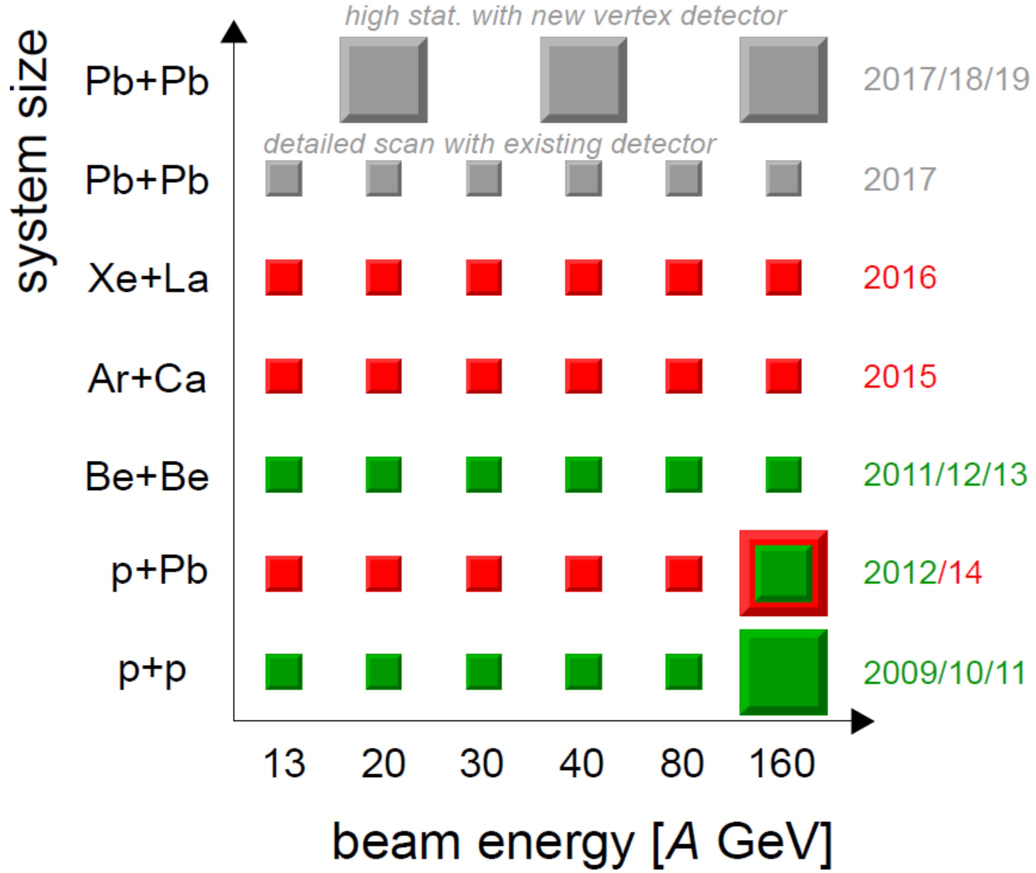
From p+p to Pb+Pb collisions:
Wounded Nucleon versus Statistical models

SPP, Bad Liebenzell, April 29, 2013

- The models
- Similarities
- Differences

The minimal model

motivation: NA61/SHINE:



The models

Wounded Nucleon Model:
(forefather of string-hadronic models)
Bialas, Bleszynski, Czyz, 1976

Particle production from independent ''decays'' of wounded nucleons W (nucleons which interacted inelastically).

Properties of wounded nucleons are independent of size of colliding nuclei, i.e. they are the same in p+p and Pb+Pb collisions at the same collision energy per nucleon

The most famous prediction:

$$\langle A\rangle/\langle W\rangle=\langle A\rangle_{NN}/2$$

<A> - mean multiplicity of hadron A
<W> - mean number of wounded nucleons

Statistical Model:
(forefather of hydrodynamical models)
Fermi, Pomeranchuk, Landau, Hagedorn, 1950-1965

All possible micro-states are equally probable.

