A Simple Holographic Model of the Conformal Window and the Techni-Dilaton

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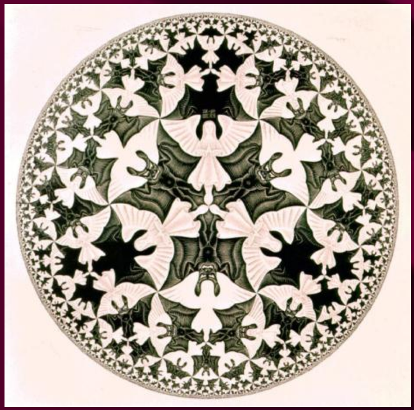
- Stability bounds in the CW
- BKT transitions
- Walking Technicolor
- Techni-dilatons
- Inflation



cf Kiritsis and Jarvinen!

Edinburgh 2013

The Conformal Window

SU(Nc) gauge theory with Nf fundamental quarks

No AF

Nf=11/2 Nc

CFT

Nf = ? Nc

χ SB

$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left\{\frac{11}{3}N_c - \frac{2}{3}N_f\right\} - \frac{g^5}{(4\pi)^4}\left\{\frac{34}{3}N_c^2 - \frac{N_f}{N_c}\left[\frac{13}{3}N_c^2 - 1\right]\right\} + \cdots$$

Using the 't Hooft coupling, and setting $\frac{N_f}{N_c} \to x$ we obtain

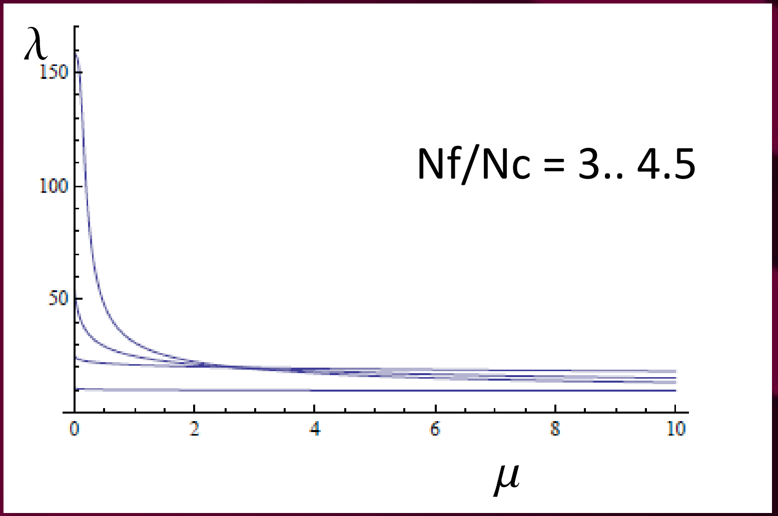
$$\lambda \equiv g^2 N_c \quad , \quad \dot{\lambda} = -b_0\lambda^2 + b_1\lambda^3 + \mathcal{O}(\lambda^4)$$

with

$$b_0 = \frac{2}{3}\frac{(11-2x)}{(4\pi)^2} \quad , \quad \frac{b_1}{b_0^2} = -\frac{3}{2}\frac{(34-13x)}{(11-2x)^2}$$

m $\bar{q}q$

$$\gamma_m^{(1)} = \mu\frac{d\ln m_q}{d\mu} = \frac{3\lambda}{(4\pi)^2}$$



If critical γ = 1…. Nf/Nc ~ 4

Appelquist, Terning, Sannino,…

Holography

We treat RG scale as a direction of space-time....

