

# *On High-Energy Charged Particle Beam Diffusion in Crystal*

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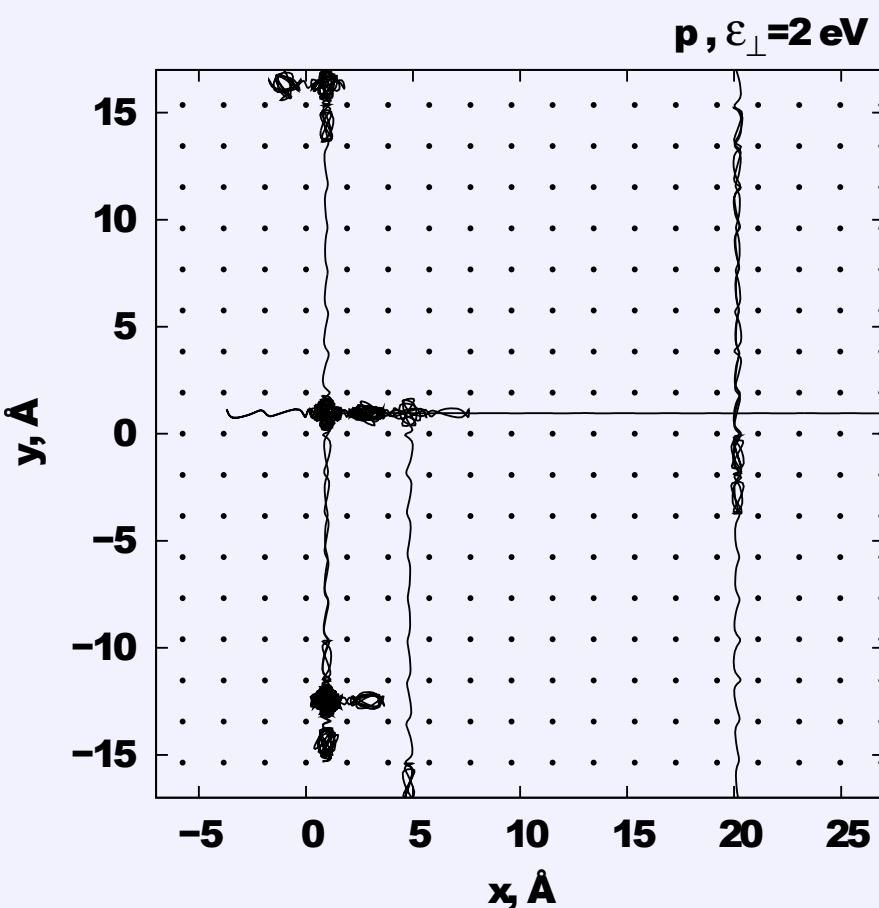
*RREPS-2013, "Blue Sevan", Armenia*

# Trajectories of charged particles in plane orthogonal to $\langle 100 \rangle$ axis of Si

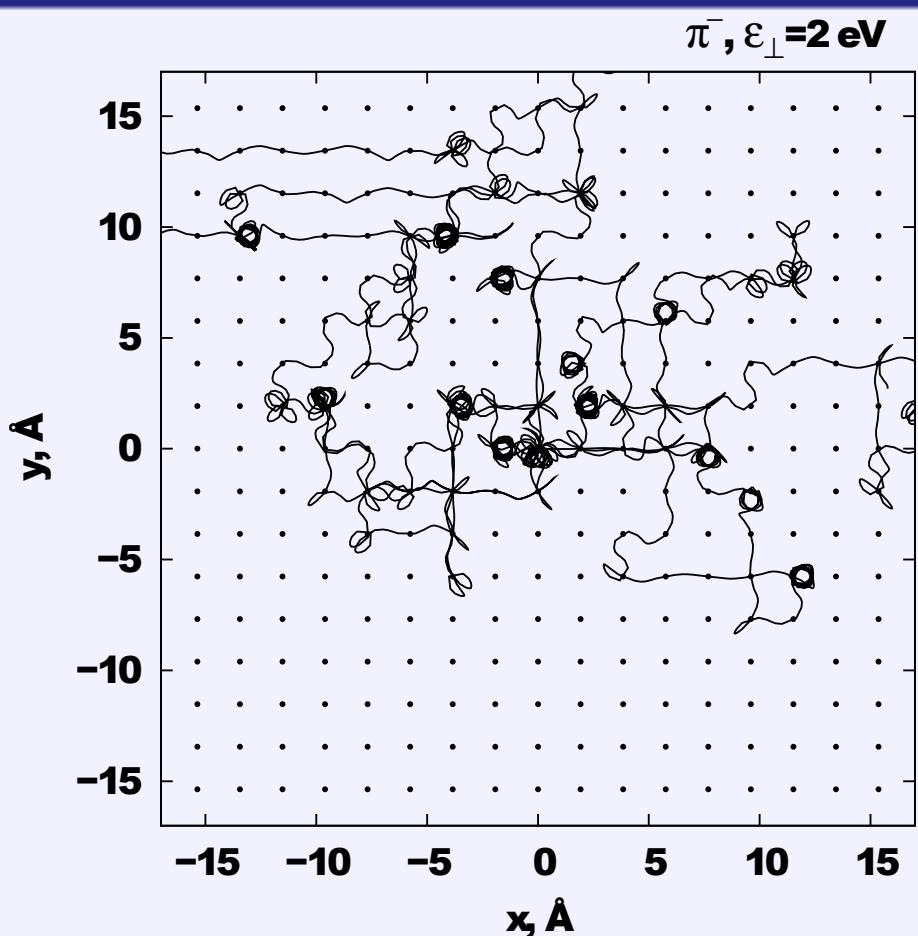
$$\frac{d^2\vec{\rho}}{dt^2} = -\frac{c^2}{E}\frac{\partial}{\partial\vec{\rho}}\sum_n U_R(\vec{\rho} - \vec{\rho}_n)$$

$$\varepsilon_{\perp} = \frac{E\dot{\vec{\rho}}^2}{2} + \sum_n U_R(\vec{\rho} - \vec{\rho}_n)$$

*protons*



*electrons*



## *Diffusion rate for a beam in crystal*

$$\langle \Delta\rho^2 \rangle(l) = \langle \Delta\rho^2 \rangle(l_0) \left( \frac{l}{l_0} \right)^\mu$$

$\Delta\rho$  – distance between beam particle and its insident point in the transverse plane

$l$  – the thickness of the crystal

$l_0$  – normalization thickness,  $l_0 \ll l$

$\mu$  – diffusion rate

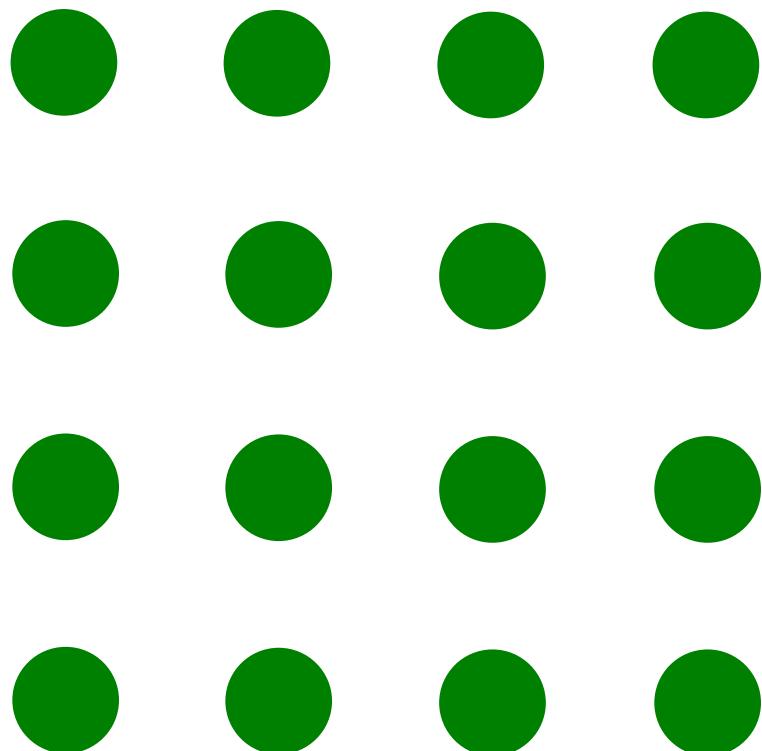
averaging is performed over all particles in the beam

$$\mu = \log_{l/l_0} \frac{\langle \Delta\rho^2 \rangle(l)}{\langle \Delta\rho^2 \rangle(l_0)} = \frac{\ln \frac{\langle \Delta\rho^2 \rangle(l)}{\langle \Delta\rho^2 \rangle(l_0)}}{\ln \frac{l}{l_0}}$$

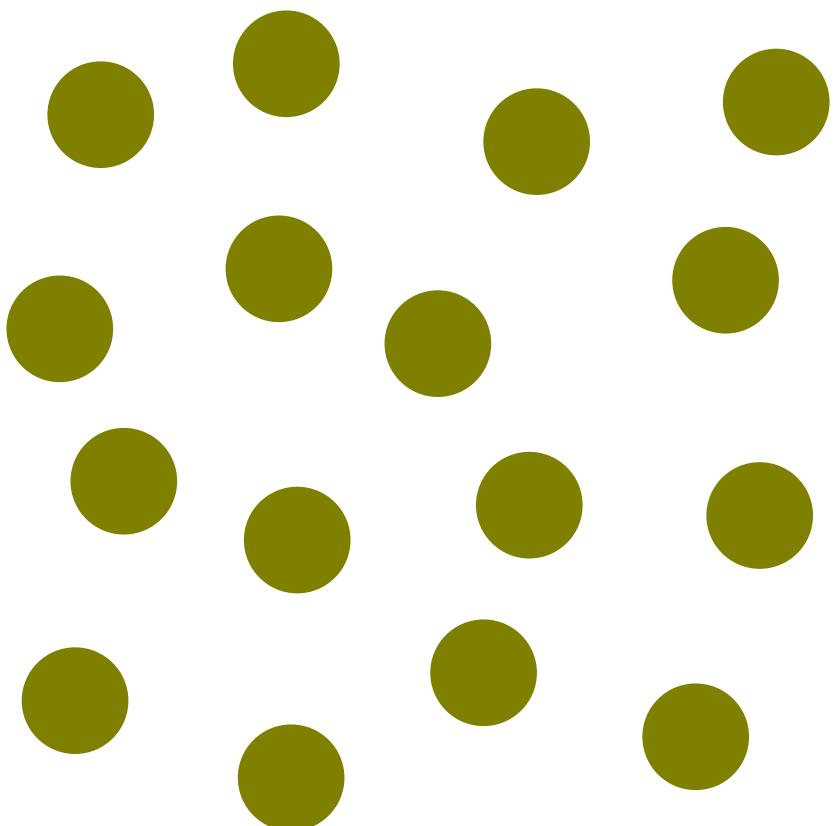
*A.A. Greenenko, A.V. Chechkin, N.F. Shul'ga. Physics Letters A 324 (2004) 82–85*