For large enough systems (e.g. Pb+Pb collisions) and popular hadrons the grand canonical ensemble can be used for mean multiplicities/spectra

The most famous prediction:

$$\frac{1}{m_T}\frac{dn}{m_T} \sim V \circ e^{-m_T/T}$$

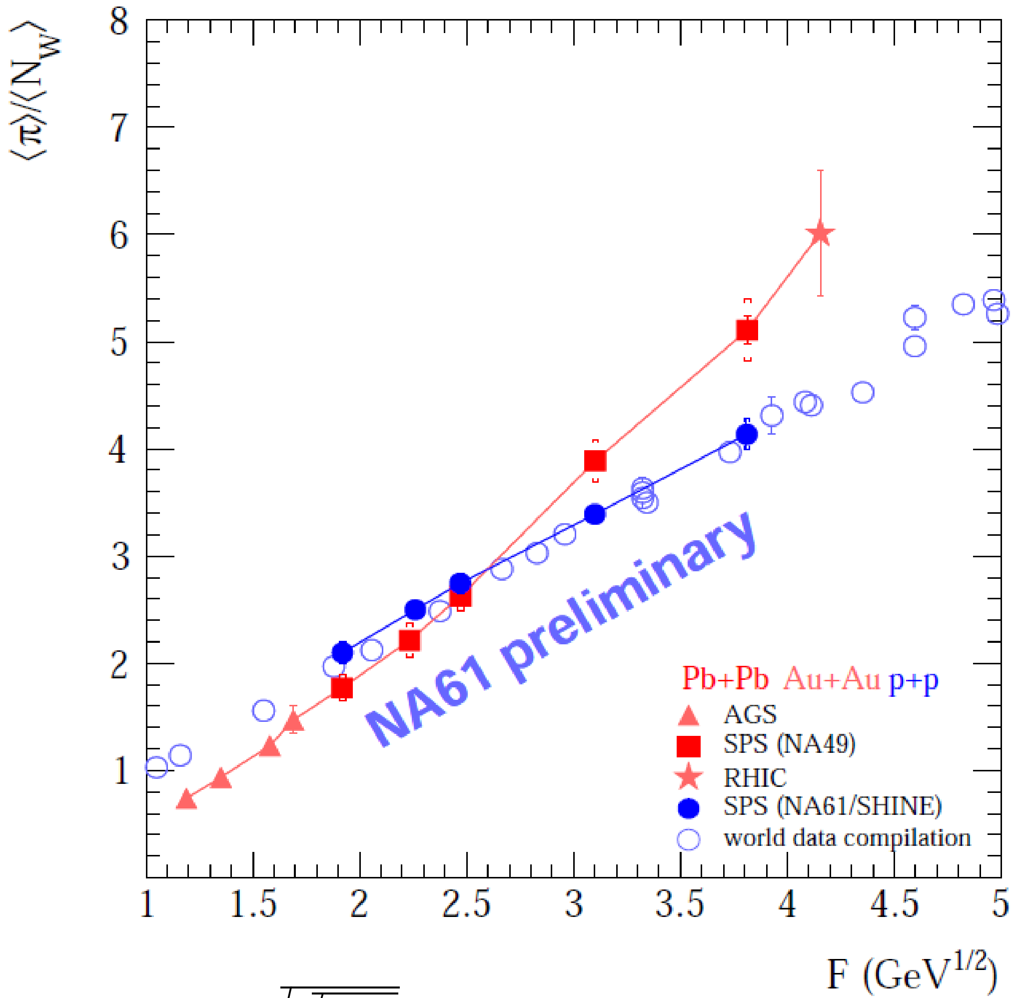
m_T – *transverse mass*
T – *temperature*
V – *volume*

Wounded Nucleon and Statistical models were formulated before the QCD period.

Their relation to QCD remains unclear.

Nevertheless they continuously serve as the basic tools to interpret results on hadron production in high energy collisions.
This is because they are simple and they approximately reproduce several basic properties of the data.

