Increase of PXR intensity due to Borrmann effect

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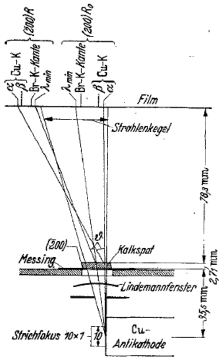

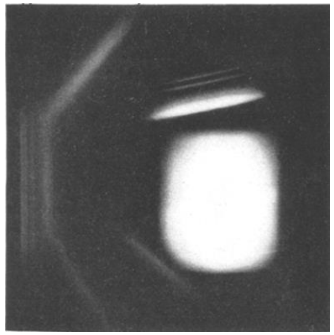

Zeitschrift für Physik, Bd. 127, S. 297—323 (1950).

Die Absorption von Röntgenstrahlen im Fall der Interferenz.

Von

G. BORRMANN.

- 1917: Dynamical diffraction theory by P.Ewald
- 1950: Borrmann effect – first time it was really used

Outline

- General expression for spectral-angular distribution of PXR
- Simplifications in thick crystal case
- Reduced density effect, increased intensity
- Outlook and conclusions

Spectral-angular distribution

In the Fraunhofer regime the Green function of Maxwell equation is

$$G(\vec{r},\vec{r}') \approx -\frac{e^{ik_0 r}}{4\pi r}\vec{E}^{(-)*}_{\vec{k}s}(\vec{r},\omega)$$

The spectral –angular distribution of emitted radiation from current

$$\vec{j}_0(\vec{r},\omega) = e_0 \int e^{i\omega t}\vec{v}(t)\delta[\vec{r}-\vec{r}(t)]dt$$

leads to

$$\boxed{\frac{\partial^2 N_{\vec{n},\omega s}}{\partial\omega\partial\Omega} = \frac{e_0^2\omega}{4\pi^2\hbar c^3}\left|\int \vec{E}^{(-)*}_{\vec{k}s}(\vec{r}(t),\omega)\vec{v}(t)e^{i\omega t}dt\right|^2}$$

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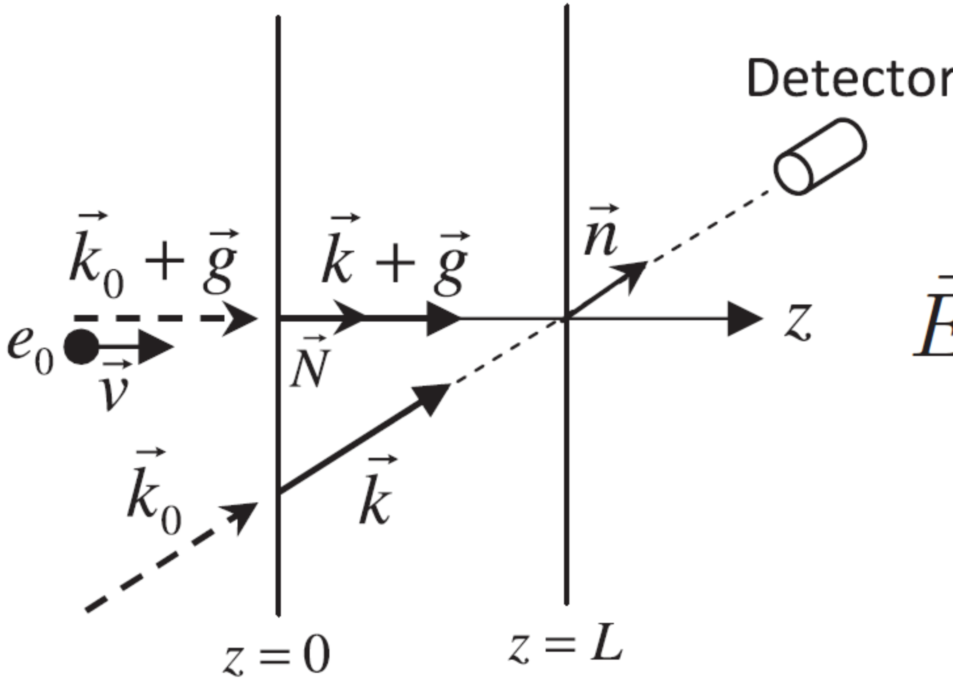
Wave field calculation: $\vec{E}^{(-)*}_{\vec{k}s}$

"Time-reversed" wave field



$$\vec{E}^{(-)*}_{\vec{k}s} = \vec{E}^{(+)}_{-\vec{k}s}$$

boundary condition at r→∞

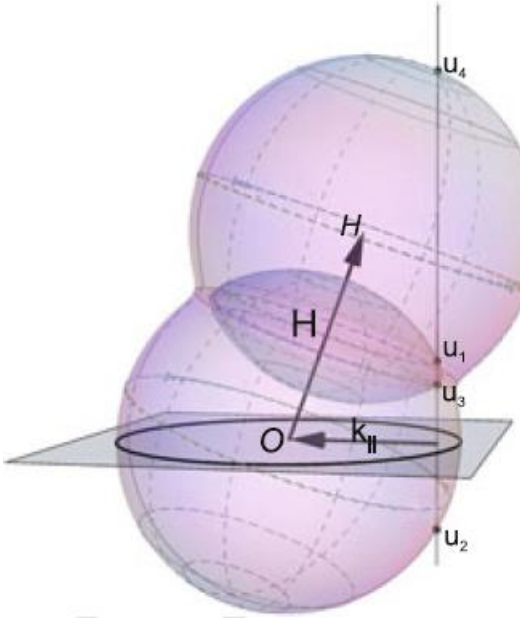
$$\vec{E}^{(-)}_{\vec{k}s}(\vec{r},\omega) \approx \vec{e}_s e^{i\vec{k}\vec{r}} + const\frac{e^{-i\vec{k}\vec{r}}}{r}$$