# *Periodically and Randomly Located Crystal Atomic Strings*

$\langle 100 \rangle$  Si



random strings approximation



## *Random string approximation (RSA)*

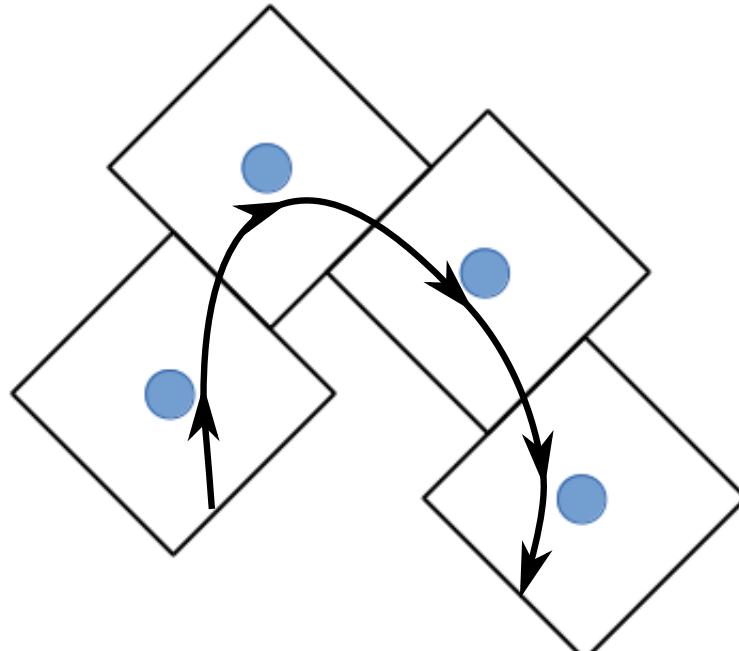
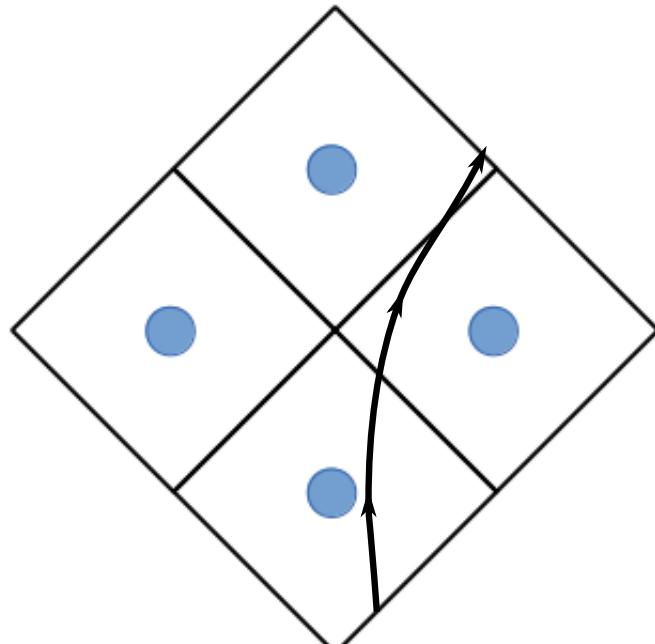
$$U(\rho) = \begin{cases} U_1 \ln \left( 1 + \frac{\beta R^2}{\rho^2 + \alpha R^2} \right) - U_2, & \rho < \bar{a}/2 \\ 0, & \rho \leq \bar{a}/2 \end{cases}$$

$\rho$  — distance from the atomic string

$\bar{a}$  — the average distance between neighboring crystal atomic strings

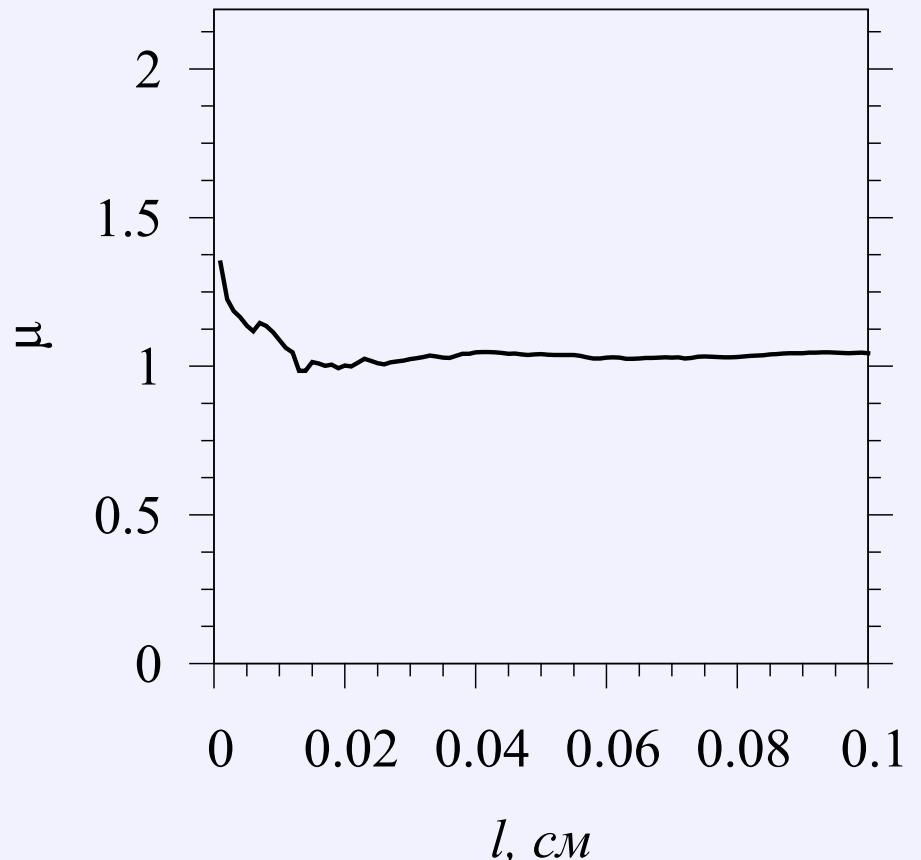
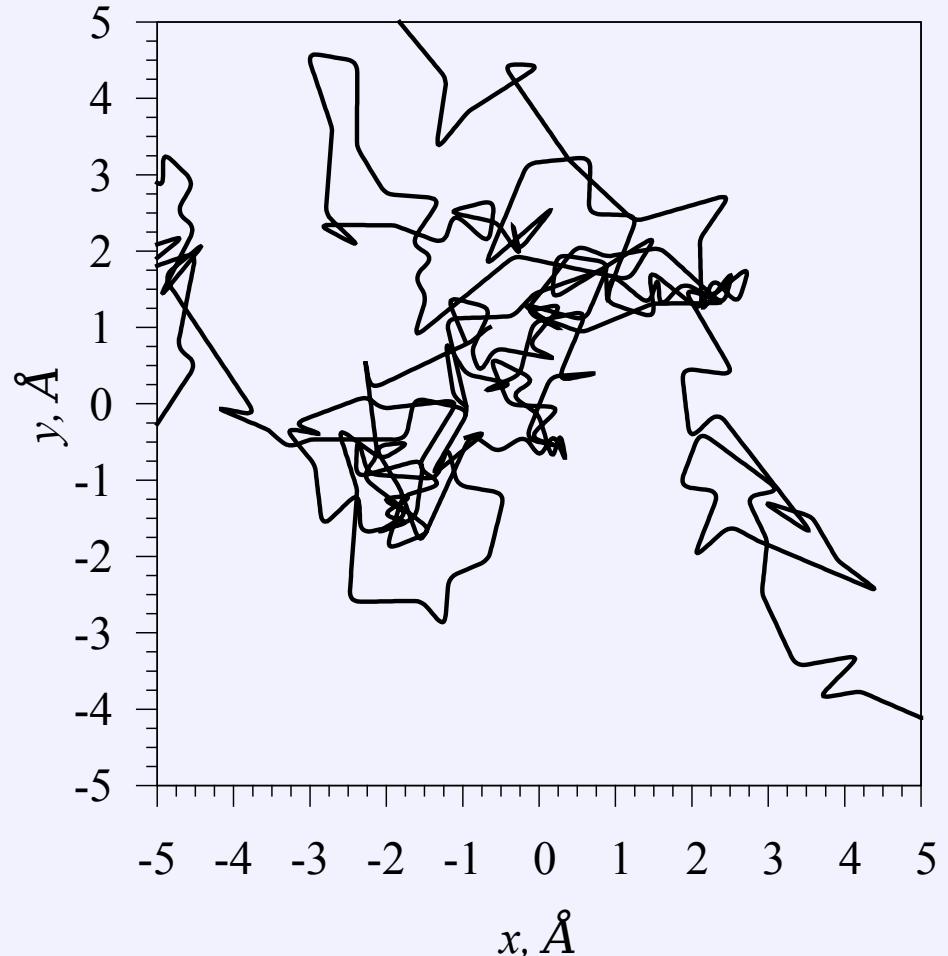
$R$  — the screening radius of the atom

$U_1$ ,  $U_2$ ,  $\alpha$  and  $\beta$  — fitting constants



# Particle motion in RSA

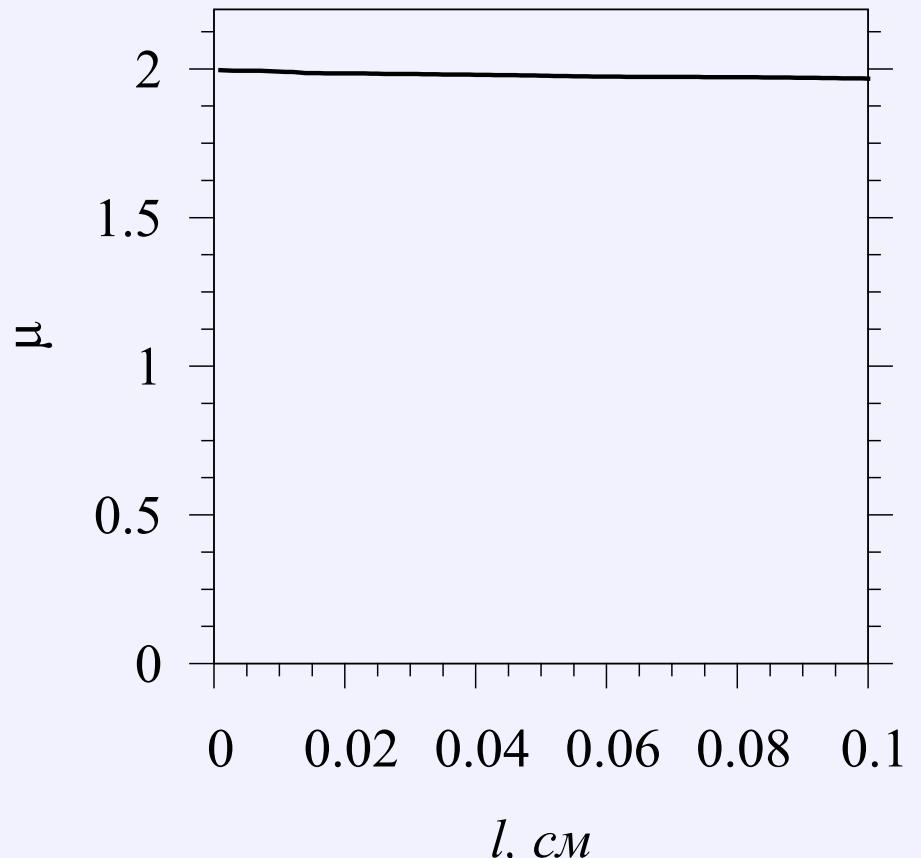
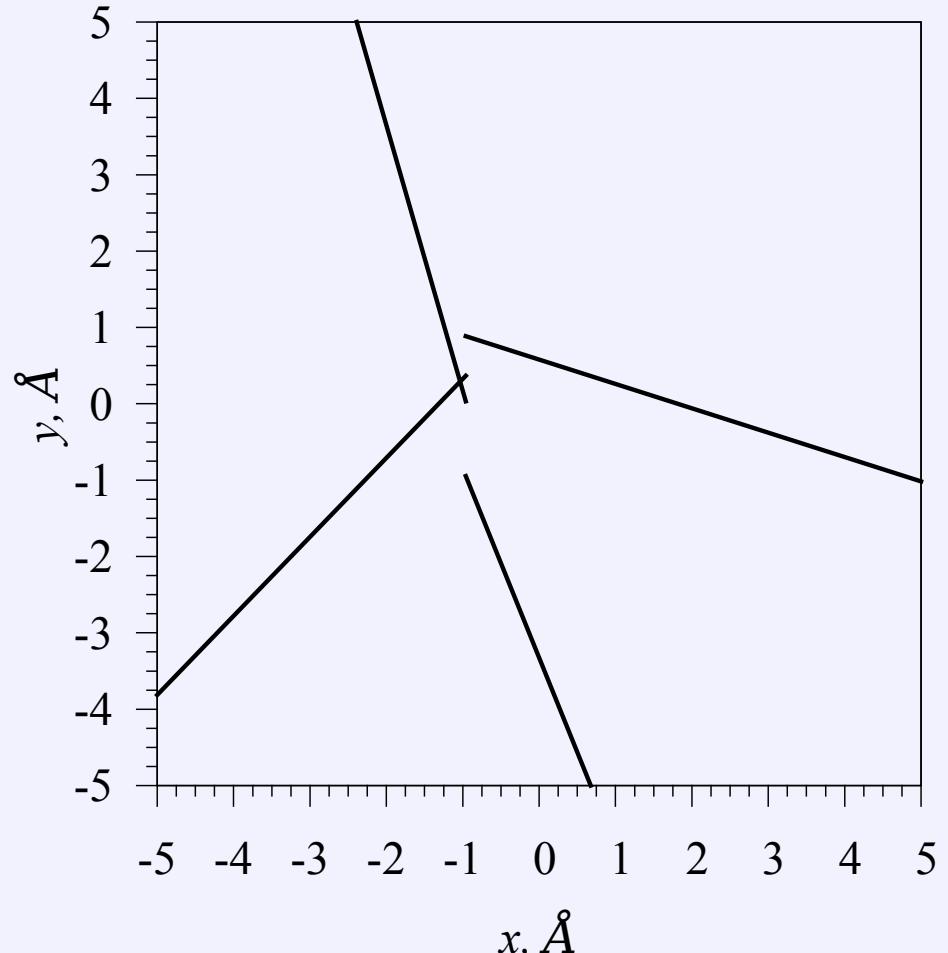
$$\varepsilon_{\perp} \ll U_{max}$$



$$E = 100 \text{ GeV}, \varepsilon_{\perp} = 1.25 \text{ eV}, l_0 = 0.01 \text{ cm}, U_{max} = 89.7 \text{ eV}$$

