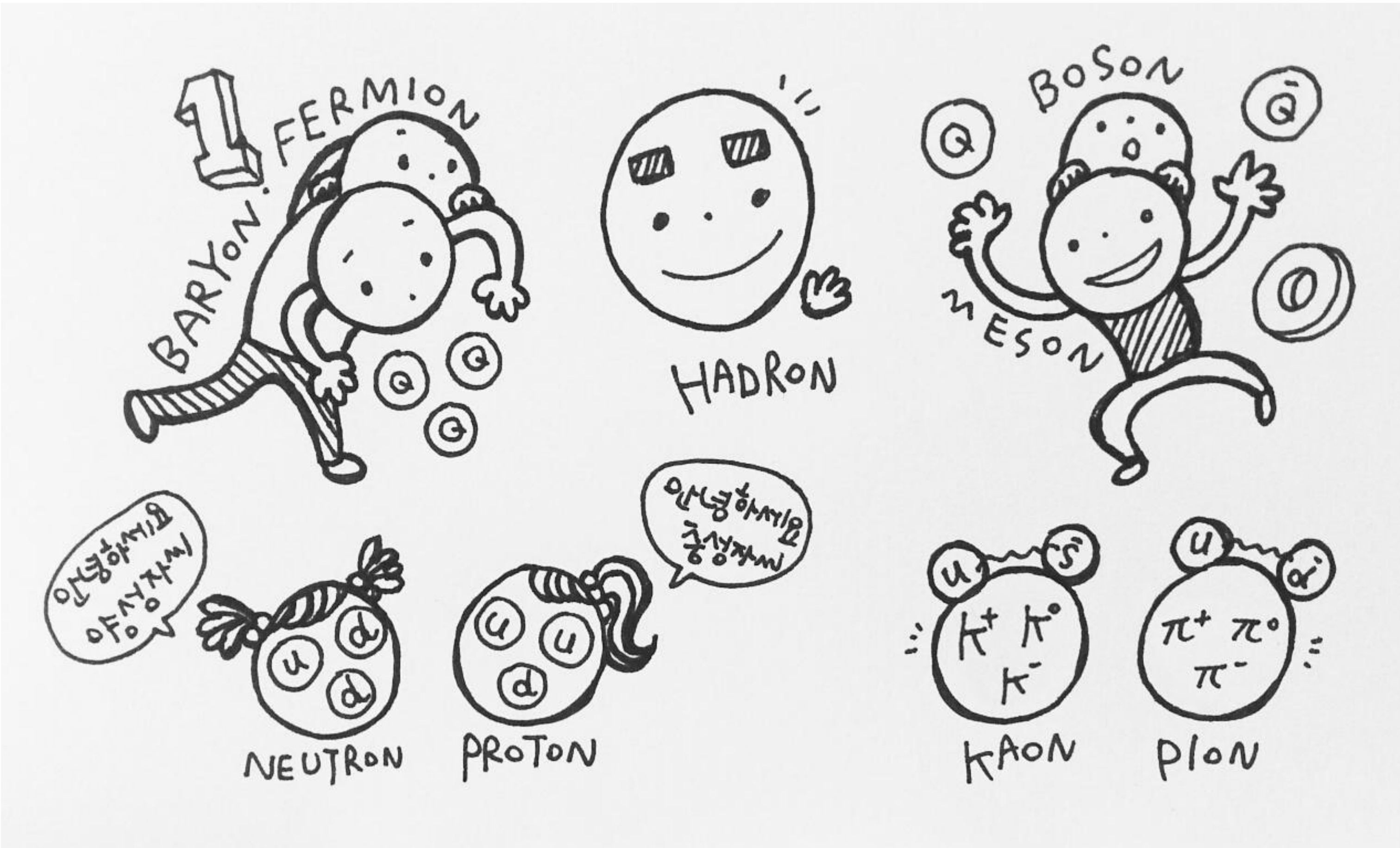
Hadron Theory Group

Inha University



FERMION
BARYON
HADRON
BOSON
MESON
안녕하세요 양성자씨
안녕하세요 중성자씨
NEUTRON
PROTON
KAON
PION

Professor

Prof. Hyun-chul Kim

hchkim@inha.ac.kr

2009년 제7회 핵물리학교 위원장
2010년 제8회 핵물리학교 운영위원

Prof. Ulugbek Yakhshiev

yakhshiev@inha.ac.kr

Members

- Ju-Hyun Jung (박사1차) rabbitx2@gmail.com
- Ji-Hye Jeong (박사1차) jhhjeong@gmail.com
- Hyeon-Dong Son (석사4차) hdson21@gmail.com
- Bobur Turimov (통합3차) bturimov@gmail.com
- Yuson Jun (통합3차) ysjun6@gmail.com
- Jehee Lee (석사1차) jhlee0727@gmail.com
- Changyoung Oh (학부3년) ocyoung1@gmail.com

Research Outlook

- Nonlocal chiral quark model from the instanton vacuum
  - Ji-Hye Jeong, Hyeon-Dong Son
- A modified pion-rho-omega mesonic Lagrangian in nuclear matter (Phys. Lett. **B** 723 442-447 (2013)
  - Ju-Hyun Jung
- Holographic QCD
  - Bobur Turimov, Changyoung Oh
- Effective Lagrangian approach
  - Yuson Jun, Jehee Lee