Wounded Nucleon Model:



$<A>/<W>=<A>_{NN}/2$

From the low SPS to the LHC energies pion multiplicity per wounded nucleon in p+p and Pb+Pb collisions is similar:

(Pb+Pb) ≈ 1.3 (p+p)

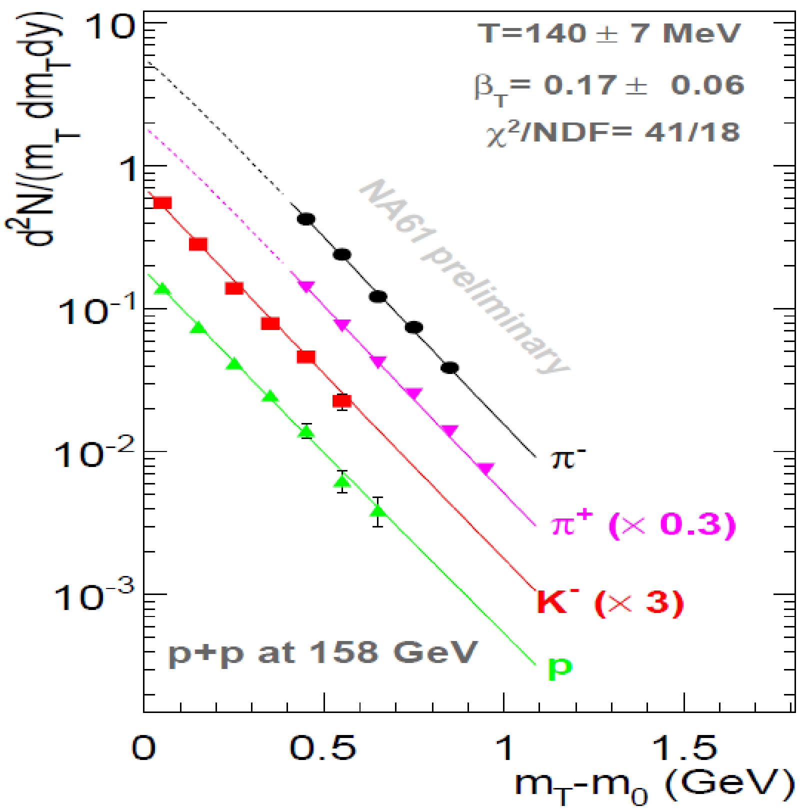
$F \approx \sqrt{\sqrt{s_{NN}}}$

$\langle \pi \rangle$ - total pion multiplicity

$\langle N_W \rangle$ - number of wounded nucleons

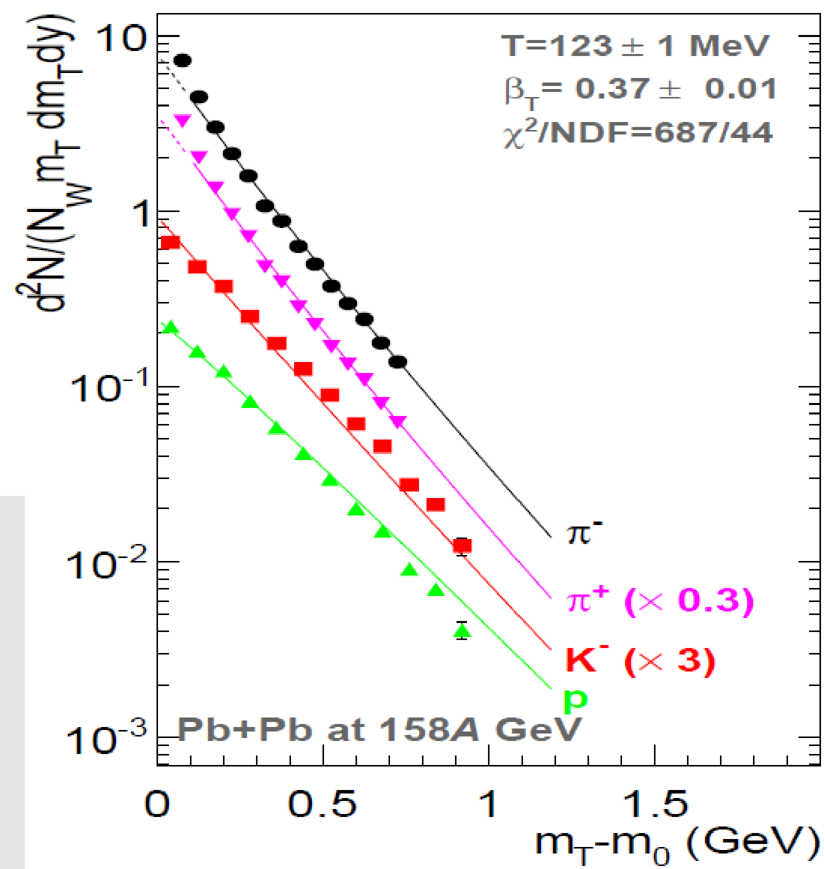
Statistical Model:

NA61: p+p (Szymon)



$$\frac{1}{m_T}\frac{d\,n}{m_T} \sim V \circ e^{-m_T/T}$$

NA49: Pb+Pb



Transverse mass spectra are approximately exponential in p+p . In central Pb+Pb the exponential dependence is modified by the transverse flow.

