

# Simulation of positron energy spectra generated by channeling radiation of GeV electrons in a tungsten single crystal

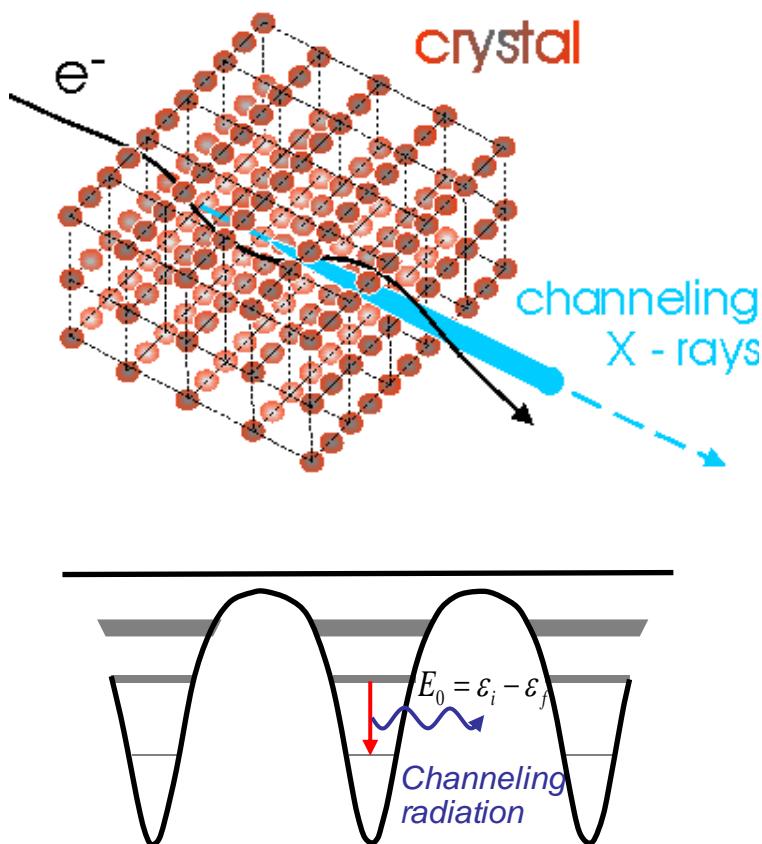
B. Azadegan, S.A. Mahdipour, W. Wagner

26.09.2013, Sevan

## Outline

- 1. Introduction**
- 2. Theory of channeling radiation in thick crystal**
- 3. Numerical results**
- 4. Solution of Fokker-Plank equation**
- 5. Positron production in a hybrid scheme**
- 6. Summary**

## 1. Introduction



$$E_x(\theta) = \frac{2\gamma^2 E_0}{(1 + \gamma^2 \theta^2)} \quad \theta < \frac{1}{\gamma}$$

$\gamma$  - Lorentz factor

$\theta$  - observation angle

### CR properties

- quasi-monochromatic
- directed
- intense
- tunable

## 2. Theory of planar channeling radiation

$$V(x) = \sum_n v_n e^{inx}$$

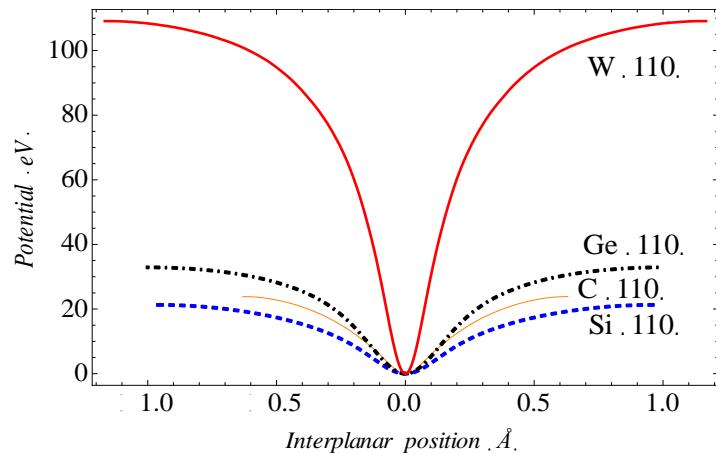
Planar Continuum potential:

$$v_n = -\frac{2\pi}{V_c} a_0^2 (e^2 / a_0) \sum_j e^{-M_j(\vec{g})} e^{-i\vec{g} \cdot \vec{r}_j} \sum_{i=1}^4 a_i e^{\left(-\frac{1}{4} \left(\frac{b_i}{4\pi^2}\right) (ng)^2\right)}$$

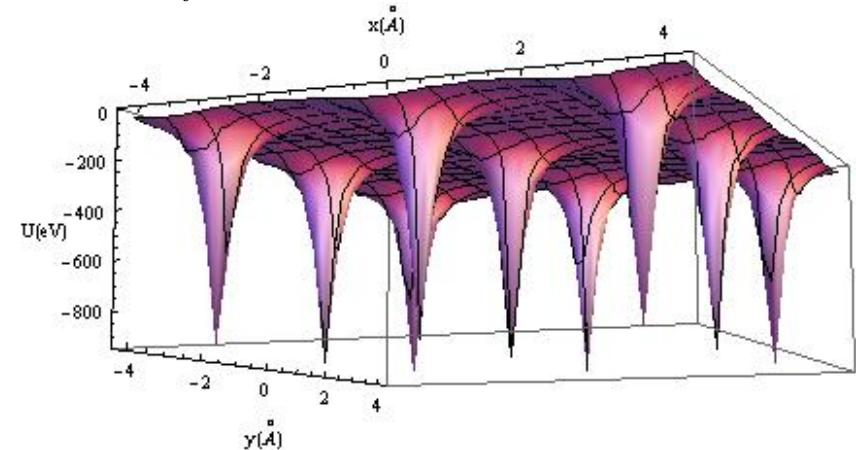
$$V(x, y) = \sum_{\vec{g}_m} v_{\vec{g}_m} e^{i\vec{g}_m \cdot \vec{r}_\perp}$$

