

# **Hadron-Resonance Gas at Freeze-out: Reminder on Importance of Repulsive Interactions**

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**V.B., M. Gazdzicki, and M.I. Gorenstein arXiv:1208.4107**



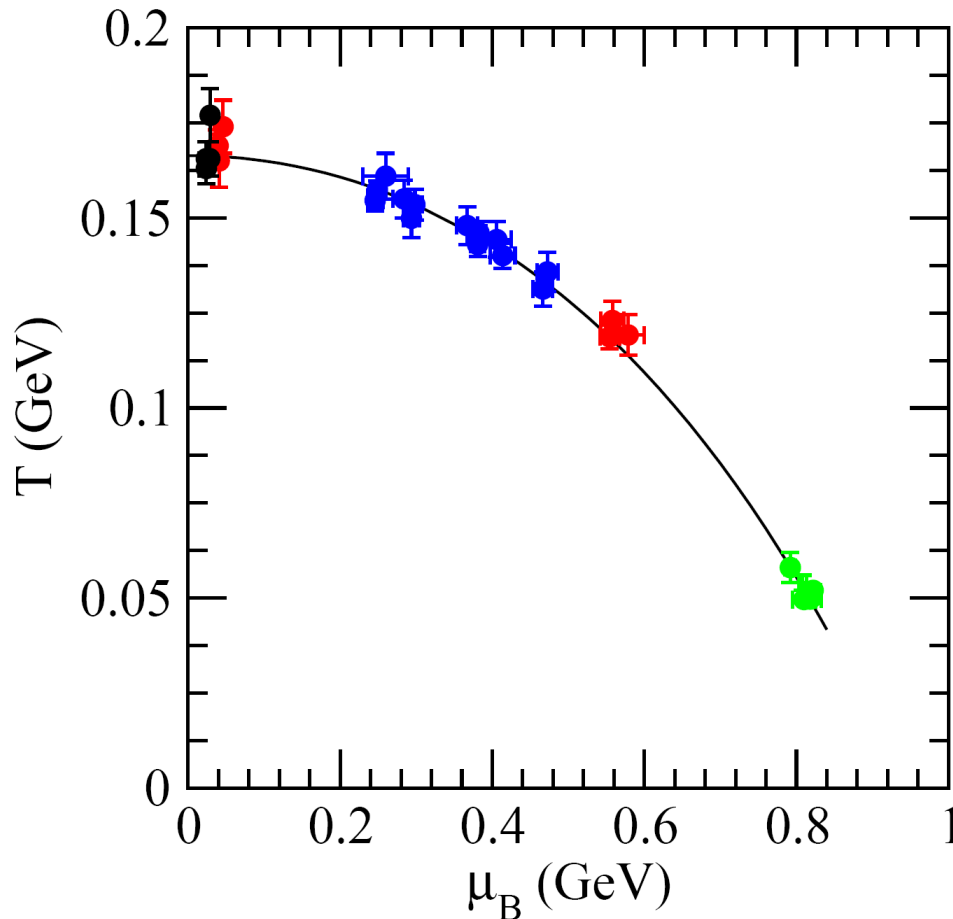
# Motivation

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- **Resonances** introduced to the ideal **Hadron Gas** take into account **attractive interactions**
- **Nucleon-nucleon potential** includes both parts – attractive at large and repulsive at small distances
- **Repulsive** interactions are accounted for by using van der Waals **excluded volume**
- **Point-like hadrons** would always become dominant phase at very high energy density due to the large number of different types of hadrons
- Just the **excluded volume** effects ensure a **phase transition** from a gas of hadrons and resonances to the quark-gluon plasma

# Hadron Gas of Point-Like particles

Hadron Gas model allows to connect the results of different experiments by the single Freeze-out line  $T(\mu_B)$



$$\langle N_i \rangle \equiv \sum_{\mathbf{p}} \langle n_{\mathbf{p},i} \rangle \simeq \frac{g_i V}{2\pi^2} \int_0^\infty p^2 dp \langle n_{p,i} \rangle,$$

$$\langle N^+ \rangle \equiv \sum_{i, q_i > 0} \langle N_i \rangle,$$

$$\langle n_{\mathbf{p},i} \rangle = \frac{1}{\exp \left[ \left( \sqrt{\mathbf{p}^2 + m_i^2} - \mu_i \right) / T \right] \pm 1}$$

