Quantum Jumps in PXRC Angular Distributions from Relativistic Channeled Electrons in a Crystal

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Two types of X-Radiation from relativistic electrons in a crystal

H.Nitta: Channeling 2008 → CLASSIFICATION

Terminology (definitions):

PXR = Parametric X-Radiation = from non-channeled electrons = diffraction of relativistic electron electromagnetic field (VIRTUAL PHOTONS) on the CRYSTALLOGRAPHIC PLANES = well known since first experimental observation (1985, Tomsk, electron synchrotron "Sirius")

PXRC = Parametric X-Radiation from channeled electrons = first predicted by H.Nitta et al (1996) = first observed and explained in 2012 at SAGA-LS (Japan) [28]

XR vs PXRC: **change of r**elativistic electrons states:
Plane waves (PXR) → transverse quantum states bound to the channeling planes (PXRC)

PXRC at planar channeling

PXRC appears when an electron penetrates through a crystal at the channeling condition: electron is in a transverse quantum bound state with a crystal plane while its longitudinal (parallel to a plane) motion is free

a) The scheme of observing the **angular distribution** of **PXRC -** 3D view

θ_B is the Bragg angle

$\vec{g}$ is the reciprocal lattice vtctor

$\vec{v} = c\vec{\beta}$ is the electron velocity

b) The schematics of mutual arrangement of the vectors

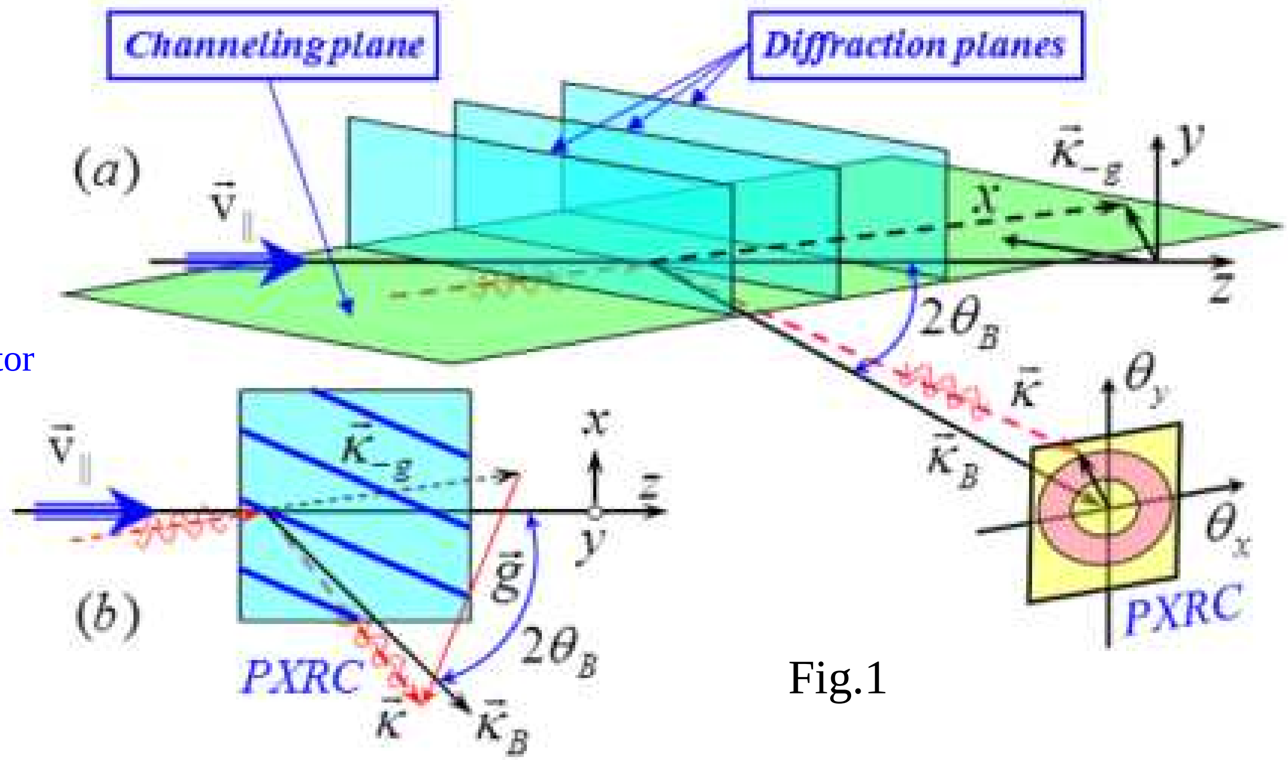

Fig.1

Experiment

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Quantum Effects for Parametric X-ray Radiation during Channeling: Theory and First Experimental Observation¶

