The energy losses of a positron in the crystalline Wiggler

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1. Introduction

Crystalline undulator radiation (CUR) is formed planar channeled relativistic positron between periodically curved crystallographic planes, what was predicted in the work [1]. Both amplitude and spatial period of the crystalline undulator are much more than amplitude and the spatial period of the positron oscillation.

In the work [2] the problem of CUR considered with taking into account the polarization of the medium, and is shown that besides the lower threshold for the energy occurs as the upper threshold for the amplitude of crystalline undulator: $A_{thr}(n) = \lambda_p/\pi\sqrt{2}n$,where λ_p is the plasma wavelength of the crystal, n is the harmonic number.

Specified in the work [1] limit on the amplitude corresponds to the first harmonic of radiation. CUR was theoretically investigated in many papers [3-5]. The works [6,7] show, using dense positron beams, one can produce stimulated CUR (SCUR). In this paper we investigate the problem of the positron radiation in the crystalline Wiggler (CW).

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2. Restrictions on positron energy and Wiggler parameters

Wiggler unlike undulator is characterized by a large value of the parameter $q = \beta_\perp \gamma$, where γ is the Lorentz factor of a positron, $\beta_\perp = 2\pi A/l$ is maximum deflection angle of CW sine-plane (A is amplitude and l is spatial period of CW). From the condition $q \gg 1$, it is followed that bottom threshold of a positron energy $\beta_\perp{}^{-1} \ll \gamma$. On the other hand, special period l of CW much more than special period $l_{ch} = \pi d\sqrt{\gamma}/\nu$ of channeled positrons, where d is interplanar distance, parameter $\nu = \sqrt{2U_0/mc^2}$ characterizes the depth of the potential well U_0, mc^2 is the rest energy positron. So, energy positron is limited, and from below, and from above we have:

$$\beta_\perp{}^{-1} \ll \gamma \ll (l\nu/\pi d)^2. \qquad (1)$$

Spatial period of the Wiggler is bounded from above with the condition $pl \le L_D$, where L_D is the dechanneling length, and p $(p \gg 1)$ is the period number of CW.

As it is known, average continuous field in the planar channel is well enough described by a parabolic potential. Then holding force, acting on a positron, equal to $4U_0/d$ and must be greater than the centrifugal force $mc^2\gamma/R_{min}$, where $R_{min} = l/2\pi\beta_\perp$ is the minimum radius of curvature planes, when the curvature describes the harmonic function. Taking into account of bottom threshold on γ (1) we have the following limitation for the spatial period CW.

$$\frac{\pi d}{\nu^2} \ll l \le \frac{L_D}{p} \qquad (2)$$

3. Total spectrum of high harmonic

For frequency-angular distribution of the number of photons n-th harmonic radiation per unit path we have

$$\frac{dN_n}{dx du^2 d\Phi dz} = \frac{\alpha}{l}\left[\left(u - \frac{n}{ux}\right)^2 + \left(\frac{n\tan\Phi}{ux}\right)^2\right] J_n{}^2(qux\cos\Phi)\delta\big(u^2 - \varphi_n(x)\big) \ , \quad (3)$$

where α is the fine structure constant, $u = \gamma\vartheta$, ϑ and Φ is the polar and azimuthal angles of radiation, $x = \omega/\Omega\gamma^2$ is a dimensionless frequency, $\Omega = 2\pi c/l$ is the oscillation frequency of the positron in CW, n is the harmonic number, J_n is a Bessel function of n-th order. Function $\varphi_n(x)$, which is included in the argument of the δ-function, is equal to

$$\varphi_n(x) = \frac{2n}{x} - Q - \frac{r^2}{x^2}, \quad (4)$$

where $Q = 1 + q^2/2,\ r = \gamma_p/\gamma\ ,\ \gamma_p = l/\lambda_p$.

The account of the polarization of the medium [2] leads to the following restriction on the frequency of the radiation:

$$\frac{n}{\alpha}\left(1 - \sqrt{1 - \frac{Qr^2}{n^2}}\right) < x < \frac{n}{\alpha}\left(1 + \sqrt{1 - \frac{Qr^2}{n^2}}\right). \quad (5)$$

Condition for the formation of radiation n-th harmonic:

$$\frac{Qr^2}{n^2} = \left(\frac{\gamma_{thr}}{\gamma}\right)^2 + \left(\frac{A}{A_{thr}}\right)^2, \quad (6)$$

where $\gamma_{thr} = \gamma_p/n$, $A_{thr} = A_1/n$, A_1 is the upper limit of the amplitude curvature for the first harmonic.

In CW provided that $A < A_{thr}$ we have $\gamma \gg \gamma_{thr}$. Consequently, unlike crystalline undulator, CW exists only on the amplitude threshold.