Analysis of the nucleon structure in a pi-rho-omega meson model

Ju-Hyun Jung
In collaboration with
Hyun-Chul Kim and Ulugbek Yakhshiev

Outline

1. Introduction
2. Theoreical framework
   a. The effective Lagrangianfor pions, rho- and omega-mesons
   b. Energy-momentum tensor form factors of nucleon
3. Results
4. Conclusion & Outlook

Introduction

The nucleon form factors of the energy-momentum tensor (EMT)

H. R. Pagels, Phys. Rev. 144, 1250 (1966)

Introduction

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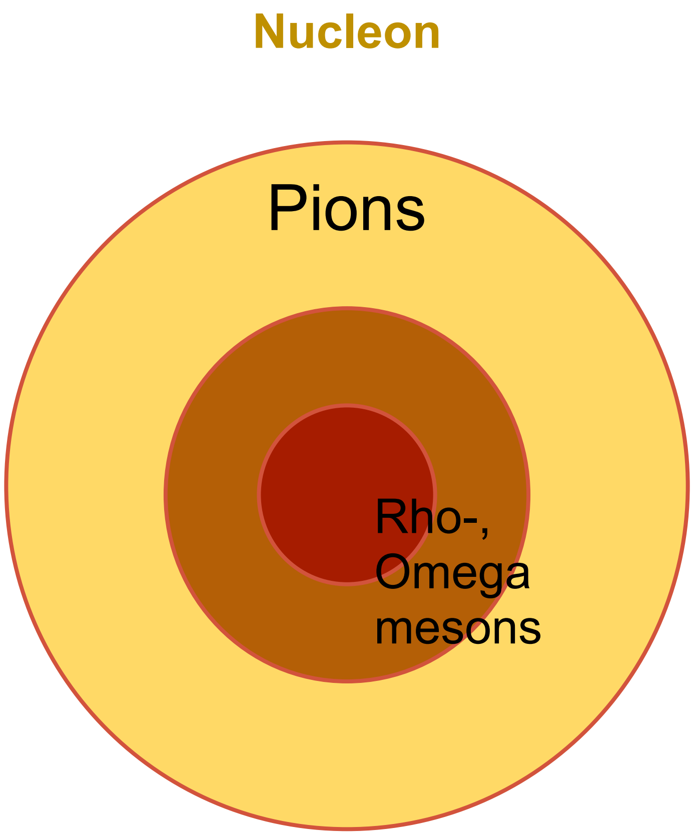
Introduction

We will present the chiral solitonic model with pion, rho and omega mesonic degrees of freedom in order to analyze EMT for nucleon

Theoreical framework



- The nucleon has a structure, not a point-like particle
- Outer shell of the nucleon is made of pions and inner core is made of another hadronic degrees of freedom, like mesons and effective pare creations.

Theoreical framework

What is EMT?

