Towards non-SUSY duality

Steven Abel (IPPP, Durham)

w/ James Barnard, ArXiV:1202.2863, JHEP 05 (2012) 044 + in progress

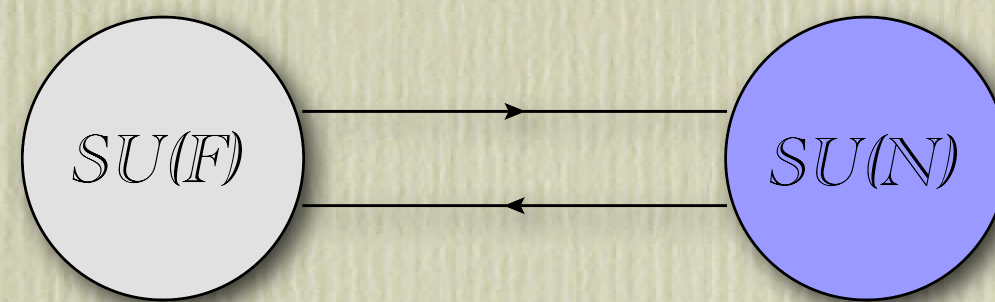
Geordie >>>

Outline

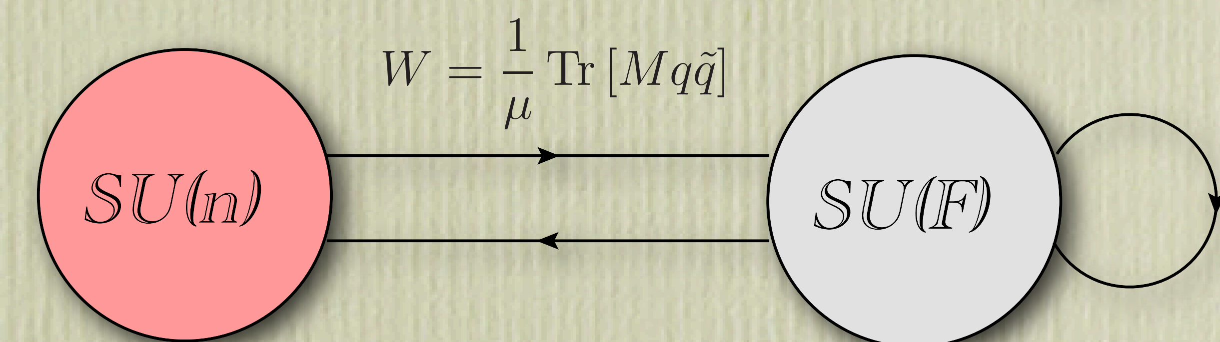
- Seiberg duality - lightning review
- Hidden local symmetries (HLS) - lightning review
- HLS in SQCD
- Special features

Lightning review of Seiberg duality

The electric model, $\mathcal{N}=1 \;\; SU(N)$ SQCD with $N+n$ flavours of quark,



Flows to a magnetic SU(n) theory



$$\begin{array}{ccc} Q\tilde{Q} & \rightarrow & M \\ Q^N & \rightarrow & q^n \end{array}$$

Harada and Yamawaki, and more recently Komargodski have argued that the SU(n) can be interpreted as a **hidden local symmetry**. Here use HLS to “derive” Seiberg duality from first principles. First step to finding similar dualities for non-SUSY theories.

Lightning review of HLS

Bando, Kugo, Uehara, Yamawaki, Yanagida

Canonical example: the phenomenology of QCD

In the absence of fermion masses QCD has $SU(3)_L \times SU(3)_R$ symmetry from u,d,s rotations. The axial part of this symmetry is spontaneously broken by strong coupling effects ....

$$SU(3)_L \times SU(3)_R \quad \rightarrow \quad SU(3)_V$$

... leading to the well-known chiral perturbation theory description, and the understanding of the light octet of $\pi^{0,\pm}, K^{0,\pm}, \bar{K}^0, \eta$ pseudo-scalar mesons as Nambu-Goldstone modes.

Lightning review of HLS

Bando, Kugo, Uehara, Yamawaki, Yanagida

The 8 pions (NGBs of the broken symmetry) can be assembled into a special unitary matrix ... $U = e^{i\pi^a (T_L^a - T_R^a)}$

transforming as $U' = g_L U g_R^\dagger$

The leading 2-derivative Lagrangian is

$$\mathcal{L} = \frac{1}{4} f_\pi^2 Tr \left(\partial_\mu U \partial^\mu U^\dagger \right)$$

Note that the vectorial symmetry $g_R = g_L$ is just a linear transformation on the pions while the axial one also has non-linear transformations in the unbroken symmetry.

Lightning review of HLS

Bando, Kugo, Uehara, Yamawaki, Yanagida

This is usually expressed by factorizing the U into representatives of left and right cosets:

$$U = \xi\tilde{\xi}^{\dagger}$$

where the flavour transformation acts (in general) non-linearly as

$$\begin{aligned} \xi &\rightarrow \xi' = g_L \xi h^{-1}(\pi, g_L, g_R) \\ \tilde{\xi} &\rightarrow \tilde{\xi}' = g_R \tilde{\xi} h^{-1}(\pi, g_L, g_R) \end{aligned}$$

