

# RAY-OPTICAL ANALYSIS OF RADIATION OF A CHARGE FLYING NEARBY A DIELECTRIC OBJECT

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## INTRODUCTION

Problems of radiation of charged particles in the presence of dielectric objects are of interests for some important applications in accelerator and beam physics. It can be mentioned, for example, a new method of bunch diagnostics offered recently.

[1] A.P. Potylitsyn, Yu.A. Popov, L.G. Sukhikh, G.A. Naumenko, M.V. Shevelev // J. Phys.: Conf. Ser. **236** (2010) 012025.

For realization of this method and for other goals, it is necessary to calculate the field of Cherenkov radiation outside a dielectric object. In a number of simple specific cases, an exact solution for the field has been obtained. However, in a majority of cases of practical interest, the complex geometry of the problem does not allow obtaining rigorous expressions for the radiation field. Therefore development of approximate methods for analyses of radiation is very actual. Some of problems with dielectric objects were considered in a series of papers, where certain approximate methods are elaborated.

[2] A.A. Tishchenko, A.P. Potylitsyn, M.N. Strikhanov // Phys. Rev. E **70** (2004) 066501.

[3] M.I. Ryazanov, M.N. Strikhanov, A.A. Tishchenko // JETP **99** (2004) 311.

[4] M.I. Ryazanov, // JETP **100** (2005) 468.

[5] D.V. Karlovets // JETP **113** (2011) 27.

## “RAY-OPTICAL” METHOD

One of methods which we develop is based on the ray optical laws. This technique is traditional in optics and applied very widely for elaboration of different optical devices. It looks natural to use such an approach in theory of particle radiation.

However, we must take into account that some of geometrical parameters are not large in comparison with the wavelength. Therefore the method under consideration is based on combination of exact solution of problem without “external” boundaries of the object and accounting of these boundaries using the ray optics.

This method concerns problems where the object size is much larger than the wavelengths under consideration. Other geometric parameters (such as the distance from the object’s border to the charge trajectory) can be arbitrary.

- [1] E.S. Belonogaya, S.N. Galyamin, A.V. Tyukhtin // Proc. of RUPAC’12, WEPPD050.
- [2] E.S. Belonogaya, A.V. Tyukhtin, S.N. Galyamin // Phys. Rev. E **87** 043201 (2013).
- [3] E.S. Belonogaya, A.V. Tyukhtin, S.N. Galyamin // Proc. of IPAC’13, MOPME065.

## “RAY-OPTICAL” METHOD

First step: the field of the charge in an infinite medium without “external” borders is calculated (calculation of “incident” field).

Second step: the approximate (ray-optics) calculation of the radiation exiting the object.

$$H_{\omega} \approx H_{\omega}^* \sqrt{D_*/D} T \exp(i \omega l / c),$$

$H_{\omega}^*$  is a value of incident field on the boundary,  $T$  is Fresnel coefficient,  
 $l$  is a ray path in vacuum,

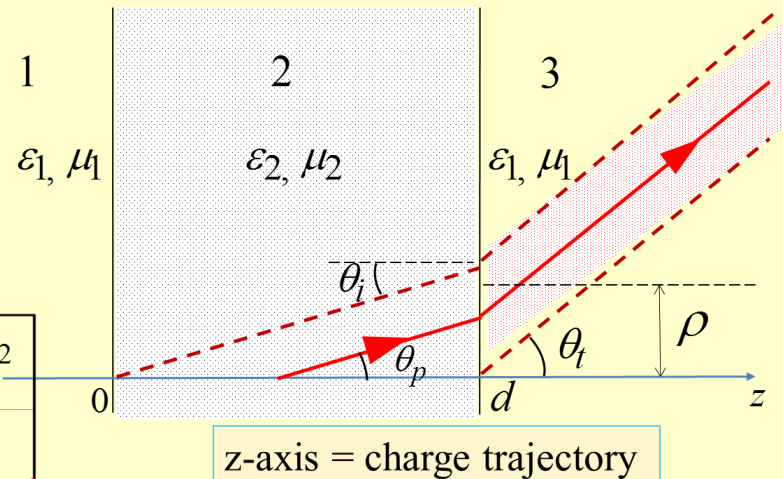
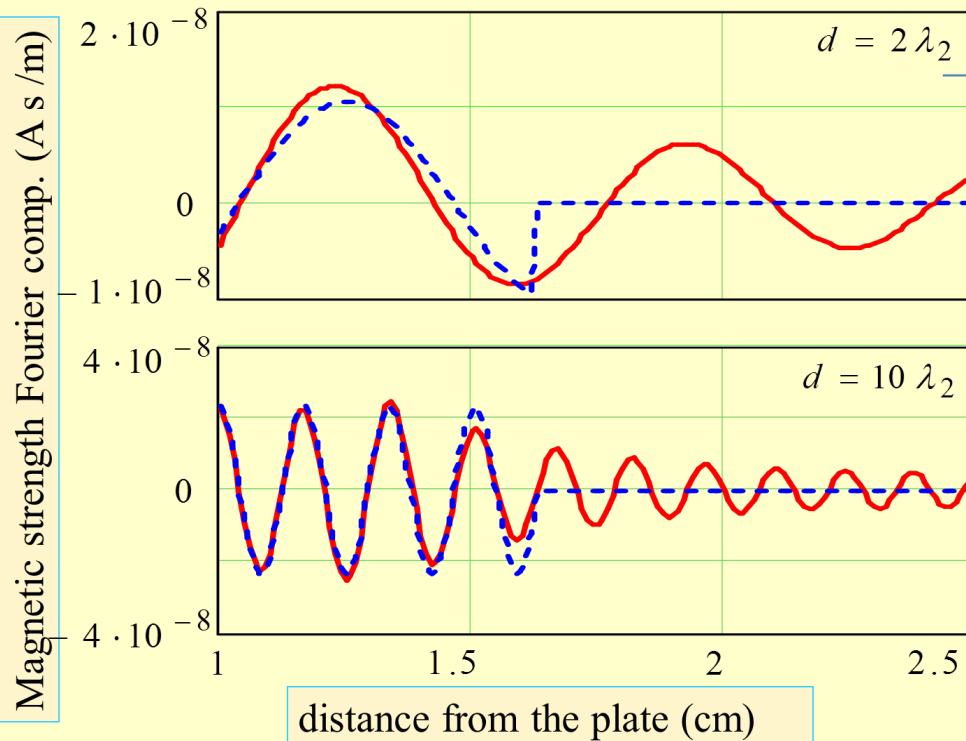
$D_*$  is a square of cross-section of ray tube on the boundary,

$D$  is a square of cross-section of ray tube on the observation point.

