Anomalous dimension of the chiral condensate from the Dirac-operator spectrum

Agostino Patella

Dirac operator

$$\boxed{\mathcal{L} = \frac{1}{2g^2}\, \mathrm{tr}\, F_{\mu\nu}^2 + \bar{\psi}\,(\not{D} + m)\,\psi}$$

(EUCLIDEAN SPACE - TIME)

NONPERTURBATIVE QUARK PROPAGATOR

$$\underbrace{\psi(x)\,\bar{\psi}(y)} = \frac{1}{\not{D} + m}\,(x,y)$$

SPECTRAL DENSITY $\qquad \not{D}^{+} = -\not{D}$

$$\not{D}\,\varphi_n = i\lambda_n[A]\,\varphi_n$$

$$\rho(\lambda) = \frac{1}{V}\left\langle \sum_n \delta(\lambda - \lambda_n) \right\rangle$$

HORRIBLY NONLOCAL

PHYSICS ?

RENORMALIZATION ?

Spectral density and quark condensate

BANKS, CASHER
DEL DEBBIO, ZWICKY
DE GRAND
[...]

$$\rho(\lambda) = \frac{1}{V}\left\langle \sum_n \delta(\lambda - \lambda_n)\right\rangle$$

$$\langle \bar{\psi}\psi \rangle = -\frac{N_f}{V}\left\langle \mathrm{Tr}\frac{1}{\not{D}+m}\right\rangle_m = -2mN_f\int_0^\infty \frac{\rho(\lambda,m)\,d\lambda}{\lambda^2+m^2}$$

CHIRAL CONDENSATE $\quad \Sigma = \frac{1}{N_f}\lim_{m\to 0}\langle\bar{\psi}\psi\rangle$

BANKS - CASHER RELATION

$$\boxed{\Sigma = -\pi \lim_{\lambda\to 0}\lim_{m\to 0}\rho(\lambda,m)}$$

IR - CONFORMALITY

$$\lim_{m\to 0}\rho(\lambda,m) \sim A\lambda^{\frac{3-\gamma_*}{1+\gamma_*}}$$

FOR SMALL λ

Renormalization

LÜSCHER, GIUSTI

RENORMALIZATION $\Longleftarrow$ ANALYSIS OF DIVERGENCIES

DIVERGENCIES OF COMPOSITE LOCAL OPERATORS
$\Longleftarrow$ OPERATOR PRODUCT EXPANSION

OPE IN PERTURBATIVE EXPANSION
$\Longleftarrow$ DIMENSIONAL ANALYSIS

STRATEGY: WRITE THE SPECTRAL DENSITY IN TERMS OF n-POINT FUNCTIONS OF LOCAL OPERATORS

Renormalization

LÜSCHER, GIUSTI

$$\rho(\lambda) = \frac{1}{V}\left\langle \sum_n \delta(\lambda - \lambda_n)\right\rangle$$

$$\langle \bar{q} q\rangle = -\frac{N_f}{V}\left\langle \mathrm{Tr}\,\frac{1}{\not{D}+m}\right\rangle_m = -2m\int_0^\infty \frac{\rho(\lambda, m)\,d\lambda}{\lambda^2 + m^2}$$

INTRODUCE A VALENCE QUARK b WITH MASS m_b
(= QUENCHED = PROBE-BRANE)

$$\langle \bar{b} b\rangle = -2m_b\int_0^\infty \frac{\rho(\lambda, m)\,d\lambda}{\lambda^2 + m_b^2}$$

$$\langle \bar{b} b\rangle(m_b = i\lambda + \varepsilon) - \langle \bar{b} b\rangle(m_b = i\lambda - \varepsilon) = -2\pi\rho(\lambda, m)$$