# Particle motion in RSA

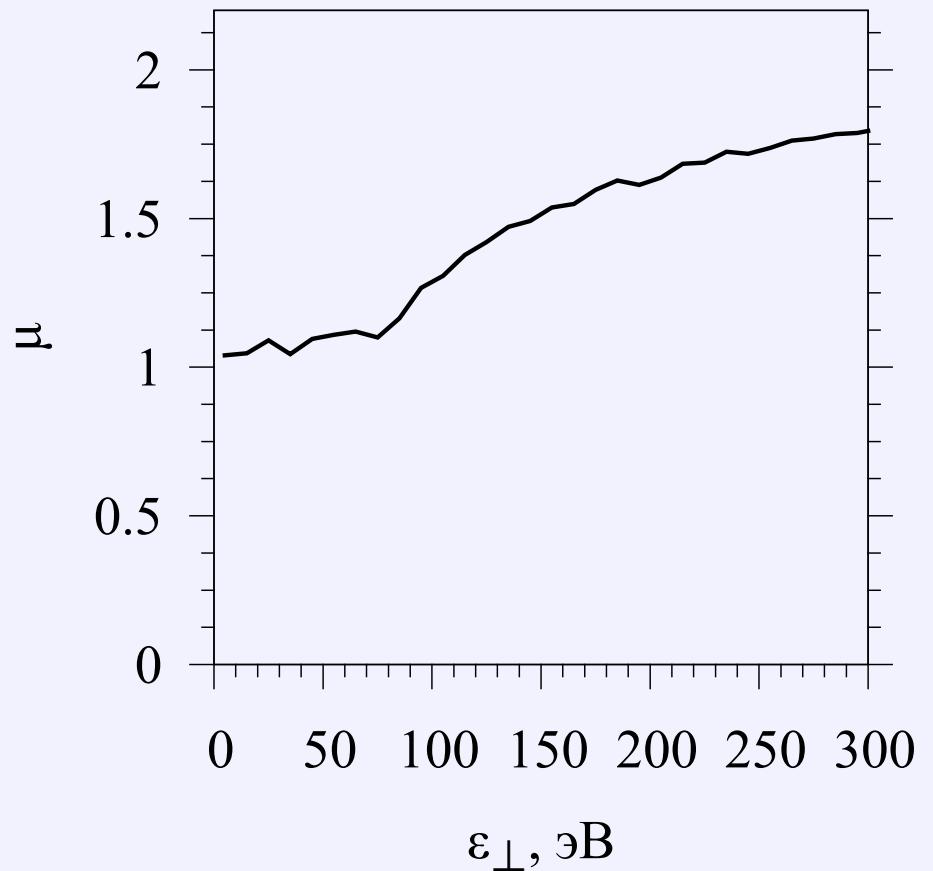
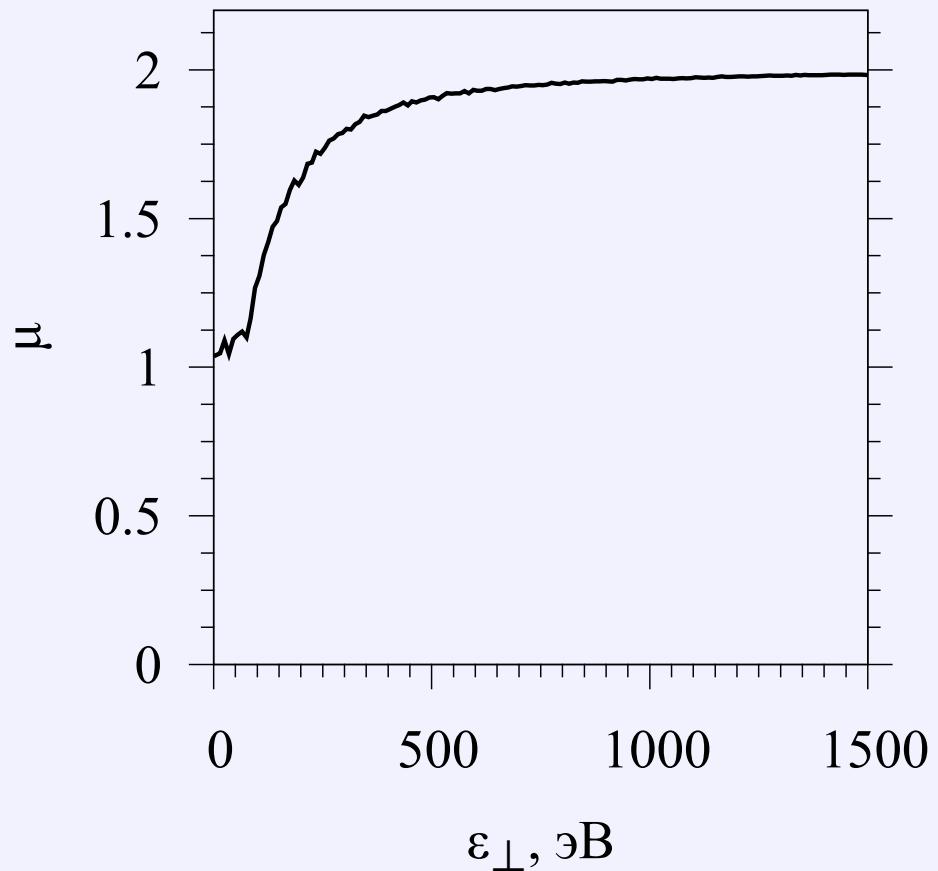
$$\varepsilon_{\perp} \gg U_{max}$$



$$E = 100 \text{ GeV}, \varepsilon_{\perp} = 897 \text{ eV}, l_0 = 0.01 \text{ cm}, U_{max} = 89.7 \text{ eV}$$

# Particle motion in RSA

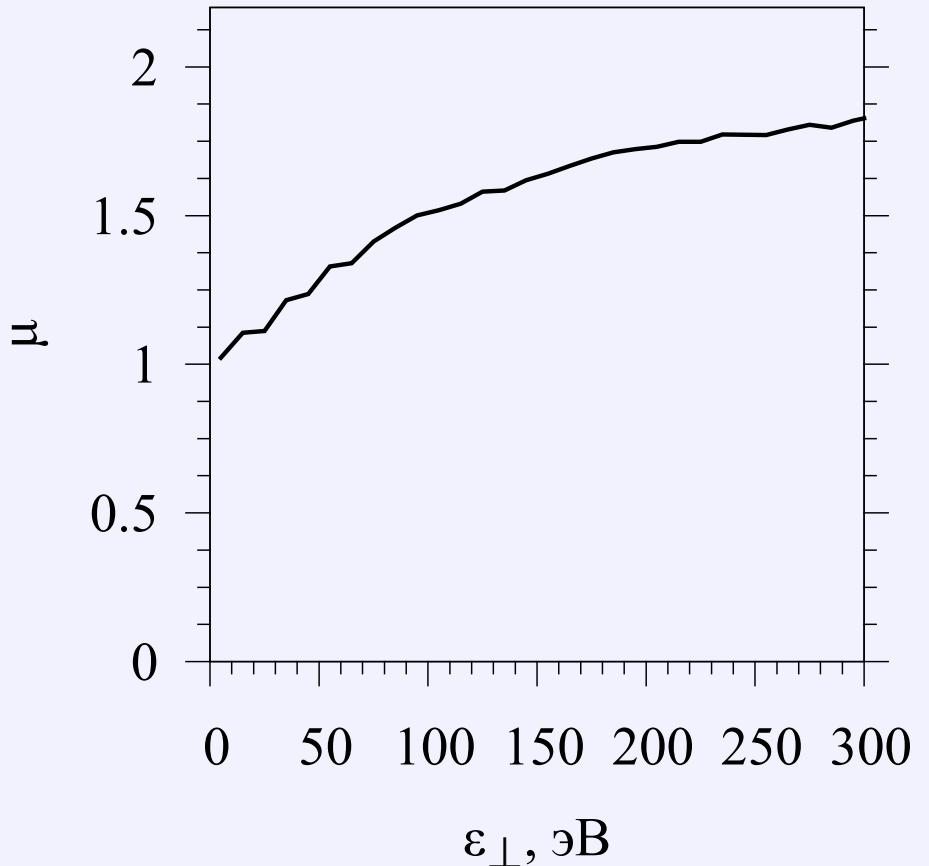
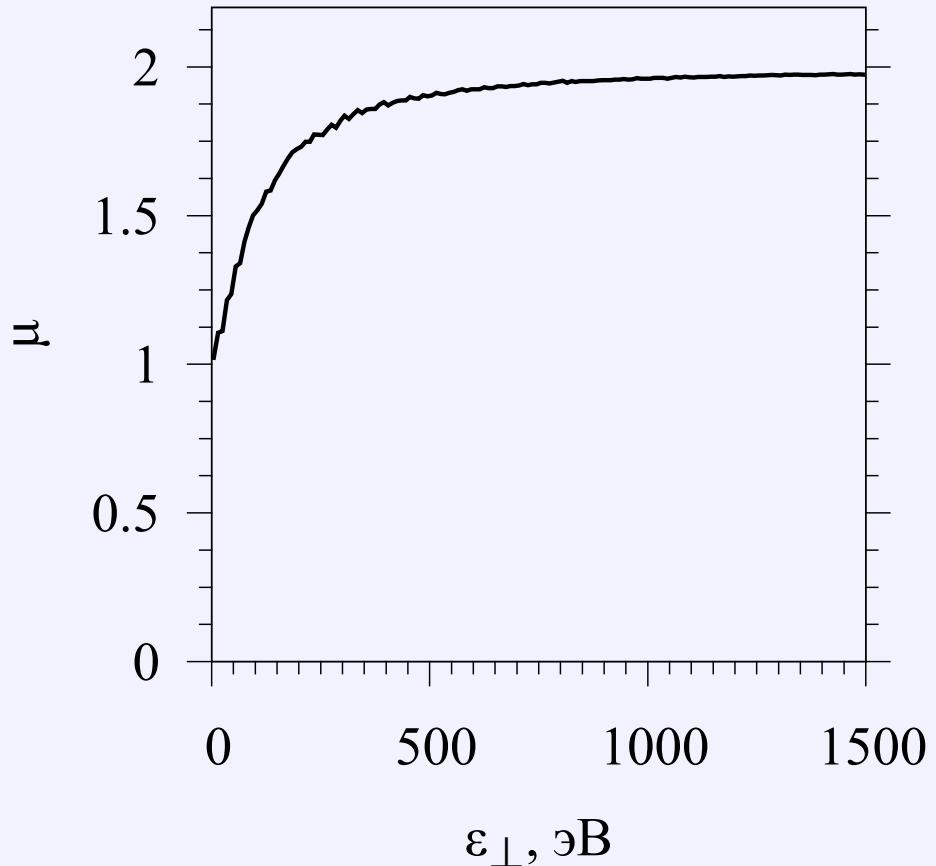
protons



$E = 100 \text{ GeV}$ ,  $l_0 = 0.01 \text{ cm}$

# Particle motion in RSA

$\pi^-$ -mesons



$E = 100$  GeV,  $l_0 = 0.01$  cm

# Particle motion in crystal

## Crystal potential

$$\text{if } \Phi_a(r) = \frac{Z|e|}{r} \exp(-r/R) \quad \text{then} \quad \Phi_s(\rho) = \frac{2Z|e|}{d} K_0(\rho/R),$$

so summation  $\sum_n U_R(\vec{\rho} - \vec{\rho}_n)$  couldn't be done analytically.