For the frequency distribution of the positron radiation from the path length nl after integration over the angle we get:

$$\frac{dN_n}{dxd\Phi} = \alpha p F_n(x) {J_n}^2(q\psi_n(x)\cos\Phi) \quad , \tag{7}$$

$$F_n(x) = \left(\varphi_n(x) - \frac{n}{x\varphi_n(x)}\right)^2 + \left(\frac{n\tan\Phi}{x\varphi_n(x)}\right)^2, \ \psi_n(x) = x\sqrt{\varphi_n(x)} \ .$$

Function $\psi_n(x)$ takes its maximum value at a frequency $x_n = n/Q$:

$$max\,\psi_n(x) = \psi_n\left(\frac{n}{Q}\right) = \frac{n}{\sqrt{Q}}\sqrt{1-\left(\frac{A}{A_{thr}}\right)^2} \ , \tag{8}$$

which with the condition $A \ll A_{thr}$ c with an accuracy of $max\left\{\frac{1}{2q^2}, \left(\frac{A}{A_{thr}}\right)^2\right\}$ equals to $n\sqrt{2}$. Therefore, the argument $n\sqrt{2}\cos\Phi$ of the Bessel function with the condition $\Phi \le \frac{\pi}{4}$ is more than n or equal n.

We use the following asymptotic formula of the Bessel function for large values of the order of n:

$$J_n(n + n^{1/3}z) = \left(\frac{2}{n}\right)^{1/3} A_i(-2^{1/3}z) + \mathrm{O}(n^{-1}), \tag{9}$$

where Airy function $A_i(-2^{1/3}z)$ with the condition $z < 0$ decreases rapidly. Here value of $z = (\sqrt{2}\cos\Phi - 1)n^{2/3}$ is positive for $\Phi \le \frac{\pi}{4}$. If $\cos\Phi$ replace the mean value $\langle\cos\Phi\rangle$ in the range $0 \le \Phi \le \pi/4$, which is equal to 0.94 , that $\sqrt{2}\langle\cos\Phi\rangle - 1 \approx 1/3$ and $z = 1/3\, n^{2/3}$.
Since the width of the Airy function order of unity, it is easy to show that the number of harmonics, whose spectra overlap each other, have almost the same intensity equal to $3n^{1/3}$.

The value of the function $F_n(x)$, when $x = n/Q$,
with accuracy up to small order $1/Q$ is equal to Q^2. On the other hand, using the asymptotic expression of the Bessel function

$${J_n}^2 = \left(\frac{2}{9}\right)^{2/3} \frac{1}{\Gamma^2\left(\frac{2}{3}\right)R^{2/3}} \backsimeq \frac{0.2}{n^{2/3}} \tag{10}$$

and passing from discrete values $x_n = n/Q$ to be continuous, for the total spectrum of photons one get:

$$\frac{dN_{tot}}{dx} = \sum_{k=n}^{n+3n^{1/3}} \frac{dN_k}{dx} = 0.6\,\alpha p\, Q^{5/7}/x^{1/3}. \tag{11}$$

The frequency spectrum of the radiation intensity is given

$\frac{dW_{tot}}{dx} = 0.6\alpha p Q^{5/3} x^{2/3}\hbar\Omega\gamma^2$. Thus, the emission spectrum of the number of photons in the CW decreases as $x^{1/3}$, and the spectrum intensity increases as $x^{2/3}$.

4. Discussion

The problem is solved in the framework of the classical theory.

We find the boundary frequency x_c , when the positron loses whole its energy:

$$\frac{W_{tot}}{\gamma mc^2} = 0.36\alpha\lambda_c \frac{p\gamma}{l}(Q\, x_c)^{5/3} \leq 1, \quad (12)$$

where λ_c is the Compton wavelength of positron. As a result:

$$x_c = 2.07 \cdot 10^7 (\frac{l}{p\gamma})^{3/5}/Q. \quad (13)$$

Such a rough assessment was necessary for the correct selection of the maximum frequency x_m:

$$(x_m/x_c)^{5/3} \ll 1 \quad (14)$$

Consider the energy loss of a positron with energy of 10 GeV in CW with parameters:

$l = 10^{-2} cm,\ A = 8 \cdot 10^{-7} cm\ (q \approx 10, Q \approx 50), L = 1cm\ (p = 10^2)$.

Using the formula (14) we get $x_c = 4$. In the frequency range with maximum photon frequency $x_m = 1$ the positron losses 4.6% of its energy, or 460 MeV energy. Since the energy loss of the positron in CW is significant, it is necessary to take them into account when solving this problem.

5. Conclusion

In the crystalline Wiggler relativistic channeling positron loss the significant part of its energy on the radiation of higher harmonics. The total frequency spectrum is formed by overlapping neighboring harmonics of radiation. Radiation is concentrated in a solid angle, whose centre is in the plane of the positron motion ($\phi = 0$). Polar angles of radiation are located around a small angle $\vartheta_m = \beta_\perp/\sqrt{2}$ with the width of the same order and the width of the azimuthal angle is of order $\Delta\phi \simeq 1/3$. Spectrum of the number of radiated photons with increasing frequency decreases by the law $x^{-1/3}$, and the spectrum of the positron energy losses is growing by the law $x^{2/3}$.

The energy loss of a positron in CW is significant.

Classical theory is applicable in a limited interval of frequencies, because when in case of the theory of radiation taking into account the photons with large frequency, the energy losses become of the order of positron energy.

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