Anomalous Dimensions of Four-Fermion Operators from Conformal EWSB Dynamics

Carlos Pena

in collaboration with:
L Del Debbio
L Keegan

outline

- motivation
  - Higgs couplings, flavour
- method
  - renormalisation of four-fermion operators
  - Schrödinger Functional techniques
- preliminary results for Minimal Walking Technicolour

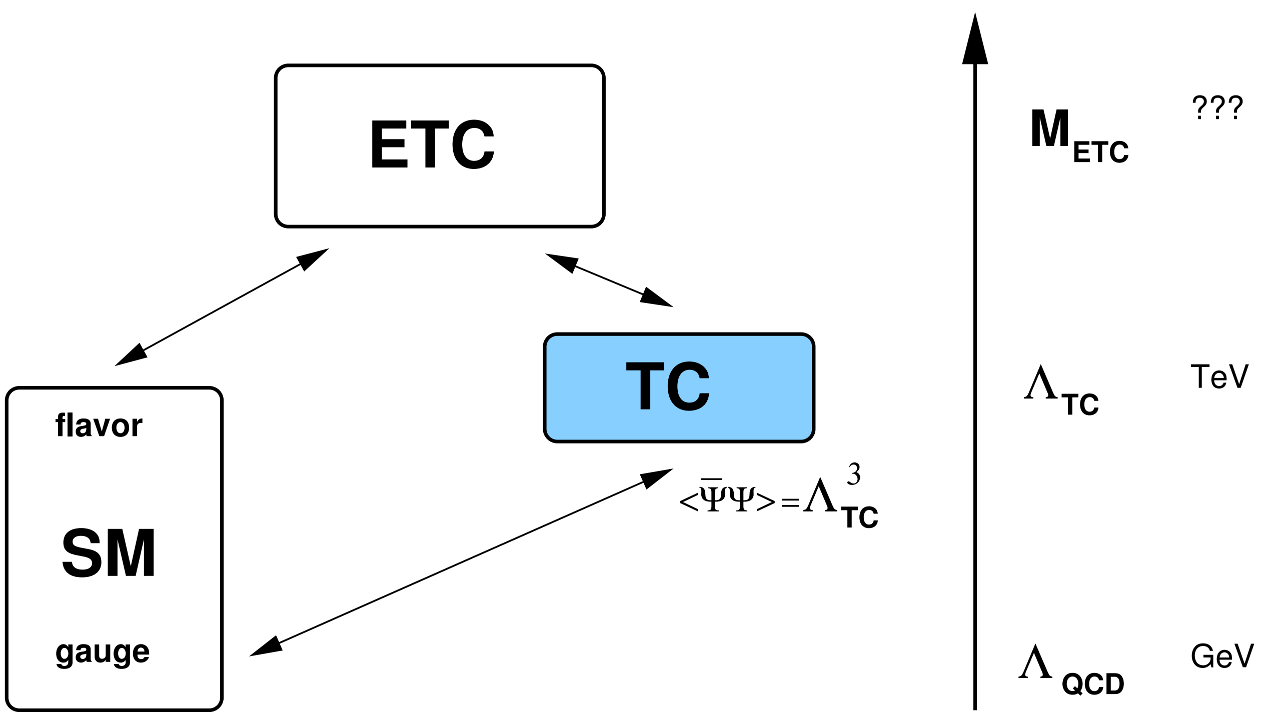
strong conformal EWSB + flavour

- strong interactions break $SU(2)_L$, generate small *W* mass ⇒ break EW symmetry with some UV QCD-like dynamics: technicolour

[Weinberg, Susskind 78]

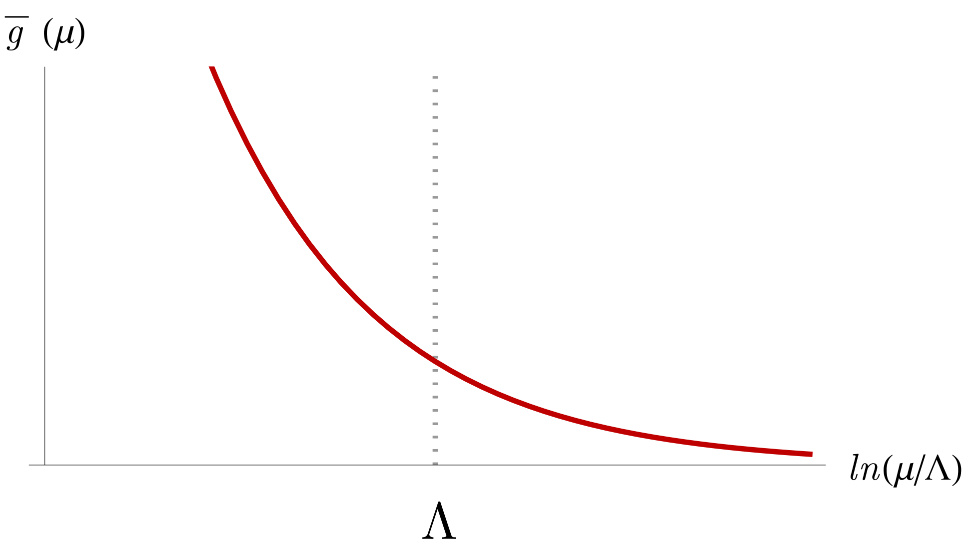
- potential to generate fermion masses and mixings, but realistic models require more elaborate dynamics (extended technicolor)

[Dimopoulos, Susskind 79; Eichten, Lane 80]



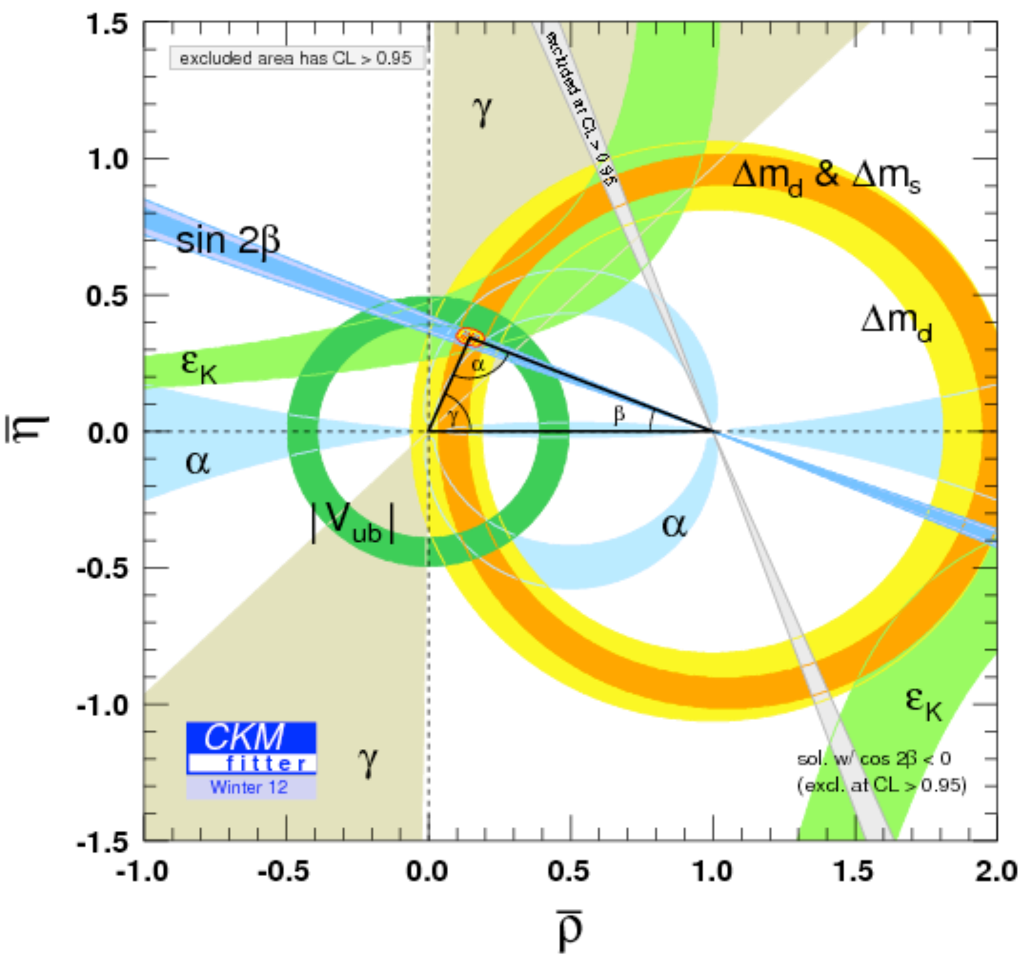
[sketch by L Del Debbio]

strong conformal EWSB + flavour

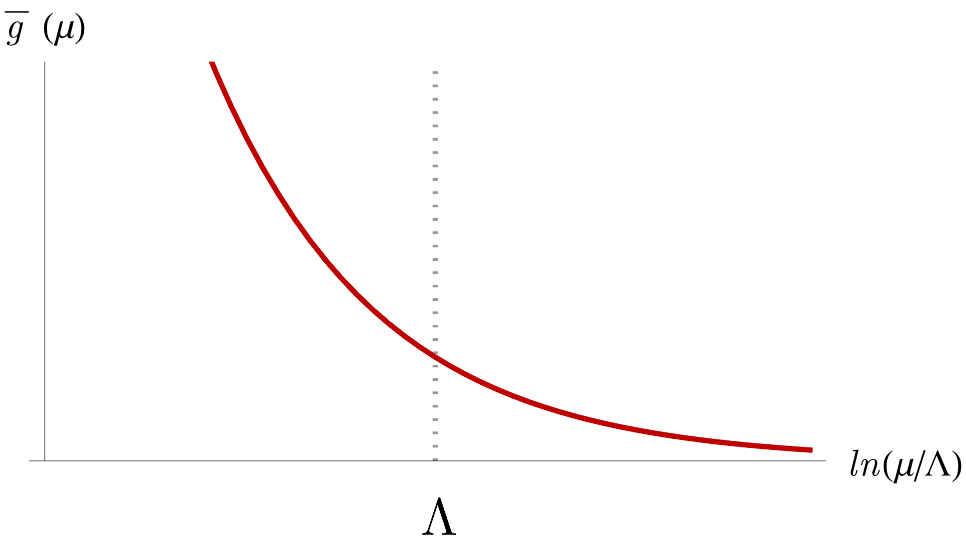


[Weinberg, Susskind 78]
[Dimopoulos, Susskind 79; Eichten, Lane 80]

- EWSB from strong dynamics
- Higgs?
- well-defined scale, log running
  ⇒ flavour hierarchies unnatural
- suppression mechanism for FCNC needed



strong conformal EWSB + flavour



[Weinberg, Susskind 78]
[Dimopoulos, Susskind 79; Eichten, Lane 80]

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today: direct CP-violation in B-decay by LHCb