This calculation is related to Fock’s method for analyzing reflection of arbitrary non-plane waves from an arbitrary surface; analogous calculations are applied to elaborate different optical systems. At the second step, the incident field is multiplied by the Fresnel transmission coefficient, and then propagation of radiation is calculated using the ray optics technique. Thus, the first of the refracted rays is obtained. If necessary, multiple reflections and refractions from the object’s borders can be taken into account.

# TESTING THE METHOD

For testing the method, we used the problem about the field of point charge flying through the dielectric layer. Such a problem has exact solution which is in a very good agreement with approximate one when the layer thickness  $> 10$  wavelength.



Exact formulas - red solid curve,  
Approximate formulas - blue dashed curve.

$$q = -\ln C \quad \beta = 0.99$$

$$\epsilon_1 = \mu_1 = \mu_2 = 1$$

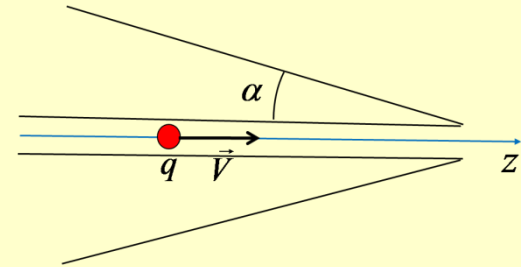
$$\epsilon_2 = 1.5$$

$$\rho = 0.6 \text{ cm}$$

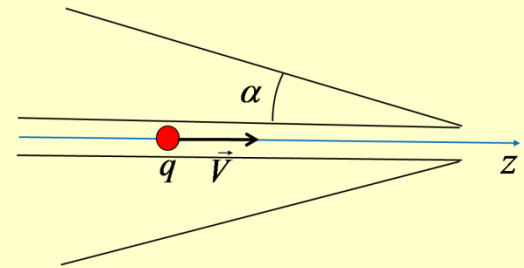
# CONE AND PRISM

We applied the method under consideration to three cases.

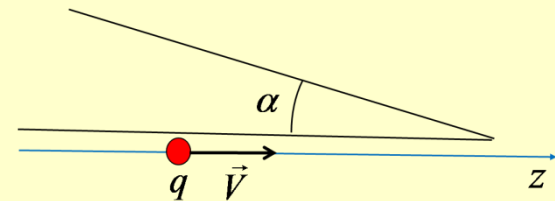
1. Dielectric cone with a vacuum channel. The charge moves along the cone axis.



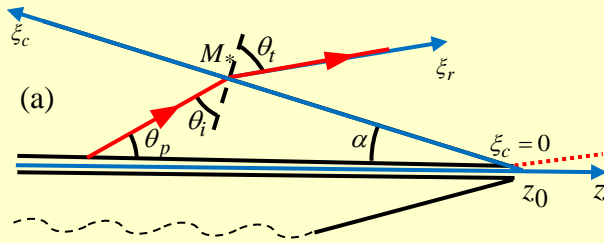
2. Dielectric prism – the case I: the charge moves in the vacuum channel (“prism-I”).  
(Transversal section is the same as for the cone)



3. Dielectric prism – the case II: the charge moves along the boundary of the prism (“prism-II”).



# THE CONE



Incident field:  $H_\phi = \int_{-\infty}^{\infty} H_{\phi\omega} e^{-i\omega t} d\omega$   $H_{\phi\omega} = \frac{iq}{2c} \eta s H_1^{(1)}(s\rho) \exp\left(i \frac{\omega}{V} z\right)$

$$\eta = -2i(\pi a)^{-1} \left[ (1 - \varepsilon\mu\beta^2)(1 - \beta^2)^{-1} \varepsilon^{-1} I_1(ka) H_0^{(1)}(sa) + s I_0(ka) H_1^{(1)}(sa) \right]^{-1}$$

$$s(\omega) = \omega V^{-1} \sqrt{\varepsilon\mu\beta^2 - 1} \quad \text{Im } s(\omega) \geq 0 \quad k(\omega) = |\omega| V^{-1} \sqrt{1 - \beta^2}$$

Point of incidence

of wave on the cone boundary:  $\rho_* = (z_0 - z_*) \tan \alpha$   $z_* = \frac{z_0 \tan \alpha + z \cot(\alpha + \theta_t) - \rho}{\tan \alpha + \cot(\alpha + \theta_t)}$

$$\sin \theta_t = \sqrt{\varepsilon\mu} \sin \theta_i \quad \theta_i = \pi/2 - \alpha - \theta_p, \quad \theta_p = \arccos(1/(\sqrt{\varepsilon\mu}\beta))$$

Ray tube widening:  $\frac{D(l)}{D(0)} = \frac{\rho}{\rho_*}$

Field outside the cone:  $H_{\phi\omega} \approx H_{\phi\omega}^{(2)*} \sqrt{\rho_*/\rho} T \exp(i \omega l / c),$

$l$  is a ray path in vacuum:  $l = (z - z_*) / \sin(\alpha + \theta_t)$

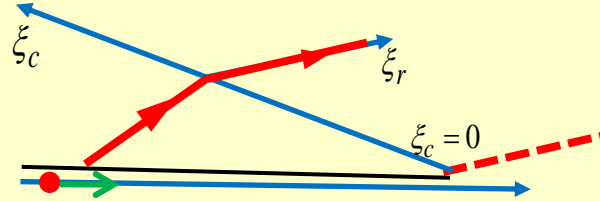
$T$  is Fresnel coefficient:  $T = \frac{2\sqrt{\mu_1/\varepsilon_1} \cos \theta_t}{\sqrt{\mu_2/\varepsilon_2} \cos \theta_i + \sqrt{\mu_1/\varepsilon_1} \cos \theta_t}$