$$\mu_i = q_i \mu_Q + b_i \mu_B + s_i \mu_S,$$

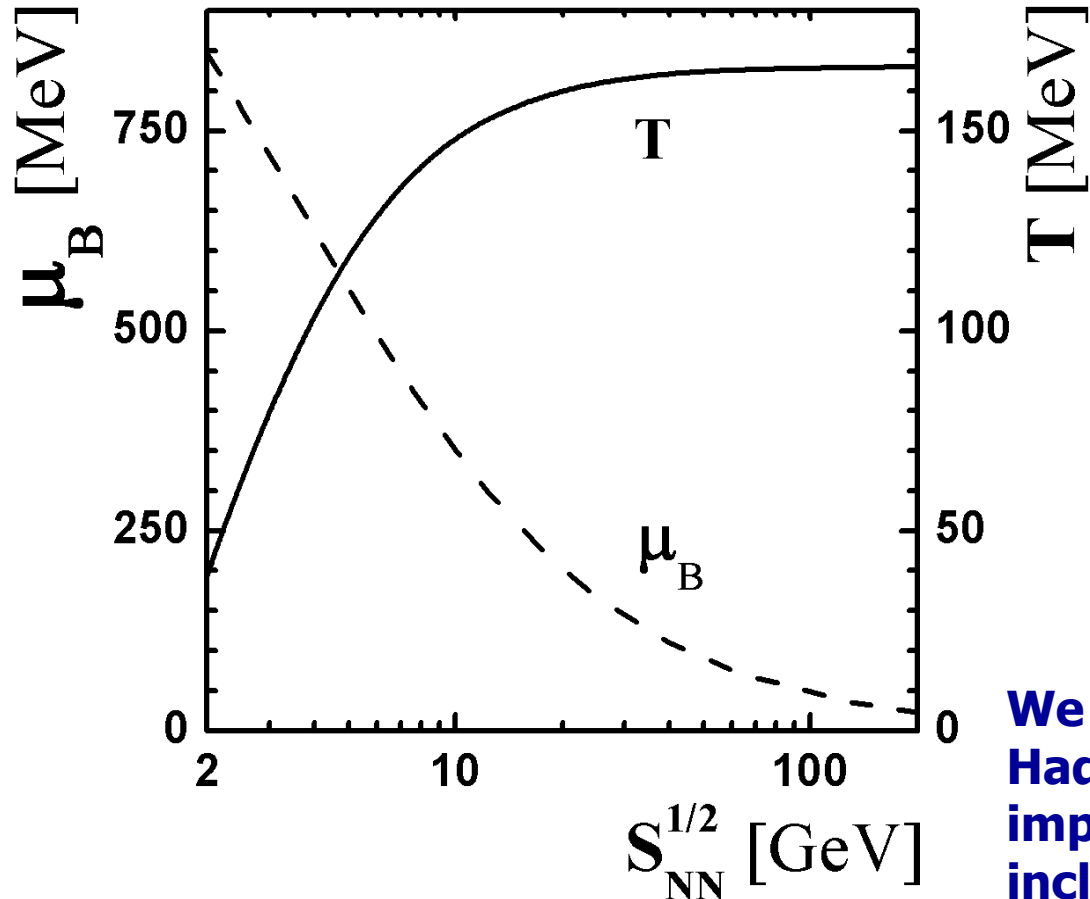
$$Q/B = 0.4 \Rightarrow \mu_Q(T, \mu_B)$$

$$S = 0 \Rightarrow \mu_S(T, \mu_B)$$

↓

$$T \quad \mu_B \quad \gamma S$$

# Hadron Gas of Point-Like particles



Cleymans et. al.:

$$\mu_B = \frac{1308}{1000 + 0.273\sqrt{s_{NN}}}$$

$$T = 166 - 139\mu_B^2 - 53\mu_B^4$$

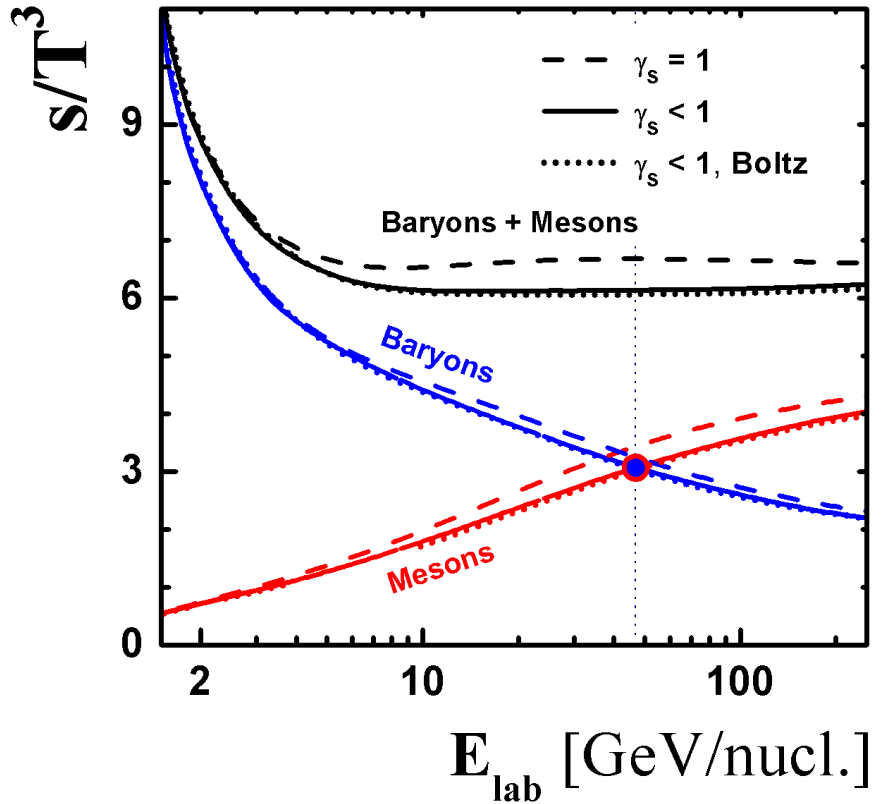
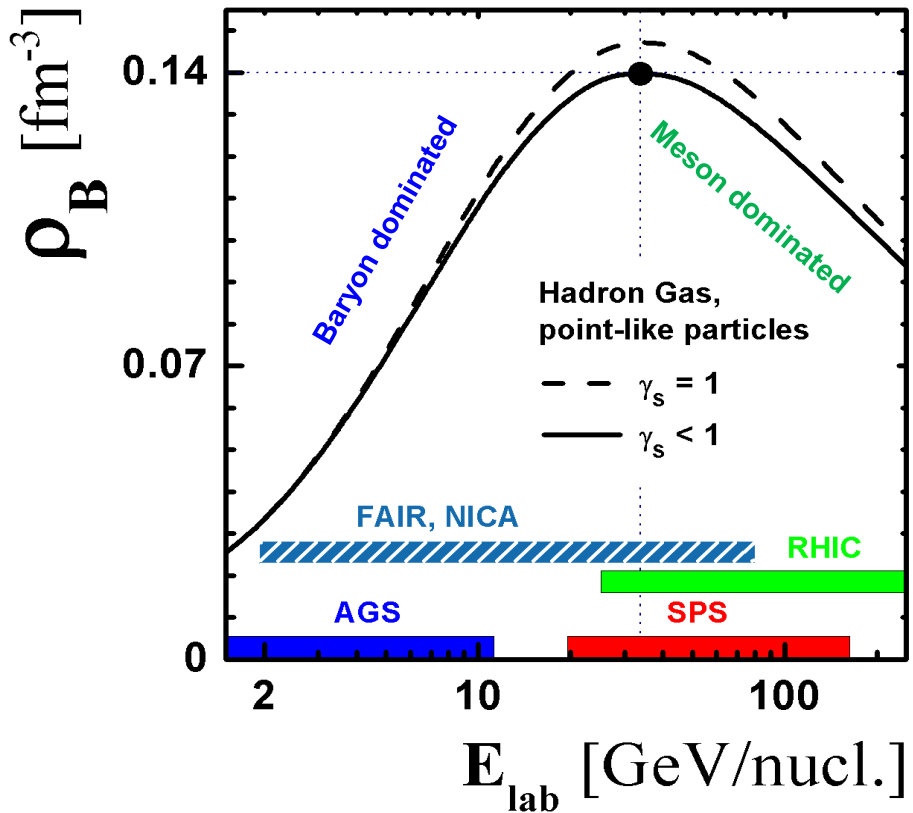
Becattini, Manninen, Gazdzicki  
PRC 73, 2006:

$$\gamma_S = 1 - 0.396 \exp\left(-1.23 \frac{T}{\mu_B}\right)$$

**We use this parameterization in Hadron Resonance Gas model implemented in THERMUS that includes all particles and resonances up to  $K_4^*(2045)$  meson and  $\Omega^-$  baryon, quantum statistics and width of resonances**

THERMUS: Wheaton, Cleymans  
Comp. Phys. 2009 [hep-ph/0407174]

# Net-Baryon and Entropy Densities



- Net-baryon density has a **maximum at 34A GeV**, where NA49 observed the signals of the onset of deconfinement
- With increasing collision energy the **baryon-dominated** matter changes to **meson-dominated** matter at **46A GeV**

# Excluded Volume HRG

The system volume should be substituted by the available volume  $V \rightarrow V - \sum_i v_i N_i$

where  $v_i$  is the excluded volume parameter  $v_i = 4 \cdot \frac{4}{3} \pi r_i^3$

and  $r_i$  is the corresponding hard sphere radius of a particle  $i$

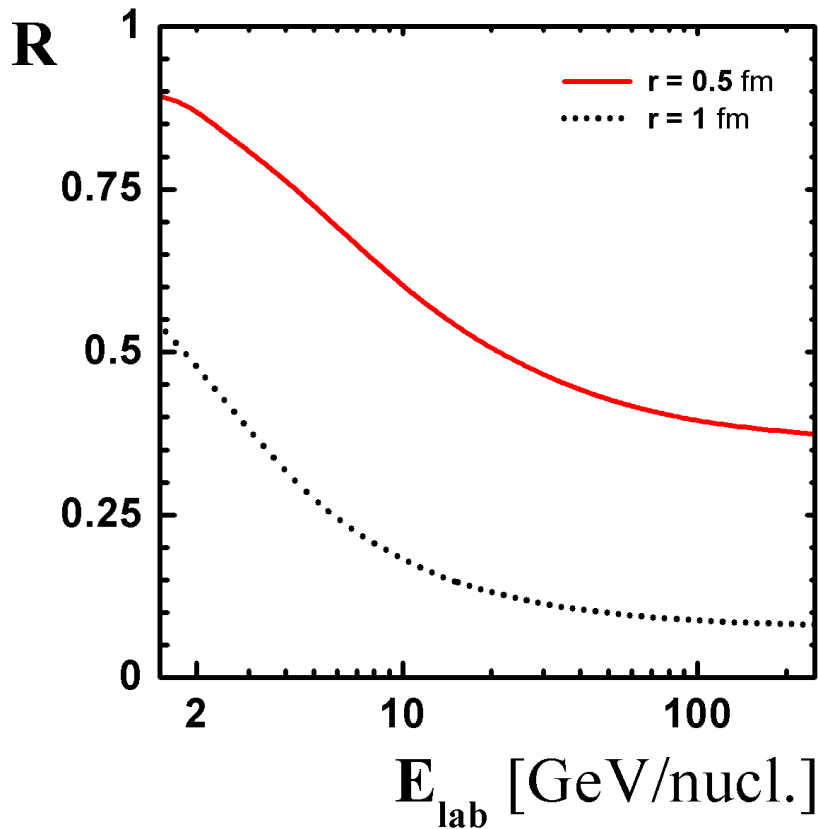
In the grand canonical ensemble, this substitution leads to a transcendental **equation for the pressure**