Lightning review of HLS

Bando, Kugo, Uehara, Yamawaki, Yanagida

This is usually expressed by factorizing the U into representatives of left and right cosets:

$$U = \xi \tilde{\xi}^\dagger$$

where the flavour transformation acts (in general) non-linearly as

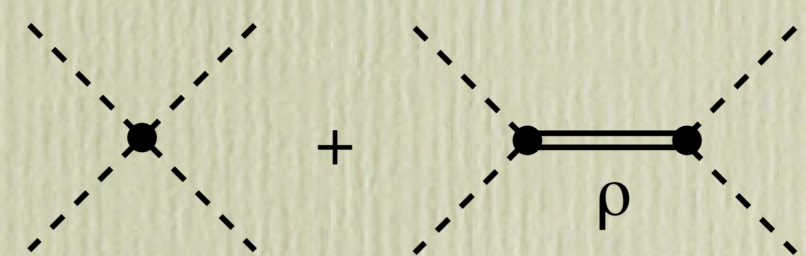
$$\begin{aligned} \xi &\rightarrow \xi' = g_L \xi h^{-1}(\pi, g_L, g_R) \\ \tilde{\xi} &\rightarrow \tilde{\xi}' = g_R \tilde{\xi} h^{-1}(\pi, g_L, g_R) \end{aligned}$$

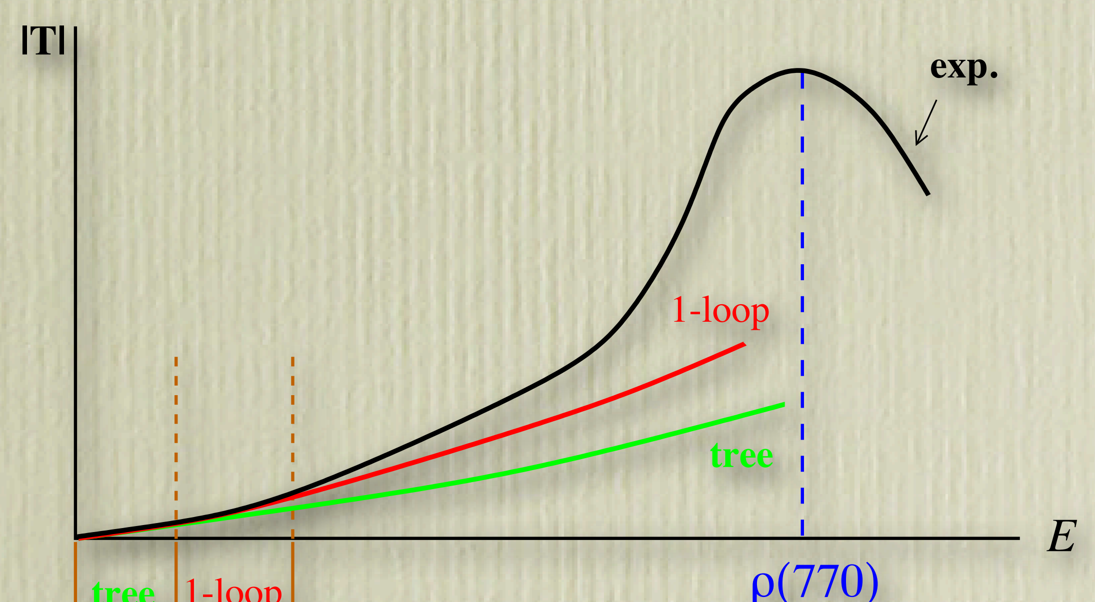
Rotations in unbroken global symmetry

Lightning review of HLS

Bando, Kugo, Uehara, Yamawaki, Yanagida

The next well defined resonance is the rho-meson (770 MeV). It has a crucial role to play in various processes: e.g. it unitarises pi-pi scattering beyond the range of validity of ChPT ...





from review of Harada, Yamawaki

Lightning review of HLS

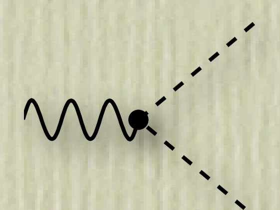
Bando, Kugo, Uehara, Yamawaki, Yanagida

But experiment hints at a deeper structure. The most famous can be summarised in terms of two parameters, g_H and a:

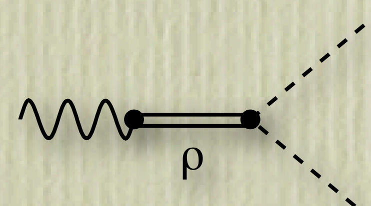
$$g_H \simeq 6, \quad a \simeq 2.$$

$$m_\rho^2 = a g_H^2 f_\pi^2$$

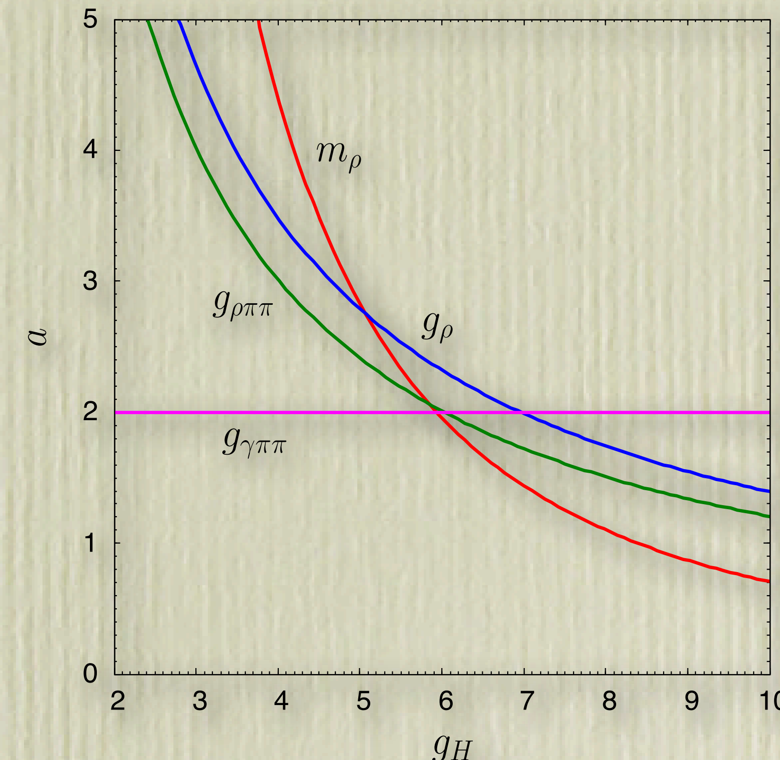
$$g_{\gamma\pi\pi} = -\frac{a-2}{2} e$$



$$g_{\rho\pi\pi} = \frac{a}{2} g_H$$



$$g_\rho = a g_H f_\pi^2$$



from Kitano 1109.6158

Lightning review of HLS

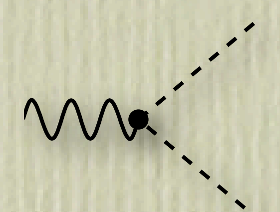
Bando, Kugo, Uehara, Yamawaki, Yanagida

But experiment hints at a deeper structure. The most famous can be summarised in terms of two parameters, g_H and a:

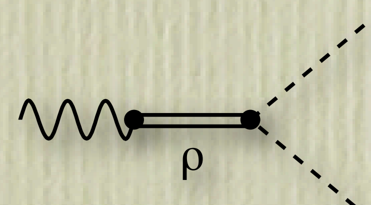
$$g_H \simeq 6, \quad a \simeq 2.$$

$$m_\rho^2 = a g_H^2 f_\pi^2$$

$$g_{\gamma\pi\pi} = -\frac{a-2}{2} e$$

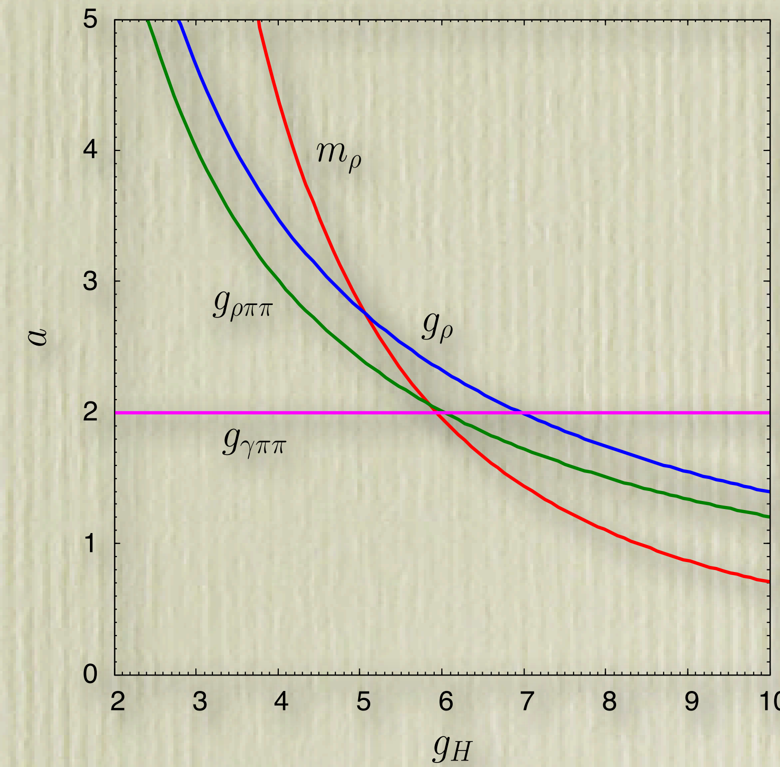


$$g_{\rho\pi\pi} = \frac{a}{2} g_H$$



$$g_\rho = a g_H f_\pi^2$$

Universality



from Kitano 1109.6158

Lightning review of HLS

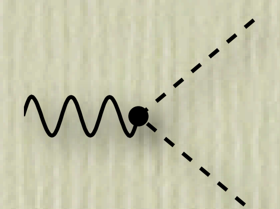
Bando, Kugo, Uehara, Yamawaki, Yanagida

But experiment hints at a deeper structure. The most famous can be summarised in terms of two parameters, g_H and a:

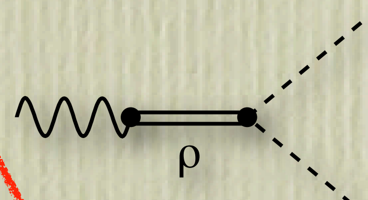
$$g_H \simeq 6, \quad a \simeq 2.$$

$$m_\rho^2 = a g_H^2 f_\pi^2$$

$$g_{\gamma\pi\pi} = -\frac{a-2}{2} e$$

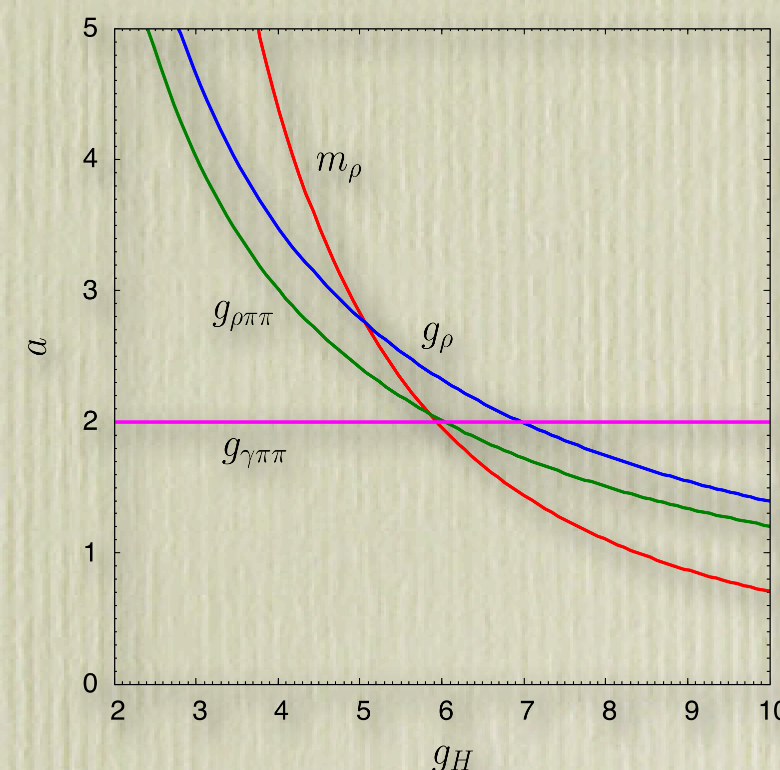


$$g_{\rho\pi\pi} = \frac{a}{2} g_H$$



$$g_\rho = a g_H f_\pi^2$$

Universality



from Kitano 1109.6158

a=2 gives "vector meson dominance": rho exchange dominates many scattering processes. (however a is subject to large RG effects between 1.1 GeV and 0)

Lightning review of HLS

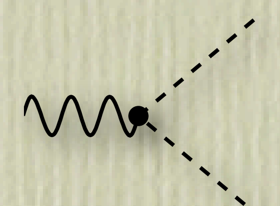
Bando, Kugo, Uehara, Yamawaki, Yanagida

But experiment hints at a deeper structure. The most famous can be summarised in terms of two parameters, g_H and a:

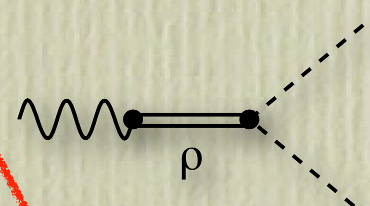
KSRFII relation

$$m_\rho^2 = a g_H^2 f_\pi^2$$

$$g_{\gamma\pi\pi} = -\frac{a-2}{2} e$$



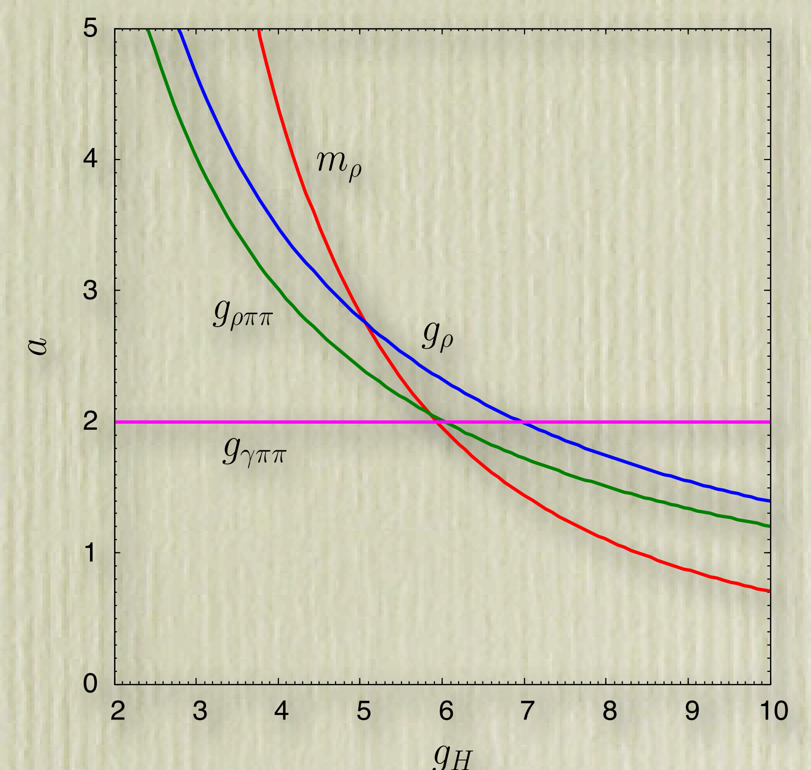
$$g_{\rho\pi\pi} = \frac{a}{2} g_H$$



$$g_\rho = a g_H f_\pi^2$$

Universality

$$g_H \simeq 6, \quad a \simeq 2.$$



from Kitano 1109.6158

a=2 gives "vector meson dominance": rho exchange dominates many scattering processes. (however a is subject to large RG effects between 1.1 GeV and 0)

Lightning review of HLS

Bando, Kugo, Uehara, Yamawaki, Yanagida

Now for HLS:

Thm. (Bando et al): *a theory with global symmetry broken as* $G \to H$ *is gauge equivalent to a theory with linearly realised global symmetry,* G*, and spontaneously broken local symmetry,* H*.*

The breaking of the HLS theory looks like $G_{global} \times H_{local} \to H$ and the sigma-model variables now transform linearly;

$$\xi(x) \longrightarrow g\xi(x)h^{-1}(x)$$

The full linearised theory with $G_{global} \times H_{local}$ symmetry can be thought of as a UV completion of the original non-linear theory.

Lightning review of HLS

Bando, Kugo, Uehara, Yamawaki, Yanagida

For example in the case of SU(3) QCD, the HLS theory is broken as

$$SU(3)_L \times SU(3)_R \times SU(3)_{V_{local}} \quad \rightarrow \quad SU(3)_V$$

Lightning review of HLS

Bando, Kugo, Uehara, Yamawaki, Yanagida

For example in the case of SU(3) QCD, the HLS theory is broken as

$$SU(3)_L \times SU(3)_R \times SU(3)_{V_{local}} \quad \rightarrow \quad SU(3)_V$$

This description explains the universality and KSFR relations. The rho's are identified as the massive gauge bosons of HLS. They gain their mass through the higgs mechanism, the pion decay constant is given by the VEV of a "higgs" (some bound state of quarks).