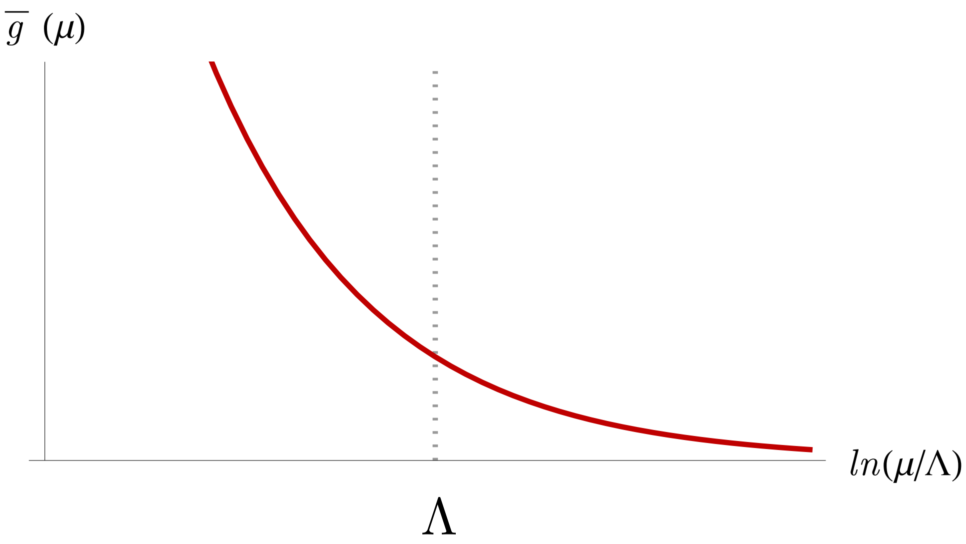
$$A_{CP}(B_s^0 \to K^-\pi^+) = 0.27 \pm 0.04\,(\text{stat}) \pm 0.01\,(\text{syst})$$

$$A_{CP}(B^0 \to K^+\pi^-) = -0.080 \pm 0.007\,(\text{stat}) \pm 0.003\,(\text{syst})$$

$$\Delta = \frac{A_{CP}(B^0 \to K^+\pi^-)}{A_{CP}(B_s^0 \to K^-\pi^+)} + \frac{\mathcal{B}(B_s^0 \to K^-\pi^+)}{\mathcal{B}(B^0 \to K^+\pi^-)}\frac{\tau_d}{\tau_s} = 0$$

$$\Delta = -0.02 \pm 0.05 \pm 0.04$$

strong conformal EWSB + flavour



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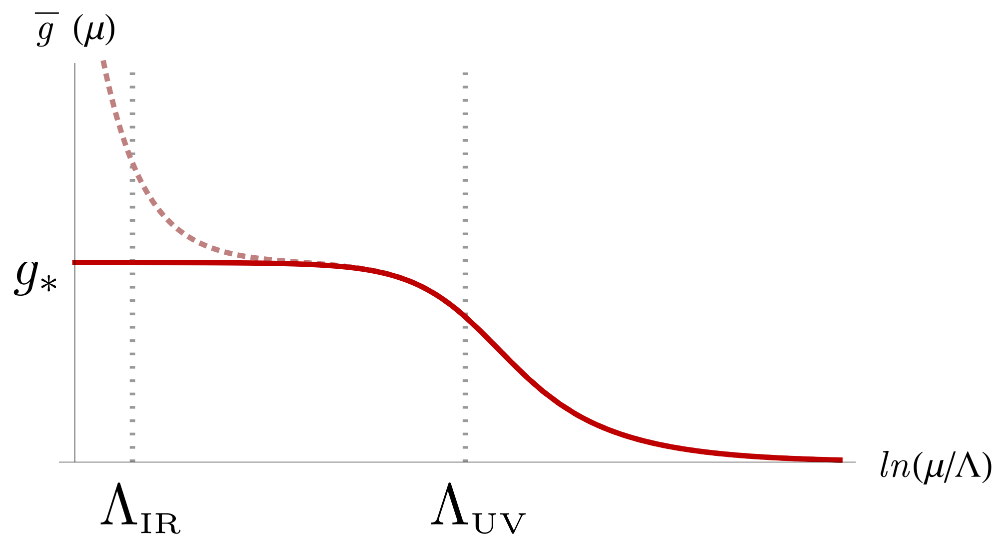


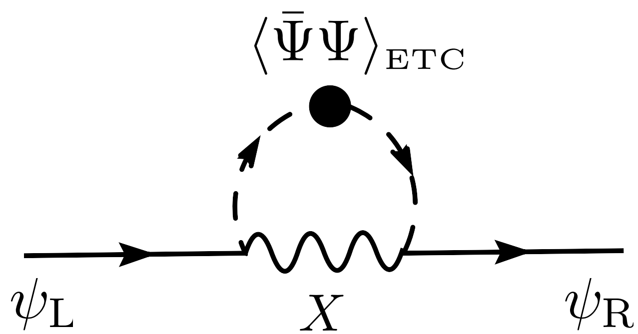
[Holdom 81, 85]
[Yamawaki et al. 86; Akiba, Yanagida 86]
[Appelquist, Wijewardhana 87]
[Appelquist, Sannino 99]

- EWSB from strong dynamics
- composite Higgs: dilaton?
- scale-invariant window ⇒
  flavour hierarchies
- flavour dynamics?

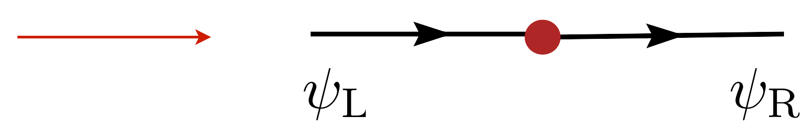
model building blocks

fermion masses

- TC condensates ⇒ masses



$$m_q \sim c(\mu/M_X)\,\frac{\Sigma_{\mathrm{ETC}}(\mu)}{M_X^2}$$



- (moderately large) anomalous dimensions + conformality ⇒ hierarchies

$$\frac{\Sigma(\Lambda_{\mathrm{UV}})}{\Sigma(\Lambda_{\mathrm{IR}})} \sim \log\left(\frac{\Lambda_{\mathrm{UV}}}{\Lambda_{\mathrm{IR}}}\right)^{\gamma} \qquad \text{vs.} \qquad \frac{\Sigma(\Lambda_{\mathrm{UV}})}{\Sigma(\Lambda_{\mathrm{IR}})} \sim \left(\frac{\Lambda_{\mathrm{UV}}}{\Lambda_{\mathrm{IR}}}\right)^{\gamma}$$

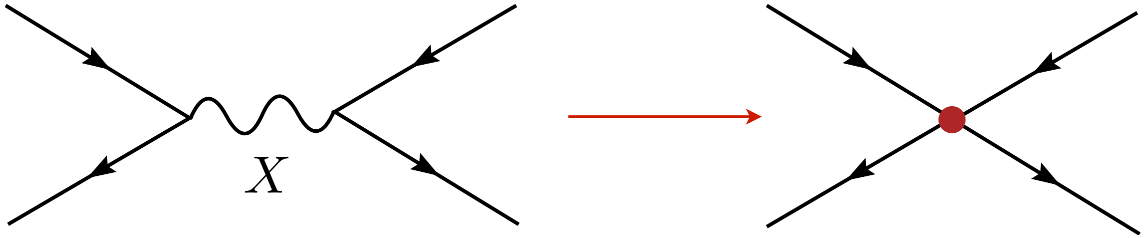
model building blocks

fermion couplings

- matrix elements ⇒ modify CKM (resp. PMNS), Higgs couplings, ...

  respect experimental constraints, suppress FCNC

$$\mathcal{L}_{\rm SM} + \frac{c(\mu/M_X)}{M_X^2}\,[\bar\psi\Gamma\psi][\bar\psi\Gamma'\psi](\mu)$$



- anomalous dimensions ⇒ hierarchies

$$\frac{\mathcal{O}(\Lambda_{\rm UV})}{\mathcal{O}(\Lambda_{\rm IR})} \sim \log\left(\frac{\Lambda_{\rm UV}}{\Lambda_{\rm IR}}\right)^{\gamma} \qquad \text{vs.} \qquad \frac{\mathcal{O}(\Lambda_{\rm UV})}{\mathcal{O}(\Lambda_{\rm IR})} \sim \left(\frac{\Lambda_{\rm UV}}{\Lambda_{\rm IR}}\right)^{\gamma}$$

example 1: Higgs-Yukawa sector

[cf. Rattazzi, Rychkov, Tonni, Vichi 08]
[cf. A Pomarol's talk]

leading correction to CFT action in top-Higgs sector [SM + strongly coupled CFT with (composite $H \sim \bar{\Psi}\Psi$) Higgs doublet]:

$$\mathcal{L}_{tH} = \frac{1}{16\pi^2}\lambda_t H\bar{Q}_L t_R + \text{h.c.} + \underbrace{\left(\frac{1}{16\pi^2}\right)^2 \lambda_t^2 \int \mathrm{d}^4x\mathrm{d}^4y\, H(x)^\dagger H(y)\bar{Q}_L t_R(x)\bar{t}_R Q_L(y)}$$

$$\approx \frac{1}{16\pi^2}\lambda_t^2 \Lambda_{\text{UV}}^{2+2d-\Delta_S} \int \mathrm{d}^4x\, H(x)^\dagger H(x)$$

$3 < \Delta_S < 4$ leads to relevant deformation of the CFT, strongly self-coupled Higgs at IR (EW) scale; scale dependence of V_{eff} determined by the value of Δ_S

example 2: flavour corrections

[cf. Rattazzi, Rychkov, Tonni, Vichi 08]
[cf. A Pomarol's talk]

same scenario: flavour violation parametrised by correction to quark propagation

$$\mathcal{L}_f = \frac{1}{16\pi^2}\left[\bar{q}\not{D}q + \underbrace{\lambda_t^2\frac{\Lambda_{\rm UV}^{2+2d-\Delta_S}}{\Lambda_{\rm UV}^2}}_{\equiv \frac{c}{\Lambda_F^2}}(\bar{q}q)^2\right]$$

Δ_S controls size of flavour couplings at IR (EW) scale $\Rightarrow$ FCNC suppression requires largish anomalous dimension

model building: determine anomalous dimensions of four-fermion operators in candidate EWSB theories

renormalisation of four-fermion operators

four-fermion operators (engineering dimension *d*=6) will mix under renormalisation with all other *d*≤6 operators with same transformation properties under all symmetries

$$\mathcal{O} = (\bar{\psi}\Gamma T\psi)(\bar{\psi}\Gamma' T'\psi)$$

spin flavour / colour

$\{\mathcal{O}_i\}$ operator basis in given symmetry sector

$$\bar{\mathcal{O}}_i(\mu) = \sum_j Z_{ij}(\mu)\mathcal{O}_j$$

mass-independent renormalisation schemes: mixing with lower-dimensional operators involves coefficients that do not depend on renormalisation scale — only on bare couplings and masses

[Testa 98]

renormalisation of four-fermion operators

$\Rightarrow$ scale dependence (anomalous dimensions) can be obtained by considering operators with four distinct flavours in chiral limit

[Donini, Giménez, Martinelli, Talevi, Vladikas 99]

full basis (after Fierzing):

$$Q_1^{\pm} \equiv \frac{1}{2}\left[(\bar{\psi}_1\gamma_\mu(\mathbf{1}-\gamma_5)\psi_2)(\bar{\psi}_3\gamma_\mu(\mathbf{1}-\gamma_5)\psi_4) \pm (2 \leftrightarrow 4)\right] \,,$$

$$Q_2^{\pm} \equiv \frac{1}{2}\left[(\bar{\psi}_1(\mathbf{1}-\gamma_5)\psi_2)(\bar{\psi}_3(\mathbf{1}-\gamma_5)\psi_4) \pm (2 \leftrightarrow 4)\right] \,,$$

$$Q_3^{\pm} \equiv \frac{1}{2}\left[(\bar{\psi}_1\gamma_\mu(\mathbf{1}-\gamma_5)\psi_2)(\bar{\psi}_3\gamma_\mu(\mathbf{1}+\gamma_5)\psi_4) \pm (2 \leftrightarrow 4)\right] \,,$$

$$Q_4^{\pm} \equiv \frac{1}{2}\left[(\bar{\psi}_1(\mathbf{1}-\gamma_5)\psi_2)(\bar{\psi}_3(\mathbf{1}+\gamma_5)\psi_4) \pm (2 \leftrightarrow 4)\right] \,,$$

$$Q_5^{\pm} \equiv \frac{1}{2}\left[(\bar{\psi}_1\sigma_{\mu\nu}(\mathbf{1}-\gamma_5)\psi_2)(\bar{\psi}_3\sigma\mu\nu(\mathbf{1}-\gamma_5)\psi_4) \pm (2 \leftrightarrow 4)\right] \,,$$

renormalisation of four-fermion operators

⇒ scale dependence (anomalous dimensions) can be obtained by considering operators with four distinct flavours in chiral limit

[Donini, Giménez, Martinelli, Talevi, Vladikas 99]

full basis (after Fierzing):

$$\begin{aligned}
Q_1^{\pm} &= Q_{\rm VV+AA}^{\pm} - Q_{\rm VA+AV}^{\pm} \\
Q_2^{\pm} &= Q_{\rm SS+PP}^{\pm} - Q_{\rm SP+PS}^{\pm} \\
Q_3^{\pm} &= Q_{\rm VV-AA}^{\pm} + Q_{\rm VA-AV}^{\pm} \\
Q_4^{\pm} &= Q_{\rm SS-PP}^{\pm} + Q_{\rm SP-PS}^{\pm} \\
Q_5^{\pm} &= Q_{\rm TT}^{\pm} - Q_{\rm T\tilde{T}}^{\pm}
\end{aligned}$$

parity-even parity-odd

renormalisation of four-fermion operators

⇒ scale dependence (anomalous dimensions) can be obtained by considering operators with four distinct flavours in chiral limit

