# Hydrodynamics for Relativistic Heavy-Ion Collisions


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JET Collaboration


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References:
arXiv:nucl-th/0305084 (Kolb, UH)
arXiv:hep-ph/0407360 (UH)
arXiv:nucl-th/0512049 (UH)
PRC 73 (2006) 034904 (UH, Song, Chaudhuri)
PLB 658 (2008) 278 (Song, UH)
PRC 77 (2008) 064901 (Song, UH)
PRC 78 (2008) 024902 (Song, UH)
arXiv:0901.4355 (UH)
PRC 81 (2009) 024905 (Song, UH)
PRL 106 (2011) 192301 (Song, Bass, UH, Hirano, Shen)
PRC 83 (2011) 024912 (Song, Bass, UH)
arXiv:1108.5323 (UH, Shen, Song)
arXiv:1204.1473 (Martinez, Ryblewski, Strickland)
arXiv:1301.2826 (UH, Snellings)

# Motivation

- Relativistic viscous hydrodynamics is the backbone of dynamical modeling for heavy ion collisions at RHIC and LHC
- Needed to describe the space time evolution of the matter produced in a heavy ion collision
- Application is justified a priori by the smallness of the shear viscosity of the plasma as measured in RHIC and LHC experiments
- Canonical viscous hydrodynamics relies on a linearization around an isotropic equilibrium state
- Anisotropic viscous hydrodynamics generalizes this to a linear expansion around a spheroidally deformed (anisotropic) local momentum distribution

# Three Lecture Plan

**Lecture 1**

- Motivation and Introduction
- Kinetic Theory vs. Hydrodynamics
- Ideal Fluid Dynamics

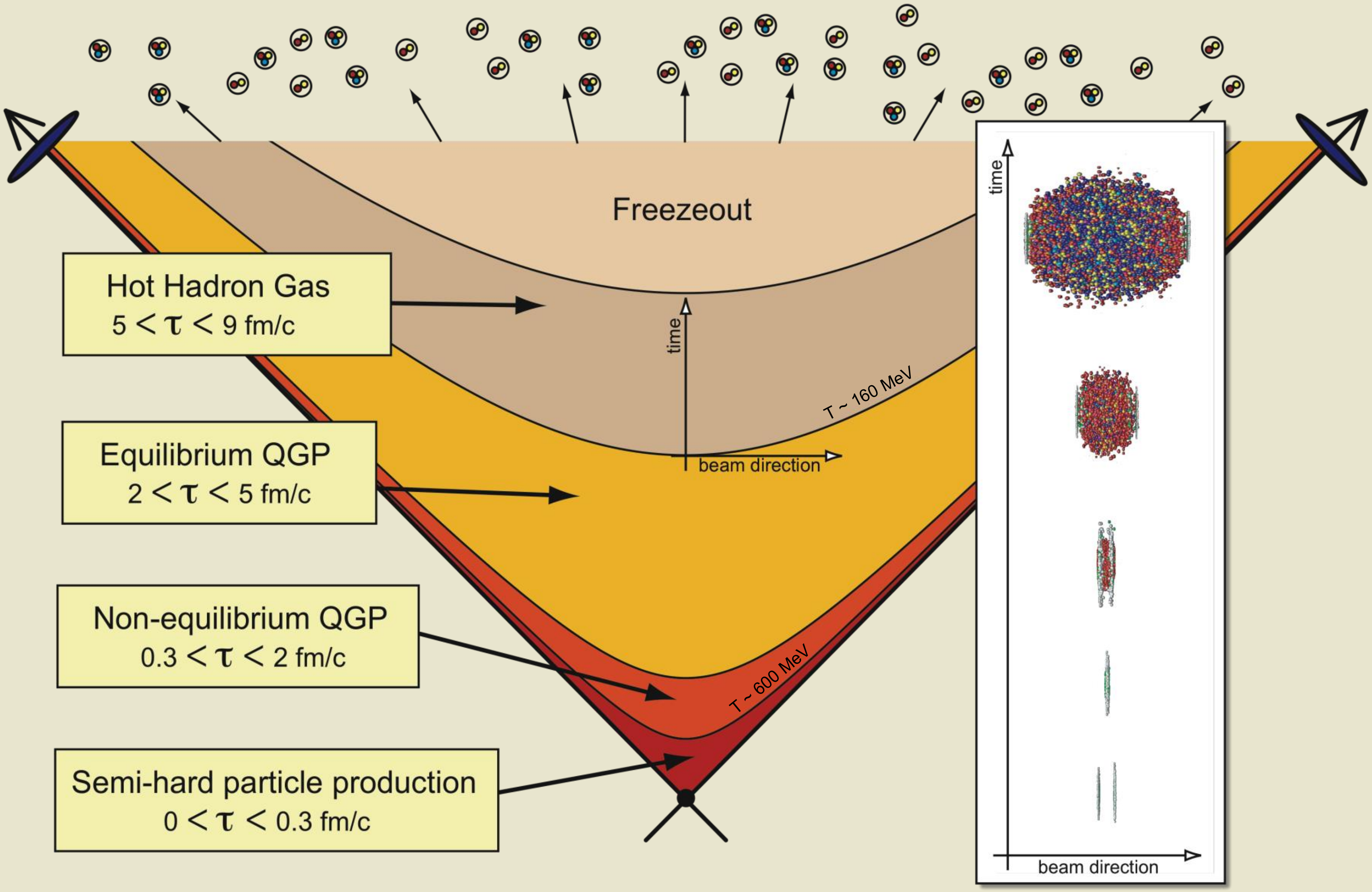
**Lecture 2**

- Ideal Fluid Equations of Motion
- 0+1d Boost-Invariant Transversely Homogeneous Systems (Bjorken Solution)
- Viscous Hydrodynamics

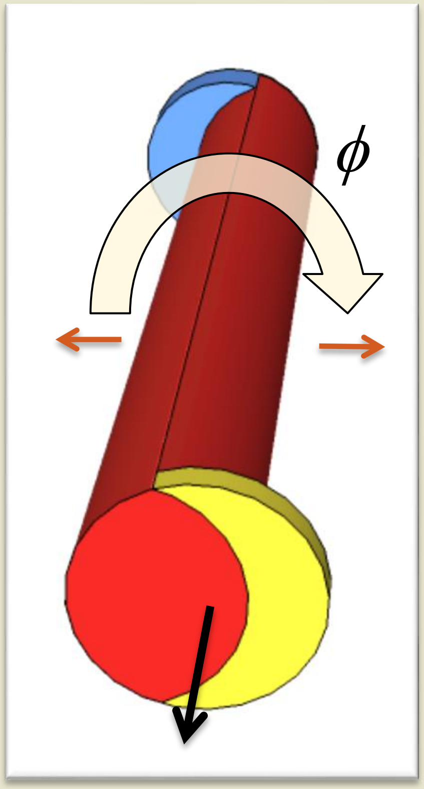
**Lecture 3**

- Non-ideal $T^{\mu\nu}$
- Viscous fluid equations
- Israel-Stewart Theory
- Anisotropic hydrodynamics
- Some applications

# Heavy Ion Collision Timescales



# Hydro for collective flow




- During non-central collisions overlap region breaks azimuthal symmetry

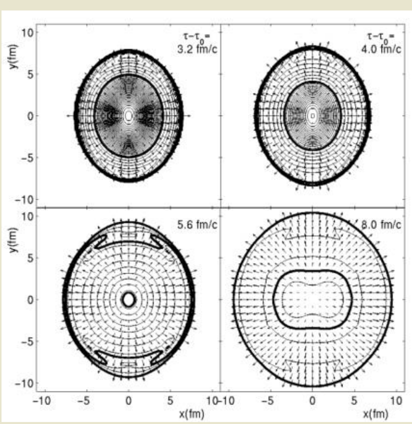
$$\frac{dN}{d\phi} = \frac{1}{2\pi}\left(v_0 + 2v_1\cos\phi + 2v_2\cos 2\phi + 2v_3\cos 3\phi + \ldots\right)$$

Directed Flow

Elliptic Flow

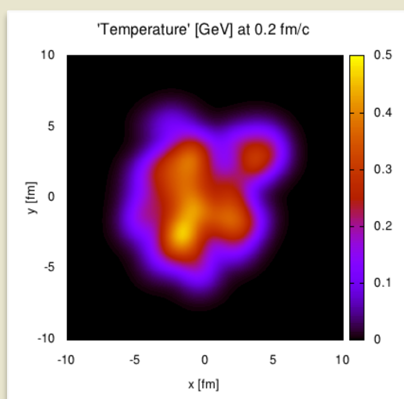
Triangular Flow

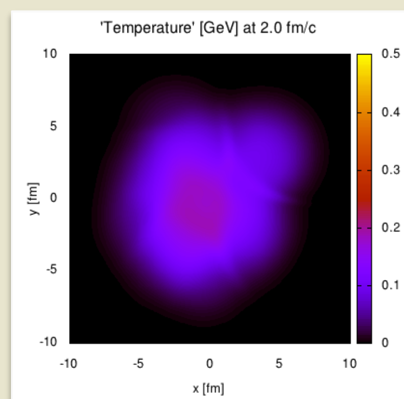
"Average" analysis

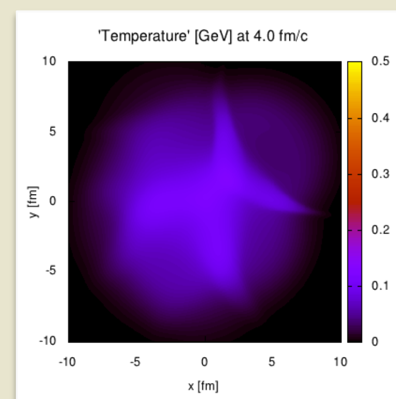



Kolb, Sollfrank, Heinz, Phys. Rev. C 62, 054909 (2000).

