Fluid Dynamics in High Energy Collisions

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Statistical Particle Production (SPP)
29 April - 03 May 2013 ,Bad Liebenzell

Earliest FD application – Landau Model (for p+p coll.)

$$T^{\mu\nu\ (0)} = e u^\mu u^\nu - P\Delta^{\mu\nu} = (e+P)u^\mu u^\nu - P g^{\mu\nu}$$

$$T^{\mu\nu}{}_{,\mu} = \sum_{k=1}^{N} T_k^{\mu\nu}{}_{,\mu} = 0.$$

$$\left(N^{\mu}{}_{,\mu} = \sum_{k=1}^{N} N_k^{\mu}{}_{,\mu} = 0.\right)$$

Landau L.D. Landau, Izv. Akad. Nauk SSSR **17** (1953) 51.

Rapidity coordinate

$$\alpha \equiv \frac{1}{2}\ln\left(\frac{t+z}{t-z}\right)$$

Proper time' coordinate

$$\beta \equiv \ln\left(\frac{\tau}{r_0}\right)$$

where $r_0 \approx d/\gamma^{c.m.}$

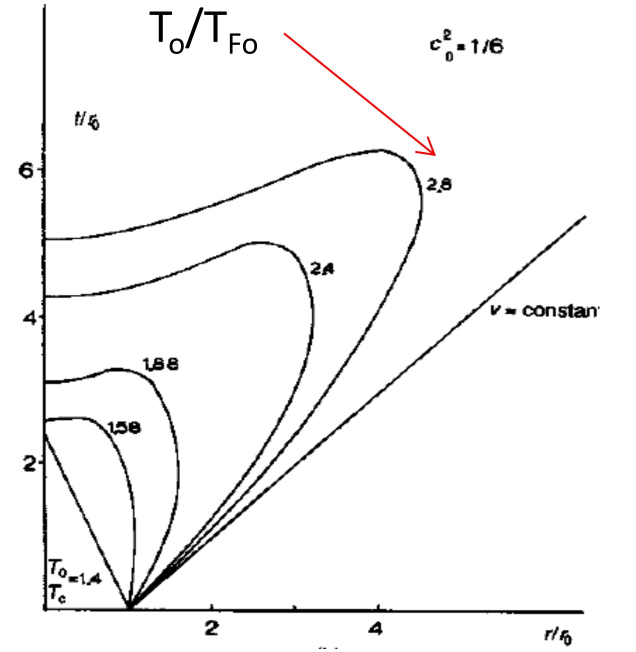
$$\partial_\beta \ln T + \lambda c_0^2 = 0, \qquad \partial_\alpha \ln T = 0,$$

or

$$\frac{\partial e}{\partial \tau} = -\lambda\frac{(e+P)}{\tau},$$

where

$$\lambda = \begin{cases} 1, & \text{linear one} - \text{dimensional expansion} \\ 3, & \text{spherical expansion} \end{cases}$$

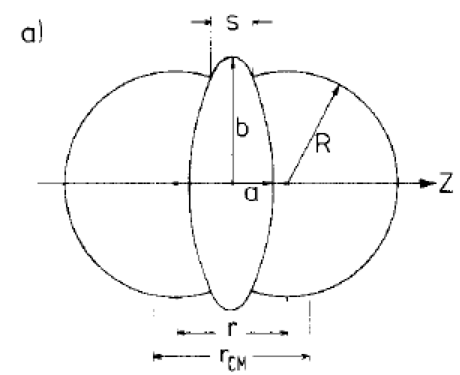


F. Cooper, G. Frye and E. Schonberg, Phys. Rev. **D11** (1975) 192.

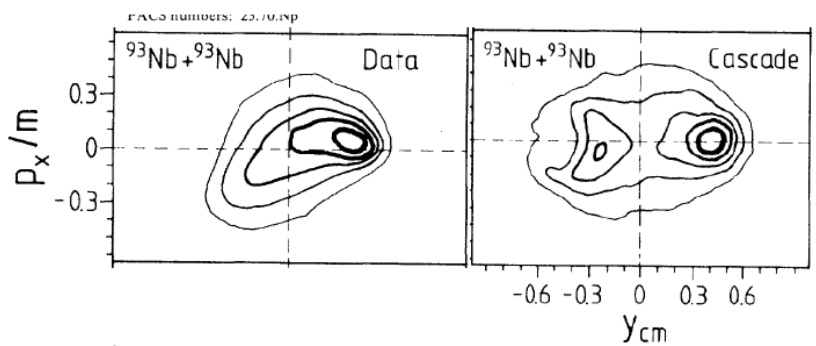
[from L.P. Cs.: Introduction to Heavy Ion Collisions (Wiley 1994-2004)
http://www.csernai.no/Csernai-textbook.pdf **]**

FD became applied later for A+A and p+A collisions
in p+p quantum effects dominate (HBT)
initial configuration is problematic
multiplicity is small

In contrast in high energy A+A collisions the applicability of FD is more realistic
Theoretically shock waves, directed flow (bounce off, side splash, …)
[W. Scheid, H. Mueller, W. Greiner; Phys.Rev.Lett. 32 (1974) 741]
[W. Scheid, W. Greiner; Z. Phys. 226 (1969) 364]
[G.F. Chapline, M.H. Johnson, E. Teller, and M.S. Weiss, Phys. Rev. D8 (1973) 135]

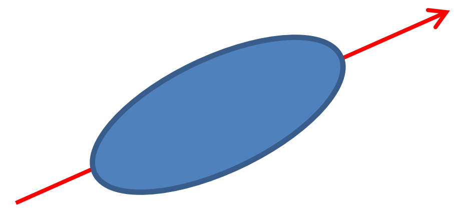


Experimantally:
[Gustafsson, H.A. et al., Phys. Rev. Lett. 52, 1590 (1984)]
[Danielewicz, P., Odyniecz, G., Phys. Lett. B 157, 146 (1985)]



Global Dynamics versus Fluctuations

Central Collisions (A+A)



- ❑ Global Symmetries
- ❑ One symmetry axis: z-axis – given by the beam direction
  - ❑ Azimuthal symmetry
  - ❑ Longitudinal, +/- z symmetry ➔ rapidity – even
  - ❑ Spherical or ellipsoidal flow, expansion
  - ❑ Global v_1, v_2, v_3, … v_n = 0 !!

- ❑ Fluctuations
- ❑ Perfect conditions for fluctuation studies
  - ❑ Azimuthal fluctuations - no interference - **perfect, odd & even harmonics**
  - ❑ Longitudinal fluctuations - global rapidity-even flow interference
    ➔ (slight) dominance for rapidity-even fluctuations
  - ❑ Best for critical fluctuation studies :

$$\frac{d^3N}{dy\,dp_t\,d\phi} = \frac{1}{2\pi}\frac{d^2N}{dy\,dp_t}\left[1 + 2v_1(y,p_t)\cos(\phi - \Psi_1^{EP}) + 2v_2(y,p_t)\cos(2(\phi - \Psi_2^{EP})) + \cdots\right]$$

Global Dynamics:

$$\frac{d^3N}{dy\,dp_t\,d\varphi} = A \sum_{n=1}^{\infty} v_n \cos(\varphi - \Psi_{RP})$$

and for odd n:

$$v_{2n+1}(-y) = -\,v_{2n+1}(y)$$

➔ We need: y_{cm} and Ψ_{RP} !

Fluctuations:

$$\frac{d^3N}{dy\,dp_t\,d\varphi} = A \sum_{n=1}^{\infty} v_n \cos(\varphi - \Psi_n)$$

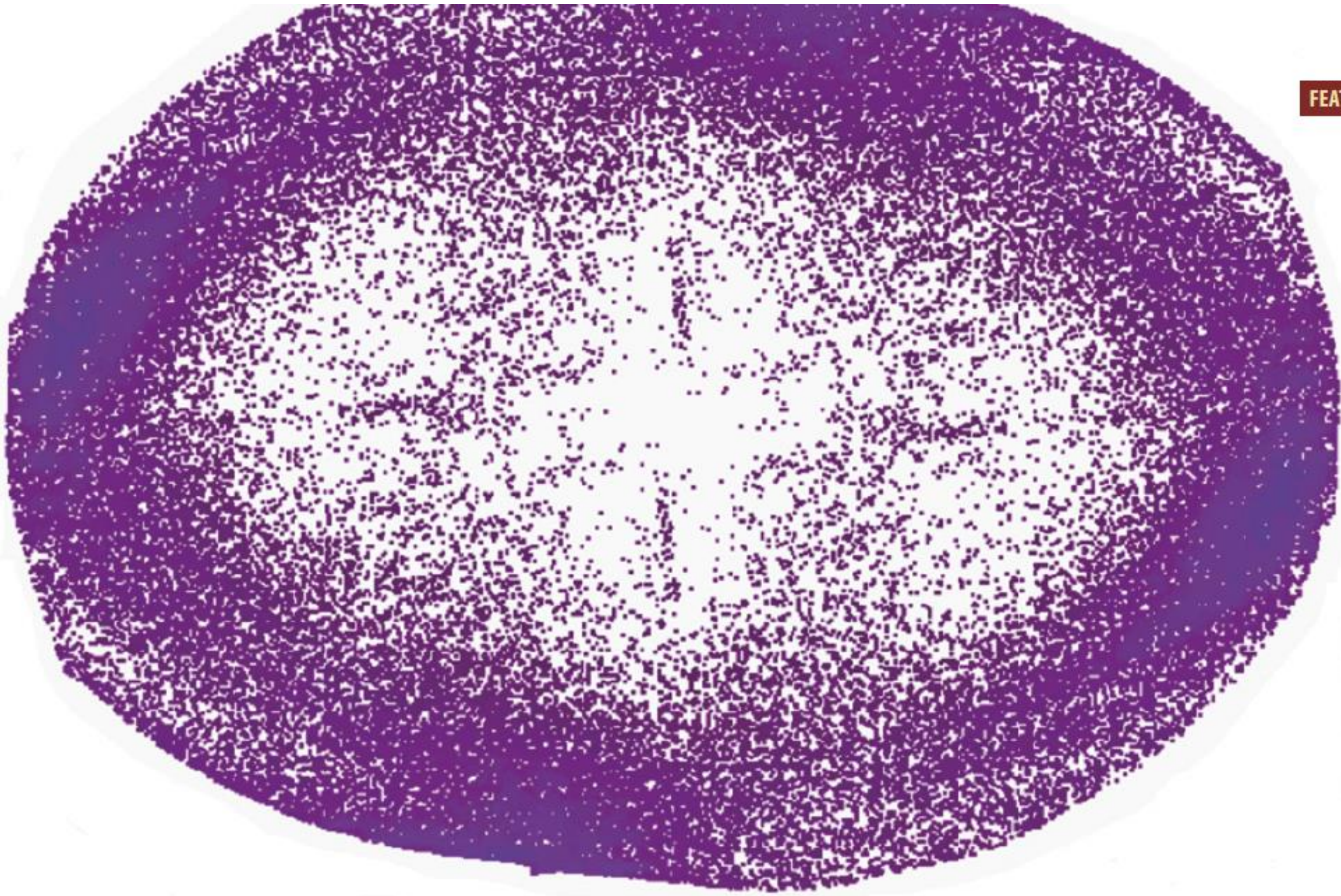
$$\frac{d^3N}{dy\,dp_t\,d\varphi} = A \sum_{n=1}^{\infty} v_n[\cos(\varphi)\cos(\Psi_n) + \sin(\varphi)\sin(\Psi_n)]$$

$$\frac{d^3N}{dy\,dp_t\,d\varphi} = A \sum_{n=1}^{\infty} v_n \cos(\Psi_n)\cos(\varphi) + v_n \sin(\Psi_n)\sin(\varphi)$$

$$\frac{d^3N}{dy\,dp_t\,d\varphi} = A \sum_{n=1}^{\infty} V_n \cos(\varphi) + U_n \sin(\varphi)$$

➔ We need: y_{cm} or integrate for y

FEATURES

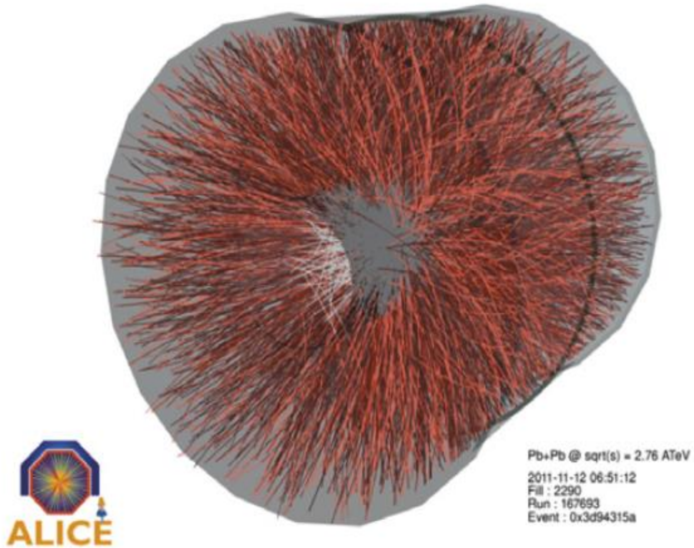


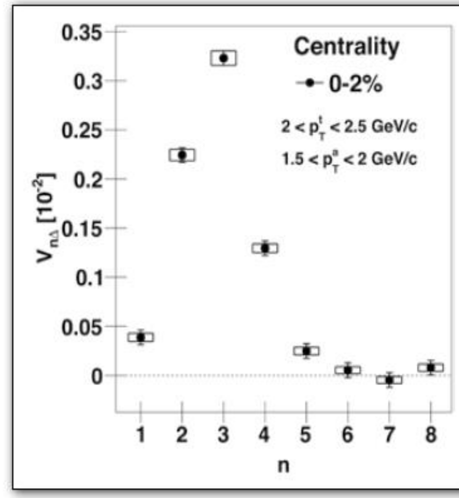
The Quark-Gluon Plasma, a nearly perfect fluid

L. Cifarelli [1], L.P. Csernai [2] and H. Stöcker [3] - DOI: 10.1051/epn/2012206

europhysicsnews
THE MAGAZINE OF THE EUROPEAN PHYSICAL SOCIETY

This is a direct proof of low viscosity !





▴ **FIG 3:** The final particles created in a lead on lead collision as reconstructed by the Time Projection Chamber of the ALICE detector. The chamber is filled in by the charged particle tracks rather evenly and densely in a near central collision. As flow fluctuation studies indicate the multipole moments up to 5 can be significantly identified. At higher energy and so higher charged particle multiplicity one can expect to see even higher multipole moments.

◂ **FIG 4:** Multipole moments of the azimuthal distribution of emitted particles in central lead on lead collisions detected by the Time Projection Chamber of the ALICE detector. From ref. [2b].

and according to present expectations it is around the low RHIC and the SPS energies. The present CERN studies could be well complemented by studying a system where the QGP is just created and critical fluctuations in dense hadronic of baryonic matter can be studied. Apart from the drop of collective directed flow due to the rapid softening of the matter at the critical point, there are many other observables, which open new ways of studies. The revolutionary fluctuation studies have an effect on the studies at the critical point also, with many new results coming from

References

[1] K. Aamodt *et al.*, (ALICE Collaboration), *Phys. Rev. Lett.* 105, 252302 (2010)

[2] K. Aamodt *et al.*, (ALICE Collaboration) arXiv:1105.3865v1 [nucl-ex], and CERN Courier, October 2011, p. 6

[3] P.K. Kovtun, D.T. Son and A.O. Starinets, *Phys. Rev. Lett.* 94, 111601 (2005)

[4] L.P. Csernai, J.I. Kapusta, L.D. McLerran, *Phys. Rev. Lett.* 97, 152303 (2006)

▾ **FIG 5:** Yields of anti-particle clusters in the mid rapidity region (|y|<0.5) of most central collisions of Pb+Pb/Au+Au as a function of the center-of-mass beam energy.

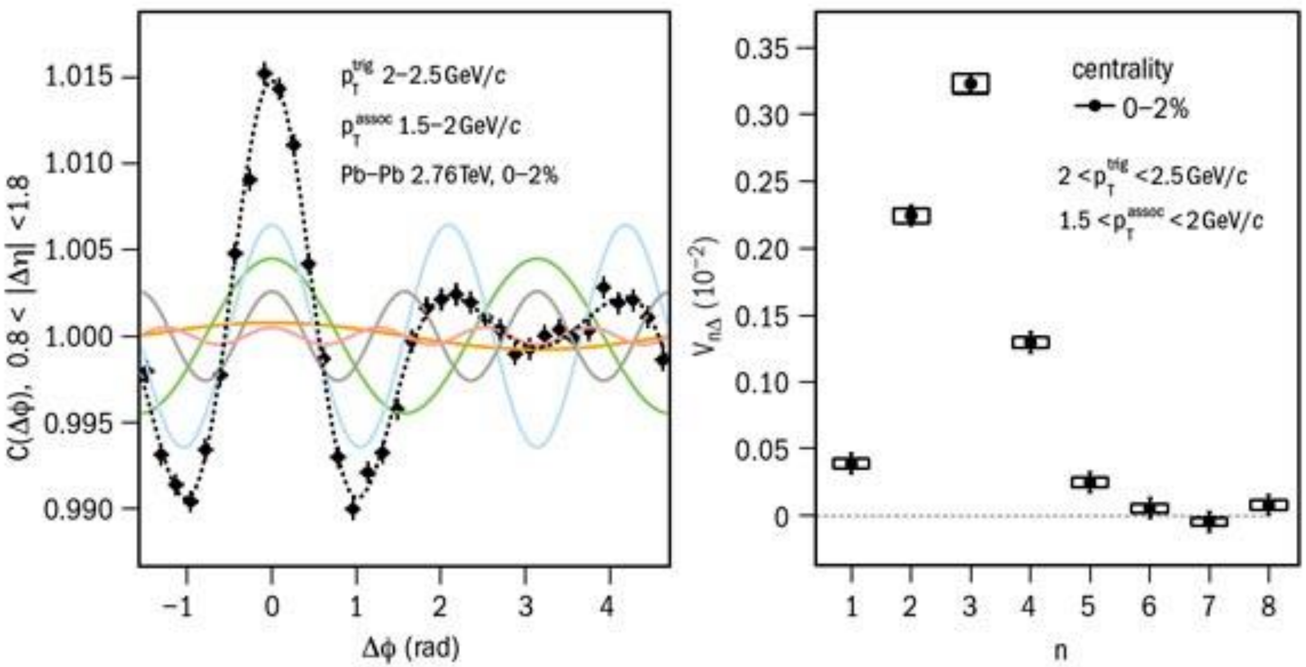
L.P. Csernai: Inroduction to Relativistic Heavy IonCollisions (Wiley, 1994)

CERN COURIER

Sep 23, 2011

Oct. 2011, p. 6

ALICE measures the shape of head-on lead-lead collisions

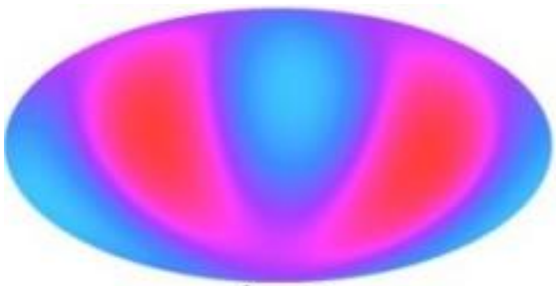


Flow originating from initial state fluctuations is significant and dominant in central and semi-central collisions (where from global symmetry no azimuthal asymmetry could occur) !