[Donini, Giménez, Martinelli, Talevi, Vladikas 99]

renormalisation pattern (dictated by discrete + flavour symmetries):

$$\bar{Q}_i^{\pm}(\mu) = \sum_{j=1}^{5} Z_{ij}^{\pm}(\mu) Q_j^{\pm}$$

$$Z^{\pm} = \underbrace{\begin{pmatrix} Z_{11}^{\pm} & 0 & 0 & 0 & 0 \\ 0 & Z_{22}^{\pm} & Z_{23}^{\pm} & 0 & 0 \\ 0 & Z_{32}^{\pm} & Z_{33}^{\pm} & 0 & 0 \\ 0 & 0 & 0 & Z_{44}^{\pm} & Z_{45}^{\pm} \\ 0 & 0 & 0 & Z_{54}^{\pm} & Z_{55}^{\pm} \end{pmatrix}}_{=f(\mu)} \underbrace{\begin{pmatrix} 1 & \Delta_{12}^{\pm} & \Delta_{13}^{\pm} & \Delta_{14}^{\pm} & \Delta_{15}^{\pm} \\ \Delta_{21}^{\pm} & 1 & 0 & \Delta_{24}^{\pm} & \Delta_{25}^{\pm} \\ \Delta_{31}^{\pm} & 0 & 1 & \Delta_{34}^{\pm} & \Delta_{35}^{\pm} \\ \Delta_{41}^{\pm} & \Delta_{42}^{\pm} & \Delta_{43}^{\pm} & 1 & 0 \\ \Delta_{51}^{\pm} & \Delta_{52}^{\pm} & \Delta_{53}^{\pm} & 0 & 1 \end{pmatrix}}_{\neq f(\mu)}$$

renormalisation of four-fermion operators

⇒ scale dependence (anomalous dimensions) can be obtained by considering operators with four distinct flavours in chiral limit

[Donini, Giménez, Martinelli, Talevi, Vladikas 99]

renormalisation pattern (dictated by discrete + flavour symmetries):

$$\bar{Q}_i^{\pm}(\mu) = \sum_{j=1}^{5} Z_{ij}^{\pm}(\mu) Q_j^{\pm}$$

$$\Delta_{ij}^{\pm} = 0$$

in parity-even sector only if chiral symmetry exactly preserved

in parity-odd sector *always*, even if chiral symmetry broken by regularisation [⇒ useful for lattice studies with Wilson fermions]

renormalisation of four-fermion operators

$\Rightarrow$ scale dependence (anomalous dimensions) can be obtained by considering operators with four distinct flavours in chiral limit

[Donini, Giménez, Martinelli, Talevi, Vladikas 99]

renormalisation pattern (dictated by discrete + flavour symmetries):

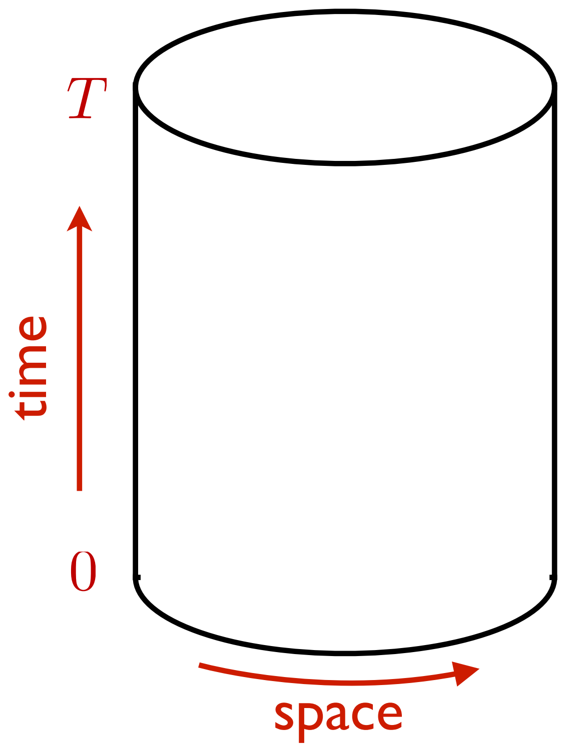
$$\bar{Q}_i^{\pm}(\mu) = \sum_{j=1}^{5} Z_{ij}^{\pm}(\mu) Q_j^{\pm}$$

aim: determine anomalous dimensions of four-fermion operators in candidate conformal QFTs of interest, using lattice field theory techniques

non-perturbative RG running

non-perturbative renormalisation: Schrödinger Functional in Euclidean space

[Lüscher, Jansen, Narayanan, Sint, Sommer, Weisz, Wolff 91-96]
[ALPHA, 96-]



$$e^{-\Gamma} = \int D[A, \bar{\psi}, \psi] \exp\{-S[A, \bar{\psi}, \psi]\}$$

Dirichlet boundary conditions in time; abelian background gauge field controlled by parameter η

periodic boundary conditions in space (up to global phase)

renormalised coupling: response to change in bkg field

$$\bar{g}^2(\mu = L^{-1}) = k \left(\frac{\partial \Gamma}{\partial \eta} \right)^{-1}$$

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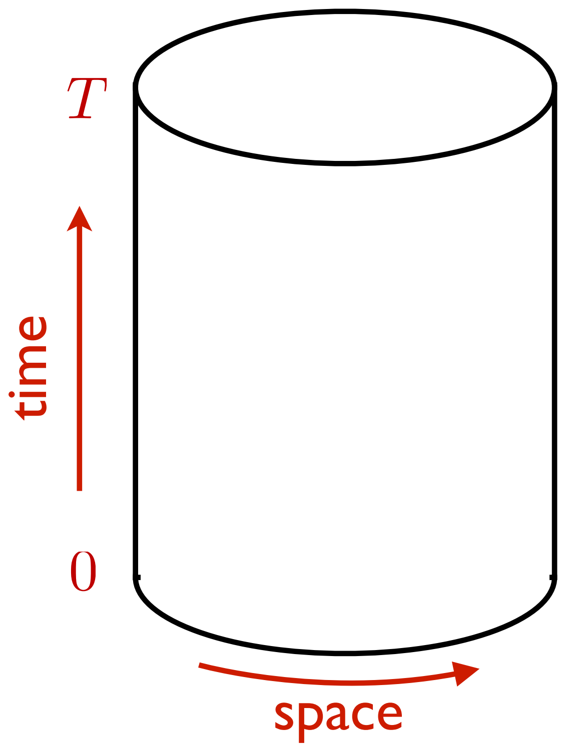

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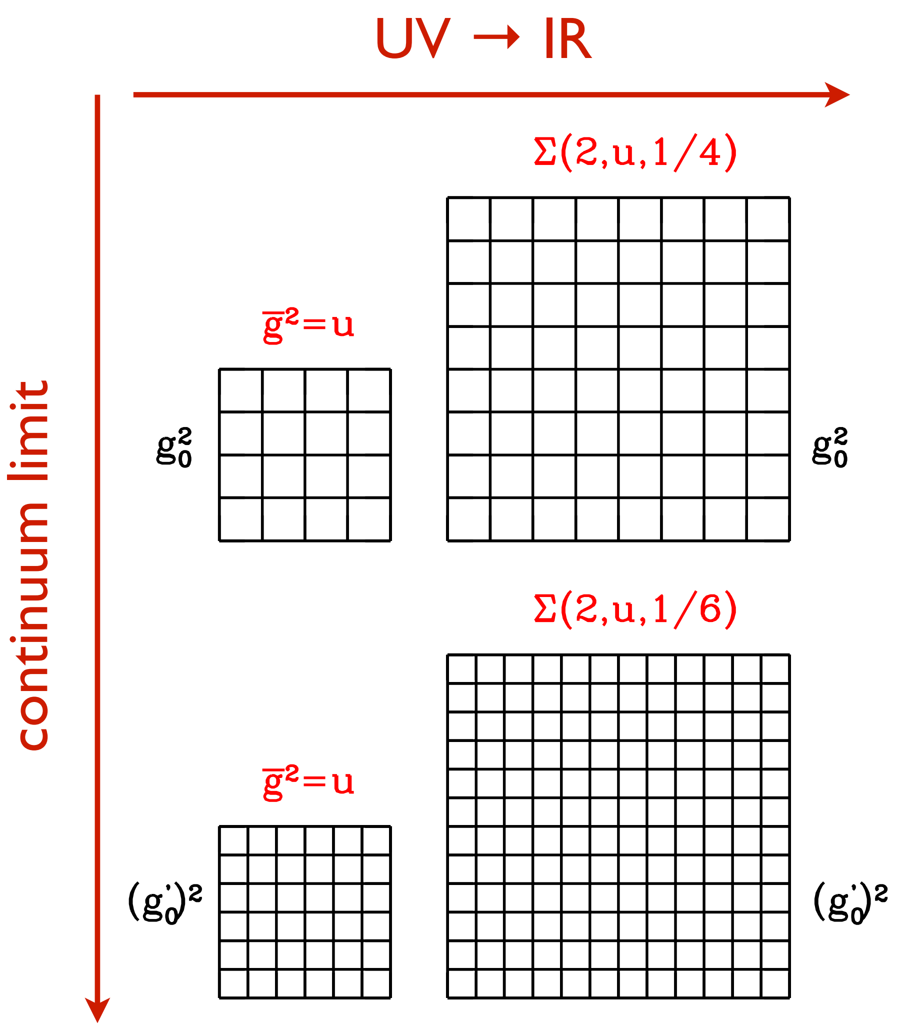
advantages:

- fully non-perturbative, easily implemented on the lattice
- allows to compute with massless fermions ⇒ mass-independent schemes
- allows to compute RG non-perturbatively via step-scaling technique

step-scaling: running coupling

- compute at fixed value of the coupling ⇔ *L* for several lattice spacings, take continuum limit
- change coupling such that *L* changes in fixed steps of *s* and iterate



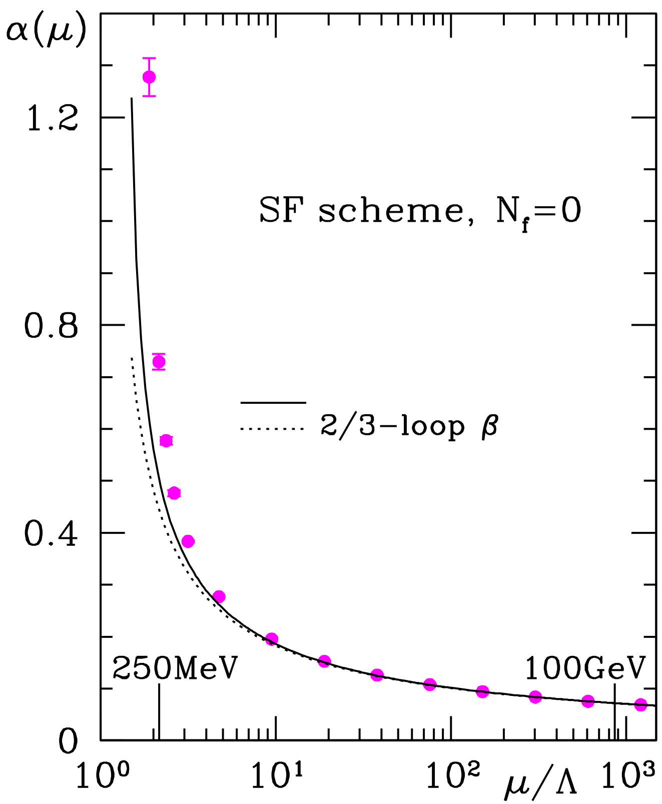
step-scaling function:

$$\sigma(s, \bar{g}^2(L^{-1})) = \bar{g}^2((sL)^{-1})$$

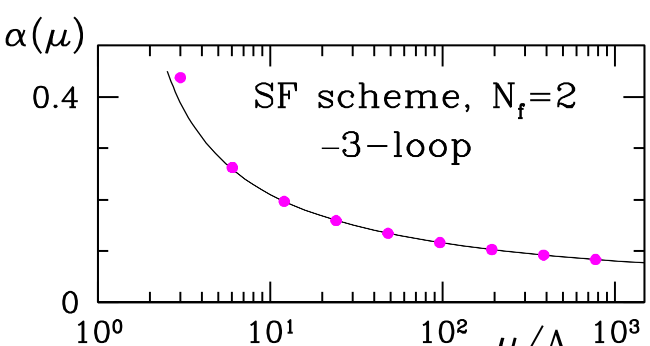
$$\Updownarrow$$

$$\log s = \int_{\bar{g}^2(L^{-1})}^{\sigma(s,\bar{g}^2(L^{-1}))} \mathrm{d}g^2 \, \frac{1}{\beta(g^2)}$$