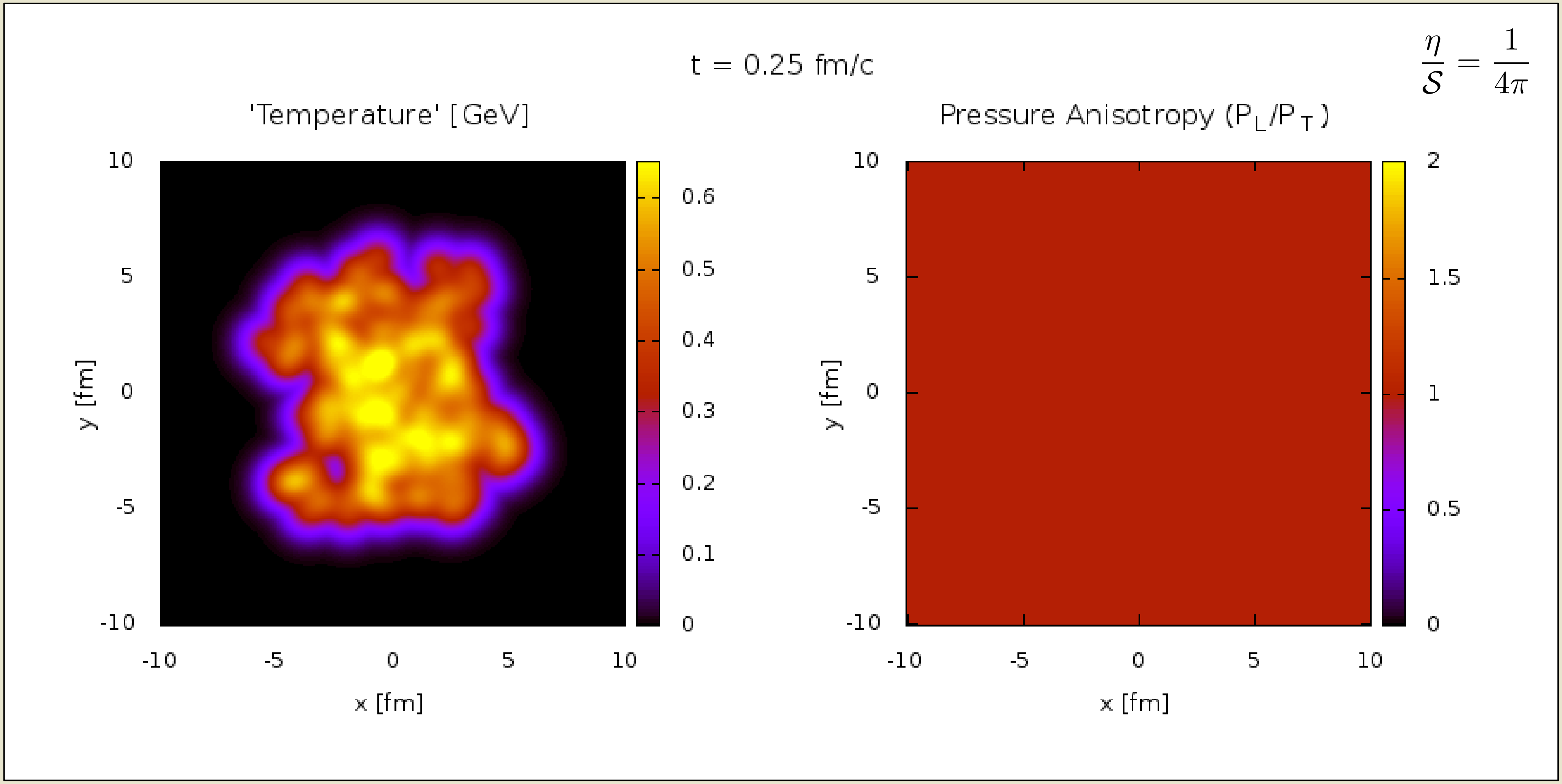
Event-by-Event analysis



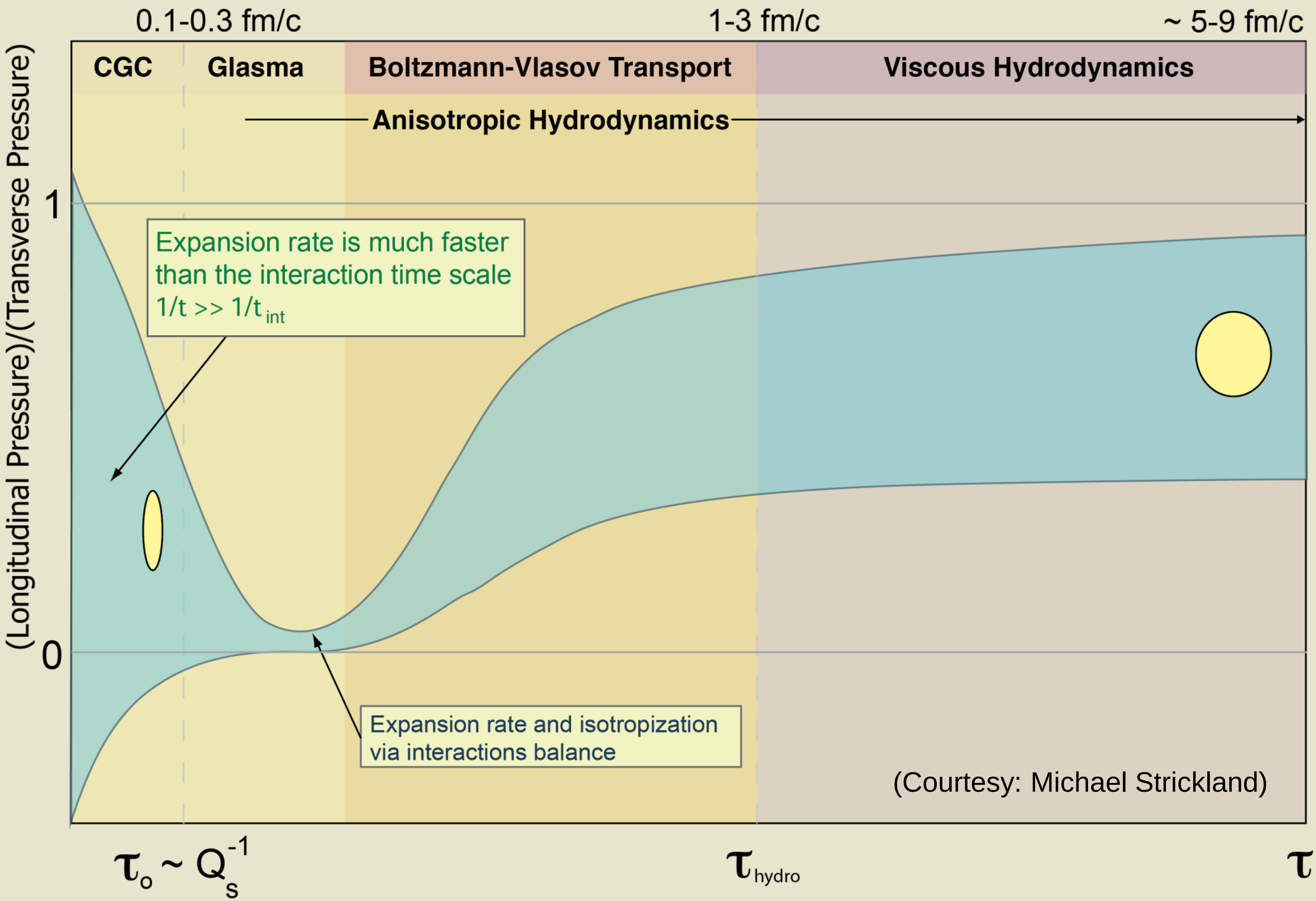



# Spatiotemporal Evolution




- Pb-Pb, b = 7 fm collision with Monte-Carlo Glauber initial conditions
  $T_0$ = 600 MeV @ $\tau_0$ = 0.25 fm/c
- Left panel shows temperature and right shows pressure anisotropy