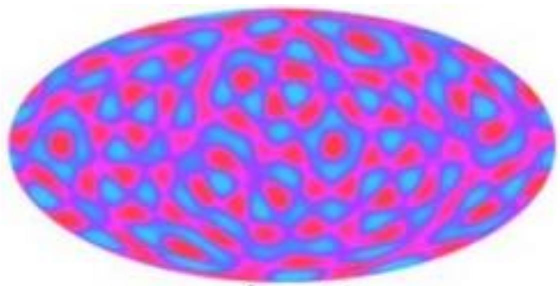
Low viscosity → Fluctuations

oil

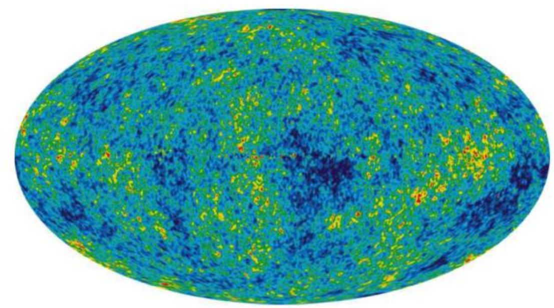
water

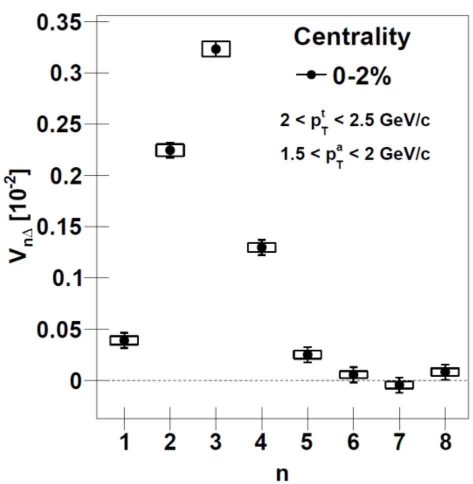


$\ell = 2$

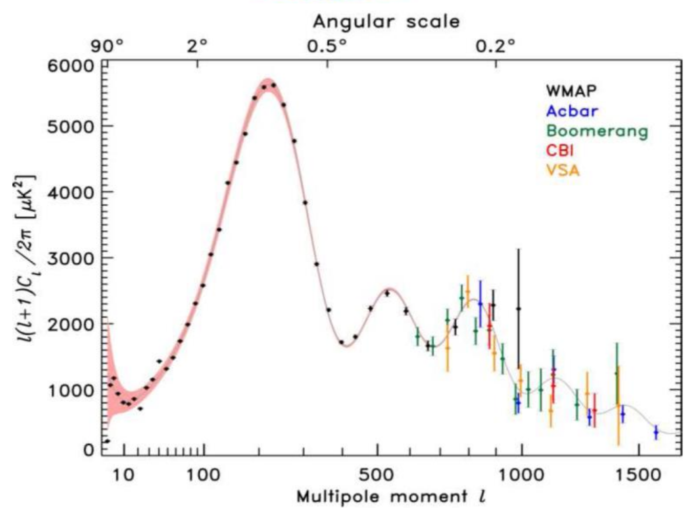


$\ell = 16$





Measurable azimuthal fluctuations up to n=8 are evidence for low viscosity



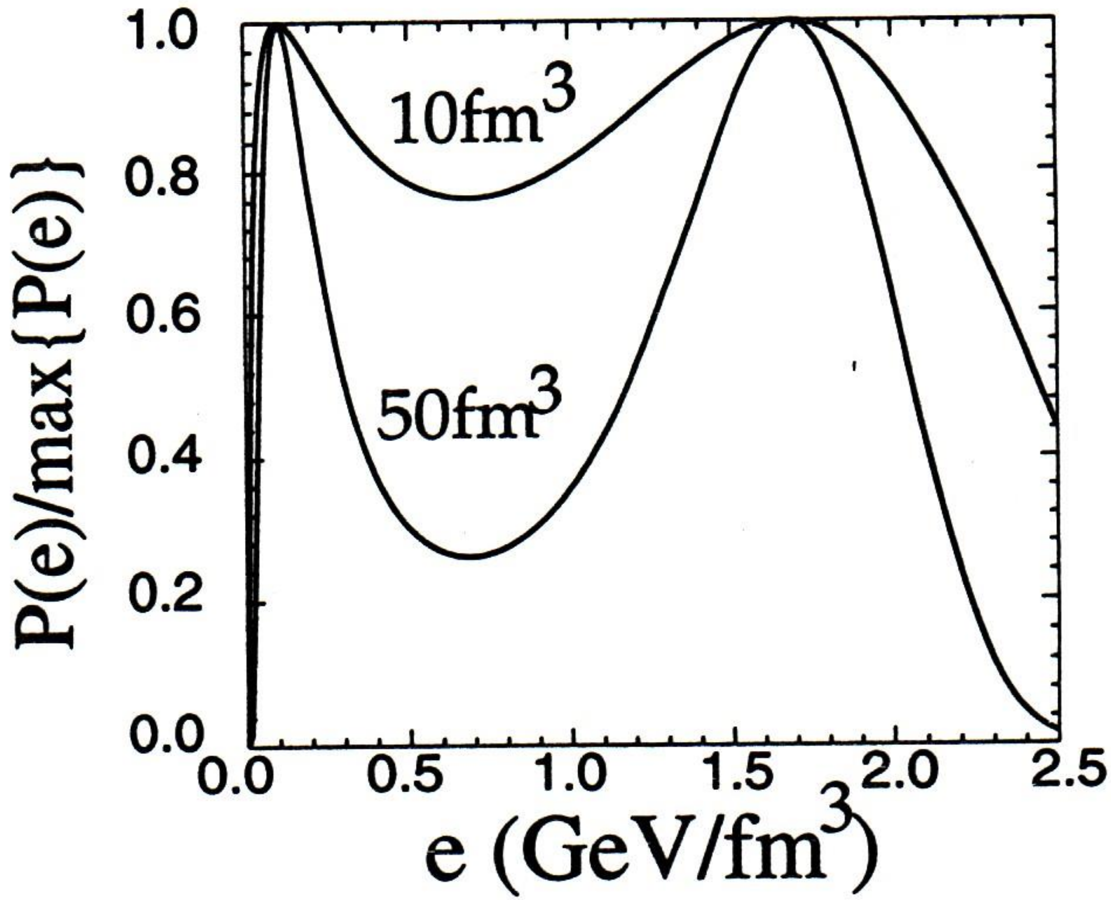


Fig. 2. *The relative probability of finding a state of a given energy density, e, in a system of given volume,* $\Omega = 10, 50\ fm^3$, *at a constant temperature,* $T = T_c$.

L.P. Csernai: Inroduction to Relativistic Heavy IonCollisions (Wiley, 1994)

Fluctuations in Hadronizing QGP

L.P. Csernai[1,2], G. Mocanu[3] and Z. Néda[3]

PHYSICAL REVIEW C **85**, 068201 (2012)

Higher order moments can be obtained from fluctuations around the critical point. → Skewness and Kurtosis are calculated for the QGP → HM phase transition

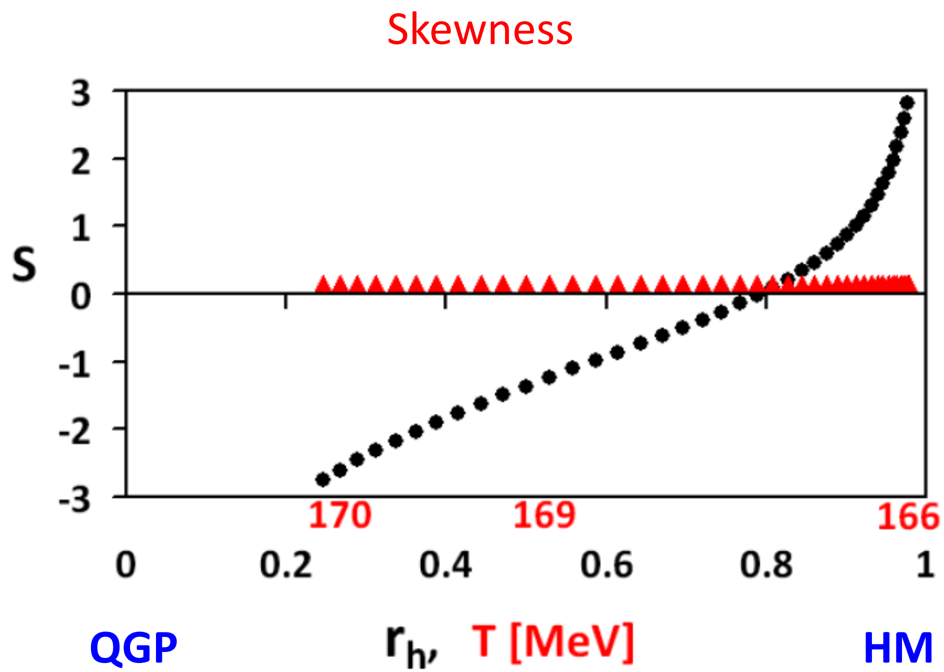
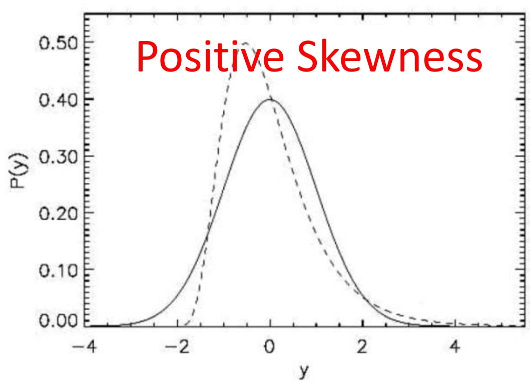
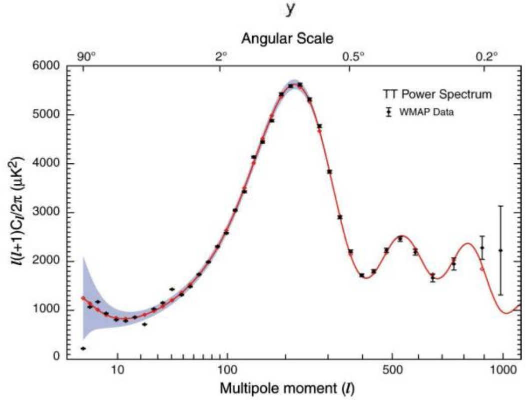


FIG. 4: (color online) Skewness as a function of the volume abundance of the hadronic matter (denoted as r_h, where 1 represents complete hadronization). The temperature scale is also indicated for clarity, the identifiers represent increments of 0.1 MeV in T. Results for $\Omega = 500 fm^3$.





Negative Skewness indicates Freeze-out mainly still on the QGP side.

Fluctuations form initial states

[1] Gardim FG, Grassi F, Hama Y, Luzum M, Ollitraut PHYSICAL REVIEW C **83**, 064901 (2011); (v_1 also)
[2] Qin GY, Petersen H, Bass SA, Mueller B PHYSICAL REVIEW C **82**, 064903 (2010)

QIN, PETERSEN, BASS, AND MÜLLER

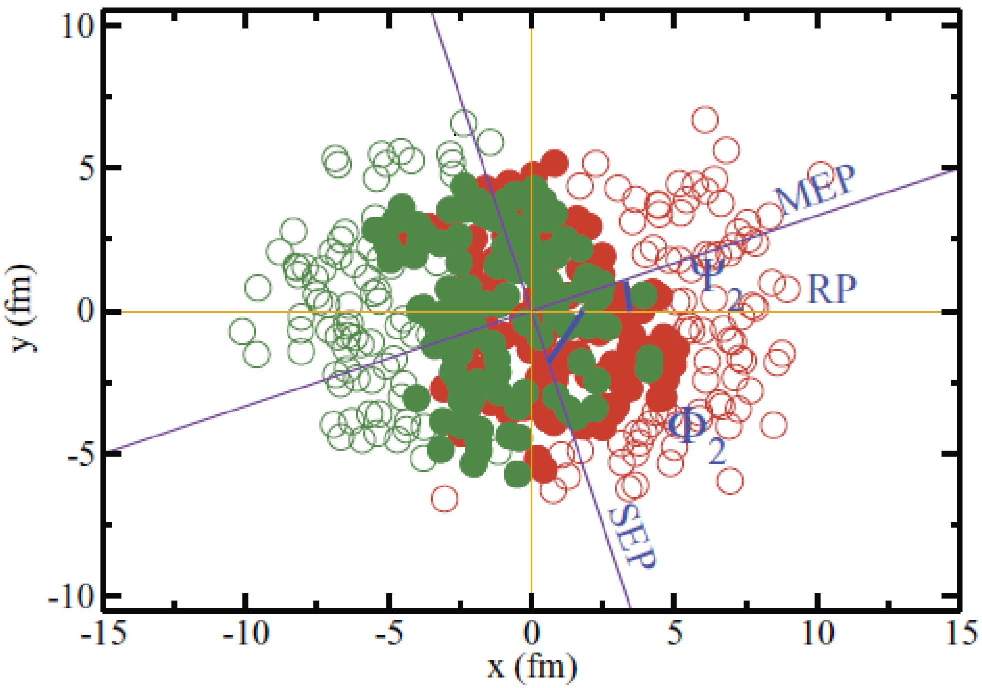


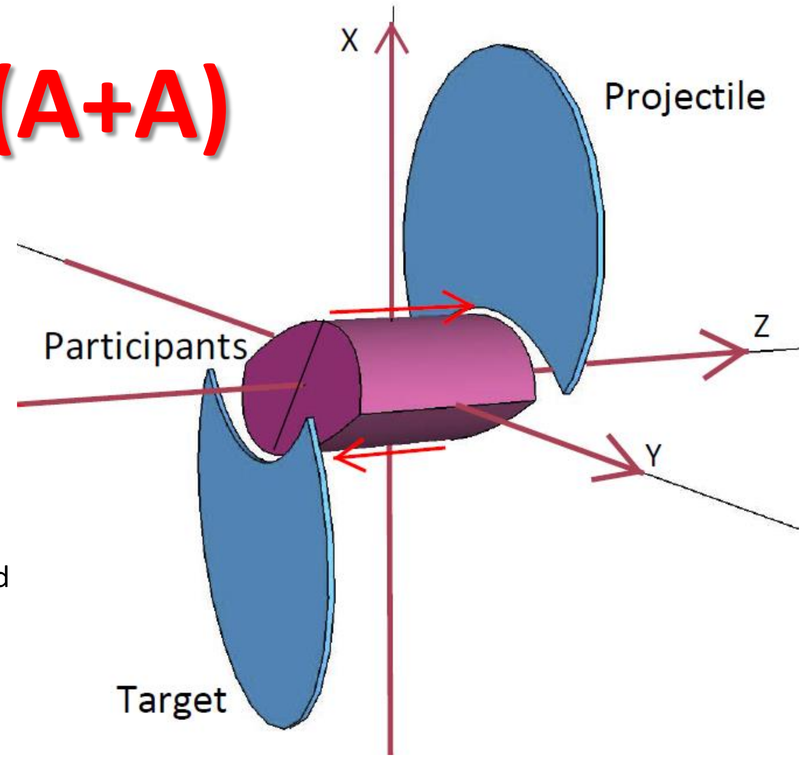
FIG. 3. (Color online) The transverse plane for one typical collision event, where the circles represent nucleons from two nuclei, with shaded ones for participating nucleons. Also shown are the locations of different planes: the reaction plane (RP), the spatial event plane (SEP), and the momentum event plane (MEP) for $n = 2$.

Cumulative event planes show weak correlation with the global collective reaction plane (RP).

If the MEP is set to zero (by definition) then CM rapidity fluctuations do not appear, and v_1 by definition is zero.

In [2] v1(pt) is analyzed (for RHIC) and the effect is dominated by fluctuations. (Similar to later LHC measurements.)

Peripheral Collisions (A+A)



- Global Symmetries
- Symmetry axes in the global CM-frame:
  - (y ←→ -y)
  - (x,z ←→ -x,-z)
  - Azimuthal symmetry: ϕ-even (cos nϕ)
  - Longitudinal z-odd, (rap.-odd) for v_odd
  - Spherical or ellipsoidal flow, expansion

$$\frac{d^3N}{dydp_td\phi} = \frac{1}{2\pi}\frac{d^2N}{dydp_t}\left[1 + 2v_1(y,p_t)\cos(\phi) + 2v_2(y,p_t)\cos(2\phi) + \cdots\right]$$

$$\frac{d^3N}{dydp_td\phi} = \frac{1}{2\pi}\frac{d^2N}{dydp_t}\left[1 + 2v_1(y-y_{CM},p_t)\cos(\phi-\Psi_{RP}) + 2v_2(y-y_{CM},p_t)\cos(2(\phi-\Psi_{RP})) + \cdots\right]$$

- Fluctuations
- Global flow and Fluctuations are simultaneously present → ∃ interference
  - Azimuth - Global: even harmonics - Fluctuations : odd & even harmonics
  - Longitudinal – Global: v1, v3 y-odd - Fluctuations : odd & even harmonics
  - The separation of Global & Fluctuating flow is a must !! (not done yet)

Analysis of Global Flow:

$$\frac{d^3N}{dy dp_t d\phi} = \frac{1}{2\pi}\frac{d^2N}{dy dp_t}\left[1 + 2v_1(y,p_t)\cos(\phi) + 2v_2(y,p_t)\cos(2\phi) + \cdots\right]$$

where $\phi = 0$ is the reaction plane azimuth angle Ψ_{RP}

$$\frac{d^3N}{dy dp_t d\phi} = \frac{1}{2\pi}\frac{d^2N}{dy dp_t}\left[1 + 2v_1(y,p_t)\cos(\phi - \Psi_{RP}) + 2v_2(y,p_t)\cos(2(\phi - \Psi_{RP})) + \cdots\right]$$

$$\frac{d^3N}{dy dp_t d\phi} = \frac{1}{2\pi}\frac{d^2N}{dy dp_t}\left[1 + 2v_1(y - y_{CM},p_t)\cos(\phi - \Psi_{RP}) + 2v_2(y - y_{CM},p_t)\cos(2(\phi - \Psi_{RP})) + \cdots\right]$$

Ψ_{RP} and y_{CM} can be determined experimentally

L. P. Csernai,[1] G. Eyyubova,[2,3] and V. K. Magas[4]
PHYSICAL REVIEW C 86, 024912 (2012)

$$\frac{d^3N}{dy dp_t d\phi} = \frac{1}{2\pi}\frac{d^2N}{dy dp_t}\left[1 + 2v_1(y,p_t)\cos(\phi - \Psi_1^{EP}) + 2v_2(y,p_t)\cos(2(\phi - \Psi_2^{EP})) + \cdots\right]$$

and Fluctuating Flow:

Ψ_n^{EP} maximizes $v_2(y,p_t)$ or $v_2(p_t)$ or v_2

$\Psi_n^{EP}\{2\}$ or $\Psi_n^{EP}\{4\}$ ...

$$v_n \cos[n(\phi - \Psi_n)] = (v_n \cos(n\Psi_n)) \cdot \cos(n\phi) + (v_n \sin(n\Psi_n)) \cdot \sin(n\phi)$$

ortho-normal series expansion for both ϕ-even and ϕ-odd functions

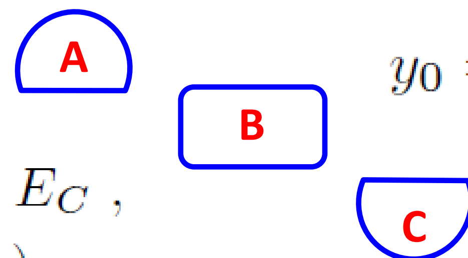
Method to compensate for C.M. rapidity fluctuations

1. Determining experimentally EbE the C.M. rapidity
2. Shifting each event to its own C.M. and evaluate flow-harmonics there

L.P. Csernai[1,2], G. Eyyubova[3] and V.K. Magas[4] arXiv:1204.5885v1 [hep-ph]

Determining the C.M. rapidity

The rapidity acceptance of a central TPC is usually constrained (e.g for ALICE $|\eta| < \eta_{lim} = 0.8$, and so: $|\eta_{C.M.}| << \eta_{lim}$, so it is not adequate for determining the C.M. rapidity of participants.



$$y_0 = 7.986$$

Participant rapidity from spectators

$$E_B = A_B\ m_{B\perp}\ \cosh(y^B) = E_{tot} - E_A - E_C\ ,$$
$$M_B = A_B\ m_{B\perp}\ \sinh(y^B) = -(M_A + M_C)$$

$$E_{tot} = 2A_{Pb}\, m_N\ \cosh(y_0)$$

$$E_A = A_P\, m_N\ \cosh(y_0),$$
$$E_C = A_T\, m_N\ \cosh(-y_0),$$

give the spectator numbers, A_P and A_T,

$$M_A = A_P\, m_N\ \sinh(y_0),$$
$$M_C = A_T\, m_N\ \sinh(-y_0),$$

$$y_E^{CM} \approx y^B = \text{artanh}\left(\frac{M_A + M_C}{E_{tot} - E_A - E_C}\right)$$

Interplay between global collective flow and fluctuating flow

PHYSICAL REVIEW C **86**, 014907 (2012)