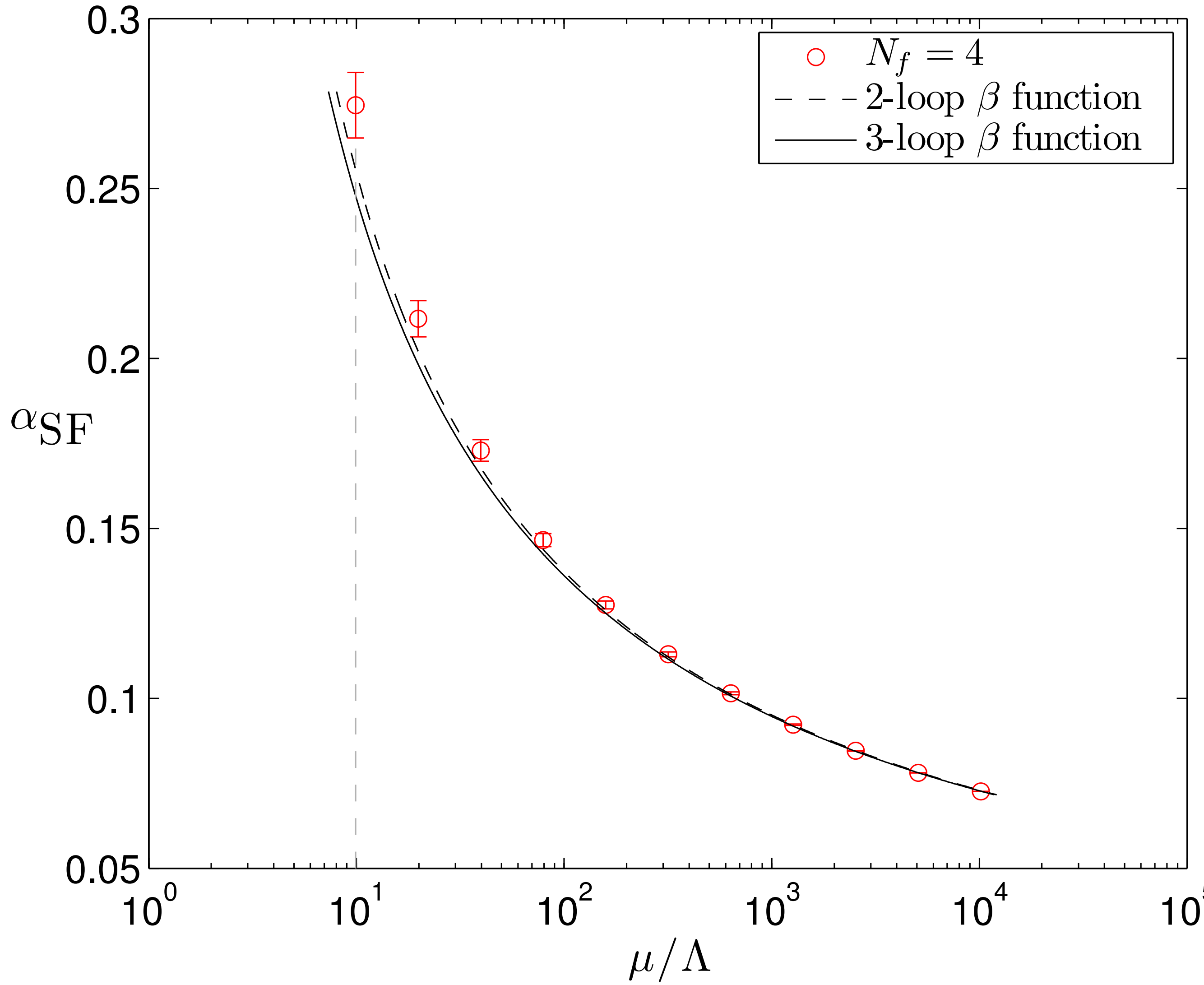
non-perturbative running coupling



[ALPHA Collaboration, 2001]



[ALPHA Collaboration, 2005]

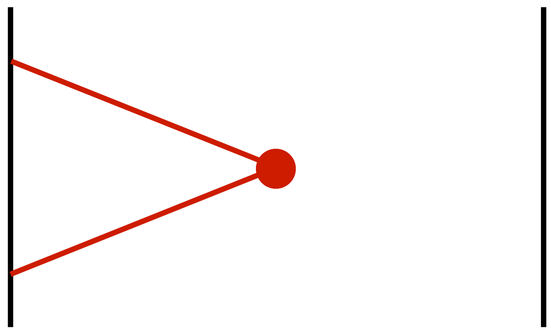


[ALPHA Collaboration, 2010]

[plots from R Sommer]

non-perturbative RG running of composite operators

renormalisation conditions: construct correlation functions of composite operator in bulk with source operators on 3d “branes” at Euclidean time *0,T*



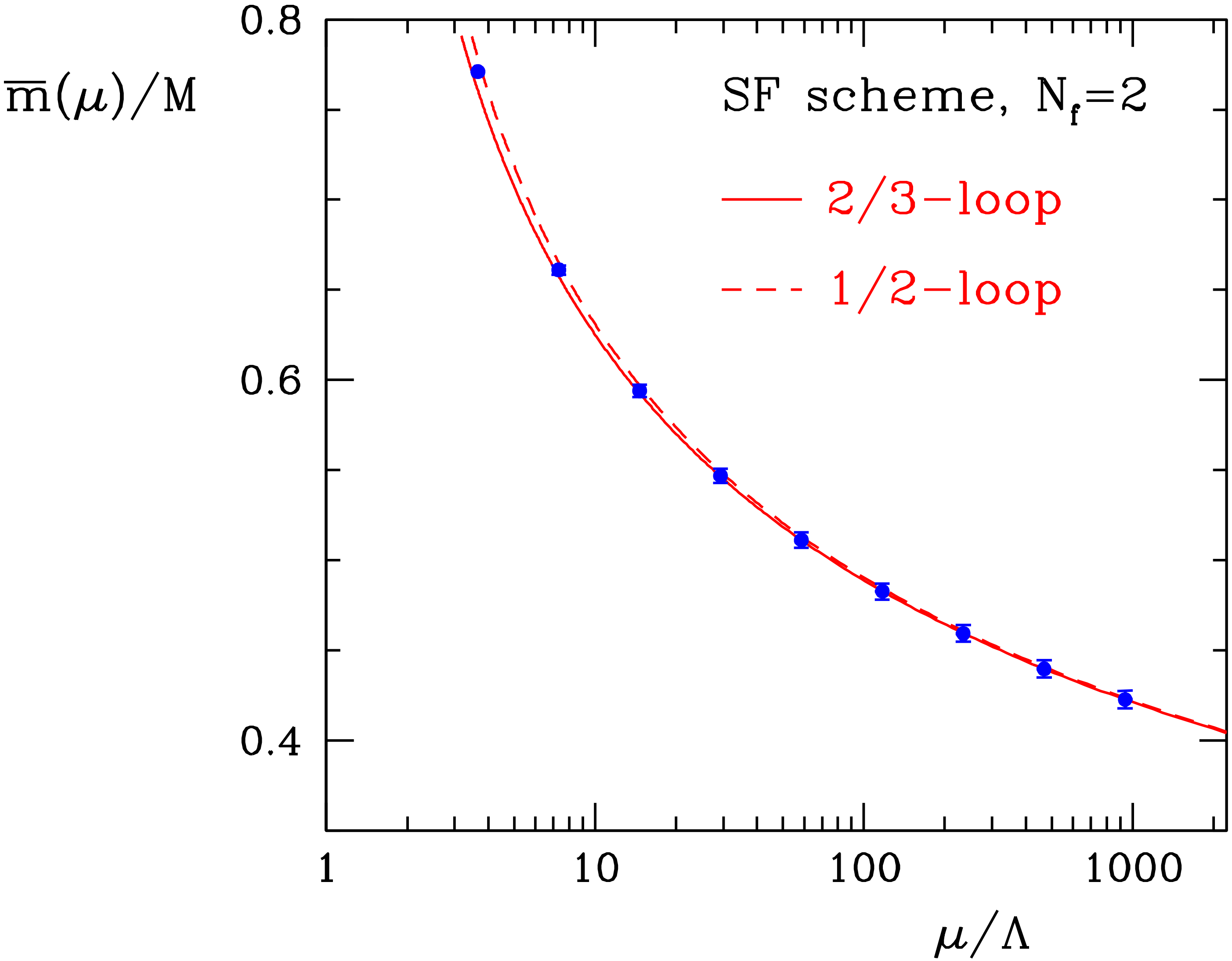
$$f_\Gamma(x_0) = \langle \mathcal{O}_\mathrm{b}\,(\bar\psi\Gamma\psi)(x)\rangle$$

$$f_1 = \langle \mathcal{O}_\mathrm{b}\mathcal{O}'_\mathrm{b}\rangle$$

$$Z_\Gamma(L^{-1})\,\frac{f_\Gamma(T/2)}{\sqrt{f_1}} = \left.\frac{f_\Gamma(T/2)}{\sqrt{f_1}}\right|_{\mathrm{tree\ level}}$$

running from ssf: $\sigma_\Gamma(s,\bar g^2(L^{-1})) = \exp\left\{\int_{\bar g^2(L^{-1})}^{\bar g^2((sL)^{-1})} \mathrm{d}g^2\,\frac{\gamma(g^2)}{\beta(g^2)}\right\}$

non-perturbative RG running of composite operators



[ALPHA 07]

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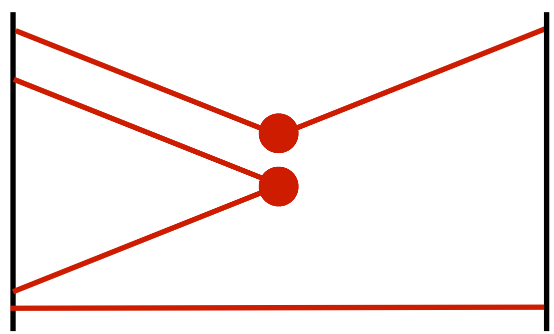
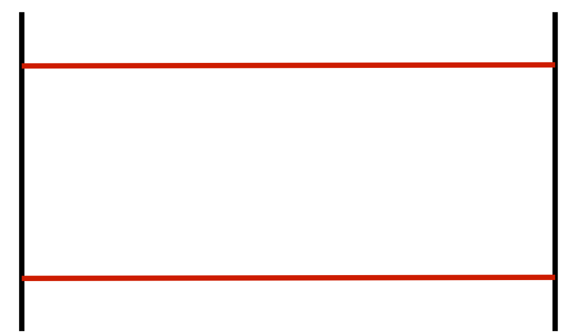


$$f_k^{\pm}(x_0) = \langle \mathcal{O}_{\rm b}\, \tilde{\mathcal{O}}_{\rm b}\, Q_k^{\pm}(x)\, \mathcal{O}'_{\rm b} \rangle$$



$$f_1 = \langle \mathcal{O}_{\rm b} \mathcal{O}'_{\rm b} \rangle$$

$$Z_k^{\pm}(L^{-1})\, \frac{f_k^{\pm}(T/2)}{f_1^{3/2}} = \left. \frac{f_k^{\pm}(T/2)}{f_1^{3/2}} \right|_{\text{tree level}}$$