# THE PRISM

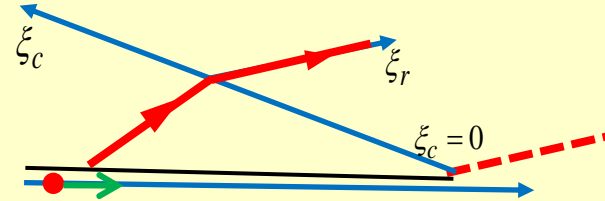
Ray tube widening in the “central” plane  $y=0$ :

$$\frac{D(l)}{D(0)} = \frac{\rho \sin(\alpha + \theta_t) - z \cos(\alpha + \theta_t) + \sqrt{\frac{\epsilon_m \mu_m}{\epsilon_v \mu_v}} \left[ z \frac{\cos \theta_t - \cos \alpha \cos(\alpha + \theta_t)}{\sin(\alpha + \theta_t)} + \rho \frac{\cos \alpha}{\cos \theta_t} \right] \frac{\sin \theta_p \cos \theta_t}{\sin \alpha}}{\rho \sin(\alpha + \theta_t) - z \cos(\alpha + \theta_t)}$$

In the case of prism with the channel (**prism -I**), the incident field is the same as for the cone.



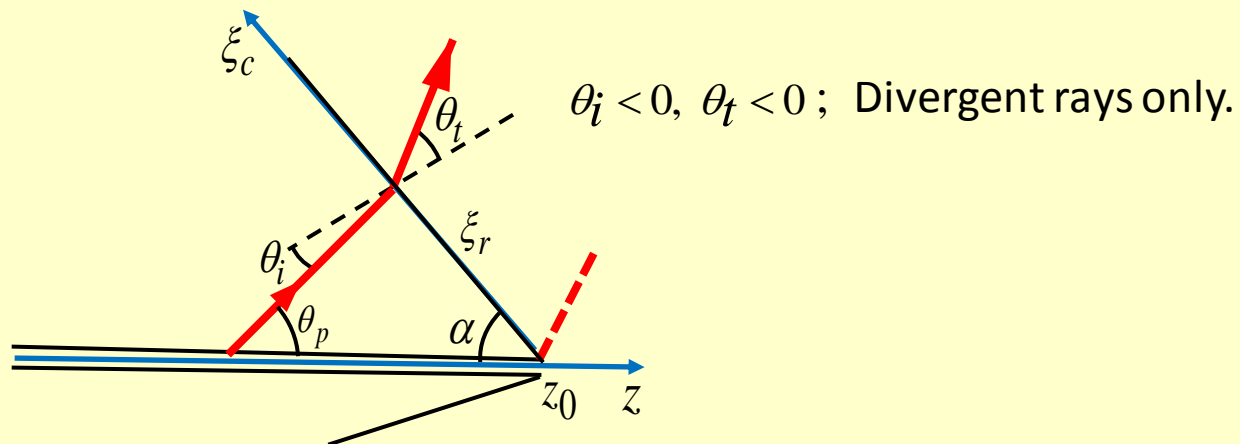
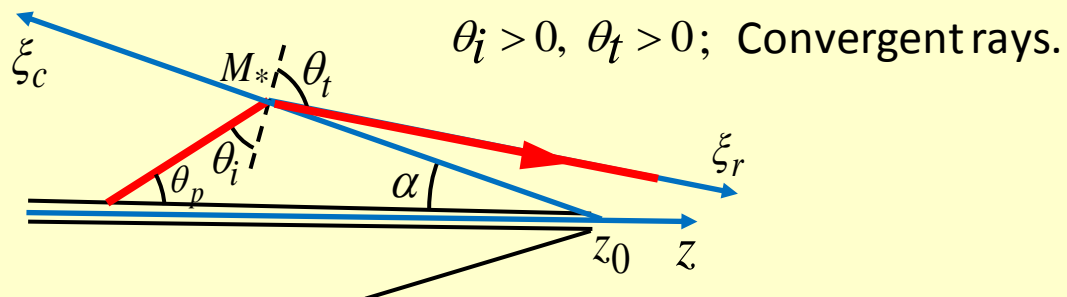
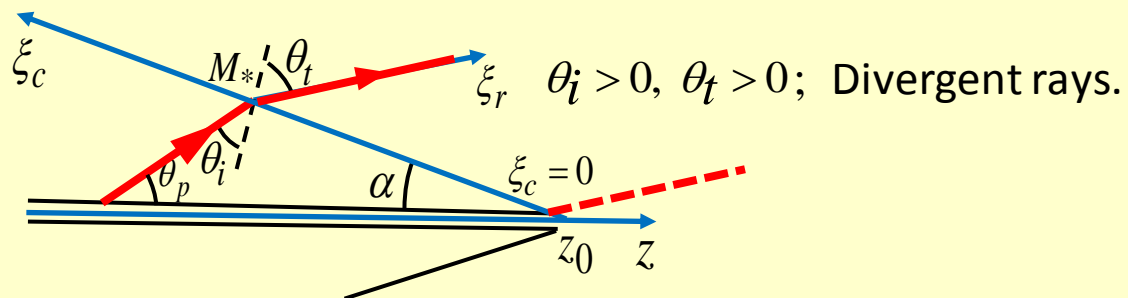
In the case of **prism-II** (when the charge moves along the boundary), the incident field is:



$$H_{\phi\omega} \approx \frac{\epsilon q \sqrt{2(1-\beta^2)} \sqrt{\epsilon\beta^2-1} \sqrt{\omega}}{c \sqrt{\pi\beta\rho} \left( \epsilon \sqrt{1-\beta^2} - i \sqrt{\epsilon\beta^2-1} \right)} \exp \left\{ -d \frac{\omega}{V} \sqrt{1-\beta^2} + i \frac{\omega}{V} z + i \frac{\omega}{V} \sqrt{\epsilon\beta^2-1} \rho - i \frac{\pi}{4} \right\}$$