HLS in SUSY QCD

A crucial point in SUSY: the Goldstone modes must be in a complex chiral superfield

$$\xi(\Pi) = e^{\Pi(x)}$$

where $\Pi(x) = \Pi^a(x)\hat{T}^a$. Note that at the superpotential level the symmetry is the complex extension G^c, so the $\hat{T}^a$ are the broken generators in $G^c/\hat{H}$ where $\hat{H} \supseteq H^c$

HLS in SUSY QCD

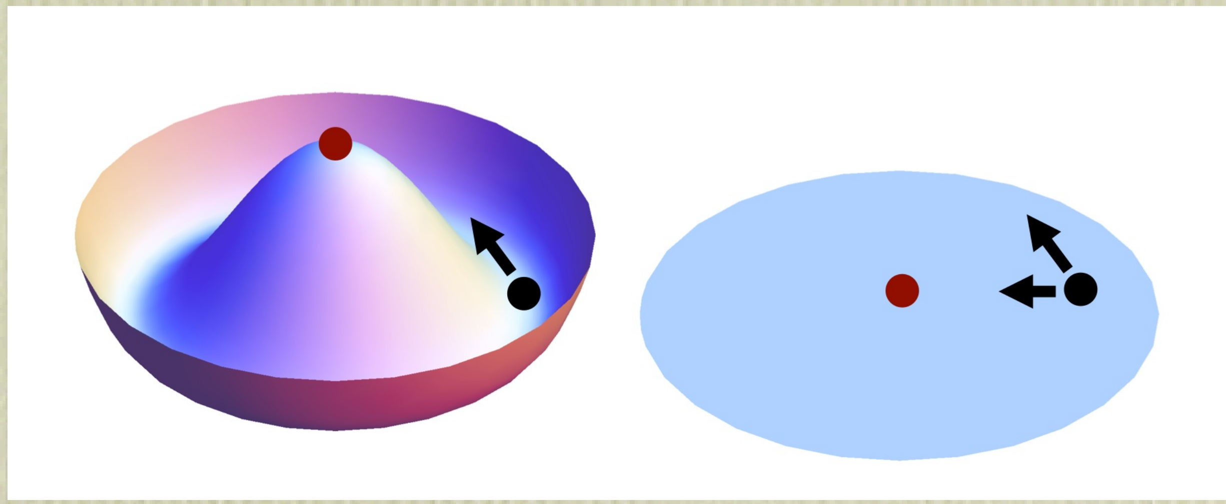
The non-NGB fields are ***moduli***, and the sigma model variables are generally of the form

$$\xi(\Pi) \supset e^{\kappa(x)} e^{i\pi(x)}$$

The anti-hermitian scalar part of Π provides the NGBs in the phase factor $e^{i\pi}$
The hermitian scalar parts provide a scaling of the symmetry breaking $\xi^\dagger \xi = e^{2\kappa}$

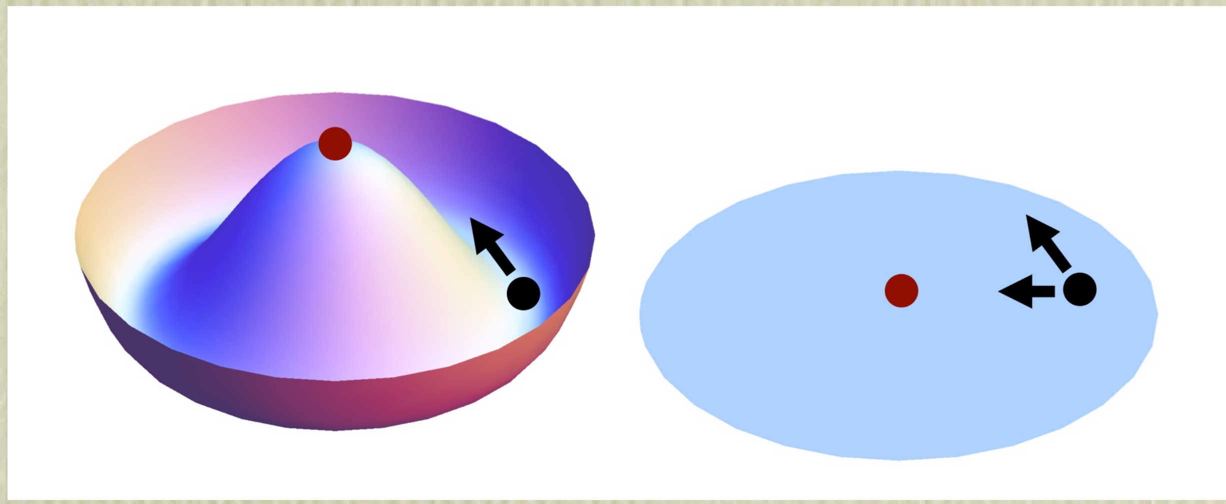
HLS in SUSY QCD

Familiar example: The mu-less MSSM... broken axial U(1), $H_{u,d} \to e^{i\alpha} H_{u,d}$ ***is promoted to a U(1)+scale symmetry*** $H_{u,d} \to e^{\Pi} H_{u,d}$