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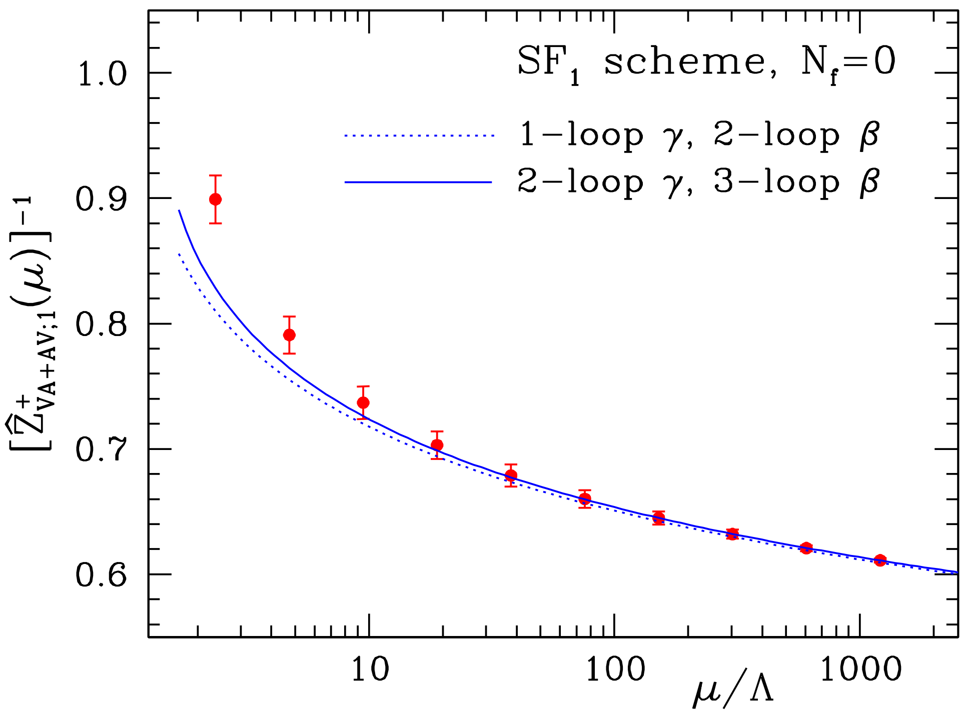
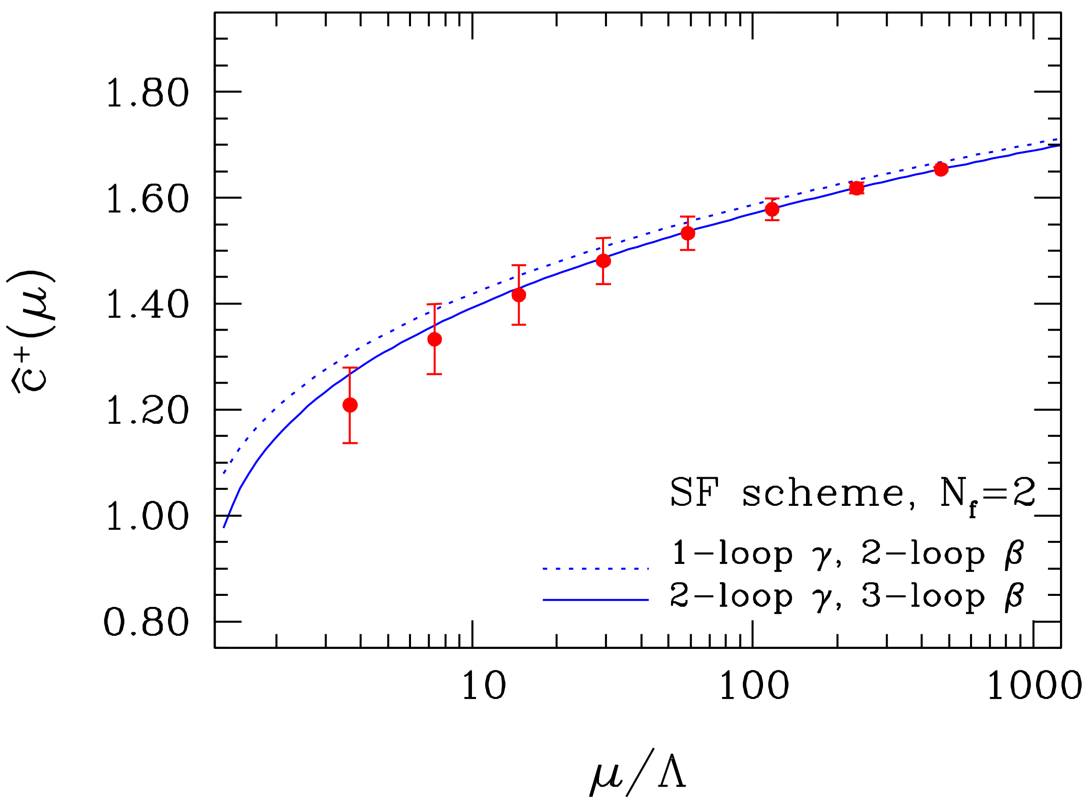


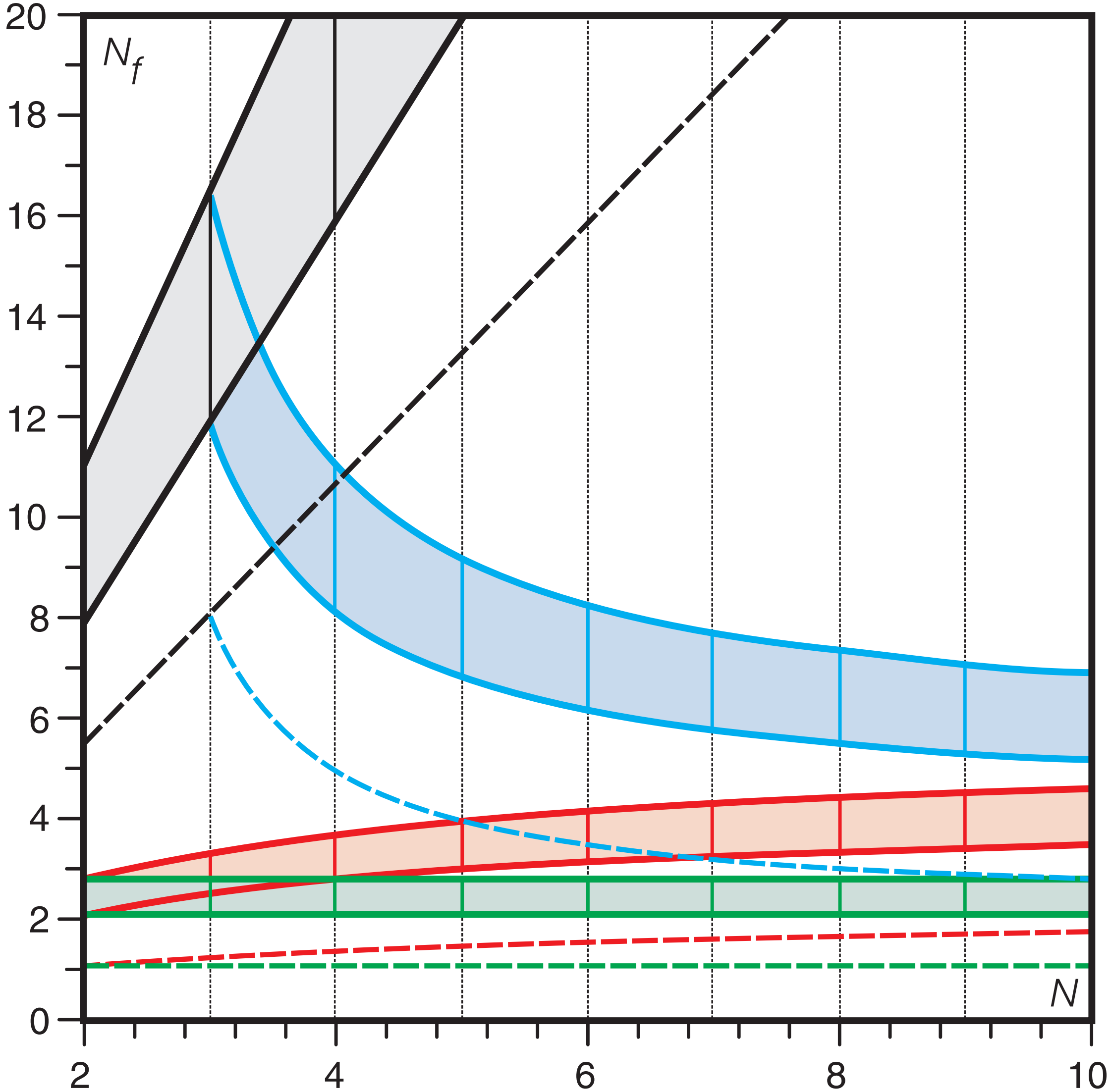
[ALPHA 05-07]



minimal walking technicolour

perturbative parameter space for walking theories:

[sketch from Dietrich, Sannino 06]

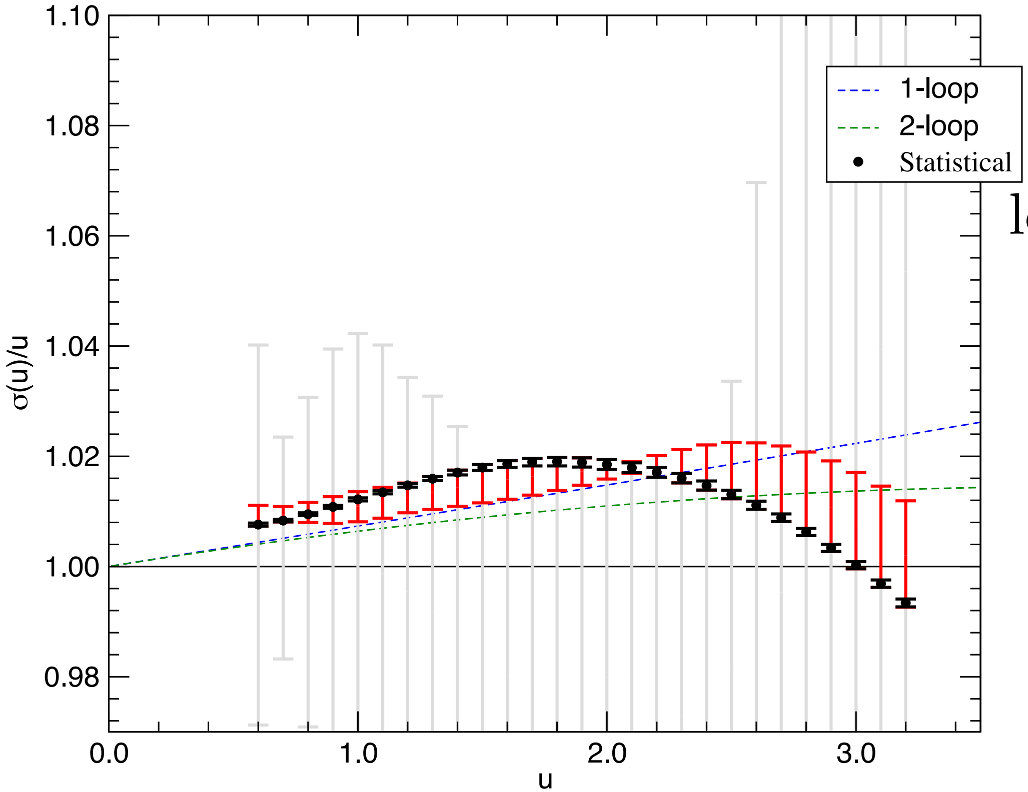


- MWTC: 2 Dirac fermions in adjoint of gauge group
- extensively studied for SU(2) gauge group
- extensive (conclusive?) evidence of conformal character
- mass anomalous dimension likely on the small side for successful phenomenology

