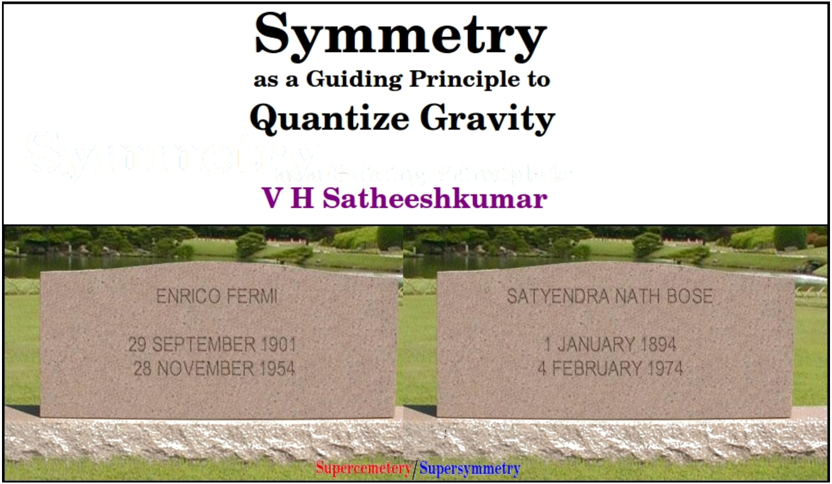
Fall 2010 - Graduate Colloquium



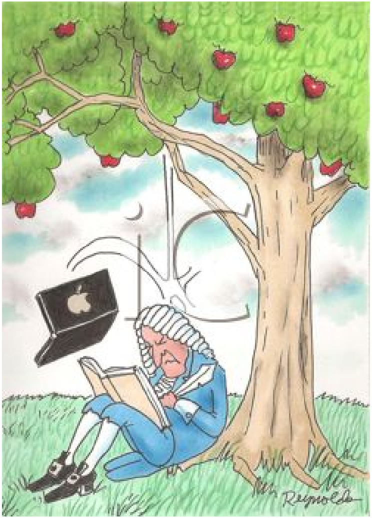
Fall 2011 - HEP Seminar



Spring 2012 - HEP Seminar



Newton discovers Gravity!



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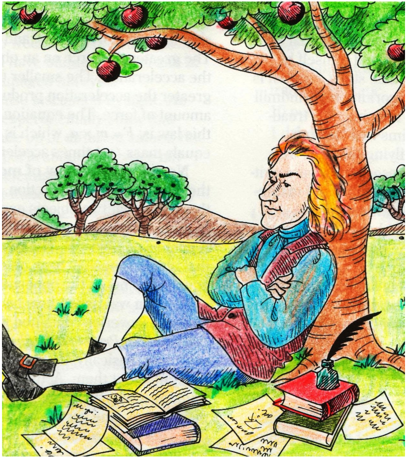
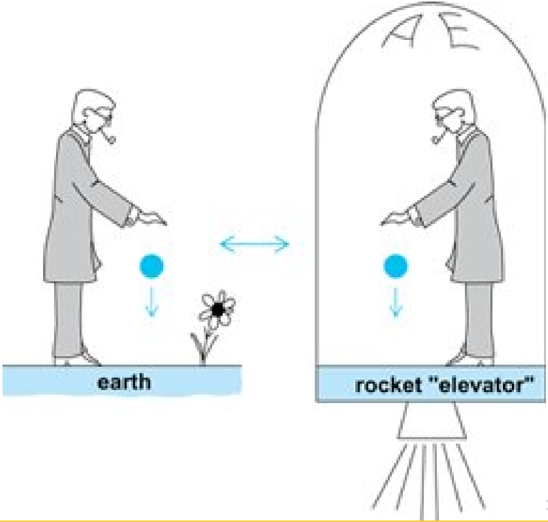
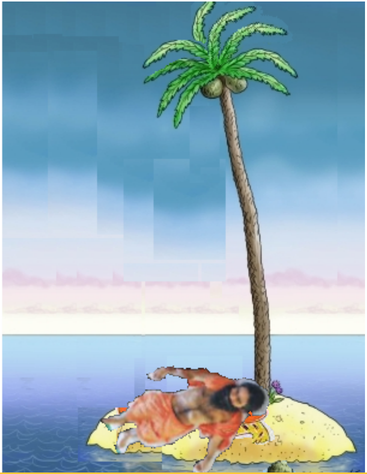


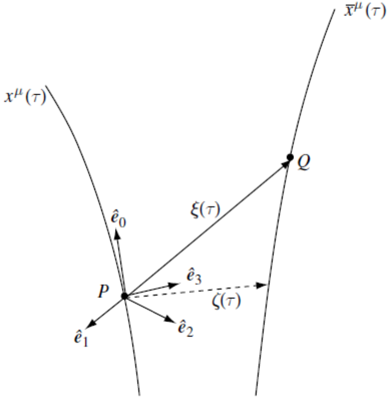
Apple doesn't fall far from the tree!