However, if we use Doyle-Terner approximation

$$\Phi_a(r) = \frac{2\pi\hbar^2}{|e|m_e} \sum_{i=1}^4 \alpha_i \left( \frac{4\pi}{\beta_i + B} \right)^{3/2} \exp\left(-\frac{4\pi^2 r^2}{\beta_i + B}\right) \quad \text{then}$$

$$\Phi_s(\rho) = \frac{1}{d_a} \int_{-\infty}^{\infty} dz \Phi_a(\rho, z) = \frac{8\pi^2\hbar^2}{|e|m_e d_a} \sum_{i=1}^4 \frac{\alpha_i}{\beta_i + B} \exp\left(-\frac{4\pi^2 \rho^2}{\beta_i + B}\right)$$

and summation could be done analytically

# Particle motion in crystal

## Crystal potential

for Si  $\langle 100 \rangle$  crystal axis

$$\begin{aligned}\langle \Phi_{\langle 100 \rangle}(\vec{\rho}) \rangle &= \sum_{n=-\infty}^{\infty} \Phi_s(\vec{\rho} - \vec{\rho}_n) = \\ &= \frac{2\pi\hbar^2}{|e|m_e d_a d_s^2} \sum_{i=1}^4 \alpha_i \theta_3 \left( -\frac{x}{d_s} \left| \frac{i(\beta_i + B)}{4\pi d_s^2} \right. \right) \theta_3 \left( -\frac{y}{d_s} \left| \frac{i(\beta_i + B)}{4\pi d_s^2} \right. \right),\end{aligned}$$

where

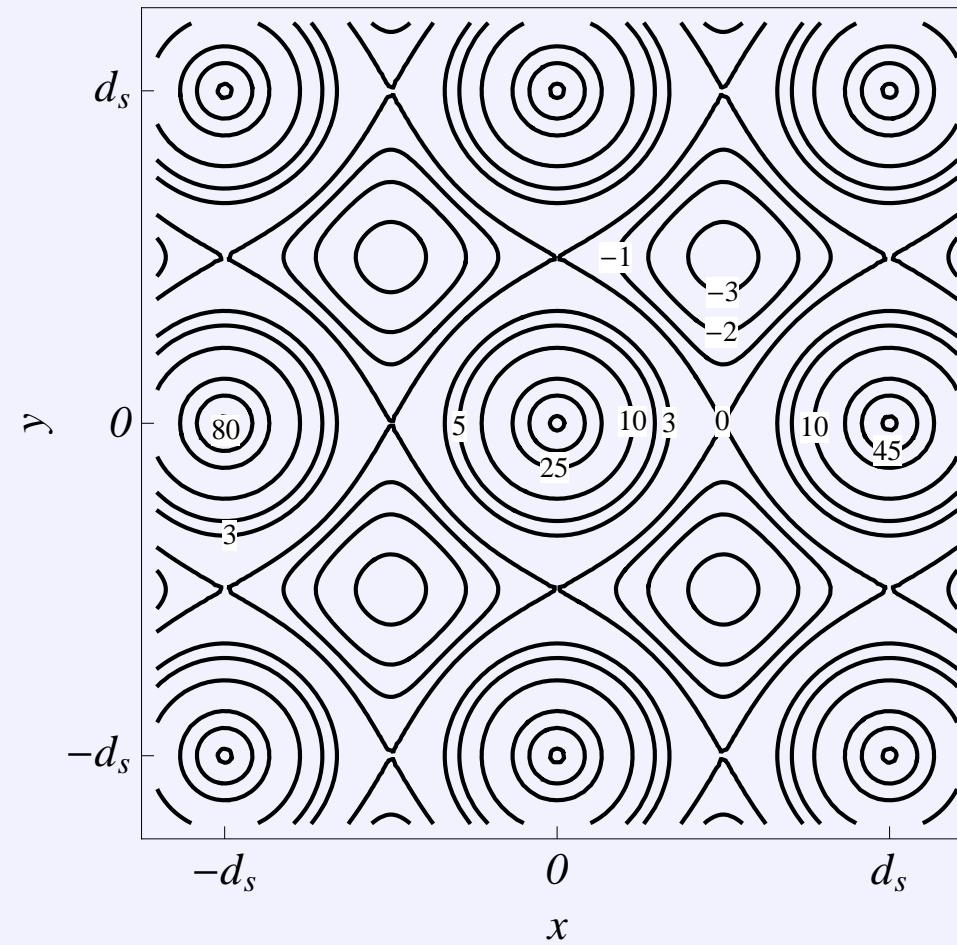
$d_s$  — distance between neighboring crystal atomic strings,

$$\theta_3(v|w) = \sum_{n=-\infty}^{\infty} \exp(\pi i w n^2) \exp(2\pi i v n) — \text{Jacobi theta function of the third type}$$

