INTERMITTENCY IN THE BLACK FOREST

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THE ISSUE

1. INTERMITTENCY IS A SIGNAL OF SELF-SIMILARITY OF THE PARTICLE SPECTRUM.
IDEAL SELF-SIMILARITY IS MANIFESTED BY FLUCTUATIONS AT ALL SCALES. IN PRACTICE THERE ARE OF COURSE LIMITS FROM BELOW AND FROM ABOVE.

2. THE SIMPLEST MECHANISM IS A SELF-SIMILAR CASCADE OF CLUSTERS: AT EACH STEP A CLUSTER SPLITS INTO TWO CLUSTERS OF HALF-SIZE. THE RATIO OF PARTICLE YIELD IN THE "DAUGTHER" TO THAT OF THE "PARENT" IS A RANDOM NUMBER w.
IF THE DISTRIBUTION OF w IS INDEPENDENT OF THE POSITION IN THE CASCADE, WE OBTAIN A SELF SIMILAR OBJECT. THE RESULTING (VERY LARGE!) FLUCTUATIONS FOLLOW A POWER LAW IN THE SCALE AT WHICH THEY ARE MEASURED - SEE NEXT SLIDES.

EXAMPLE

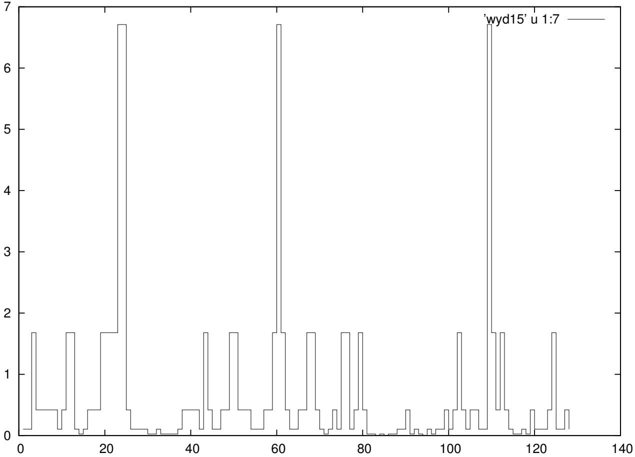

Figure: An example of the distribution obtained after 7 steps of a cascade. At each step the bin is split into two bins of half-width, their content multiplied randomly by $w_1 = 1.6$ and $w_2 = 0.4$, with probability equal $1/2$; $< w >= 1$.

SIMPLE CASCADE

SUPPOSE THAT AT EACH LEVEL OF THE CASCADE A CLUSTER IS SPLIT INTO TWO CLUSTERS OF HALF-SIZE. THEN AFTER k STEPS WE FIND 2^k CLUSTERS OF SIZE $\delta = \Delta/2^k$ WITH THE (RANDOM) AVERAGE MULTIPLICITY GIVEN BY

$$\bar{n}(k) \equiv \bar{n}(0)W(k) = \bar{n}(0)w_1 w_2 ... w_k; \quad k = \log(\Delta/\delta)/\log 2 \qquad (1)$$

WHERE w_i GIVE THE CONTENT OF THE "DAUGHTER" AT THE LEVEL i WITH RESPECT TO THE "PARENT".

IF DISTRIBUTION OF w AT EACH BRANCHING IS THE SAME, THE MOMENTS OF THIS DISTRIBUTION ARE

$$< [W(k)]^r >=< w_1^r > ... < w_k^r >=< w^r >^k; \quad < w >= 1 \qquad (2)$$

IN TERMS OF δ WE HAVE

$$< [W(k)]^r >= [\Delta/\delta]^{f_r}; \quad f_r = \log[< w^r >]/\log 2 \qquad (3)$$