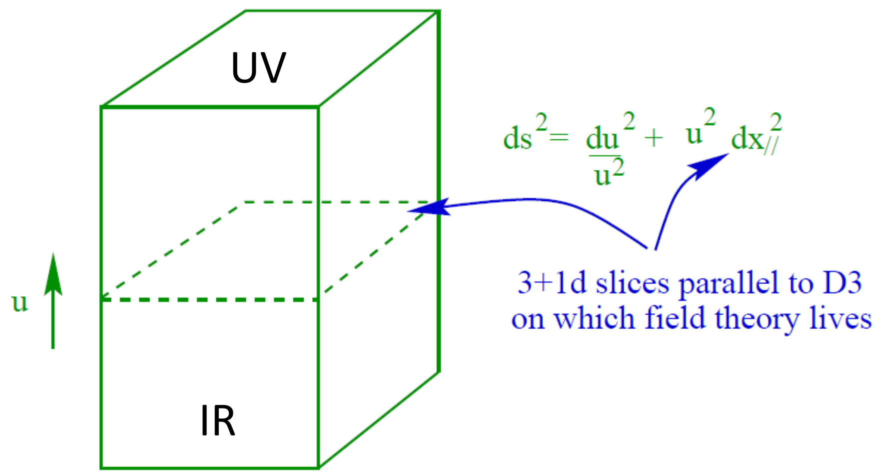
Conformal theories eg

$$\int d^4x \, (\partial\phi)^2$$

have a dilatation symmetry

$$\phi \to e^{\alpha} \, \phi$$

$$x_{3+1} \to e^{-\alpha} \, x_{3+1}$$



Reproduced in AdS metric when

$$u \to e^{\alpha} \, u$$

So u can represent an energy scale...

Holography

Now solve the Klein Gordon equation in AdS....

C and C' are objects in the gauge theory... they have dimension

Δ and $4\text{-}\Delta$

They have the same symmetry properties (as ϕ)

We associate them with gauge invariant J and O

$$\int d^4x \; JO$$

$$S \sim \int d^4x \; du \; \sqrt{-g}(g^{MN}\partial_M\phi\partial_N\phi + M^2\phi^2)$$

Consider spatially homogenous solutions where $\phi = \phi(u)$

$$S \sim \int d^4x \; du \; u^3(u^2\partial_u\phi\partial_u\phi + M^2\phi^2)$$

E-L equation:

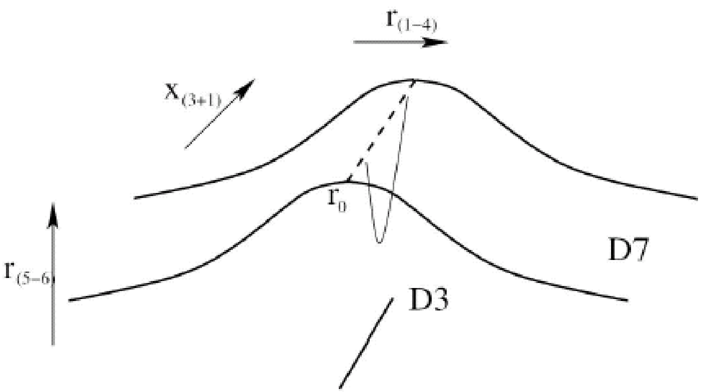
$$\partial_u \left[u^5\partial_u\phi\right] - u^3M^2\phi = 0$$

has solution

$$\phi = \frac{C}{u^\Delta} + \frac{C'}{u^{(4-\Delta)}}, \qquad \Delta(\Delta - 4) = M^2$$

AdS/CFT has taught us that AMAZINGLY the bulk is weakly coupled when N=4 SYM is strongly coupled...

Top-Down Under-pinning



Chiral Symmetry Breaking and Pions in Non-Supersymmetric Gauge/Gravity Duals

J. Babington [a], J. Erdmenger [a], N. Evans [b], Z. Guralnik [a] and I. Kirsch [a*]

Towards a holographic dual of large-N_c QCD

Martín Kruczenski,[a] David Mateos,[b] Robert C. Myers [b,c] and David J. Winters [b,d]

Mesons in Gauge/Gravity Duals A Review

Johanna Erdmenger [a], Nick Evans [bc], Ingo Kirsch [d] and Ed Threlfall [b] *

Flavoured Large N Gauge Theory in an External Magnetic Field

Veselin G. Filev*, Clifford V. Johnson*, R. C. Rashkov[†1] and K. S. Viswanathan[†]

Towards a Holographic Model of the QCD Phase Diagram

Nick Evans,[*] Astrid Gebauer,[†] and Maria Magou[‡]
School of Physics and Astronomy, University of Southampton, Southampton, SO17 1BJ, UK
Keun-Young Kim[§]

Low Energy Hadron Physics in Holographic QCD

Tadakatsu SAKAI[1,*)] and Shigeki SUGIMOTO[2,**)]

Brutally Simple Model of the CW

We model the $\bar{q}q$ condensate by a scalar in AdS

$$S \sim \int d^4x \, du \, \sqrt{-g}(g^{MN}\partial_M\phi\partial_N\phi + M^2\phi^2)$$

$$\phi = \frac{\bar{q}q}{u^3} + \frac{m}{u}, \qquad \Delta(\Delta - 4) = -3$$

We've described a dimension 3 operator…

But in the IR Δ should change…

$$u \rightarrow \rho \qquad \phi = \rho L$$

$$\mathcal{L} \sim \frac{1}{2}\left[\rho^3(\partial_\rho L)^2 + \rho\Delta m^2(\rho, L)L^2\right]$$