# Particle motion in crystal

## Crystal potential

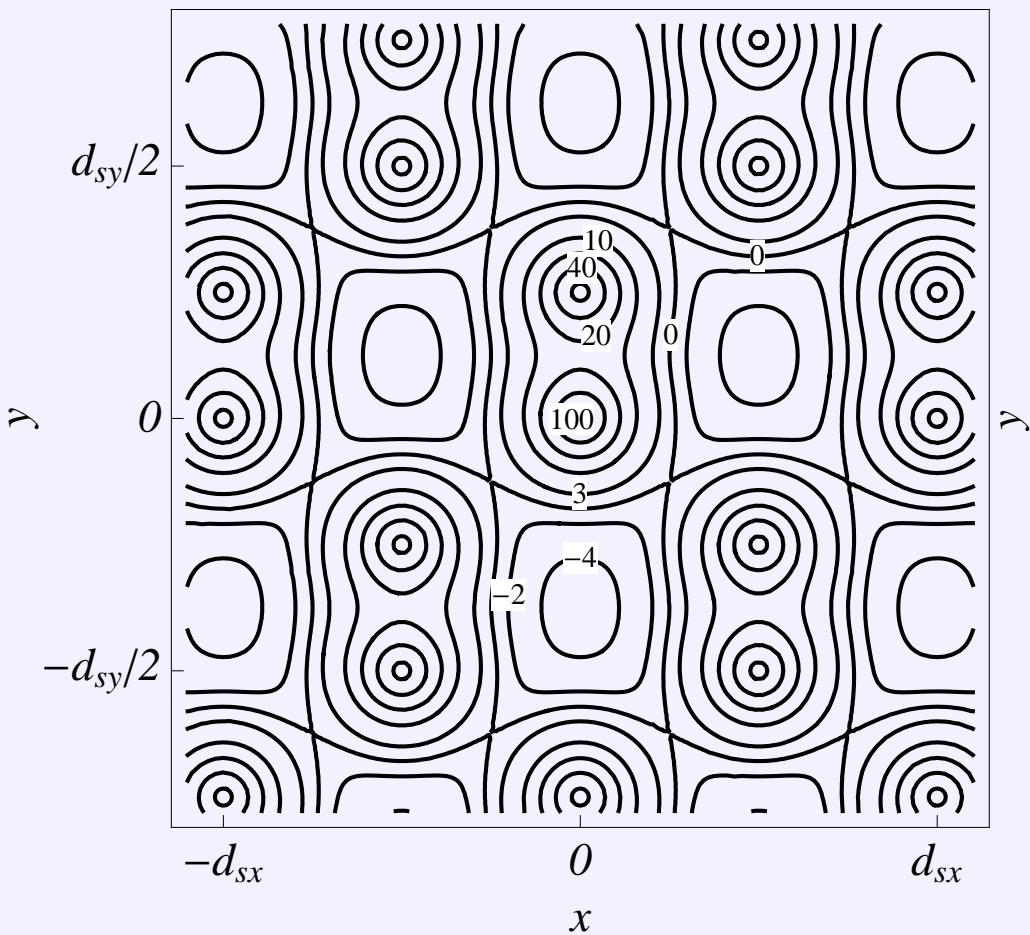
Si  $\langle 100 \rangle$



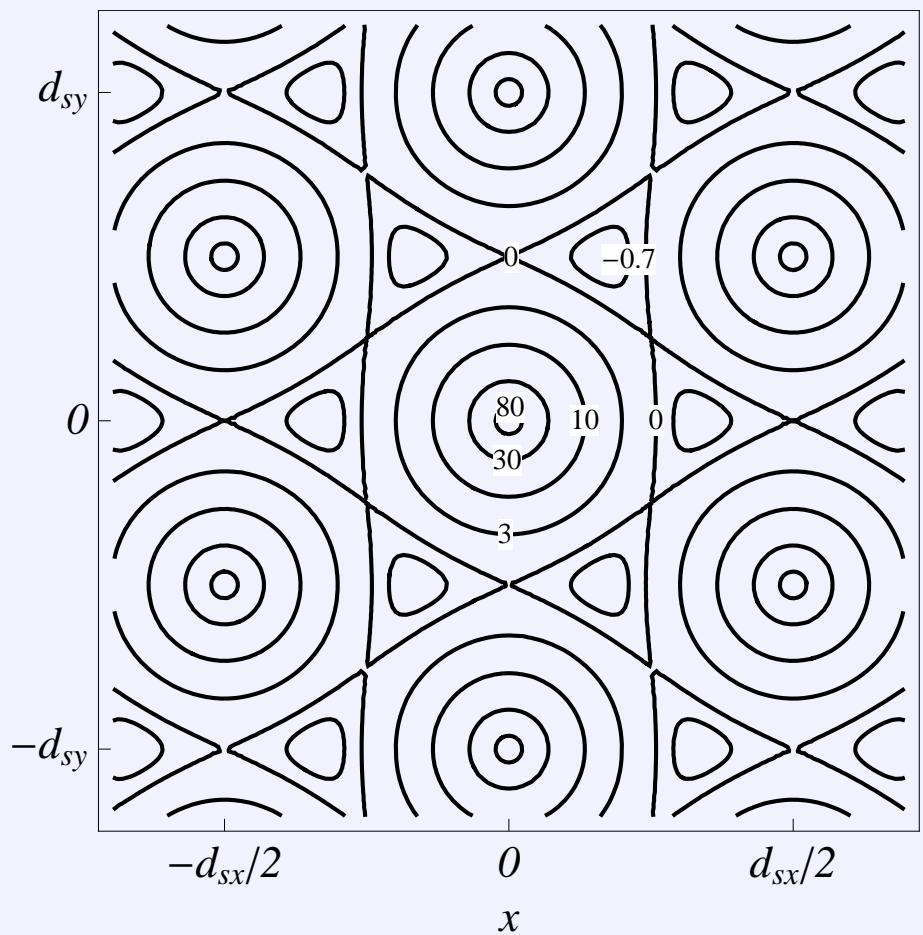
# Particle motion in crystal

## Crystal potential

Si  $\langle 110 \rangle$

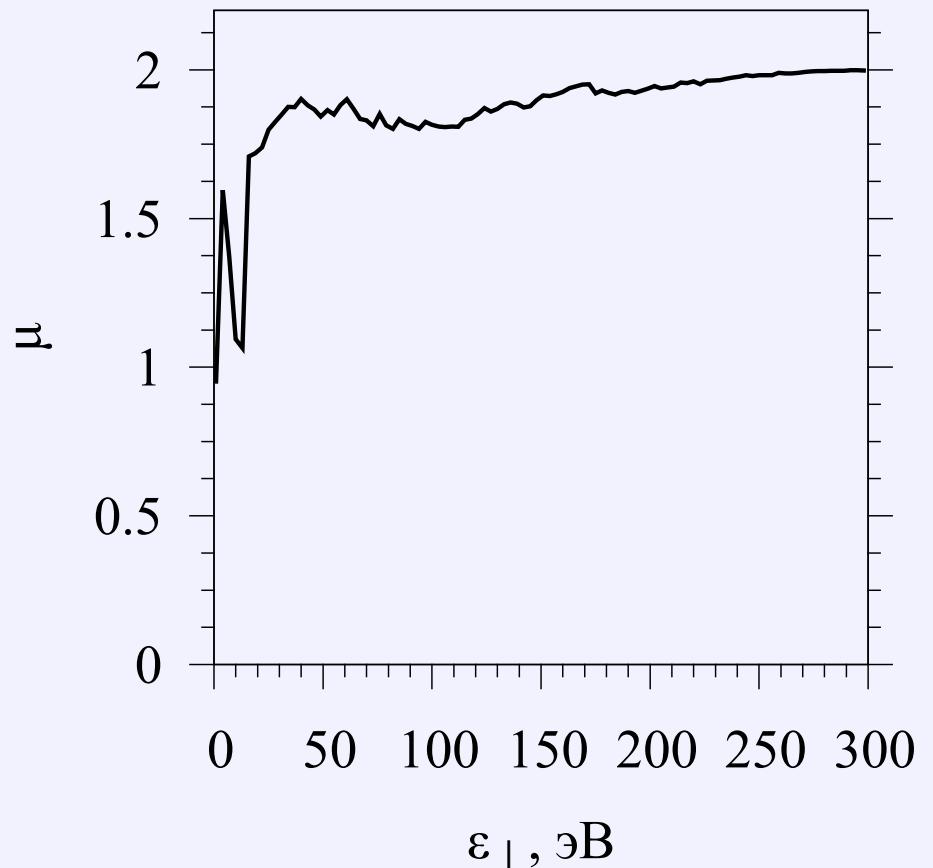
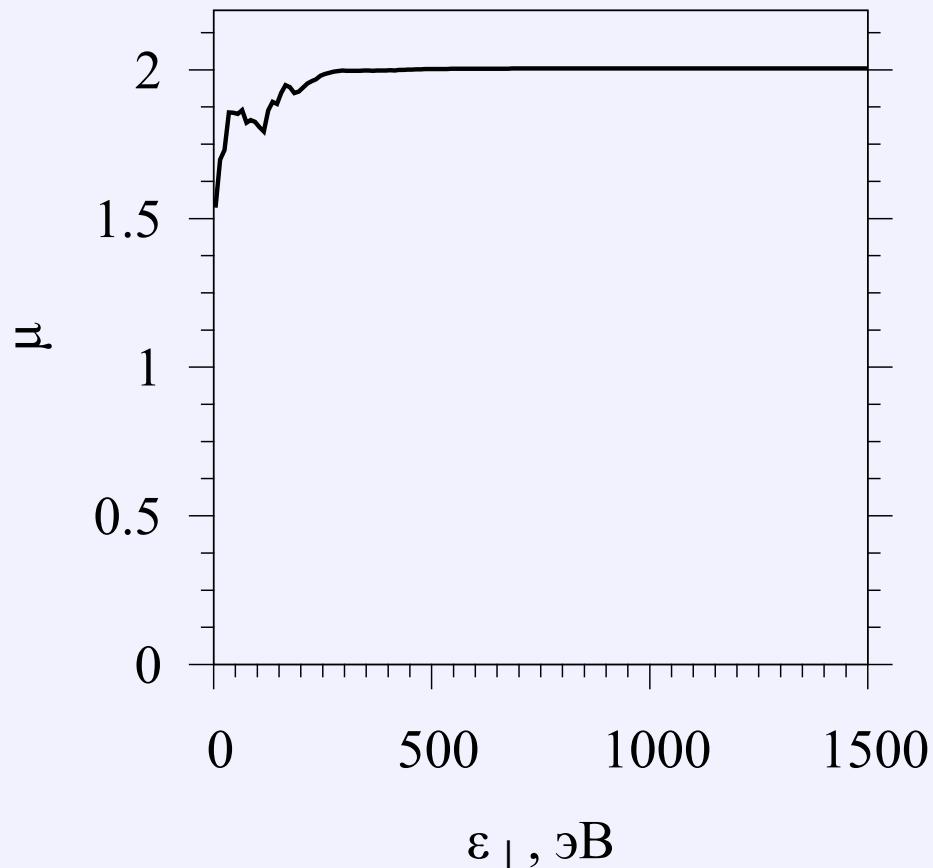


Si  $\langle 111 \rangle$



# Particle motion in crystal

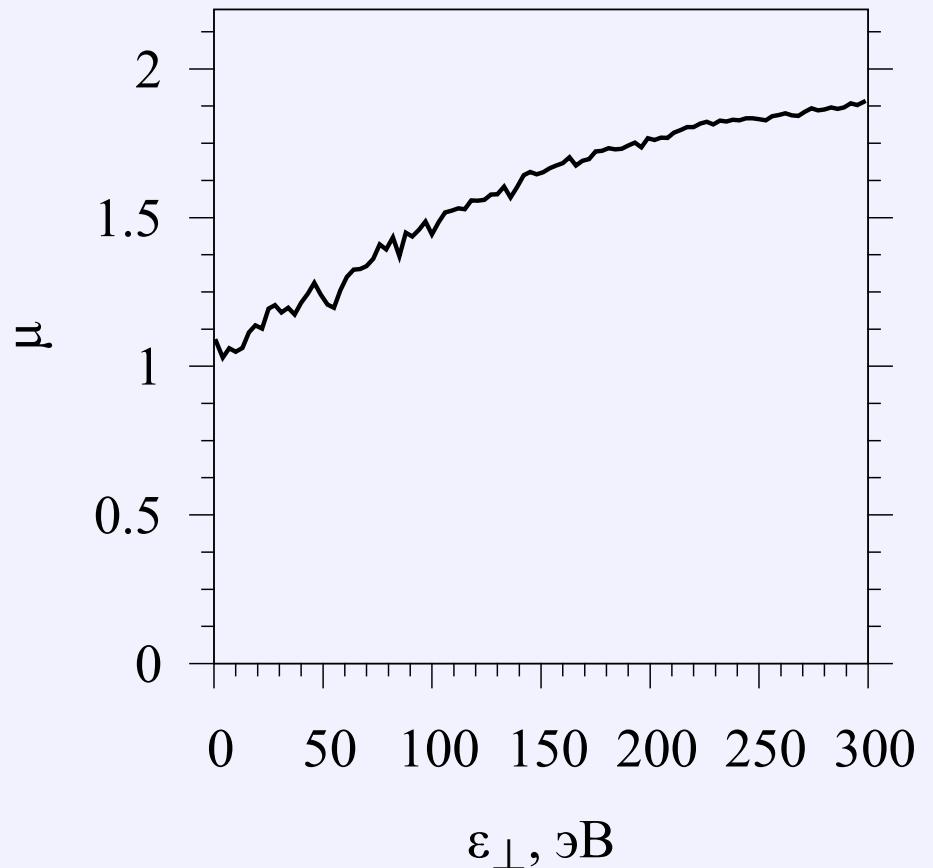
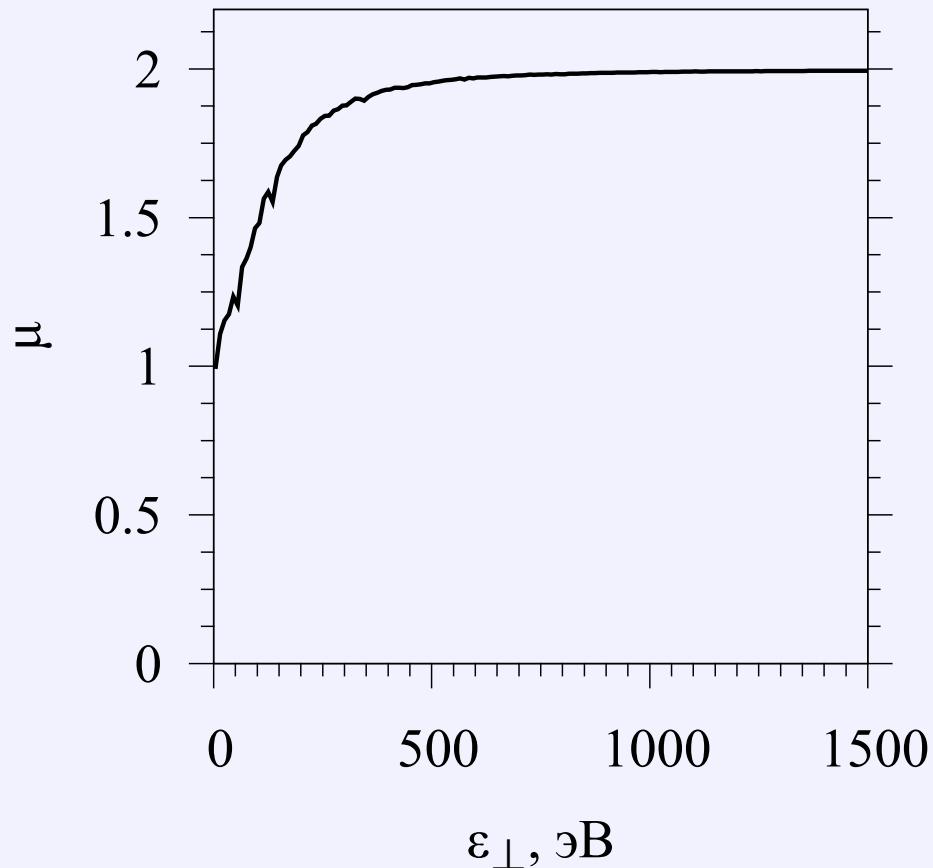
protons