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The theory of X-ray radiation from relativistic channeled electrons at the Bragg angles—parametric X-ray radiation (PXR) during channeling (PXRC)—is developed while accounting for two quantum effects: the initial population of bound states of transverse motion and the transverse "form-factor" of channeled electrons. An experiment was conducted using a 255 MeV electron beam from a linac at the SAGA Light Source. We have identified a difference in the angular distributions of PXR and PXRC and obtained a fairly good agreement between the theoretical and experimental results.

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Experiment

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BEAM INTERACTIONS WITH MATERIALS AND ATOMS

Experimental and theoretical study of PXRC (Parametric X-Radiation at Channeling) from 255 MeV electrons in Si

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The X-radiation from relativistic channeled electrons at the Bragg angles – Parametric X-Radiation at Channeling (PXRC) – is studied both experimentally and theoretically. The experiment was carried out using a 255 MeV electron beam from a Linac at newly constructed beam line for the study of interactions between a relativistic electron beam and crystals at the SAGA Light Source. The observed asymmetry of PXRC angular distribution at (220) planar channeling in a 20 μm Si is explained taking account of two quantum effects: initial populations and transverse form-factors of the quantum states of planar channeled electrons. Further perspectives for PXRC studies at SAGA-LS are analyzed.

PXRC from planar channeled electrons

Angular distribution of PXRC from channeled electron being in a quantum state n of transverse motion

$$I^n_{\rm PXRC} = \frac{d^3 N_{nn}}{d\theta_x d\theta_y dz} = I_{\rm PXR} |F_{nn}(q)|^2$$

$$I_{\rm PXR} = \frac{\alpha\omega_B}{16\pi c \sin^2\theta_B}\left[\frac{\theta_x^2}{1+W_\pi^2} + \frac{\theta_y^2}{1+W_\sigma^2}\right]$$

$$|F_{nn}|^2 = \left|\int_{-d/2}^{d/2} \phi_n^*(y)\exp(-i\omega_B\theta_y y/c)\phi_n(y)dy\right|^2$$

$$W_\tau = \frac{1}{2|\chi_g|P_\tau}\left(R - \frac{|\chi_g|^2 P_\tau^2}{R}\right), \ \tau = (\pi,\sigma),$$

$$R = \left[\theta_x - \frac{\Omega_{if}}{\omega_B}\cos\theta_B\right]^2 + \theta_y^2 + R_\circ,$$

$$R_\circ = \theta_{kin}^2 - 2\frac{\Omega_{if}}{\omega_B}, \ \theta_{kin}^2 = \gamma^{-2} + |\chi_0|,$$

the **form-factor** of the transverse quantum channeling state with the number n.

Here $\hbar\omega$ is the energy of PXRC photon, $\phi_n(y)$ is the wave function for planar channeled electrons and d is the distance between the channeling planes, $\alpha = e^2/c\hbar$

PXRC from planar channeled electrons

The PXRC angular distribution

$$I_{\text{PXRC}} = \sum_{n}^{N} I^{n}_{\text{PXRC}} P_n(\theta_0) = I_{\text{PXR}} \sum_{n}^{N} P_n(\theta_0) \, | F_{nn}(q) |^2$$

N - is the number of quantum channeling states

Initial population of the *n*-th quantum channeling state

$$P_n(\theta_0) = \frac{1}{d} \left| \int_{-d/2}^{d/2} e^{ip\theta_0 y/\hbar} \varphi_n(y) dy \right|^2$$

θ_o - is an angle of incidence of electron beam with respect to the channeling planes (*p* is initial momentum of electrons)