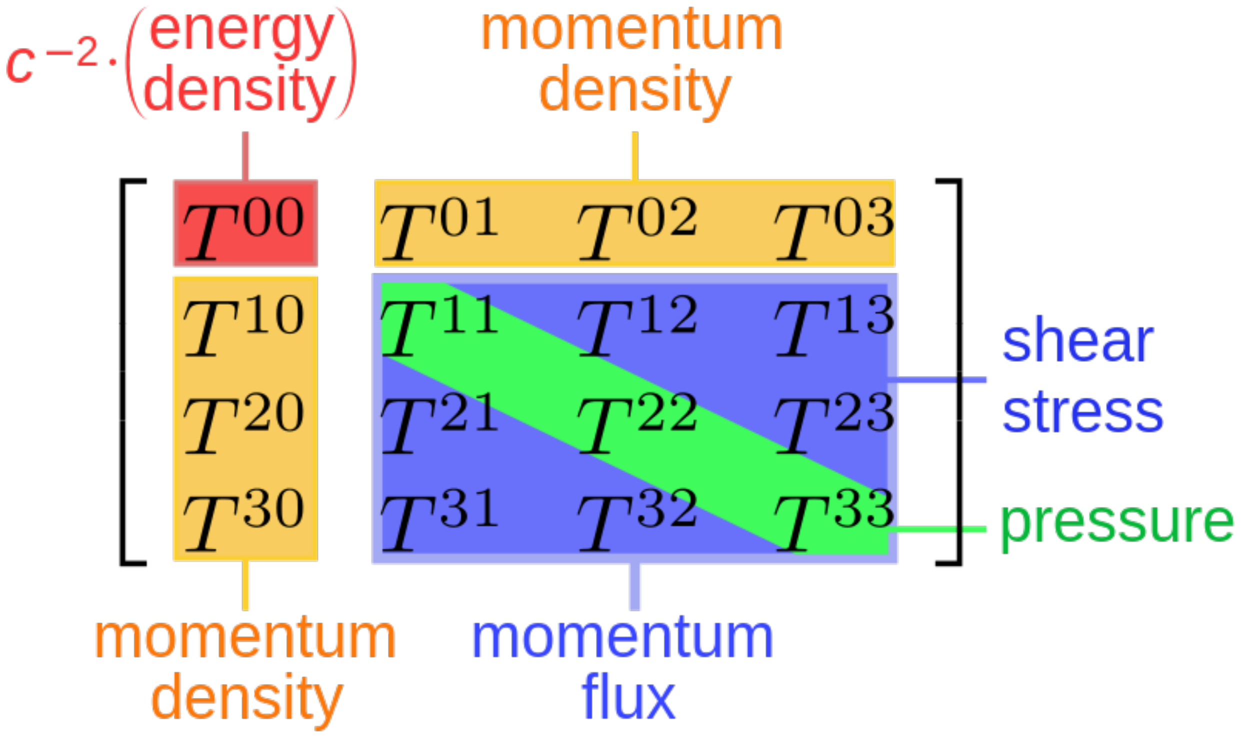
$$x^\mu \rightarrow x^\mu - a^\mu$$

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial\left(\partial_\mu \phi_a\right)} \partial^\nu \phi_a - g^{\mu\nu} \mathcal{L}$$

$$\partial_\mu T^{\mu\nu} = 0$$

Noether's theorem : If a system has a continuous symmetry property, then there are corresponding quantities whose values are conserved in time.

Theoreical framework



The effective Lagrangian for pions, rho- and omega-mesons

$$\mathcal{L} = \mathcal{L}_A + \mathcal{L}_{\chi SB} + \mathcal{L}_A + \mathcal{L}_{kin} + \mathcal{L}_{WZ}$$

$$\begin{aligned}
\mathcal{L}_A &= \frac{f_\pi^2}{4} Tr\left(\partial_\mu U \partial^\mu U^\dagger\right) \\
\mathcal{L}_{\chi SB} &= \frac{f_\pi^2 m_\pi^2}{2} Tr\left(U + U^\dagger - 2\right) \\
\mathcal{L}_V &= -a\frac{f_\pi^2}{4} Tr\left(D_\mu\xi \cdot \xi^\dagger + D_\mu\xi^\dagger \cdot \xi\right) \\
\mathcal{L}_{kin} &= -\frac{1}{2g^2} Tr\left(F_{\mu\nu}^2\right) \\
\mathcal{L}_{WZ} &= \left(\frac{N_c}{2} g\right) \omega_\mu \frac{\epsilon^{\mu\nu\alpha\beta}}{24\pi^2} Tr\left\{\left(U^\dagger\partial_\nu U\right)\left(U^\dagger\partial_\alpha U\right)\left(U^\dagger\partial_\beta U\right)\right\}
\end{aligned}$$

$$\xi = \sqrt{U}$$

$$\begin{aligned}
D_\mu &= \partial_\mu - iV_\mu \\
F_{\mu\nu} &= \partial_\mu V_\nu - \partial_\nu V_\mu - i\left[V_\mu, V_\nu\right] \\
V_\mu &= \frac{g}{2}\left(\tau^a \rho_\mu^a + \omega_\mu\right)
\end{aligned}$$

The effective Lagrangian for pions, rho- and omega-mesons

Skyrme's spherically symmetric Ansatz for $U(x)$

$$U(x) = \exp\left[i\hat{x}_a \tau_a F(r)\right], \qquad (\hat{x} = x_a/r)$$

Wu-Yang-t'Hooft-Polyakov Ansatz for $\rho_\mu(r)$

$$\begin{aligned} \frac{g}{2}\tau^a \rho_0^a(r) &= 0, \\ \frac{g}{2}\tau^a \rho^{i,a}(r) &= \frac{\epsilon^{ika}}{2r}\hat{x}^k \tau^a G(r). \end{aligned}$$

$$\omega_0(r) = \omega(r)$$

The effective Lagrangian for pions, rho- and omega-mesons

$$M_H = \left[\frac{f_\pi^2}{2}\left(F'^2 + 2\frac{\sin^2 F}{r^2}\right) - f_\pi^2 m_\pi^2 (\cos F - 1) - \frac{a}{2} g^2 f_\pi^2 \omega^2 + a\frac{f_\pi^2}{r^2}\left[G - (1 - \cos F)\right]^2 + \frac{1}{2g^2 r^4}\left\{2r^2 G'(r)^2 + G(r)^2 (G(r) - 2)^2\right\} + \frac{1}{2}\omega'^2 + \left(\frac{N_c}{2} g\right)\frac{1}{2\pi^2 r^2}\omega \sin^2 F\, F'\right]$$

- Calculated soliton mass for the static fields has the following form

Equations of motion

$$F'' = \frac{\sin F}{4r^2}\left[-\frac{3g\sin F\omega'}{\pi^2 f_\pi^2} + 8\left(2-\cos F\right) - 16G\right] - \frac{2F'}{r} + m_\pi^2 \sin F$$

$$G'' = 2f_\pi^2 g^2\left(\cos F + G - 1\right) + \frac{\left(G-2\right)\left(G-1\right)G}{r^2}$$

$$\omega'' = 2f_\pi^2 g^2 \omega - \frac{3gF'\sin^2 F}{4\pi^2 r^2} - \frac{2\omega'}{r}$$

$F\left(0\right) = n\pi = \pi$

$F\left(\infty\right) = G\left(\infty\right) = \omega\left(\infty\right) = 0$

$g\left(0\right) = 1 - \left(-1\right)^n = 2$

$\omega'\left(0\right) = 0$

$B = n = 1$

- The solutions of the equations gives the static solitonic configuration.