WHEN WE LOOK AT THE SPECTRAL DENSITY WE JUST LOOK AT n-POINT FUNCTIONS OF LOCAL BILINEARS OF VALENCE QUARKS

Renormalization

LÜSCHER, GIUSTI

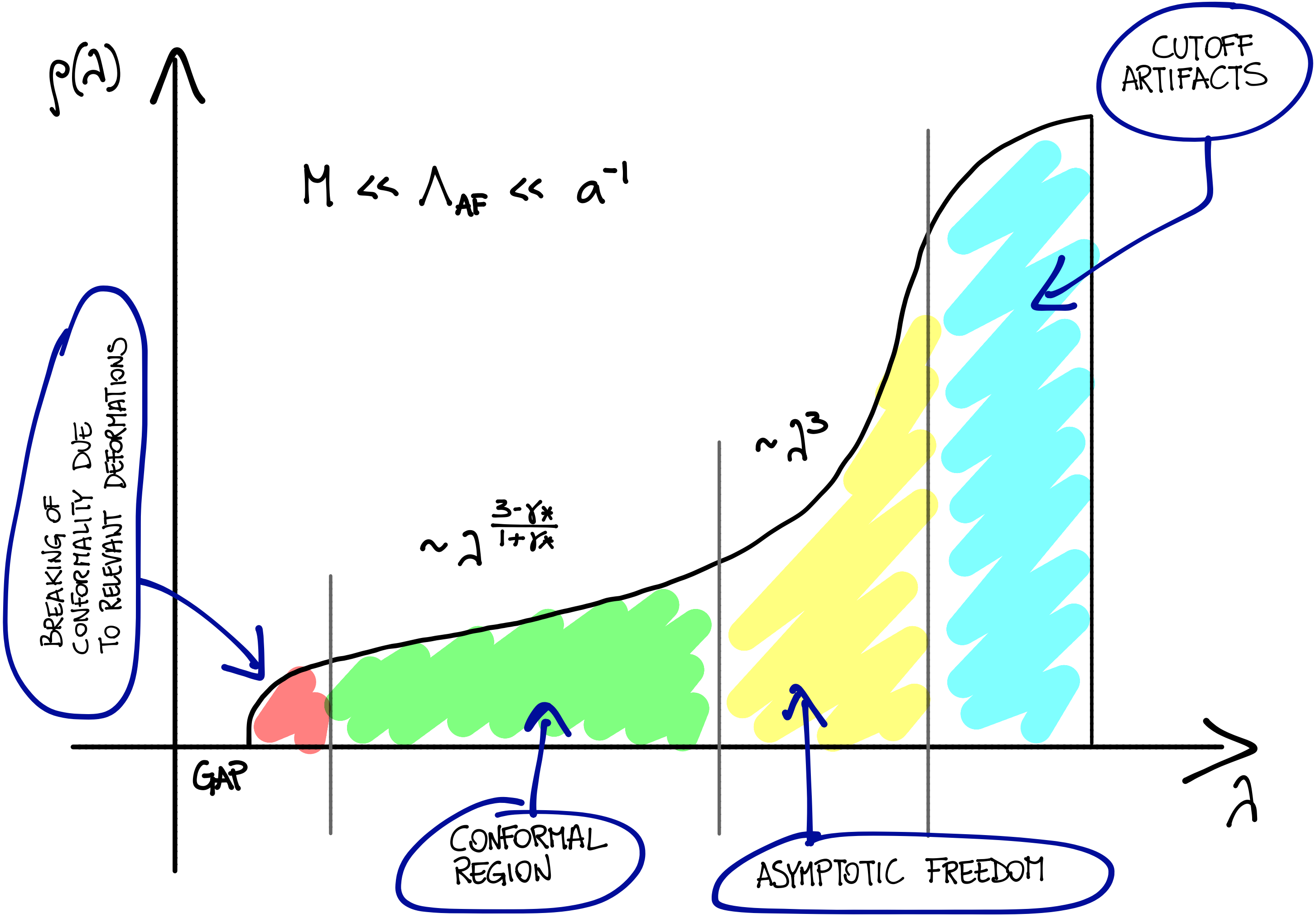
$$\rho_R(\lambda_R, m_R) = \lim_{\varepsilon \to 0} Z_m \, \rho(Z_m \lambda_R, Z_m m_R)$$

FINITE @ ALL ORDERS OF PERTURBATIVE EXP.

CONFORMAL THEORY

$$\rho_R(\lambda_R) = s^{-3+\gamma_*} \rho(s^{1+\gamma_*} \lambda_R)$$

$$\rho_R(\lambda_R) = \left(\frac{\lambda_R}{\lambda_0}\right)^{\frac{3-\gamma_*}{1+\gamma_*}} \rho(\lambda_0)$$