Indian scholar discovered gravity before Newton!



Tidal Forces



GEODESIC DEVIATION

In GR, the tidal force on body by a gravitating body is give by the equation for geodesic deviation. Such an equation that is valid in any coordinate system is given by

$$\frac{D^2\xi^a}{Du^a} + R^a{}_{cbd}\xi^b \dot{x}^c \dot{x}^d = 0. \tag{1}$$

Tidal Force

The tidal force experienced by a freely falling observer is given by

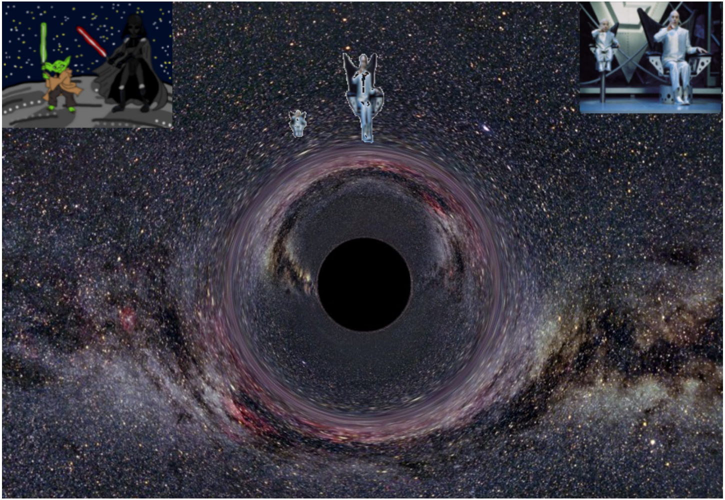
$$\frac{d^2\zeta^{\hat{\alpha}}}{d\tau^2} = c^2 R^{\hat{\alpha}}{}_{\hat{0}\hat{0}\hat{\delta}}\,\zeta^{\hat{\delta}} \tag{2}$$

where

$$R^{\hat{\alpha}}{}_{\hat{\beta}\hat{\gamma}\hat{\delta}} = R^{\mu}{}_{\sigma\nu\rho}\left(\hat{e}^{\alpha}\right)_{\mu}\left(\hat{e}_{\beta}\right)^{\sigma}\left(\hat{e}_{\gamma}\right)^{\nu}\left(\hat{e}_{\delta}\right)^{\rho} \tag{3}$$

is the Riemann tensor in the tetrad frame.

Think inside the box!



Our Results

We consider all the four possible asymptotically flat Black Hole solutions in GR.

- Schwarzschild [1916]
- Reissner-Nordström [1918]
- Kerr [1963]
- Kerr-Newman [1965]

Schwarzschild Black Hole

The Schwarzschild metric in the usual coordinates is given by

$$ds^2 = \left(1 - \frac{2\mu}{r}\right) c^2 dt^2 - \left(1 - \frac{2\mu}{r}\right)^{-1} dr^2 - r^2 (d\theta^2 + sin\theta^2 d\phi^2)$$

These lead to the following components of the mixed Riemann tensor in the tetrad basis.

$$\begin{aligned} \frac{d^2\zeta^{\hat{r}}}{d\tau^2} &= +\frac{2\mu}{r^3} c^2 \zeta^{\hat{r}} \\ \frac{d^2\zeta^{\hat{\theta}}}{d\tau^2} &= -\frac{\mu}{r^3} c^2 \zeta^{\hat{\theta}} \\ \frac{d^2\zeta^{\hat{\phi}}}{d\tau^2} &= -\frac{\mu}{r^3} c^2 \zeta^{\hat{\phi}} \end{aligned} \quad (4)$$

REISSNER-NORDSTRÖM BLACK HOLE

The Reissner-Nordström metric is given by

$$ds^2 = \left(1 - \frac{2\mu}{r} + \frac{q^2}{r^2}\right) c^2 dt^2 - \left(1 - \frac{2\mu}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 - r^2(d\theta^2 + sin\theta^2 d\phi^2) \quad (5)$$

where $q^2 = \frac{GQ^2}{4\pi\varepsilon_0 c^4}$ These lead to the following components of the mixed Riemann tensor in the tetrad basis.