HLS in SUSY QCD

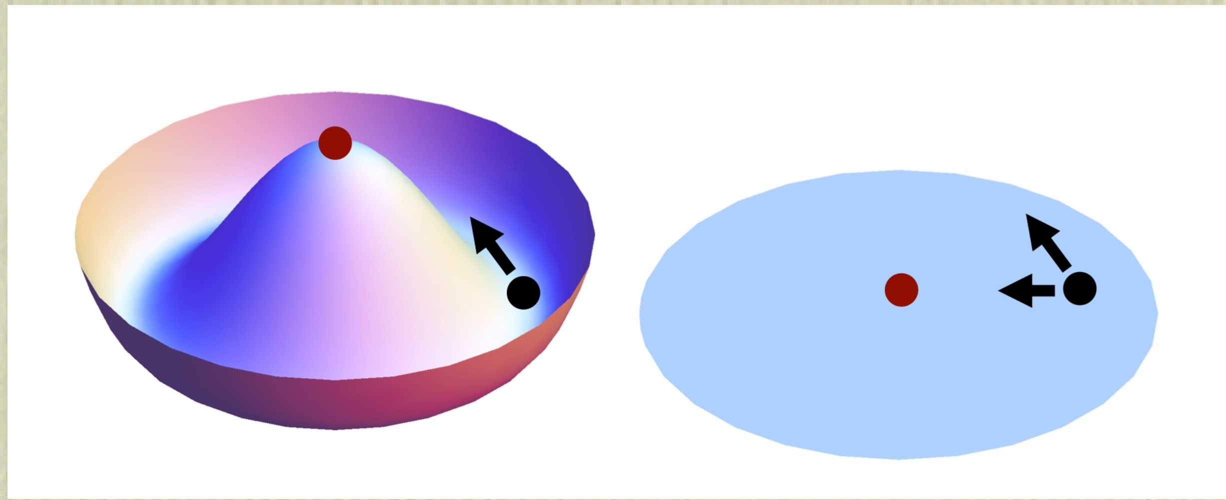
Familiar example: The mu-less MSSM... broken axial U(1), $H_{u,d} \to e^{i\alpha} H_{u,d}$ ***is promoted to a U(1)+scale symmetry*** $H_{u,d} \to e^{\Pi} H_{u,d}$



The higgs of the MSSM is also a pseudo-dilaton (aka pseudo-NGB of scale invariance) until we explicitly break this scaling symmetry with a mu-term

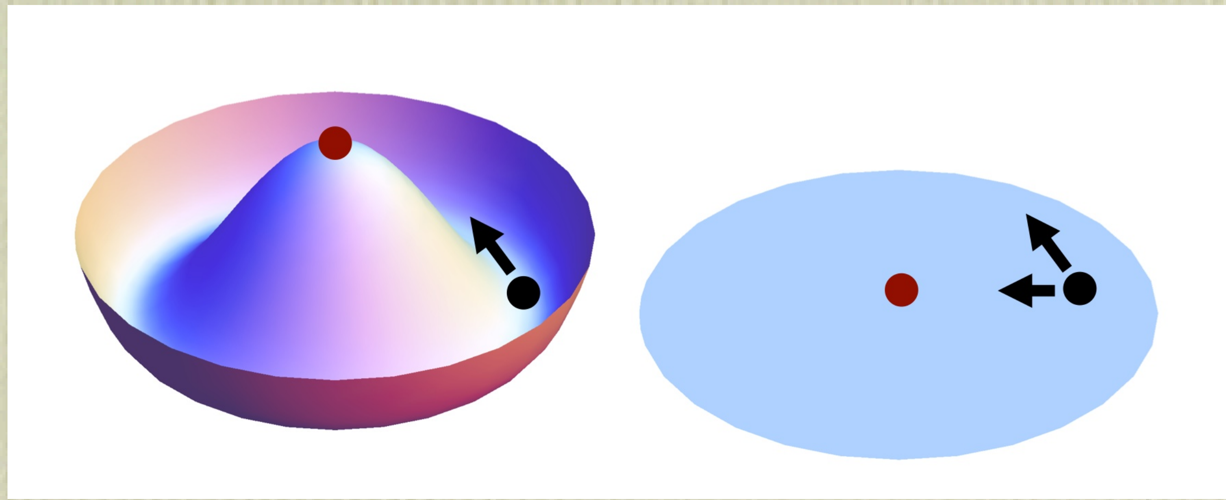
HLS in SUSY QCD

BOTTOM LINE: A dialling down of symmetry breaking is possible in SUSY because some partners of NGBs corresponds to moduli. ***Connected with R-symmetry***.

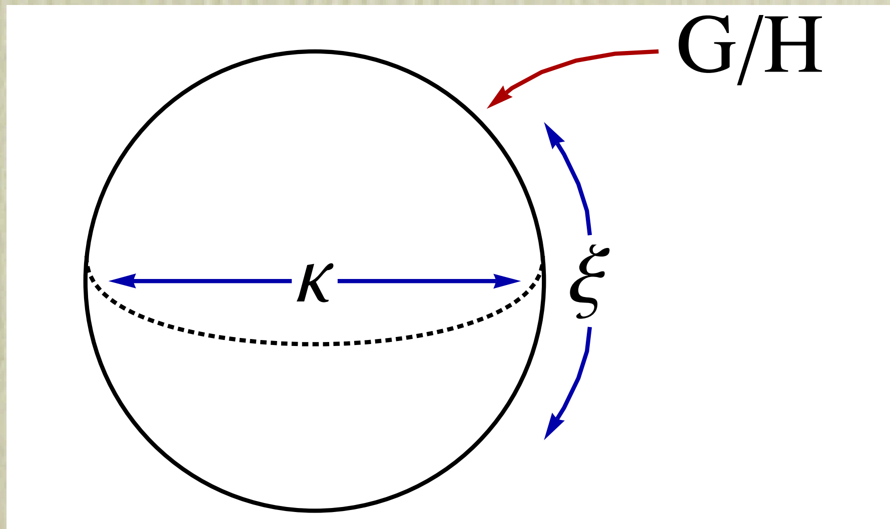


HLS in SUSY QCD

BOTTOM LINE: A dialling down of symmetry breaking is possible in SUSY because some partners of NGBs corresponds to moduli. ***Connected with R-symmetry***.