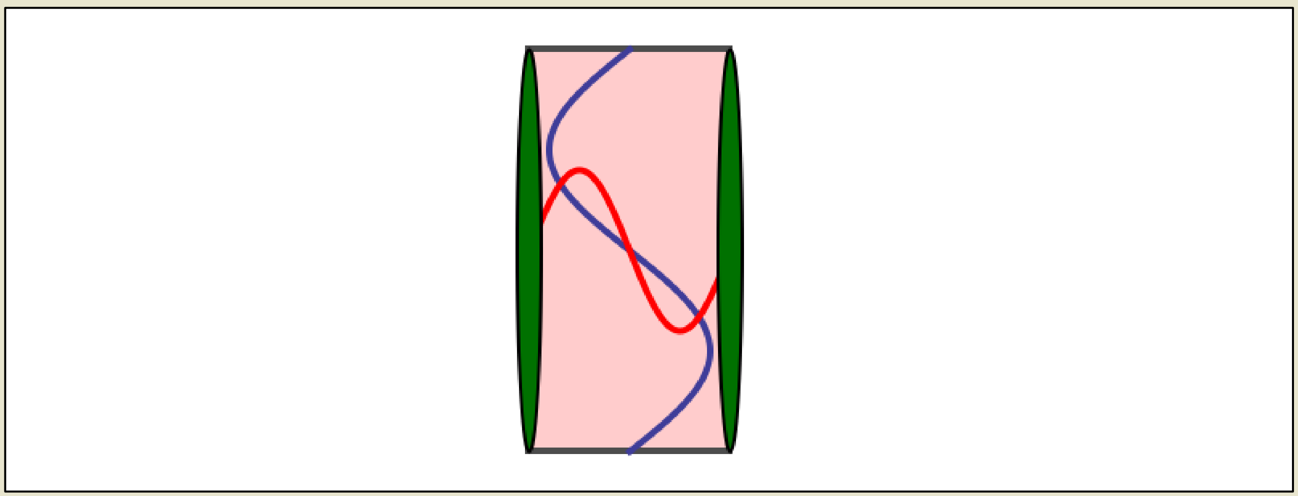
# QGP momentum anisotropy



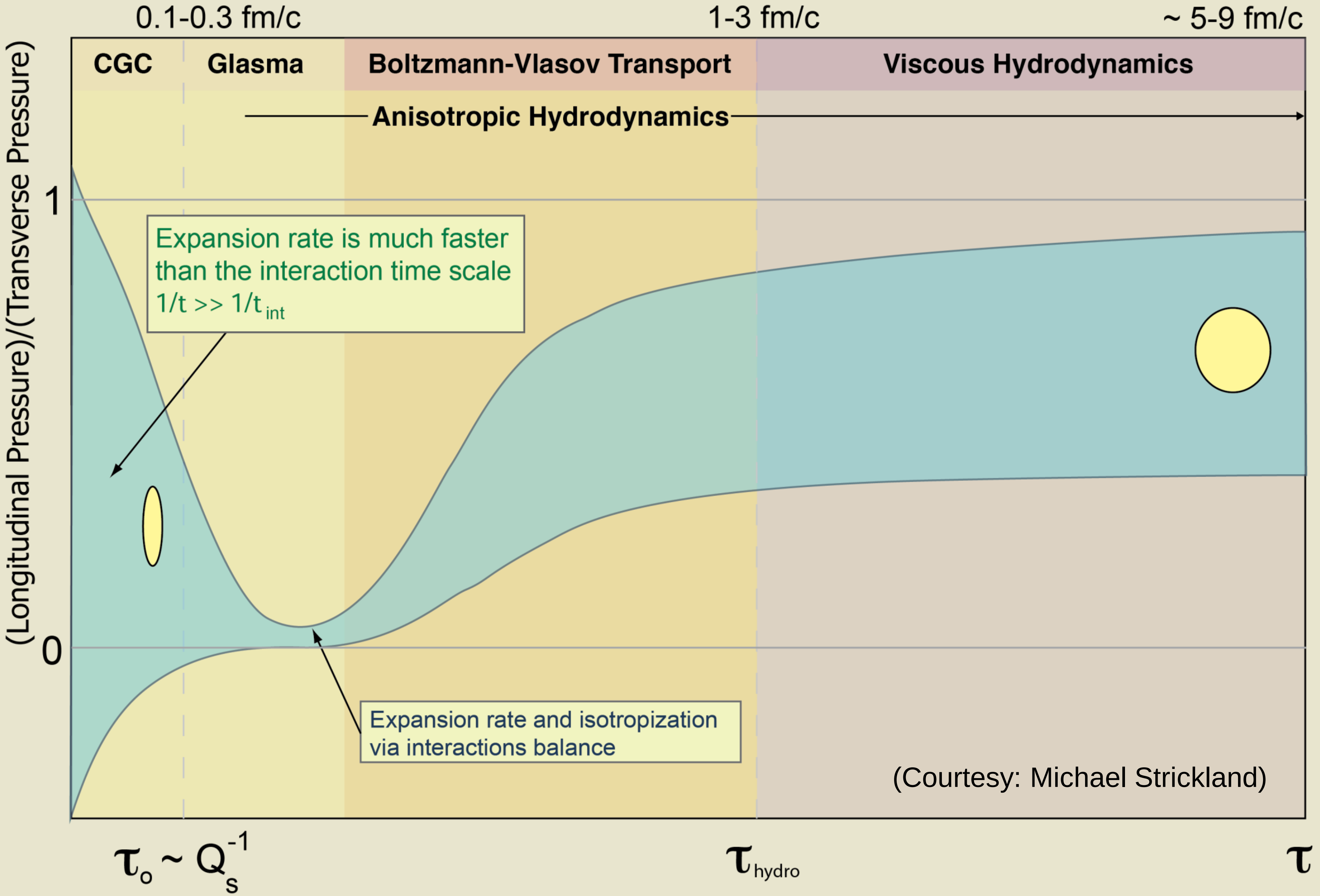
# Longitudinal Expansion

- The nearly speed of light expansion of the quark gluon plasma along the beamline direction causes “longitudinal cooling” of the plasma during the first few fm/c of the plasma’s lifetime.

- One can think of the system as a tiny one-dimensionally expanding universe in which momenta are red-shifted along the beamline direction.



# QGP momentum anisotropy



# - Hydro From Transport –

(see handwritten lecture notes)

# - Ideal fluid dynamics –

(see handwritten lecture notes)

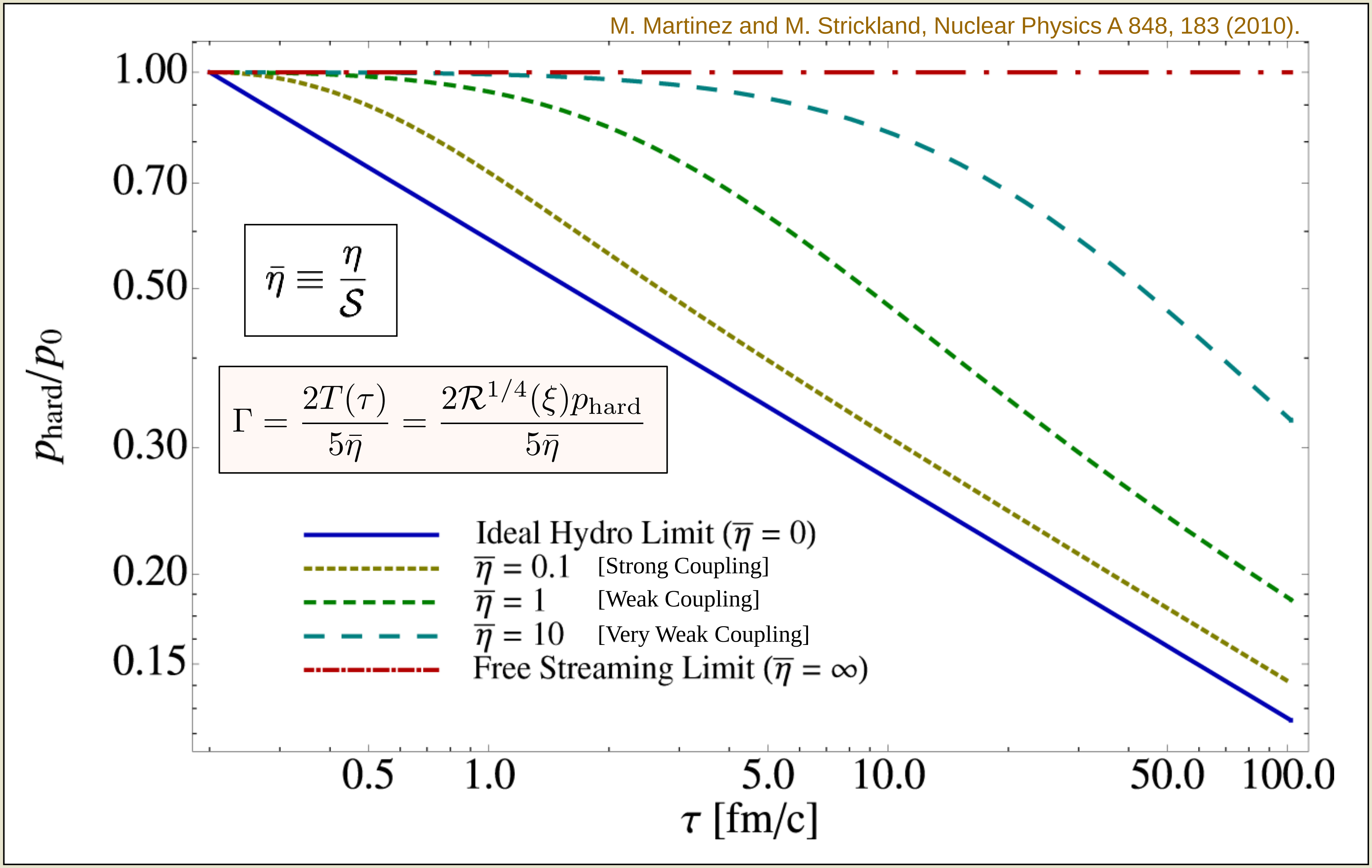
# - Anisotropic Hydrodynamics -

# “Temperature” vs. Time

$$\frac{1}{1+\xi}\partial_\tau\xi - \frac{2}{\tau} - 6\,\partial_\tau\log\Lambda = 2\Gamma\left[1-\mathcal{R}^{3/4}(\xi)\sqrt{1+\xi}\right]$$

$$\frac{\mathcal{R}'(\xi)}{\mathcal{R}(\xi)}\partial_\tau\xi + 4\,\partial_\tau\log\Lambda = \frac{1}{\tau}\left[\frac{1}{\xi(1+\xi)\mathcal{R}(\xi)} - \frac{1}{\xi} - 1\right]$$

