# Challenges in observing IRFP on the lattice

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Strongly interacting dynamics beyond the Standard Model, 25/4/2013

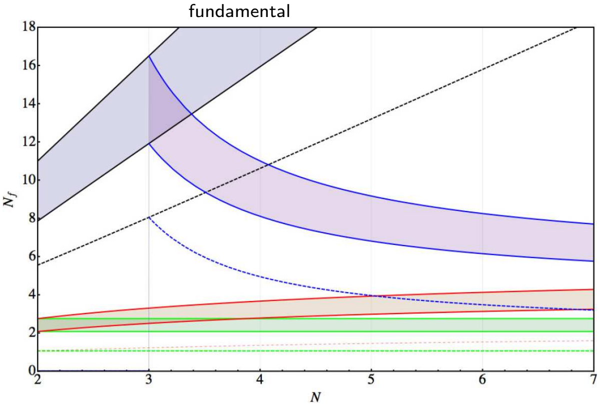
# Introduction:

- **Schrödinger functional** is a powerful tool
  - running coupling $g^2$, fixed pt.
  - anomalous exponent $\gamma(g^2)$
  - $m_{\text{fermion}} = 0$; continuum scheme
- Big problem: coupling runs very slowly $\rightarrow$ *Strong bare coupling*
  - $\rightarrow$ Stability? Bulk transitions?
  - $\rightarrow$ Large lattice artifacts? (Irrelevant operators) How quickly these vanish with growing $L/a$?
  - (Is taste breaking w. staggered fermions relevant?)
  - small, noisy signal

$\rightarrow$ **Improvement:**
unimproved Wilson
$\rightarrow$ improved Wilson-clover
$\rightarrow$ highly "improved" hypercubic stout smeared Wilson-clover ("HEX").

- Theories:
  - SU(2) with $N_f = 4, 6$ and 10 fundamental rep. fermions
  - SU(2) with $N_f = 2$ adjoint rep. fermions

# Conformal window in SU(N) gauge

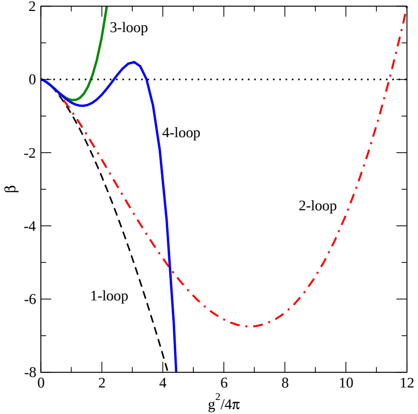



[Appelquist, Lane, Mahanta, Cohen, Georgi, Yamawaki, Schrock, Sannino, Tuominen, Dietrich]

- Upper edge of band: asymptotic freedom lost
- Lower edge of band: ladder approximation
- Walking can be found near the lower edge of the conformal window: large coupling, non-perturbative - lattice simulations needed!
- In higher reps it is easier to satisfy EW constraints [Sannino,Tuominen,Dietrich] $\rightarrow$ lot of recent activity!

# Existence of the IRFP essentially non-perturbative

Example: Perturbative $\beta$-function of SU(2) gauge with $N_f = 6$ fundamental rep fermions

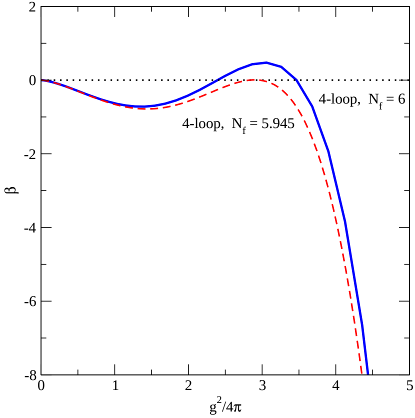



[4-loop MS: Ritbergen, Vermaseren, Larin]

Results from lattice: existence of IRFP inconclusive [Karavirta et al]