Breitenlohmer-Freedman Bound

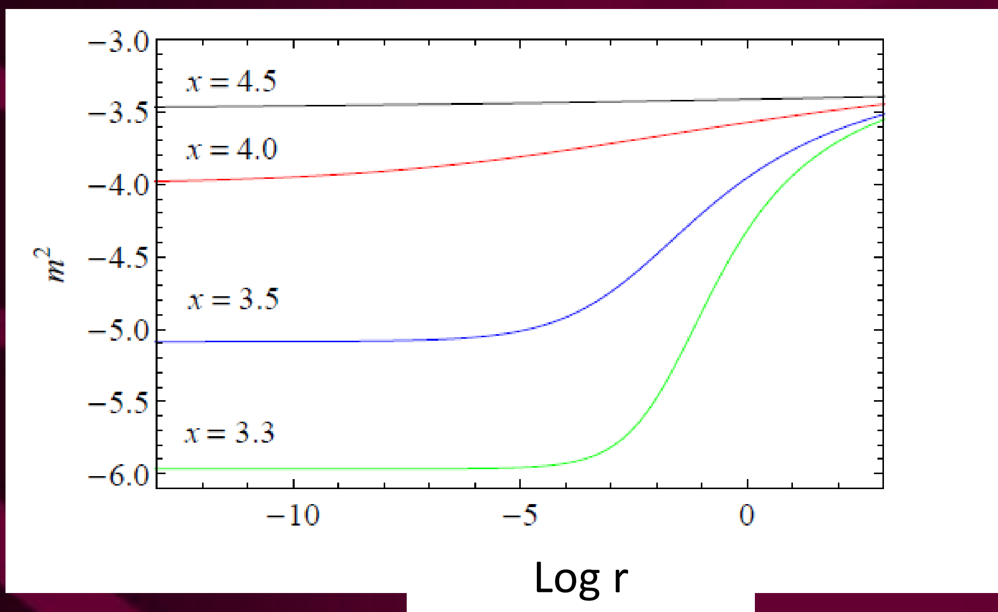
A scalar in AdS is stable until $m^2 < -4$

ie $\Delta < 2$

Brutally Simplified Model

$$\mathcal{L} \sim \frac{1}{2}\left[\rho^3(\partial_\rho L)^2 + \rho\Delta m^2(\rho, L)L^2\right]$$

EG $m^2 = \Delta(\Delta - 4)$ $\gamma_m^{(1)} = \mu\frac{d\ln m_q}{d\mu} = \frac{3\lambda}{(4\pi)^2}$ $\delta m_*^2 \sim 2\gamma_{m*}^{(1)} = \frac{6\lambda_*}{(4\pi)^2}$



With the perturbative result for two loop running of qq dimension the transition occurs at

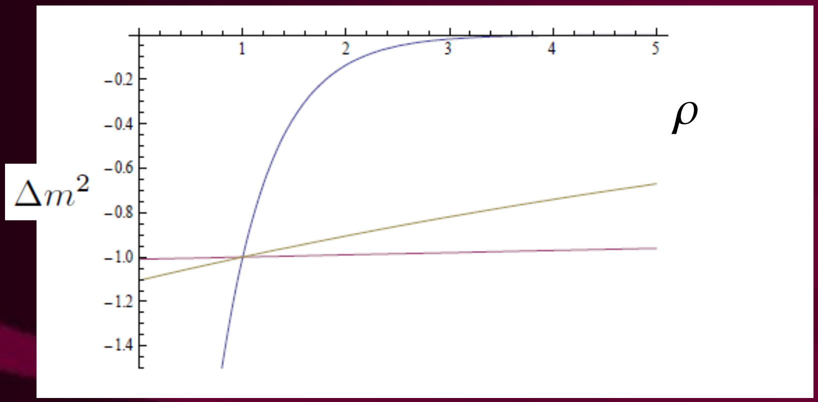
Nf/Nc = 4

If r = ρ then complete instability at small r… to stabilize solution at small r use $r = \sqrt{\rho^2 + L^2}$

Example of Numerics

Let's work with

$$\Delta m^2 = -\Delta_{IR} e^{-\frac{\alpha}{\Delta_{IR}}\left(\sqrt{\rho^2+L^2}-\Lambda\right)}$$