Axial Continuum potential:

$$v_{\vec{g}_m} = -\frac{2\pi}{V_c} a_0^2 (e^2 / a_0) \sum_j e^{-i\vec{g} \cdot \vec{r}_j} \sum_{i=1}^4 a_i e^{\left(-\frac{1}{4} \left(\frac{b_i}{4\pi^2} + 2\langle u_j^2 \rangle\right) |\vec{g}_m|^2\right)}$$



Planar continuum potential of (110) plane of C, Si, Ge and W single crystal



Axial continuum potential of <100> axis of W single crystal

## 2. Theory of planar channeling radiation (classical)

Classical model

$$E_e > 100 \text{ MeV}$$

Planar :

$$\gamma m \ddot{x}(t) = F = -\frac{\partial V(x)}{\partial x}$$

$$\gamma m \ddot{x} = -\frac{\partial V(x, y)}{\partial x}$$

Axial :

$$\gamma m \ddot{y} = -\frac{\partial V(x, y)}{\partial y}$$

$$\gamma m \ddot{z} = 0$$

Angular energy distribution:

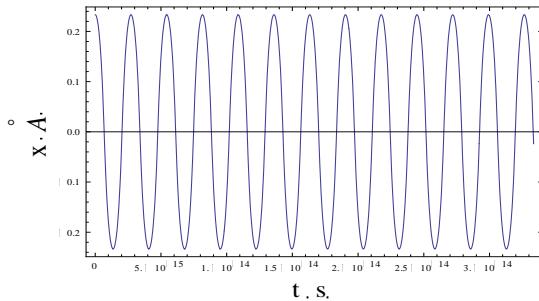
$$\frac{d^2 E}{d\omega d\Omega} = \frac{e^2}{4\pi^2 c} \left| \int_0^\tau e^{i(\omega t - \vec{k} \cdot \vec{r})} \frac{\vec{n} \times ((\vec{n} - \vec{\beta}) \times \dot{\vec{\beta}})}{(1 - \vec{\beta} \cdot \vec{n})^2} dt \right|^2$$

Total radiated energy  
in thick crystal:

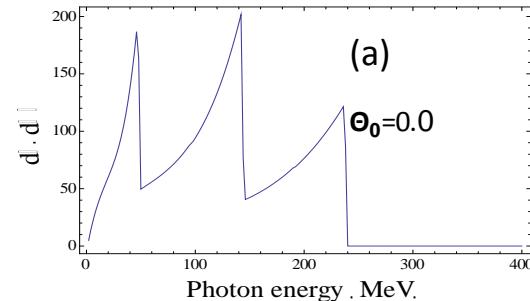
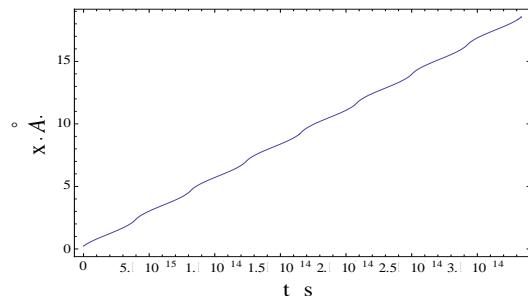
$$\frac{dE}{d\omega \Delta z} = \frac{e^2}{c^4 T^2} \sum_{n=1}^{\infty} \Theta[1 - \eta_n] \left( \eta_n^2 - \eta_n + \frac{1}{2} \right) \cdot |\dot{x}_{\tilde{\omega}}|^2$$

$$\eta_n = \frac{T\omega}{4\pi\gamma^2 n}; \quad \tilde{\omega} = \frac{2\pi n}{T}; \quad \dot{x}_{\tilde{\omega}} = \int_0^T \dot{x} e^{i\tilde{\omega}t} dt$$

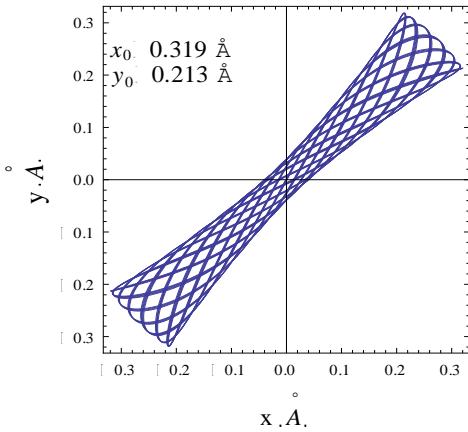
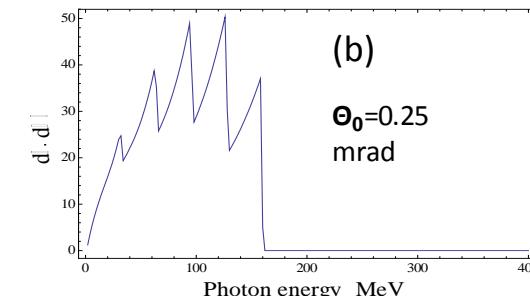
### 3. Numerical results



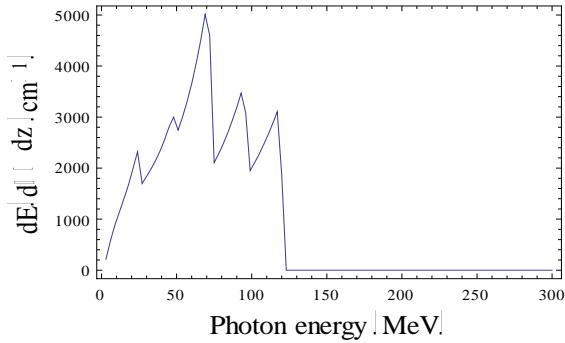
Planar:



Trajectories and CR spectra for one incident point with a)  
a) zero  
b) 0.25 mrad incidence angles  
for 2 GeV electron channeled along the (110) plane of a tungsten single crystal.



