Review of Synchrotron Radiation effects in the FFS

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Beamsize

We are interested in the horizontal beamsize at the IP.

Horizontal plane

$$\sigma^2 = \sigma_0^2 + \sigma_i^2 + \sigma_{rad}^2$$

$$\sigma_0 \equiv \text{zero}^{\text{th}} \text{ order approx.}$$
$$\sigma_i \equiv \text{result from aberrations}$$
$$\sigma_{rad} \equiv \text{interaction with magnets}$$

Evaluated by:

- tracking of particles
- mathematical approximations

Beam Radiation Model

x describes the displacement of a particle at ($s = L$, e.g. IP) due to radiation, where all other effects are ignored.

$$x = \sum_{i=1}^{N(T)} \Delta x_{i,total} - x_0 \tag{1}$$

- $\Delta x_{i,total}$: is the total deviation due to the i^{th} photon radiated
- x_0: is $\langle \sum_{i=1}^{N(T)} \Delta x_{i,total} \rangle$,
  in order to make $\langle x \rangle = 0$, and $\sigma_{rad}^2 = \langle x^2 \rangle$
- N: is the number of photons radiated
- T: time to cross the bending magnet

Finding Δx_i

Δx_i is the effect at $(s = L)$ due to a photon of energy u radiated at $s = s_i$, therefore it has to be propagated from s_i to L.

$$\Delta x_i = (u/E) R_{16}(s_i, L) \tag{2}$$

It becomes:

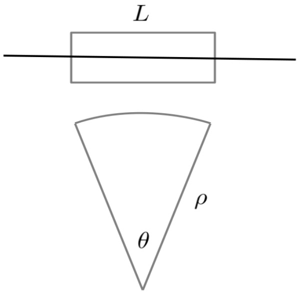
$$\sigma^2_{rad} \approx C_2 \int \frac{E^5}{\rho^3} \left\{ \sqrt{\frac{\beta_L}{\beta_s}} \left[\eta \cos \Delta\phi(s, L) + (\alpha\eta + \beta\eta') \sin \Delta\phi(s, L)\right] - \eta_L \right\}^2 ds$$

- $C_2 = 4.13 \times 10^{-11} [\mathrm{m}^2 \cdot \mathrm{GeV}^{-5}]$
- E: is the beam energy

included in MAPCLASS2.

One dipole (theoretical expression)

Using $R_{16} = \rho(1 - \cos\theta)$



$$\begin{aligned}
\sigma^2_{rad} &= C_2 \int_0^L \frac{E^5}{\rho^3} R^2_{16}(s, L) ds \\
&= C_2 \int_0^\theta \frac{E^5}{\rho^3} [\rho(1 - \cos(\theta - \chi))]^2 \rho d\chi \\
&= C_2 E^5 \left[\frac{1}{4}(6\theta - 8\sin\theta + \sin(2\theta))\right] \quad {}^{1} \\
&= C_2 E^5 \left(\frac{\theta^5}{20} - \frac{\theta^7}{168} + \frac{\theta^9}{2880} - \frac{17\theta^{11}}{1330560} + O(\theta^{13})\right)
\end{aligned}$$

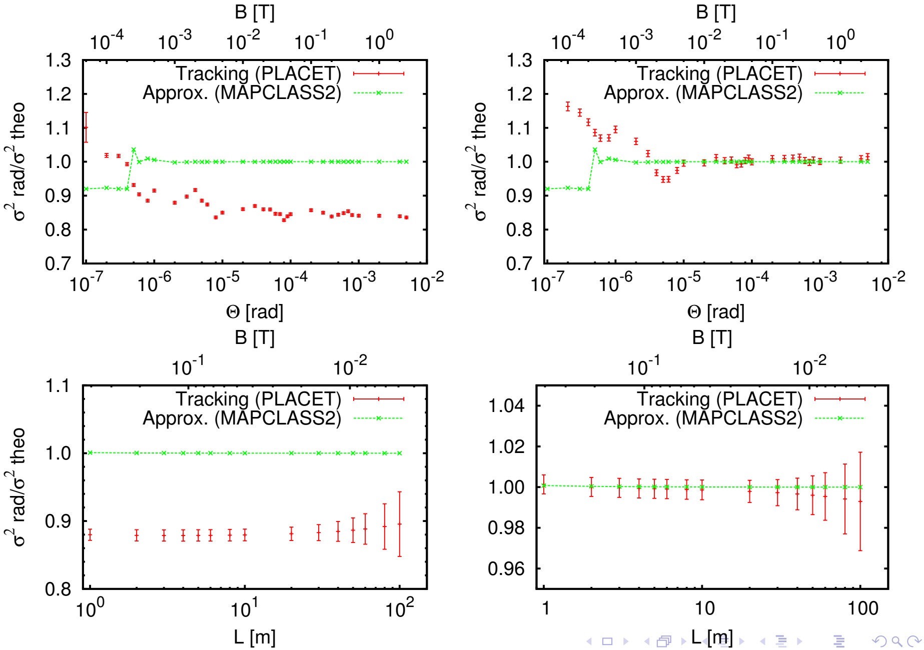
Theoretical expression for a drift has been also derived. Now, the theoretical expression, the approximated result and tracking with PLACET could be compared.
Some care should be taken when using the expression above due to numerical precision.
MAPCLASS2 uses on MAD-X twiss table results.
PLACET has two different radiation methods "Default" and "six_dim".
Radiation beamsize has been normalized to the theoretical value.