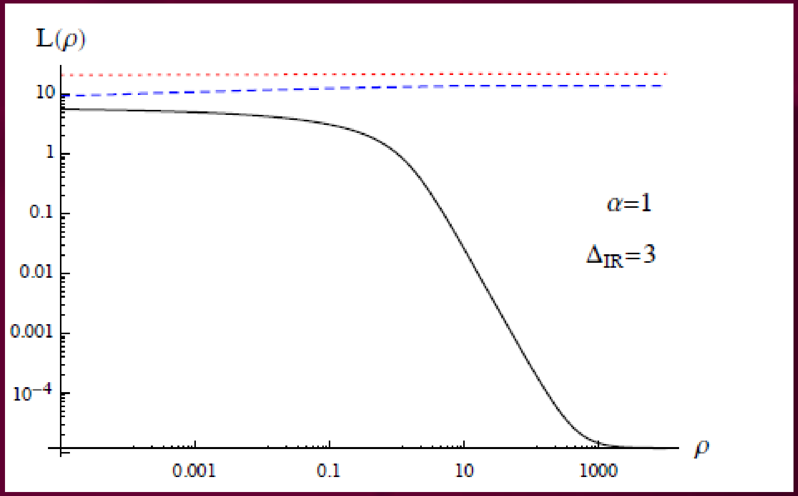
Δ = 1..... BF bound violated

Λ = 1..... at scale Λ

α determines slope at Λ

We numerically solve for L(ρ) by shooting from an IR boundary condition.... L' (ρ) = 0 seems sensible...

L(0) is the IR quark mass...



Meson Masses

We can now include the x-dependent term in the action…

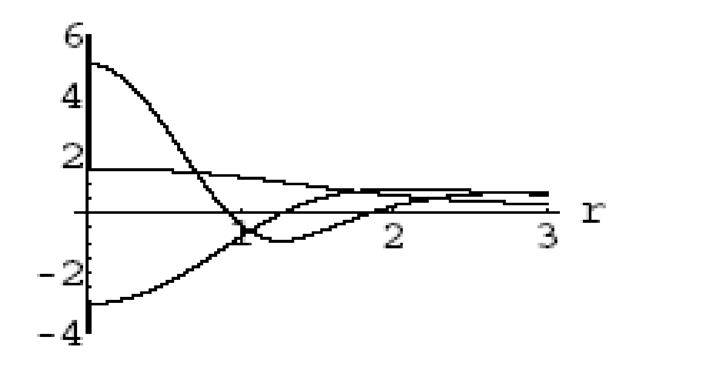
$$\mathcal{L} \sim \frac{1}{2}\left[\rho^3(\partial_\rho L)^2 + \rho\Delta m^2(\rho, L)L^2 + \frac{\rho^3}{(L^2+\rho^2)^2}(\partial_x L)^2\right]$$

L is again cutting off the IR here…

$$L = L_0 + \delta(\rho)e^{ikx}$$

$$k^2 = -M^2$$

We can find L embeddings and then the mesonic qq scalar states…



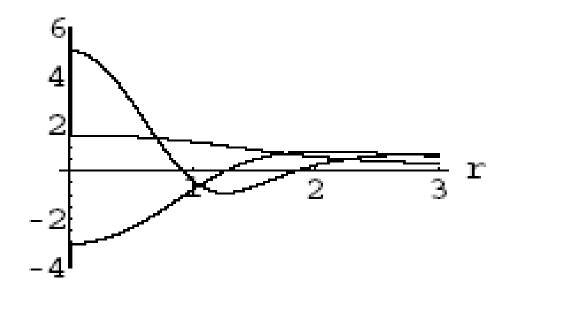
BKT transition

arXiv:0905.4752[hep-th]; arXiv:1002.3159 [hep-th]

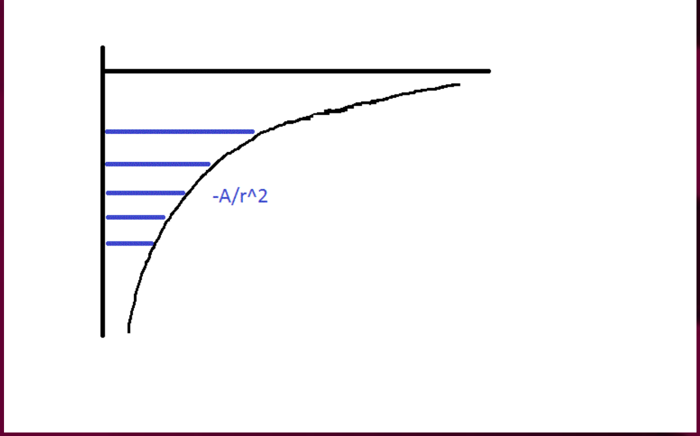
A transition due to a violation of the BF bound in the deep IR is of holographic BKT type…