Axial:



Trajectory (rosette motion) and CR spectra for one incident point for 1 GeV electron channeled along the  $\langle 100 \rangle$  axis of W single crystal.

## 4. Solution of Fokker-plank equation

Fokker-Plank equation:

$$\frac{\partial F(z, E_{\perp})}{\partial z} = \frac{\partial^2}{\partial E_{\perp}^2} [D^2(E_{\perp})F(z, E_{\perp})] - \frac{\partial}{\partial E_{\perp}} [D^1(E_{\perp})F(z, E_{\perp})]$$

Boundary conditions:

$$\frac{\partial F(z, 0)}{\partial z} = 0, \frac{\partial F(z, E_{\perp,c})}{\partial z} = 0$$

Drift coefficient:

$$D^1(E_{\perp}) = \aleph \int_{-x_d}^{x_d} \frac{\exp(-x^2/2u_1^2)}{\sqrt{2(E_{\perp} - U(x))/E}} dx$$

Diffusion coefficient:

$$D^2(E_{\perp}) = \aleph \int_{-x_d}^{x_d} \frac{2(E_{\perp} - U(x))\exp(-x^2/2u_1^2)}{\sqrt{2(E_{\perp} - U(x))/E}} dx$$

$$\aleph = \frac{E_s^2}{4EvX_0} \frac{4}{T(E_{\perp})} \frac{d_p}{\sqrt{2\pi u_1}} \quad E_s = 13.6 \text{ MeV}$$

Time parameter:

$$T(E_{\perp}) = 2 \sqrt{\frac{E}{2c^2}} \int_{-x_d}^{x_d} \frac{dx}{\sqrt{(E_{\perp} - U(x))}}$$

## 4. Solution of Fokker-plank equation

For the numerical solution of the Fokker-Plank equation, a uniform distribution of the electron across the transverse  $x$  coordinate, and a Gaussian scattering distribution tilted by an angle  $\theta_0$ , and with standard deviation  $\sigma_y$  for the angular divergence were assumed.

$$F_0(E_\perp) = \frac{1}{4\sqrt{2\pi}(\sigma_{y/\theta_c})U(\frac{dp}{2})} \int_{-x_d}^{x_d} \frac{1}{\theta(E_\perp)} \left( \exp \left[ -\frac{(\theta(E_\perp) - \theta_0)^2}{2(\sigma_y)^2} \right] + \exp \left[ -\frac{(-\theta(E_\perp) - \theta_0)^2}{2(\sigma_y)^2} \right] \right)$$

$$\theta(E_\perp) = \sqrt{\frac{2(E_\perp - U(x))}{E}}$$

$$\theta_c = \sqrt{2U(d_p/2)/E}$$

(a) Time parameter  $c.T$

(b) Drift coefficient  $D^1$ ,

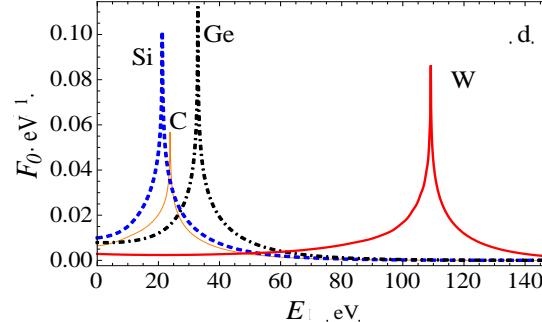
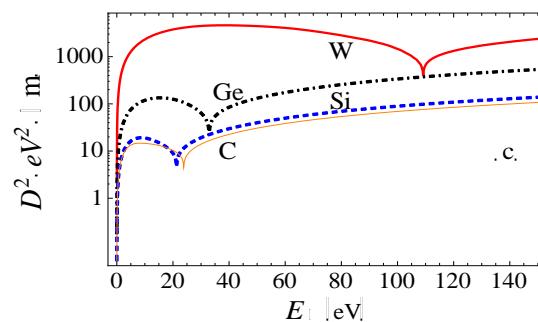
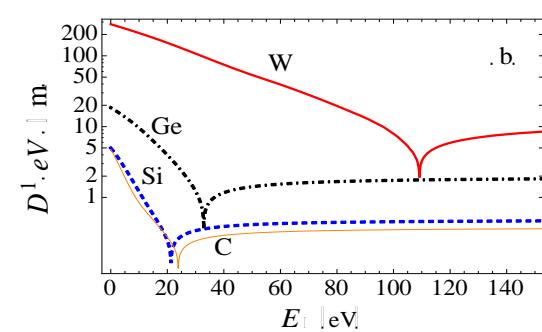
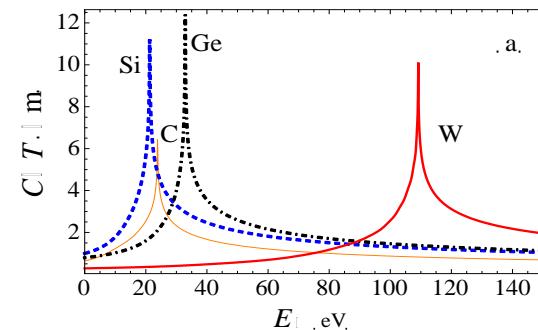
(c) Diffusion coefficient  $D^2$