$$\frac{d^2\zeta^{\hat{r}}}{d\tau^2} = \left(\frac{2\mu}{r^3} - \frac{3q^2}{r^4}\right) c^2 \zeta^{\hat{r}} \quad (6)$$

$$\frac{d^2\zeta^{\hat{\theta}}}{d\tau^2} = -\left(\frac{\mu}{r^3} - \frac{q^2}{r^4}\right) c^2 \zeta^{\hat{\theta}}$$

$$\frac{d^2\zeta^{\hat{\phi}}}{d\tau^2} = -\left(\frac{\mu}{r^3} - \frac{q^2}{r^4}\right) c^2 \zeta^{\hat{\phi}}$$

Kerr Black Hole

The Kerr metric in the Boyer-Lindquist coordinates is given by

$$
\begin{aligned}
ds^2 \;&=\; \frac{\Delta}{\rho^2}\left(c\,dt - a\,sin\theta^2\,d\phi\right)^2 - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2 \\
&\quad - \frac{(r^2+a^2)^2 sin\theta^2}{\rho^2}\left(d\phi - \frac{a}{r^2+a^2}c\,dt\right)^2
\end{aligned}
\tag{7}
$$

where

$$
\begin{aligned}
\rho^2 &= r^2 + a^2 cos^2\theta \\
\Delta &= r^2 - 2\mu r + a^2 \\
\Sigma^2 &= (r^2+a^2)^2 - a^2\Delta\sin^2\theta.
\end{aligned}
\tag{8}
$$

Kerr Black Hole

These lead to the following components of the mixed Riemann tensor in the tetrad basis.

$$\frac{d^2\zeta^{\hat{r}}}{d\tau^2} = \frac{2\mu r}{\rho^{10}}(r^2 - 3a^2\cos^2\theta)\left[(r^2+a^2)^2 + a^4\sin^4\theta - 2a^2(r^2+a^2)\sin^2\theta\right] c^2\zeta^{\hat{r}} \qquad (9)$$

$$\frac{d^2\zeta^{\hat{\theta}}}{d\tau^2} = -\frac{\mu r}{\rho^{10}}(r^2 - 3a^2\cos^2\theta)\left[(r^2+a^2)^2 + a^4\sin^4\theta - 2a^2(r^2+a^2)\sin^2\theta\right] c^2\zeta^{\hat{\theta}}$$

$$\frac{d^2\zeta^{\hat{\phi}}}{d\tau^2} = -\frac{\mu r}{\rho^{10}\Delta}(r^2 - 3a^2\cos^2\theta)\left[\Delta(r^2+a^2)^2 + a^4\Delta\sin^4\theta - 2a^2(r^2+a^2)^2\sin^2\theta\right] c^2\zeta^{\hat{\phi}}$$

Kerr-Newman Black Hole

In the Boyer-Lindquist coordinates, the Kerr-Newman metric is given by

$$ds^2 = \frac{\Delta}{\rho^2}\left(c\,dt - a\,sin\theta^2\,d\phi\right)^2 - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2 - \frac{(r^2+a^2)^2 sin\theta^2}{\rho^2}\left(d\phi - \frac{a}{r^2+a^2}c\,dt\right)^2 \quad (10)$$

where

$$\rho^2 = r^2 + a^2 cos^2\theta \quad (11)$$
$$\Delta = r^2 - 2\mu\,r + a^2 + q^2$$
$$\Sigma^2 = (r^2+a^2)^2 - a^2\Delta\sin^2\theta.$$

The symbols have their usual meanings, $q^2 = \frac{GQ^2}{4\pi\varepsilon_0 c^4}$.

Kerr-Newman Black Hole

The tidal forces of a Kerr-Newman black hole are

$$\frac{d^2\zeta^{\hat{r}}}{d\tau^2} = \frac{1}{\rho^{10}}\left[2\mu r(r^2-3a^2\cos^2\theta)-3q^2\left(r^2-a^2\cos^2\theta\right)\right] \quad (12)$$
$$\left[(r^2+a^2)^2+a^4\sin^4\theta-2a^2(r^2+a^2)\sin^2\theta\right]c^2\zeta^{\hat{r}}$$

$$\frac{d^2\zeta^{\hat{\theta}}}{d\tau^2} = \frac{-1}{\rho^{10}}\left[\mu r\left(r^2-3a^2\cos^2\theta\right)-q^2\left(r^2-a^2\cos^2\theta\right)\right]$$
$$\left[(r^2+a^2)^2+a^4\sin^4\theta-2a^2(r^2+a^2)\sin^2\theta\right]c^2\zeta^{\hat{\theta}}$$

$$\frac{d^2\zeta^{\hat{\phi}}}{d\tau^2} = \frac{-1}{\rho^{10}\Delta}\left[\mu r\left(r^2-3a^2\cos^2\theta\right)-q^2\left(r^2-a^2\cos^2\theta\right)\right]$$
$$\left[\Delta(r^2+a^2)^2+a^4\Delta\sin^4\theta-2a^2(r^2+a^2)^2\sin^2\theta\right]c^2\zeta^{\hat{\phi}}$$