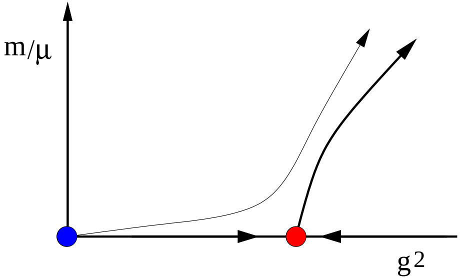
# "Walking" at $N_f \lesssim 6$

Interestingly, the fixed point vanishes from 4-loop MS beta function if $N_f$ is slightly lowered from 6:



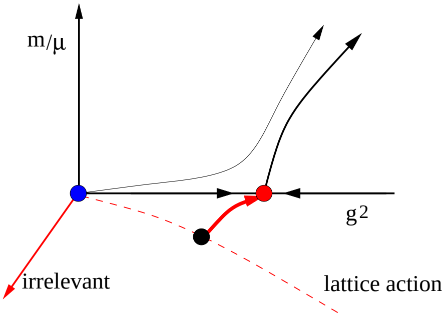
# RG flow in the conformal case

- Relevant parameters at UV: $g^2$ and $m_Q$




- Only $m_Q$ is relevant at the IRFP.
- Scaling near IRFP: masses of physical particles $M \propto (m_Q)^{1/(1+\gamma)}$

# RG flow on the lattice




- Irrelevant operators (cutooff effects) die out as $(a/L)^{d-\gamma'}$ ($L$: IR scale, $\gamma'$: some anomalous exponent)
- Evolution of $g^2$ along the physical axis *very slow*

$\Rightarrow$ irrelevant operators can (and do!) mask the physical evolution

- Need either:
  - Very large lattices (large $L/a$) – impractical
  - Very high quality lattice action – small cutoff effects

# Measuring the coupling

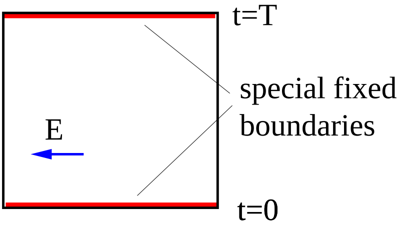
**Schrödinger functional**: Generate a *background* chromoelectric field using non-trivial fixed boundary conditions, parametrised with a **twist angle** $\eta$

At the classical level, we have

$$\frac{dS_{\text{class.}}}{d\eta} = \frac{A}{g^2}$$

where $A(\eta)$ is a known constant.
At the quantum level, we define the coupling through

$$\frac{1}{g^2} \quad = \quad \left\langle \frac{1}{A}\frac{dS}{d\eta} \right\rangle$$



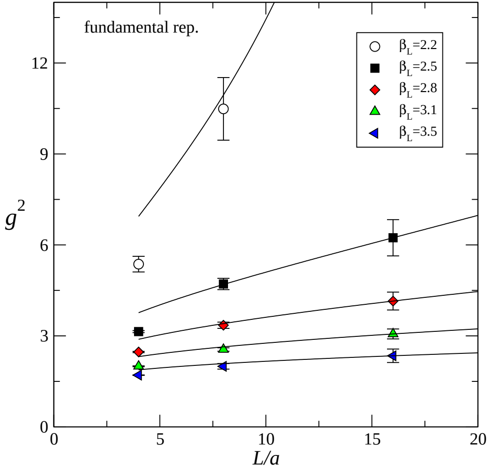

- Evaluates $g^2$ directly at scale $\mu = 1/L$, the lattice size
- Can use $m_Q = 0$
- Has been used very succesfully in QCD by the Alpha collaboration

# The idea, roughly:

Measure $g^2$ at different $\beta_L$ (lattice spacing $a$) and lattice sizes

Example: fundamental representation SU(2), $N_f = 2$:

- $L/a$ grows, $k \sim a/L$ decreases, $g^2(L)$ increases: *asymptotic freedom*, OK!
- Large $\beta_L \to$ small lattice spacing $\to$ small volume
- Continuous line: coupling evaluated from 2-loop perturbative $\beta$-function (fixed to measurement at $L/a = 16$)