In sigma model language …



HLS in SUSY QCD

Now, for the electric SQCD theory:

	$SU(N)$	$SU(N+n)_L$	$SU(N+n)_R$	$U(1)_B$	$U(1)_R$
Q	$\square$	$\tilde{\square}$	$\mathbf{1}$	$1/N$	$n/(N+n)$
$\tilde{Q}$	$\tilde{\square}$	$\mathbf{1}$	$\square$	$-1/N$	$n/(N+n)$

Break the flavour symmetry with squark VEVs which can be flavour rotated to

$$Q = \begin{pmatrix} \boldsymbol{v} & 0 \end{pmatrix} \quad \text{where} \quad \boldsymbol{v} = \mathrm{diag}(v_1, \ldots, v_N).$$

Since the quarks are rank N, the unbroken flavour group is

$$H = SU(n)_L \times SU(n)_R \times U(1)_{B'} \times U(1)_{R'}$$

HLS in SUSY QCD

Considering e.g. $SU(N+n)_L$, the broken and unbroken generators are of the form

$$\hat{T}_L = \begin{pmatrix} \hat{T}_{L,N} + \mathbb{1} & \hat{T}_u \\ 0 & 0 \end{pmatrix} \qquad \hat{S}_L = \begin{pmatrix} 0 & 0 \\ \hat{S}_l & \hat{S}_{L,n} + \mathbb{1} \end{pmatrix}$$

The corresponding goldstone modes are given by $\xi = e^{\kappa_R} \begin{pmatrix} e^{\kappa_B} \xi_N & \xi_u \\ 0 & \mathbb{1} \end{pmatrix}$

HLS in SUSY QCD

An invariant non-linear sigma model Lagrangian can be constructed by applying a ***projection*** onto the H covariant subspace: if the sigma-model variables are in the same coset (i.e. left coset) as the original quarks then they transform as ...

$$\xi_\eta = e^{\kappa_R}\begin{pmatrix}\xi_u \\ \mathbb{1}\end{pmatrix} \longrightarrow g_L \xi_\eta \hat{h}^{-1}_{L,n}$$

The “standard” Kahler potential including the tilde’d fields is

$$K^S = \mathrm{Tr}\left[v^2 \ln\left(\xi_\eta^\dagger \xi_\eta\right) + \tilde{v}^2 \ln\left(\tilde{\xi}_\eta \tilde{\xi}_\eta^\dagger\right)\right].$$

HLS in SUSY QCD

Now for the HLS version:

We match the unbroken ***flavour*** symmetry with gauge factors: the Kahler potential is found to be unique at leading order up to an arbitrary parameter, *a;*

$$K = \mathrm{Tr}\left[(1-a)v^2 \ln(\xi_\eta^\dagger \xi_\eta) + (1-\tilde{a})\tilde{v}^2 \ln(\tilde{\xi}_\eta \tilde{\xi}_\eta^\dagger)\right] +$$
$$\mathrm{Tr}\left[av^2\left(\left(\frac{\xi_\eta^\dagger \xi_\eta}{d_\eta^\dagger d_\eta}\right)e^{V_{B'}-V_L} - V_{B'}\right) + \tilde{a}\tilde{v}^2\left(\left(\frac{\tilde{\xi}_\eta \tilde{\xi}_\eta^\dagger}{d_\eta^\dagger d_\eta}\right)e^{V_R-V_{B'}} + V_{B'}\right)\right]$$

The Lagrangian (integrate K with respect to theta/theta-bars) is invariant under flavour transformations. (The factors remove ***moduli associated with broken U(1)'s, eg R-symm***.)

- Physically *a* has the same meaning as *a* previously.
- Eliminating V's by their EoM's, the terms proportional to *a* cancel and we recover the previous non- HLS sigma-model Lagrangian.

HLS in SUSY QCD

Next we define $V = \frac{1}{2}\left(V_L + V_R\right), \quad V' = \frac{1}{2}\left(V_L - V_R\right)$. Since only V is anomaly-free - we need gauge only this group, leaving V' as a modulus and making the identifications:

$$V' = -\ln\left(\sigma\sigma^\dagger\right) \qquad q = \sqrt{a} v \xi_\eta \sigma \qquad \tilde{q} = \sqrt{\tilde{a}}\tilde{v}\sigma\tilde{\xi}_\eta$$

This gives the *quarks and gauge group of the magnetic Seiberg dual*, with Kahler potential

$$K = \mathrm{Tr}\left[q^\dagger q e^{-V} + \tilde{q}\tilde{q}^\dagger e^V\right] + v^2 \ln\left(\frac{\det\left(q^\dagger q\right)}{b^\dagger b}\right) + \tilde{v}^2 \ln\left(\frac{\det(\tilde{q}\tilde{q}^\dagger)}{\tilde{b}\tilde{b}^\dagger}\right)$$

HLS in SUSY QCD

The magnetic mesons come from the "flipped" coset assignment (i.e. the sigma model variables are in the opposite coset compared to that of the original quarks).

Finally we note that if variables of both types of coset are included we have double counted. The potential of the Seiberg magnetic theory removes precisely the superfluous ones:

$$W = \frac{1}{\mu}\,\mathrm{Tr}\,[Mq\tilde{q}]$$

Features

Gauge symmetry restoration: so far (and usually) the entire HLS discussion is in the broken flavour theory. But an important difference in SQCD is that we can *restore the gauge symmetry* to get the unbroken Seiberg dual: taking $e^{\bar{\kappa}_R} \to 0$ we recover the canonical smooth Kahler potential for the quarks. (Also note that a=1 gives enhanced symmetry - c.f. Georgi.) This underlies the appearance of full Seiberg duality rather than just HLS.

Features

Prediction for a: We have colour-flavour locking of the H-factors. This allows us to equate the V determined from its equation of motion, with the V determined from mapping of conserved flavour currents from electric to magnetic theories: this determines *a=1* when $\tilde{v} = v$ and *a=2* when $\tilde{v} = 0$.