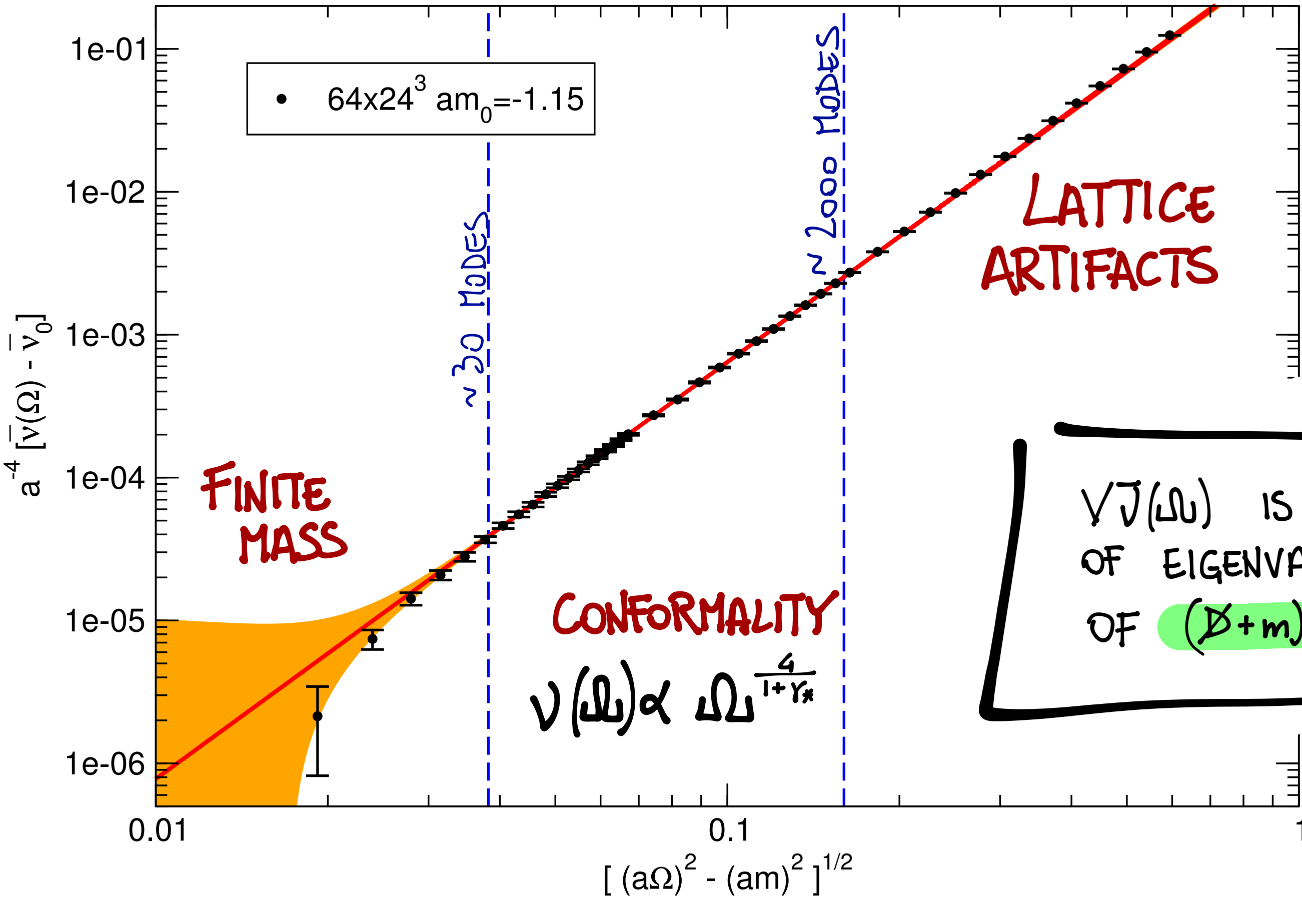
$\rho(\lambda)$
$M \ll \Lambda_{AF} \ll a^{-1}$
CUTOFF ARTIFACTS
$\sim \lambda^3$
$\sim \lambda^{\frac{3-\gamma_*}{1+\gamma_*}}$
BREAKING OF CONFORMALITY DUE TO RELEVANT DEFORMATIONS
GAP
λ
CONFORMAL REGION
ASYMPTOTIC FREEDOM

Mode number

$$\bar{\nu}(\Omega) = 2\int_0^{\sqrt{\Omega^2-m^2}} \rho(\lambda,m)\,d\lambda$$

SU(2) + 2 ADJ DIRAC FERMIONS

$\gamma_* = 0.371(20)$



$V\bar{J}(\Omega)$ IS THE NUMBER OF EIGENVALUES $\leq \Omega^2$ OF $(\not{D}+m)^{\dagger}(\not{D}+m)$

Conclusions

* THE SPECTRAL DENSITY RENORMALIZES MULTIPLICATIVELY

* IT PROBES THE PROPAGATION OF (POINT-LIKE) VALENCE QUARKS

* ITS ANOMALOUS DIMENSION IS EQUAL TO THE MASS ANOMALOUS DIMENSION

* IT PROVIDES A CONVENIENT WAY TO CALCULATE γ_* FROM NUMERICAL SIMULATIONS (BUT LOTS OF EIGENVALUES ARE REQUIRED!)