1. The Roman Pots of the TOTEM experiment

Proton-proton elastic scattering has been measured by the TOTEM experiment at the CERN Large Hadron Collider at $\sqrt{s} = 7$ TeV in dedicated runs. The small scattering angle (down to a few percent) is detected with the movable near-beam insertions (Roman Pots) equipped with stacks of silicon microstrip detectors, installed on the outgoing LHC beams.

The Roman Pots of TOTEM
- 4 stations at $s = 1.147$ m and $s = 2.20$ m
- 6 Pots per station (4 vertical + 2 horizontal devices)
- a total of 24 Roman Pots

High angular and spatial resolution of track reconstruction:
- Precise detector alignment:
  - Beam touching alignment around 50 μm
- Track based alignment $\delta x < 10 \mu m$, $\delta y = 10 \mu m$

2. How to transport protons from the interaction point to the detectors?

Scattered protons are detected after having moved through a segment of the LHC lattice containing 29 magnets per beam.

At IPs they are characterized by transverse position $(x, y)$ and scattering angle $(\delta x, \delta y)$ and they are observed with transverse positions $(x_{IP}, y_{IP})$ and angles $(\delta x_{IP}, \delta y_{IP})$. Proton trajectories are described by a transport matrix:

$$ T = \begin{pmatrix} x_{IP} & y_{IP} \\ \delta x_{IP} & \delta y_{IP} \end{pmatrix} \begin{pmatrix} T_{xx} & T_{xy} \\ T_{yx} & T_{yy} \end{pmatrix} \begin{pmatrix} x & y \\ \delta x & \delta y \end{pmatrix} $$

with $x_{IP}$ magnification and $L_{IP}$ effective length being particularly important, which can be expressed in terms of the betatron amplitude $\chi$ as:

$$ L_{IP} = \frac{1}{\chi} \left( \frac{1}{T_{xx}} \right) $$

In case of a perfect machine at $m_3$ would be 0. In reality, they are close to zero and approximately $\delta x_{IP} = \frac{1}{\chi} \left( \frac{1}{T_{xx}} \right)$.

As the values of the reconstructed angles are directly depend on the optical functions, the accuracy of optics defines the systematic errors of the final physics results.

3. Optics errors induced by LHC imperfections

The proton transport matrix $T(x, M)$ is defined by the machine settings $M$. Therefore it is calculated with the MAD-X code for each group of runs with identical optics based on some data sources:

- TIMBER: actual currents of the magnets
- FIDEL: LSA: current to strength conversion curves
- WISE: measured imperfections (harmonics, displacements, rotations)

The lattice is subject to additional $\Delta M$ imperfections; not measured per se but so far which can alter the transport matrix

$$ T(x, M) \rightarrow T(x, M + \Delta M) = T(x, \Delta M) + \Delta T $$

$\Delta T$ being measurable with 5–10% accuracy is not enough for the TOTEM physics program.

4. Constraints for optics estimation from distributions measured with Roman Pots for $\beta_{IP} = 3.5$ m

Elastically scattered proton collinearity constraints

$$ \frac{S_1}{L_1} \text{ and } \frac{S_2}{L_2} \text{ can be expressed with measurables:} $$

$$ R_1 = \frac{S_1}{L_1} \quad \text{ and } \quad R_2 = \frac{S_2}{L_2} \quad \text{ or } \quad R_3 = \frac{S_3}{L_3} $$

and $R_N$ follow this pattern for beam 2. Their precision is 0.5%.

5. Real optics estimation: matching the machine parameters for $\beta_{IP} = 3.5$ m

From detailed sensitivity studies it is known that there are 6 relevant magnets per beam segments between IPs and Roman Pots:

- the inner triplet (2 MQXA and 2 MQXB magnets)
- the MQLM, MQY magnets (less important, but not negligible)

In total 36 constraints were applied:

- 10 constraints from measurements
- 24 optics constraints (2 constraints: strength, rotation) = 24 magnet design constraints
- 2 beam momentum constraints

26 parameters optimized, magnet strengths, rotations, beam momenta.

The phase space is $\sigma = \theta^2$ and the $\chi^2 = \frac{1}{2} \cdot \sigma^2$ function is minimized, where

$$ \chi^2 = \frac{1}{2} \cdot \left( \frac{1}{T_{xx}} \right)^2 \quad \text{ and } \quad \sigma^2 = \frac{1}{2} \cdot \left( \frac{1}{T_{xx}} \right)^2 $$

The design part defines the nominal machine as an attractor $A \in \tau$, while the measured part “pushes” the place of minimum from $A$ to a nearby $P$ to meet the measured constraints.

In practice both the CERN MICRON and Windows Solve were used to estimate $P$. Finally, $\sigma_{\Delta}$ and $\lambda$ were used for data analysis.

6. Matching results

- Before matching
  - Beam 1: $L_{1P} = 22.4 \text{ m, } \Delta L_{1P}/\text{Opt}= -3.21 \pm 10^{-1}$
  - Beam 2: $L_{2P} = 16.4 \text{ m, } \Delta L_{2P}/\text{Opt}= -3.29 \pm 10^{-1}$
- After matching
  - Beam 1: $L_{1P} = 22.6 \text{ m, } \Delta L_{1P}/\text{Opt}= -3.12 \pm 10^{-1}$
  - Beam 2: $L_{2P} = 20.7 \text{ m, } \Delta L_{2P}/\text{Opt}= -3.15 \pm 10^{-1}$

7. Monte-Carlo validation of the method for $\beta_{IP} = 3.5$ m

In the study the following LHC parameters were perturbed to simulate the effect of $\Delta M$:

- The strength of relevant magnets
- Beam momenta
- Magnet displacements, rotations
- Risers, harmonics
- Scattered protons angular distribution

Distribution of optical function errors resulting from imperfections:

- Bias of $\beta_{IP}/\beta$ = 0.73% RMS of $\beta_{IP}/\beta$ = 1.6% (Mean $\Delta L_{1P}/\text{Opt}= -3.21 \pm 10^{-1}$)
- Bias of $\delta L_{1P}/\Delta L_{1P} = 0.39$, RMS$(\delta L_{1P}/\Delta L_{1P}) = 4.2$% (Mean $\Delta L_{1P}/\text{Opt}= -3.21 \pm 10^{-1}$)
- Optics imperfections after matching with Roman Pot

Bias of $\beta_{IP}/\beta = 0.13%$ RMS of $\beta_{IP}/\beta = 0.17%$

Bias of $\delta L_{IP}/\Delta L_{1P} = 0.083%$ RMS$(\delta L_{IP}/\Delta L_{1P}) = 0.16%$

---

Frigyes Janos Nemes@cern.ch and Hubert Niewiadomski@cern.ch