- **Key question:** *what we are looking for comparing PXR and PXRC at different electron beam energies ?*
- **Answer:** *appearance of quantum jumps connected with increase of the number N of quantum channeling states when electron beam energy (relativistic factor) increases*

Separate channeling plane approximation

Pöschl-Teller potentilal $U(y) = -U_0 \cosh^{-2} \lambda y$

γ=40 → N = 4 (3 even state)
γ=39 → N = 3 (2 even state)

γ=24 → N = 3 (2 even state)
γ=23 → N = 2 (1 even state)

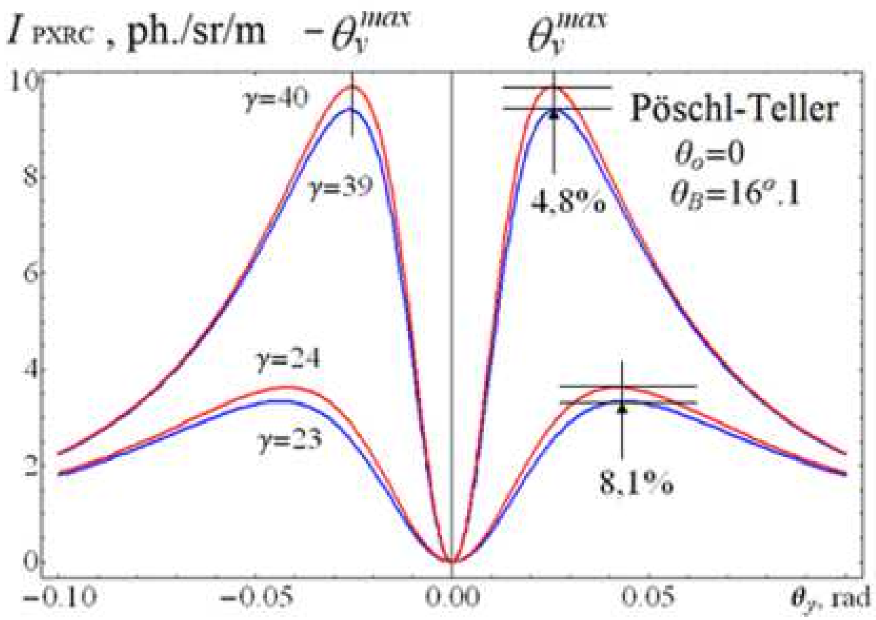


Fig.2

The maximums of I_{PXRC} observed at $\theta_y = \pm\, \theta_y^{max}$

The change in the number of bound states N leads to the more prominent change in I_{PXRC} only when the next quantum channeling state becomes populated

Beyond the separate plane approximation

PXRC angular distribution calculated with the "true" periodic planar (220) Si channeling potential I_{PXRC}