System size dependence: Similarities

In WNM hadrons are produced from “decays”of independent wounded nucleons

In SM(GCE) hadrons are produced from “decays”of independent volume elements

Similar predictions concerning system size dependence of hadron production properties

Mean multiplicities

WNM:

$\langle A \rangle \sim W$

$\langle B \rangle \sim W$

SM(GCE):

$\langle A \rangle \sim V$

$\langle B \rangle \sim V$

(for T and μ_B = const(V))

Ratios of mean multiplicities are independent of system size and its fluctuations:

$\langle A \rangle / \langle B \rangle$ is independent of $P(W)$ in WNM and of $P(V)$ in SM(GCE) (it is strongly intensive)

(Experimental results (e.g. the so-called strangeness enhancement) contradict this prediction.)

Multiplicity Fluctuations: the second moments

$(\omega[x] \equiv \mathrm{Var}[x]/\langle x\rangle, \quad \mathrm{Var}[x] = \langle (x-\langle x\rangle)^2\rangle)$

WNM:

$$\omega[A] = \omega^*[A] + \langle A\rangle/\langle W\rangle\, \omega[W]$$
$$\omega[B] = \omega^*[B] + \langle B\rangle/\langle W\rangle\, \omega[W]$$

$$\langle AB\rangle = \langle AB\rangle^* \langle W\rangle + \langle A\rangle\langle B\rangle/\langle W\rangle^2(\langle W^2\rangle-\langle W\rangle)$$

SM(GCE):

$$\omega[A] = \omega^*[A] + \langle A\rangle/\langle V\rangle\, \omega[V]$$
$$\omega[B] = \omega^*[B] + \langle B\rangle/\langle V\rangle\, \omega[V]$$

$$\langle AB\rangle = \langle AB\rangle^* \langle V\rangle + \langle A\rangle\langle B\rangle/\langle V\rangle^2(\langle V^2\rangle-\langle V\rangle)$$

where $\omega^*[A]$, $\omega^*[B]$ and $\langle AB\rangle^*$ are quantities calculated for any fixed value of W or V

Properly weighted **sums** of second moments
of joint multiplicity distribution of two hadrons are
independent of system size and its fluctuations
(P(W) and P(V))
(they are strongly intensive)

MG, Mrowczynski 1992
Gorenstein, MG, 2011
MG, Gorenstein, Mackowiak-Pawlowska, 2013

Properly weighted **sums** of second moments:

$$\Delta[A,B] = (\langle B\rangle\omega[A] - \langle A\rangle\omega[B])/(\langle B\rangle - \langle A\rangle)$$

$$\Sigma[A,B] = (\langle B\rangle\omega[A] + \langle A\rangle\omega[B] - 2(\langle AB\rangle - \langle A\rangle\langle B\rangle))/(\langle B\rangle + \langle A\rangle)$$

(Σ is a reincarnation of the Φ fluctuation measure)