1

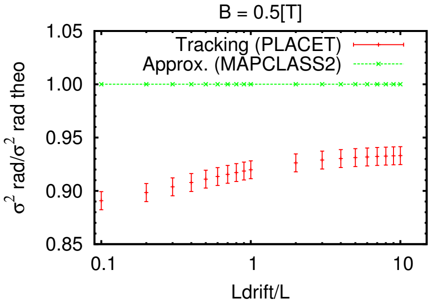
[1]If E is considered constant.

Default Synrad

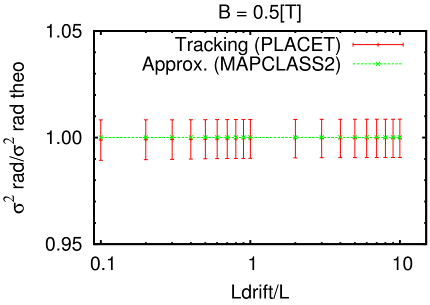
Flag "-six_dim 1"

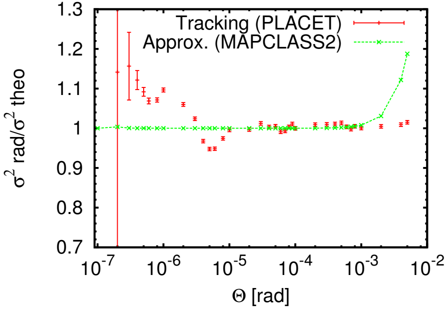


Default Synrad

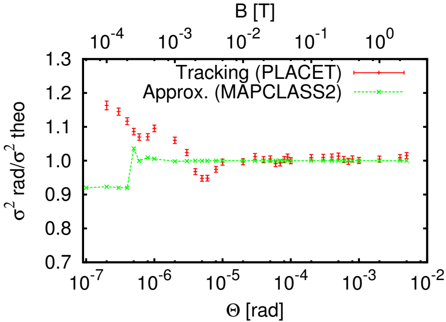


Flag "-six_dim 1"