$$p = \sum_i p_i^{\text{id}}(T, \tilde{\mu}_i) \quad \tilde{\mu}_i = \mu_i - v_i p$$

and modified density

$$n_i = \frac{n_i^{\text{id}}(T, \tilde{\mu}_i)}{1 + \sum_i v_i n_i^{\text{id}}(T, \tilde{\mu}_i)}$$

# Suppression factor



If all particles have the same volume then in Boltzmann approximation:

$$R(T, \mu_B; r) = \frac{\exp(-v p/T)}{1 + v \sum_j n_j^{\text{id}}(T, \tilde{\mu}_j)}$$

Particle ratios cancel this factor

Mean multiplicities absorb it in volume

However densities change:

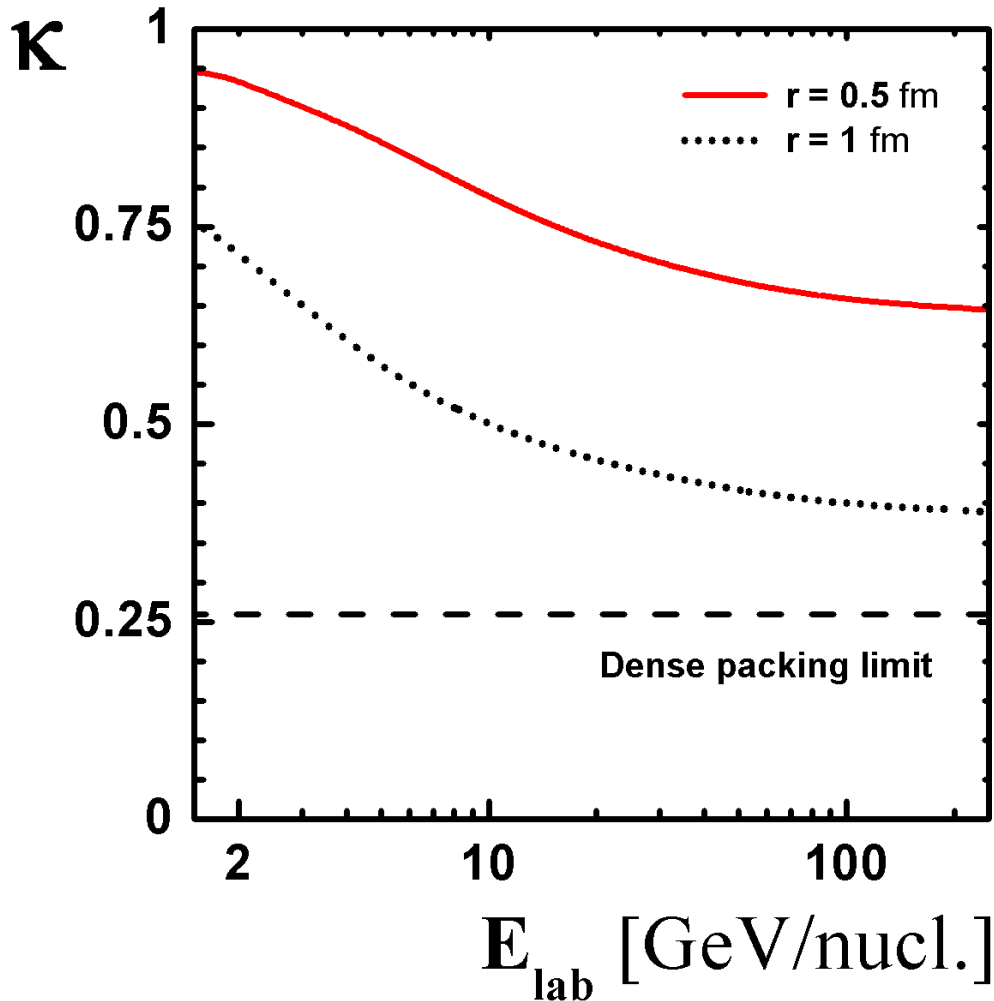
$$\rho_B(T, \mu_B) = R \rho_B^{\text{id}}(T, \mu_B)$$

$$s(T, \mu_B) = R s^{\text{id}}(T, \mu_B)$$

$$\varepsilon(T, \mu_B) = R \rho_B^{\text{id}}(T, \mu_B)$$

Typical values of hard-core radii considered in the literature are  $r = 0.3 - 0.8$  fm. One may therefore expect a decrease of the density maximum and a shift of its position to a smaller collision energy