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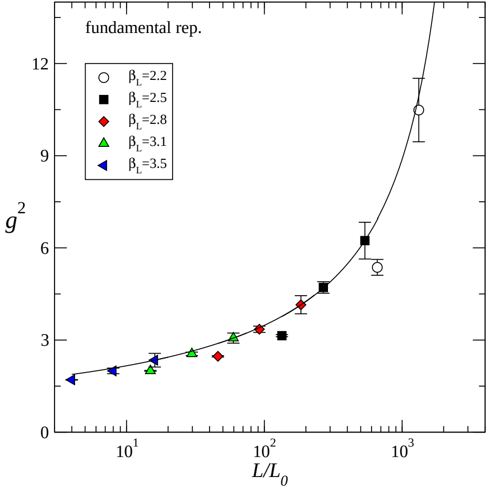


# Step scaling function

- Step scaling: coupling when the lattice size is (e.g.) doubled

$$\Sigma(u, L/a) = g^2(g_0^2, 2L/a)_{u=g^2(g_0^2, L/a)}$$

- Continuum limit:

$$\sigma(u) = \lim_{a/L\to 0} \Sigma(u, L/a)$$

- Step scaling is related to $\beta$-function:

$$-2\ln 2 = \int_u^{\sigma(u)} \frac{dx}{\sqrt{x}\beta(\sqrt{x})}$$

- Close to the fixed point:

$$\beta(g) \approx \frac{g}{2\ln 2}\left(1 - \frac{\sigma(g^2)}{g^2}\right)$$

- 1-loop analysis indicates that finite lattice spacing effects large ($\sim 50\%$ at $L/a = 10$) $\Rightarrow$ improvement! [Alpha; Karavirta et al.]

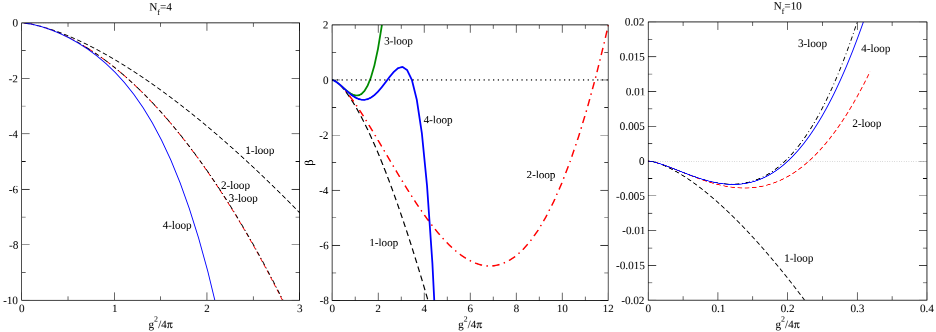
# SU(2) fundamental representation at $N_f = 4, 6, 10$

# Fundamental rep SU(2) with $N_f = 4, 6$ and 10

- Measure coupling using SF
- Measure $\gamma$ also using SF (different boundary conditions)
- Choose:
  - $N_f = 4$: QCD-like, chiral symmetry breaking
  - $N_f = 6$: $\sim$ lower edge of conformal window
  - $N_f = 10$: upper edge of conformal window
- We use 1-loop perturbative $c_{\rm SW}$, with perturbative boundary improvement coefficients

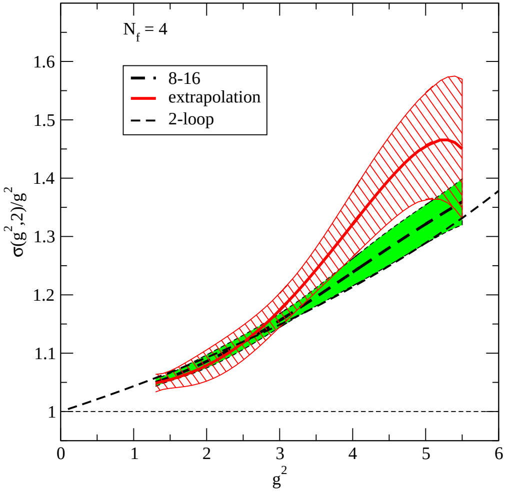
# Fundamental rep: perturbation theory

Perturbative $\beta$-function w. $N_f = 4, 6, 10$ [3,4-loop MS: Ritbergen, Vermaseren, Larin]