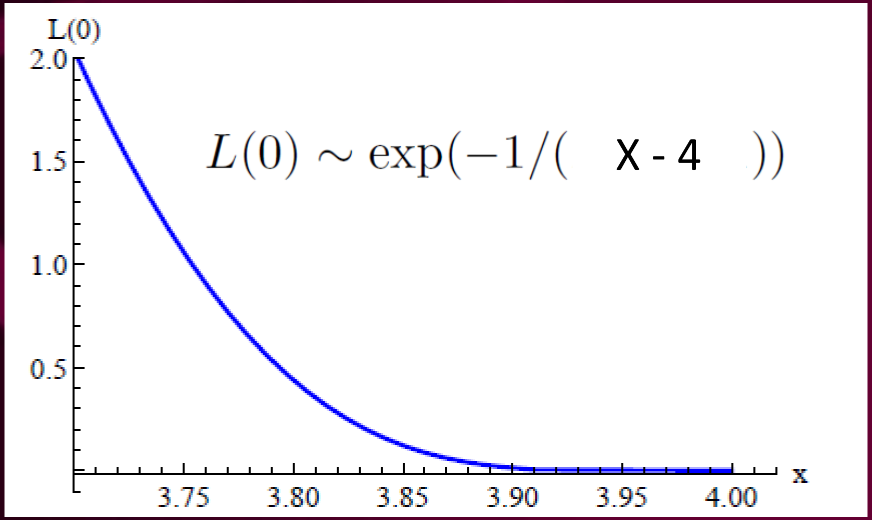
$$L = L_0 + \delta(\rho) e^{ikx}$$

$$k^2 = -M^2$$



The Schroedinger equation for the mesonic fluctuations at $m^2 = -4$ has an infinite number of unstable modes…





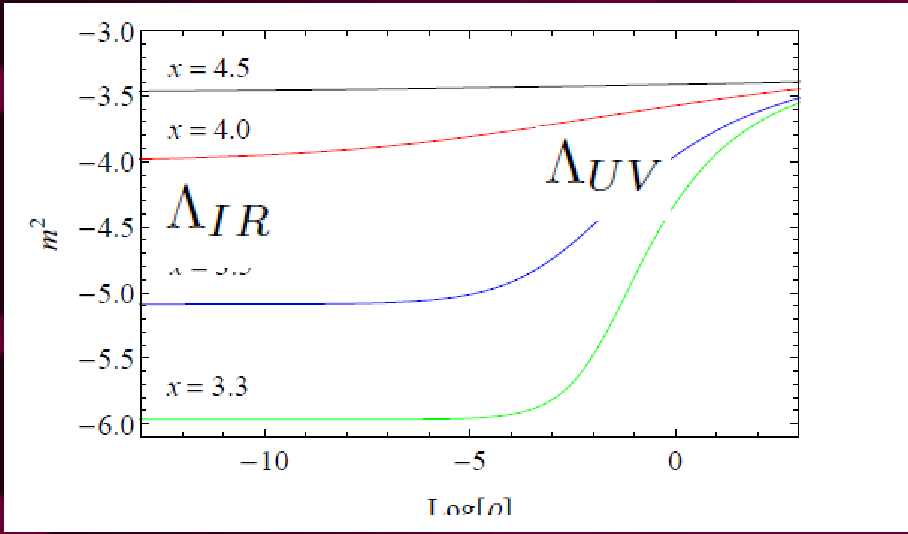
(Miransky scaling)