ATLAS

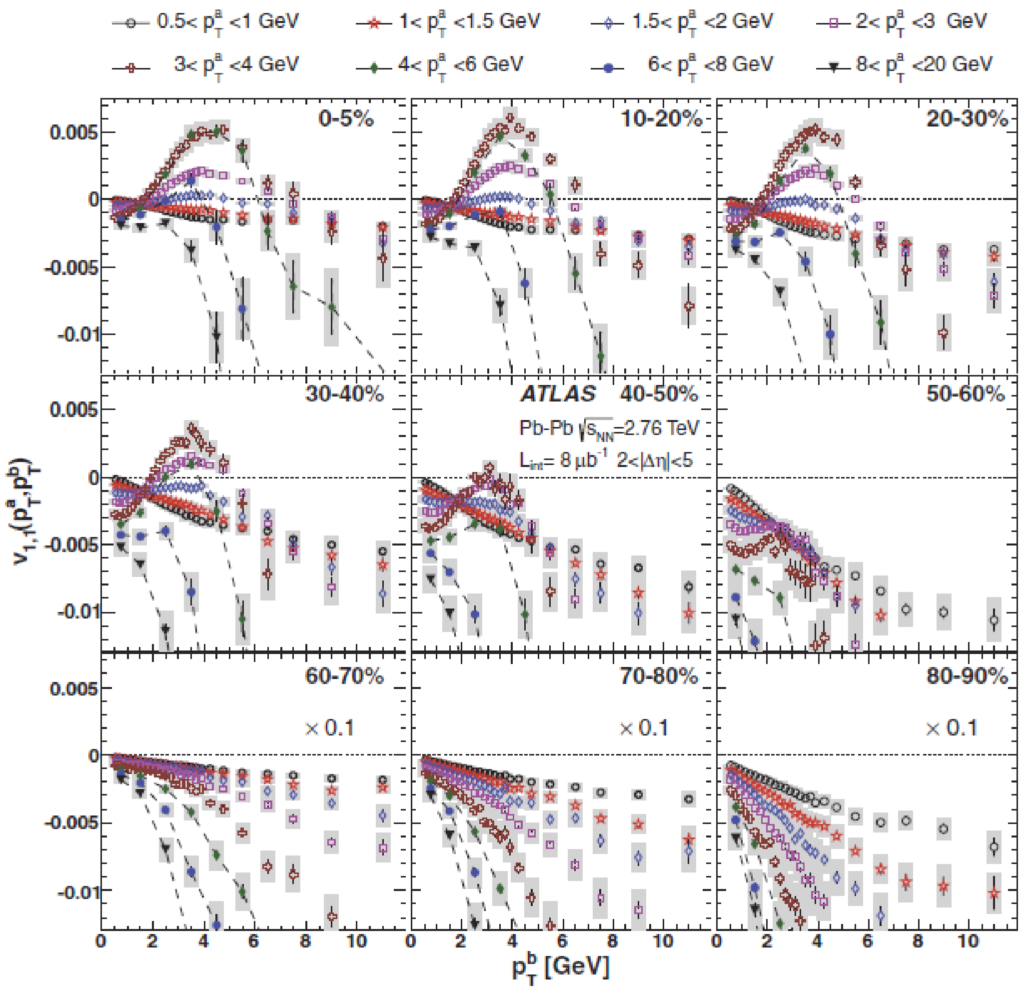
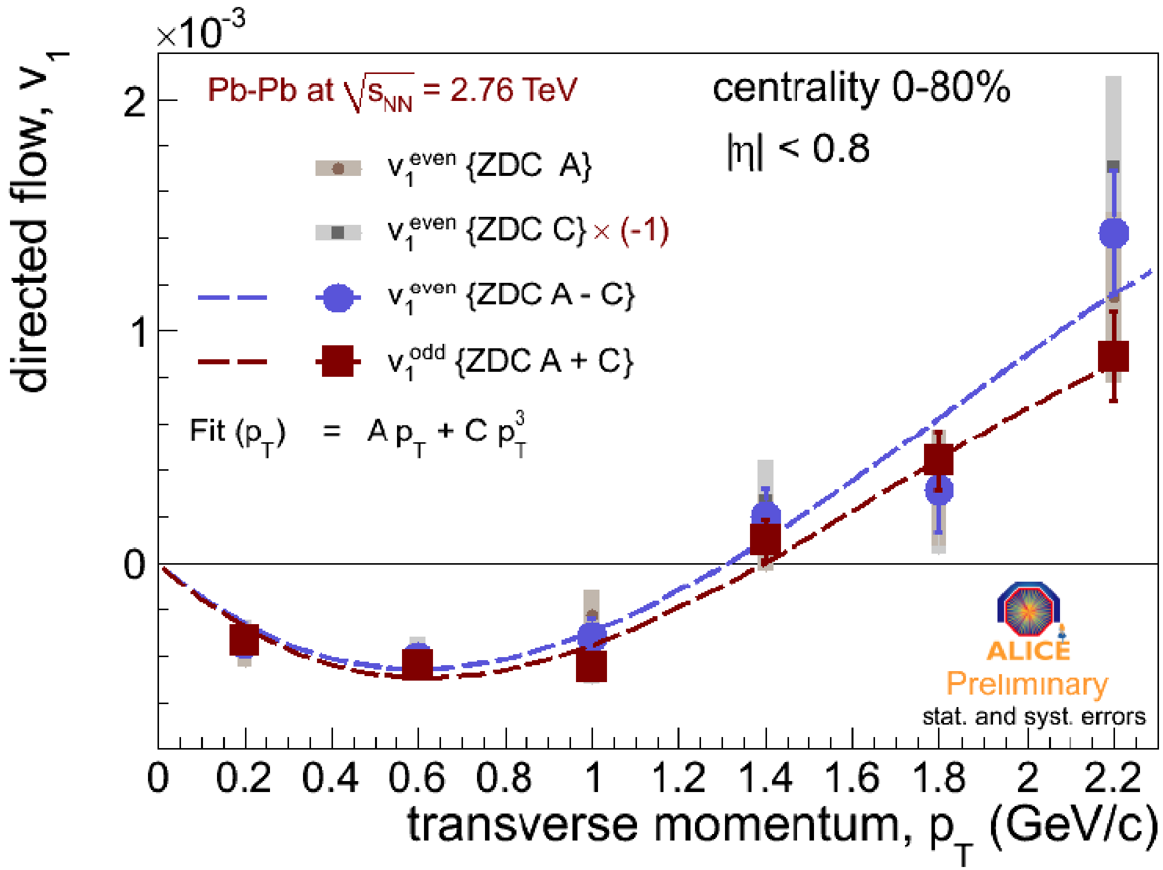


FIG. 19. (Color online) $v_{1,1}(p_T^a, p_T^b)$ for $2 < |\Delta\eta| < 5$ vs p_T^b for different p_T^a ranges. Each panel presents results in one centrality interval. The error bars and shaded bands represent statistical and systematic uncertainties, respectively. The data points for the three highest p_T^a intervals have coarser binning in p_T^b, hence are connected by dashed lines to guide the eye. The data in the bottom three panels are scaled down by a factor of ten to fit within the same vertical range.

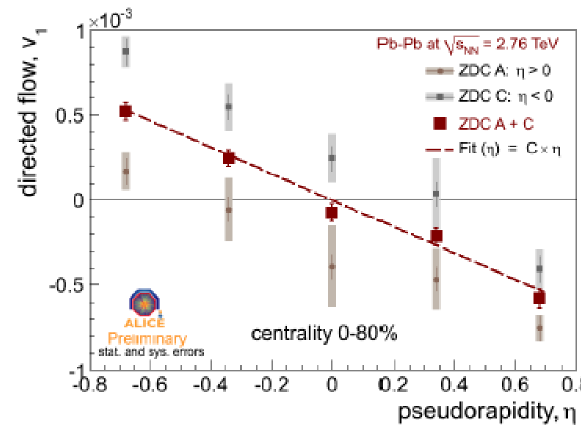
Global Flow in Peripheral Collisions (A+A)

- Many interesting phenomena:
  - Historically: Bounce off / Side splash; Squeeze out → pressure & EoS
  - 3^{rd} flow or Anti-flow (QGP), Rotation, KHI, Polarization, etc
  - These occur only if viscosity is low! → viscosity
  - With increasing energy flow becomes strongly F/B directed & v_1 decreases

Extracting even and odd parts of v_1



what we expect for even and odd projections:



V_1 from Global Collective flow ➔ $v_1(p_t) = 0$!!!

Even and odd parts of v_1 have similar shape and magnitude

Initial States

Collective flow

- There are alternative origins:
- (a) Global collective flow (RP from spectators)
- (b) Asymmetries from random <u>I.S.</u> fluctuations
- (c) Asymmetries from <u>Critical Point</u> fluctuations

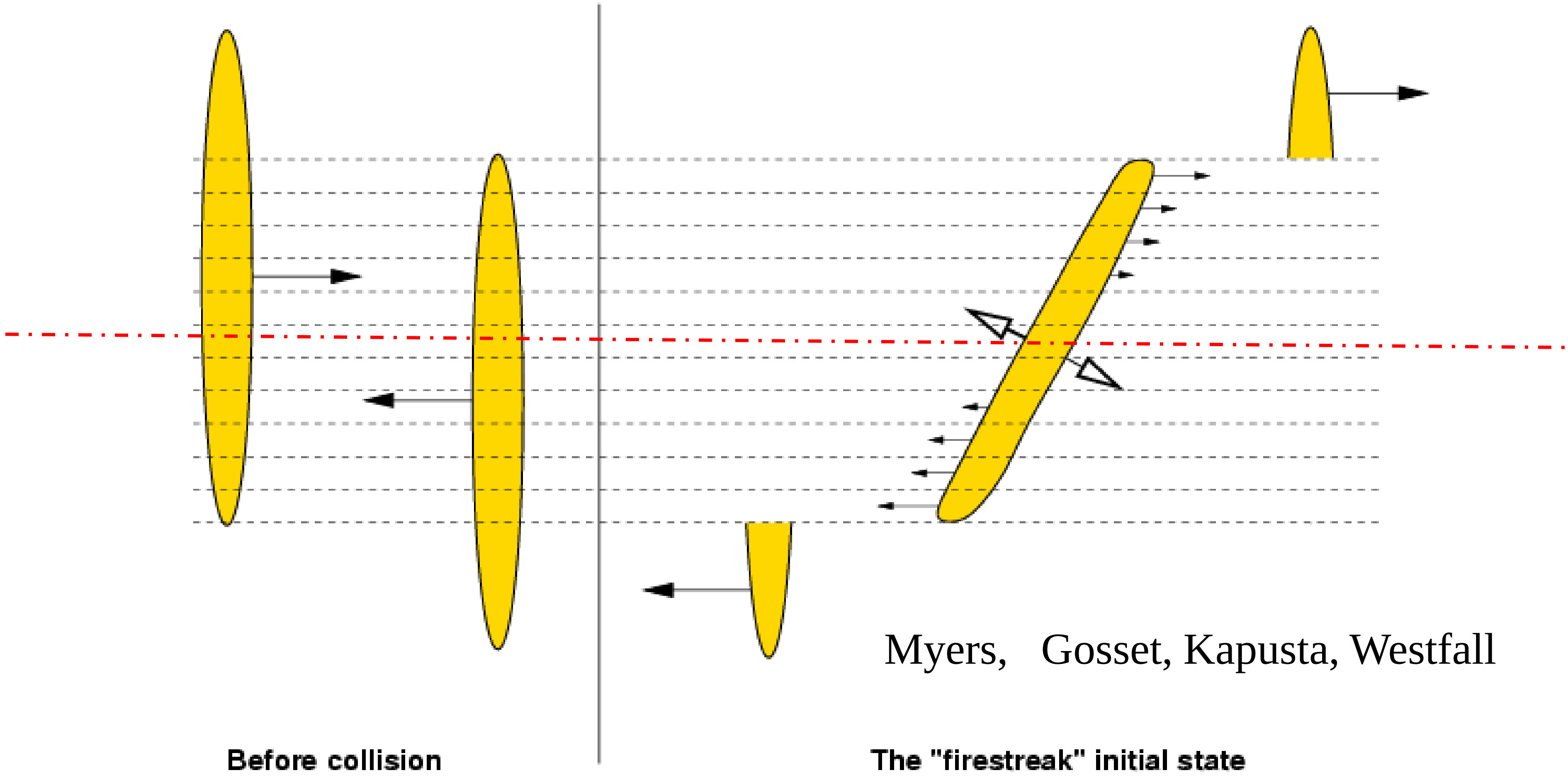
- **Goal is to separate the these**

  ➔ This provides more insight

- How can we see the flow of QGP?

  ➔ Rapid hadronization and freeze-out

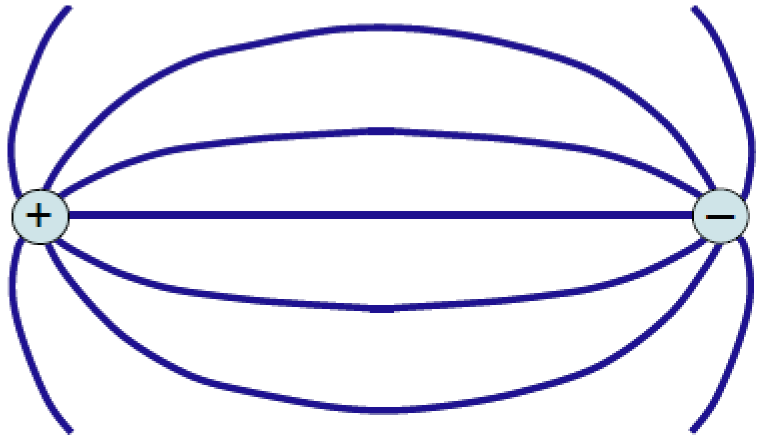
„Fire streak" picture – 3 dim.



Symmetry axis = z-axis. Transverse plane divided into streaks.

Flux – tubes

ED or QED:

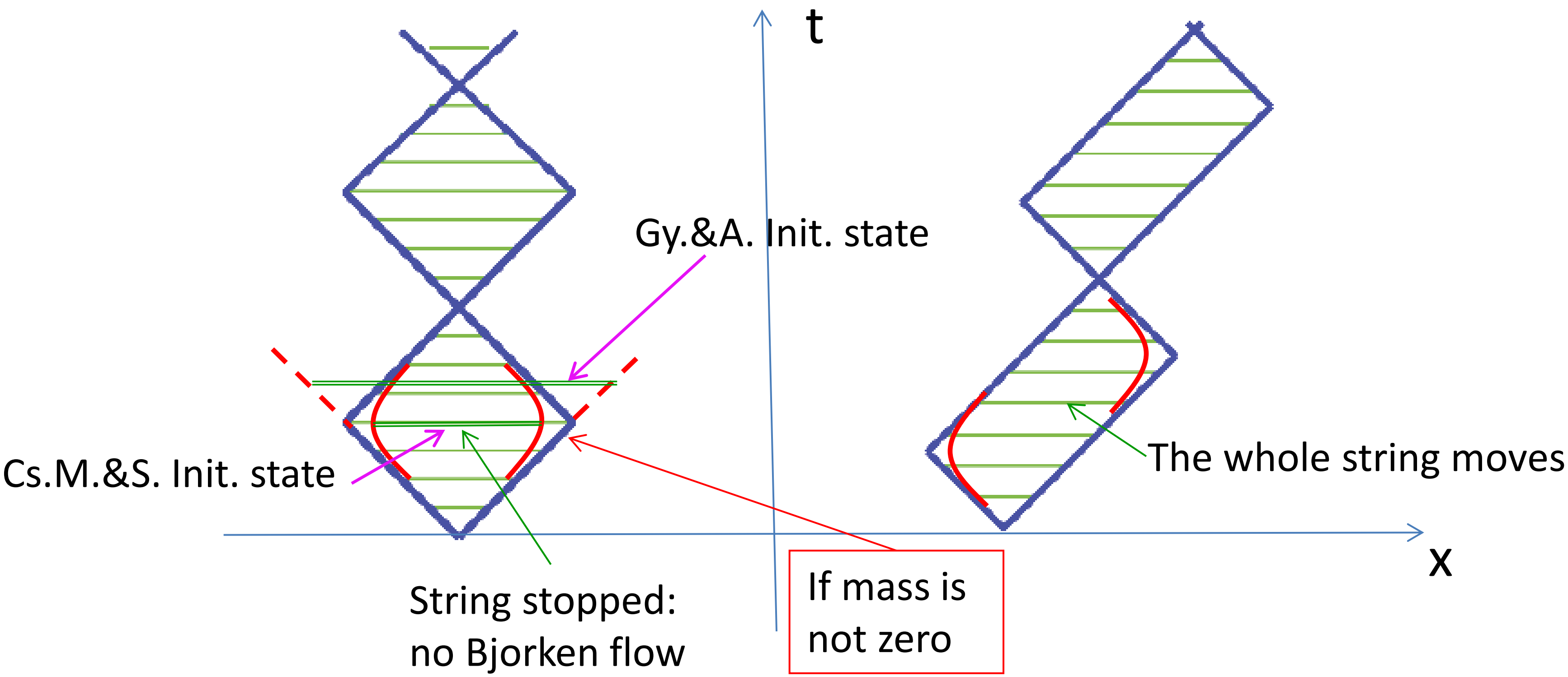


Gluon self-interaction makes field lines attract each other. →
QCD:

→ linear potential → confinement

String model of mesons / PYTHIA

Light quarks connected by string → mesons have '**yo-yo**' modes:



[T. Sjostrand & H.U. Bengtsson, 1984-1987]
PYTHIA

Nuclear Physics A460 (1986) 723–754
North-Holland, Amsterdam

BARYON RECOIL AND THE FRAGMENTATION REGIONS IN ULTRA-RELATIVISTIC NUCLEAR COLLISIONS*

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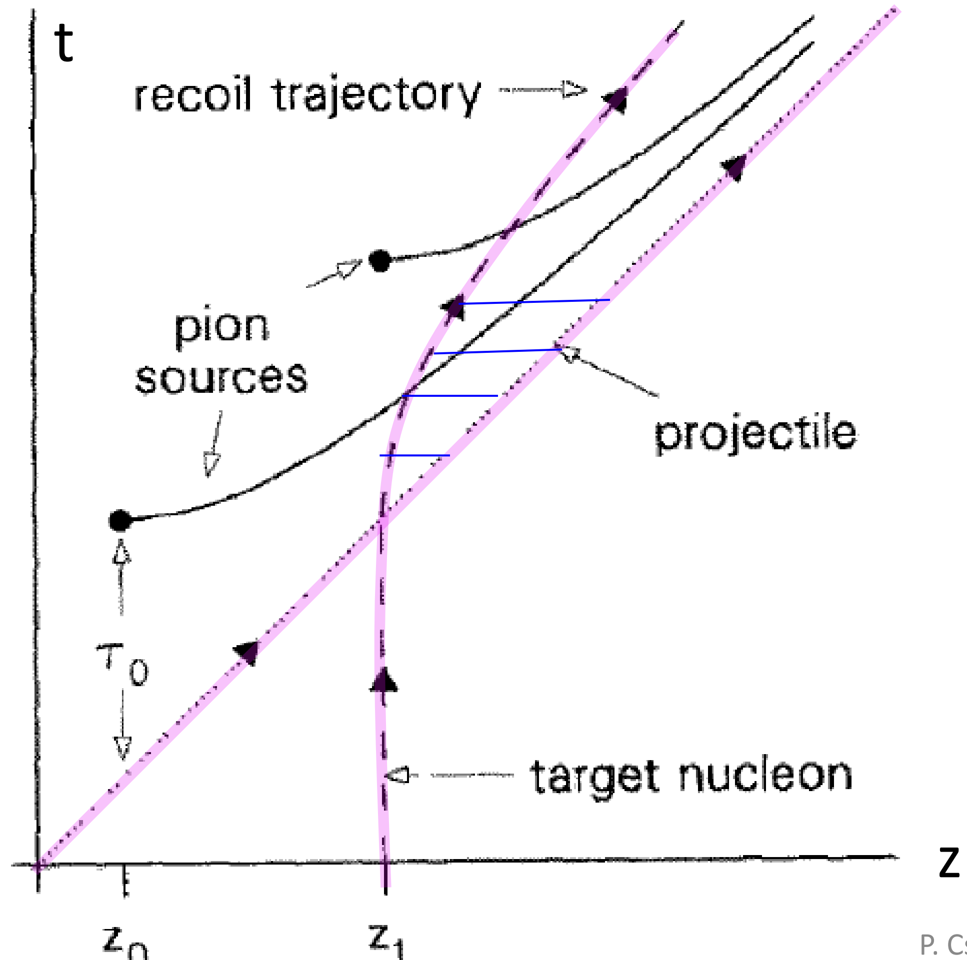
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA

Received 11 June 1986

Yo-yo in the fixed target frame → target recoil → density and energy density increase in the "fragmentation region"



Initial stage: Coherent Yang-Mills model

[Magas, Csernai, Strottman, Pys. Rev. C ‘2001]