(d) Initial probability density distribution

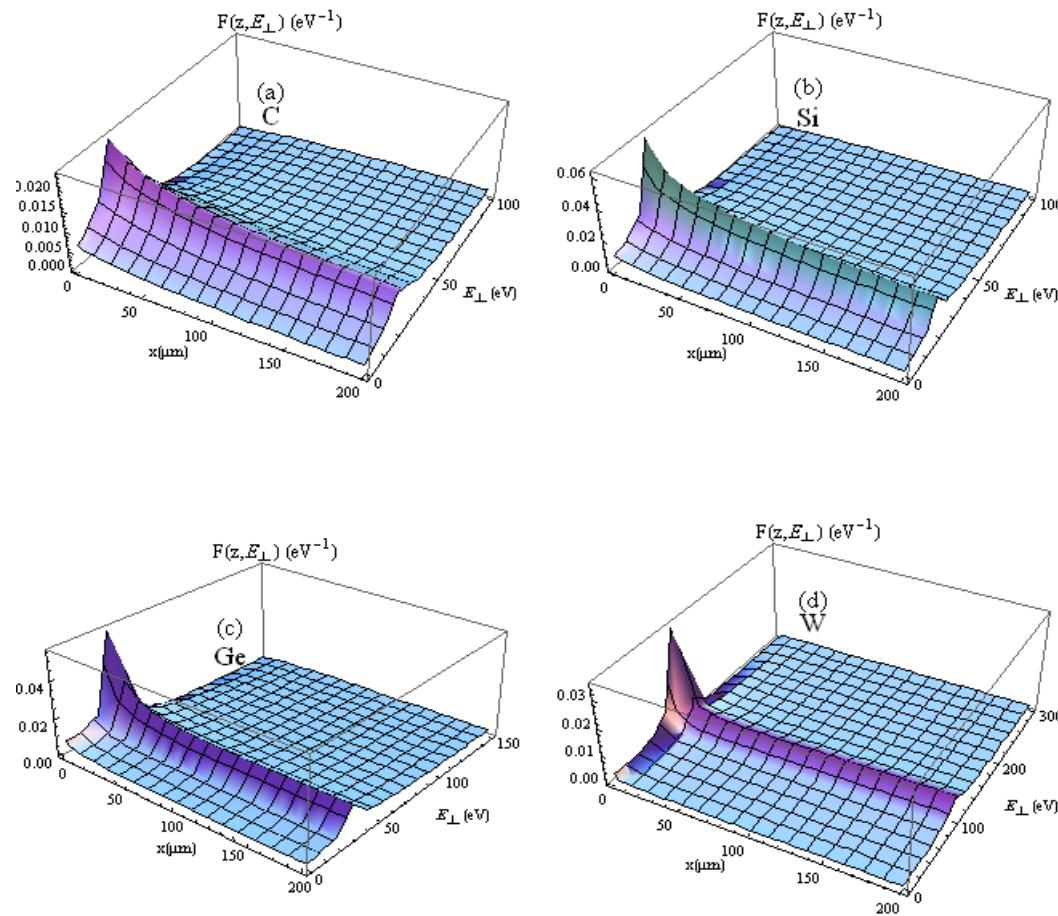
$F_0$  at  $z=0$

calculated with standard deviation

$\sigma_y=100 \text{ } \mu\text{rad}$  and  $\theta_0=0$  all as function of transverse energy for 2 GeV electrons.



## 4. Solution of Fokker-plank equation

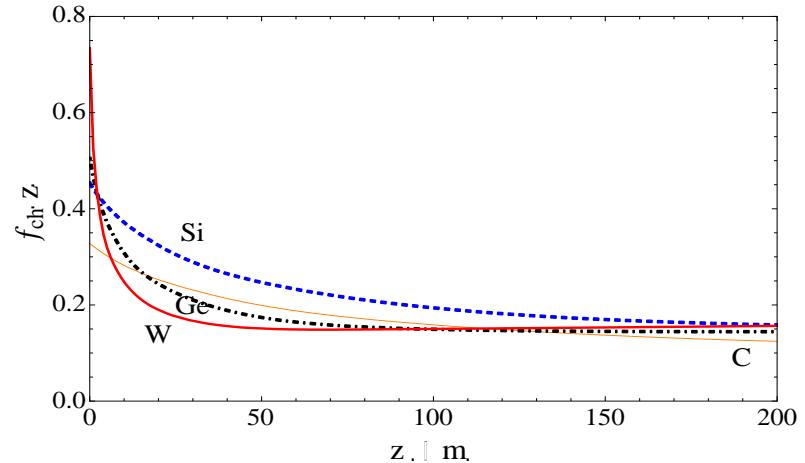


probability density for 2 GeV electrons channeled along (110) plane

## 4. Solution of Fokker-plank equation

Electron dechanneling function:

$$f_{ch}(z) = \int_0^{U(\frac{dp}{2})} F(z, E_{\perp}) dE_{\perp}$$



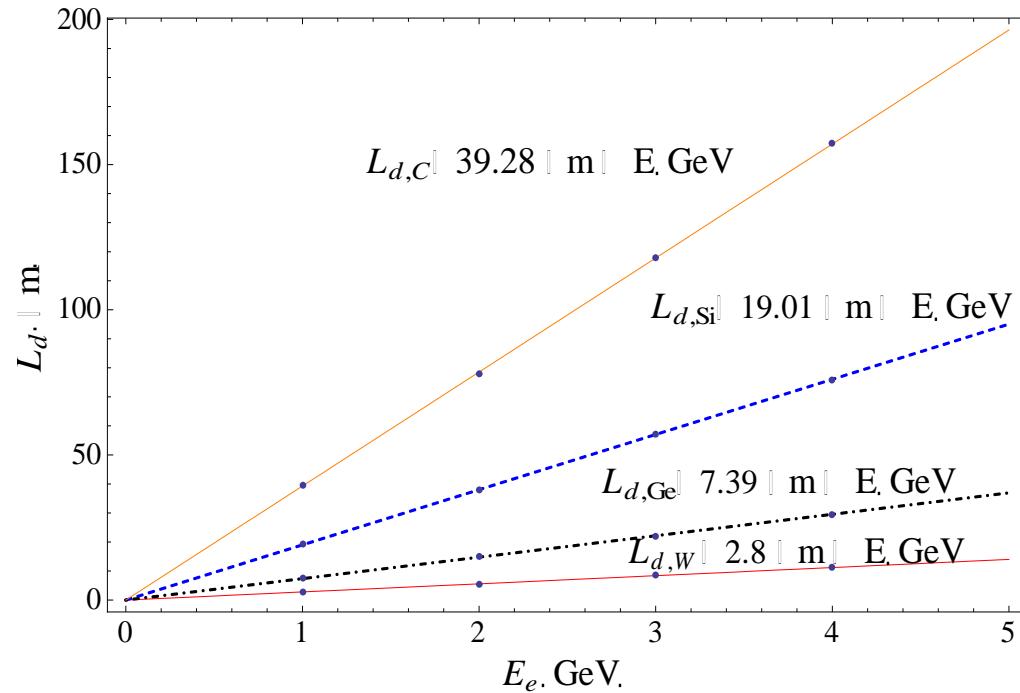
$$J(z, E_{\perp}) = -\frac{\partial}{\partial E_{\perp}} [D^2(E_{\perp})F(z, E_{\perp})] + D^1(E_{\perp})F(z, E_{\perp}) = J_{diff} + J_{drift}$$

Dechanneling length:

$$L_d(z) = \frac{f_{ch}(z)}{J_{drift}\left(z, E_{\perp} = U(\frac{dp}{2})\right)}$$

$$f_{ch}(z) = \frac{1}{e} f_{ch}(0)$$

## 4. Solution of Fokker-plank equation

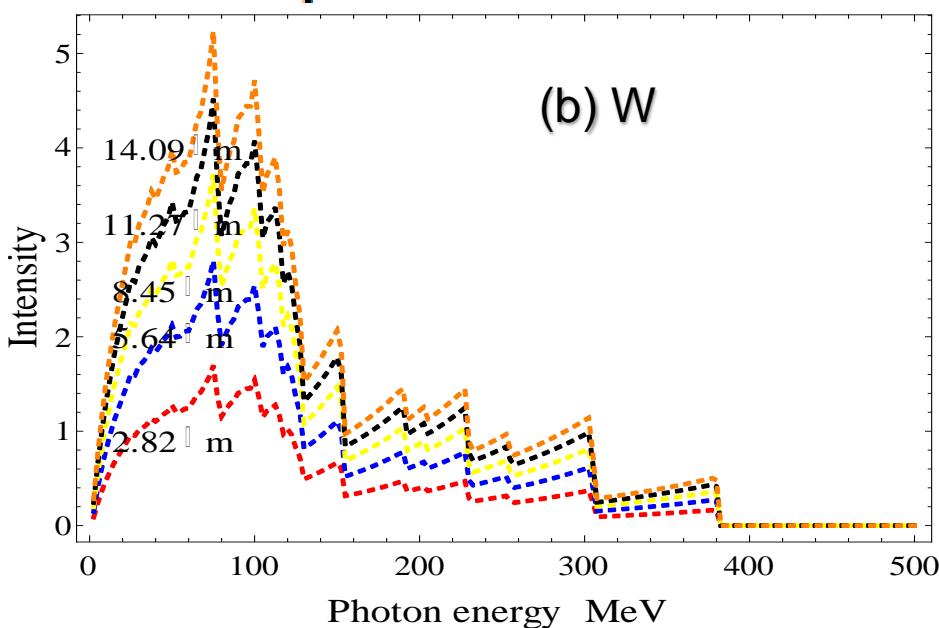
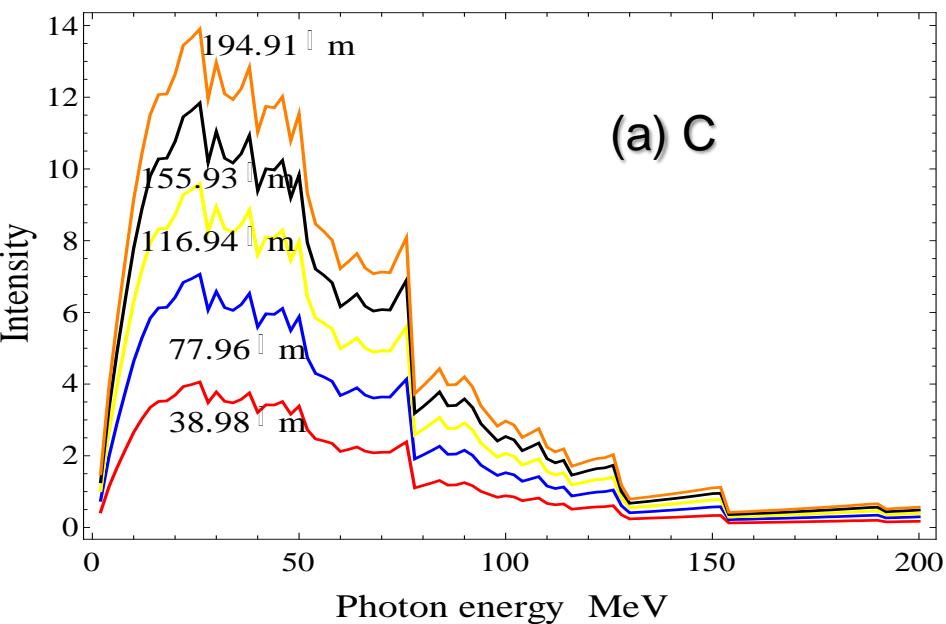