# Available Volume



$$\kappa = \frac{V - v \sum_i N_i}{V} = \exp\left(\frac{v p}{T}\right) R$$

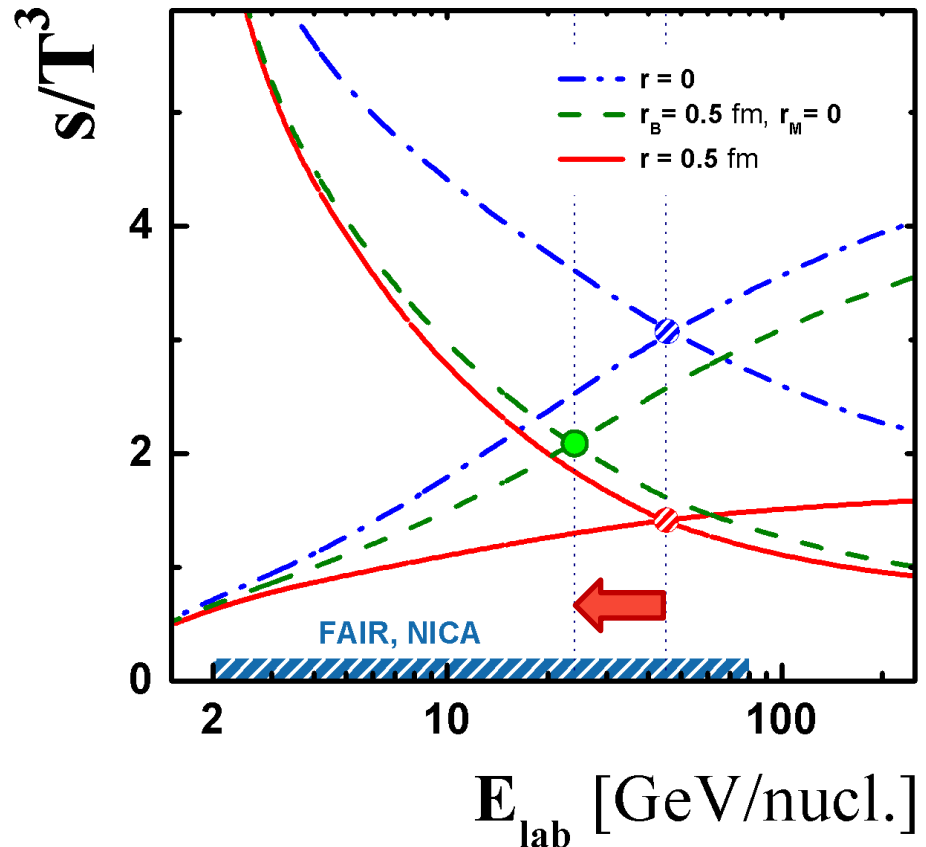
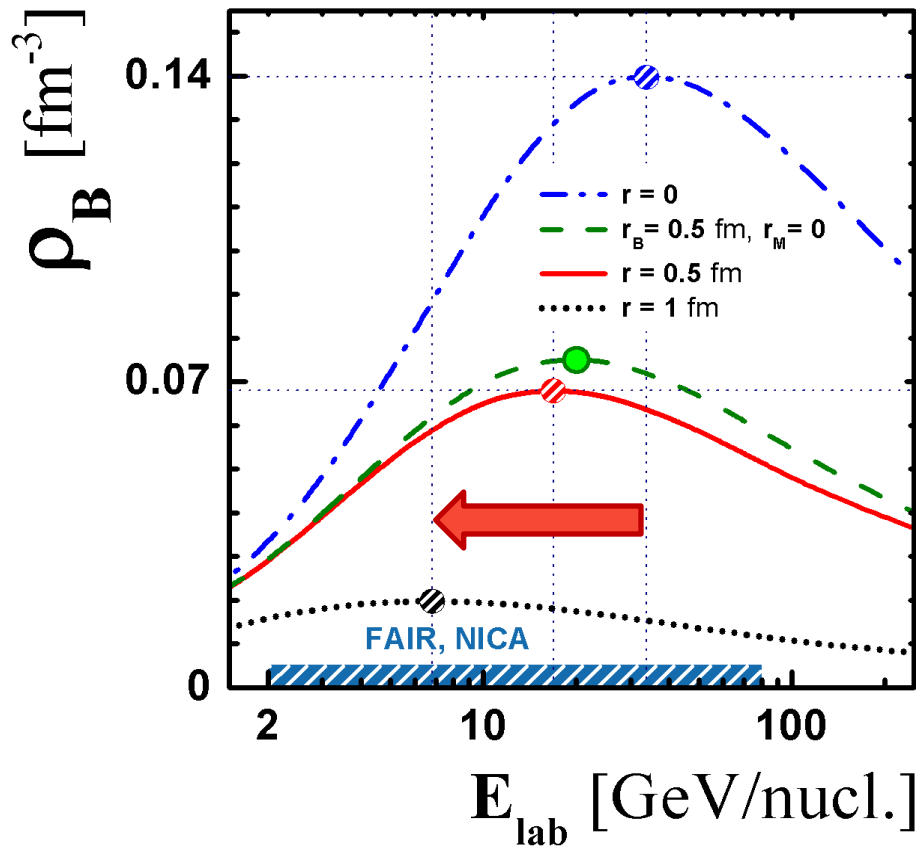
It is always larger than the dense packing limit for hard spheres.