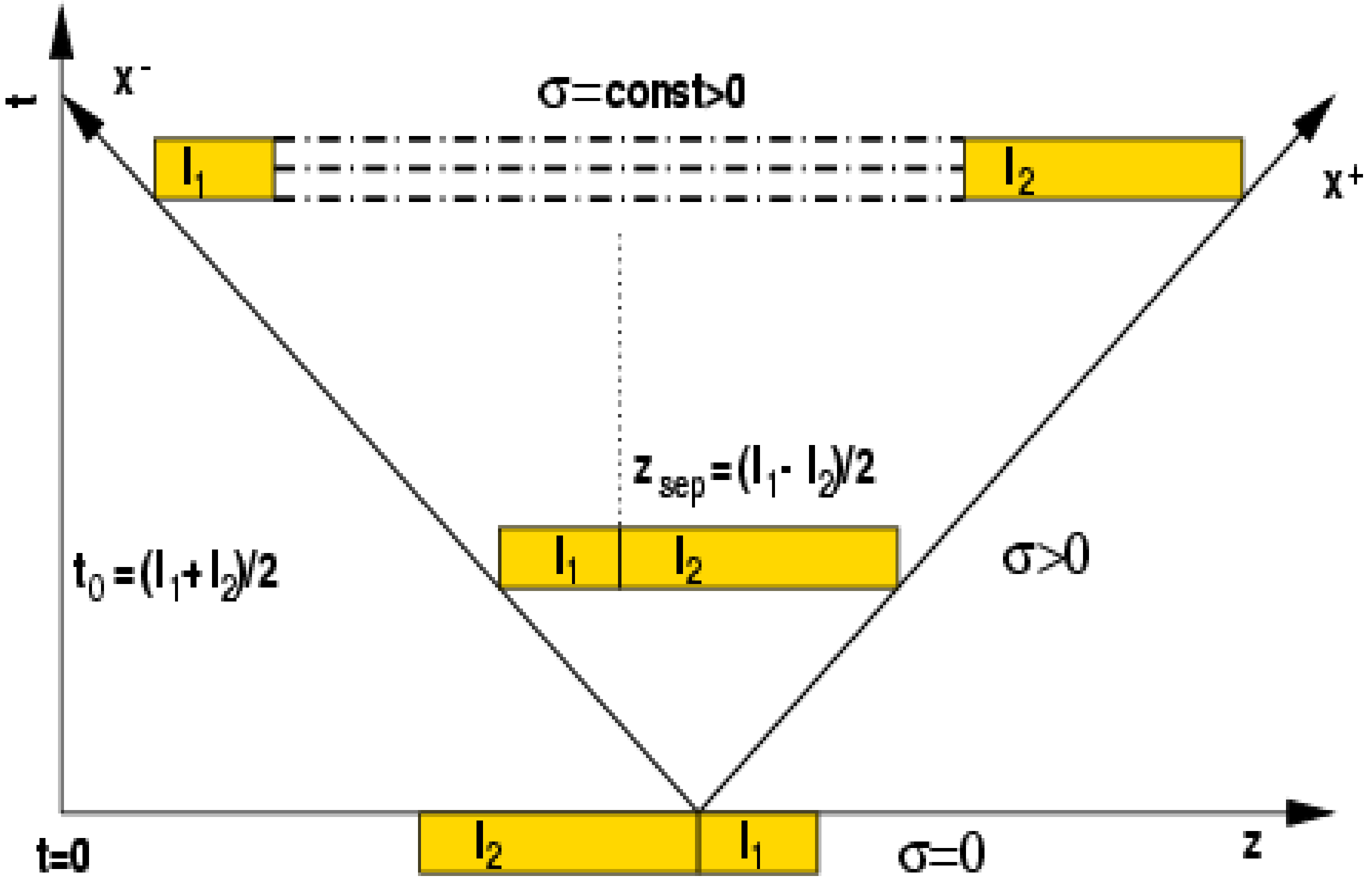
M. Gyulassy, L. Csernai Nucl. Phys. A660 (1986) 723-754.

$$\partial_\mu T^{\mu\nu} = F^{\nu\mu} n_\mu + \Sigma_\pi^\nu$$
$$\partial_\mu n^\mu = 0$$

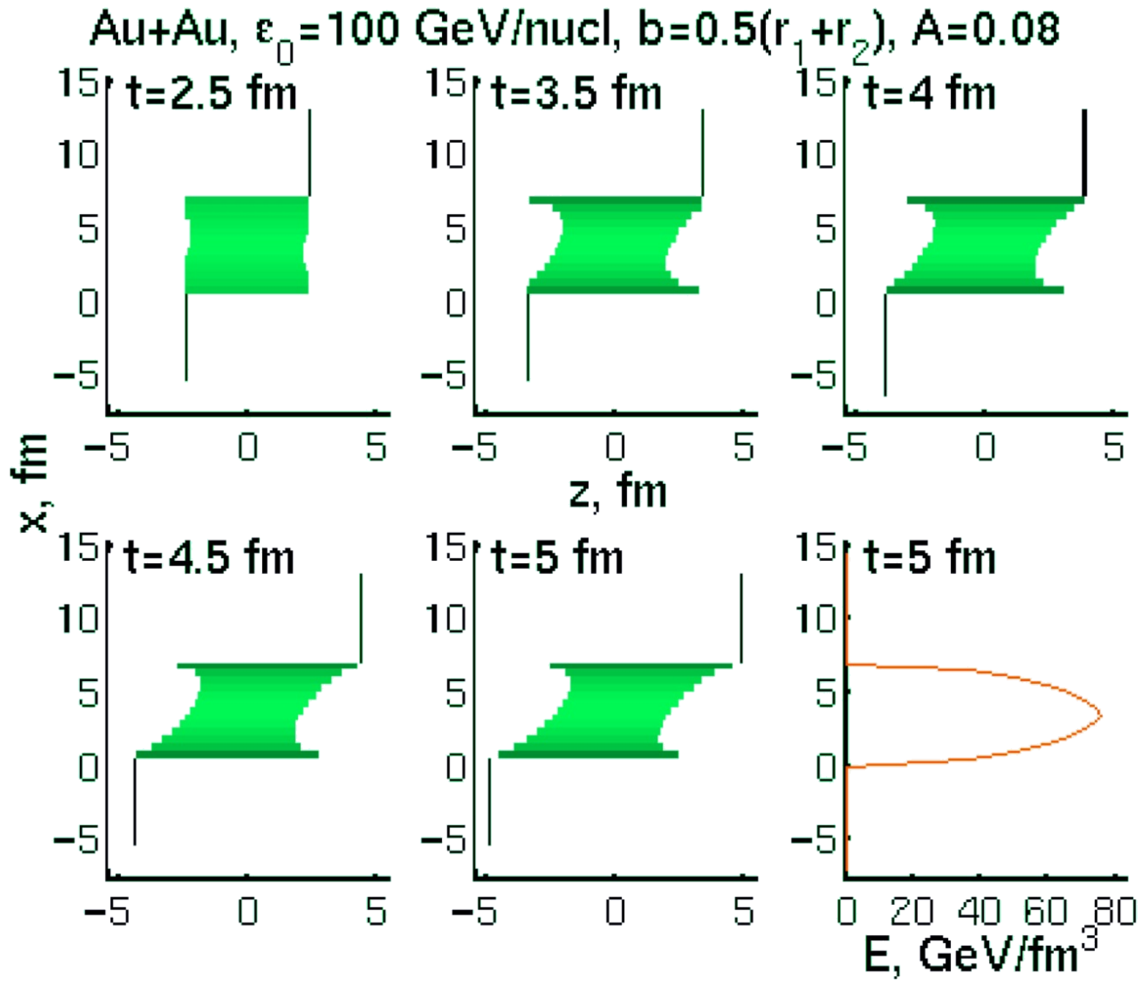
- $T^{\mu\nu} = e_t \left(\left(1 + c_0^2\right) u_t^\mu u_t^\nu - c_0^2 g^{\mu\nu} \right)$
- Σ_π^ν – pion source term.
- $F^{\mu\nu}$ – effective field, describes interaction between target and projectile.

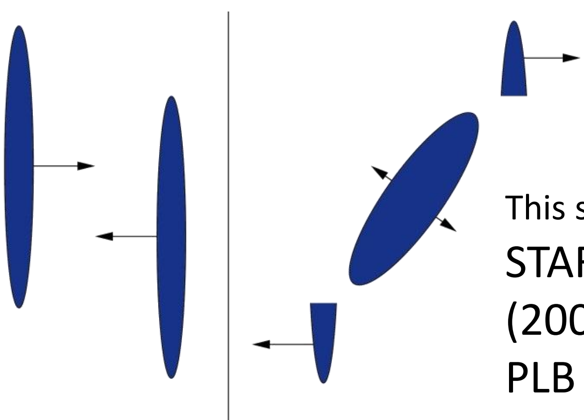
$$F^{\mu\nu} = \begin{pmatrix} 0 & -\sigma \\ \sigma & 0 \end{pmatrix} \quad ,$$

String rope --- Flux tube --- Coherent YM field



Initial State





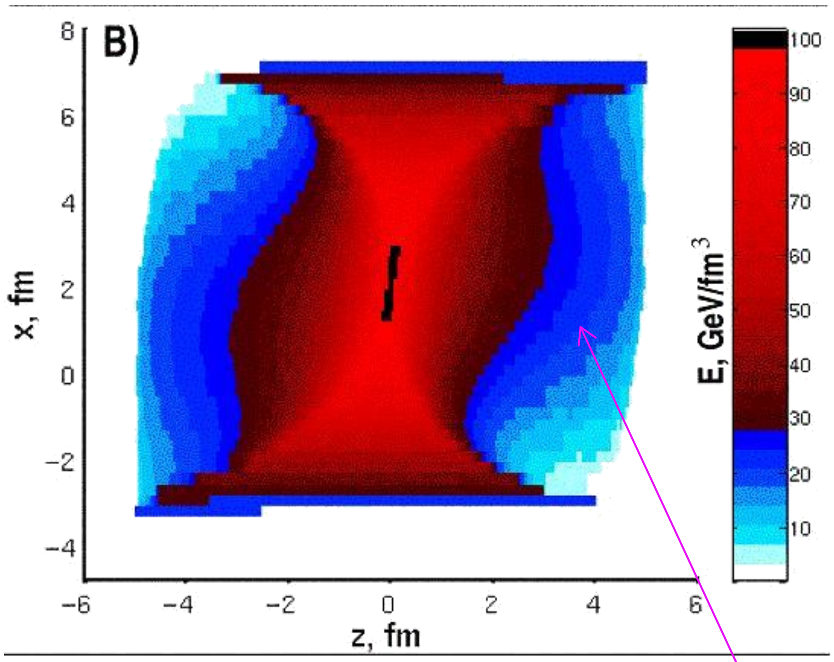
Before collision

Tilted initial Landau's disk

This shape is confirmed by STAR HBT: PLB496 (2000) 1; & M.Lisa &al. PLB 489 (2000) 287.

3rd flow component

Initial state – reaching equilibrium

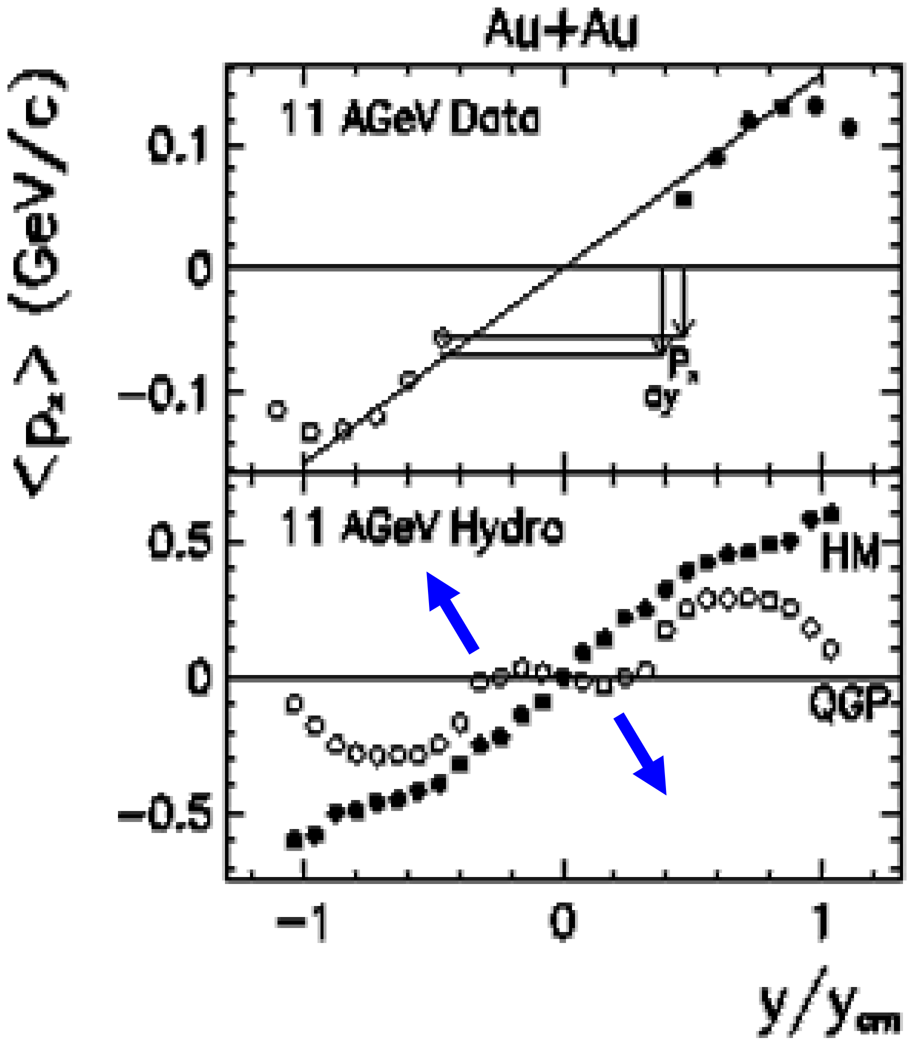


Initial state by V. Magas, L.P. Csernai and D. Strottman
Phys. Rev. C64 (01) 014901

Relativistic, 1D Riemann expansion is added to each stopped streak

M1

3rd flow component

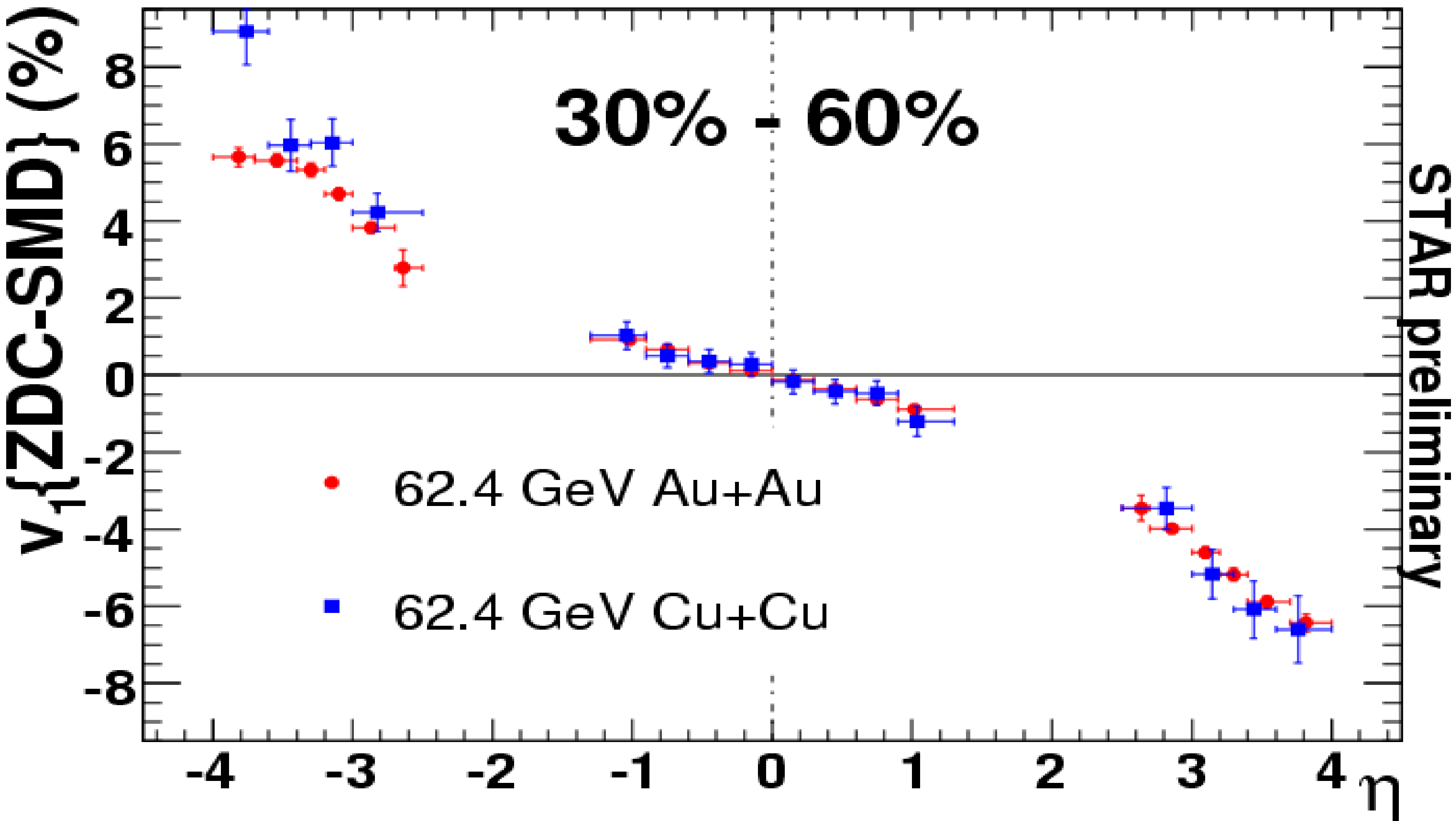


Hydro

[Csernai, HIPAGS'93] &
[Csernai, Röhrich, 1999]

G. Wang / STAR
QM 2006 :

$v_1(\eta)$: system-size dependence

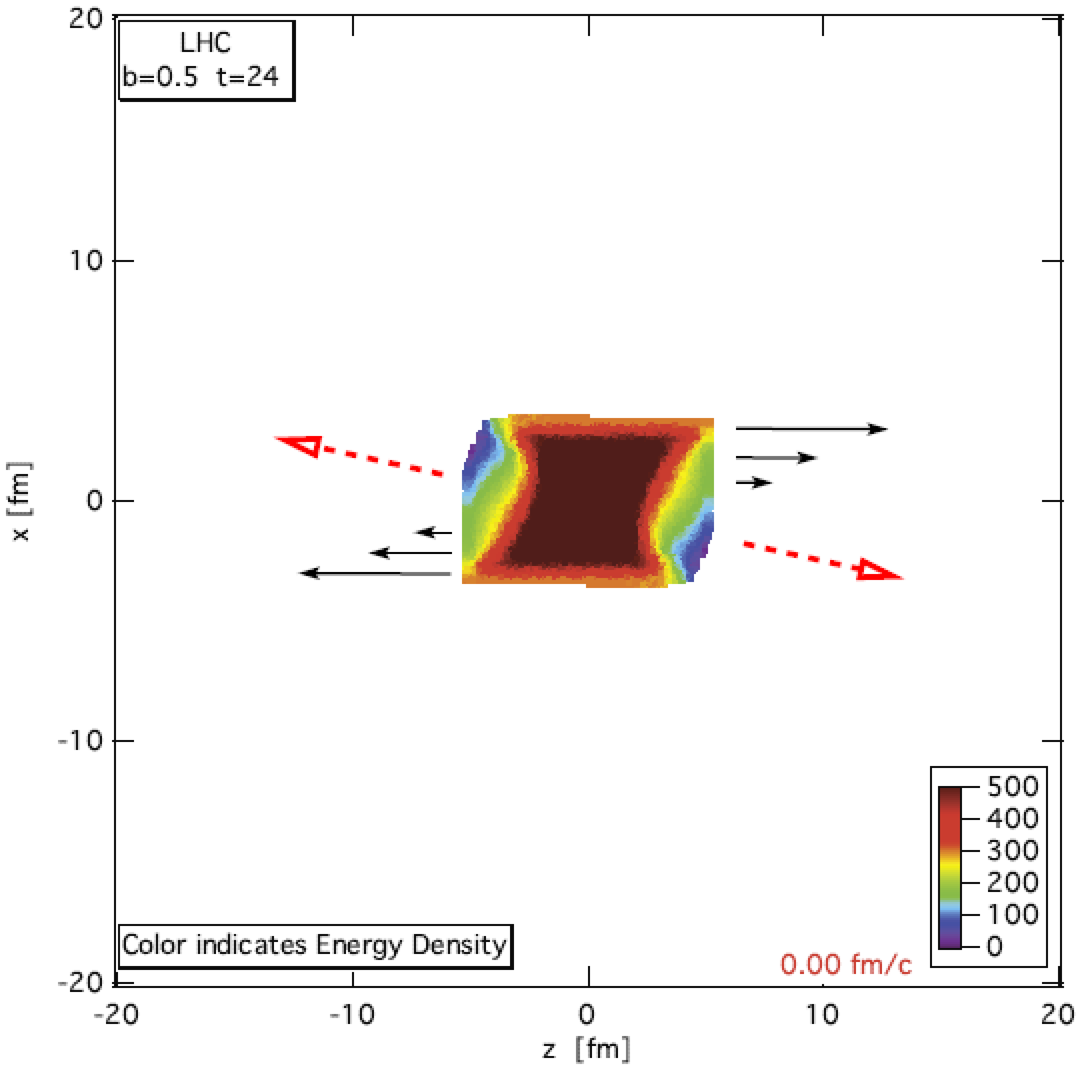


System size doesn't seem to influence $v_1(\eta)$.