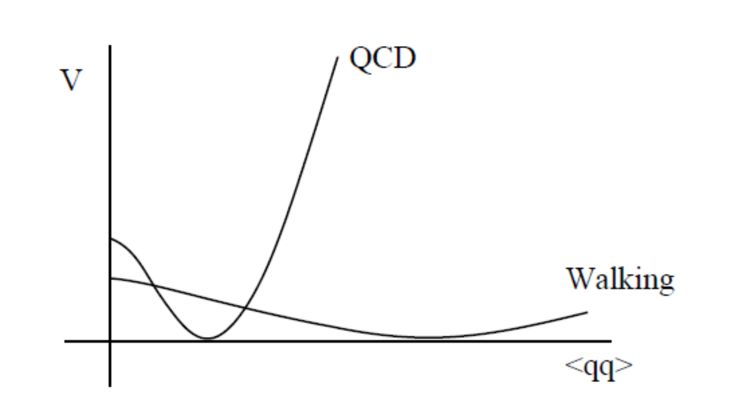
Walking Dynamics

Just above the CW regime theories have an enhanced UV quark condensate

$$\langle \bar{q} q \rangle_{UV} \sim \Lambda_{UV} \langle \bar{q} q \rangle_{IR} \sim \Lambda_{UV} \Lambda_{IR}^2$$

$$f_\pi \sim \Lambda_{IR}$$





- Is the sigma particle light – a techni-dilaton?