We also remind that the excluded volume parameter  $v$  is assumed to be 4 times larger than the hadron volume

This ensures a consistency of the excluded volume approach at all collision energies even for the largest considered radius  $r = 1$  fm

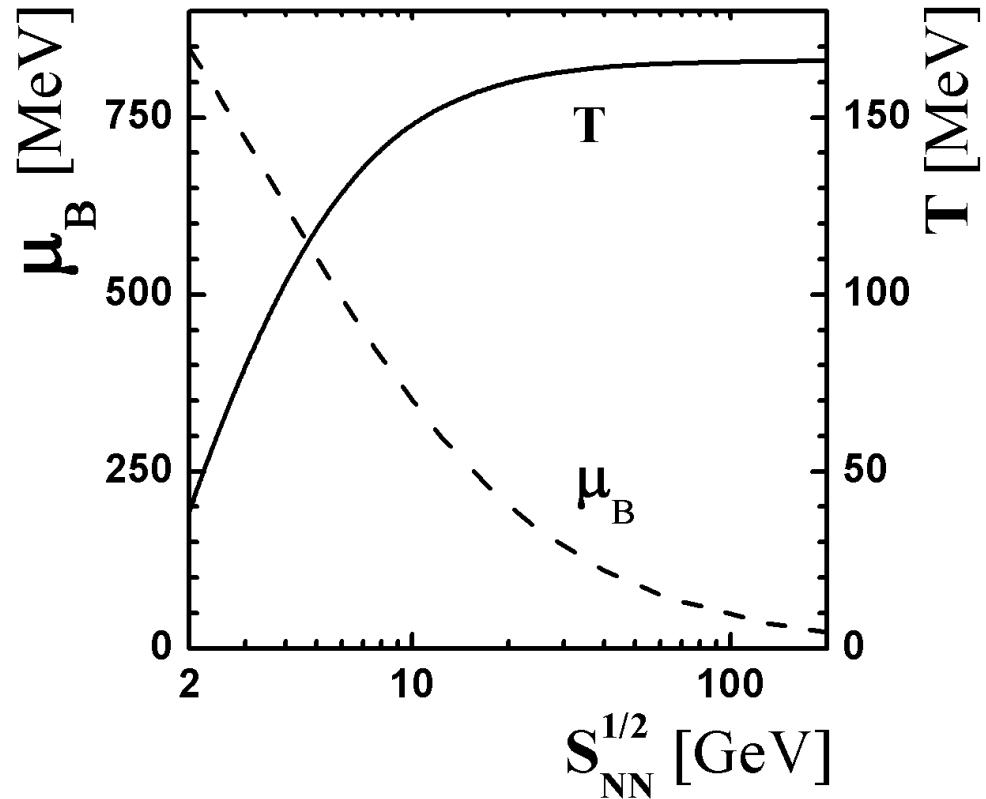
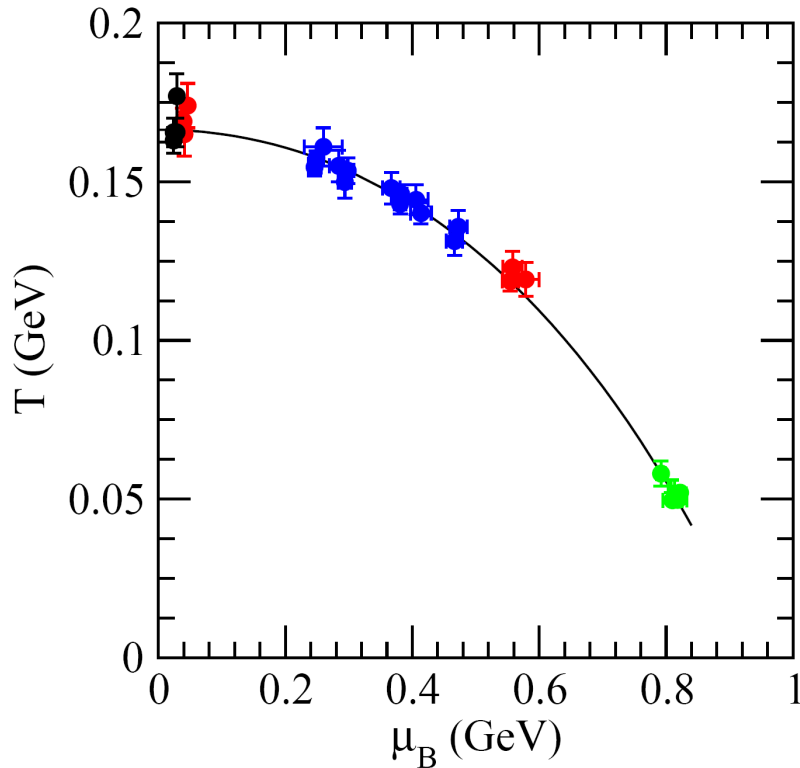


# Excluded Volume Effects



- **Density maximum moves from 34 to 17 and 7 AGeV**
- **Transition point of baryon/meson domination moves from 46 to 23 AGeV**
- **One should refit data and obtain new freeze-out line to conclude about effects arising due to different baryon and meson radiuses**