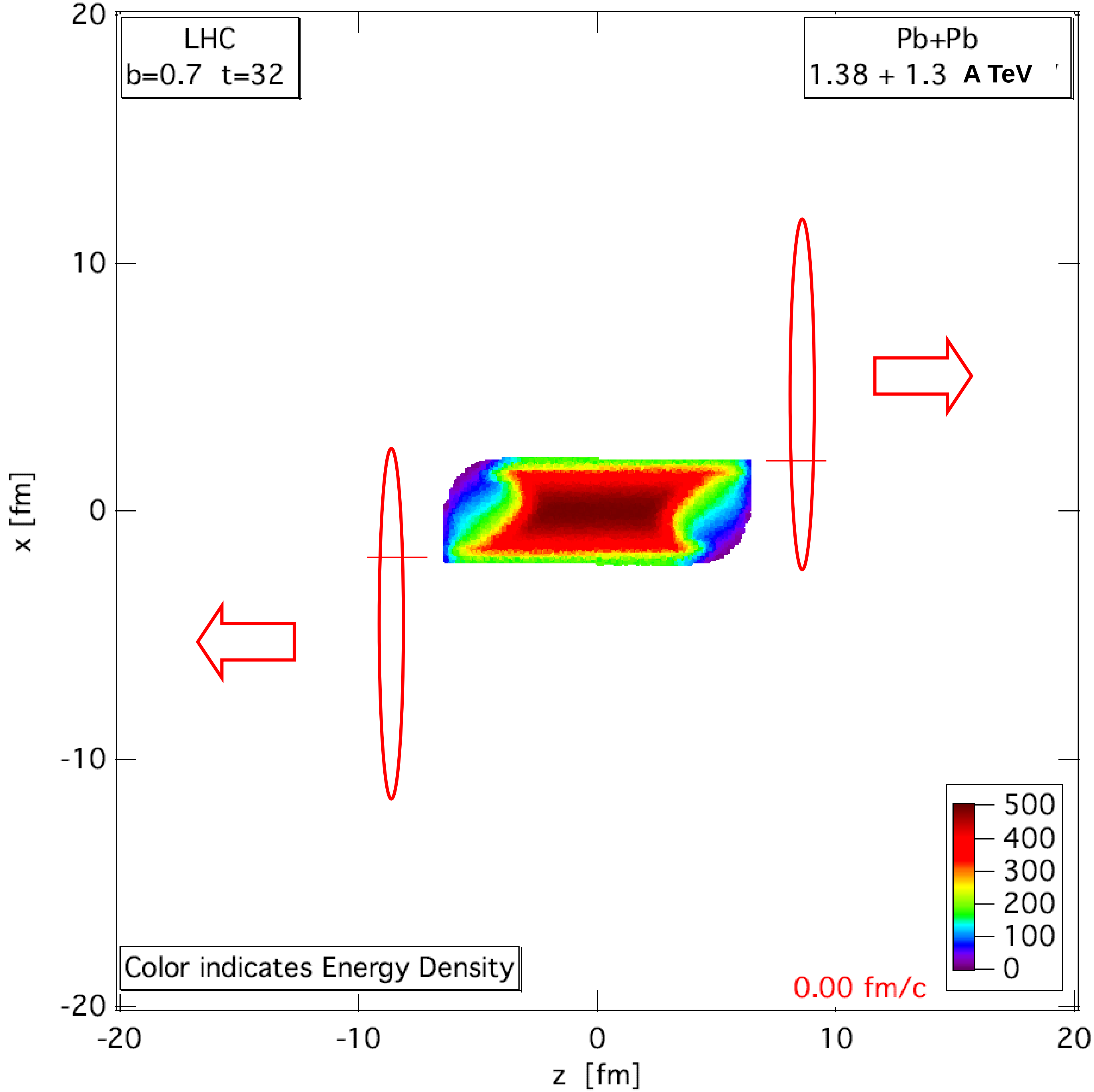
L. P. Csernai,[1,2,3] V. K. Magas,[4] H. Stöcker,[3] and D. D. Strottman[1,3]

PHYSICAL REVIEW C 84, 024914 (2011)

Anti-flow (v1) at LHC



Initial energy density [GeV/fm3] distribution in the reaction plane, [x,y] for a Pb+Pb reaction at 1.38 + 1.38 ATeV collision energy and impact parameter b = 0.5_bmax at time 4 fm/c after the first touch of the colliding nuclei, this is when the hydro stage begins. The calculations are performed according to the effective string rope model. This tilted initial state has a flow velocity distribution, qualitatively shown by the arrows. The dashed arrows indicate the direction of the largest pressure gradient at this given moment.



PIC-hydro

Pb+Pb 1.38+1.38 A TeV, b= 70 % of b_max

Lagrangian fluid cells, moving, ~ 5 mill.

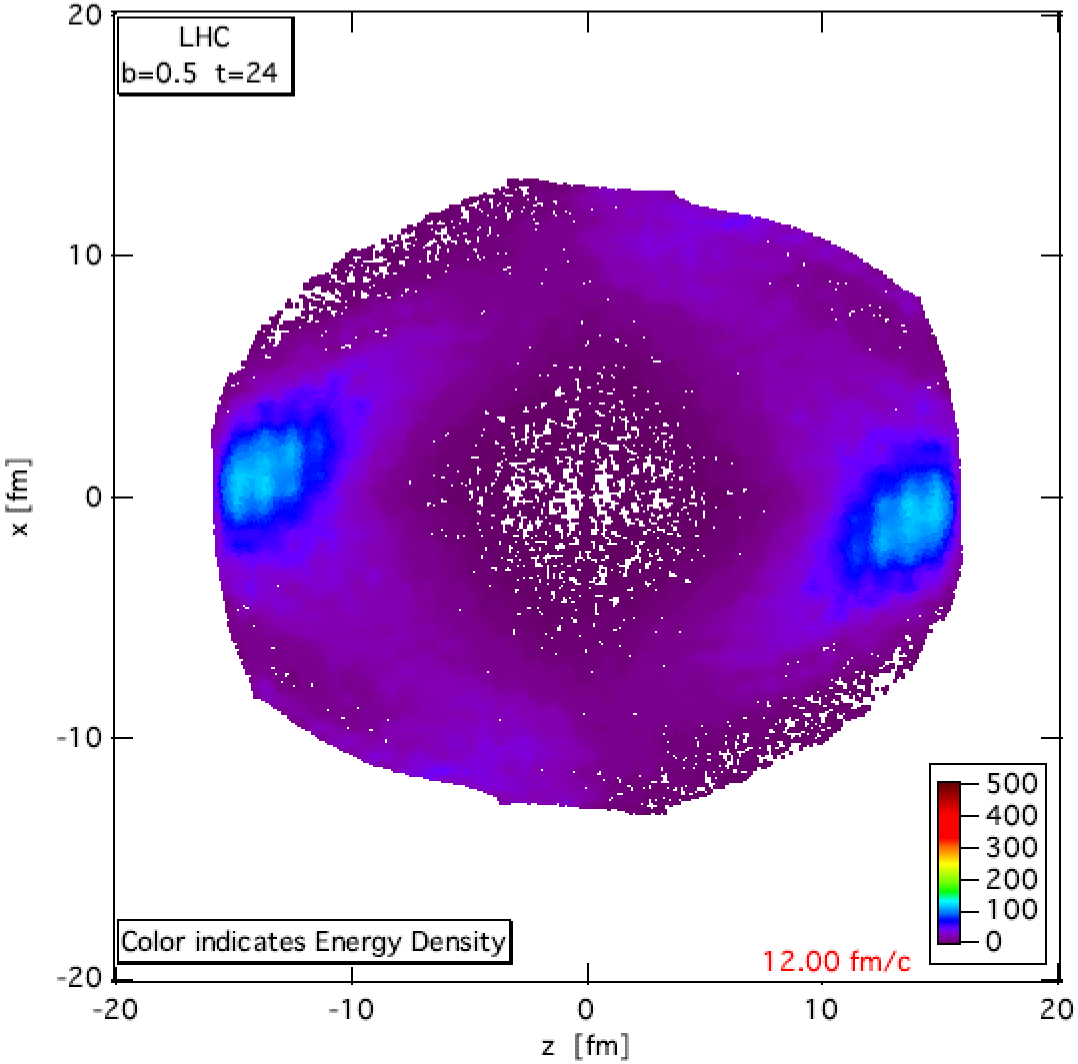
MIT Bag m. EoS

FO at T ~ 200 MeV, but calculated much longer, until pressure is zero for 90% of the cells.

Structure and asymmetries of init. state are maintained in nearly perfect expansion.

..\zz-Movies\LHC-Ec-1h-b7-A.mov

Anti-flow (v1)

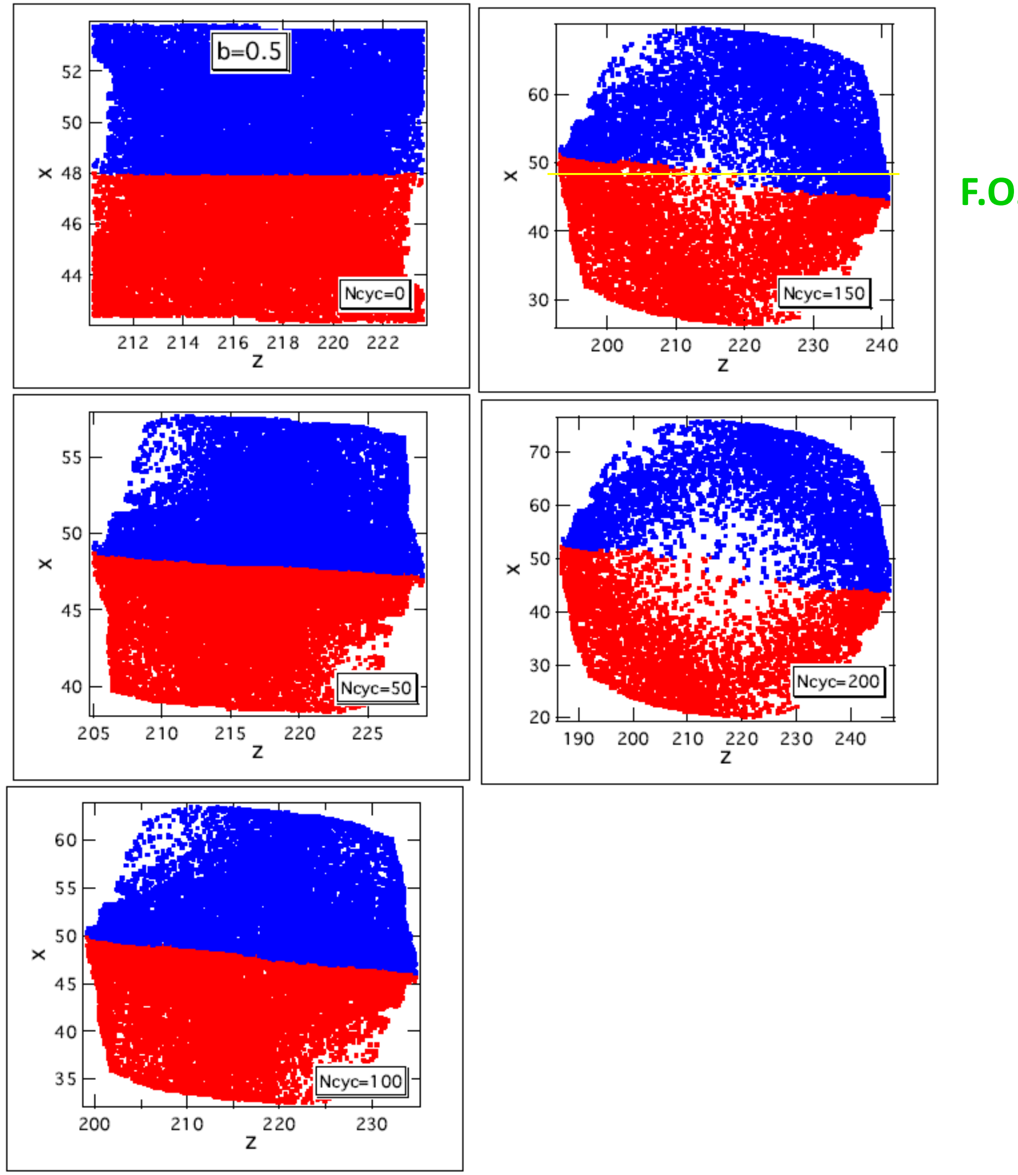


The energy density [GeV/fm3] distribution in the reaction plane, [x,z] for a Pb+Pb reaction at 1.38 + 1.38 A.TeV collision energy and impact parameter b = 0.5b_max at time 12 fm/c after the formation of the hydro initial state. The expected physical FO point is earlier but this post FO configuration illustrates the flow pattern.

[LP. Csernai, V.K. Magas, H. Stöcker, D. Strottman, Phys. Rev. **C84** (2011) 02914]

Rotation

The rotation is illustrated by dividing the upper / lower part (blue/red) of the initial state, and following the trajectories of the marker particles.

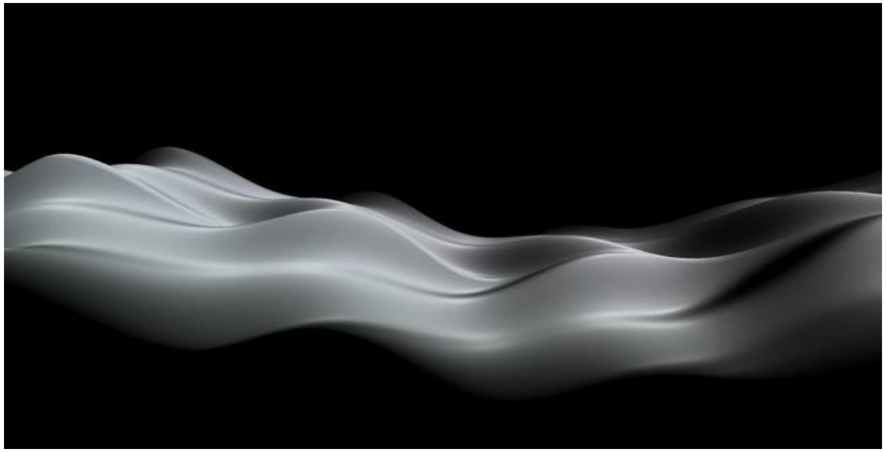


Turbulence ?

Kelvin-Helmholtz Instability (KHI)

- Turbulent fluctuations are common in **air*** and **water***
- Usually ∃ source*
- Usually damped, but weakly
- ∃ quasi-stationary and developing instabilities
- For KHI the source is shear-flow

Low viscosity → Turbulence



oil

water

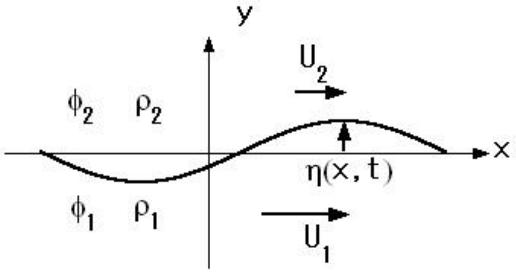
Viscous liquid shows smooth sinusoidal waves, while a non-viscous fluid has sharp, non-sinusoidal waves, leading to turbulence.

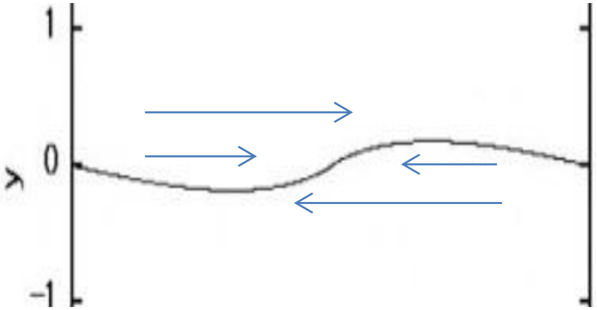
A typical turbulent phenomenon is the Kelvin-Helmholtz instability

KHI in air from above

The Kelvin – Helmholtz instability

- Initial, almost sinusoidal waves



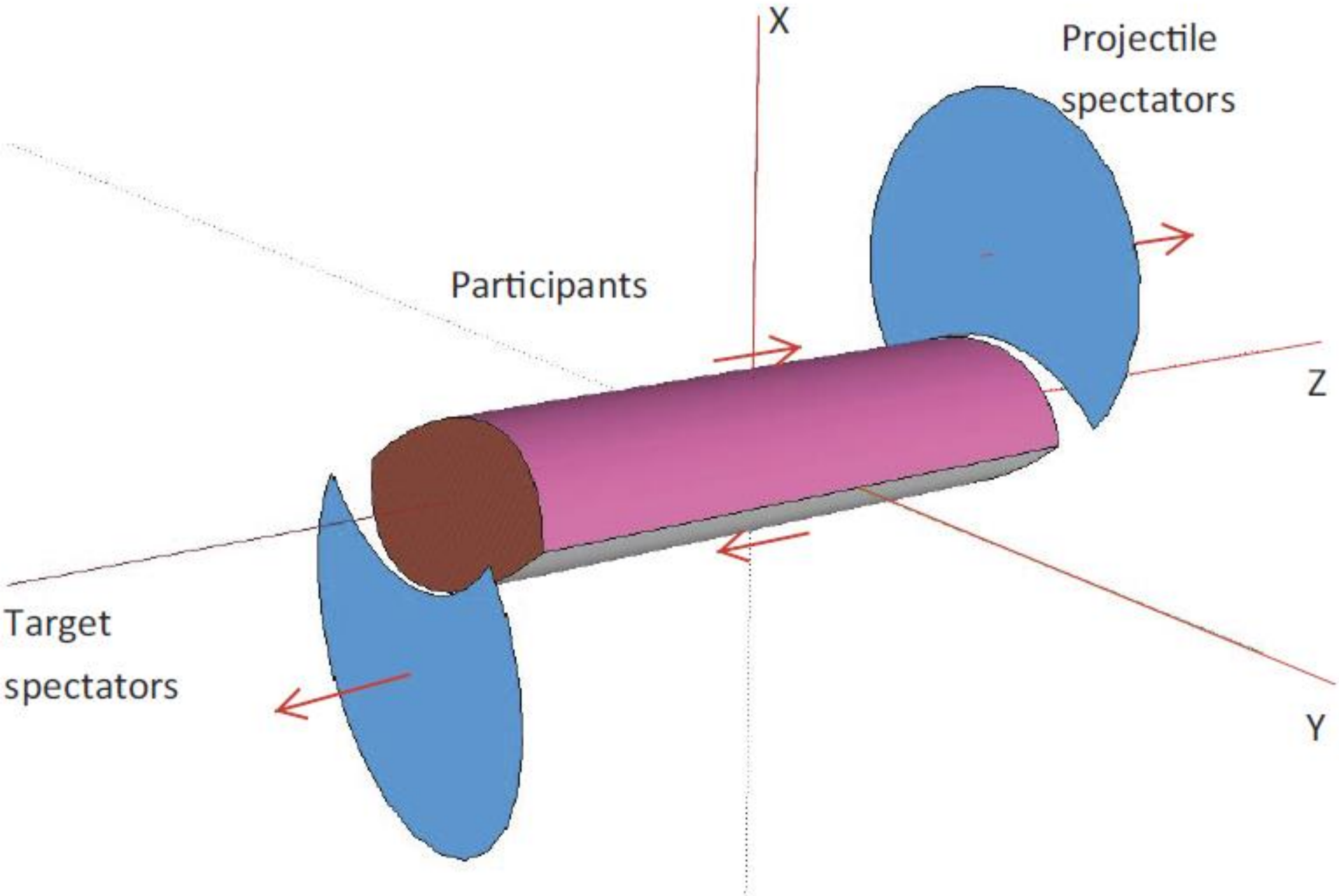


- **Well developed, non-linear wave**

The interface is a layer with a finite thickness, where viscosity and surface tension affects the interface. Due to these effects singularity formation is prevented in reality. The roll-up of a sheet is observed

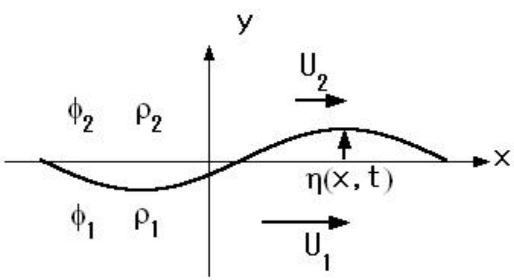
[Chihiro Matsuoka, Yong Guo Shi, Scholarpedia]

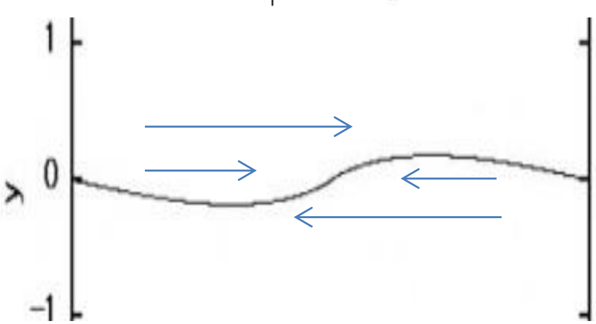
Initial geometry at ultra-relativistic energies