Equations of motion

In the limit $r \to \infty$, we can derive the asymptotic behavior of the profiles.

$$\begin{cases} F(r) & = \beta_F \dfrac{e^{-m_\pi r}}{r^2}(1 + m_\pi r) \\ G(r) & = \beta_G \dfrac{e^{-2m_\pi r}}{r^2} \qquad\qquad \text{as} r \to \infty \\ \omega(r) & = \beta_\omega \dfrac{e^{-3m_\pi r}}{r^3} \end{cases}$$

Quantization

We use rigid body quantization approximation. in this scheme nucleon appears as rotational state of classical skyrmion

$$U(\boldsymbol{r},t) = A(t)U(\boldsymbol{r})A^{\dagger}(t)$$

$$\frac{g}{2}\boldsymbol{\tau}\cdot\boldsymbol{\rho}_0(\boldsymbol{r},t) = \xi_1(\boldsymbol{r})A(t)\tau_j K_j A^{\dagger}(t) + \xi_2(\boldsymbol{r})K_k\hat{x}_k A(t)\tau_j\hat{x}_j A^{\dagger}(t)$$

$$\boldsymbol{\tau}\cdot\boldsymbol{\rho}_i(\boldsymbol{r},t) = A(t)\boldsymbol{\tau}\cdot\boldsymbol{\rho}_i(\boldsymbol{r})A^{\dagger}(t)$$

$$\boldsymbol{\omega}(\boldsymbol{r},t) = \frac{\Phi(\boldsymbol{r})}{r}\boldsymbol{K}\times\hat{\boldsymbol{x}} = \Omega(\boldsymbol{x})\boldsymbol{K}\times\hat{\boldsymbol{x}}$$

$$A^{\dagger}(t)\,\partial_0 A(t) = A^{\dagger}\dot{A} \equiv i\boldsymbol{\tau}\cdot\boldsymbol{K}$$

$$H[F,G,\omega,\xi_1,\xi_2,\Phi] = \int \mathcal{L}d^3r = -M_H + \lambda Tr\left(\dot{A}\dot{A}^{\dagger}\right)$$

- In order to ascribe spin and isospin quantum number to soliton, we will consider time-dependent matrix A

Quantization

$$
\begin{aligned}
\lambda \;=\; 4\pi \int dr \Bigg[& \frac{2}{3} f_\pi^2 r^2 \sin^2 F - \frac{a}{6} (g f_\pi)^2 \Phi^2 \\
& + \frac{a}{3} f_\pi^2 r^2 \left(8 \sin^4 \frac{F}{2} - 8\xi_1 \sin^2 \frac{F}{2} + 3\xi_1^2 + 2\xi_1\xi_2 + \xi_2^2 \right) \\
& + \frac{1}{3g^2} \left\{ 4G^2 \left(\xi_1^2 + \xi_1\xi_2 - 2\xi_1 - \xi_2 + 1 \right) \right. \\
& \left. + 2 \left(G^2 - 2G + 2 \right) \xi_2^2 + 3r^2 \left(\xi_1' \right)^2 + r^2 \left(\xi_2' \right)^2 + 2r^2 \xi_1' \xi_2' \right\} \\
& - \frac{1}{6} \left[\Phi'^2 + \frac{2\Phi^2}{r^2} \right] + g \frac{\Phi}{2\pi^2} \sin^2 F F' \Bigg]
\end{aligned}
$$

- Here Lambda is moment of inertia of the rotating skyrmion. Rotating skyrmion describes the nucleon.

Quantization

$$
\begin{aligned}
\xi_1'' &= 2f_\pi^2 g^2 \left(\cos F + \xi_1 - 1\right) \\
&\quad + \frac{G^2\left(\xi_1 - 1\right) + 2(G-1)\xi_2}{r^2} - \frac{2\xi_1'}{r} \\
\xi_2'' &= 2f_\pi^2 g^2 \left(-\cos F + \xi_2 + 1\right) \\
&\quad + \frac{G^2\left(\xi_1 - 1\right) + 2\left\{(G-3)\,G + 3\right\}\xi_2}{r^2} - \frac{2\xi_2'}{r} \\
\Phi'' &= 2f_\pi^2 g^2 \Phi - \frac{3F' \sin^2 F}{2\pi^2} + \frac{2\Phi}{r^2}
\end{aligned}
$$

Energy-momentum tensor form factors of nucleon

$$\left\langle p' \left| \hat{T}_{\mu\nu}(0) \right| p \right\rangle = \overline{u}(p') \left[M_2(t) \frac{P_\mu P_\nu}{M_N} + J(t) \frac{i\left(P_\mu \sigma_{\nu\mu} + P_\nu \sigma_{\mu\nu}\right)\Delta^\rho}{2M_N} + d_1(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N} \right] u(p)$$

where $P = (p + p')/2,\ \Delta = (p' - p)$ and $t = \Delta^2$

$$\langle p' | p \rangle = 2p^0 (2\pi)^3 \delta^{(3)}(\mathbf{p}' - \mathbf{p}) \qquad \bar{u}(p)u(p) = 2M_N$$