$E = 100 \text{ GeV}$ ,  $l_0 = 0.01 \text{ cm}$

# Particle motion in crystal

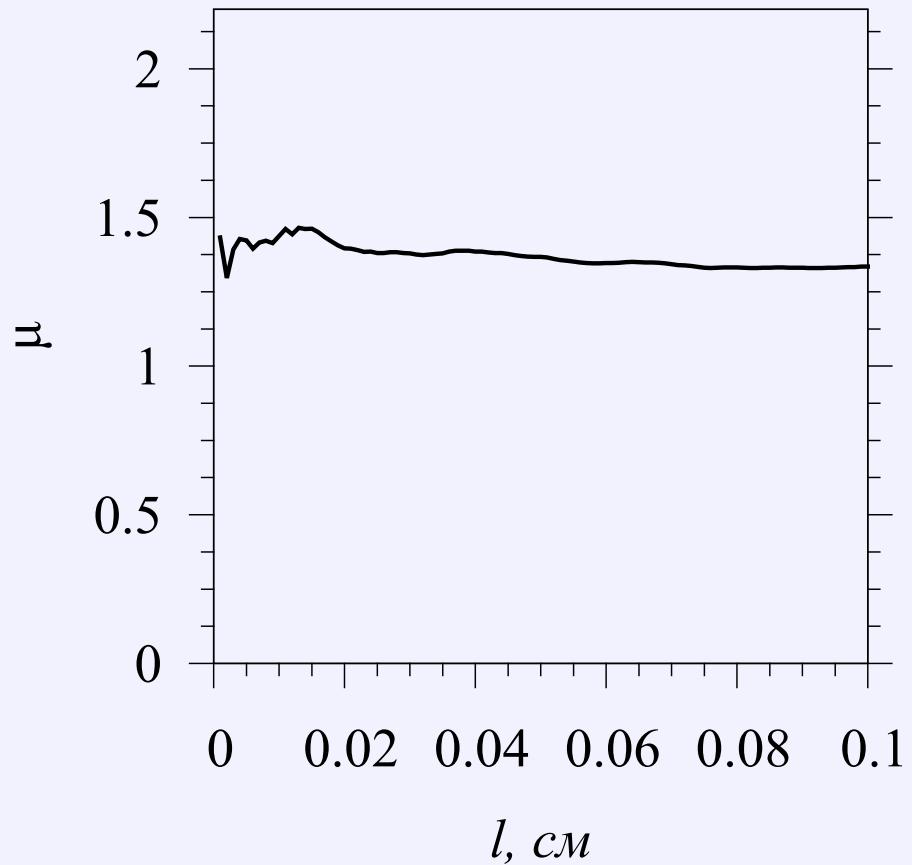
$\pi^-$ -mesons



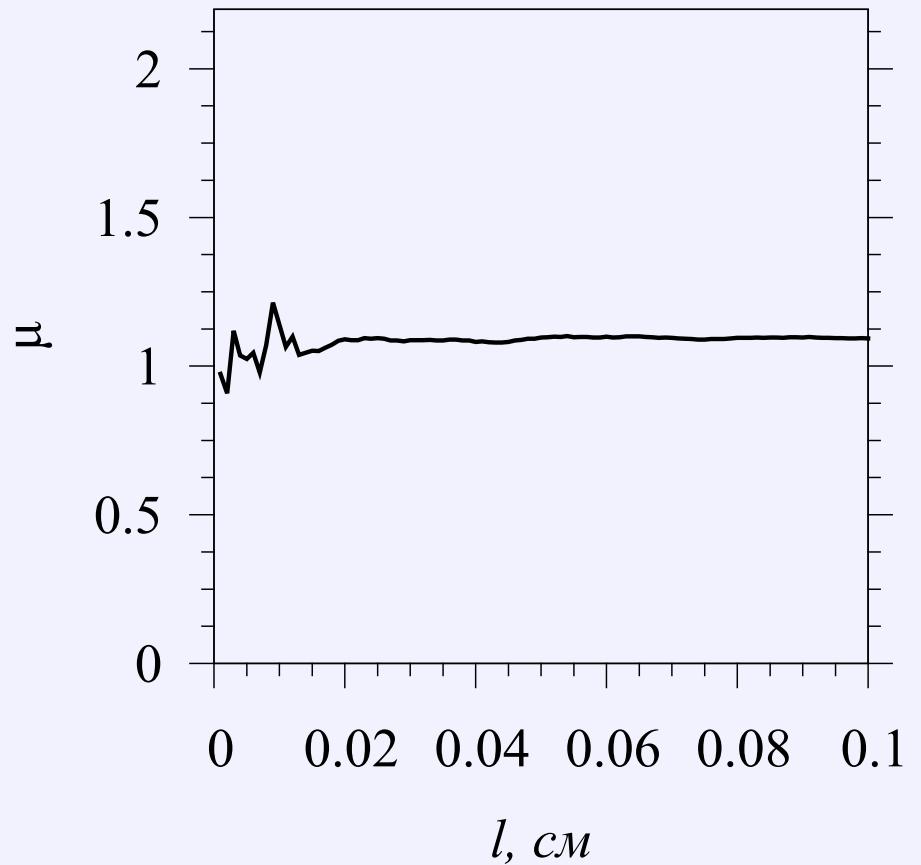
$E = 100$  GeV,  $l_0 = 0.01$  cm

# *Particle motion in crystal*

*protons*



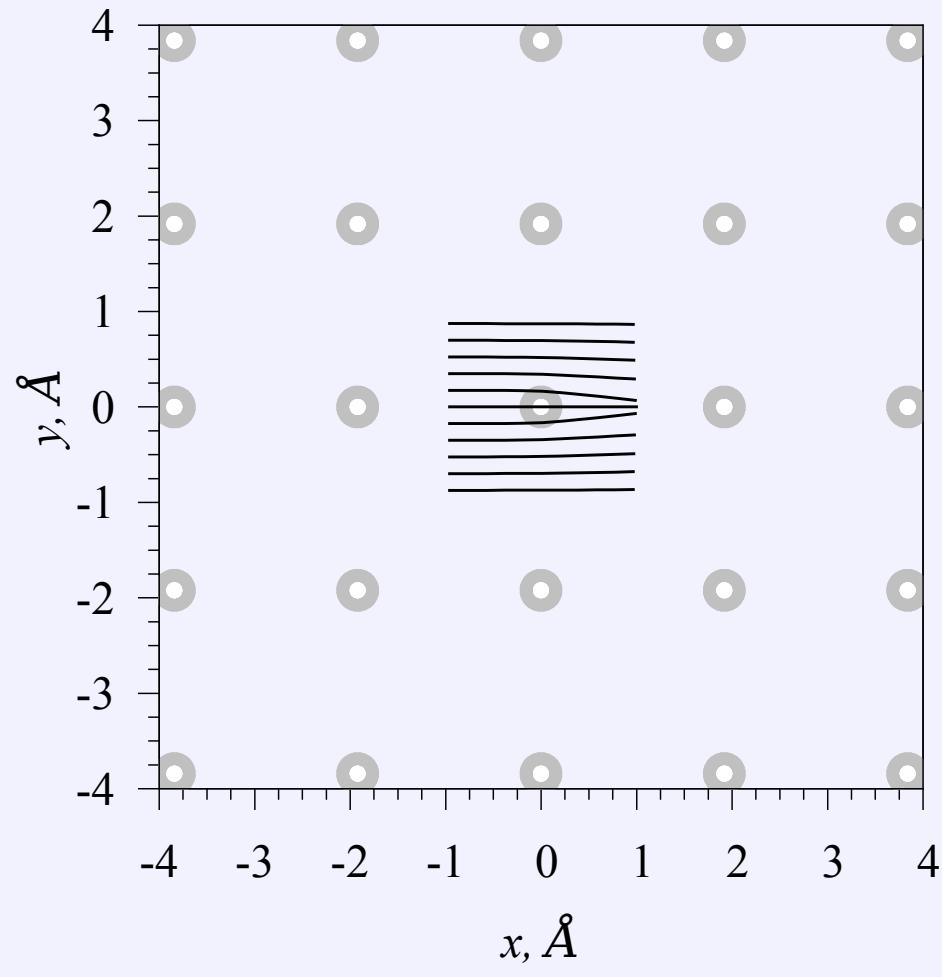
$\pi^-$ -mesons



$$E = 100 \text{ GeV}, l_0 = 0.01 \text{ cm}$$

# Rainbow scattering

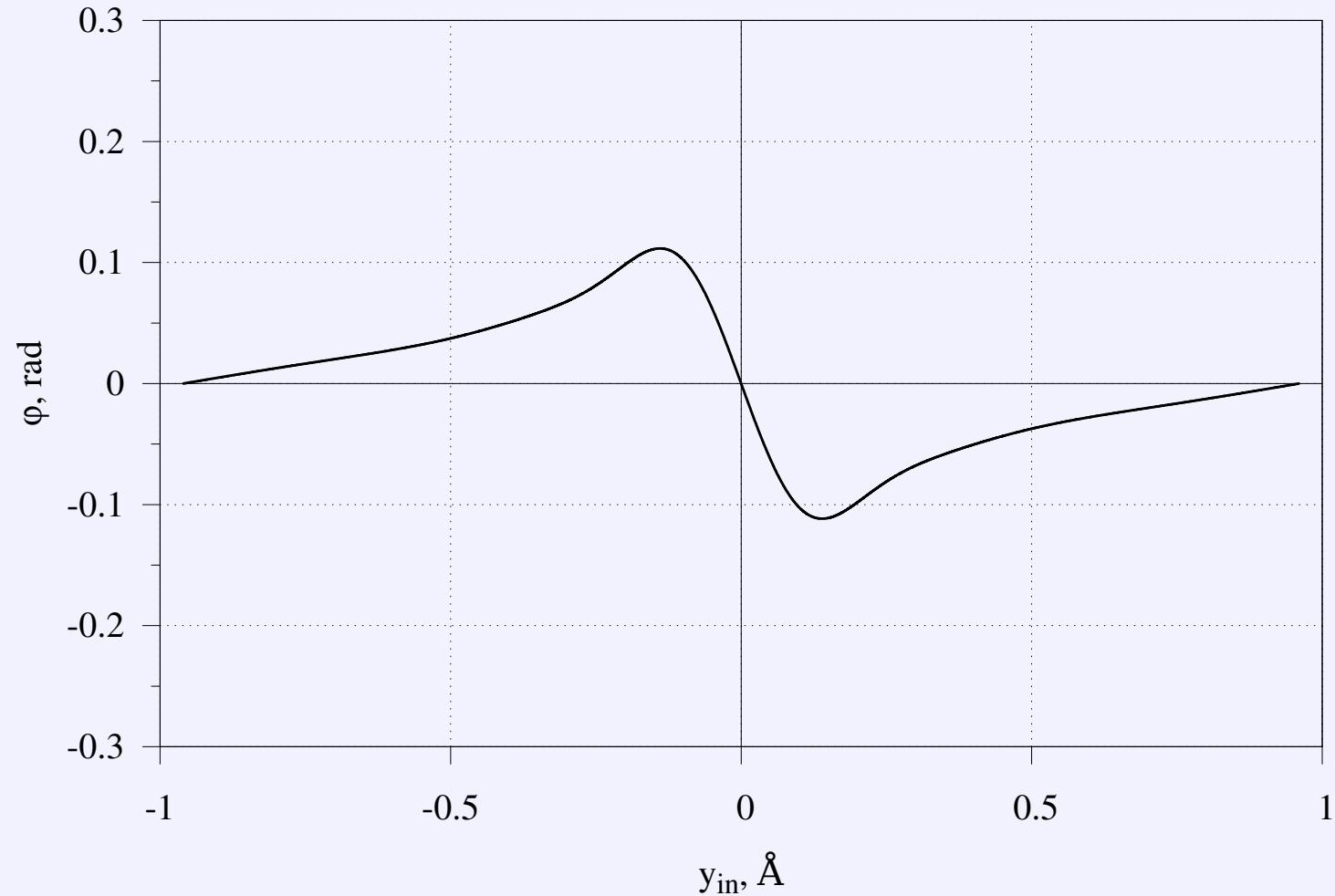
After several discussions with Prof. Guidi, Andrea Mazzolari and Laura Bandiera...



$e^-$ ,  $E = 1 \text{ GeV}$ ,  $v_y^{in}=0$ ,  $v_x^{in} = v \sin(d_s/l)$

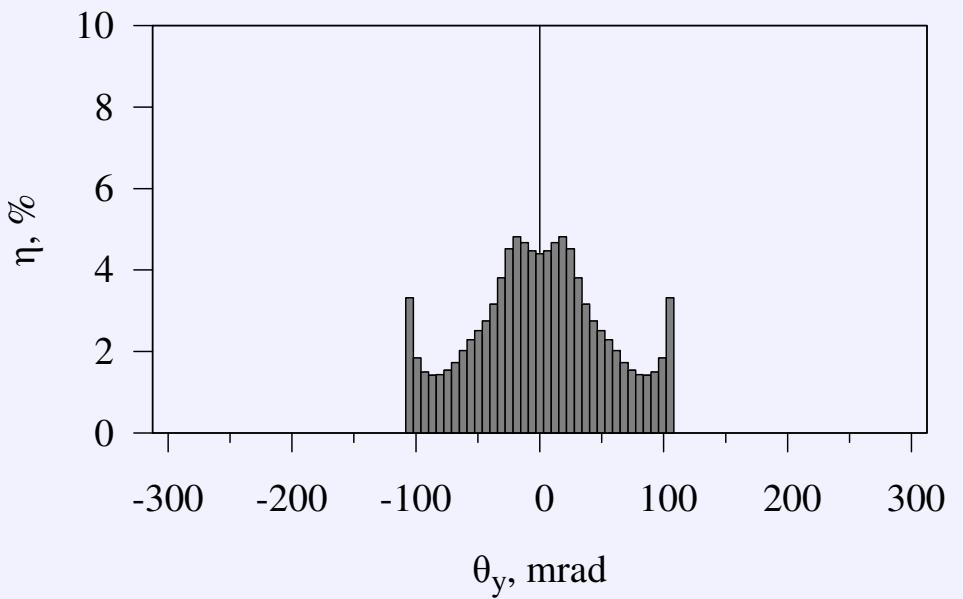
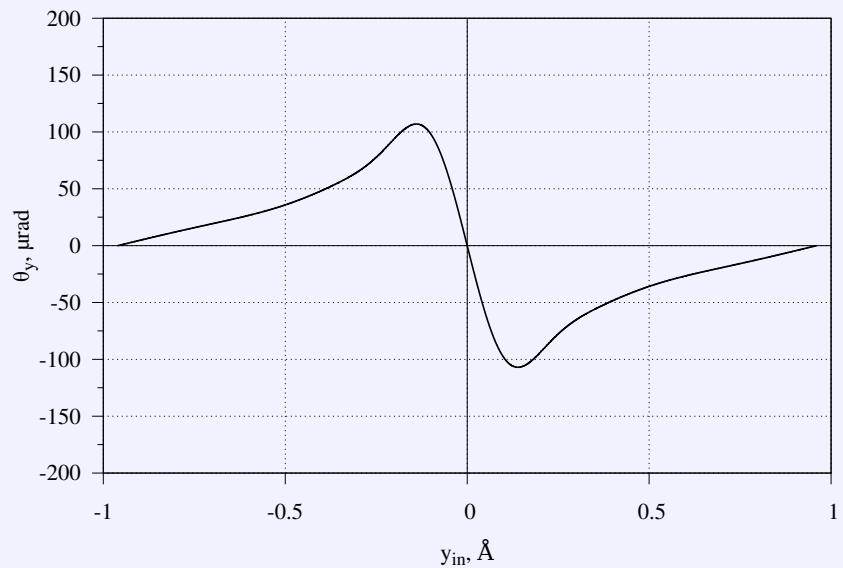
## Rainbow scattering