- Do these make good inflation theories?

Techni-dilaton

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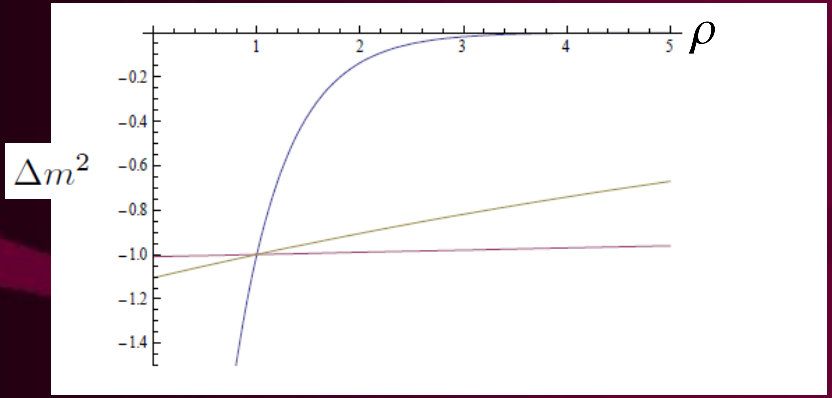

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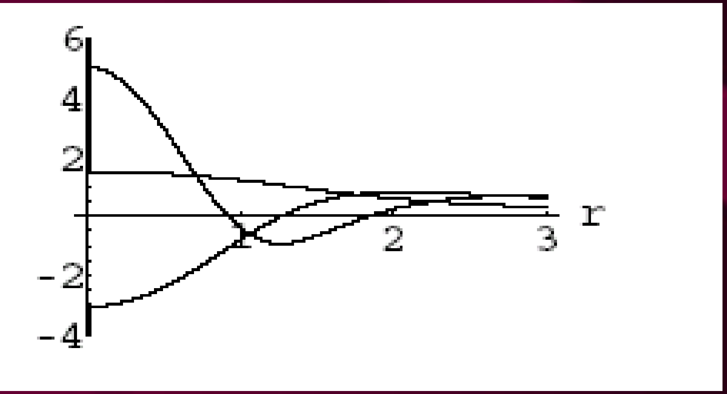
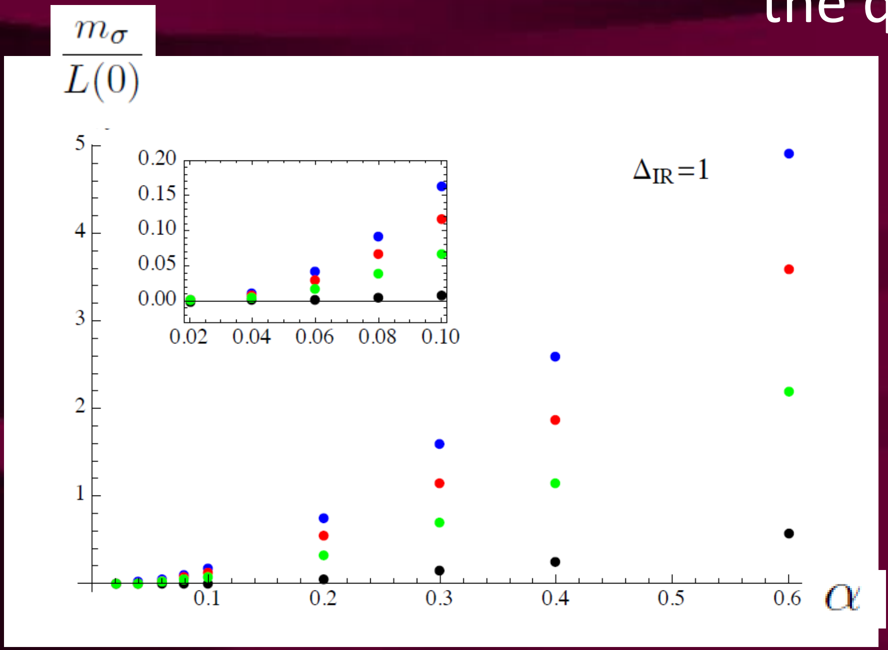
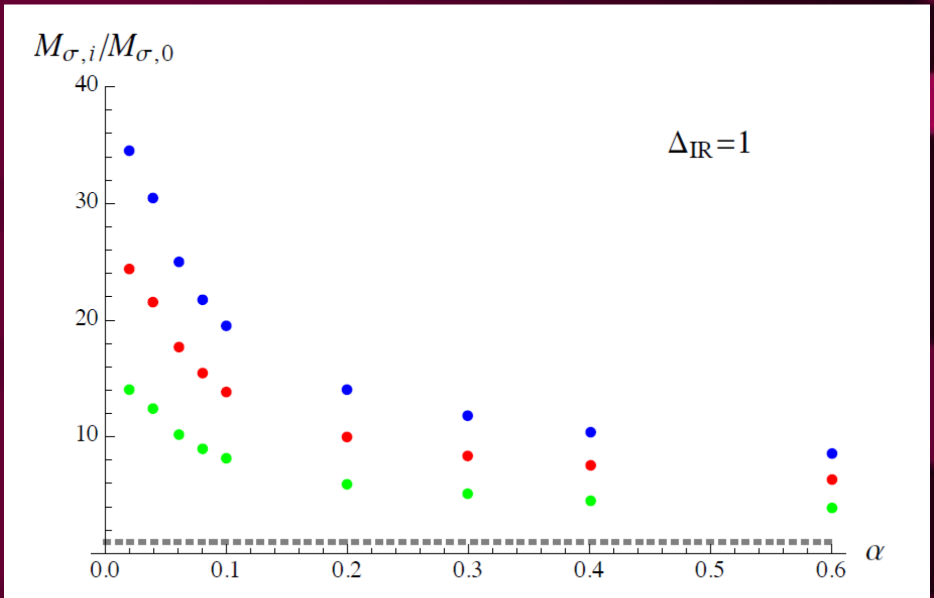