- $M_2(t)$ and $J(t)$ desdribe the nucleon momentum and angular momentum
- Interpretation of $d_1(t)$ less trivial, but also gives information about nucleon structure

Energy-momentum tensor form factors of Nucleon

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_a)} \partial^\nu \phi_a - g^{\mu\nu} \mathcal{L}$$

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_a)} \partial^\nu \phi_a + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \rho_a)} \partial^\nu \rho_a + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \omega_a)} \partial^\nu \omega_a - g^{\mu\nu} \mathcal{L}$$

$$F_\pi = \mathcal{O}(N_c^{1/2}), \quad e = \mathcal{O}(N_c^{-1/2}), \quad m_\pi = \mathcal{O}(N_c^0)$$

$$M_2(t) = \mathcal{O}(N_c^0), \quad J(t) = \mathcal{O}(N_c^0), \quad d_1(t) = \mathcal{O}(N_c^2).$$

Energy-momentum tensor form factors of Nucleon

$$T^{00}(r) = \frac{f_\pi^2}{2}\left(2\frac{\sin^2 F}{r^2} + F'^2\right) + f_\pi^2 m_\pi^2 (1-\cos F)$$
$$+\frac{2f_\pi^2}{r^2}(1-\cos F + G)^2 - g^2 f_\pi^2 \omega^2$$
$$+\frac{1}{2g^2r^2}\left\{2r^2G'^2 + G^2(G+2)^2\right\} - \frac{1}{2}\omega'^2$$
$$+\left(\frac{3}{2}g\right)\frac{1}{2\pi^2r^2}\omega \sin^2 F\, F'$$

$$T^{0i}(\mathbf{r},\mathbf{s}) = \frac{e^{ilm}r^l s^m}{(\mathbf{s}\times\mathbf{r})^2}\rho_J(r)$$

$$T^{ij}(r) = s(r)\left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right) + p(r)\,\delta^{ij}$$

Energy-momentum tensor form factors of Nucleon

$$\rho_J(r) = \frac{f_\pi^2}{3\Lambda}\left[\sin^2 F + 8\sin^4\frac{F}{2} + 4\sin^2\frac{F}{2}G - 4\sin^2\frac{F}{2}\xi_1 - 2\xi_1 G\right]$$
$$+\frac{1}{3g^2r^2\Lambda}\left[-r^2\xi_1' G' - (\xi_1 G - G - \xi_2)\left(2G + G^2\right)\right]$$
$$+\frac{g}{8\pi^2\Lambda}\Phi\sin^2 F F'$$

$$p(r) = -\frac{1}{6}f_\pi^2\left(F'^2 + 2\frac{\sin^2 F}{r^2}\right) - f_\pi^2 m_\pi^2(1-\cos F)$$
$$-\frac{2}{3r^2}f_\pi^2(1-\cos F + G)^2 + f_\pi^2 g^2\omega^2$$
$$+\frac{1}{6g^2r^2}\left\{2r^2G'^2 + G^2(G+2)^2\right\} + \frac{1}{6}\omega'^2$$