Twiss



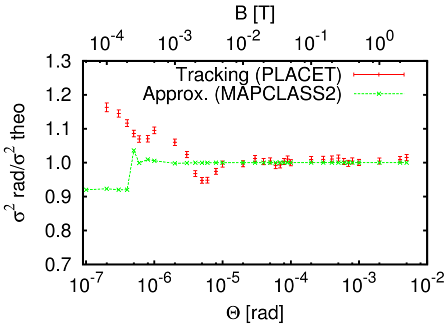
ptc_twiss

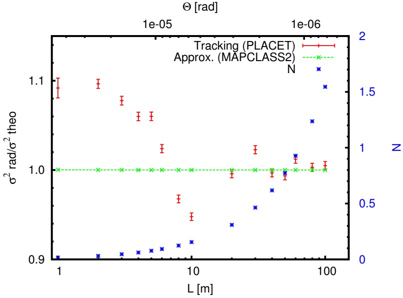
Low number of photons

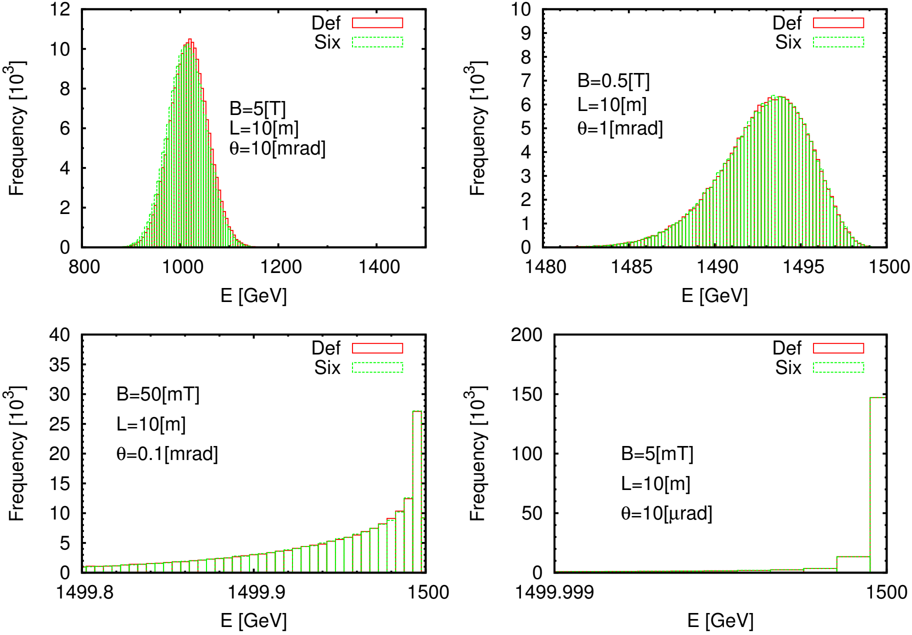
Average number of photons emitted is

$$\langle N \rangle = \frac{1}{c} \int_0^L ds \int_0^\infty du \; n(u,s) \qquad (3)$$

$$\approx C_1 E \theta \qquad ; C_1 = 20.61[\mathrm{GeV}]^{-1} \qquad (4)$$



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Six
B=5[T]
L=10[m]
θ=10[mrad]
Frequency [10^3]
E [GeV]
0 2 4 6 8 10 12
800 1000 1200 1400
Def
Six
B=0.5[T]
L=10[m]
θ=1[mrad]
Frequency [10^3]
E [GeV]
0 1 2 3 4 5 6 7 8 9 10
1480 1485 1490 1495 1500
Def
Six
B=50[mT]
L=10[m]
θ=0.1[mrad]
Frequency [10^3]
E [GeV]
0 5 10 15 20 25 30 35 40
1499.8 1499.9 1500
Def
Six
B=5[mT]
L=10[m]
θ=10[μrad]
Frequency [10^3]
E [GeV]
0 50 100 150 200
1499.999 1500

Conclusions

- Mapclass2 agrees with theory, some care should be taken with twiss file.
- "six_dim" in PLACET radiation model is more accurate than default radiation calculation. Both shows differences with theoretical model for bends causing low photon emission.
- Theoretical model for low photon emission stills in check.

References

Sands, Matthew. Emittance growth from radiation fluctuations. SLAC/AP – 47. December, 1985.

Renier, Ives. Implementation and validation of the linear collider final focus prototype : ATF2 at KEK (Japan). Doctoral Thesis, LAL10-91. June 2010.

Additional slides

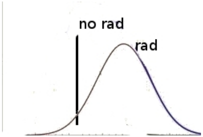
Dispersion function

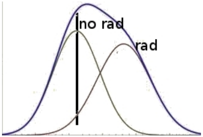
$$\begin{pmatrix} \eta(s_2) \\ \eta'(s_2) \\ 1 \end{pmatrix} = \begin{pmatrix} C(s_1, s_2) & S(s_1, s_2) & R_{16}(s_1, s_2) \\ C'(s_1, s_2) & S'(s_1, s_2) & R_{26}(s_1, s_2) \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \eta(s_1) \\ \eta'(s_1) \\ 1 \end{pmatrix}$$

PLACET error bars

$$f = \frac{x_{rad}^2 - x_{norad}^2}{x_0^2}$$

$$\delta f = \frac{2}{\sqrt{N_{part}}} \frac{(x_{rad}^2 + x_{norad}^2)}{x_0^2}$$

Higher angle: most particles radiate. 

Lower angle: some particles radiate, others don't. 

$$un(u)du \equiv P(u/\hbar)du/\hbar = P(\omega)d\omega \tag{5}$$