The Kelvin – Helmholtz instability

- Initial, almost sinusoidal waves



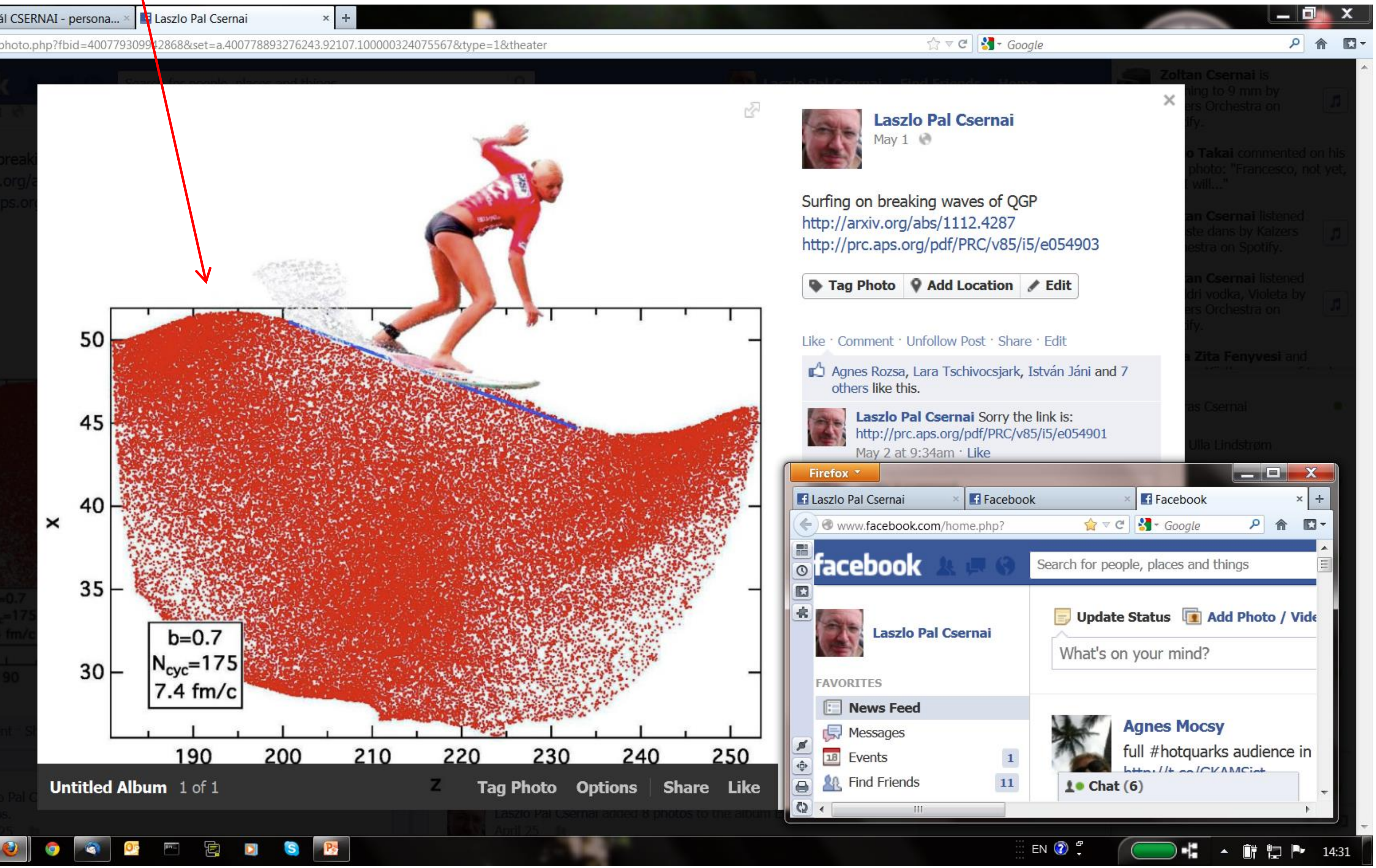


- Well developed, non-linear wave

The interface is a layer with a finite thickness, where viscosity and surface tension affects the interface. Due to these effects singularity formation is prevented in reality. The roll-up of a sheet is observed

[Chihiro Matsuoka, Yong Guo Shi, Scholarpedia]

Kelvin-Helmholtz Instability (KHI)



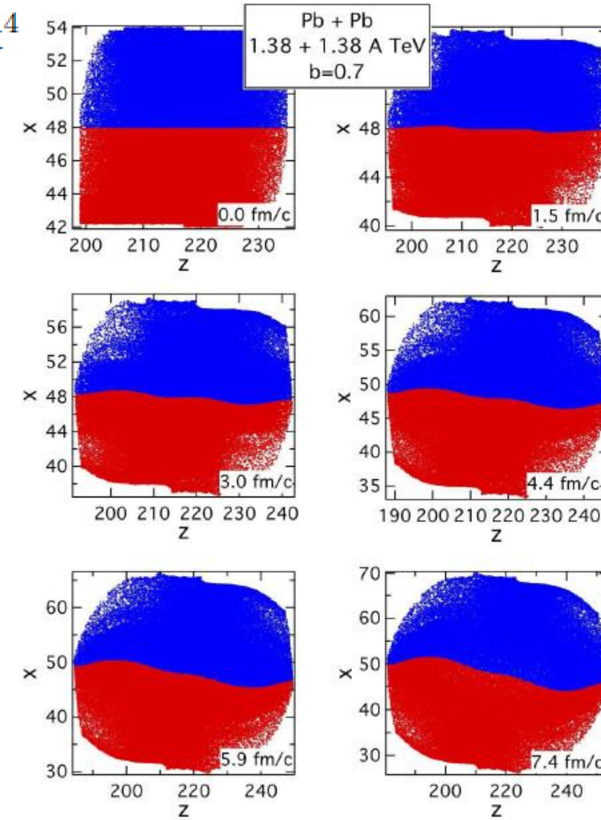
Kelvin-Helmholtz instability in high-energy heavy-ion collisions

L.P. Csernai[1,2,3], D.D. Strottman[2,3], and Cs. Anderlik[4]

PHYSICAL REVIEW C **85**, 054901 (2012)

arXiv:1112.4287v3 [nucl-th]

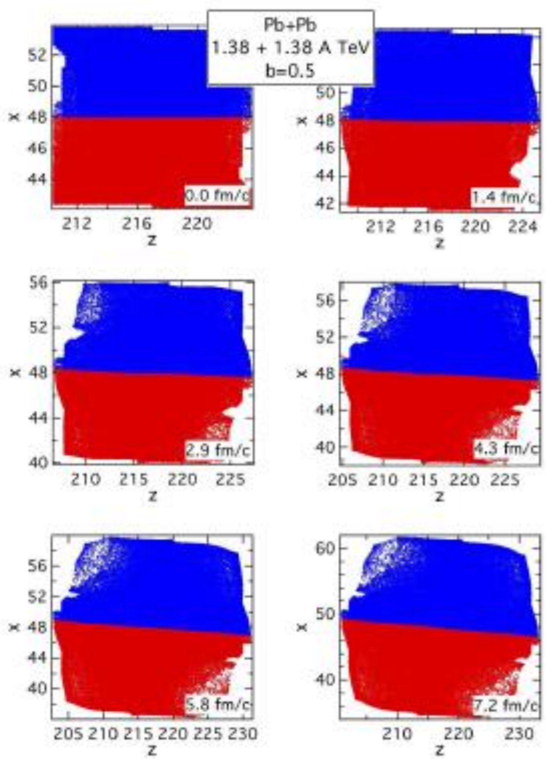
KHI →



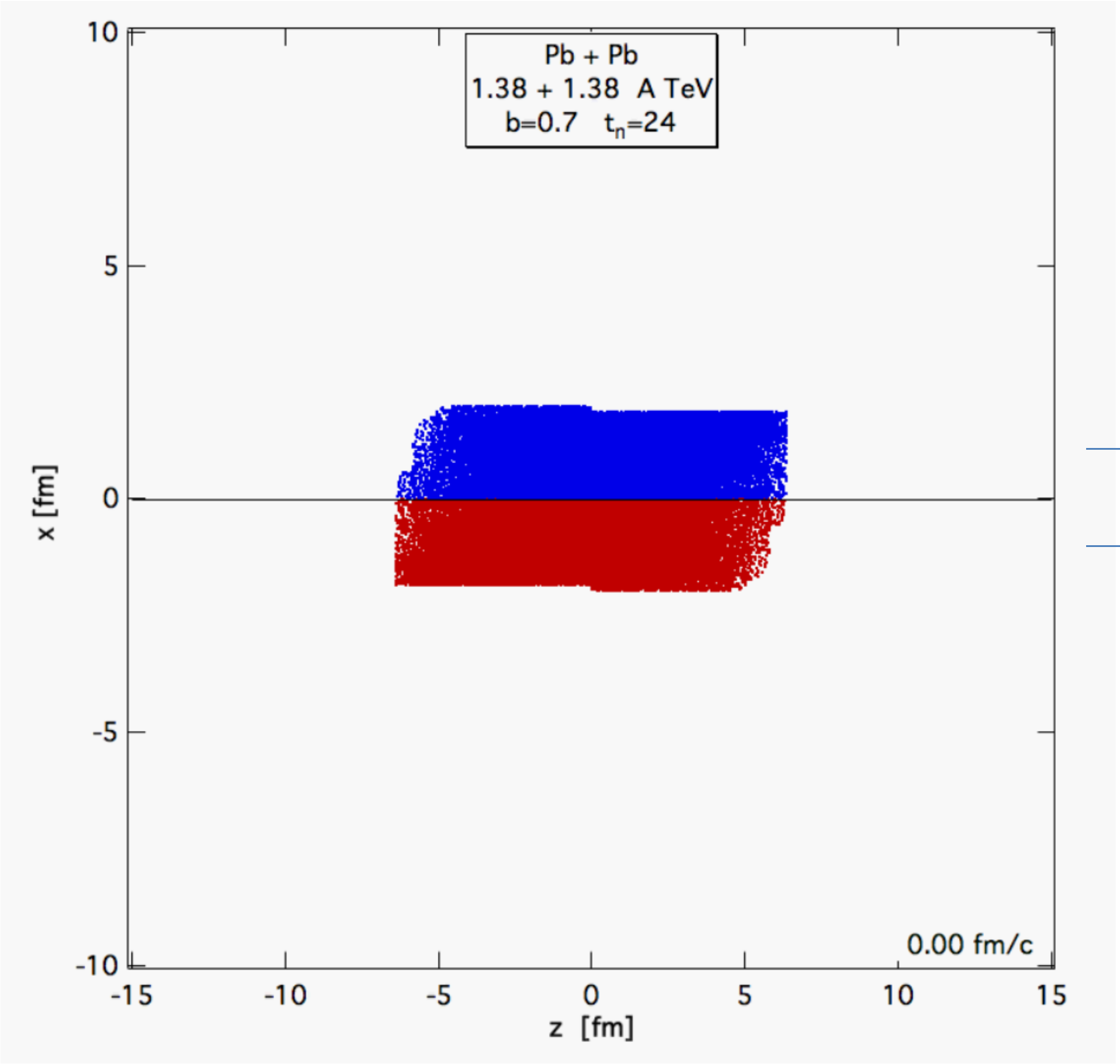
2.4 fm

FIG. 1: (color online) Growth of the initial stage of Kelvin-Helmholtz instability in a 1.38A + 1.38A TeV peripheral, $b = 0.7b_{\max}$, Pb+Pb collision in a relativistic CFD simulation using the PIC-method. We see the positions of the marker particles (Lagrangian markers with fixed baryon number content) in the reaction plane. The calculation cells are $dx = dy = dz = 0.4375$fm and the time-step is 0.04233 fm/c The number of randomly placed marker particles in each fluid cell is 8^3. The axis-labels indicate the cell numbers in the x and z (beam) direction. The initial development of a KH type instability is visible from $t = 1.5$ up to $t = 7.41$ fm/c corresponding from 35 to 175 calculation time steps).

ROTATION

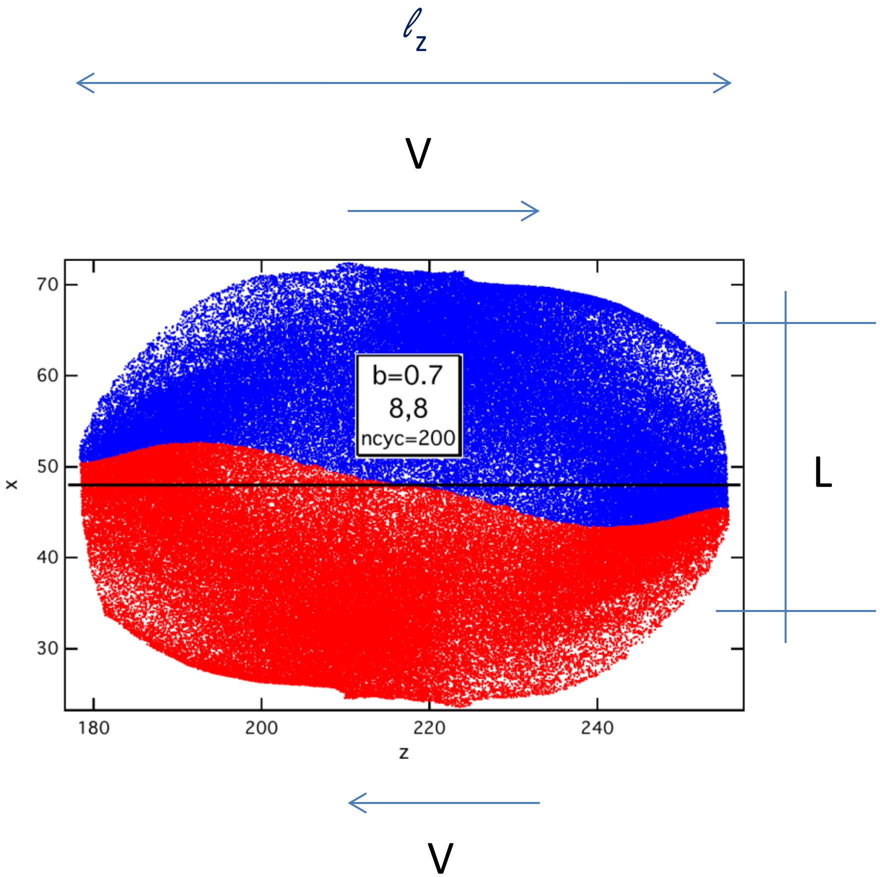


KHI



2.1 fm

The Kelvin – Helmholtz instability (KHI)



Our resolution is $(0.35\text{fm})^3$ and 8^3 markers/fluid-cell → ~ 10k cells & 10Mill m.p.-s

- Shear Flow:
- L=(2R-b) ~ 4 – 7 fm, init. profile height
- ℓ_z =10–13 fm, init. length (b=.5-.7b_{max})
- V ~ ±0.4 c upper/lower speed →
- Minimal wave number is k = .6 - .48 fm^{-1}
- KHI grows as $\propto \exp(st)$, where $s = kV$ →
- Largest k or shortest wave-length will grow the fastest.
- The amplitude will double in 2.9 or 3.6 fm/c for (b=.5-.7b_{max}) without expansion, and with favorable viscosity/Reynolds no. Re=LV/ν .
- → this favors large L and large V

The Kelvin – Helmholtz instability (KHI)

- **Formation of critical length KHI (Kolmogorov length scale)**
- ∃ critical minimal wavelength beyond which the KHI is able to grow. Smaller wavelength perturbations tend to decay. (similar to critical bubble size in homogeneous nucleation).
- **Kolmogorov**: $\lambda_{Kol} = [\nu^3/\epsilon]^{1/4}.$
- Here $\epsilon = \dot{e}/\rho \quad \propto \quad T\dot{\sigma}/\rho \quad \propto \quad \nu,$ is the specific dissipated flow energy.
- We estimated: $\lambda_{Kol} = \begin{cases} 2.1 \div 5.4 \text{ fm for } b = 0.5 b_{max} \\ 1.4 \div 3.6 \text{ fm for } b = 0.7 b_{max} \end{cases}$
- It is required that $l_z > \lambda_{Kol}$. → we need b > 0.5 b_{max}
- Furthermore
  Re = 0.3 – 1 for "$\eta/s = 1$" and
  Re = 3 – 10 for "$\eta/s = 0.1$"

[Du-Juan Wang, Bergen, et al.,]

$$\Omega(z,x) \equiv w(z,x)\omega(z,x)$$

$$w_{ik} \equiv (N_{cell}/E_{tot}) \;\; E_{ik}.$$

Relativistic

$$\Theta \equiv \nabla_\mu u^\mu = \partial_\mu u^\mu,$$

Classical

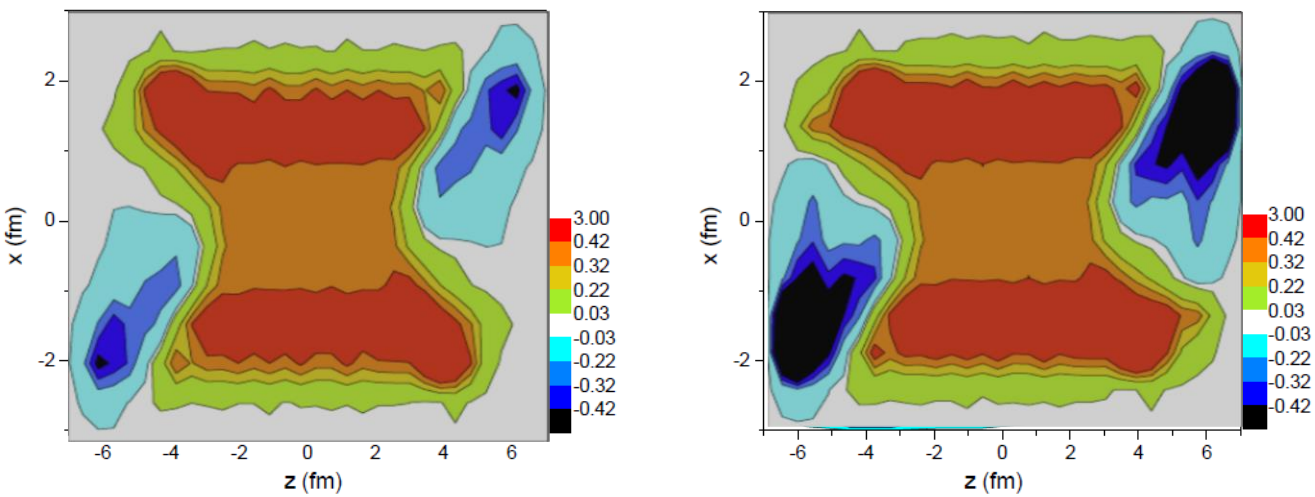
$$\omega^\mu_\nu \equiv \frac{1}{2}(\nabla_\nu u^\mu - \nabla^\mu u_\nu),$$

$$\omega_y \equiv \omega_{xz} \equiv \frac{1}{2}(\partial_z v_x - \partial_x v_z)$$

If $\partial_\tau u^\mu \equiv \dot{u}^\mu = u^\alpha \partial_\alpha u^\mu$ is negligible

$$\omega^x_{\ z} = -\omega^z_{\ x} = \frac{1}{2}(\partial_z \gamma v_x - \partial_x \gamma v_z) = \frac{1}{2}\gamma(\partial_z v_x - \partial_x v_z) + \frac{1}{2}(v_x \partial_z \gamma - v_z \partial_x \gamma)$$

$\Omega(z,x)$



Max = 3. c/fm

FIG. 1: The classical (left) and relativistic (right) weighted vorticity, Ω_{zx}, calculated in the reaction, [x-z] plane at t=0.17 fm/c. The collision energy is $\sqrt{s_{NN}} = 2.76$ TeV and $b = 0.7 b_{max}$, the cell size is $dx = dy = dz = 0.4375 fm$. The average vorticity in the reaction plane is 0.1434 / 0.1185 for the classical / relativistic weighted vorticity respectively.