Energy-momentum tensor form factors of Nucleon

At the zero momentum transfer $t = 0$,

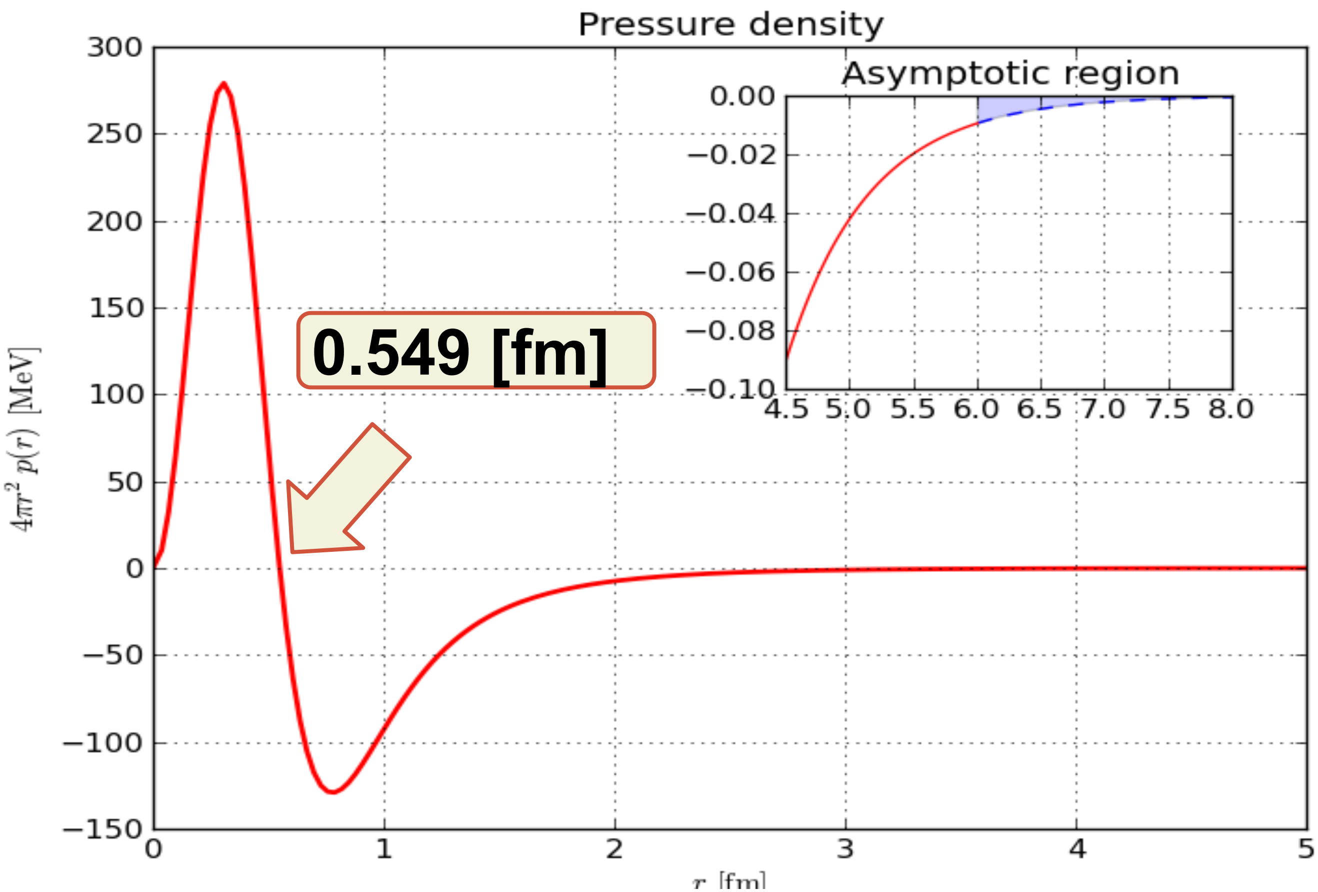
$$M_2^*(0) \; = \; \frac{1}{M_N^*} \int \mathrm{d}^3\boldsymbol{r}\, T_{00}^*(r) = 1 \,, \qquad J^*(0) \; = \; \int \mathrm{d}^3\boldsymbol{r}\, \rho_J^*(r) = \frac{1}{2} \,.$$

$$d_1^* \; = \; 5\pi M_N^* \int_0^\infty dr\, r^4\, p^*(r) \; = \; -\frac{4\pi M_N^*}{3} \int_0^\infty dr\, r^4\, \boldsymbol{s}^*(r)$$

stability condition

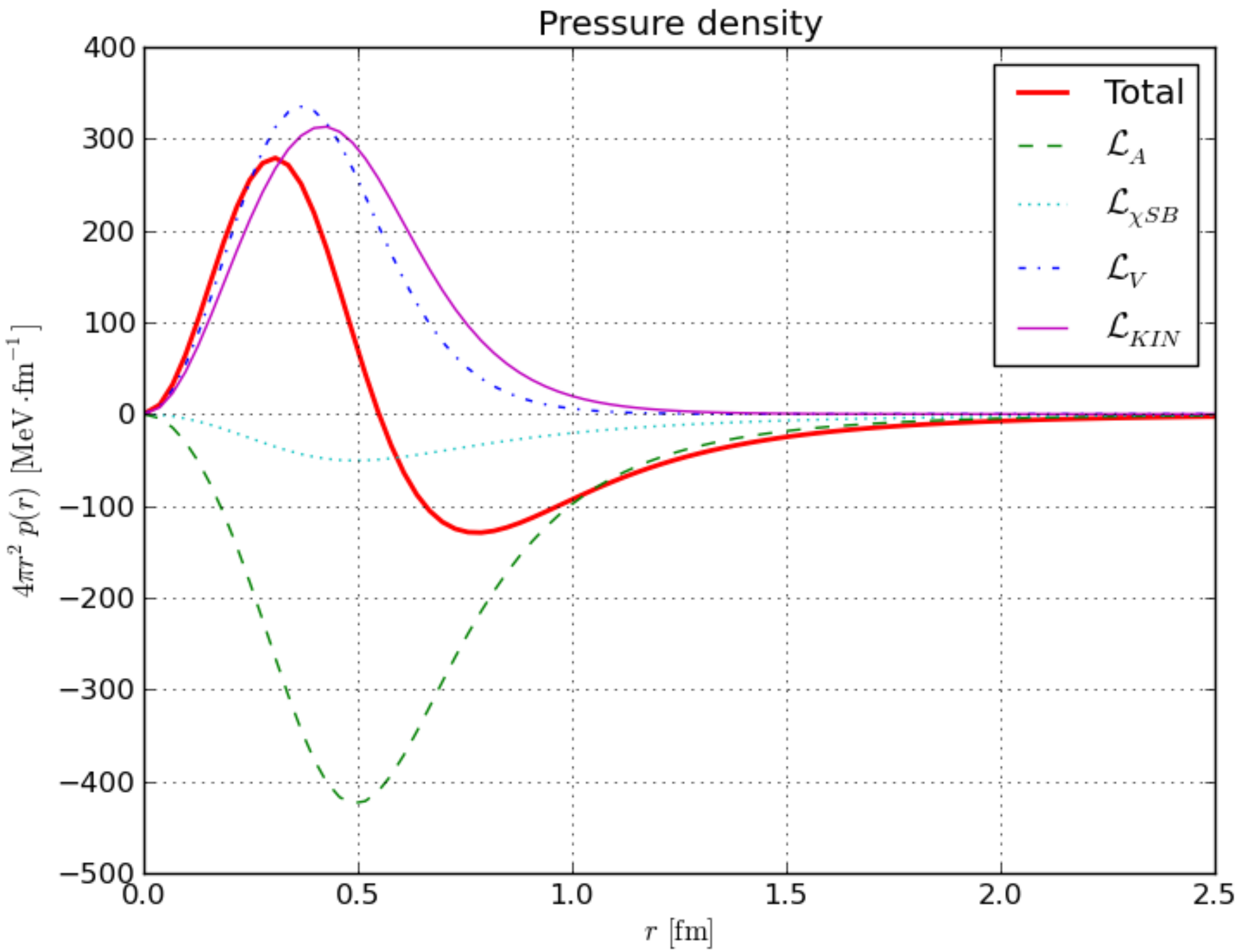
$$\int_0^\infty dr\, r^2\, p^*(r) \; = 0 \,.$$