$p_{\text{hard}} \leftrightarrow \Lambda$

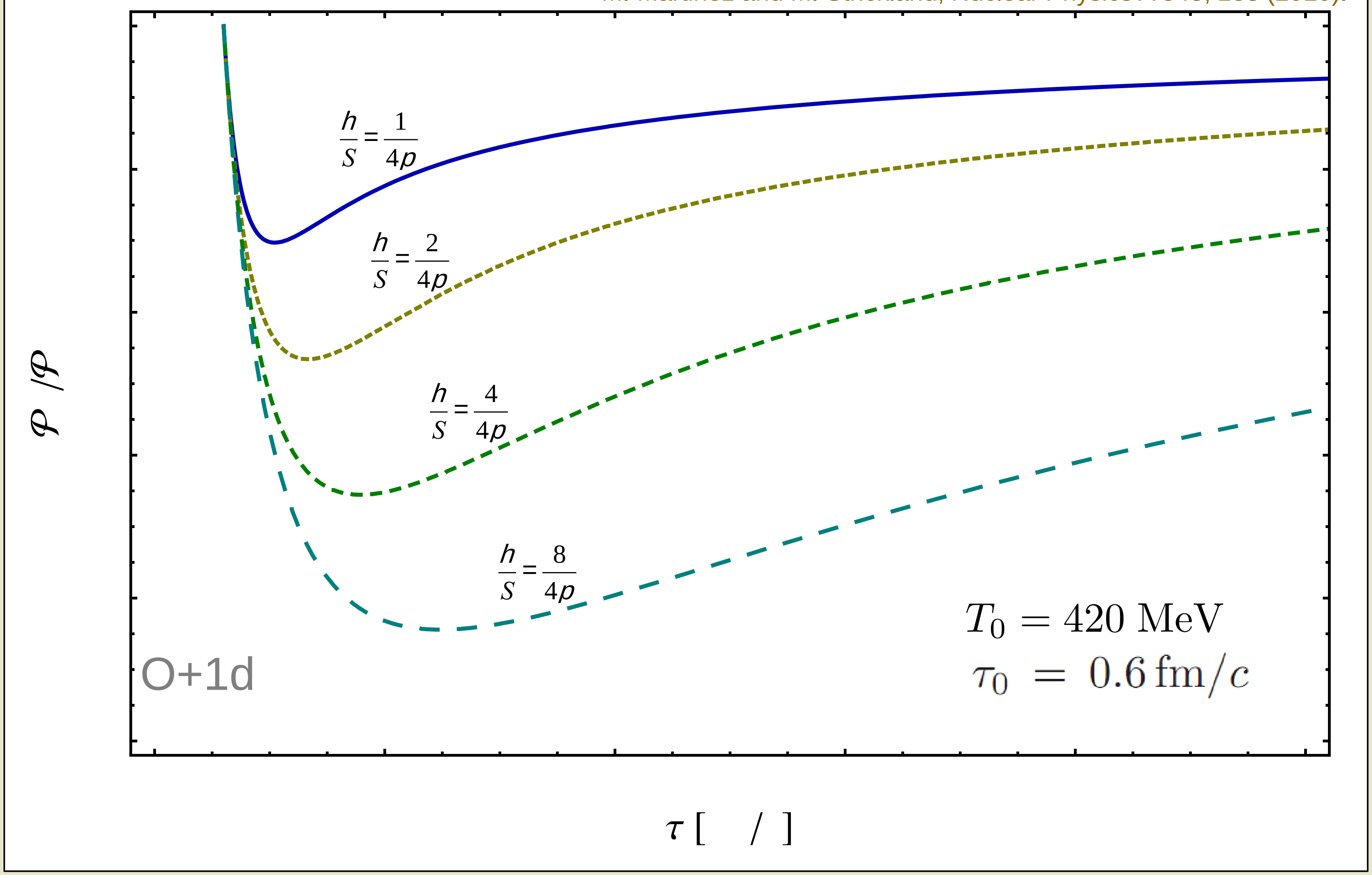


# Pressure Anisotropy

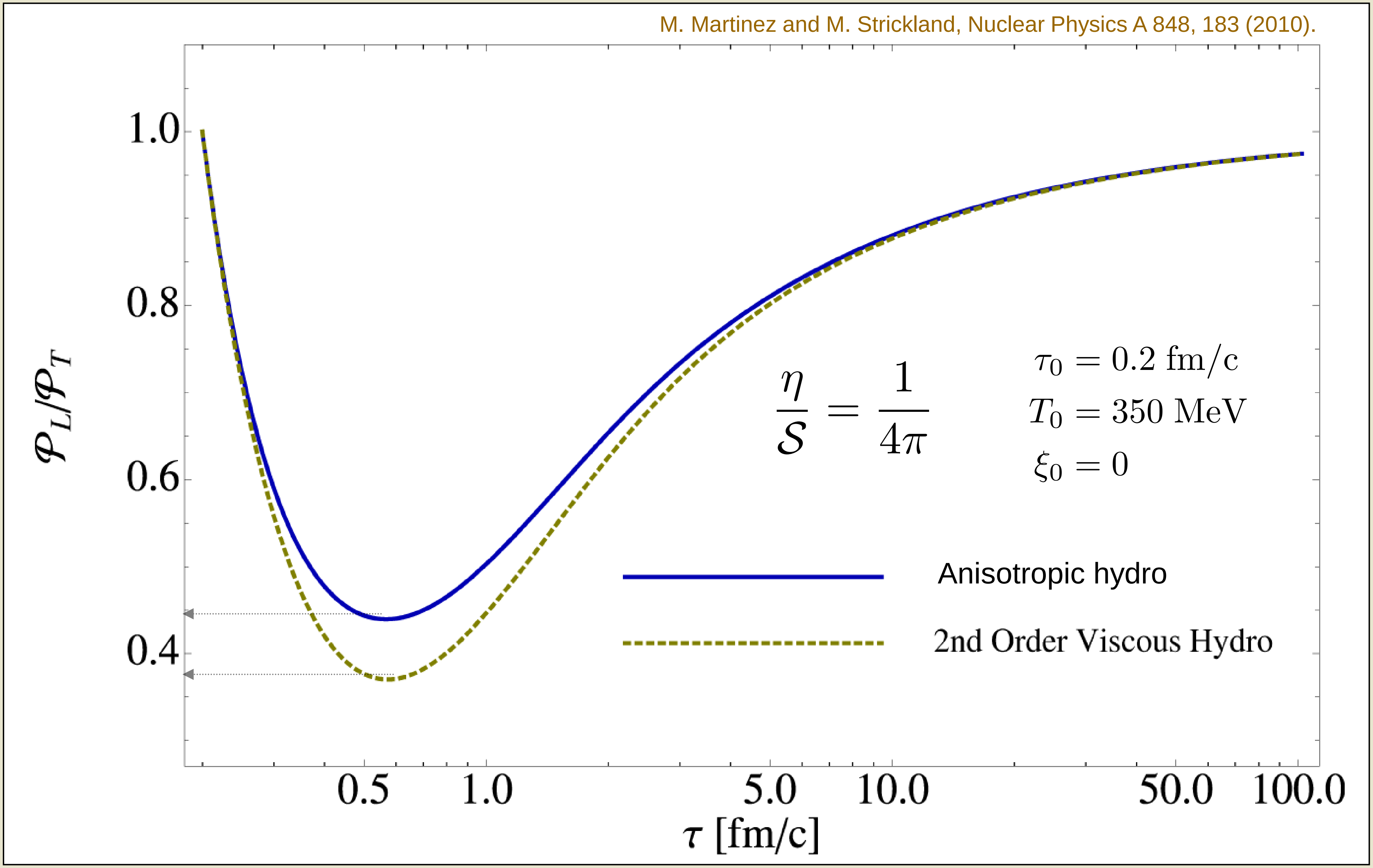
$$\frac{1}{1+\xi}\partial_\tau \xi - \frac{2}{\tau} - 6\,\partial_\tau \log \Lambda = 2\Gamma\left[1 - \mathcal{R}^{3/4}(\xi)\sqrt{1+\xi}\right]$$

$$\frac{\mathcal{R}'(\xi)}{\mathcal{R}(\xi)}\partial_\tau \xi + 4\,\partial_\tau \log \Lambda = \frac{1}{\tau}\left[\frac{1}{\xi(1+\xi)\mathcal{R}(\xi)} - \frac{1}{\xi} - 1\right]$$