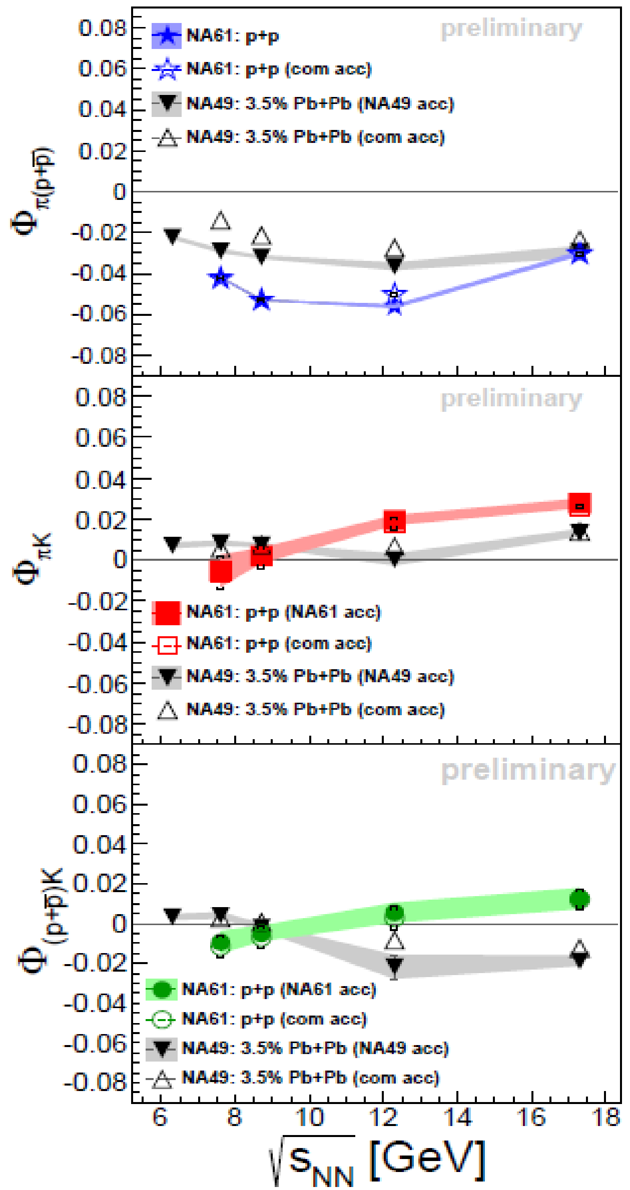
are independent of P(W) in WNM
and of P(V) in SM(GCE)

MG, Mrowczynski 1992
Gorenstein, MG, 2011
MG, Gorenstein, Mackowiak-Pawlowska, 2013

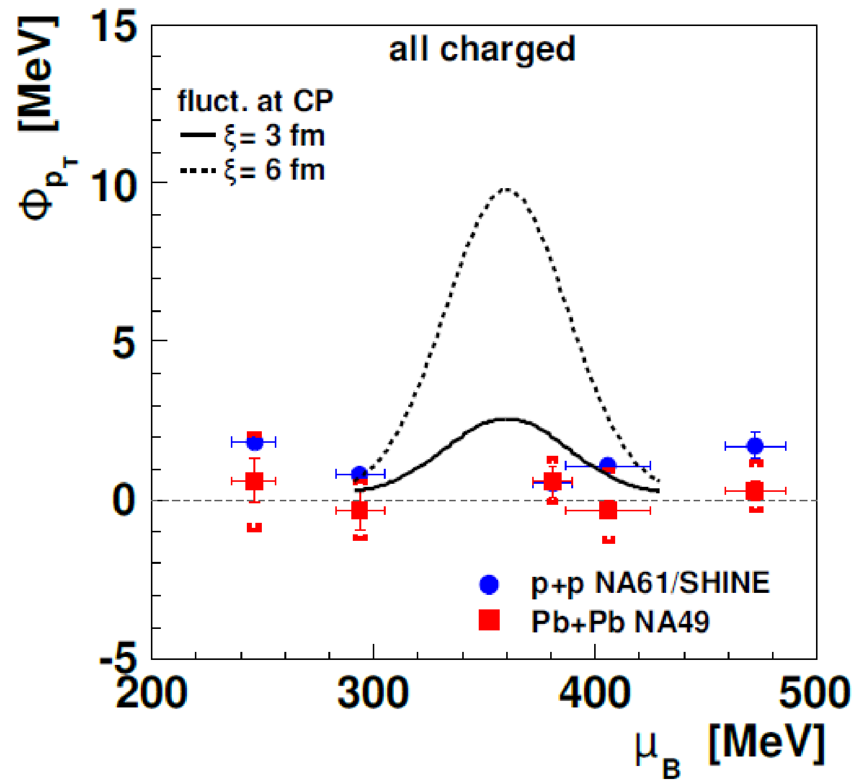
(This prediction is under experimental test
by NA61 (p+p ... Xe+La) and NA49 (Pb+Pb).)

First test results shown at CPOD 2013:
p+p (NA61) vs central Pb+Pb (NA49)

Maja, Anar:



Tobiasz:



Fluctuations in p+p and central Pb+Pb are similar (if properly quantified).