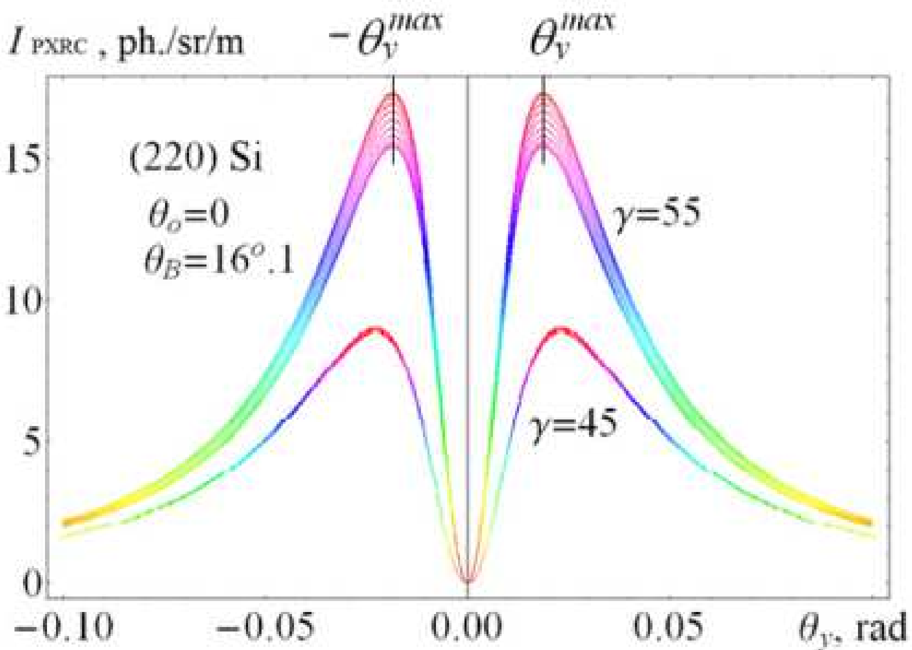
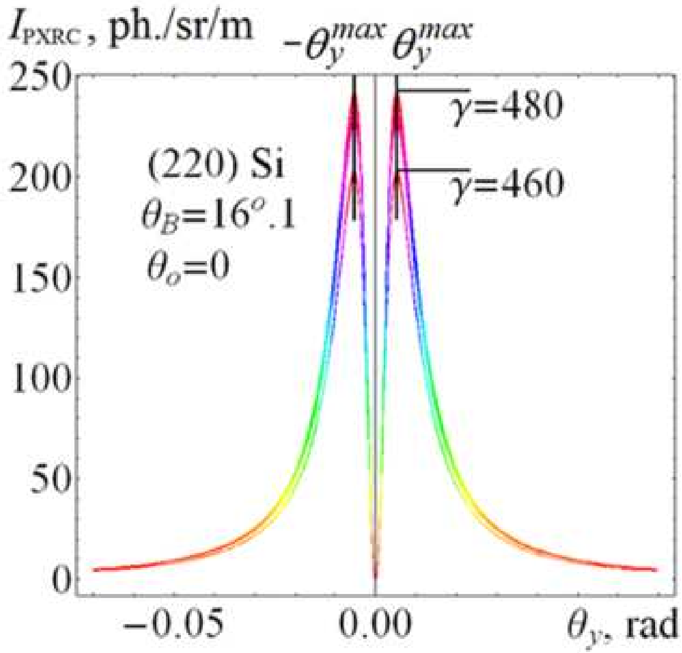


Fig.3

γ=55 → N = 5
γ=45 → N = 4



γ=480 → N = 15
γ=460 → N = 14

Fig.4

The maximums of I_{PXRC} observed at $\theta_y = \pm\,\theta_y^{max}$

Beyond the separate plane approximation

The maximums of PXRC and PXR (dashed line) as the function of γ in the (220) Si (in the plane $\theta_x = 0$)