M. Martinez and M. Strickland, Nuclear Physics A 848, 183 (2010).



# IS Hydro vs. aHydro : strong coupling



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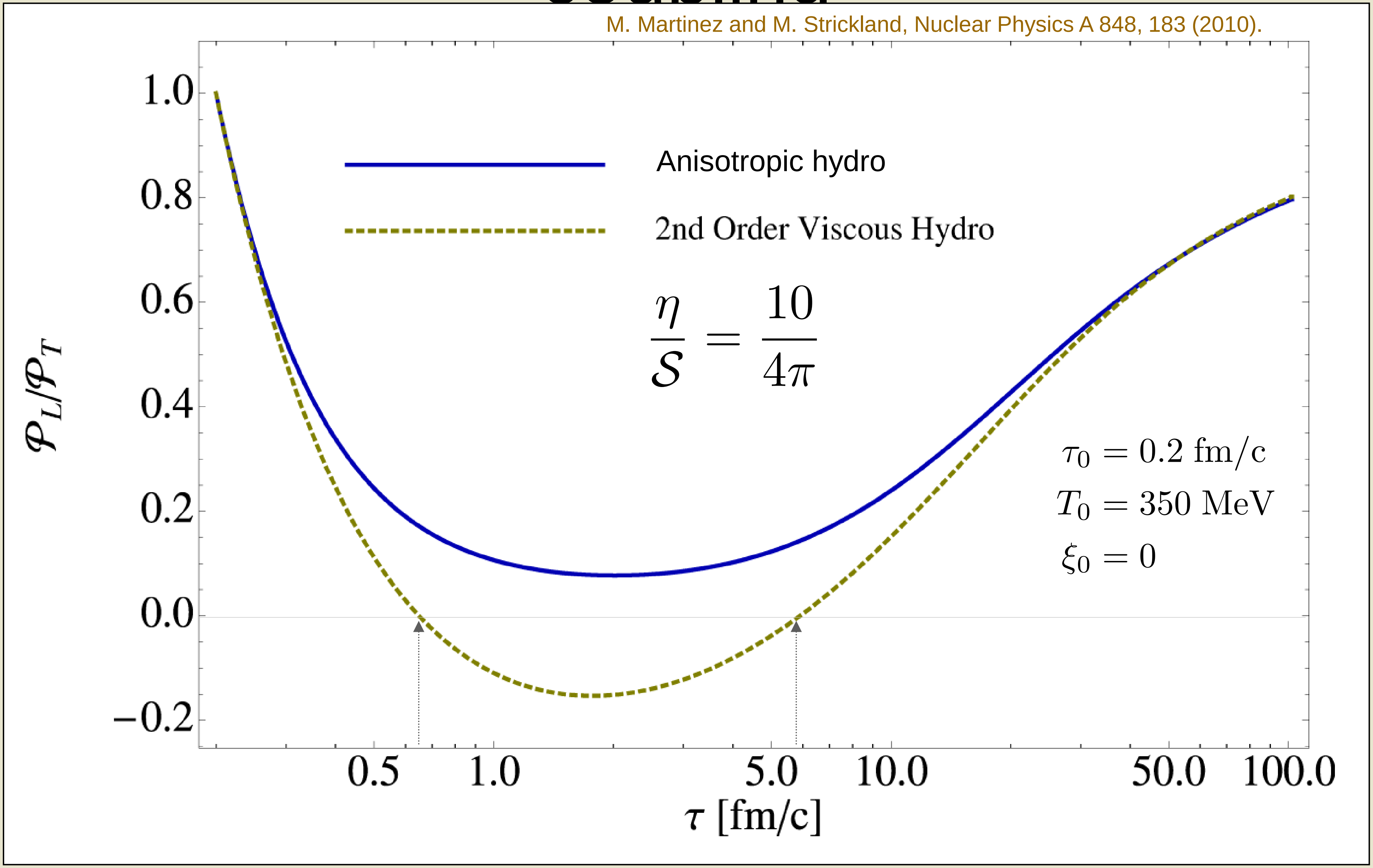


# - An Exactly Solvable Case -

# Exact 0+1d RTA Solution

- It is possible to solve the 0+1d Boltzmann equation in the relaxation time approximation (RTA) exactly
- Can be reduced to solving an integral equation numerically
- We can then compare different approximations to the exact solution

Boltzmann EQ
$$p^\mu \partial_\mu f(x,p) = C[f(x,p)]$$

RTA
$$C[f] = \frac{p_\mu u^\mu}{\tau_{\rm eq}}\left[f_{\rm eq}\Big(p_\mu u^\mu, T(x)\Big) - f(x,p)\right]$$

Solution for the energy density

W. Florkowski, R. Ryblewski, and M. Strickland, arXiv:1304.0665 and arXiv:1305.7234

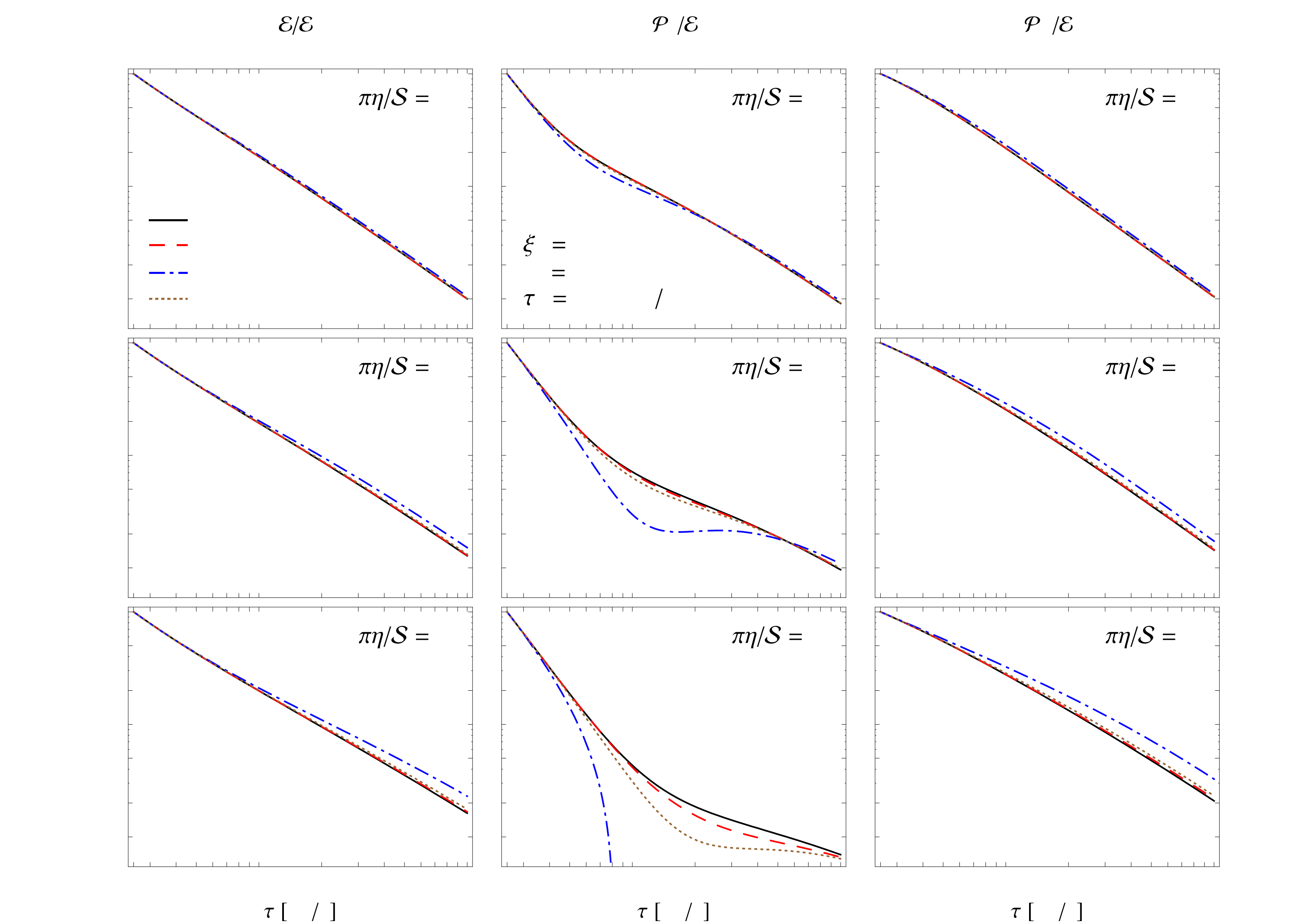
$$\bar{\mathcal{E}}(\tau) = D(\tau,\tau_0)\,\frac{\mathcal{R}\big(\xi_{\rm FS}(\tau)\big)}{\mathcal{R}\left(\xi_0\right)} + \int_{\tau_0}^{\tau}\frac{d\tau'}{\tau_{\rm eq}(\tau')}\,D(\tau,\tau')\,\bar{\mathcal{E}}(\tau')\,\mathcal{R}\left(\left(\frac{\tau}{\tau'}\right)^2 - 1\right),$$

$$\tau_{\rm eq}(\tau) = \frac{5\bar\eta}{T(\tau)}$$

$$D(\tau_2,\tau_1) = \exp\left[-\int_{\tau_1}^{\tau_2} d\tau\,\tau_{\rm eq}^{-1}(\tau)\right]$$