$$\phi(y_{in}) = \pi - 2y_{in} \int_{r_{min}}^{\infty} \frac{d\rho}{\rho^2 \sqrt{1 - \frac{U_s(\rho)}{\varepsilon_{\perp}} - \frac{b^2}{\rho^2}}}$$



$e^-$ ,  $E = 1$  GeV,  $v_y^{in}=0$ ,  $v_x^{in} = v \sin(d_s/l)$

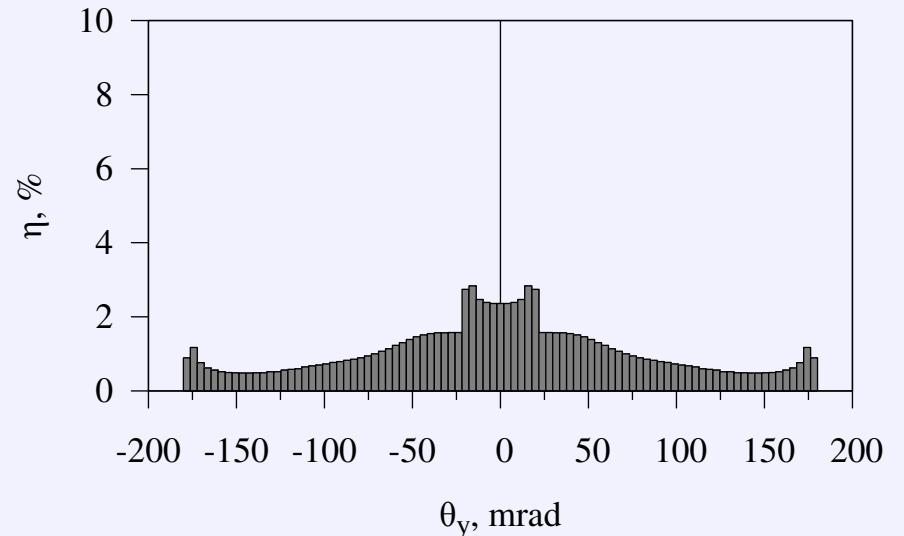
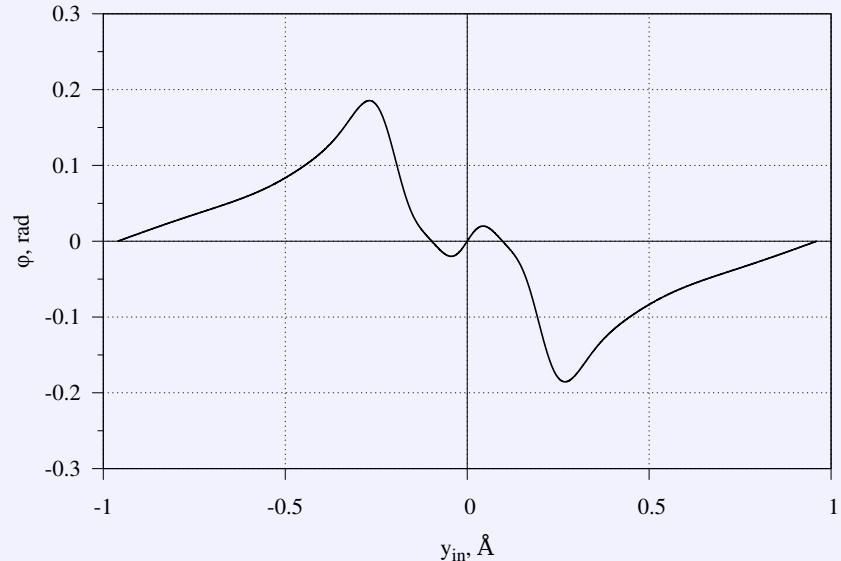
# Rainbow scattering



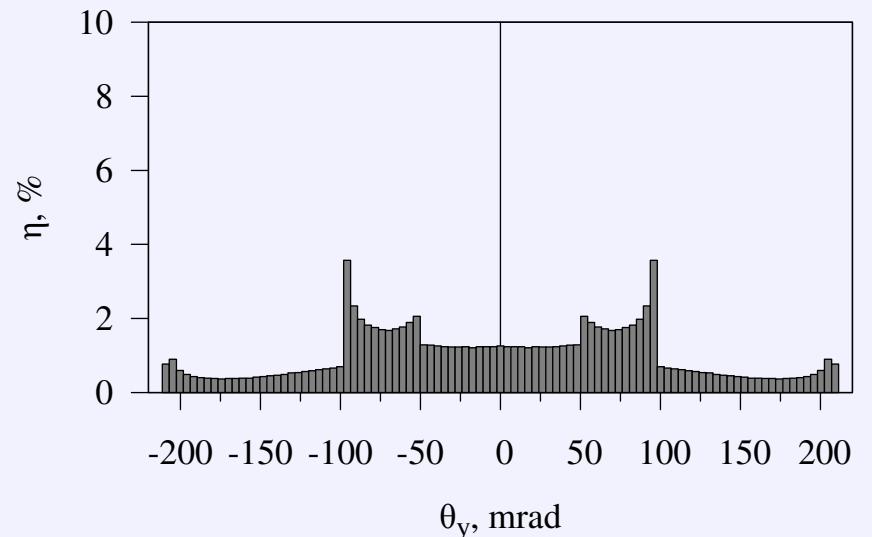
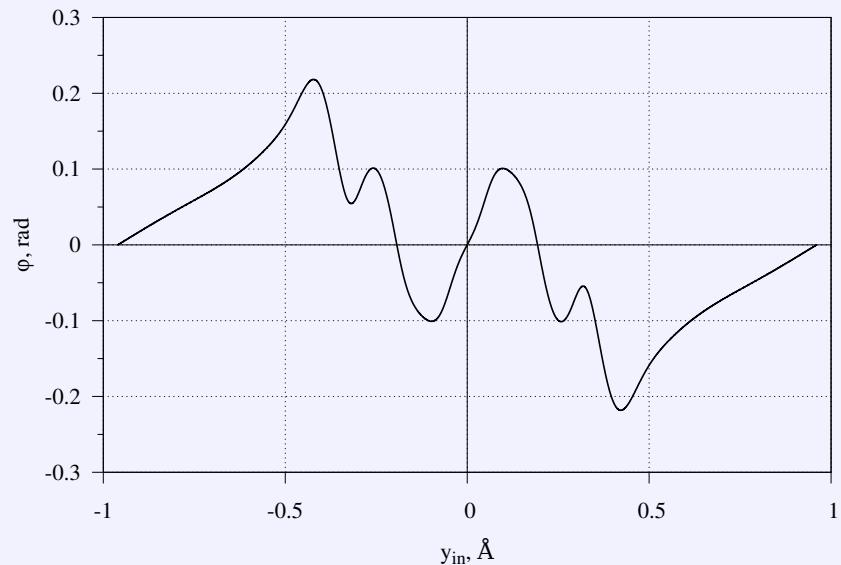
$e^-$ ,  $E = 1$  GeV,  $v_y^{in}=0$ ,  $v_x^{in} = v \sin(d_s/l)$

## Rainbow scattering

*the same conditions, but two times larger crystal thickness*

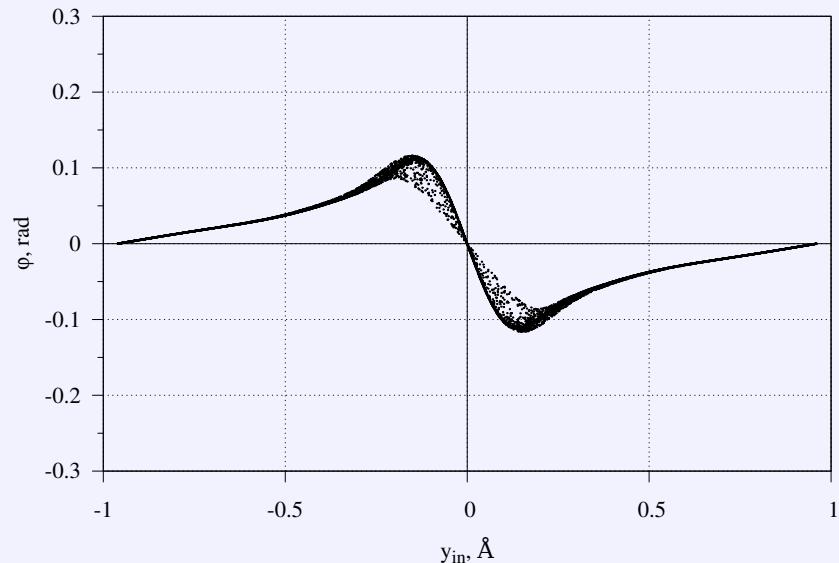


*the same conditions, but three times larger crystal thickness*

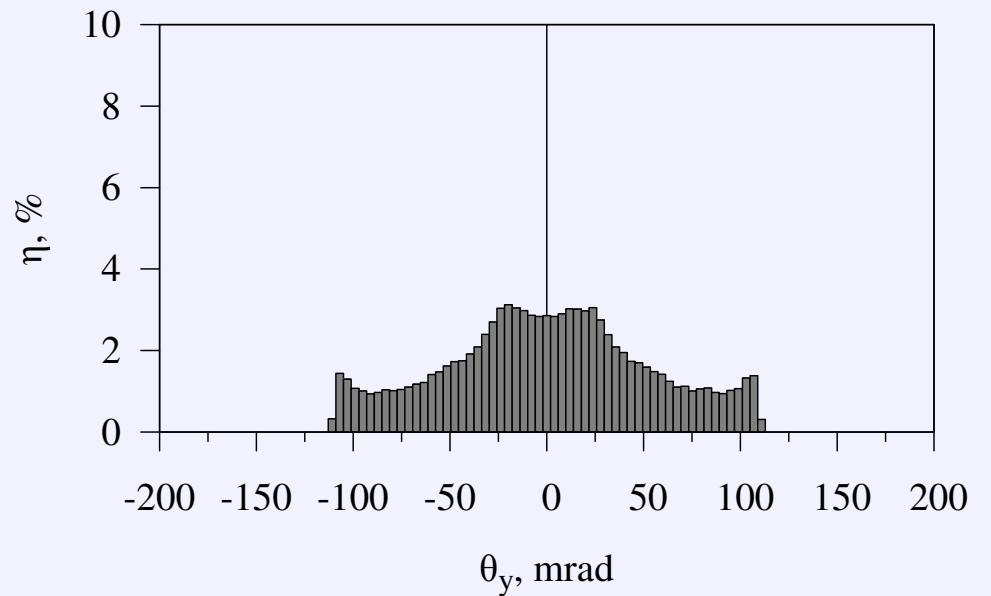


# Rainbow scattering

*electrons scattering on one atomic string for the beam that impinges on the crystal uniformly filling elementary cell in the  $(x, y)$  plane*