THE "THEORETICAL" MOMENTS

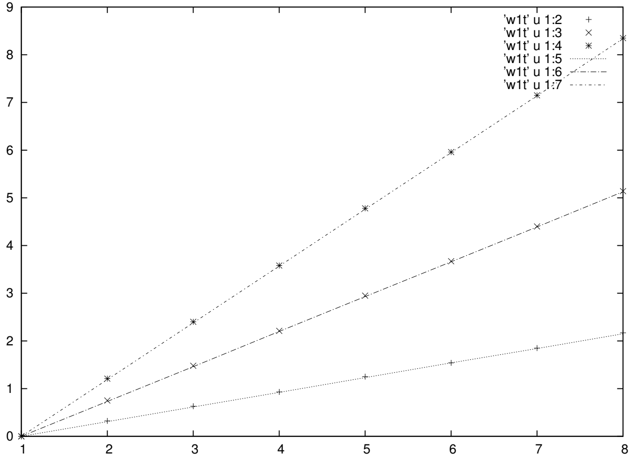

Figure: "Theoretical" moments evaluated from 1000 events

THE "EXPERIMENTAL" MOMENTS

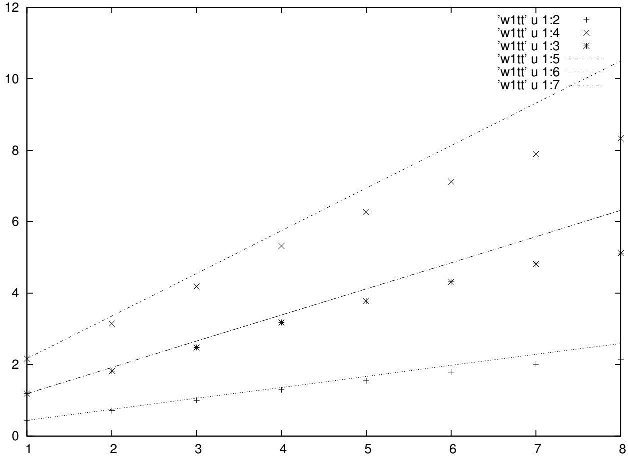

Figure: "Experimental" moments evaluated from 1000 events

STATISTICAL NOISE

1. $W(k)$ **GIVES THE AVERAGE NUMBER OF PARTICLES IN A GIVEN BIN.**

$$W(k) = \bar{n}(k)/\bar{n}(0) \tag{4}$$

2. **HOWEVER, THE ACTUALLY MEASURED NUMBER OF PARTICLES, BEING AN INTEGER, FLUCTUATES AROUND** $\bar{n}_i$**;**

3. **THIS "STATISTICAL NOISE" CAN BE APPROXIMATED BY THE POISSON DISTRIBUTION WITH THE AVERAGE** $\bar{n}_i$**.**

4. **THEREFORE THE ACTUAL DISTRIBUTION OF PARTICLES IN A BIN** i **IS**

$$P(n_i; \bar{n}_i) = e^{-\bar{n}_i} \frac{\bar{n}_i^{n_i}}{n_i!} \tag{5}$$

FACTORIAL MOMENTS

1. FROM THE KNOWN PROPERTY OF THE POISSON DISTRIBUTION WE GET AT A FIXED $\bar{n}$:

$$F_r \equiv \sum_{n=0}^{\infty} n(n-1)...(n-r+1)e^{-\bar{n}}\frac{\bar{n}^n}{n!} = \bar{n}^r \tag{6}$$

THUS THE AVERAGE OVER FLUCTUATING $\bar{n}$ AT THE LEVEL k GIVES

$$< [\bar{n}(k)]^r >=< F_r(k) > \tag{7}$$

2. CONCLUSION: THE FACTORIAL MOMENT OF THE OBSERVED PARTICLE DISTRIBUTION GIVE DIRECTLY THE MOMENT OF THE DISTRIBUTION OF $\bar{n}$.

THE STATISTICAL NOISE IS REMOVED!
(Bialas-Peschanski)

CORRELATION FUNCTIONS

1. FACTORIAL MOMENTS ARE INTEGRALS OF THE CORRELATION FUNCTIONS

$$f_2(\delta) = \int_\delta \frac{d^3p_1}{E_1}\frac{d^3p_2}{E_2}\rho_2(p_1,p_2)\left[\int_\delta \frac{d^3p}{E}\rho_1(p)\right]^{-2} \qquad (8)$$

TO OBTAIN SINGULARITY IN $F_2(\delta)$, $\rho_2(p_1,p_2)$ MUST ALSO BE SINGULAR. TAKING

$$\rho_2(p_1,p_2) \sim \rho_1(p_1)\rho_1(p_2)|p_1-p_2|^{-\alpha} \qquad (9)$$

WE HAVE: $f_2(\delta) \sim \delta\delta^{1-\alpha}/\delta^2 \sim \delta^{-\alpha}$.

CONCLUSION: POWER LAW SINGULARITY IN f_2 IMPLIES POWER LAW SINGULARITY IN THE CORRELATION FUNCTION. IT MAY THUS SIGNAL A PHASE TRANSITION !

Ochs, Bialas-Seixas: Three-dimensional studies are necessary!