Small relative to what? We compare the meson masses to the quark mass L(0)





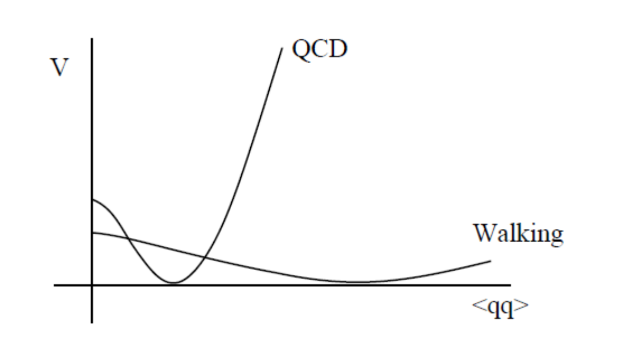
As α falls to zero so do the meson masses… preparing to become a conformal spectrum at α=0…

The lightest state becomes anomalously light – a techni-dilaton (?)…

One could imagine tuning m_higgs < 10 m_techni-quark this way… seems quite easy…

Inflation

arXiv:1009.5678 [hep-th]; arXiv:1208.3060 [hep-ph]



$$\frac{\bar{q}q}{V_0^{3/4}} \to \infty$$

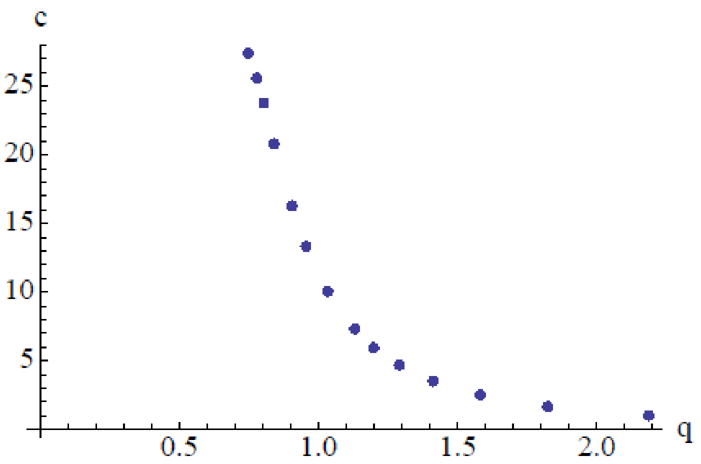
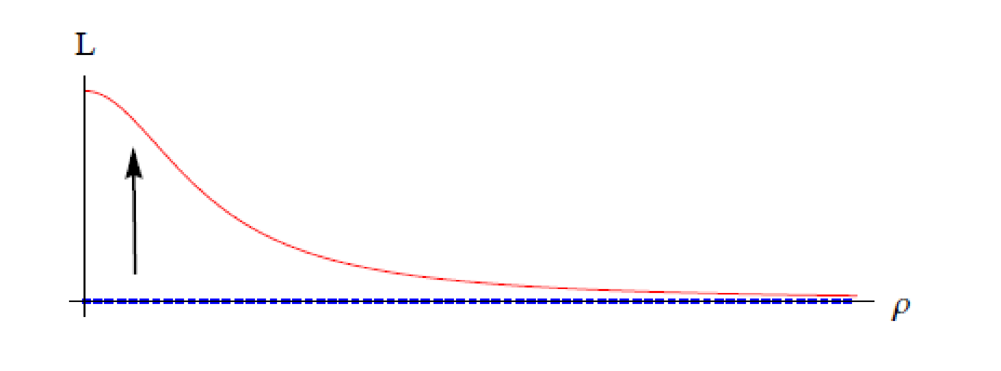
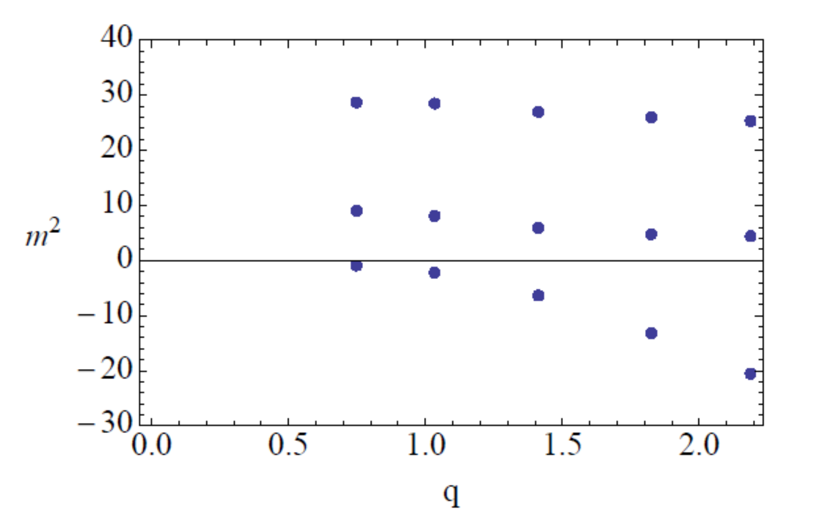


FIG. 3: The quark condensate $\langle \bar{q}q \rangle$ against A in the model of (7) and q in (11) in each case for fixed vacuum energy ΔV.

$$V \simeq \frac{\Delta V}{v^4}\left(\phi^2 - v^2\right)^2$$

The roll from false vacuum to true…





The meson masses of the unstable vacuum.. There is a single inflaton… whose mass falls to zero as one approaches the transition…

You can also time the roll in an explicit time dependent computation and show it increases in length as one approaches the CW transition…

If you're willing to fine tune to the transition you can get an arbitrarily flat potential/ long roll time…

Conclusions

- The CW and exit can be sensibly modelled by a single scalar for $\bar{q}q$ in AdS with dynamics input via a radially dependent mass term
- Holography predicts the conformal window becomes unstable when γ=1
- Holography predicts the transition is of BKT nature
- If the gradient of γ falls to zero at and around γ=1 then a techni-dilaton is observed
- Tuning to the exit point gives a very flat effective potential for $\bar{q}q$ suitable for naïve slow roll inflation

Jarvinen & Kiritsis Holographic Model

arXiv:1112.1261 [hep-ph]

- 5d supergravity $ds^2 = e^A dr^2 + dx_4^2$
- λ scalar to represent running coupling
- V(λ, A) – impose preferred IR and UV behaviour

Nf/Nc ~ 4

- ϕ scalar to represent $\bar{q}q$ condensate
- V(ϕ, λ, A) – to determine if $\bar{q}q$ condenses