## CONE AND PRISM



## CONE AND PRISM

$$\Sigma = \int_{-\infty}^{\infty} S dt = \int_0^{\infty} \sigma d\omega$$

$\Sigma$  is a total energy passing unity square  
 $\sigma$  is a spectral density of the radiation energy passing through unity square

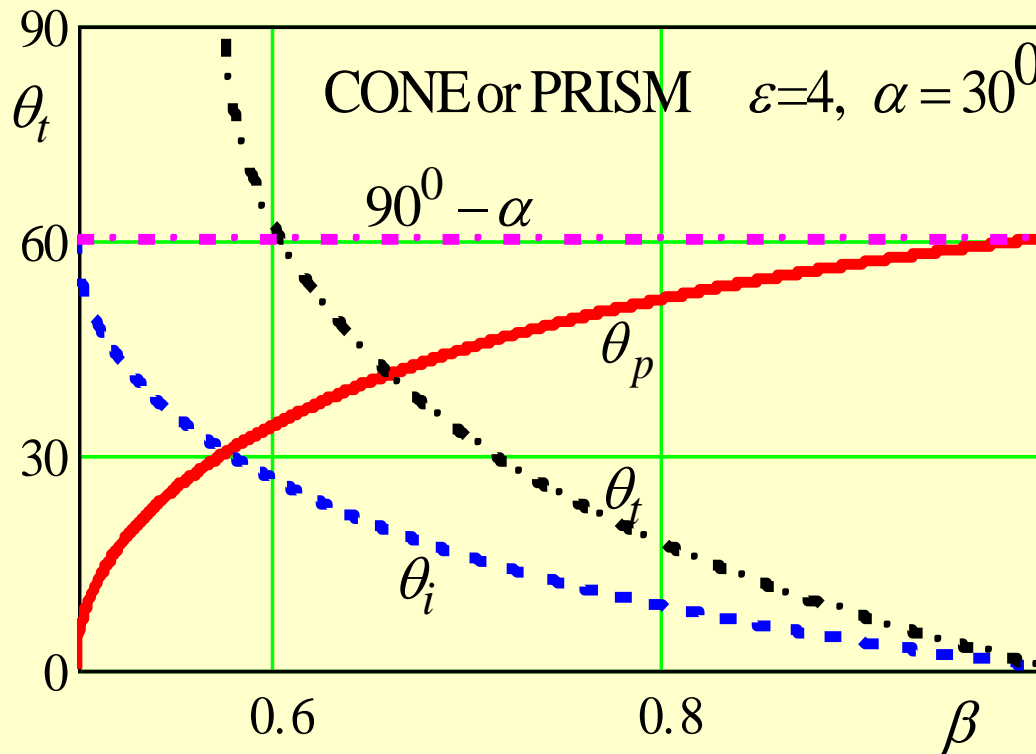
Parameters for computations:

$$q = -1 \text{ nC}, \quad \varepsilon = 4, \quad \mu = 1, \quad a = 2 \text{ mm}, \quad \omega = 2\pi \cdot 3 \cdot 10^{10} \text{ s}^{-1}.$$

$\sigma$  is given in  $\text{J} \cdot \text{s} / \text{m}^2$ , distances are given in cm.