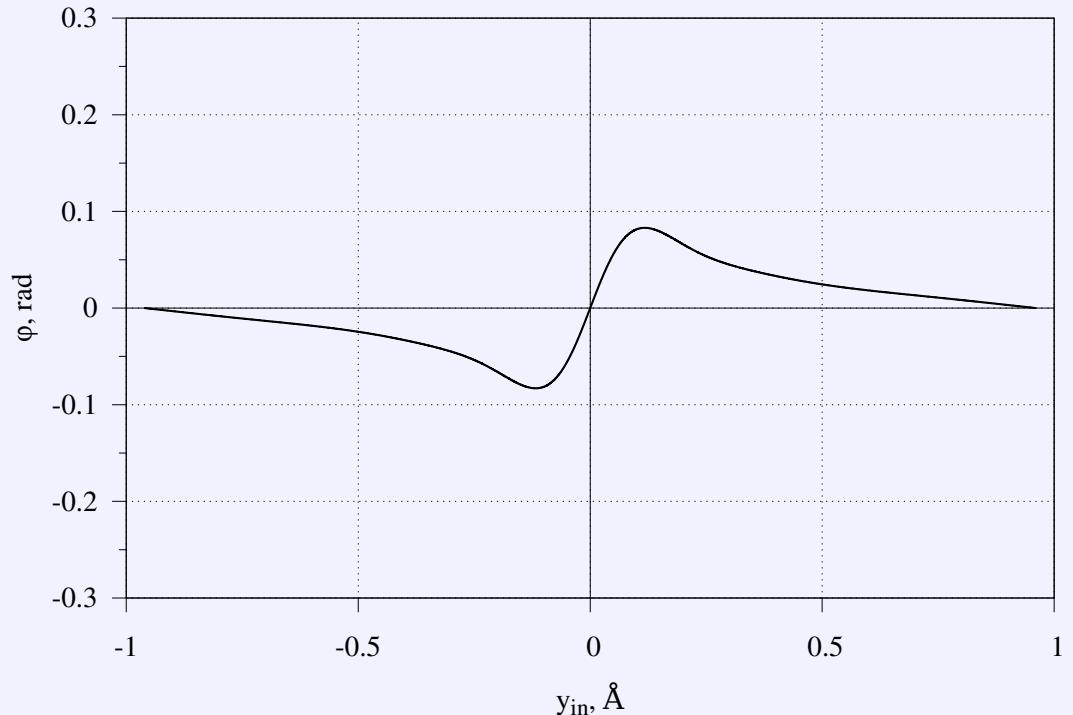
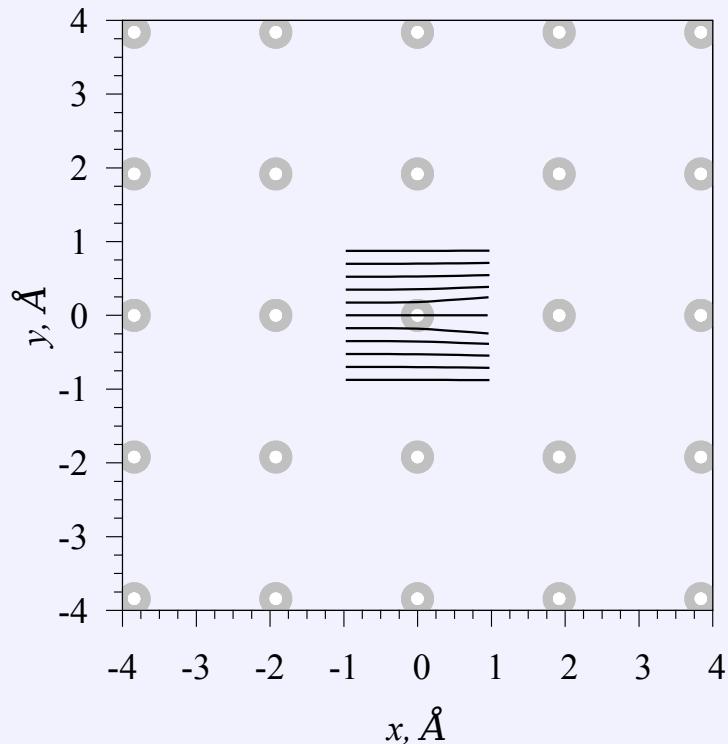


$e^-$ ,  $E = 1$  GeV,  $v_y^{in}=0$ ,  $v_x^{in} = v \sin(d_s/l)$



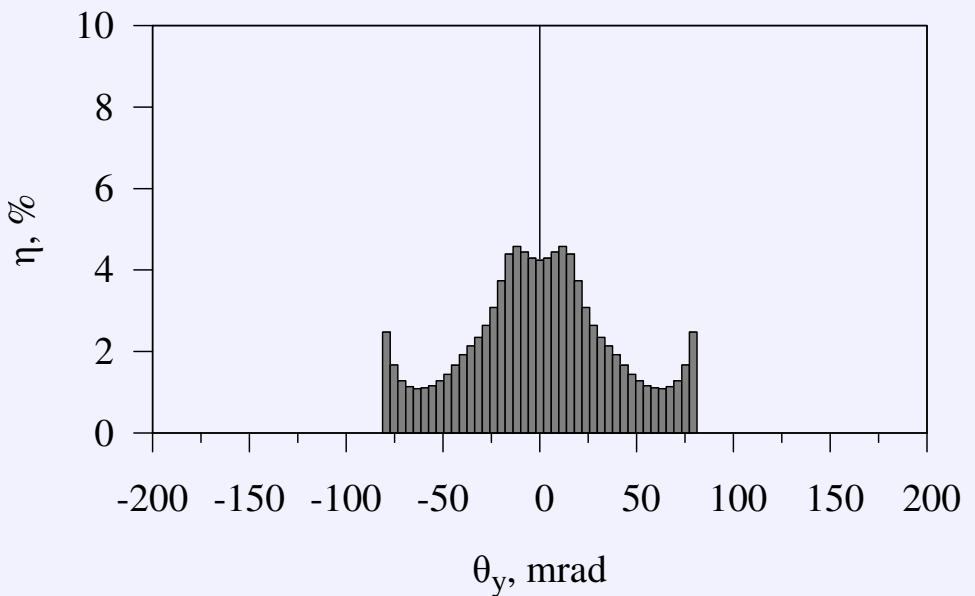
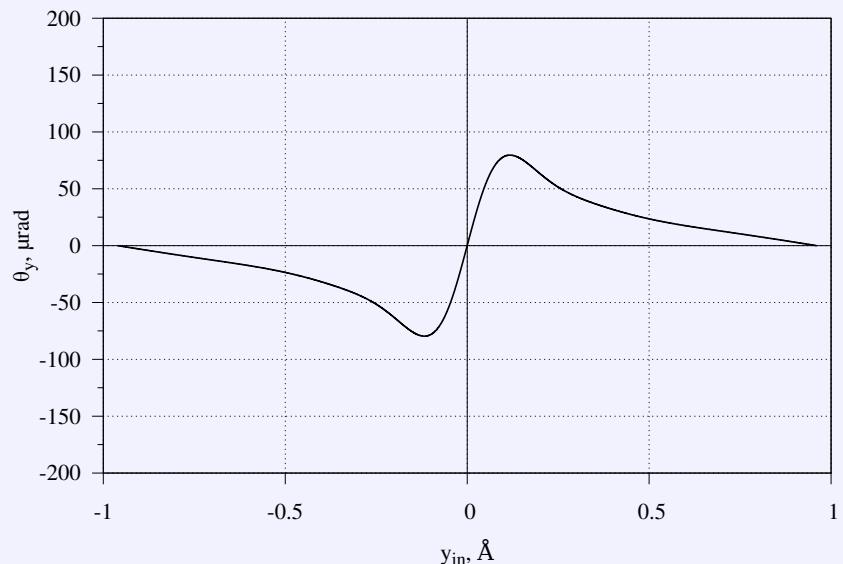
# Rainbow scattering

protons



$$p, E = 1 \text{ GeV}, v_y^{in}=0, v_x^{in} = v \sin(d_s/l)$$

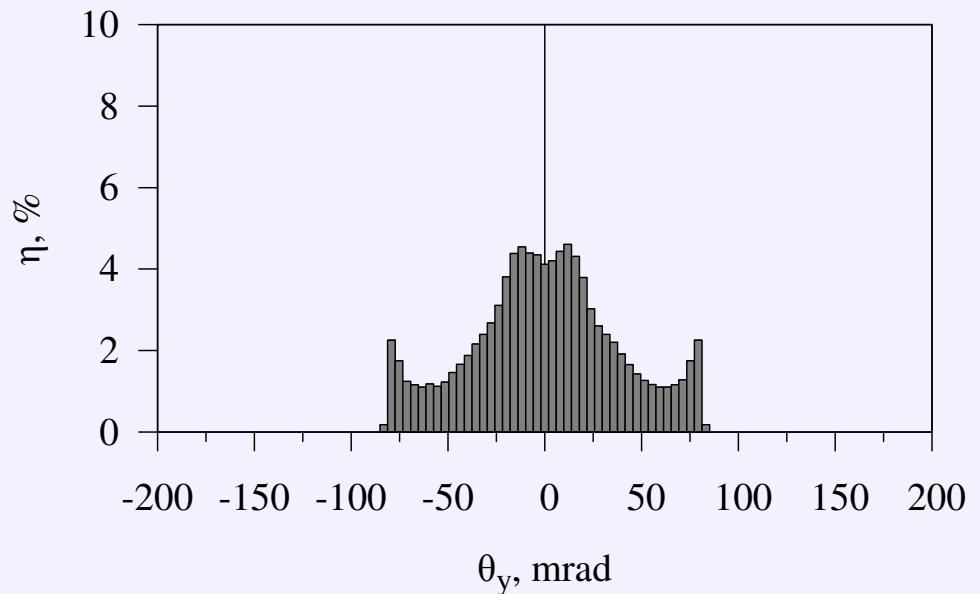
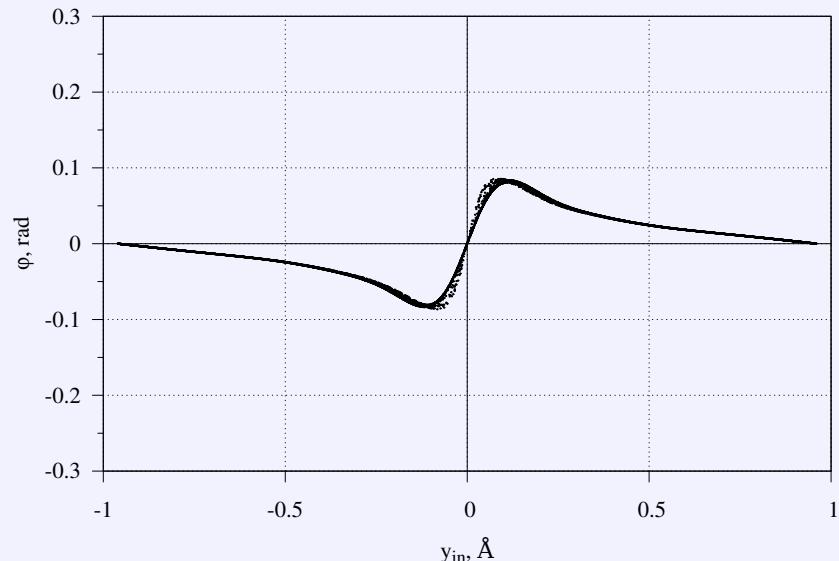
# Rainbow scattering



$p, E = 1 \text{ GeV}, v_y^{in}=0, v_x^{in} = v \sin(d_s/l)$

# Rainbow scattering

*protons scattering on one atomic string for the beam that impinges on the crystal uniformly filling elementary cell in the  $(x, y)$  plane*



$$p, E = 1 \text{ GeV}, v_y^{in}=0, v_x^{in} = v \sin(d_s/l)$$

*THANK YOU FOR ATTENTION!*