For the most general (Kerr-Newman) black hole, the radial tidal force is given by

$$\frac{d^2\zeta^{\hat{r}}}{d\tau^2} = \frac{1}{\rho^{10}}\left[2\mu r(r^2-3a^2\cos^2\theta)-3q^2\left(r^2-a^2\cos^2\theta\right)\right] \left[(r^2+a^2)^2+a^4\sin^4\theta-2a^2(r^2+a^2)\sin^2\theta\right]c^2\zeta^{\hat{r}} \quad (13)$$

Let L be the length scale at which the tidal forces kick in

$$\frac{\rho^{10}}{L^2} = \left[2\mu r(r^2-3a^2\cos^2\theta)-3q^2\left(r^2-a^2\cos^2\theta\right)\right] \left[(r^2+a^2)^2+a^4\sin^4\theta-2a^2(r^2+a^2)\sin^2\theta\right] \quad (14)$$

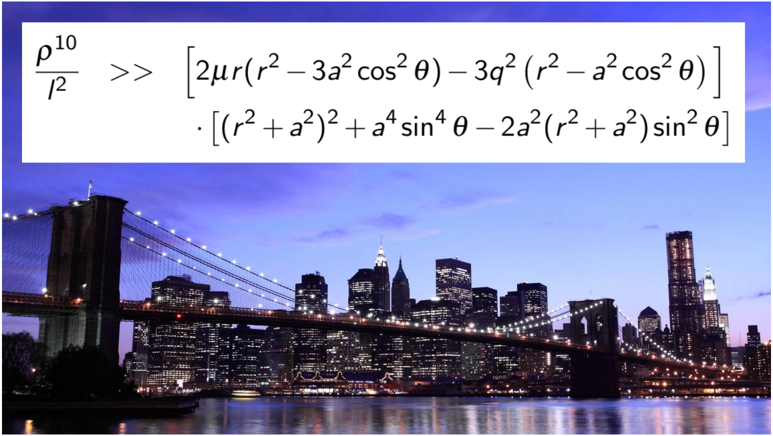
If l is the length scale of the theory, then for tidal forces not have an effect on the experiment, we should have

$$l << L$$

.

A new inequality!

$$\frac{\rho^{10}}{l^2} >> \left[2\mu r(r^2 - 3a^2\cos^2\theta) - 3q^2\left(r^2 - a^2\cos^2\theta\right)\right] \cdot \left[(r^2+a^2)^2 + a^4\sin^4\theta - 2a^2(r^2+a^2)\sin^2\theta\right]$$



For the Schwarzschild black hole

$$\frac{2GM}{c^2 r^3} = \frac{1}{L^2}$$

For $l << L$, this implies

$$r^3 >> \frac{2GMl^2}{c^2}$$

An inequality!

$$\frac{r^3}{Ml^2} >> \frac{2G}{c^2}$$

Looks a lot like $\Delta x \Delta p \geq \hbar$

Death by Black Hole!

- Assuming that the limit of tolerance to stretching or compression of a human body is an acceleration gradient of $\approx 400\, ms^{-2}/m$, for a human to survive the tidal forces at the Schwarzschild radius requires a very massive black hole with $M \geq 10^5 M_{Sun}$
- If you fell towards a supermassive black hole, with say $M \approx 10^9 M_{Sun}$
- If you fell towards a "small" black hole, of mass say $10 M_{Sun}$, you would be shredded by the tidal forces of the hole well before you reached the event horizon.

For $l = l_{Pl}$, our result says $r_{our}^3 >> 2\mu l_{Pl}^2$.
Quantum Gravity effects become important when the curvature is of the order of Planck's scale.

$$R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} = \frac{48\mu^2}{r^6} \tag{15}$$

This says $r_{conventional}^3 = 4\sqrt{3}\mu l_{Pl}^2$.
We are in total agreement with the accepted scale but only deeper and more precise!
$[r_{conventional} = 1.5 r_{our}]$

Quantum Field Theory near Black Hole Singularity

- QED
- QCD
- EW

Some Remarks

- How good is the semi-classical approximation in the Inflationary paradigm?
- Should one need Lorentz Invariance in a theory of Quantum Gravity?
- How good are the Black Hole Thermodynamics results?

THANKS!

- I have been benefited by the discussions and the classes I took with Dr. Gerald Cleaver, Dr. Anzhong Wang and Dr. B F L Ward.
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- Thanks to Dr. J Dittmann and Dr. K Hatakayama for the opportunity.

Thank You for coming!