Dechanneling lengths for (110)-planar channeling  
of electrons as function of the electron energy

## 4. Solution of Fokker-plank equation

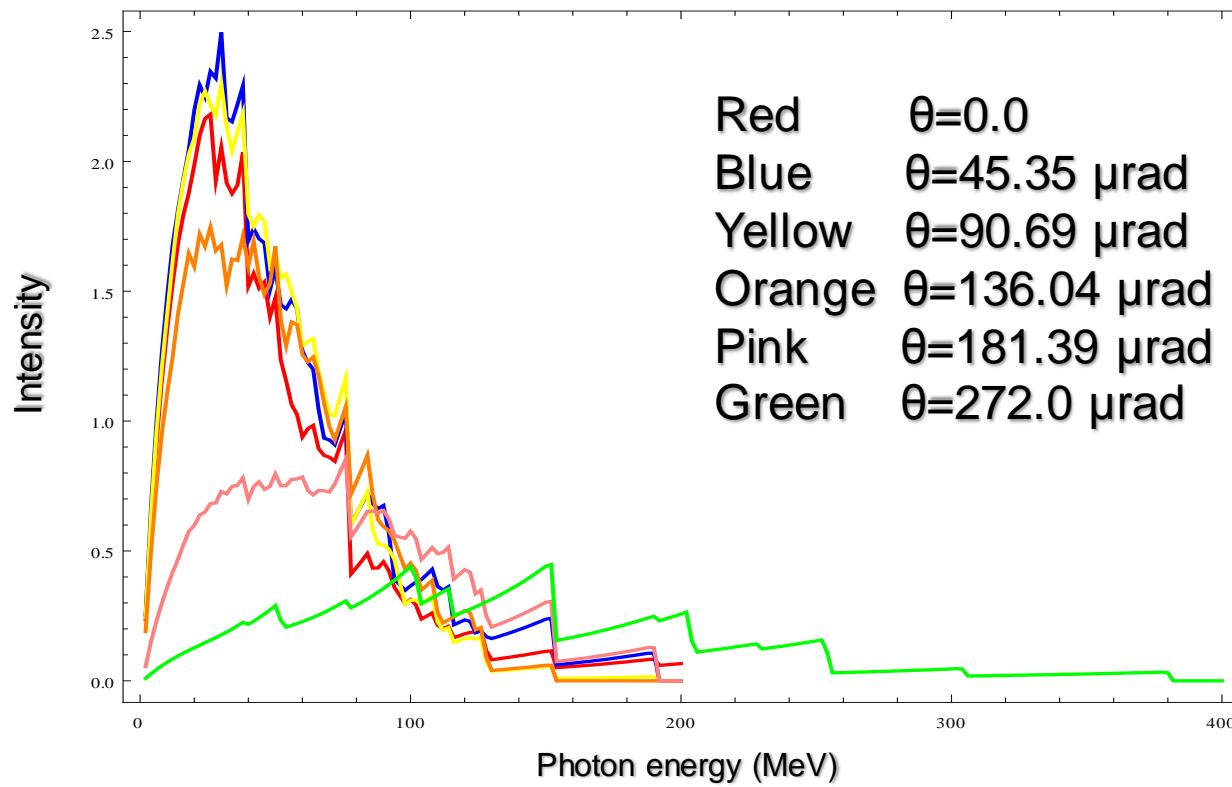
Radiation intensity in thick crystals:

$$I_{\text{thick}}(\hbar\omega, E_{\perp}) = \frac{1}{L} \int_0^L \frac{dE}{d\hbar\omega \Delta z} F(z, E_{\perp}) dz$$



Thickness dependence of radiation spectrum for 2 GeV electrons channeled along (110) plane of a) diamond b) Tungsten

#### 4. Solution of Fokker-plank equation

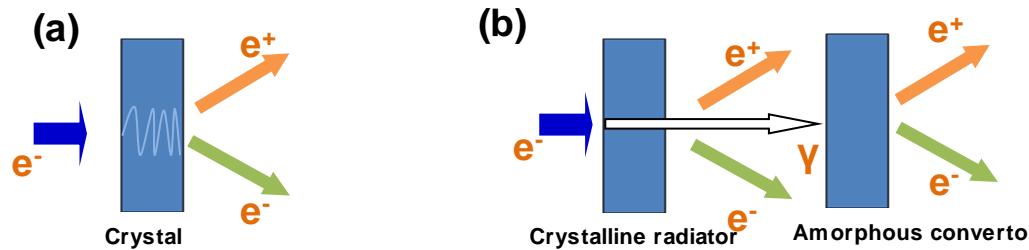


Dependence of radiation spectrum on incidence angle of electrons for 2 GeV electrons channeled along (110) plane of Ge