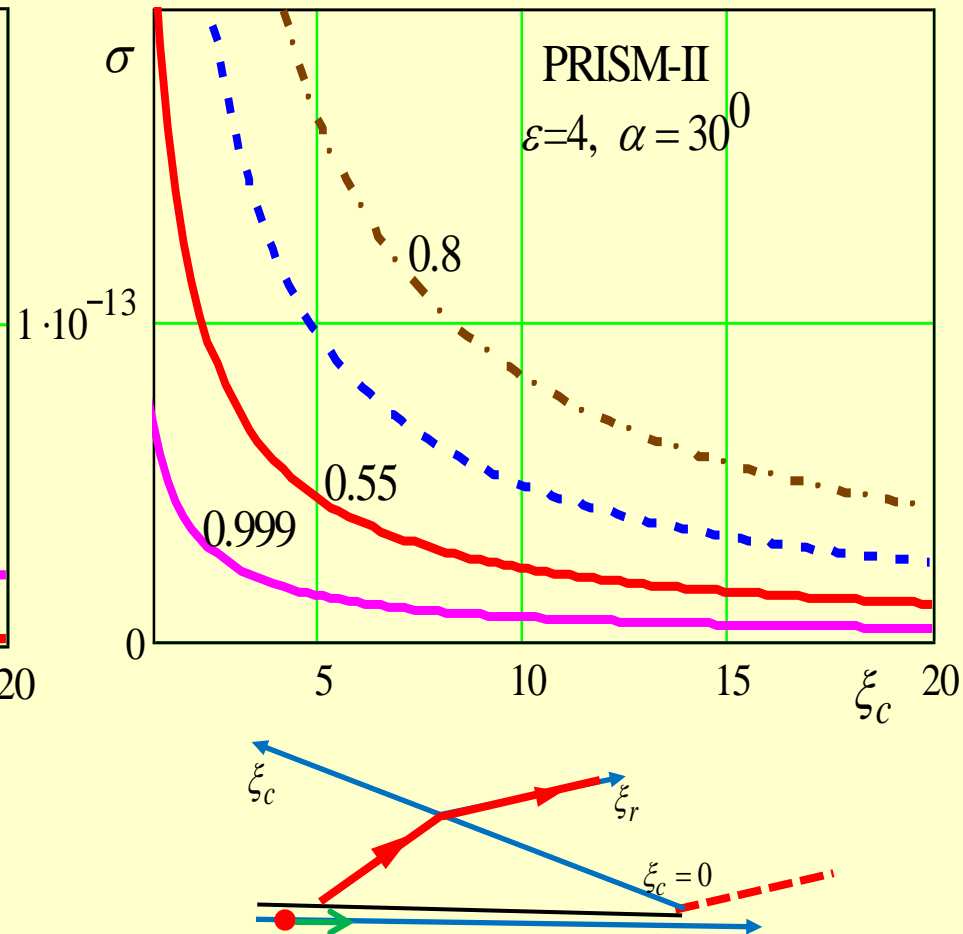
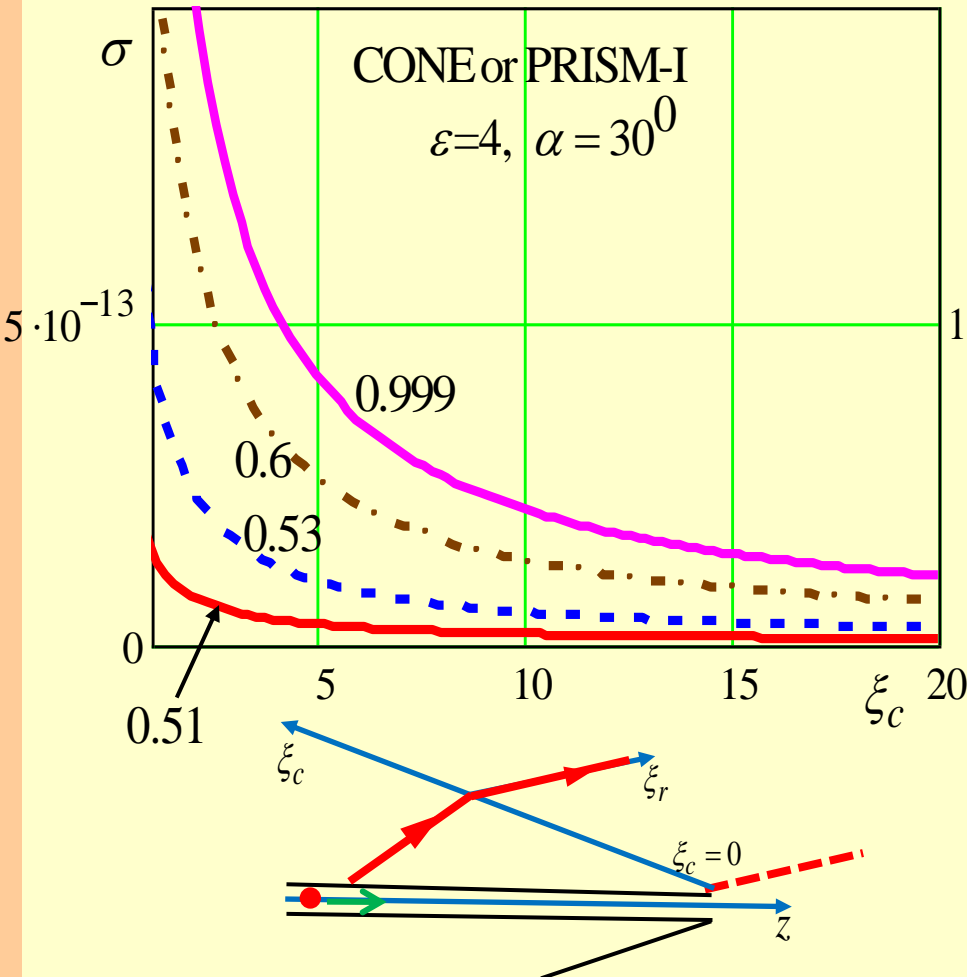
## RESULTS FOR CONE AND PRISM

The angle of the Cherenkov radiation, angle of incidence and angle of refraction (in degrees) as a function of the charge velocity.



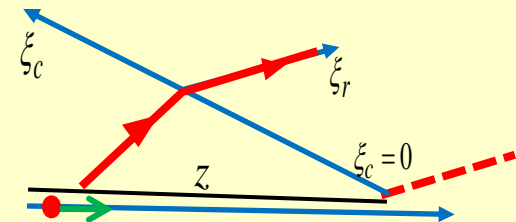
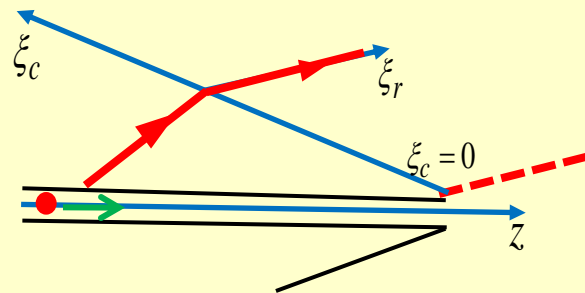
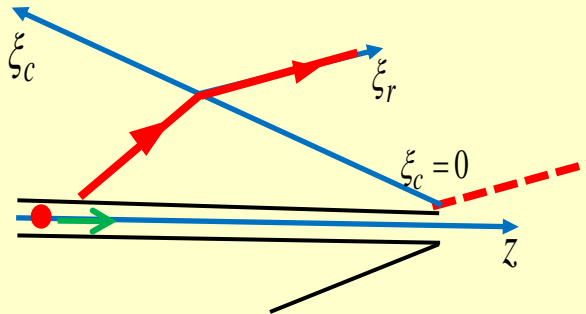
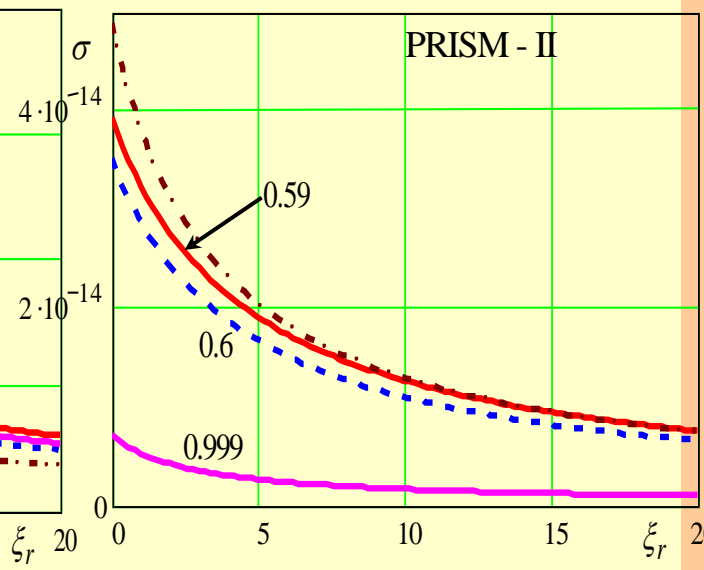
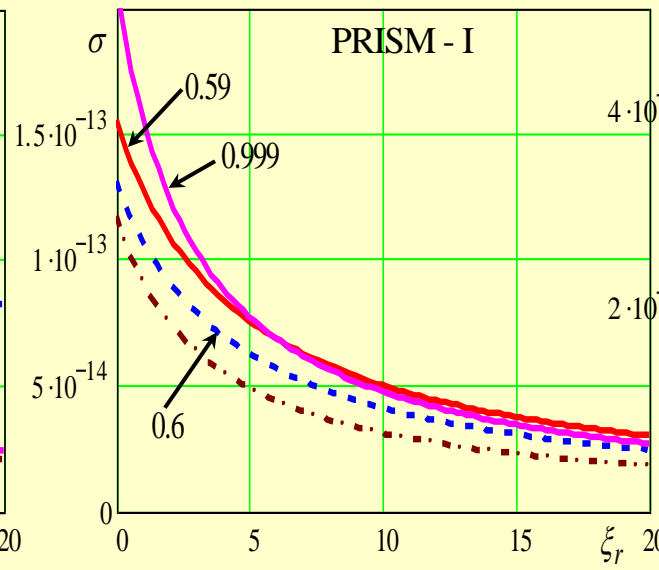
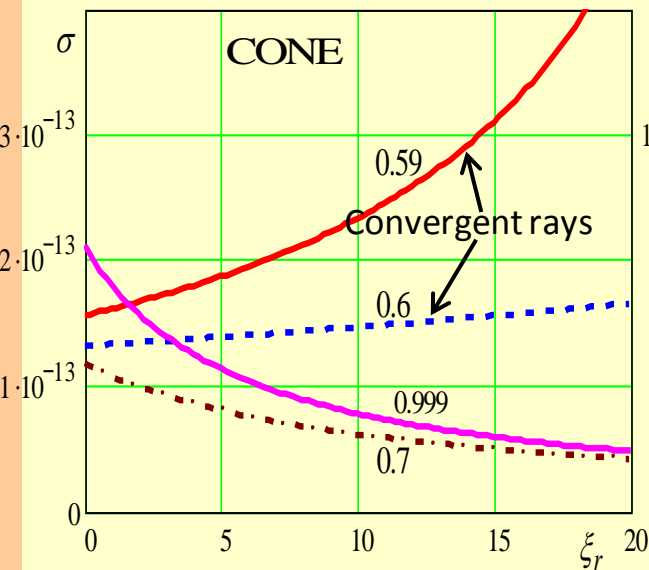
# RESULTS FOR CONE AND PRISM