Classical

All y-layers

Relativistic

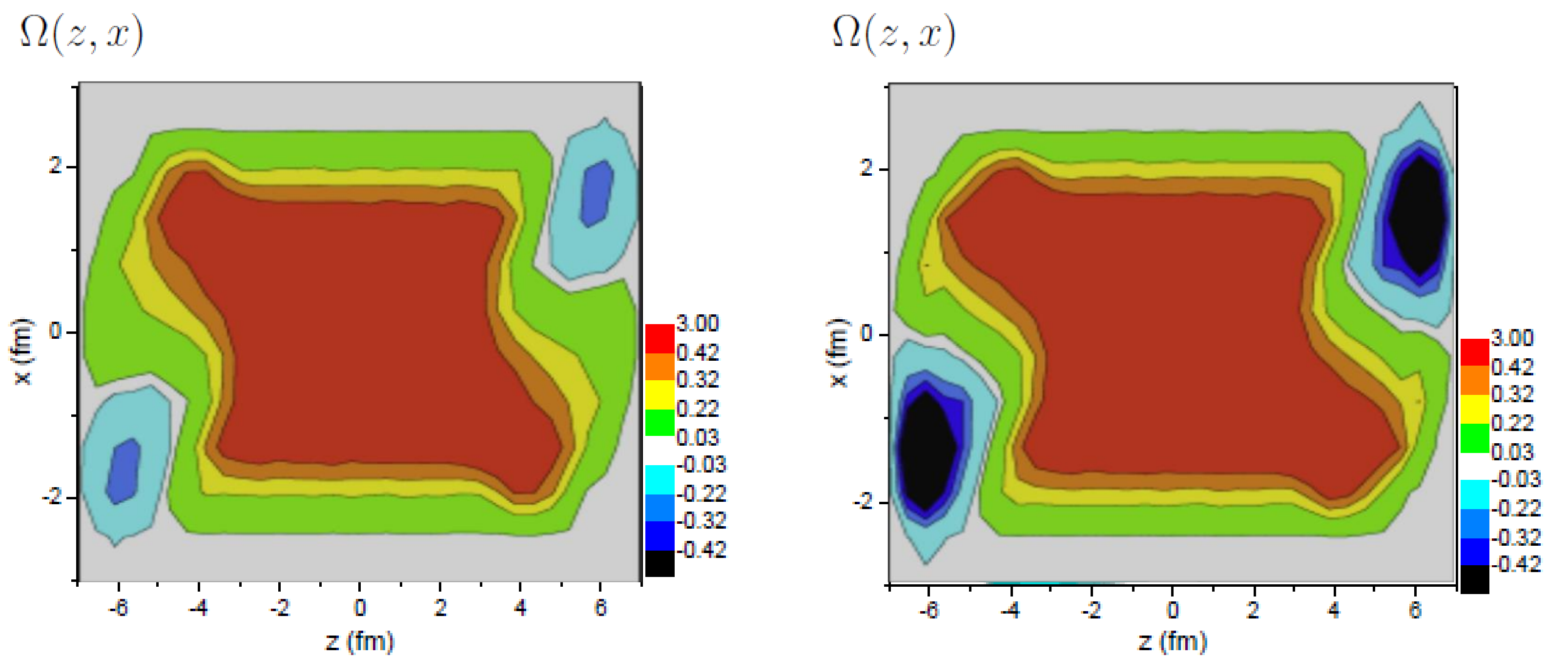


FIG. 4: The classical (left) and relativistic (right) weighted vorticity calculated for all [x-z] layers at t=0.17 fm/c. The collision energy is $\sqrt{s_{NN}}$ = 2.76 TeV and $b = 0.7b_{max}$, the cell size is $dx = dy = dz = 0.4375 fm$.

Classical

Relativistic

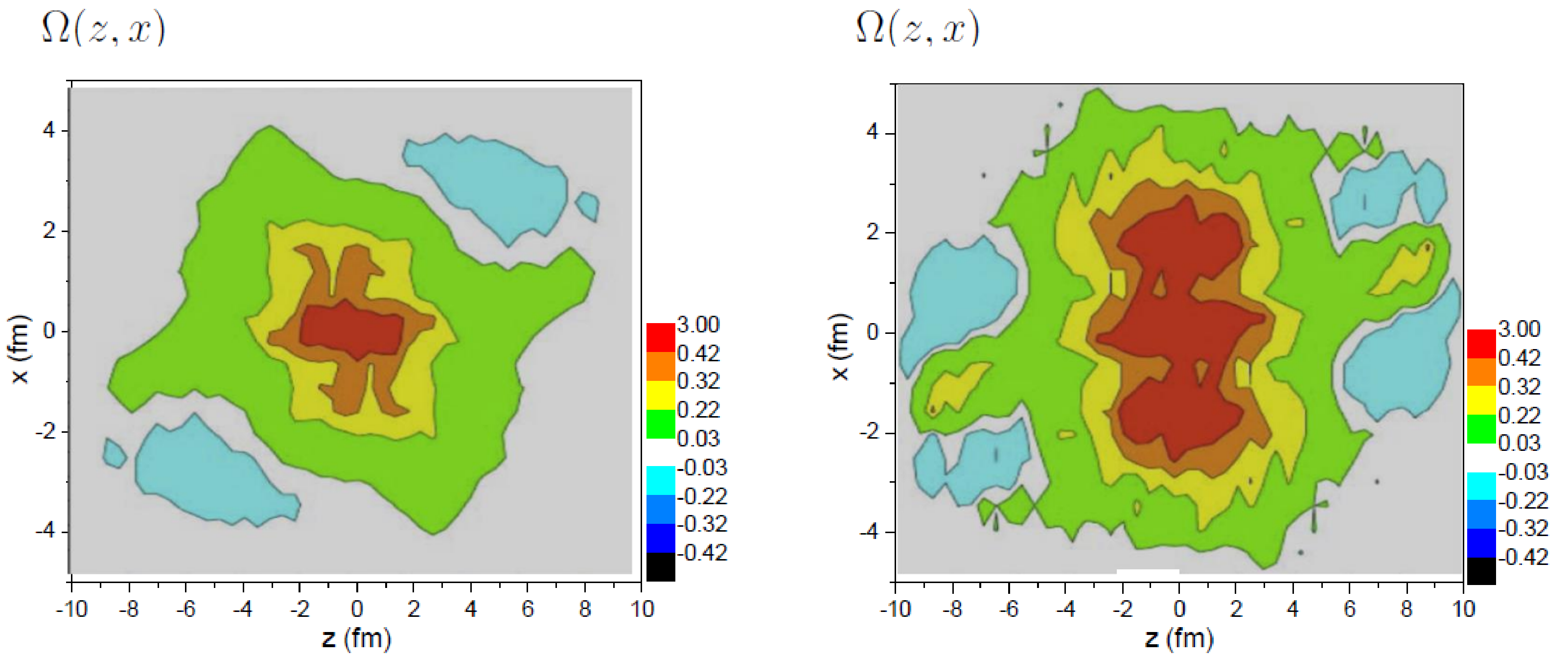


FIG. 5: The classical (left) and relativistic (right) weighted vorticity calculated for all [x-z] layers at t=3.56 fm/c. The collision energy is $\sqrt{s_{NN}} = 2.76$ TeV and $b = 0.7 b_{max}$, the cell size is $dx = dy = dz = 0.4375 fm$. The average vorticity in the reaction plane is 0.0538 / 0.10685 for the classical / relativistic weighted vorticity respectively.

the surface element $S(t)$. Then we can describe the *circulation* along

$$\Gamma(C(t)) = \oint_{C(t)} \mathbf{v} \cdot d\mathbf{l} = \int\int_{S(t)} \vec{\omega} \cdot \mathbf{n}\, dS$$

where ω is the vorticity

$$\vec{\omega} = \ \mathbf{r}ot\ \mathbf{v}$$

The circulation is conserved for perfect incompressible classical fluids.

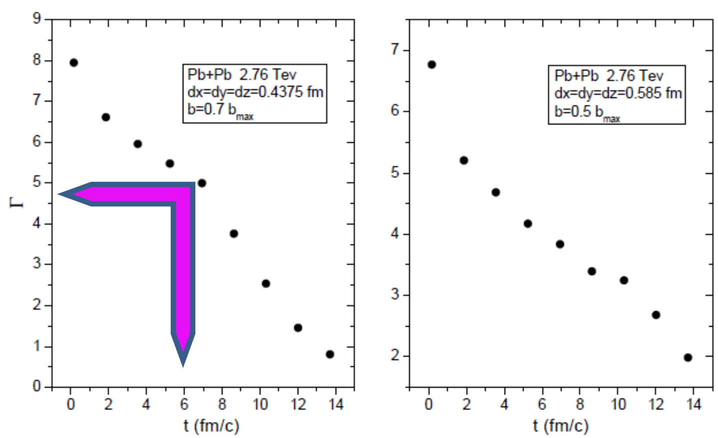
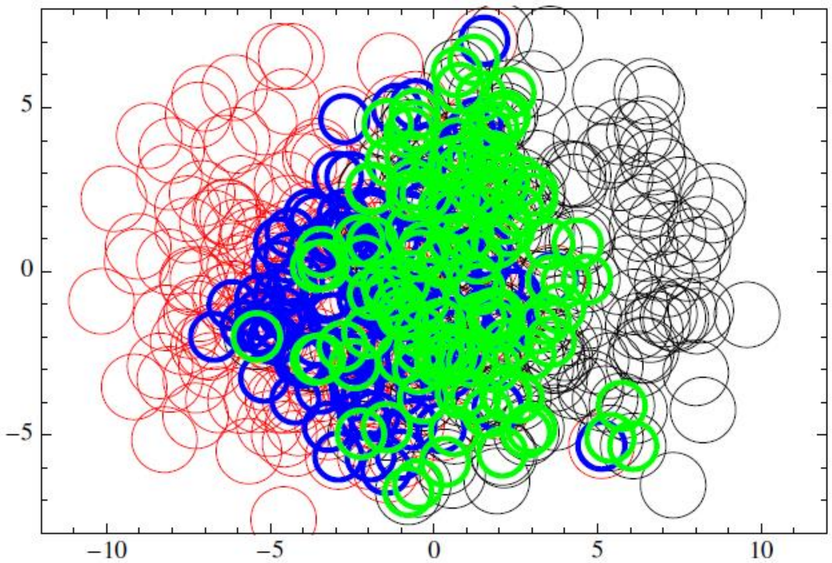


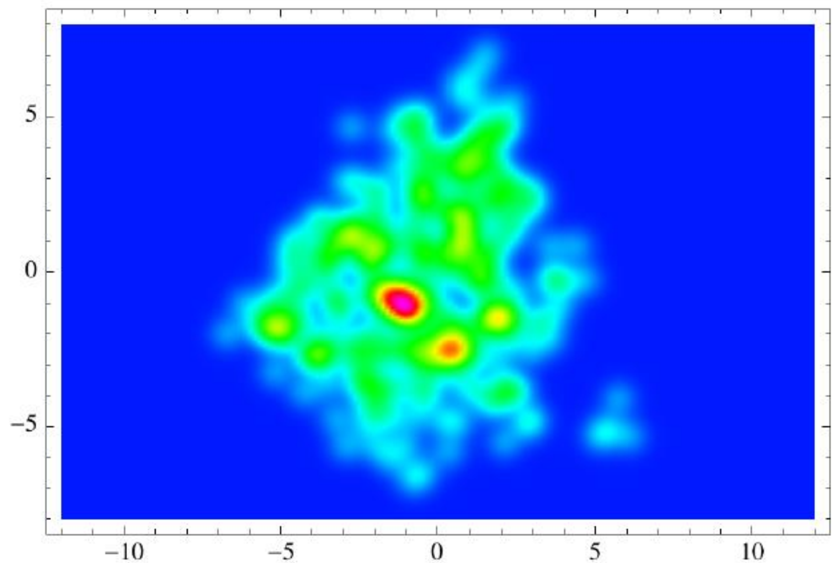
FIG. 7: The time dependence of classical circulation, $\Gamma(t)$, in units of [fm c], calculated for all [x-z] layers and then taking the average of the circulations for all layers. The collision energy is $\sqrt{s_{NN}} = 2.76$ TeV and $b = 0.7b_{max}$, the cell size is $dx = dy = dz = 0.4375 fm$ (left). For comparison another initial state configuration was also tested for the same collision energy but $b = 0.5b_{max}$, the cell size is $dx = dy = dz = 0.585 fm$ (right). This configuration shows also the rotation, but due to its less favorable parameters it does not show the KHI. Although at this impact parameter, which is less peripheral the reaction plane has a larger area filled with matter, nevertheless the initial classical circulation is less by about 15%. For the more peripheral case with smaller numerical viscosity the circulation decreases with time faster and the circulation for the two cases becomes equal around $t = 10$ fm/c.

Onset of turbulence around the Bjorken flow

S. Floerchinger & U. A. Wiedemann, JHEP 1111:100, 2011; arXiv: 1108.5535v1

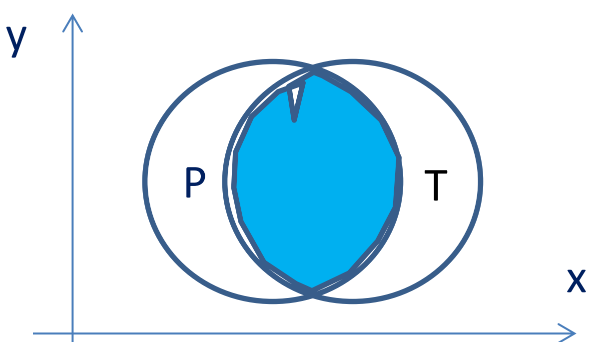
y





nucleons x [fm] energy density x [fm]

- Transverse plane [x,y] of a Pb+Pb HI collision at $\sqrt{s_{NN}}$=2.76TeV at b=6fm impact parameter
- Longitudinally [z]: **uniform** Bjorken flow, (expansion to infinity), depending on τ only.

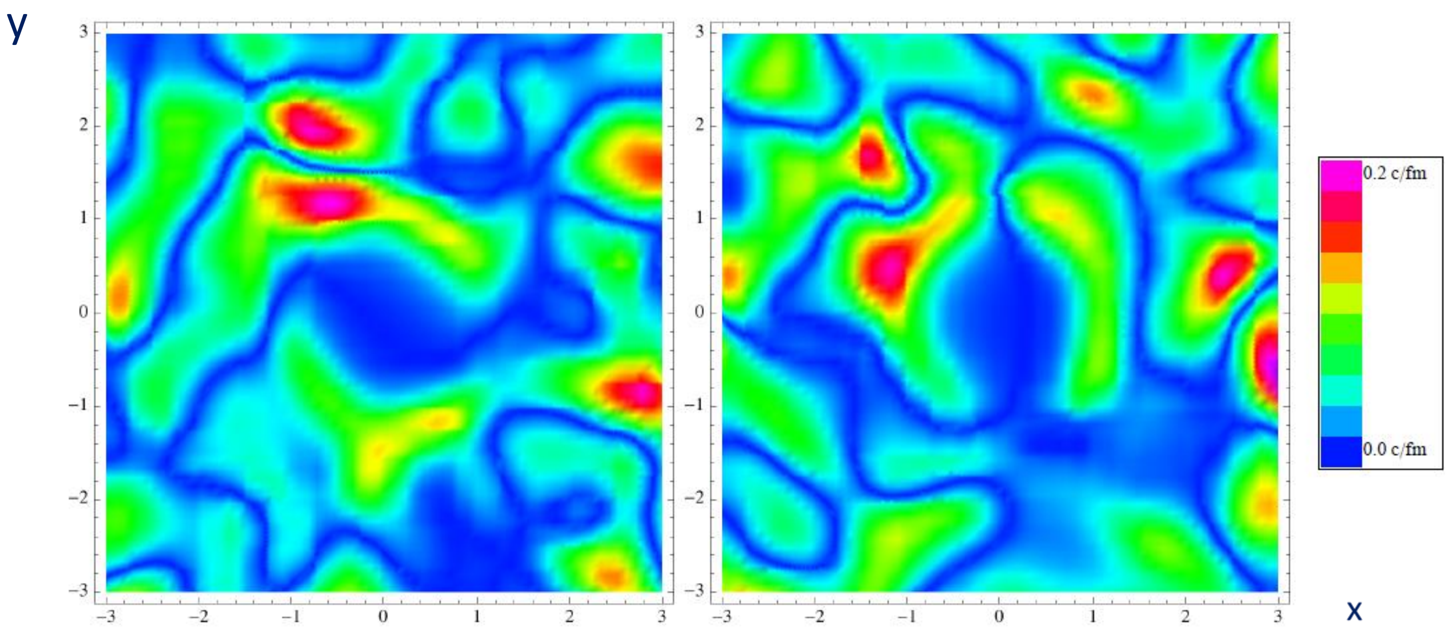


Green and **blue** have the same longitudinal speed (!) in this model. Longitudinal shear flow is omitted.

Onset of turbulence around the Bjorken flow

S. Floerchinger & U. A. Wiedemann, JHEP 1111:100, 2011; arXiv: 1108.5535v1

Max = 0.2 c/fm



absolute value of vorticity $|\partial_1 u^2 - \partial_2 u^1|$ divergence $|\partial_1 u^1 + \partial_2 u^2|$

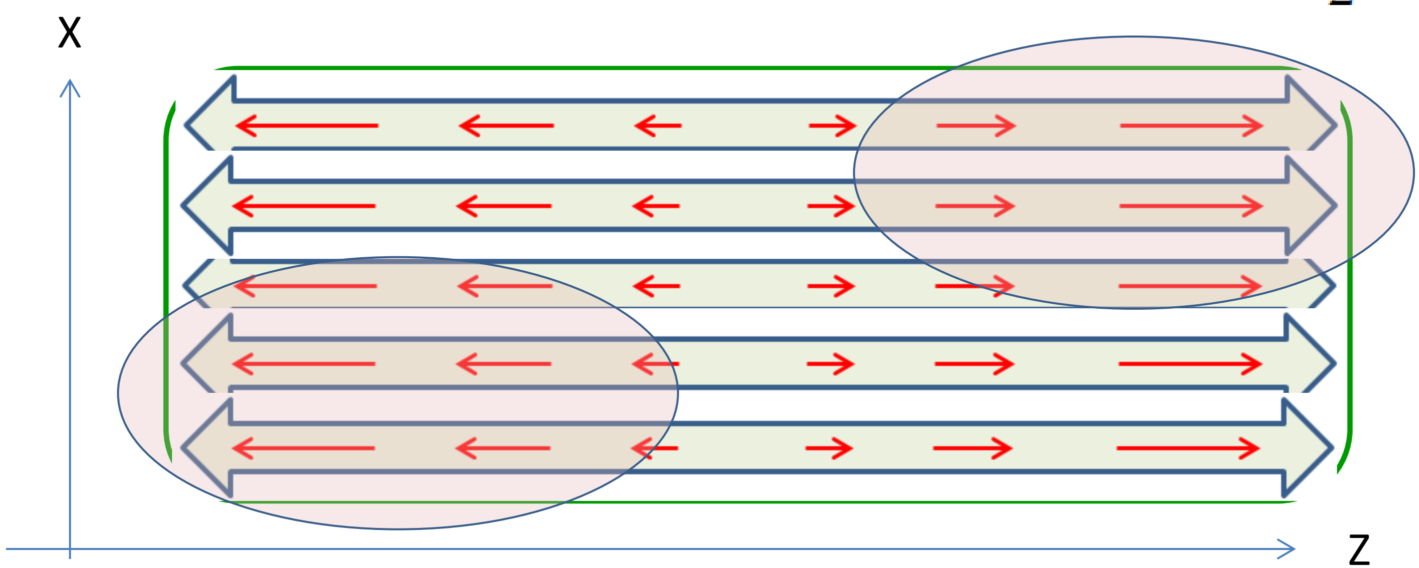
- Initial state Event by Event vorticity and divergence fluctuations.
- Amplitude of random vorticity and divergence fluctuations are the same
- In dynamical development viscous corrections are negligible (→ no damping)
- Initial transverse expansion in the middle (±3fm) is neglected (→ no damping)
- High frequency, high wave number fluctuations **may feed** lower wave numbers

Typical I.S. model – scaling flow

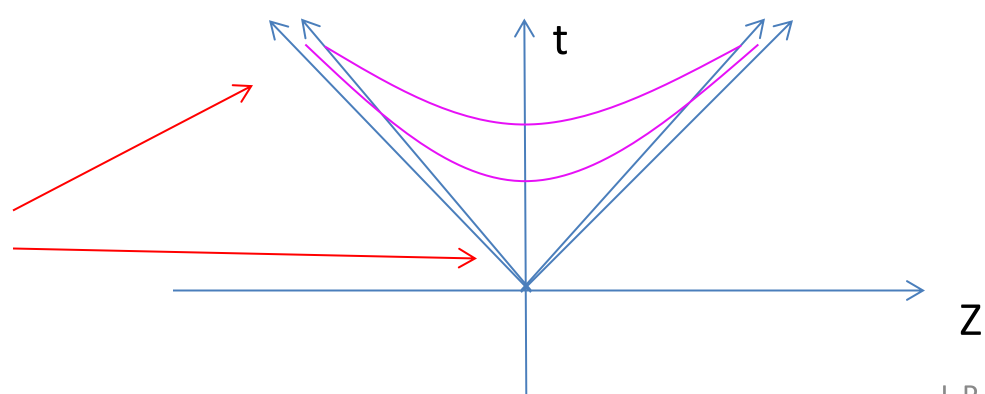
The same longitudinal expansion velocity profile in the whole [x,y]-plane !
No shear flow. **No string tension**! Usually **angular momentum is vanishing**!

$$\omega_y \equiv \omega_{xz} \equiv -\omega_{zx} \equiv \frac{1}{2}(\partial_z v_x - \partial_x v_z)$$

$$\boldsymbol{\omega} \equiv \frac{1}{2}\mathrm{rot}\ \boldsymbol{v} = \frac{1}{2}\nabla \times \boldsymbol{v}$$



Zero vorticity
&
Zero shear!