Features

Importance of confinement: Consider turning on the original quark VEVs. As we saw right at the beginning, the meson VEV has rank N and appears in the superpotential:

$$W = \frac{1}{\mu}\,\mathrm{Tr}\,[Mq\tilde{q}]$$

Due to the meson VEV, N flavours of magnetic quark get masses and are integrated out. This leaves only n flavours of SU(n) quarks which can be assembled into baryons. Classically these would obey the constraint $b^{N+1\ldots N+n}\tilde{b}_{N+1\ldots N+n}$ = 0.

But this can't be the whole story because SU(n) would be unbroken and the HLS description wouldn't work (the broken electric theory certainly contains no massless gauge bosons).

In order to get around this, an SU(n) theory must confine whenever #flavours = #colours:

$$b^{N+1\ldots N+n}\tilde{b}_{N+1\ldots N+n} = m_q^N \Lambda_{\mathrm{mg}}^{2n-N} = (v\tilde{v})^N \mu^{-N} \Lambda_{\mathrm{mg}}^{2n-N}$$

Features

Prediction for matching scale: if we choose $v = \tilde{v}$ then $b = \tilde{b} = v^n$ and we can solve the above and get an extra piece of information for mu;

$$\mu^N = v^{2(N-n)} \Lambda_{\rm mg}^{2n-N}$$

In conjunction with the usual SQCD matching relation $\mu^{N+n} = \Lambda_{\rm el}^{2N-n} \Lambda_{\rm mg}^{2n-N}$ this determines the magnetic dynamical scale completely in terms of vevs ...

$$\Lambda_{\rm mg} = \Lambda_{\rm el} \left(\frac{v}{\Lambda_{\rm el}} \right)^{2(N^2-n^2)/n(N-2n)}$$

One possible application

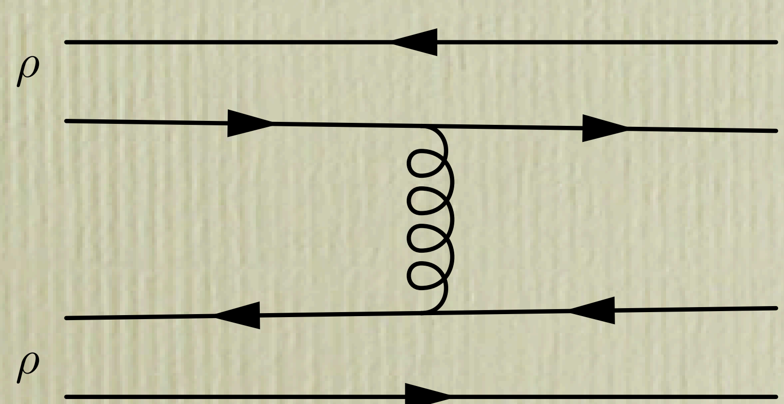
Partially emergent SU(2): if SU(2) is a hidden local symmetry, can go continuously from higgsed SU(2) to technicolour by adding electric quark masses, $m > \Lambda_{\rm el}$.

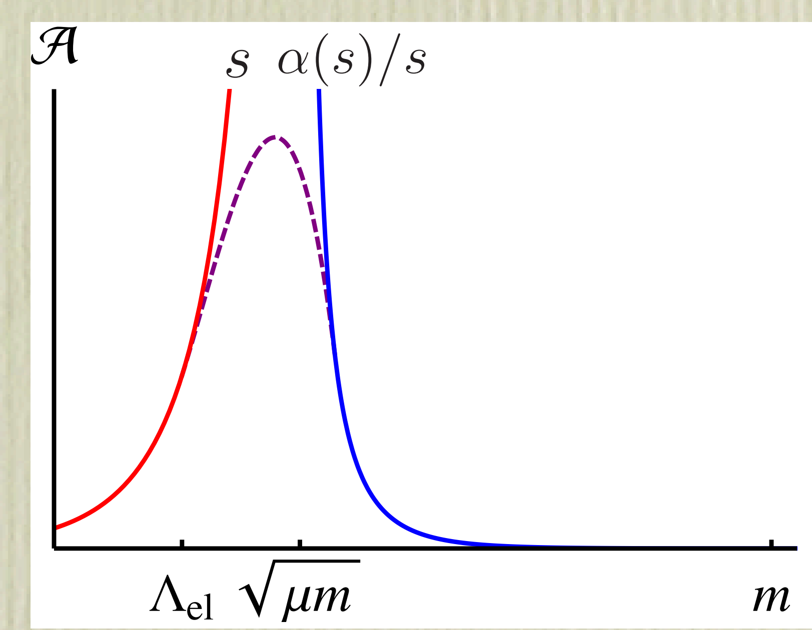
(e.g. Maekawa, Takahashi; SAA, Khoze; SAA Gherghetta; Sannino; Craig, Stolarski, Thaler; Csaki, Shirman, Terning; Csaki Randall Terning).

$$Q = \begin{pmatrix} v\mathbb{1} + \delta Q & P \end{pmatrix}$$

$$V_{\rm mg}^{\alpha} \approx \frac{1}{\mu m}\,\mathrm{Tr}\left[S^{\alpha}(P^{\dagger}P - \tilde{P}\tilde{P}^{\dagger})\right] \qquad \sqrt{\mu m} = \Lambda_{\rm el}\left(\frac{m}{\Lambda_{\rm el}}\right)^{n/2N}$$

At low energies VV scattering is unitarized by the composite higgs. At higher scales the bosons open up:





Summary

- HLS sheds light on why Seiberg duality exists
- Results: leading order Kahler potential; determination of "a" (Vector-meson-dominance)
- Determination of matching scales in certain cases
- Other possibilities: Non-supersymmetric dualities maybe based on theories with scale invariance? (Previous ideas in this direction by Harada-Yamawaki).