The spectral density of energy  $\sigma$  as a function of the distance (cm) from the cone and prism vertex along the cone (prism) surface; values of  $\beta$  are given near the curves.



# RESULTS FOR CONE AND PRISM

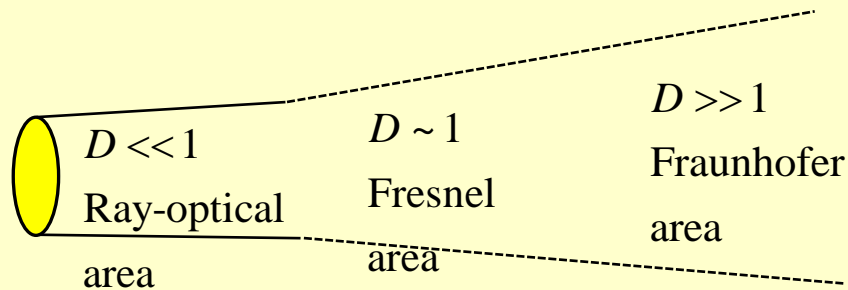
The spectral density of energy  $\sigma$  as a function of the distance (cm) along the ray from the cone (prism) surface;  
the initial point  $M_*$  is situated at  $\rho_* = 5\text{cm}$ ; values of  $\beta$  are given near the curves.  
 $\varepsilon=4$ ,  $\alpha=30^\circ$



# “RAY-OPTICAL” AND “KIRCHHOFF” (“ANTENNA”) METHODS

If a radiating surface is finite then ray-optical method can be used for limit distances only.

Wave parameter:  $D \equiv \frac{\text{Distance} \times \text{Wave Length}}{\text{Square of Radiating Surface}}$



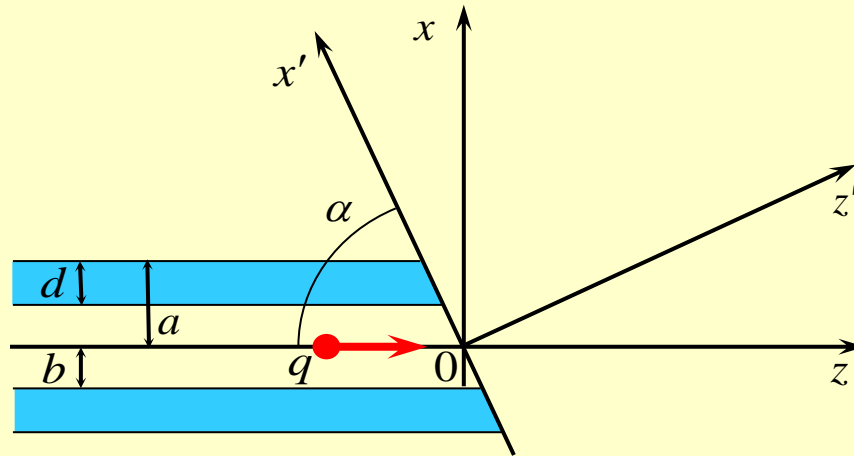
Often we need to know the radiation for large distances where ray optics is not true. In this situation, we can use the method which is like the “Kirchhoff technique” or its generalization which is known in the antenna theory.

This technique is true if wavelength is much less than the size of the boundary surface. According to Kirchhoff method the field on the boundary surface is approximately equal to the incident field. This method is traditional for solution of different problems, especially in optics.