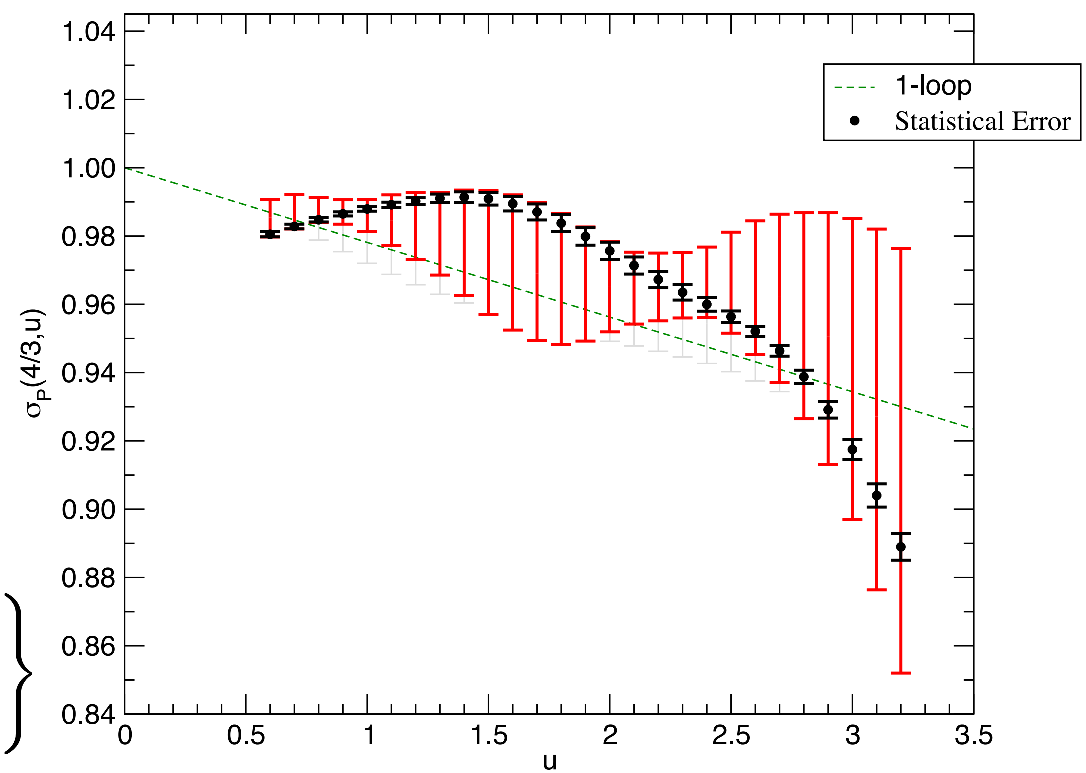
[Bursa et al., Del Debbio et al. 09-10]
[Hietanen, Rummukainen, Tuominen 09]
[Catterall, Del Debbio, Giedt, Keegan 10-11]
[DeGrand, Shamir, Svetitsky 11]
[Giedt, Weinberg 11-12]
[Patella 12]
[Karavirta et al. 12]

minimal walking technicolour

[Bursa, Del Debbio, Keegan, Pica, Pickup 10]



$$\log s = \int_{\bar{g}^2(L^{-1})}^{\sigma(s,\bar{g}^2(L^{-1}))} \mathrm{d}g^2 \, \frac{1}{\beta(g^2)}$$



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preliminary results MWTC

- determine four-fermion operator anomalous dimensions via SF non-perturbative RG running
- Wilson fermion regularisation ⇒ explicit breaking of chiral symmetry, work in parity-odd sector
- define several renormalisation schemes — scheme independence of anomalous dimensions at fixed point provides strong constraint

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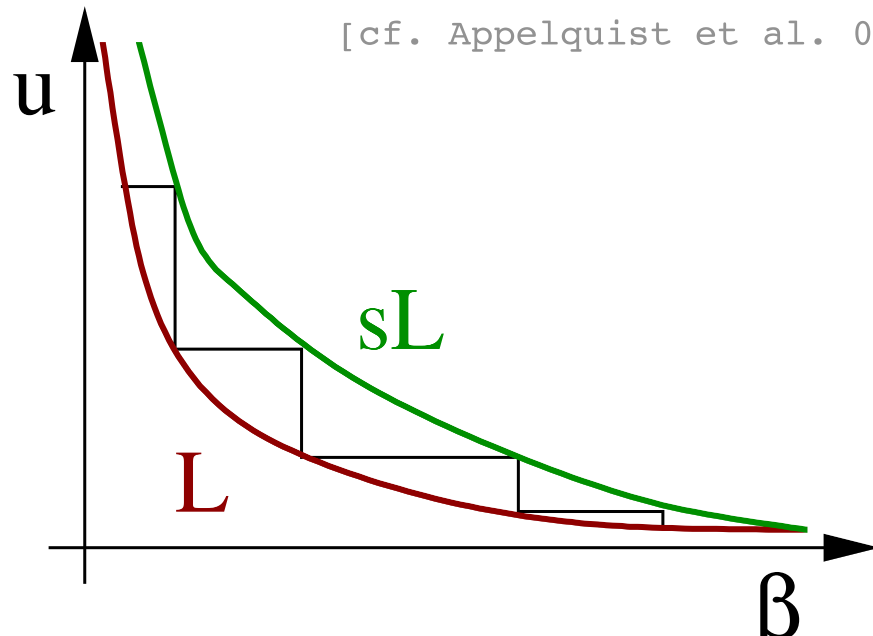
β	$L=8$	$L=10$	$L=12$	$L=16$
2.05	$20k$ (88%)	$20k$ (87%)	$20k$ (85%)	$20k$ (86%)
2.20	$20k$ (88%)	$20k$ (86%)	$20k$ (86%)	$20k$ (85%)
2.50	$20k$ (90%)	$20k$ (88%)	$20k$ (89%)	$20k$ (83%)
3.00	$20k$ (95%)	$20k$ (89%)	$20k$ (88%)	$20k$ (86%)
3.50	$20k$ (95%)	$20k$ (89%)	$20k$ (86%)	$20k$ (87%)
4.50	$20k$ (96%)	$20k$ (91%)	$20k$ (88%)	$20k$ (85%)
8.00	$20k$ (96%)	$20k$ (92%)	$20k$ (90%)	$20k$ (87%)
16.00	$20k$ (96%)	$20k$ (90%)	$20k$ (87%)	$20k$ (83%)

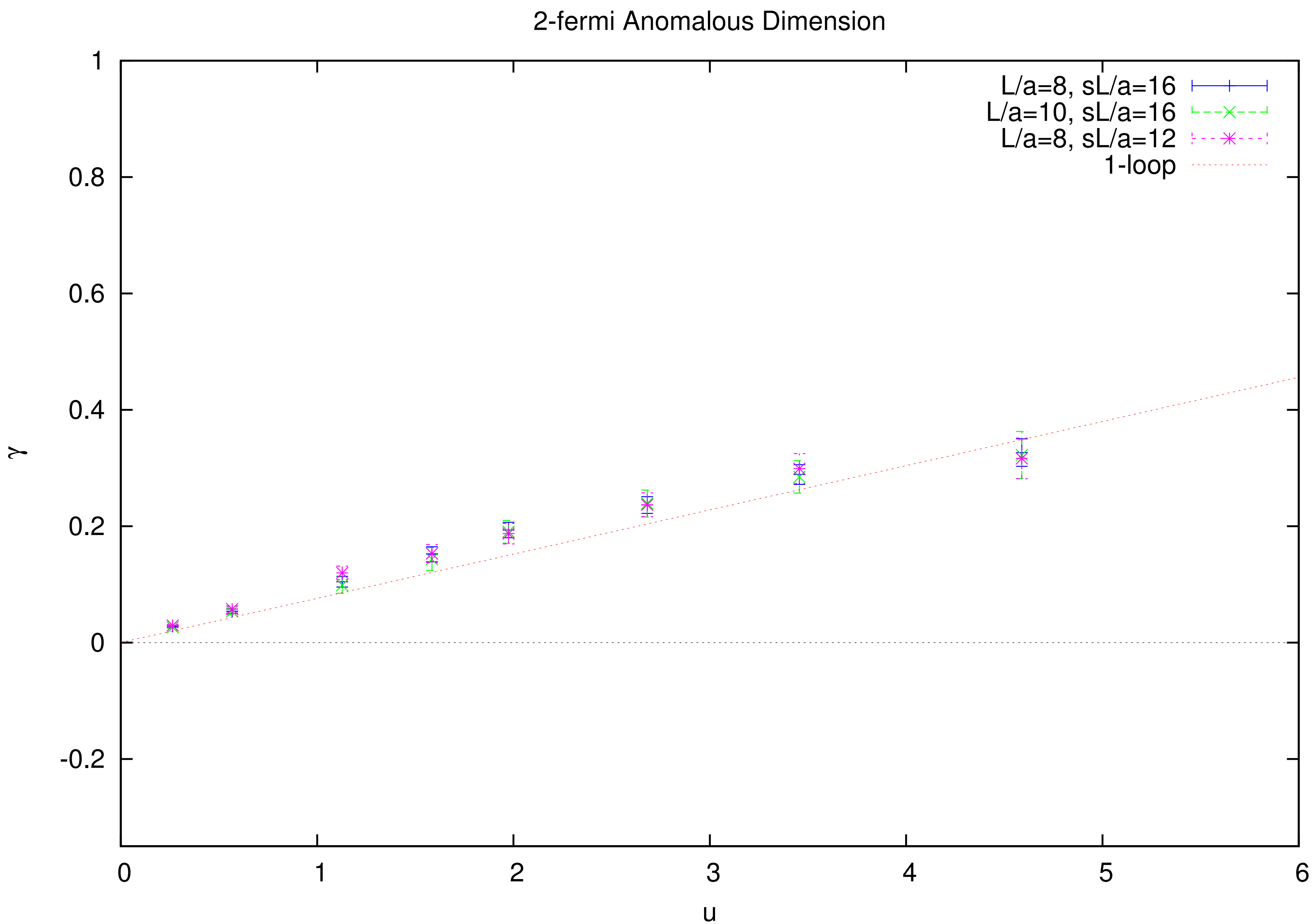
alleviate numerical cost by using mild scale dependence to perform interpolations

[cf. Appelquist et al. 09]