Results



$p(r) > 0$ (repulsion) in the inner region, and $p(r) < 0$ (attraction) in the outer region

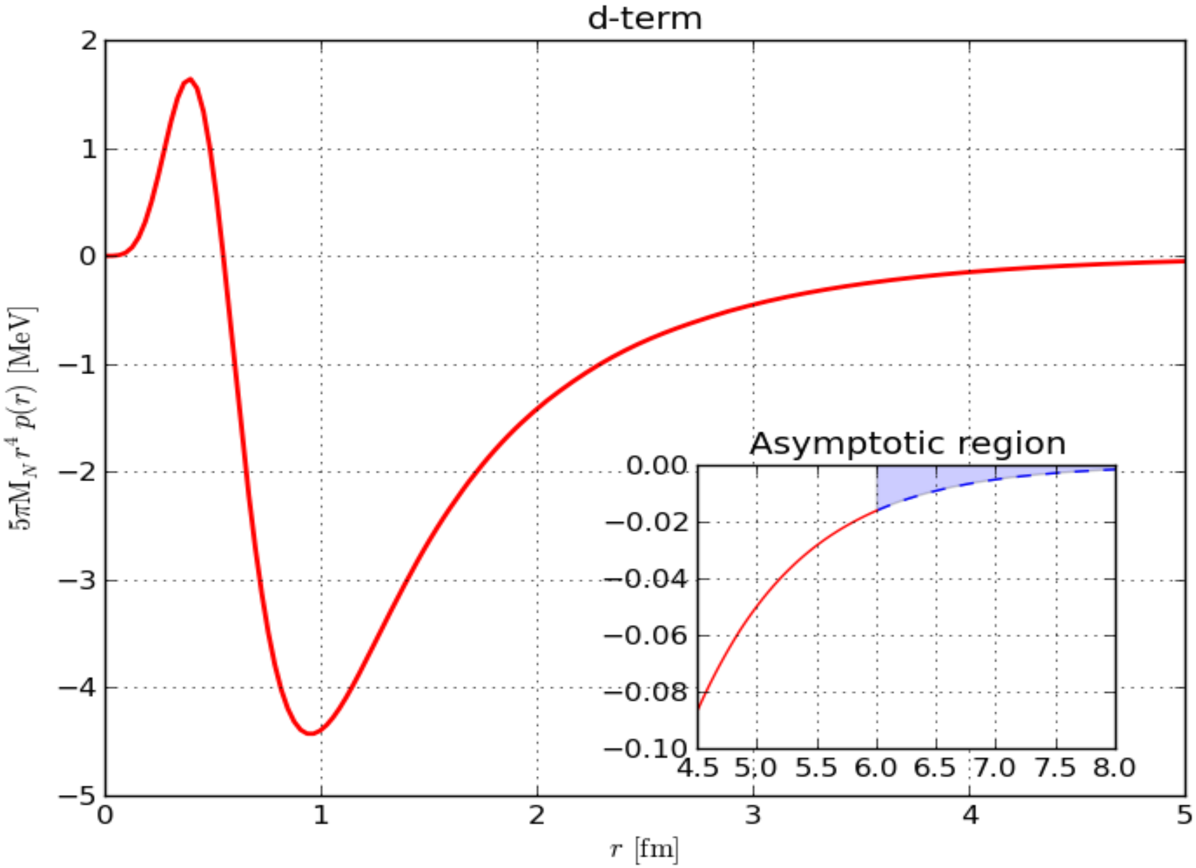
Results



Pressure density
400
300
200
100
0
−100
−200
−300
−400
−500
0.0
0.5
1.0
1.5
2.0
2.5
$4\pi r^2\, p(r)$ [MeV · fm^{-1}]
r [fm]
Total
$\mathcal{L}_A$
$\mathcal{L}_{\chi SB}$
$\mathcal{L}_V$
$\mathcal{L}_{KIN}$