## 5. Positron production in a hybrid scheme

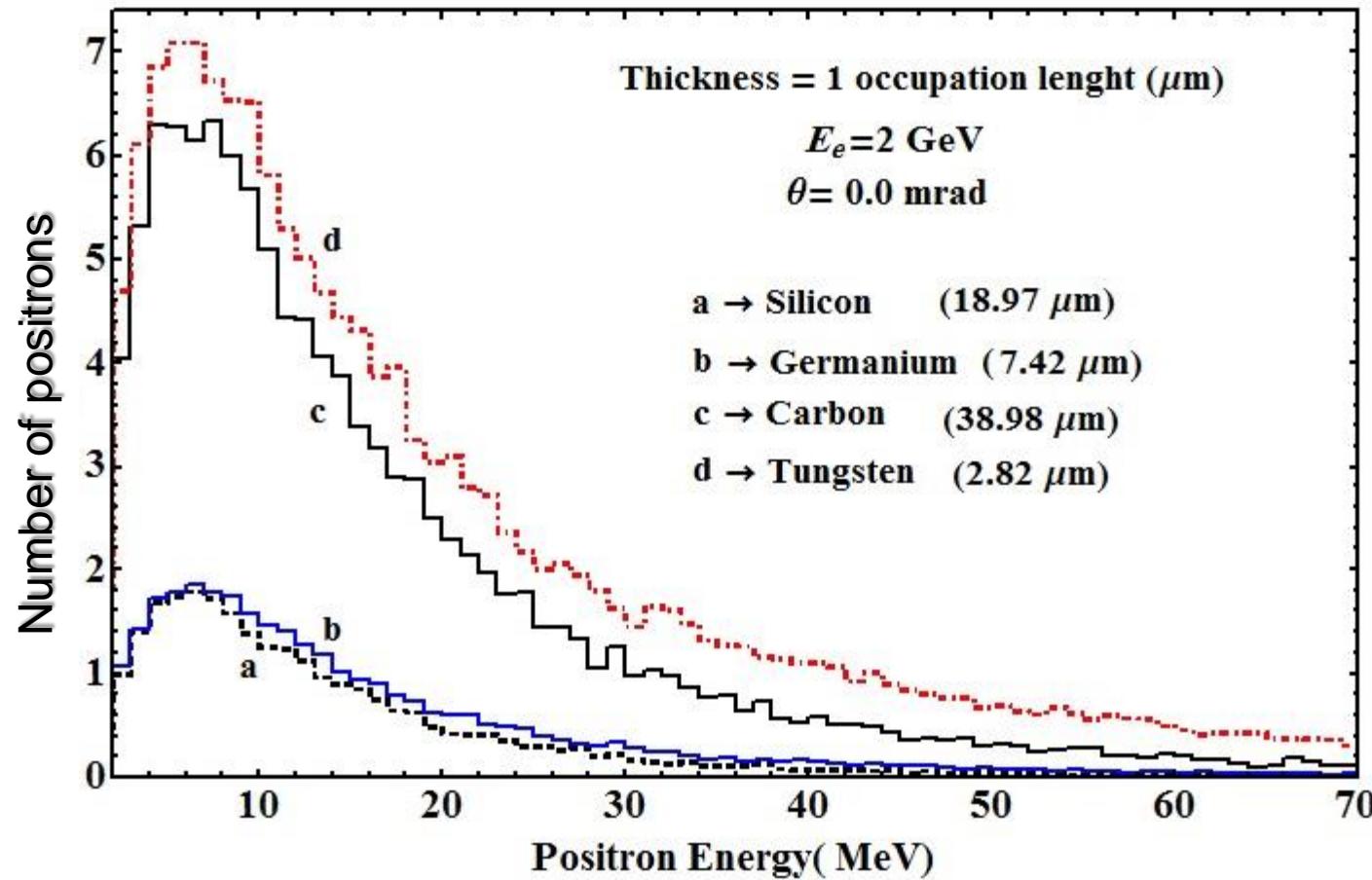
Schemes of non-conventional positron sources.

- a) One single crystal.
- b) Crystalline target combined with an amorphous convertor.



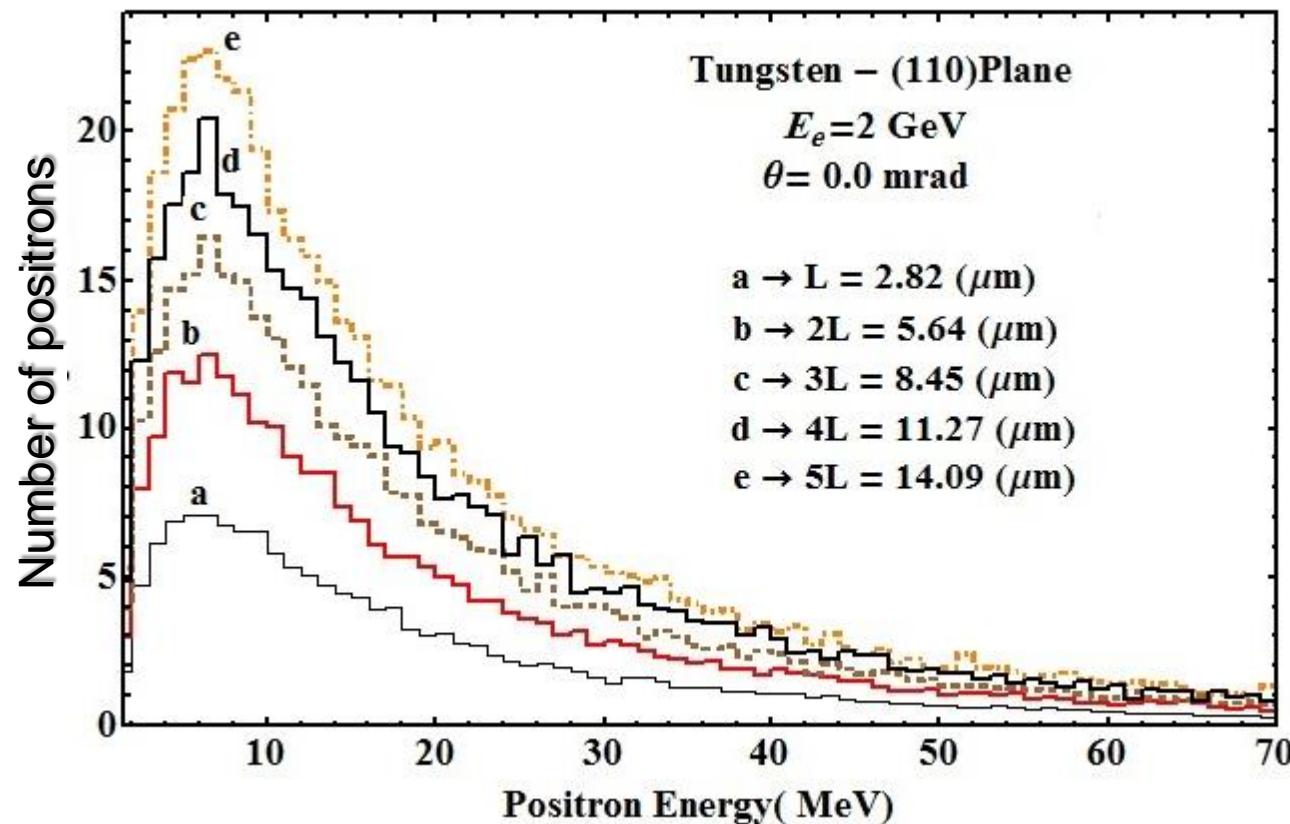
Positron spectra are simulated by means of **GEANT4** Monte Carlo code taking the CR/CB spectra as input data