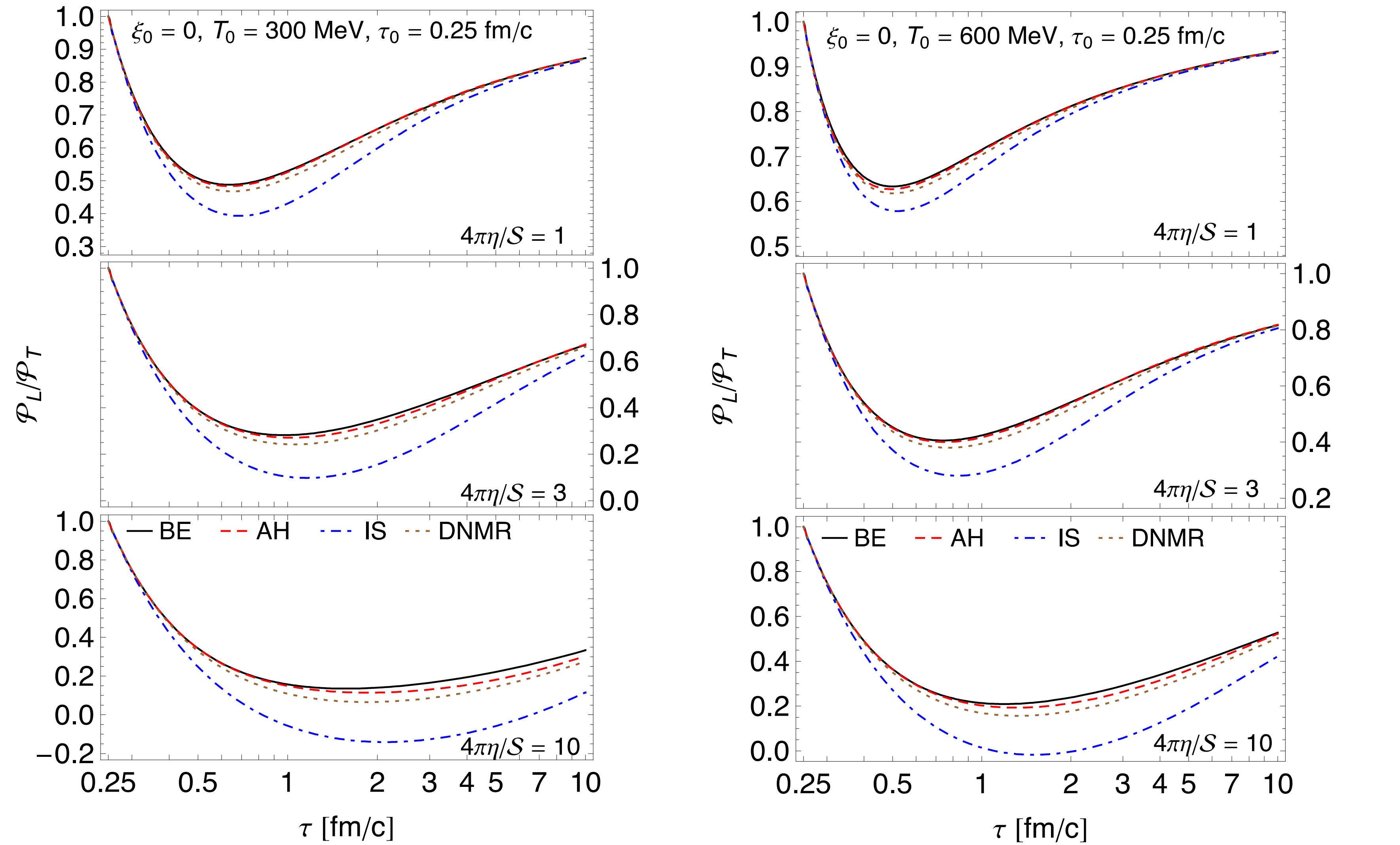
$$\frac{\partial D(\tau_2,\tau_1)}{\partial\tau_2} = -\frac{D(\tau_2,\tau_1)}{\tau_{\rm eq}(\tau_2)}$$

# Exact 0+1d RTA Solution




BE = Exact Solution    AH = aHydro    IS = Israel Stewart    DNMR = Denicol et al, Phys. Rev. D 85, 114047 (2012)

# Exact 0+1d RTA Solution

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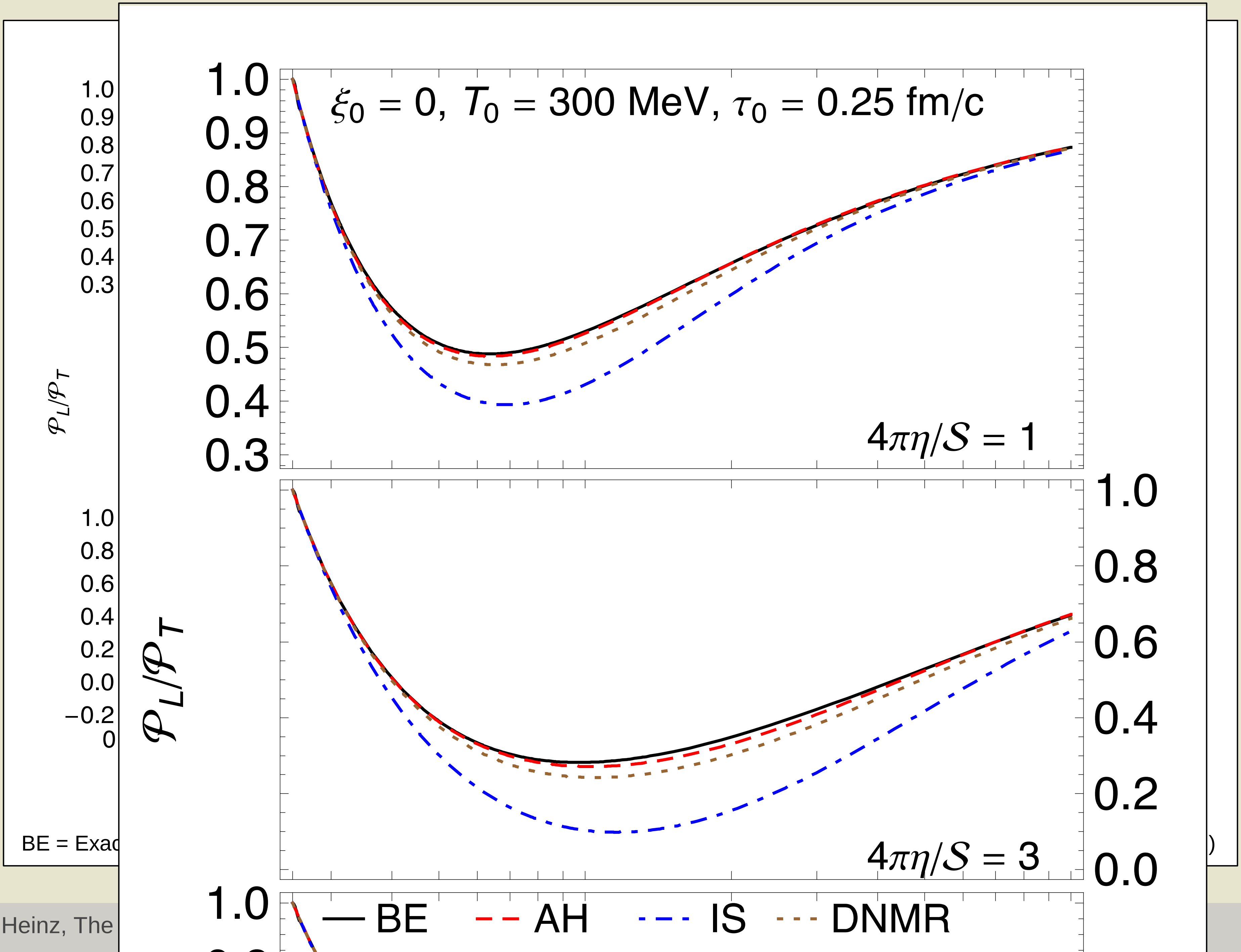
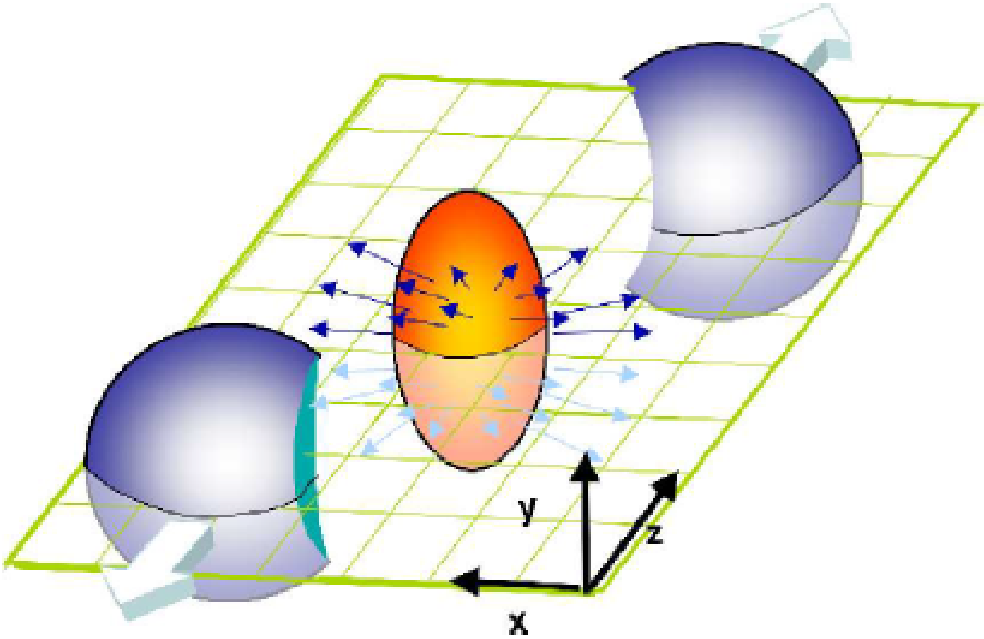
# Exact 0+1d RTA Solution