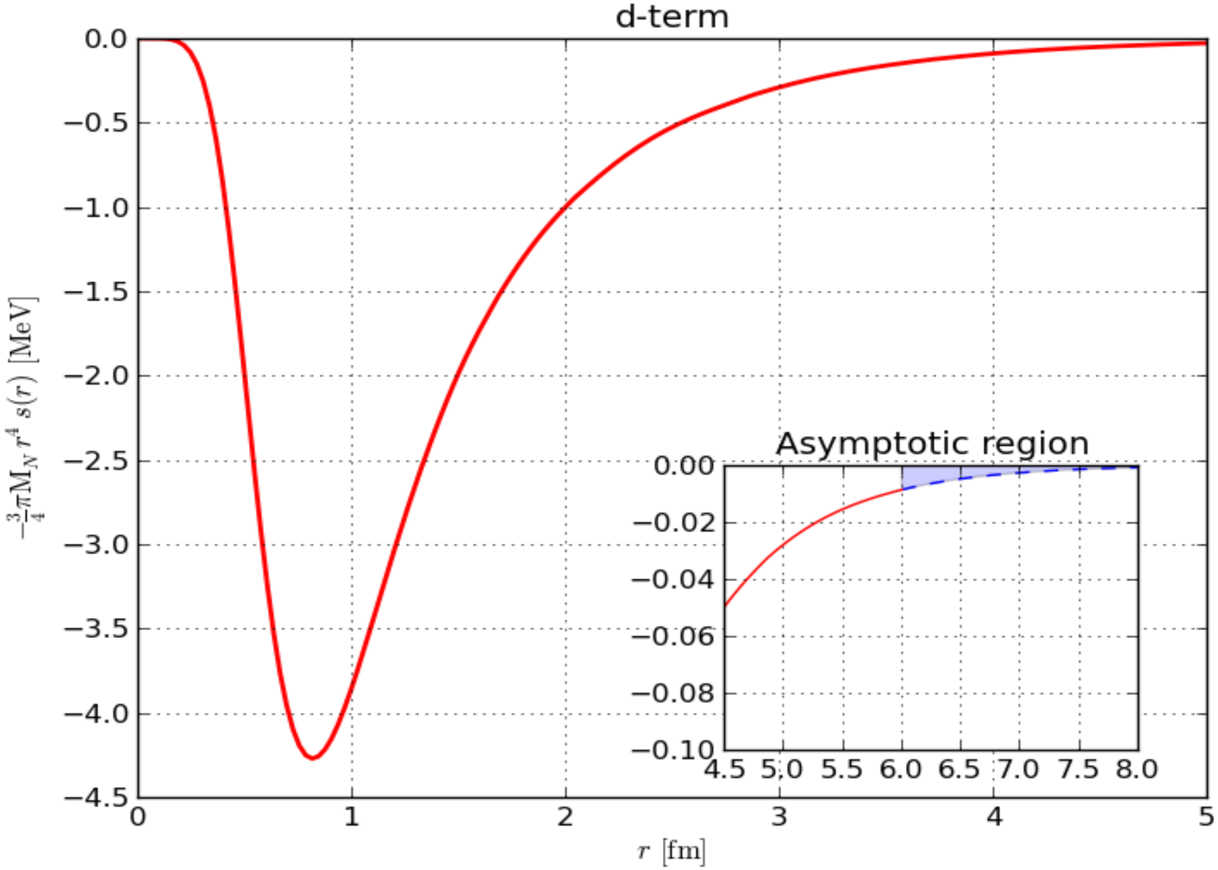
Results

$$d_1^* = 5\pi M_N^* \int_0^\infty dr\, r^4\, p^*(r) = -\frac{4\pi M_N^*}{3} \int_0^\infty dr\, r^4\, s^*(r)$$



Results

$$d_1^* = 5\pi M_N^* \int_0^\infty dr\, r^4\, p^*(r) = -\frac{4\pi M_N^*}{3} \int_0^\infty dr\, r^4\, s^*(r)$$



Results

	$T_{00}(0)$ $[\text{GeV}\cdot\text{fm}^{-3}]$	$\langle r_E^2\rangle$ $[\text{fm}^2]$	$p(0)$ $[\text{GeV}\cdot\text{fm}^{-3}]$	r_0 [fm]	$d_1(0)$
$\pi-\rho-\omega$ mesonic model	1.473	0.778	0.584	0.549	-5.032
Skyrme model [2]	1.45	0.68	0.26	0.71	-3.54
Chiral quark solton model [3]	1.70	0.67	0.232	0.57	-2.35

TABLE I. Different quantities related to the nucleon EMT densities and their form factors: $T_{00}(0)$ denotes the energy in the center of the nucleon; $\langle r_E^2\rangle$ is the mean square radii for the energy densities, respectively; $p(0)$ represents the pressure in the center of the nucleon, whereas r_0 designates the position where the pressure changes its sign; $d_1(0)$ is the value of the $d_1(t)$ form factor at the zero momentum transfer.

Conclusion & Outlook

- We studied nucleon structure within pi-rho-omega soliton model
- The results are consistent with other model (original skyrme model, chiral quark soliton model, etc...)
- It is also interesting how nucleon structure changes in nuclear medium within this framework
- Corresponding studies under the way.

Thank you