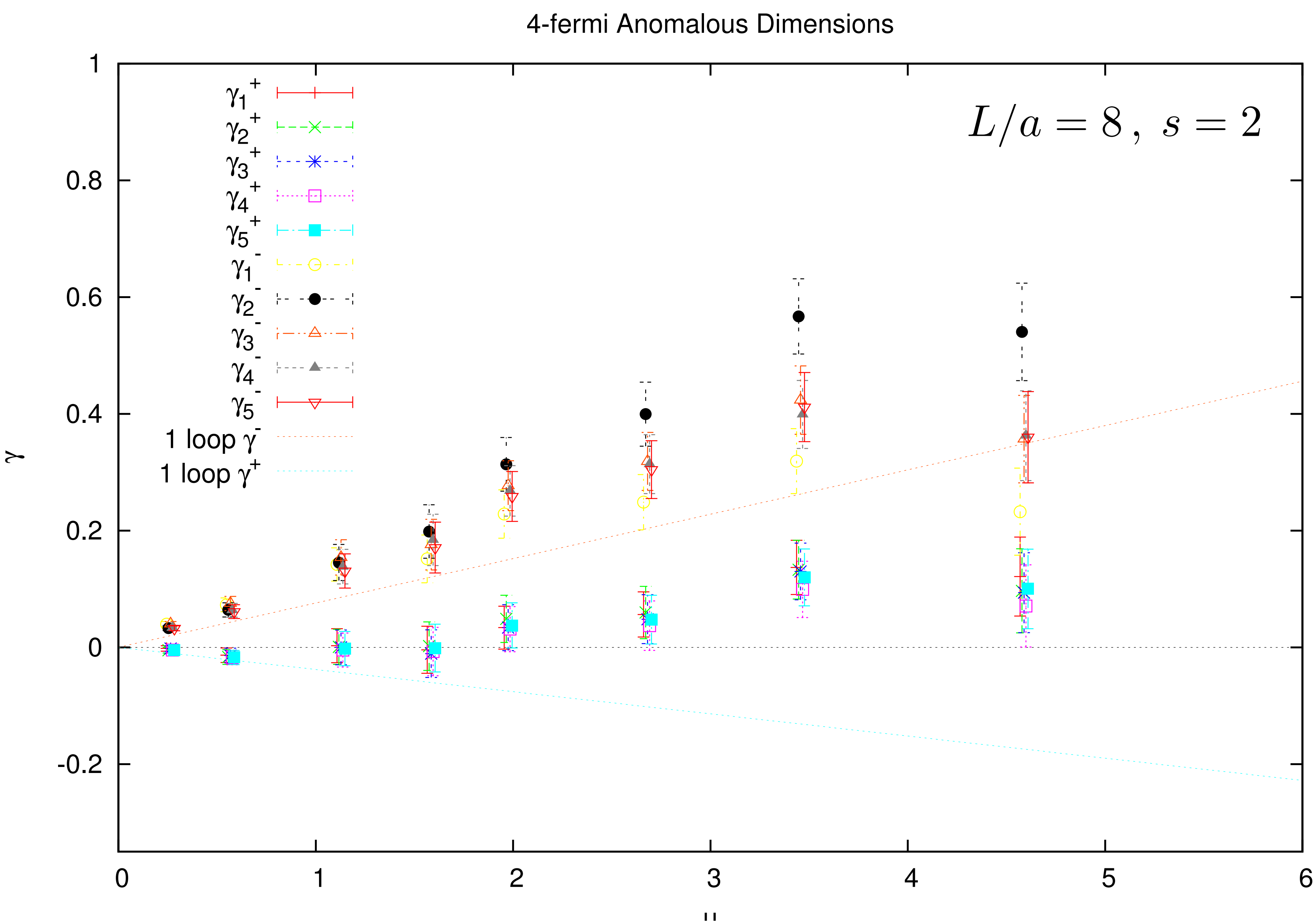
preliminary results MWTC

mass anomalous dimension redux



preliminary results MWTC

left current-left current operators, five different renormalisation schemes



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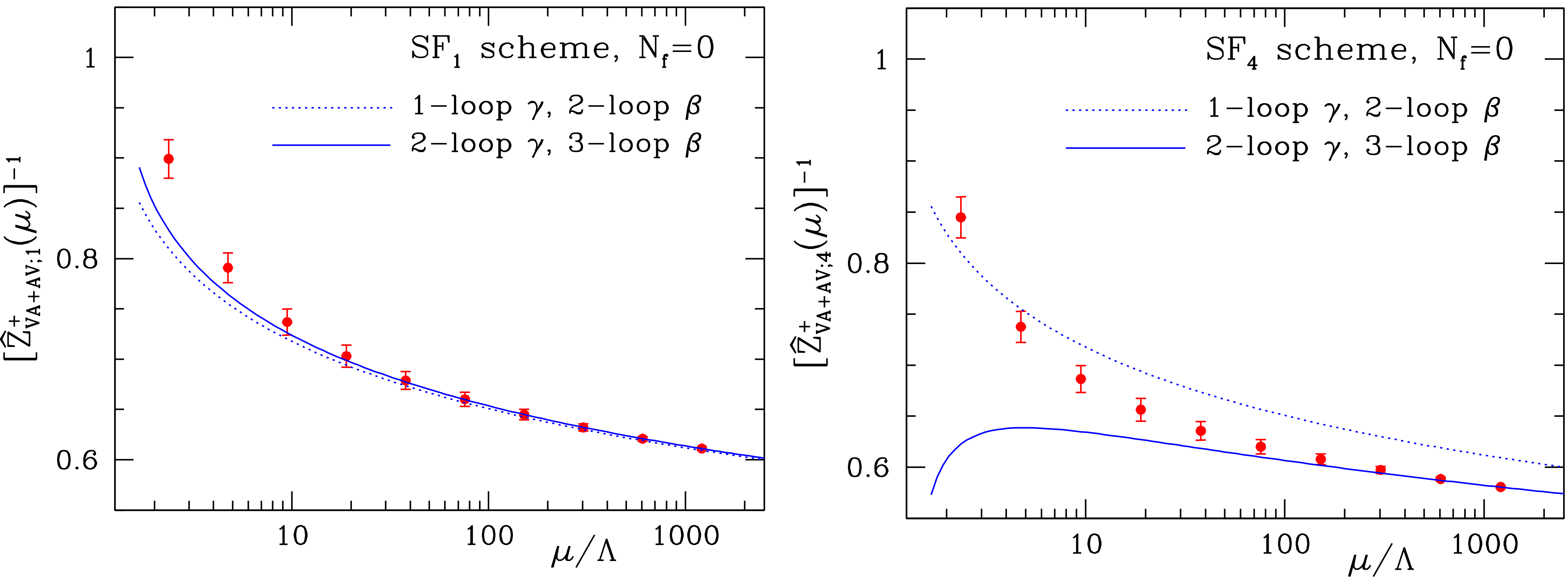
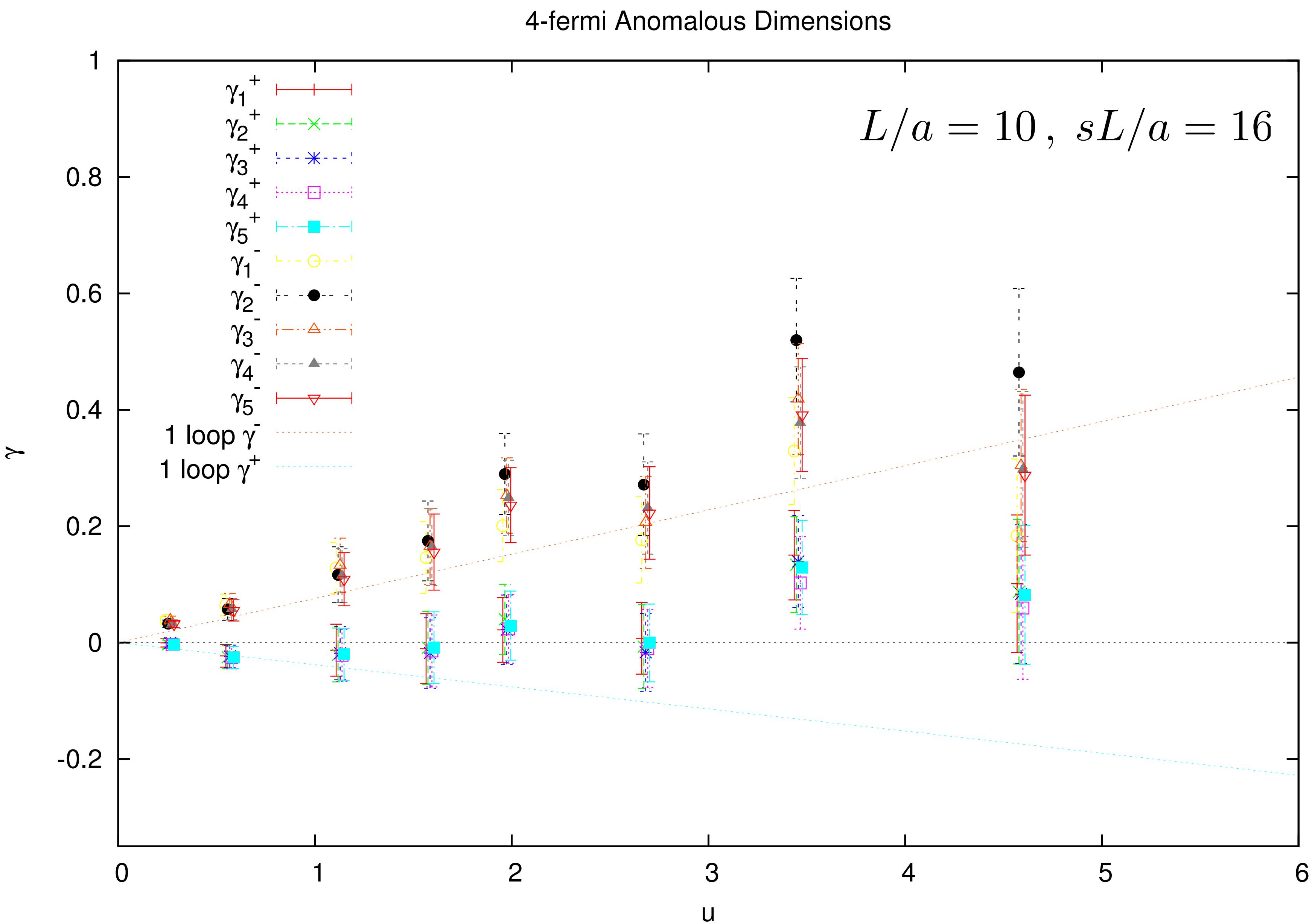


(cf. the strong scheme dependence in a running theory)

preliminary results MWTC

left current-left current operators, five different renormalisation schemes



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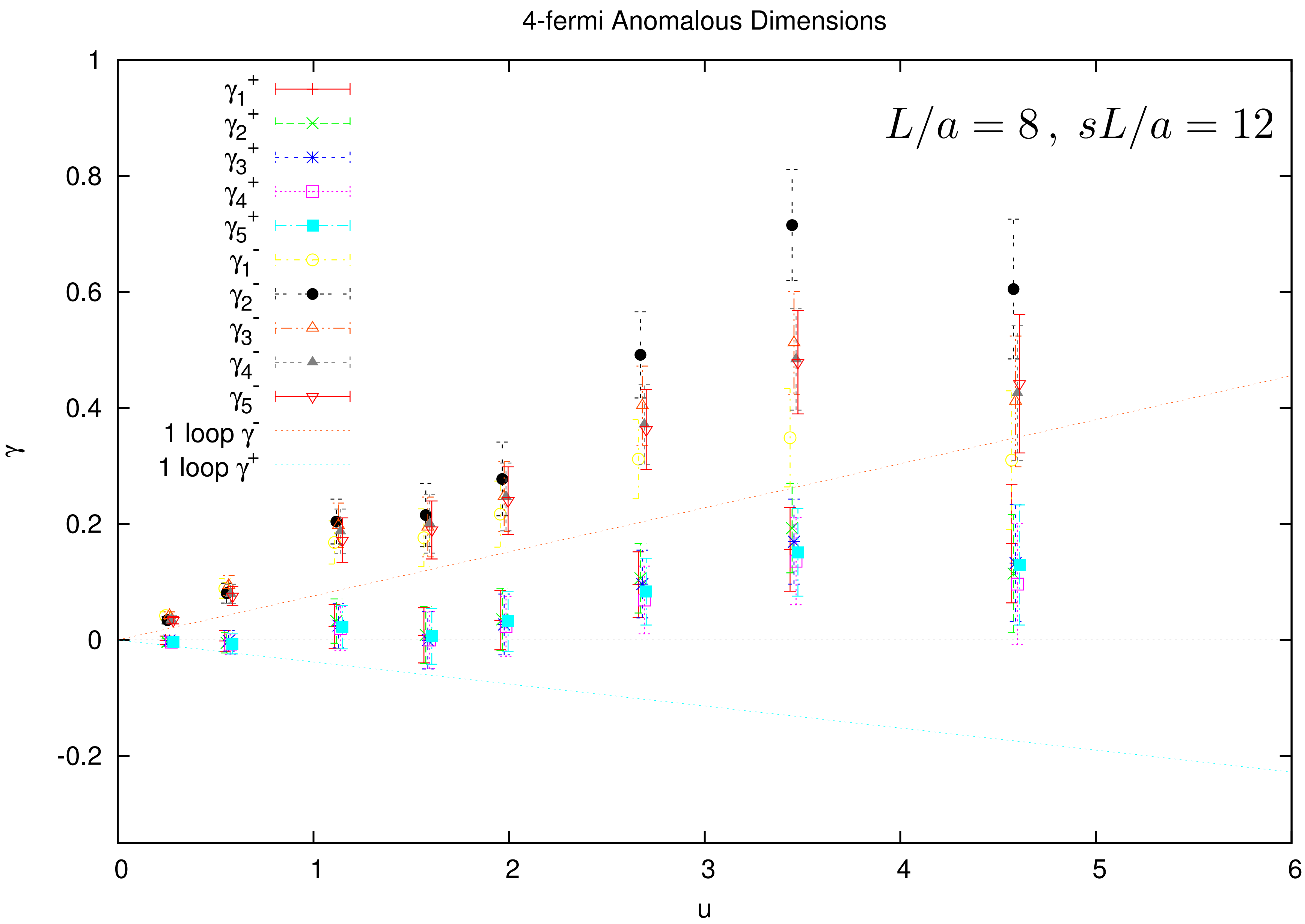


conclusions and outlook

- determine anomalous dimensions of four-fermion operators non-perturbatively in candidate conformal strong EWSB models for model building
- feasibility tested in MWTC — so far results for multiplicatively renormalisable left current-left current operator, preliminary $\gamma_* \sim 0.2 \div 0.5$
- anomalous dimensions for other operators being worked out, easy to extend to other models of interest (with vector couplings)
- many improvements possible for better precision:
  - more statistics, finer lattices
  - O(a) improvement for milder cutoff effects (chirally rotated SF?) [cf. talk by C Pica]
  - gradient flow renormalisation combined with SF [Fritzsch, Ramos 13]