$$I^{max}_{PXRC}(\gamma) = I_{PXRC}(\gamma)|_{\theta_y=\theta_y^{max}}$$

The number N of bound channeled states as the function of γ

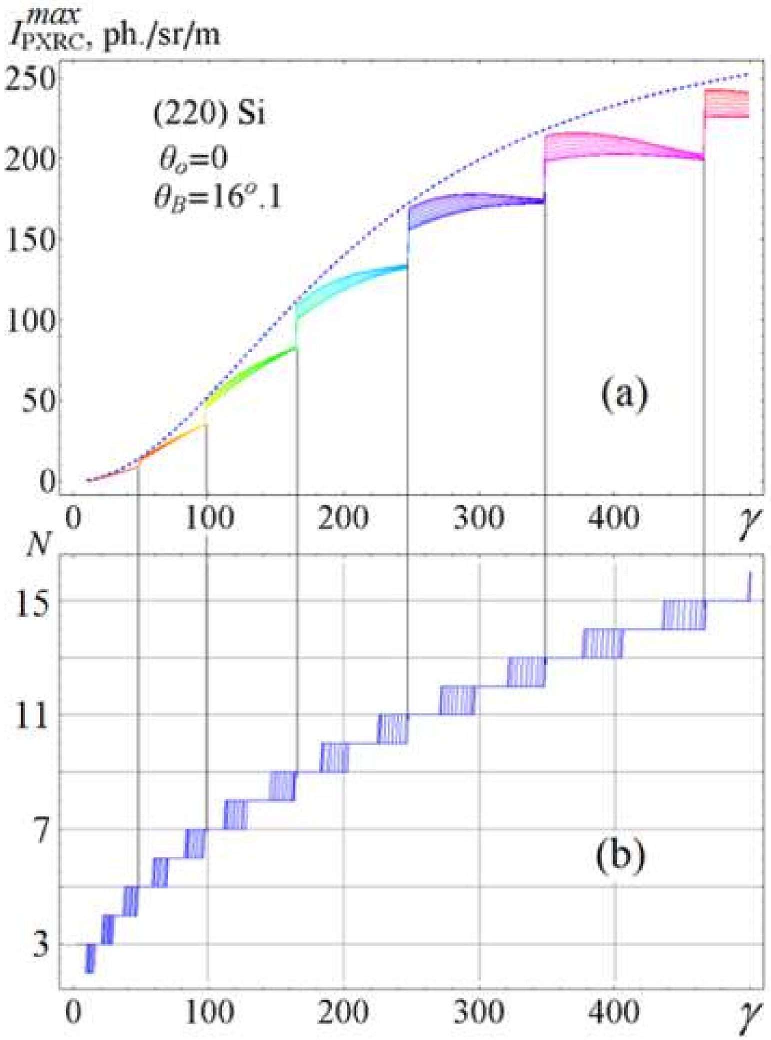


Fig.5

Comparison of PXRC and PXR

	(220) Si		Pöschl-Teller	
γ	45	55	23	24
I_{PXRC}^{max}	7.98...8.15	15.4...17.3	3.3330	3.6240
I_{PXR}^{max}	12.4	18.2	3.3331	3.6245

Table 1

Table 1 $\oplus$ Fig.5 → The "quantum jumps" in angular distributions of PXRC from channeled relativistic electrons in a crystal **really exist**

To characterize the difference in angular distributions of PXR and PXRC, we introduce the quantity

$$\delta = \delta(\theta_y^{max};\gamma) = \frac{I_{PXR}^{max} - I_{PXRC}^{max}}{I_{PXR}^{max}} \equiv$$

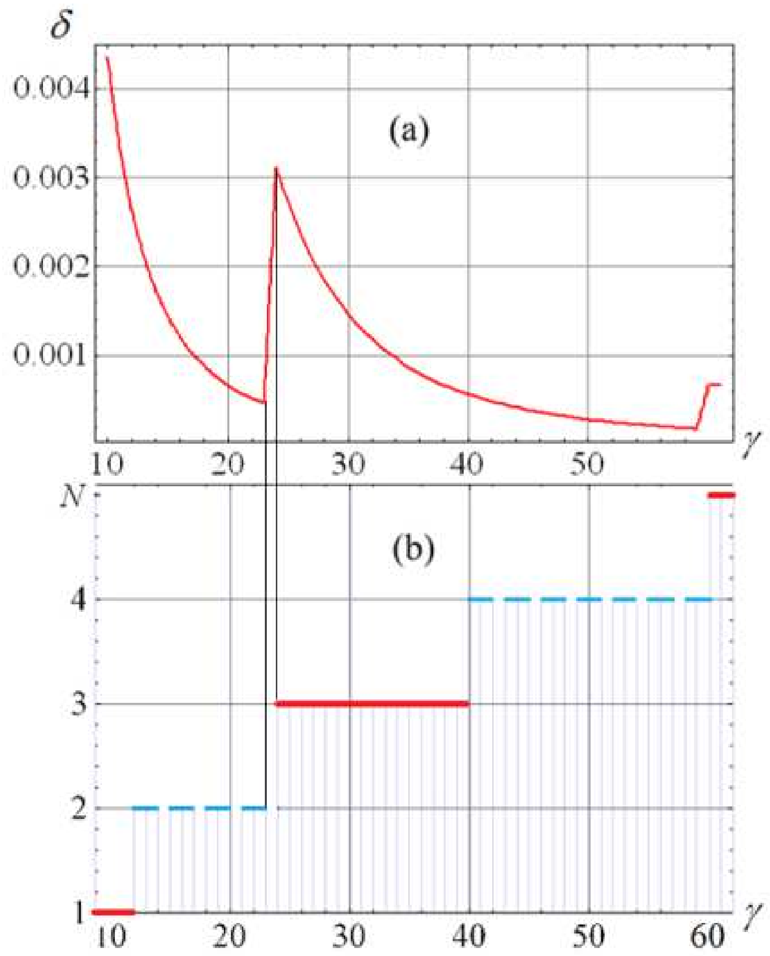
$$\equiv 1 - \sum_n P_n(\theta_0)\left|F_{nn}(\theta_y^{max})\right|^2,$$