Such a re-arrangement of the matter density is dynamically not possible in a short time!

Adil & Gyulassy (2005) initial state

x, y, η, τ coordinates → Bjorken scaling flow

PHYSICAL REVIEW C **72**, 034907 (2005)

Considering a longitudinal *"local relative rapidity slope",* based on observations in D+Au collisions:

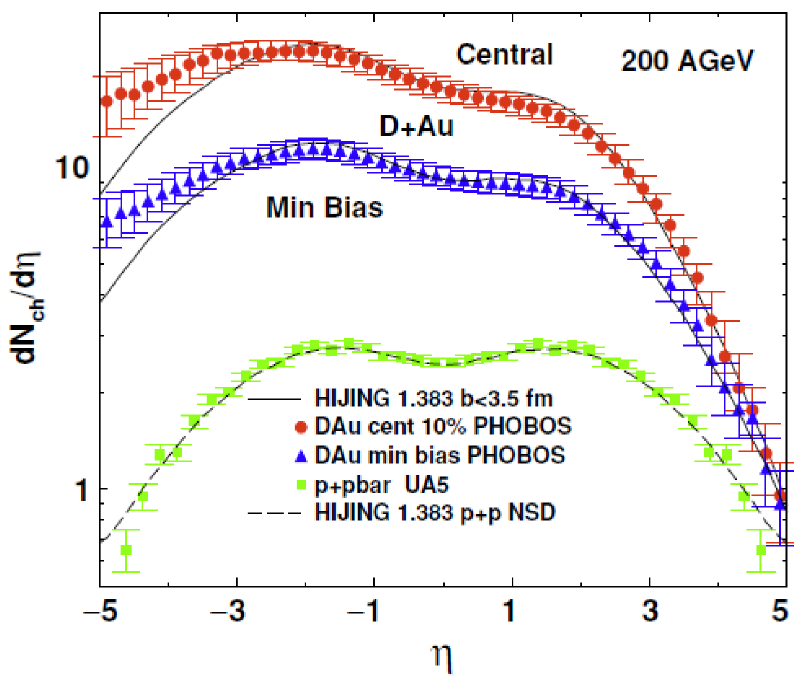
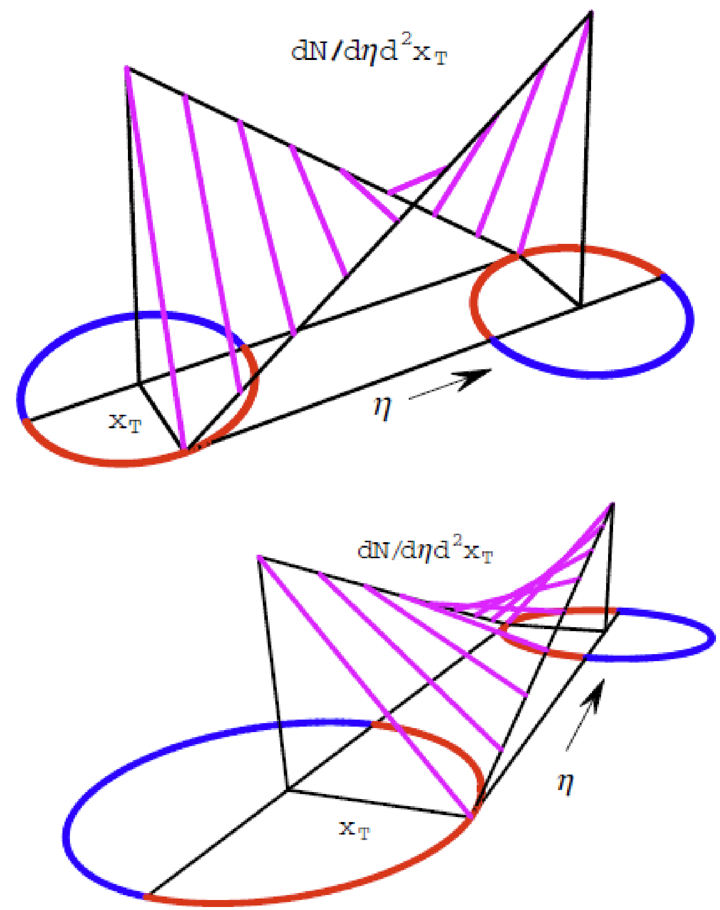
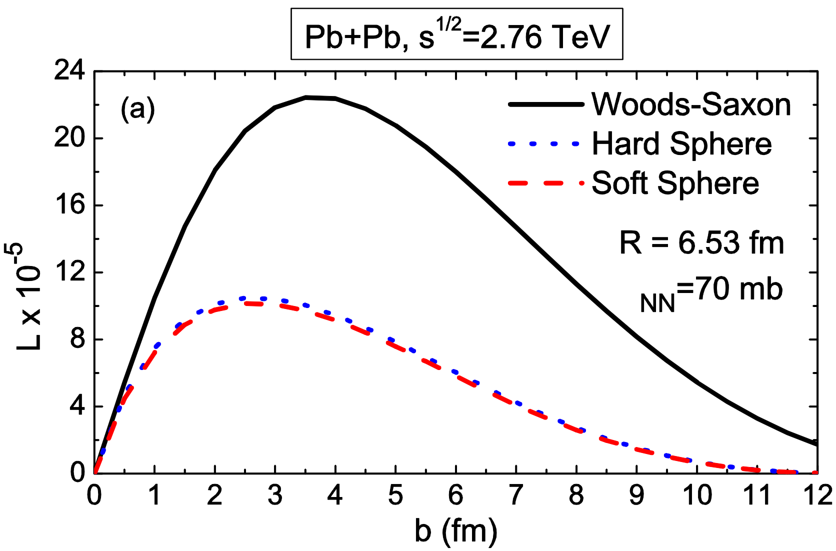


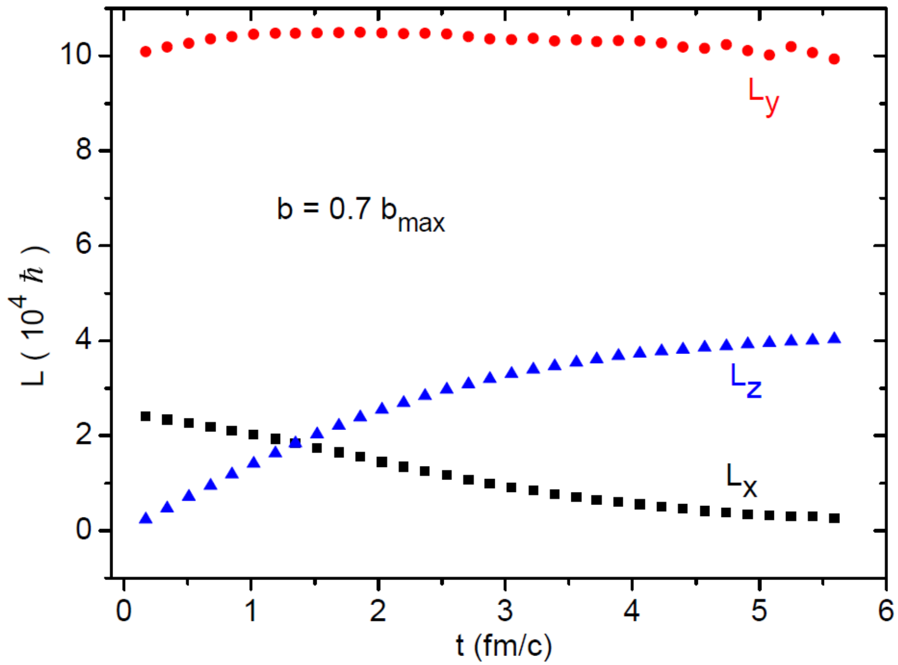
FIG. 2. (Color online) Asymmetric pseudorapidity distributions of charged hadrons produced in D+Au minimum bias and central 0–10% reactions at 200*A* GeV from PHOBOS [12] are compared to $p+\bar{p}$ data from UA5 [13]. The curves show predictions using the HIJING v1.383 code [14,15].

→



Detecting initial rotation





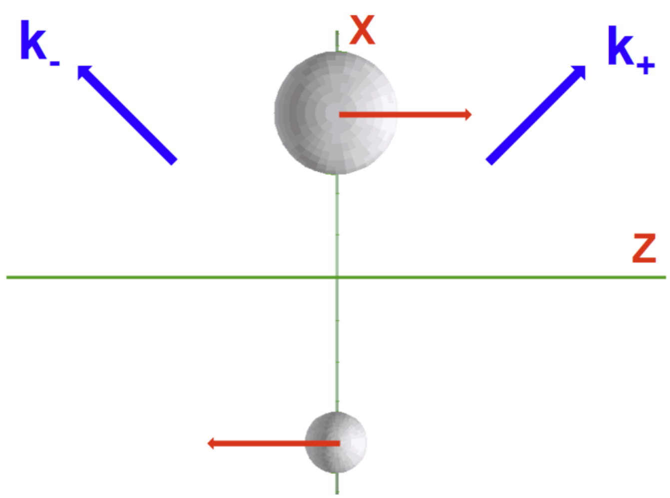
V. Vovchenko, D. Anchishkin, and L.P. Csernai, Submitted to Phys. Rev. C [CD10362]

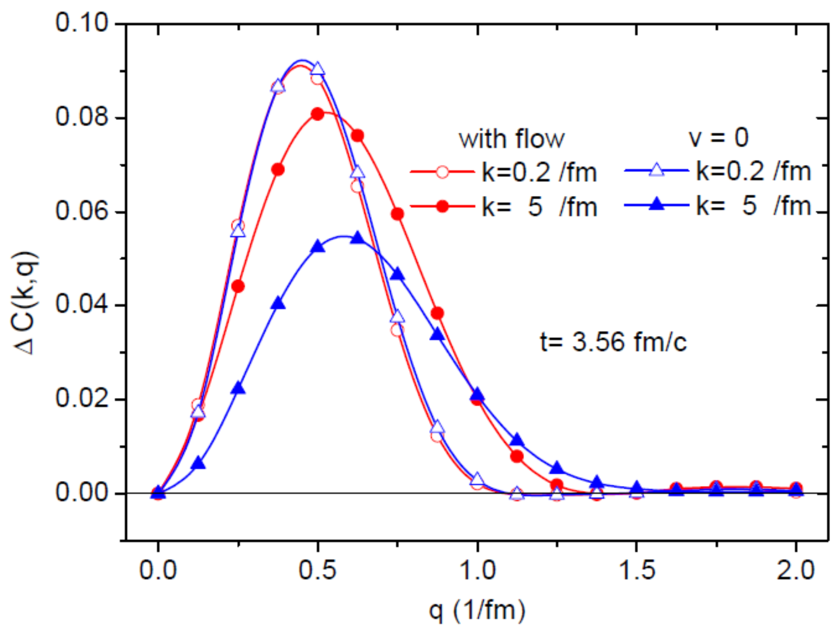
J. H. Gao, S. W. Chen, W. T. Deng, Z. T. Liang, Q. Wang and X. N. Wang, Phys. Rev. C 77, 044902 (2008).

F. Becattini, F. Piccinini, J. Rizzo, Phys. Rev. C 77, 024906 (2008).

Detecting initial rotation: Two particle correlations, Diff. HBT

$$\Delta C(k_{\pm}, q_{out}) \equiv C(k_+, q_{out}) - C(k_-, q_{out}) = \frac{4\exp(-R^2q^2)\ \epsilon \sinh\left(\frac{2u_z\, bk}{T_s}\right)\ (1{-}\epsilon^2)\left[1 - \cosh\left(\frac{u_z\, bq}{T_s}\right)\cos\left(aqd_x\right)\right]}{\left[(1{+}\epsilon^2)\cosh\left(\frac{2u_z\, bk}{T_s}\right) + (1{-}\epsilon^2)\right]^2 - 4\epsilon^2\sinh^2\left(\frac{2u_z\, bk}{T_s}\right)}.$$



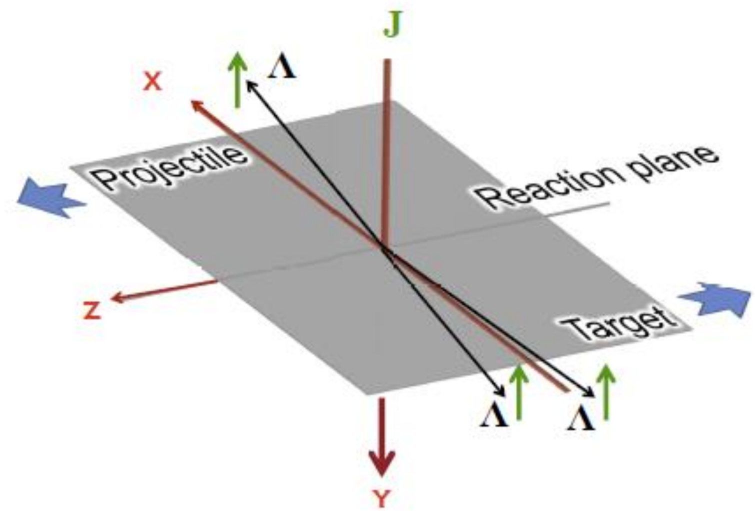


[L.P. Csernai, S. Velle, subm. to PRC]

[L.P. Csernai, S. Velle, D.J. Wang , in prep.]

FIG. 3. (color online) The flow velocity dependence of the differential correlation function at the final time.

Detecting rotation: Lambda polarization

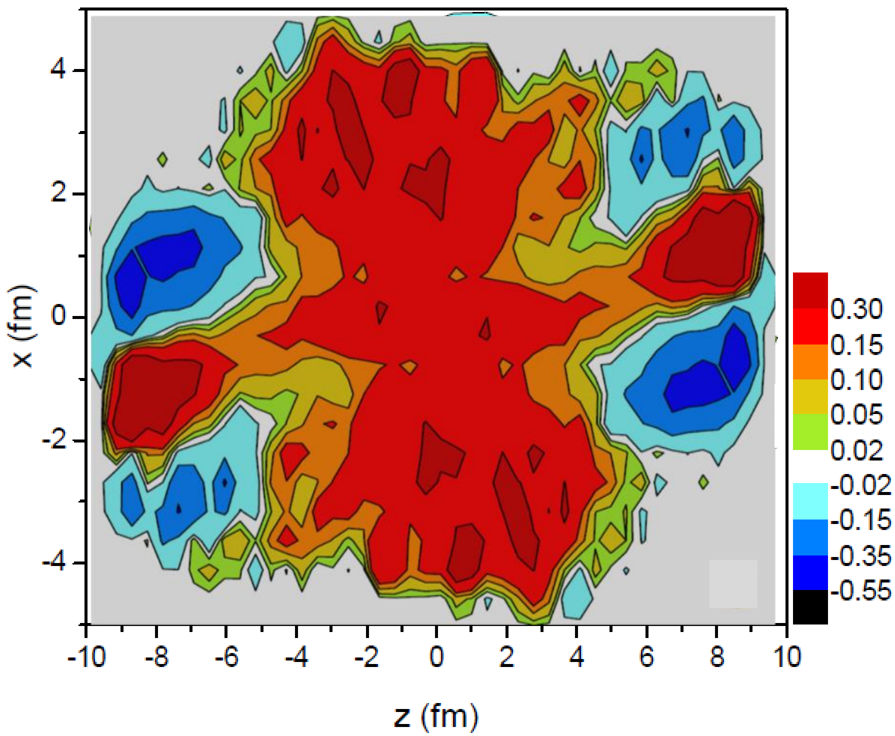


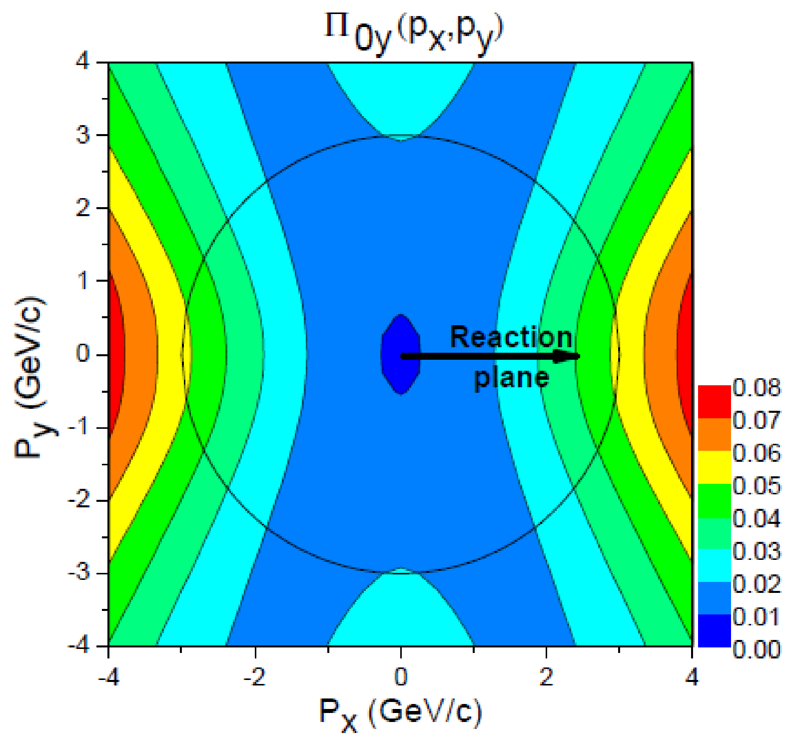
$$\mathbf{\Pi}(p) = \frac{\hbar\varepsilon}{8m}\frac{\int \mathrm{d}V\, n_F\, (\nabla \times \beta)}{\int \mathrm{d}V\, n_F}$$

$$\beta^\mu(x) = (1/T(x))u^\mu(x)$$ ← From hydro

$$\mathbf{\Pi}_0(p) = \mathbf{\Pi}(p) - \frac{\mathbf{p}}{\varepsilon(\varepsilon+m)}\mathbf{\Pi}(p)\cdot\mathbf{p}$$

[F. Becattini, L.P. Csernai, D.J. Wang, Submitted to Phys. Rev. Lett. arXiv:1304.4427v1 [nucl-th]]





Summary

- FD model: Initial State + **EoS** + Freeze out & Hadronization
- In p+p I.S. is problematic, but ∃ collective flow
- In A+A the I.S. is causing global collective flow
- Consistent I.S. is needed based on a dynamical picture, satisfying causality, etc.
- Several I.S. models exist, some of these are oversimplified beyond physical principles.
- Experimental outcome strongly depends on the I.S.

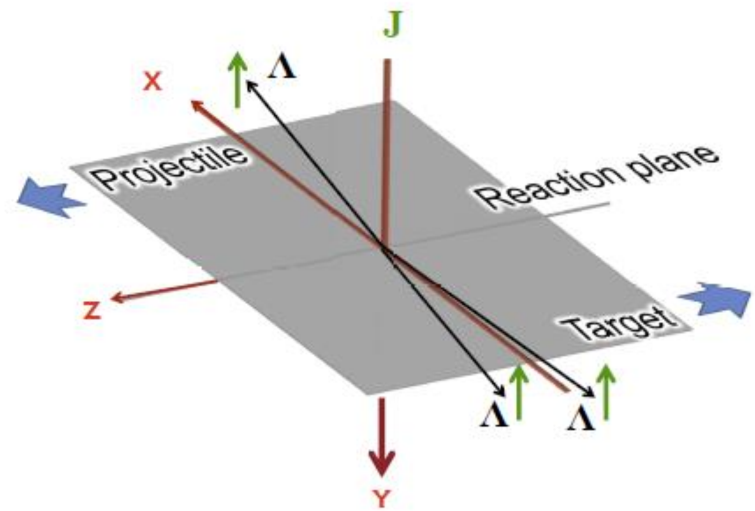
Thank you

Detecting rotation: Lambda polarization

$$\mathbf{\Pi}(p) = \frac{\hbar\varepsilon}{8m} \frac{\int \mathrm{d}V\, n_F\, (\nabla \times \beta)}{\int \mathrm{d}V\, n_F}$$

$$\beta^\mu(x) = (1/T(x))u^\mu(x)$$ ← From hydro

$$\mathbf{\Pi}_0(p) = \mathbf{\Pi}(p) - \frac{\mathbf{p}}{\varepsilon(\varepsilon+m)}\mathbf{\Pi}(p)\cdot\mathbf{p}$$



[F. Becattini, L.P. Csernai, D.J. Wang, Submitted to Phys. Rev. Lett. arXiv:1304.4427v1 [nucl-th]]