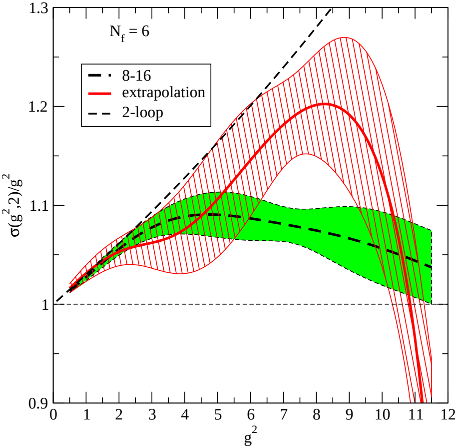

$N_f = 4$ QCD-like, confining
$N_f = 6$ completely non-perturbative
$N_f = 10$ perturbative Banks-Zaks FP, test case.

# Step scaling function: $N_f = 4$



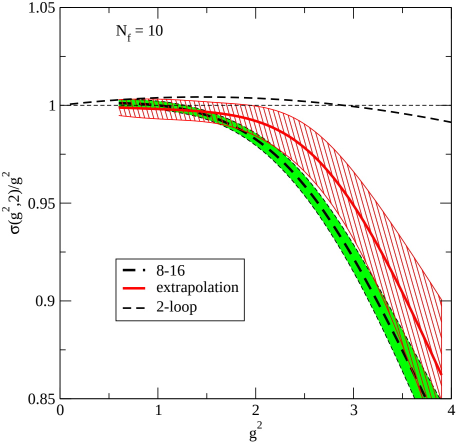

QCD-like behaviour

# Step scaling function: $N_f = 6$




- Perhaps IRFP at $g^2 \gtrsim 12$ ($\alpha \gtrsim 1$)?
- Lose control at $g^2 \sim 10 - 14$ ($\beta_L \approx 1.39$)
- Need to have actions which work there

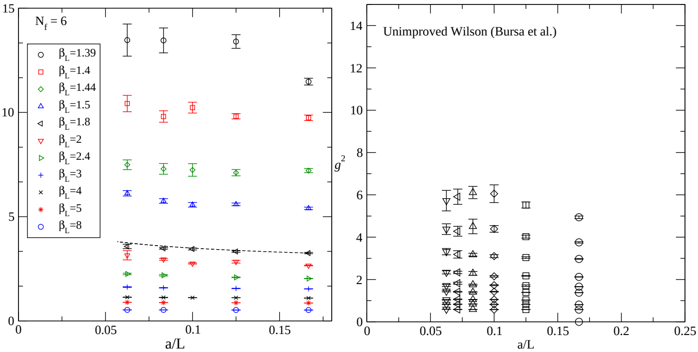
# Step scaling function: $N_f = 10$




- We see $\sim$ zero evolution below $g^2 \sim 2.5$
- Above this step scaling diverges from perturbative curve.
- It is caused by our $\beta_L = 4/g_0^2 = 1$ data set – strong coupling, lattice artefact?

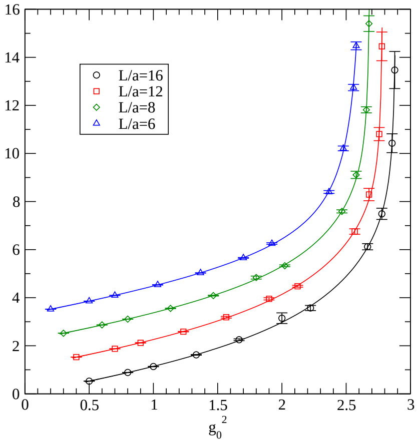
# $N_f = 6$: compare clover/Wilson

[Unimproved Wilson: Bursa et al.]