backup

determination of chiral point for mass-independence

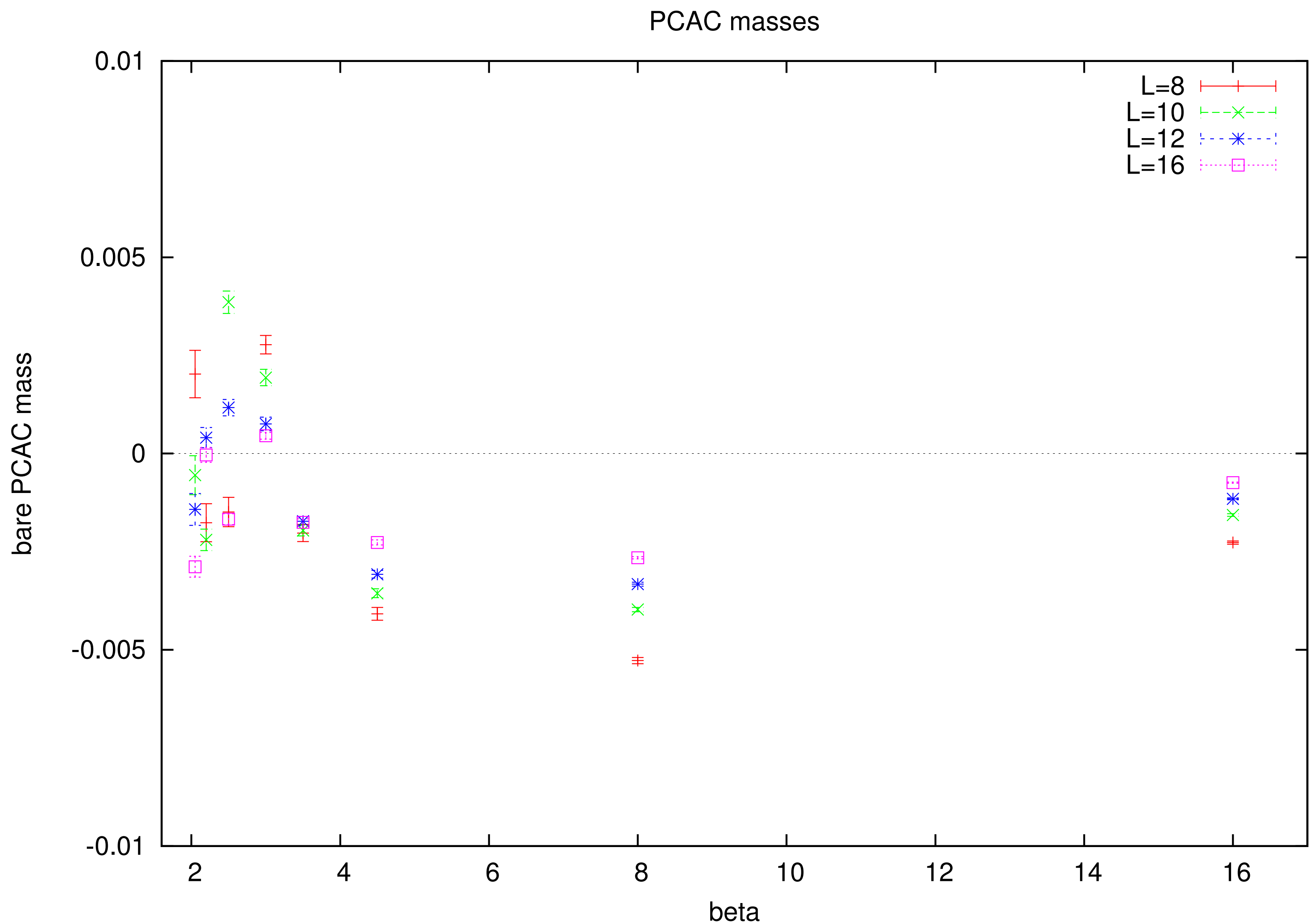


FIG. 1. Measured bare PCAC masses, in all cases $|am| \lesssim 0.005$

determination of chiral point for mass-independence

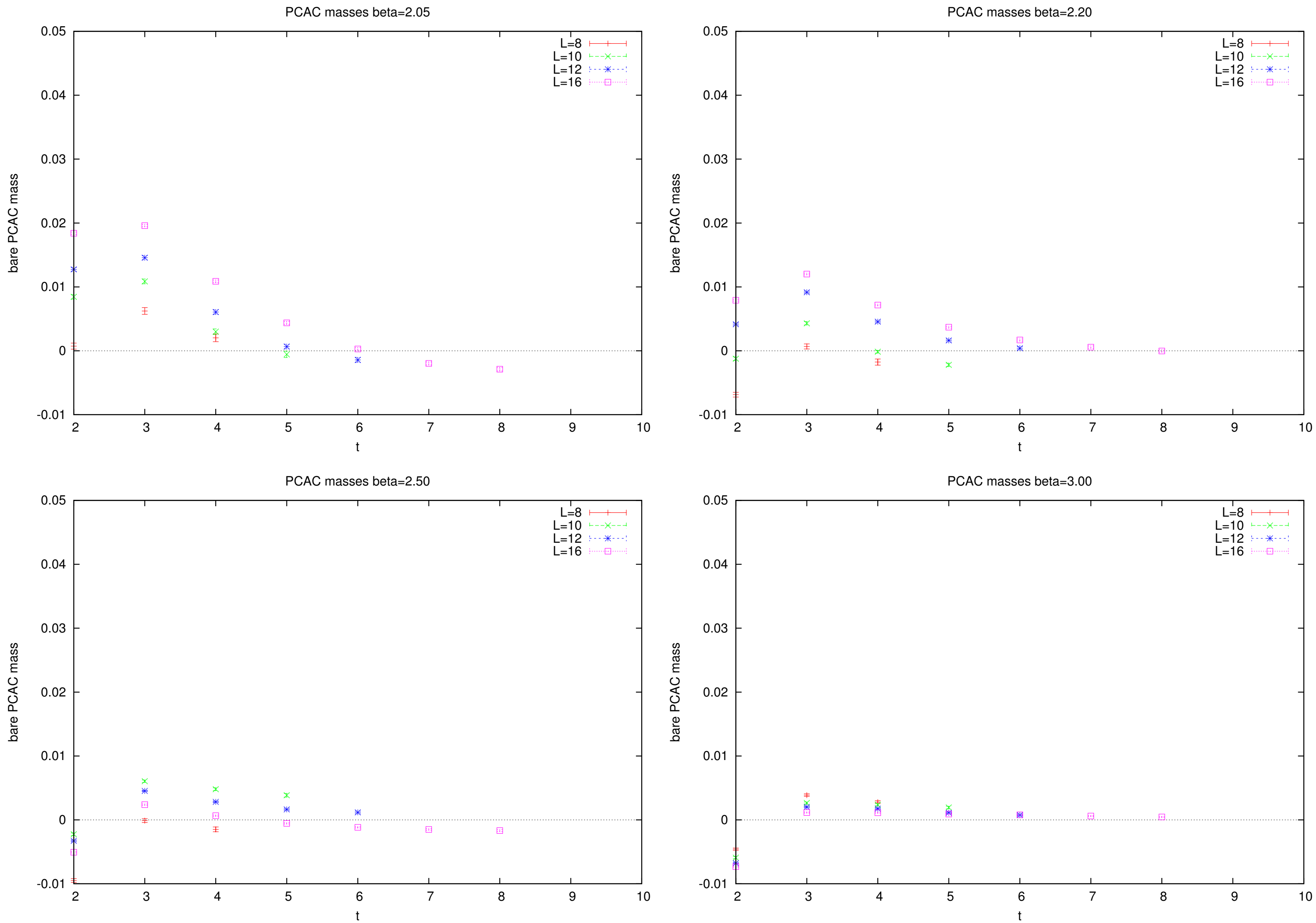
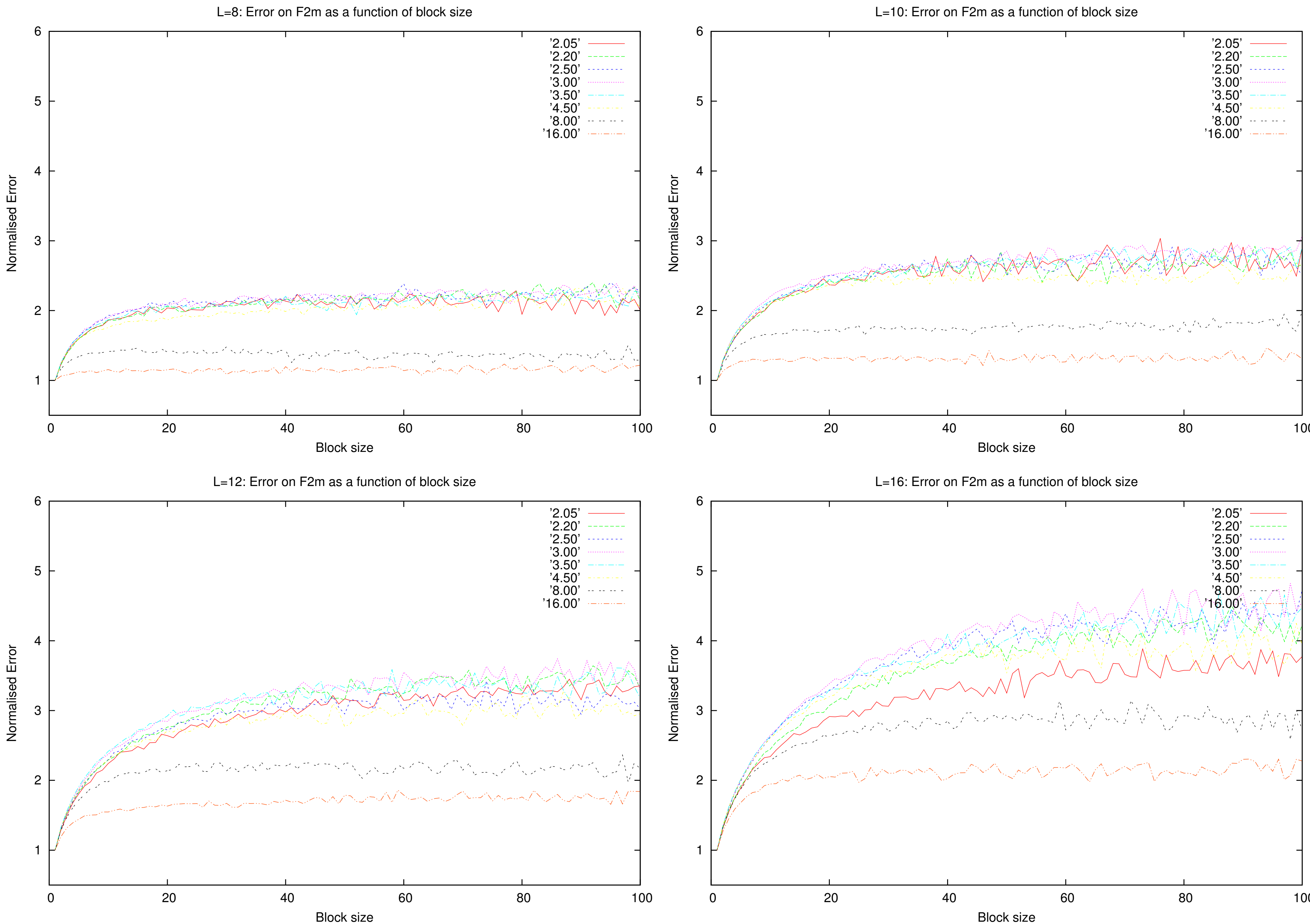


FIG. 2. Measured bare PCAC masses at each timeslice - at strong coupling we see large finite size effects.

autocorrelations



determination of anomalous dimensions

1. determine lattice ssf Σ, interpolate to improve resolution and take CL

2. construct approximant of anomalous dimension

$$\gamma_X(\bar{g}^2(L^{-1})) \equiv \frac{\log|\sigma_X(s,\bar{g}^2(L^{-1}))|}{\log|s|}$$

small beta function makes approximation reasonable — detailed analysis for ssf of course needed for extensive checks