# Freeze-out line for $r=0$ or $r_B=r_M$

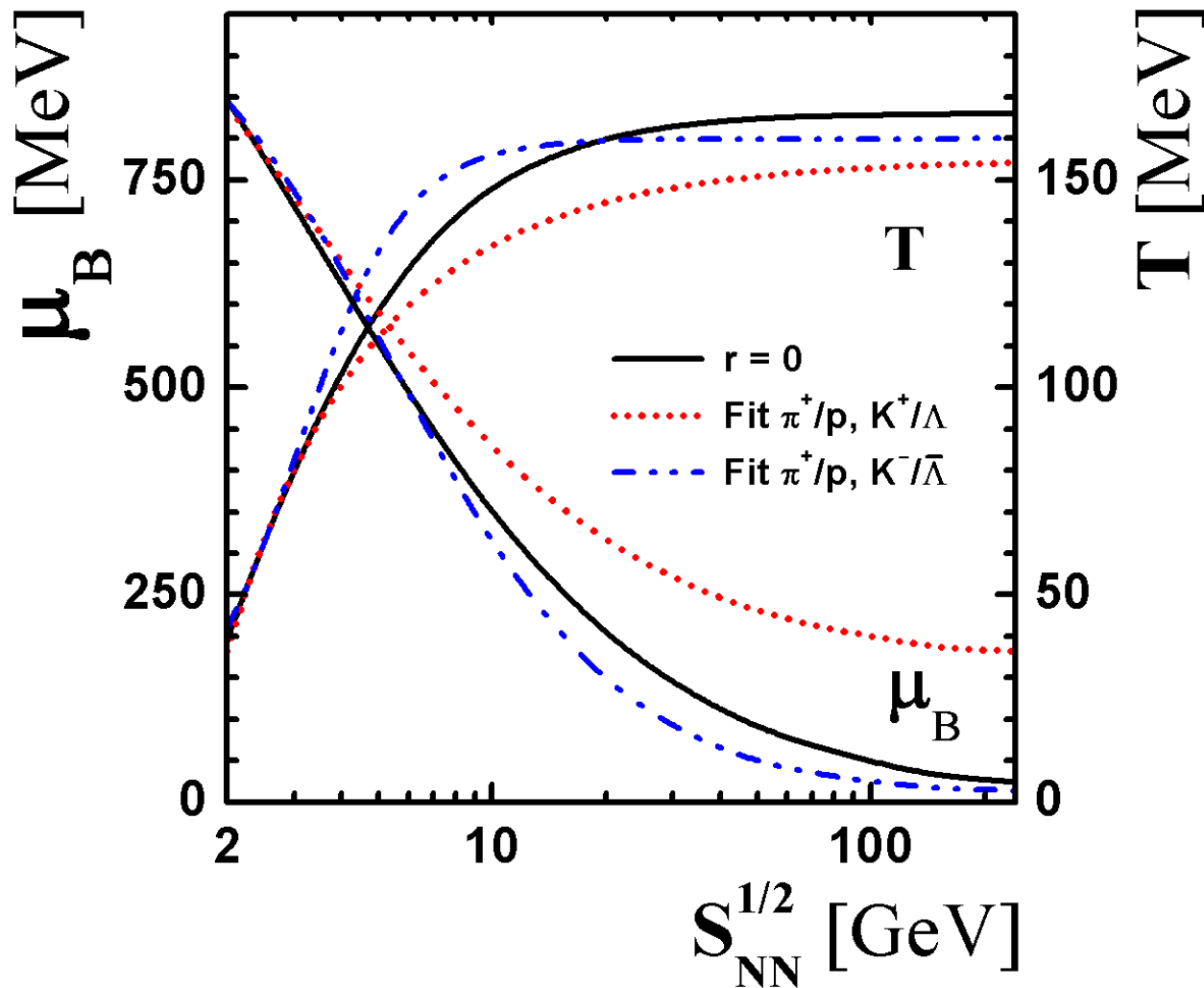


The  $r=0$  line was obtained as a fit of the  $T(\mu_B)$  from different groups.

The new fit in a single model (THERMUS) gives a different line.

For simplicity we take two ratios obtained from the  $r=0$  freeze-out line and fit them with  $r_B=0.5$  fm,  $r_M=0$