# - A few applications to RHIC and LHC collisions -

# Elliptic flow




In non-central collisions the overlap region is elliptically deformed $\Longrightarrow$ anisotropic pressure gradients $\Longrightarrow$ anisotropic ("elliptic") collective flow.

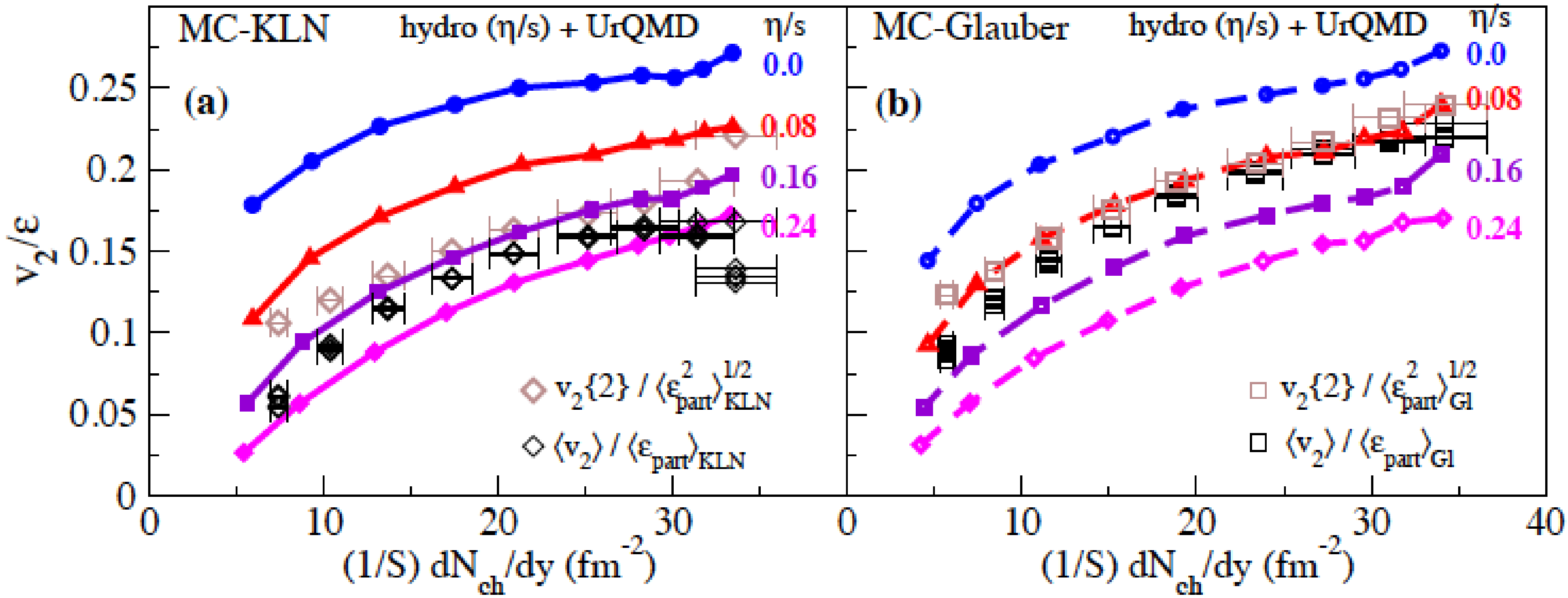
Elliptic flow

- $\longrightarrow$ peaks at midrapidity
- $\longrightarrow$ driven by spatial deformation of reaction zone at thermalization
- $\longrightarrow$ magnitude of signal probes degree and time of thermalization
- $\longrightarrow$ "self-quenching": it shuts itself off as dynamics reduces deformation (H. Sorge)
- $\longrightarrow$ sensitive to Equation of State during first $\sim 5\,\mathrm{fm}/c$

$$v_2(y, p_T, b) = \langle \cos(2\phi) \rangle_{y, p_T, b} = \frac{\int d\phi\, \cos(2\phi)\, \frac{dN}{dy\, p_T dp_T\, d\phi}(b)}{\int d\phi\, \frac{dN}{dy\, p_T dp_T\, d\phi}(b)}$$

# VISHNU – First extraction of $(\eta/s)_{\mathrm{QGP}}$ from AuAu@RHIC with quantified errors: $1 \leq 4\pi(\eta/s)_{\mathrm{QGP}} \leq 2.5$

H. Song, S.A. Bass, U. Heinz, T. Hirano, C. Shen, PRL106 (2011) 192301




VISHNU: hybrid code that simulates QGP by viscous hydro and switches to UrQMD hadronic Boltzmann cascade after hadronization

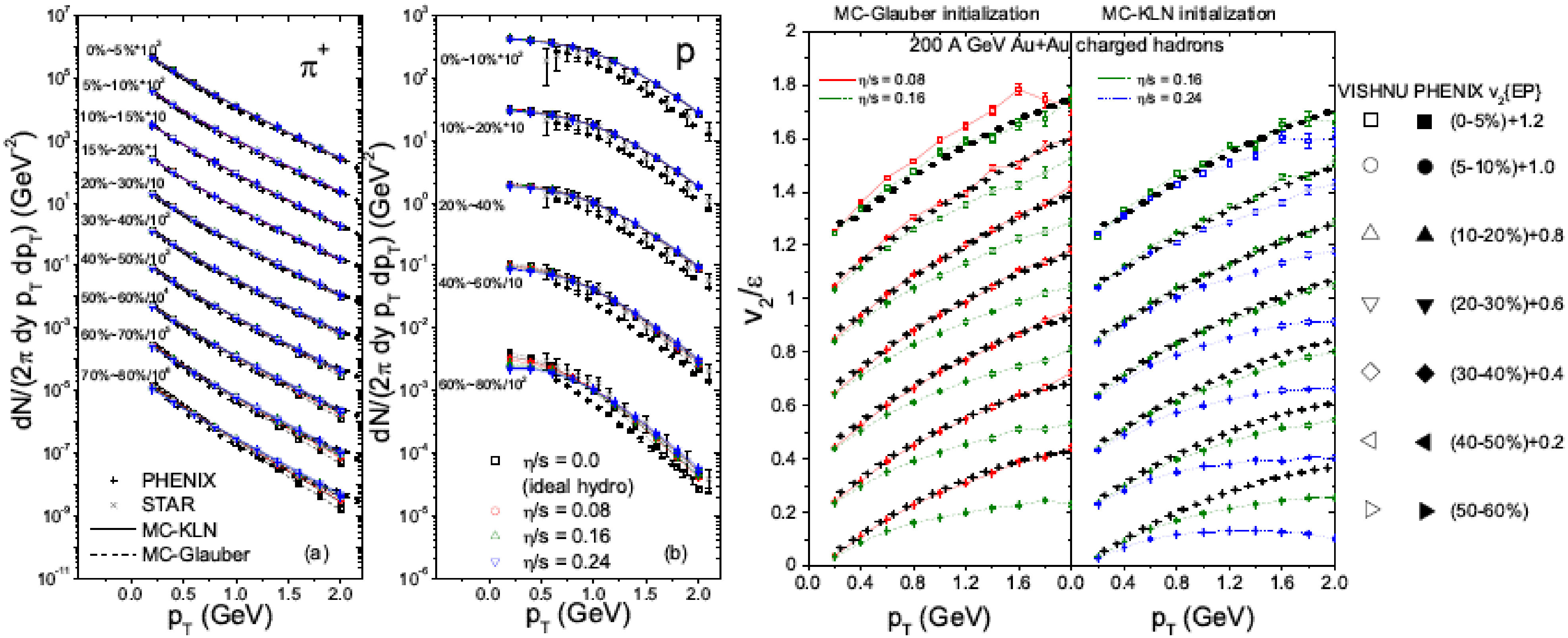
Hadronic phase is highly dissipative and cannot be described hydrodynamically with good precision (H. Song, S.A. Bass, U. Heinz, PRC 83 (2011) 024912)

VISHNU eliminates hadronic uncertainties from extraction of QGP viscosity.