Dependences on relativistic factor

maximum values of
the quantity δ

Number N of bound channeling states for (220) planar channeling in a Si crystal calculated in a separate-plane approximation
(the Pöschl-Teller potential with U_o = 21,17 eV).

Fig.6



Dependences on relativistic factor

maximum values of the quantity δ

Number N of bound channeling states for (220) planar channeling in a "real" 1D periodic potential Si crystal

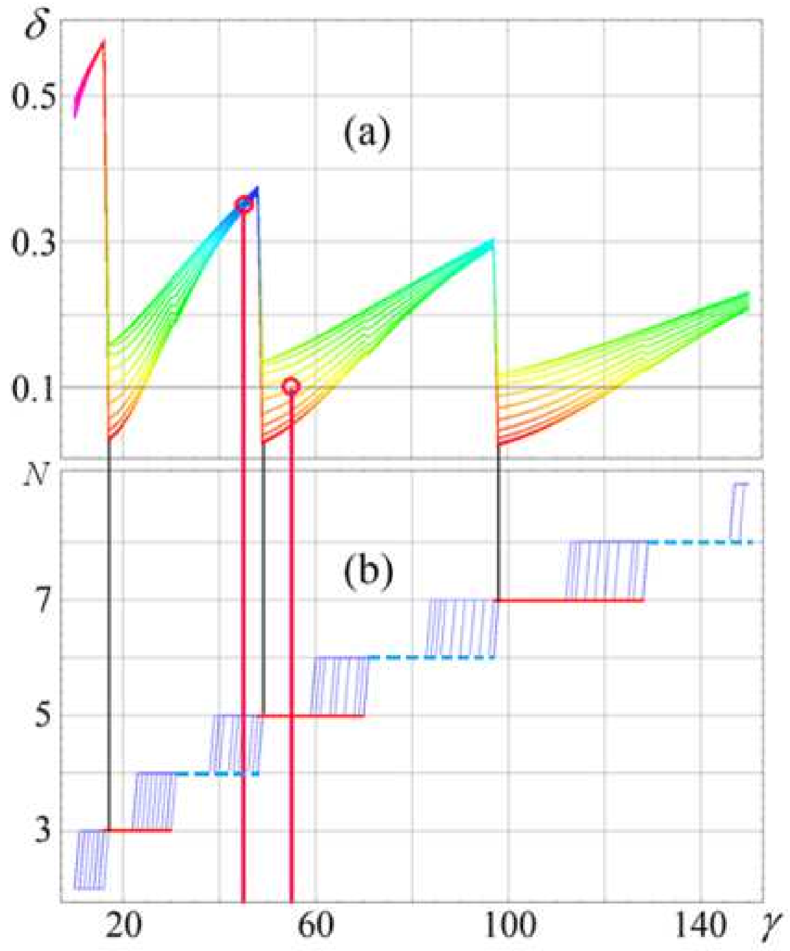


Fig.7

Conclusions

For the **parametric X-radiation at channeling** (PXRC) at zero **incident angle** of electrons with respect to the channeling planes **the difference δ** depends on the **relativistic factor** of channeled electrons, which defines the number of bound quantum channeling states.

Based on these idea **we predict:**

quantum jumps in angular distributions of parametric X-radiation from channeled relativistic electrons (PXRC)

The effect is connected with:

a)the number of quantum states of channeled electrons
b)form-factor of the transverse quantum channeling states
c)initial population of these quantum states.

Every **quantum jump** in PXRC angular distribution is connected with appearance of every new quantum channeling state with increase of the electron beam energy.

Thank you for attention