# New fit with $r_B=0.5$ fm, $r_M=0$



The new fit gives very different freeze-out lines because

$$\frac{\pi^+}{p}, \frac{K^+}{\Lambda} \sim e^{-\frac{\mu_B}{T}}$$

↓

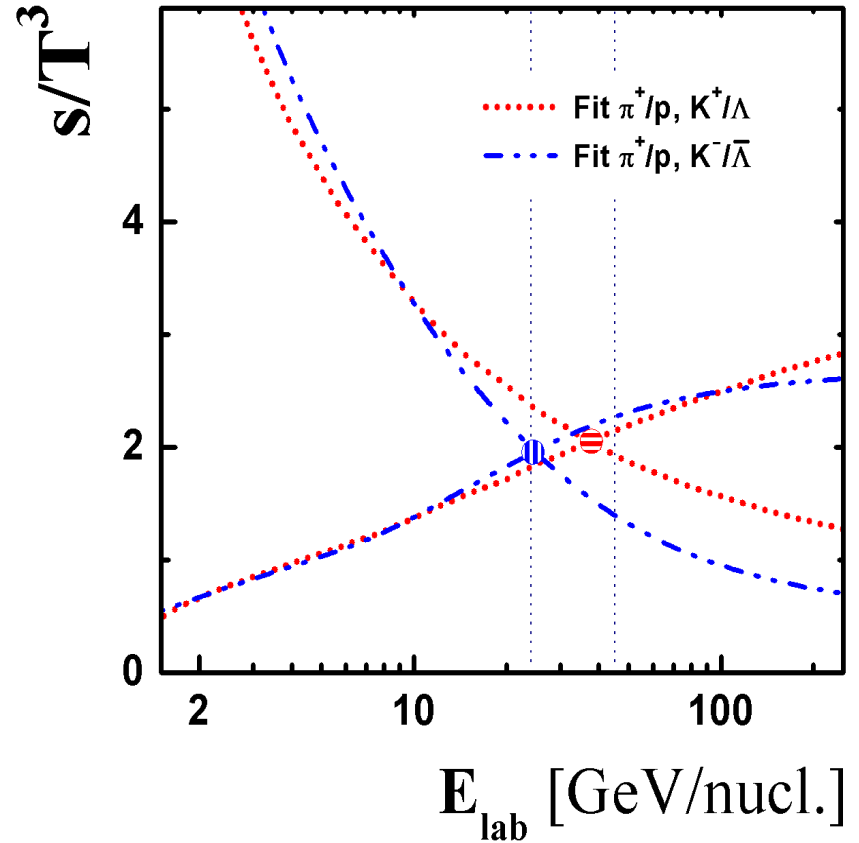
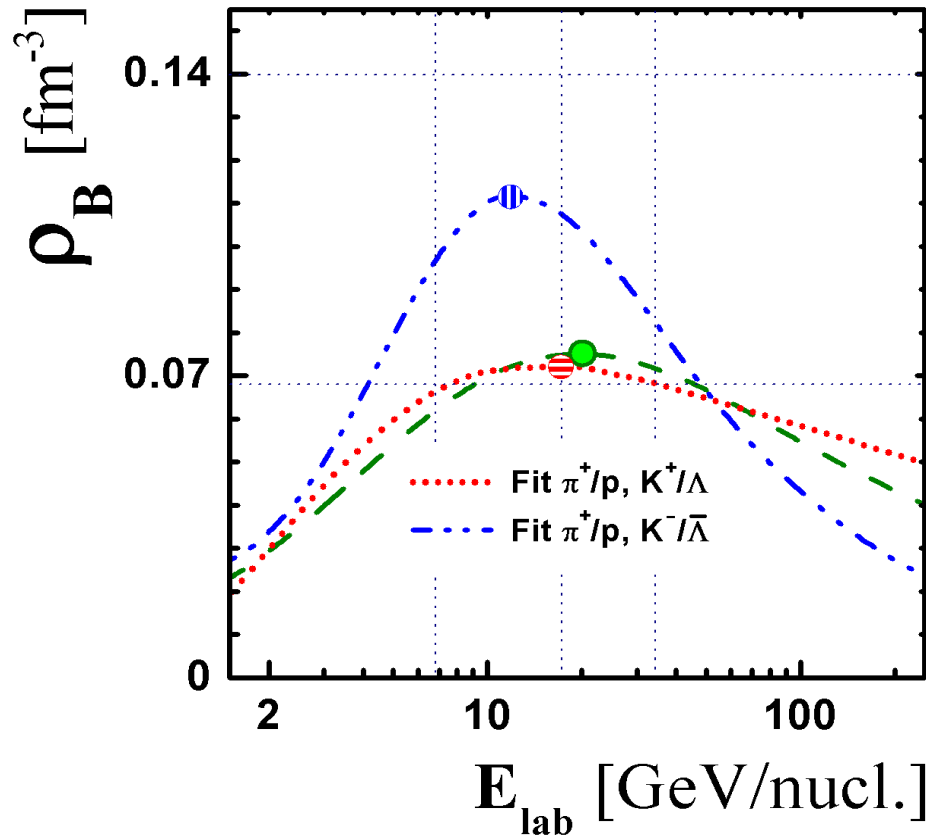
$\frac{\mu_B}{T}$  increases

$$\frac{K^-}{\bar{\Lambda}} \sim e^{+\frac{\mu_B}{T}}$$

↓

$\frac{\mu_B}{T}$  decreases

# New fit with $r_B=0.5$ fm, $r_M=0$



The new fit changes the details and preserves the main features: change in the position of density maximum and baryon/meson transition point.



# Conclusions:

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- **Excluded Volume** effects lead to **smaller** particle number and entropy **densities**
- The position of the net baryon density **maximum** moves to **lower energies**
- The **transition point** from baryon to meson domination lies at **higher energies** than **density** maximum
- The transition point **moves to lower energies** only for **different meson and baryon radiuses** in the system
- **Further research is needed**



A lush, moss-covered waterfall cascades over several tiers of large, flat rock formations in a dense forest. Sunlight filters through the trees, creating dappled light on the water and rocks. The water flows into a shallow pool at the bottom, surrounded by more mossy rocks and greenery.

**Thank you!**