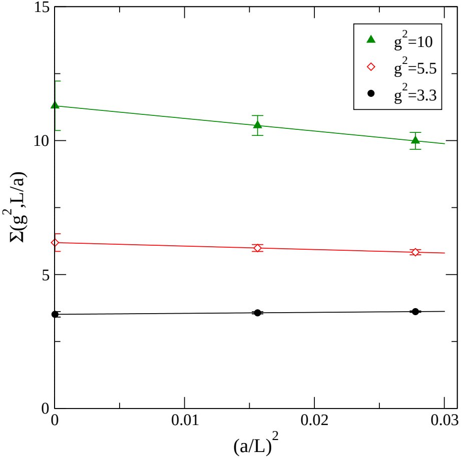
- Interpolate data with the rational function

$$\frac{1}{g^2(\beta_L, L/a)} = \frac{1}{g_0^2}[1 + \sum_{i=1}^{n} a_i g_0^{2i}]/[1 + \sum_{i=1}^{m} b_i g_0^{2i}].$$

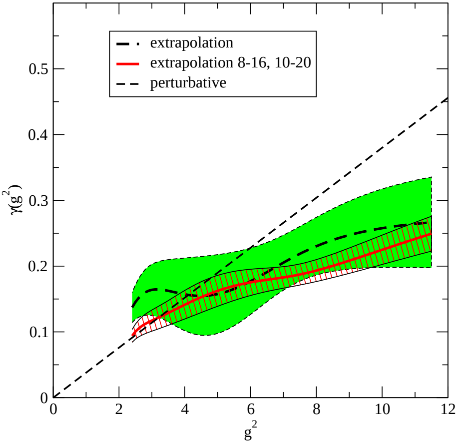



- Construct step scaling using pairs $L/a$=(6,12) and (8,16)
- $\Sigma(u, L/a) = g^2(2L/a)_{g^2(L/a)=u}$

- We use 2nd order in $(a/L)$ extrapolation to continuum
- Or, use only 8-16 (largest volume step) without interpolation
- $N_f = 6$, 3 arbitrarily chosen $u = g^2$-values:



# Result: $N_f = 6$ mass anomalous exponent




- requires simulations with different SF boundary conditions than coupling
- Easier to measure than coupling
- 6-12, 8-16, 10-20
- Smaller than perturbative at strong coupling – generic feature?

# Challenges:

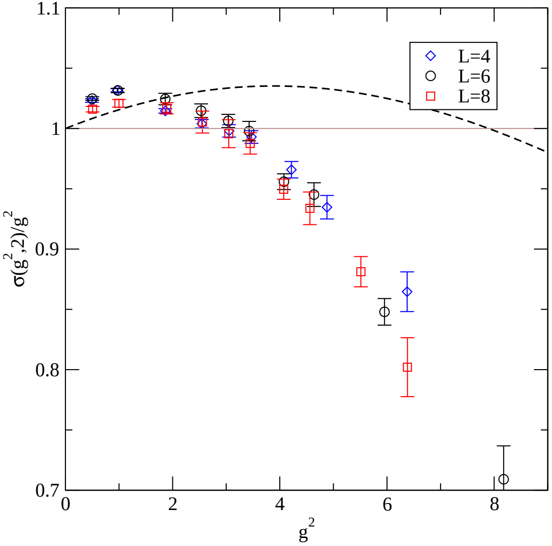
- In the previous computation we used perturbatively determined Wilson-clover coefficient and boundary improvement terms.
- Why not non-perturbative clover coefficient?
  *fails, clover coefficient becomes too large at strong bare coupling* [Karavirta et al.]

$\rightarrow$ Perturbative or tree-level clover coefficient? *OK.*

- Boundary improvement terms for SF have been calculated in pert. theory (fundamental: [Luscher, Weisz, Sint, Sommer]; higher reps: [Karavirta et al.])
  These are *not reliable at strong bare coupling*: correction $\sim 100\%$.
- Optimised background field value (higher reps: [Karavirta et al; Sint and Vilaseca]): background field becomes weaker; *noise increases significantly*.
- Recipe: use action which suppresses fluctuations: HEX smeared Wilson-clover action. [Degrand et al.]
  - improvement coefficients close to tree-level values
  - stability can improve
  - measurements much less noisy

# SU(2) with 2 adjoint fermions

*Very preliminary* step scaling function




- Discretisation effects under much better control

# Conclusions

- Measurement of the coupling constant evolution is significantly more difficult than in QCD-like theories:
  - Slow evolution $\rightarrow$ small signal
  - Slow evolution $\rightarrow$ strong bare coupling
- We should use actions which
  - can be used at strong lattice scale coupling
  - have as small as possible cutoff effects
- Further ideas: gradient flow; cooling