## “KIRCHHOFF” (“ANTENNA”) METHOD

We apply this approach to the problem on radiation from an open end of waveguide. The point charge  $q$  moves along the axis of the circular waveguide with cylindrical dielectric layer. The end of waveguide can be, in principle, oblique. We will interesting the mode with high number  $m \gg 1$ .

This problem is interesting now because such a scheme can be used for development of generator of Terahertz radiation (frequencies from hundreds Gigahertz to several Terahertz).



# GENERAL FORMULAE OF “ANTENNA” METHOD

In line with the antenna theory:

$$\vec{E}(x', y', z') \square \frac{ik_0}{4\pi} \iint_{S_a} g \left[ \vec{e}_R [[\vec{e}_{z'}, \vec{H}^a(\xi, \eta)] \vec{e}_R] \right] d\xi d\eta + \frac{ik_0}{4\pi} \iint_{S_a} g \left[ [\vec{E}^a(\xi, \eta), \vec{e}_{z'}] \vec{e}_R \right] d\xi d\eta,$$

for  $k_0 R \gg 1$  ( $k_0 = \omega / c$ )

$$g = \exp\left(ik_0 \sqrt{(x' - \xi)^2 + (y' - \eta)^2 + z'^2}\right) / \sqrt{(x' - \xi)^2 + (y' - \eta)^2 + z'^2}$$

$S_a$  is an aperture,  $\vec{E}^a(\xi, \eta)$ ,  $\vec{H}^a(\xi, \eta)$  are field components on the aperture.

$x'$ ,  $y'$ ,  $z'$  is an observation point.

In the Fraunhofer zone this result can be simplified:

$$\vec{E} = \frac{ik_0}{4\pi} g_0 \iint_{S_a} \left\{ \left[ \vec{e}_{R'}, \left[ [\vec{e}_{z'}, \vec{H}^a], \vec{e}_{R'} \right] \right] + \left[ [\vec{E}^a, \vec{e}_{z'}], \vec{e}_{R'} \right] \right\} \exp[-ik_0 (\vec{r}'_a, \vec{e}_{R'})] d\xi d\eta$$

$$g_0 = \exp(ik_0 R) / R, \quad R = \sqrt{x'^2 + y'^2 + z'^2}, \quad \vec{r}'_a = \vec{e}_{x'} \xi + \vec{e}_{y'} \eta.$$

$$k_0 R \gg 1, \quad D \equiv \frac{R \lambda_m}{(a / \sin \alpha)^2} \gg 1.$$



## RESULTS FOR HIGH WAVEGUIDE MODE

We consider the case of ultra relativistic motion of the charge.  
The waveguide mode in the infinity waveguide has simple form:

$$E_z \approx \text{Re} \left\{ -A \sqrt{n^2 - 1} \begin{cases} \sin(s_m d) & \text{for } r < b \\ \sqrt{b/r} \sin[s_m(a-r)] & \text{for } r > b \end{cases} \exp\left(\frac{i\omega_m \zeta}{V}\right) \right\},$$

$$E_r \approx \text{Re} \left\{ A \begin{cases} \varepsilon r b^{-1} \cos(s_m d) & \text{for } r < b \\ \sqrt{b/r} \cos[s_m(a-r)] & \text{for } r > b \end{cases} \exp\left(\frac{i\omega_m \zeta}{V}\right) \right\},$$

$$H_\varphi \approx \text{Re} \left\{ iA\varepsilon \begin{cases} r b^{-1} \cos(s_m d) & \text{for } r < b \\ \sqrt{b/r} \cos[s_m(a-r)] & \text{for } r > b \end{cases} \exp\left(\frac{i\omega_m \zeta}{V}\right) \right\}.$$

$$\omega_m = \frac{c\chi_m}{d\sqrt{n^2 - 1}}, \quad s_m = \chi_m/d, \quad A = \frac{8qc \cos(s_m d)}{b^2 d \sqrt{n^2 - 1} \omega_m}.$$

$$m \gg 1 \Rightarrow \chi_m \approx \pi m \gg 1,$$

Conditions of validity:

$$\gamma \gg 1 (\beta \approx 1), \quad k_1 b \approx \frac{\omega_m b}{c\gamma} \ll 1; \quad \frac{\omega_m}{c} b \ll 1, \quad \frac{\omega_m}{c} d \gg 1 \Rightarrow s_m b \approx \frac{\omega_m}{c} b \sqrt{n^2 - 1} \gg 1$$

## RESULTS FOR HIGH WAVEGUIDE MODE

We consider that:

- The field on the aperture of the vacuum channel is approximately equal to the field of the mode in infinity waveguide.
- The field on the aperture of dielectric is found in the following way:  
the mode divides into two cylindrical waves (converging and divergent), each of them divides into waves with horizontal and vertical polarizations, and then the Fresnel coefficients are used.

In the case of **orthogonal end** ( $\alpha = \pi/2$ ) the field from the vacuum part of aperture can be found analytically.

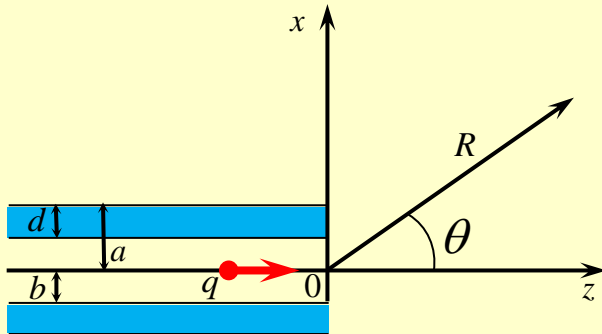
This contribution is the main. The contribution of the dielectric aperture is small.