Predictions of WNM and SM(GCE) concerning system size dependence of mean multiplicity ratios as well as Σ and Δ are almost identical.

The only difference is that the system size parameter in WNM, the number of wounded nucleons W, is discreet, whereas the system size parameter in SM(GCE), the volume V, is continuous.

Problem I:
Find predictions of WNM and SM(GCE) which are different for discreet and continuous system size parameters.

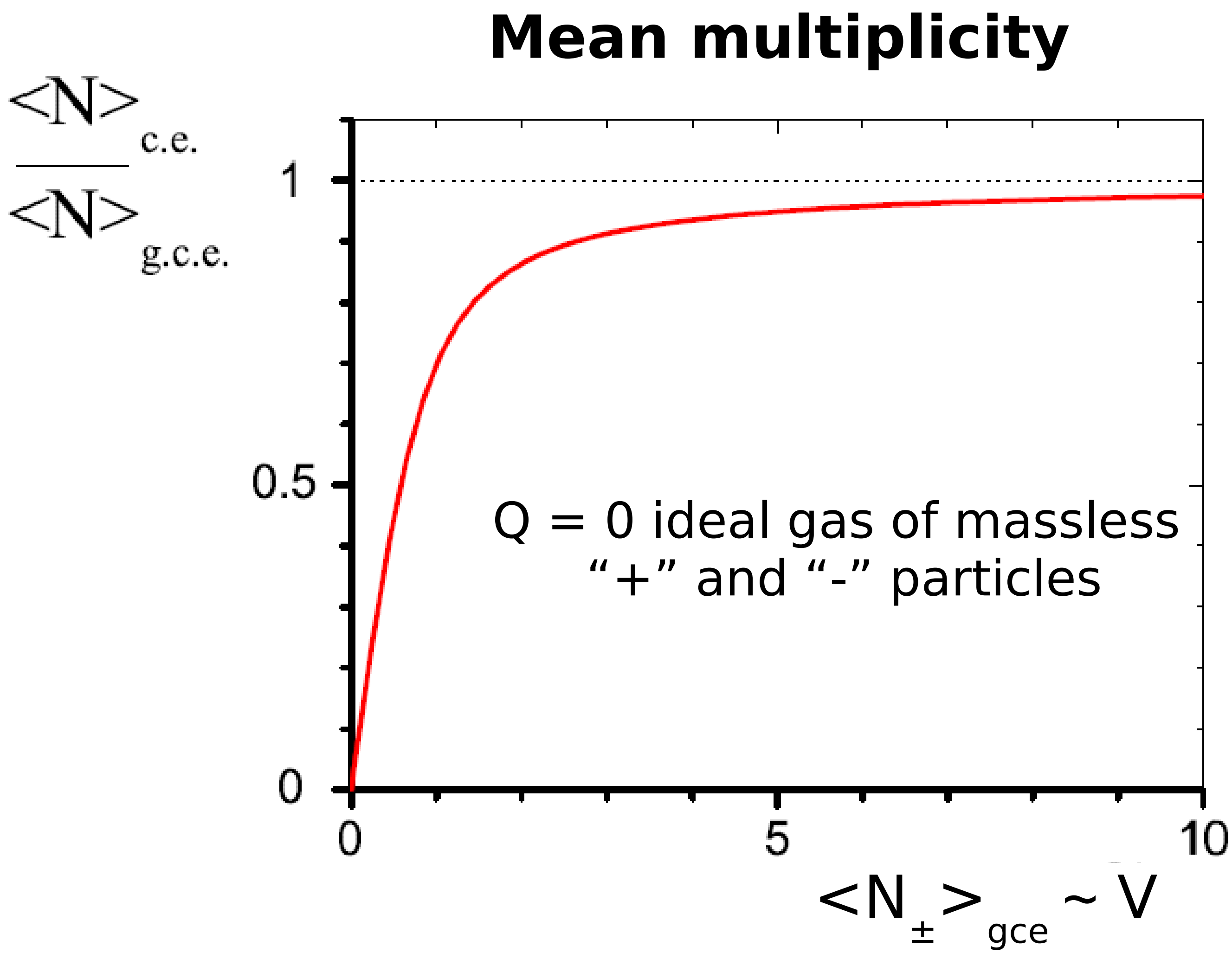
System size dependence: Differences

Predictions of the statistical model concerning system size dependence change qualitatively when material and/or motional conservation laws are introduced, i.e.
instead of the grand canonical ensemble (GCE), the canonical (CE) or micro-canonical (MCE) ensembles are used.