Helped to focus attention on initial-state model uncertainties

# VISHNU: Global description of AuAu@RHIC spectra and $v_2$

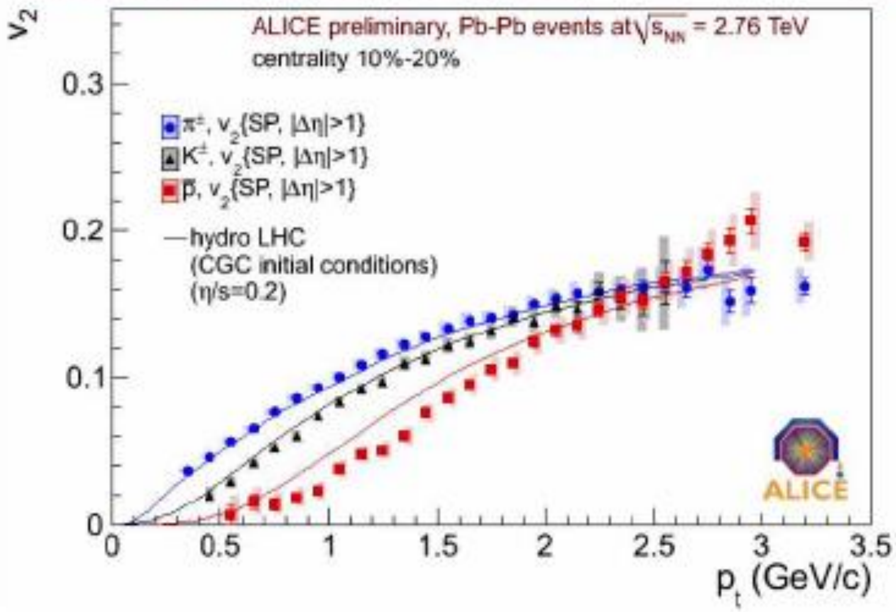
VISHNU (H. Song, S.A. Bass, U. Heinz, T. Hirano, C. Shen, PRC 83 (2011) 054910)



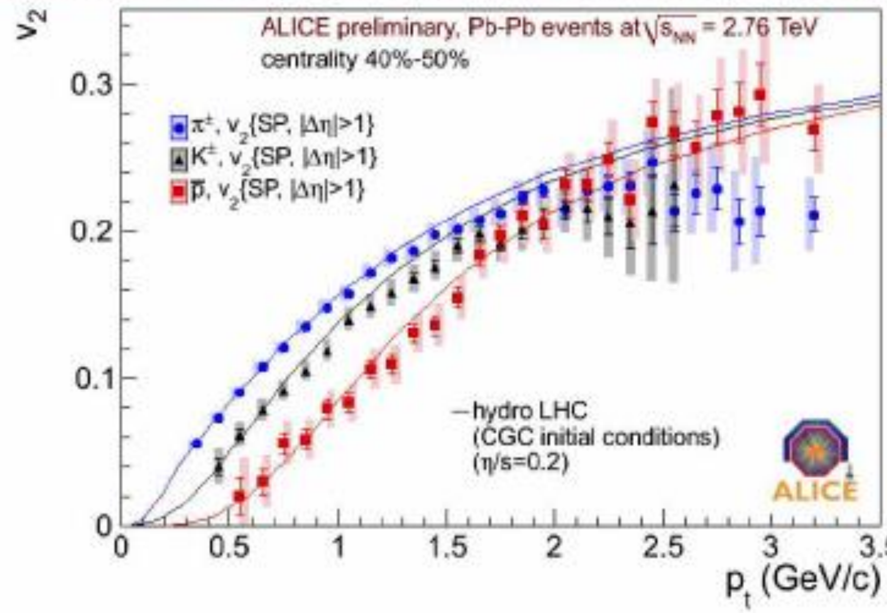

- $(\eta/s)_{\rm QGP} = 0.08$ for MC-Glauber and $(\eta/s)_{\rm QGP} = 0.16$ for MC-KLN work well for charged hadron, pion and proton spectra and $v_2(p_T)$ at all collision centralities
- experimental $v_2$ data have statistical precision to determine $\eta/s$ to $<0.04$, but initial-state model ambiguities are much larger and cannot be resolved from spectra and $v_2$ alone
  $\Longrightarrow$ **need higher-order $v_n$!**

# Successful hydrodynamic prediction of $v_2(p_T)$ for identified hadrons in PbPb@LHC

Data: ALICE, Quark Matter 2011




Prediction: Shen et al., PRC84 (2011) 044903 (VISH2+1)



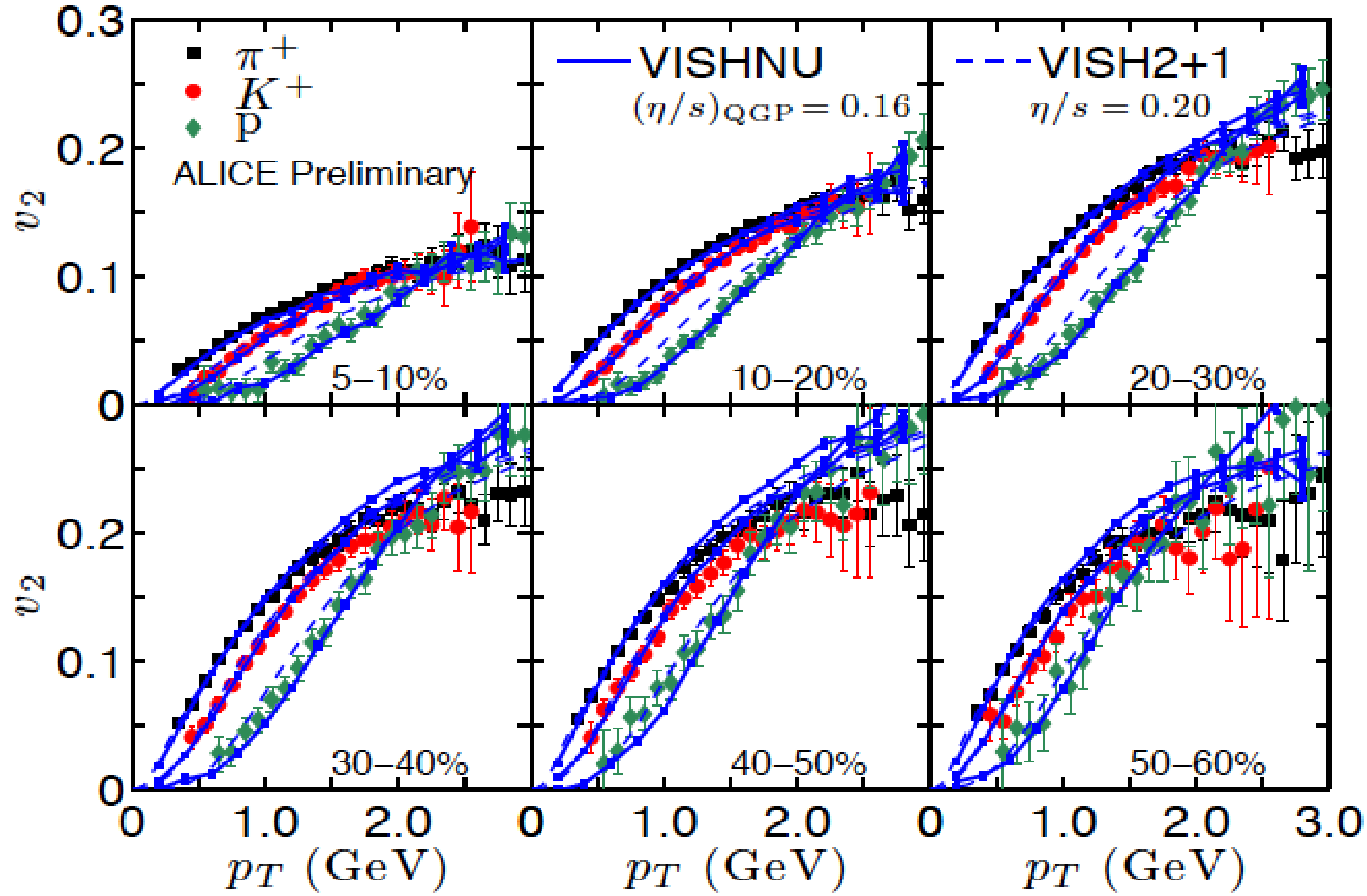

Perfect fit in semi-peripheral collisions!

The problem with insufficient proton radial flow exists only in more central collisions

Adding the hadronic cascade (VISHNU) helps:

# $v_2(p_T)$ in PbPb@LHC: ALICE vs. VISHNU

Data: ALICE, preliminary (Snellings, Krzewicki, Quark Matter 2011)
Dashed lines: Shen et al., PRC84 (2011) 044903 (VISH2+1, MC-KLN, $(\eta/s)_{QGP}$=0.2)
Solid lines: Song, Shen, UH 2011 (VISHNU, MC-KLN, $(\eta/s)_{QGP}$=0.16)




VISHNU yields correct magnitude and centrality dependence of $v_2(p_T)$ for pions, kaons **and protons!**

**Same $(\eta/s)_{QGP} = 0.16$ (for MC-KLN) at RHIC and LHC!**