The main maximum direction: 
$$\theta_{\max} \approx \frac{1}{2} \arcsin \left( \frac{5 \cdot d \sqrt{n^2 - 1}}{\chi_m b} \right)$$

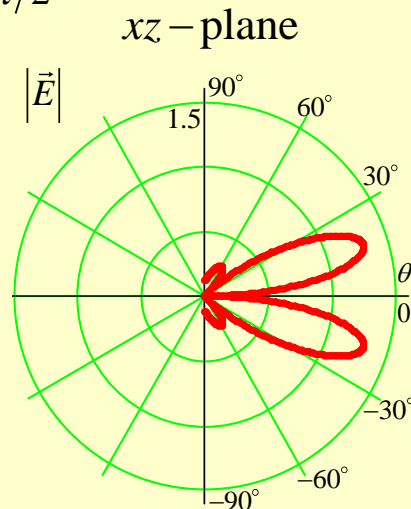
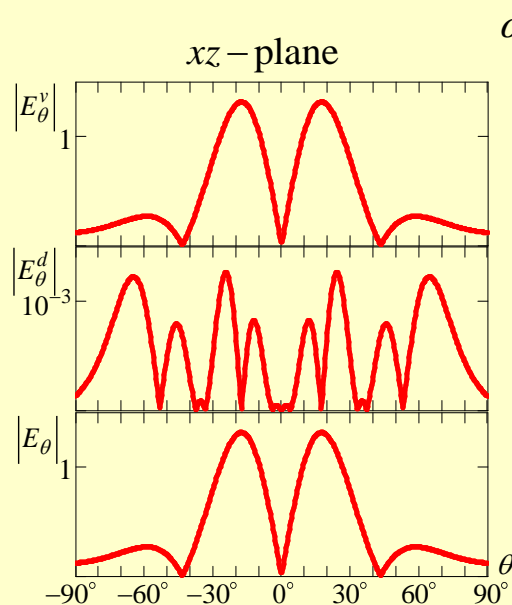
The width of radiation pattern:  $\Delta\theta \sim 2\theta_{\max}$

# NUMERICAL RESULTS FOR HIGH WAVEGUIDE MODE

The case of orthogonal end  $\alpha = \pi/2$



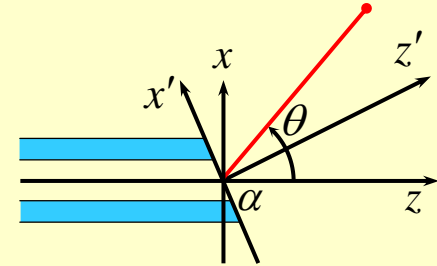
Electric field in the Fraunhofer area for the mode with number  $m=10$   
 $q=1nC$ ,  $n^2=10$ ,  
 $a=0.24cm$ ,  $b=0.1cm$ ,  $R=1cm$



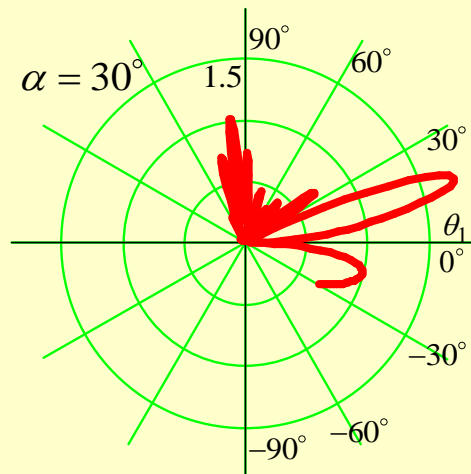
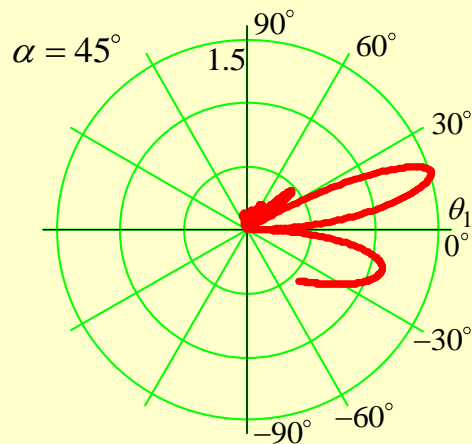
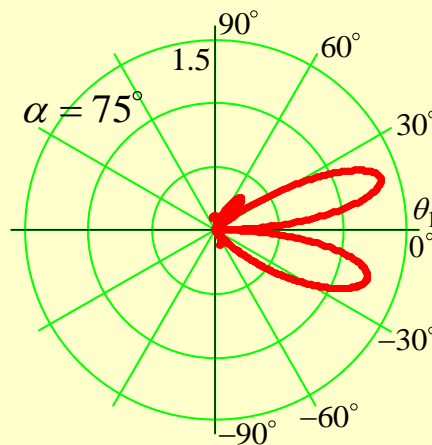
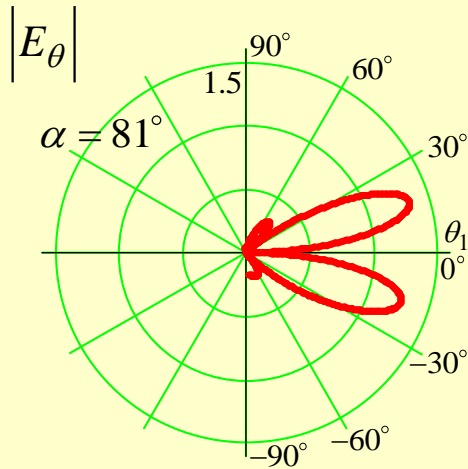
# NUMERICAL RESULTS FOR HIGH WAVEGUIDE MODE

## The case of oblique end

$xz$ -plane



$$q=1nC, \quad n^2=10, \\ a=0.24\text{cm}, \quad b=0.1\text{cm}, \quad R=1\text{cm}$$



## CONCLUSION

We develop two approximate methods for calculation of the wave field from the charge moving in the presence of non-conductive objects.

The first of them is applicable under condition that the object is much more than the wave length and the observation point is in the “ray-optical” area where the wave parameter is small. The method has been testing for the dielectric plate and applied for the cases of dielectric cone and prism.

The second method is based on the technique which is known in the antenna theory and very close to the Kirchhoff technique. This method allows calculating the field both in the ray-optical area and in the Fraunhofer area. However, this method requires calculation of some complex integrals.

Using this method we considered exiting a high waveguide mode from the waveguide with dielectric layer and open end. This problem is interesting now because such a scheme can be used for development of source of Terahertz radiation.