First moments:	*Rafelski, Danos, Turko, Redlich, 1980* *Becattini, Ferroni, 2004*
Second moments:	*Begun, Gorenstein et al., 2004+*

The SM(GCE) and WNM predictions differ qualitatively from the SM(CE) and SM(MCE) ones.

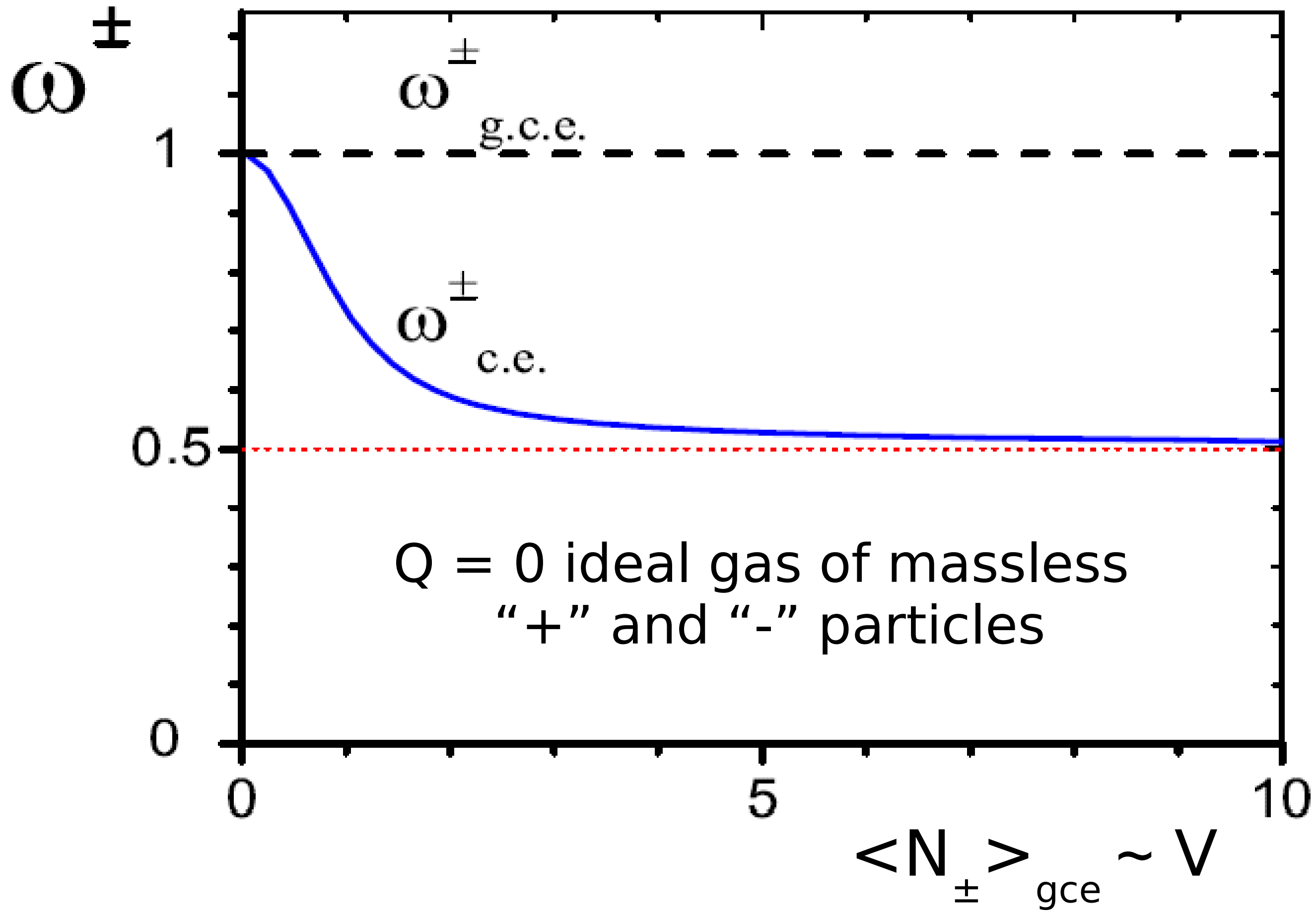
SM(GCE) (=WNM) versus SM(CE)