## 5. Positron production in a hybrid scheme



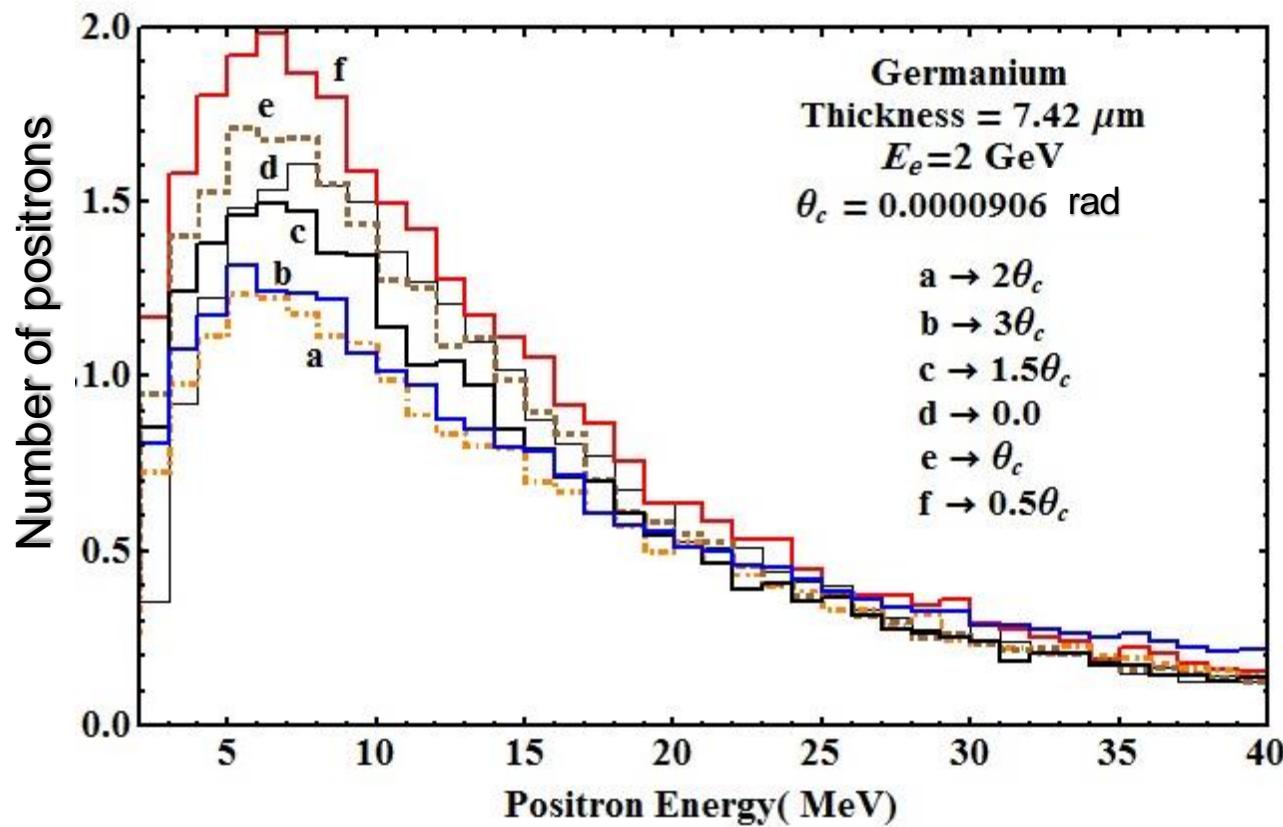
Comparison of positron energy distributions between C, Si, Ge, W radiator crystals

## 5. Positron production in a hybrid scheme



Dependence of positron energy distribution on thickness of radiator crystal  
for 2 GeV electrons channeled along (110) plane of W radiator crystal

## 5. Positron production in a hybrid scheme



Dependence of positron energy distributions on incidence angle of electrons for 2 GeV electrons channeled along (110) plane of Ge

## 6. Summary

- Planar CR emitted by electrons channeled in thick crystals has been investigated theoretically on the base of the solution of Fokker-Plank equation.
- Dependence of CR spectrum on the incidence angle of electron has been investigated.
- Dependence of positron energy distribution on the thickness of radiator crystal and the incidence angle of electron has been investigated in a hybrid positron production scheme.
- Positron energy distributions of C, Si, Ge and W radiator crystals have been compared.
- W radiator crystal with small channeling length produce more positron in comparison with C, Si and Ge.
- Comparison of positron energy distribution in planar and axial CR needs the solution of Fokker-Plank equation for axial CR in two dimentions.