CORRELATION FUNCTIONS

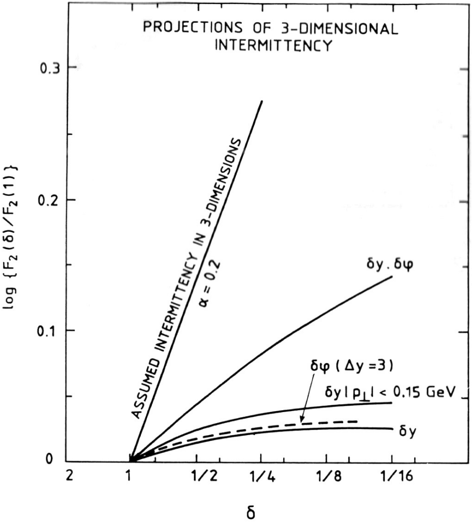

Figure: Factorial moments F_2 versus the bin size δ for various projected distributions [Bialas-Seixas (90)].

PHASE TRANSITION

CONSIDER THE NORMALIZED FACTORIAL MOMENT f_r:

$$f_r(\delta) = \int_\delta \frac{d^3p_1}{E_1} ... \frac{d^3p_r}{E_r} \rho_r(p_1, ..., p_r) \left[\int_\delta \frac{d^3p}{E} \rho_1(p) \right]^{-r} \quad (10)$$

TAKING

$$\rho_r = \rho_1(p_1)...\rho_r(p_r)\{[|p_1 - p_2||p_2 - p_3|...|p_{r-1}p_r|]^{-\alpha} + perm\} \quad (11)$$

WE HAVE

$$f_r \sim \delta[\delta^{1-\alpha}]^{r-1}\delta^{-r} \sim \delta^{-(r-1)\alpha} \equiv \delta^{-\phi_r} \quad \rightarrow \quad \phi_r = (r-1)\alpha. \quad (12)$$

CONCLUSION: IN THE SECOND ORDER PHASE TRANSITION, THE INTERMITTENCY EXPONENTS FOLLOW THE SIMPLE RULE (12) (SIMPLE FRACTAL).

IN CASCADE THIS RULE IS GENERALLY NOT VALID. [Bialas-Hwa]

BOSE-EINSTEIN CORRELATIONS

OBSERVATION OF CHARGE DEPENDENCE INDICATES THAT HBT CORRELATIONS DOMINATE THE EFFECTS. THIS MAY HAVE INTERESTING CONSEQUENCES. IF INDEED THE HBT CORRELATION FUNCTION $C(p_1 - p_2)$ SHOWS A POWER-LAW SINGULARITY, THEN THE DISTRIBUTION OF THE SOURCE OF PARTICLES

$$|\rho(x)|^2 \sim \int dq e^{iqx} C(q) \tag{13}$$

MUST ALSO EXIBIT A POWER-LAW SINGULARITY.

E.G., TAKING $C(q) \sim [1 + |q|^2 L^2/4]^{-\beta}$ WE HAVE (two dimensions for simplicity)

$$|\rho(x)|^2 \sim \hat{x}^{\beta-1} K_{\beta-1}[2\hat{x}]; \quad \hat{x} = |x|/L \tag{14}$$

FOR SMALL $\hat{x}$: $|\rho(\hat{x})|^2 \sim \hat{x}^{2(\beta-1)}$;
FOR LARGE $\hat{x}$: $|\rho(\hat{x})|^2 \sim \hat{x}^{\beta-3/2} e^{-[2\hat{x}]}$.

OUTLOOK

1. OBSERVATION OF POWER LAW DEPENDENCE OF THE (NORMALIZED) FACTORIAL MOMENTS ON THE RESOLUTION, SEEMS THE MOST EFFECTIVE WAY OF FINDING THE POWER-LAW SINGULARITY IN THE MULTI-PARTICLE SYSTEMS.

2. AT THE SECOND ORDER PHASE TRANSITION, THE SYSTEM BECOMES A SIMPLE FRACTAL. IT IS CHARACTERIZED BY THE INTERMITTENCY EXPONENTS SATISFYING THE LINEAR RELATION $\phi_r = (r-1)\alpha$ WITH α INDEPENDENT OF r.

3. IF THE EFFECT OF INTERMITTENCY IS RELATED TO THE HBT CORRELATIONS, THEN THE SPACE-TIME STRUCTURE OF THE SYSTEM SHOULD EITHER SHOW STRONG (SELF-SIMILAR) FLUCTUATIONS OF THE SIZE, OR EXIBIT THE FRACTAL NATURE (OR BOTH).