The simplest example of the statistical model with material conservation laws

SM(GCE) (=WNM) versus SM(CE)

Scaled variance

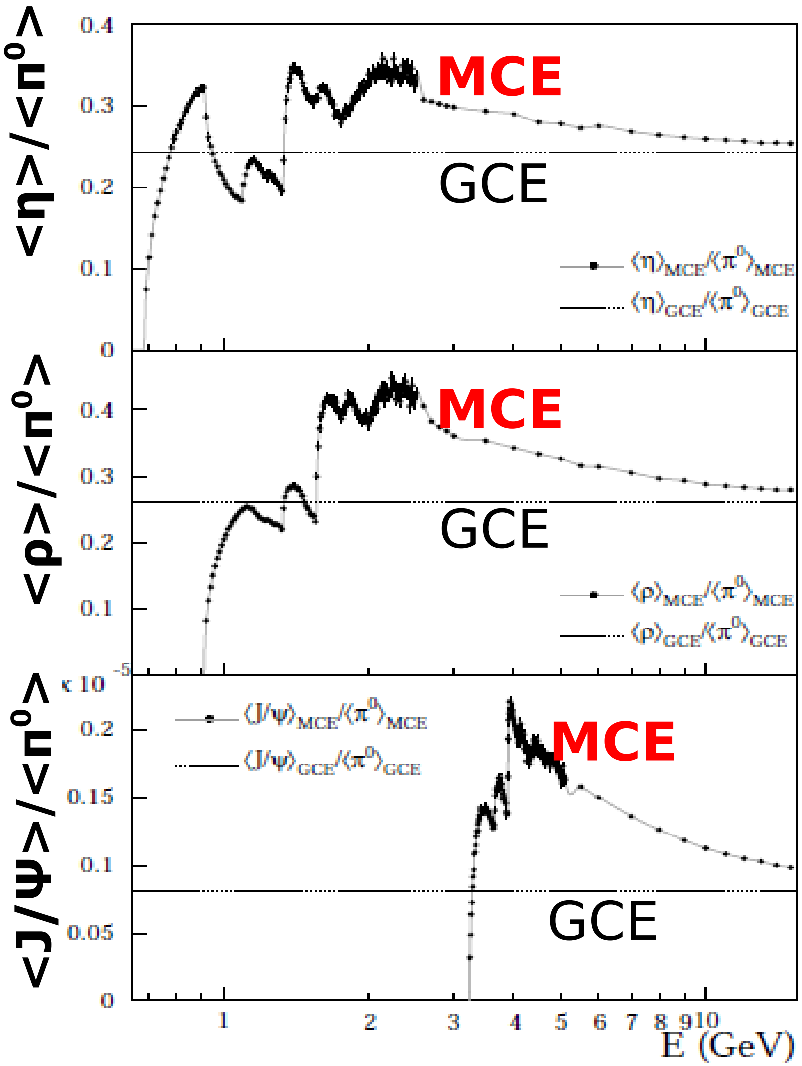


The simplest example of the statistical model with material conservation laws

Begun, MG, Gorenstein, Zozulya, 2004

SM(GCE) (=WNM) versus SM(MCE)

Mean multiplicity



~ V in GCE

Micro-canonical ensemble
for hadron-resonance gas

The most complicated example
of the statistical model with motional
conservation laws

Begun, Ferroni, Gorensetin, MG, Becattini, 2006

Problem II:
Find predictions of SM(CE) and SM(MCE) for strongly intensive measures of fluctuations Δ and Σ.

This may allow to experimentally distinguish between WNM, SM(GCE) and SM(CE), SM(MCE) at the level of fluctuations (second moments) (scaled variance is biased by W or V fluctuations).

The minimal model from p+p to Pb+Pb

+ Initial energy and volume fluctuations
  (← the KNO scaling of multiplicity distributions in p+p)

+ Strict material and motional conservation laws
  (← strangeness suppression and power law tail in p+p)

+ Hydrodynamical expansion
  (← non-thermal spectra in Pb+Pb)

+ Equation of state with phase transition
  (← kink, step, horn in Pb+Pb)