Wave field calculation: two beam dynamical diffraction

Set of equations for two strong waves

$$\left(\frac{k^2}{\omega^2} - 1 - \chi_0\right)\vec{E}^{(+)}_{\vec{k}s} - c_s\chi_{-\vec{g}}\vec{E}^{(+)}_{\vec{k}_g s} = 0$$

$$\left(\frac{k_g^2}{\omega^2} - 1 - \chi_0\right)\vec{E}^{(+)}_{\vec{k}_g s} - c_s\chi_{\vec{g}}\vec{E}^{(+)}_{\vec{k}s} = 0.$$



dispersion equation

$$\vec{k}_{\mu s} = \vec{k} + \frac{\omega}{c}\frac{\epsilon_{\mu s}}{\gamma_0}\vec{N}$$

solutions near Bragg conditions

$$\boxed{\epsilon_{\mu s} = \frac{1}{4}\{[\chi_0 + \beta\chi_0 - \beta\alpha_B] \pm \sqrt{[\chi_0 + \beta\chi_0 - \beta\alpha_B]^2 + 4\beta[\chi_0\alpha_B - (\chi_0^2 - c_s^2\chi_{\vec{g}}\chi_{-\vec{g}})]}\},}$$

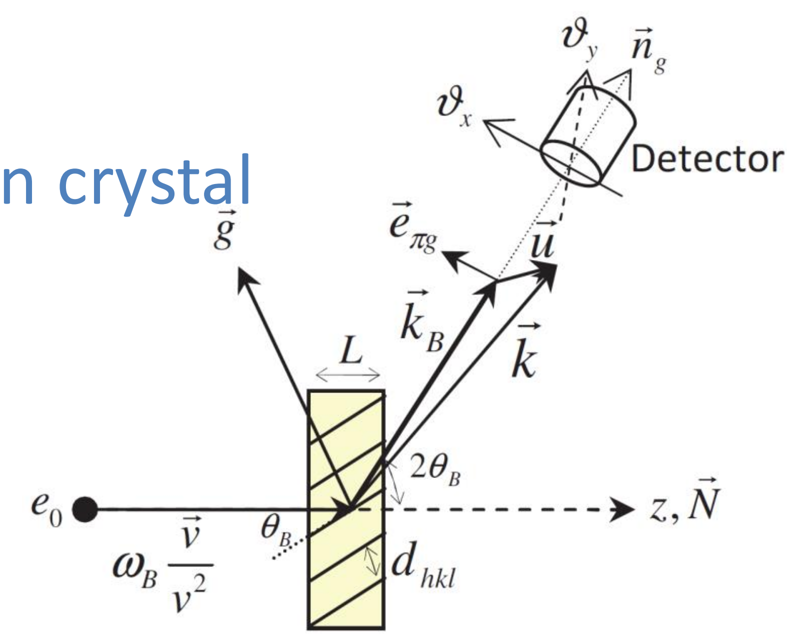
Spectral-angular distribution: general case

$$\frac{\partial^2 N_s^g}{\partial\vec{\Omega}\partial\omega} = \frac{e_0^2}{\hbar\omega\pi^2 c^3}(\vec{e}_{sg}\vec{n}_g)^2|\gamma_0|\beta^2|\sum_{\mu=1}^{2}\lambda_{\mu s}^g\boxed{\left[\frac{1}{q_0^g}-\frac{1}{q_{\mu s}^g}\right]}(1-e^{-ikLq_{\mu s}^g\gamma_0^{-1}})|^2$$

Transferred momentum and radiation coherent length

$$q_0^{(g)} = \frac{1}{k_B L_0^{(g)}} = \frac{m^2c^4}{E^2} + \theta_{(g)}^2;$$ in vacuum

$$q_{\mu s}^{(g)} = \frac{1}{k_B L_{\mu s}^{(g)}} = \frac{m^2c^4}{E^2} + \theta_{(g)}^2 - 2\epsilon_{\mu s};$$ in crystal



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Electron scattering in the medium

$$\theta_{(g)}^2 \rightarrow \tilde{\theta}_{(g)}^2 = \vartheta_x^2 + \vartheta_y^2 + \theta_s^2(L);\ \theta_s^2(L) = \frac{E_s^2}{E^2}\frac{L}{L_R},\quad E_s \approx 21\ \text{MeV}$$

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Spectral-angular distribution: when $|\chi_0''| \ll |\chi_0'|$

$$\frac{\partial^2 N_s^g}{\partial\vec{\Omega}\partial\omega} = \frac{e_0^2}{\hbar\omega\pi^2 c^3}(\vec{e}_{sg}\vec{n}_g)^2 |\cos 2\theta_B| \beta^2 \left| \sum_{\mu=1}^{2} \lambda_{\mu s}^g \frac{(1 - e^{-ikLq_{\mu s}^g / \cos 2\theta_B})}{q_{\mu s}^g} \right|^2$$

Under conditions $A \gg 1, |\Im q| \ll |\Re q|$ we can use

$$\left| \frac{1 - e^{-iAq}}{q} \right|^2 \approx \pi\delta(\Re q) \frac{1 - e^{-2A|\Im q|}}{|\Im q|}$$

at $\Re q_{\mu s}^{(g)} = 0$ we have $\Im q_{\mu s}^{(g)} = \dfrac{1}{k_B L_{abs}} \Phi_s(\eta^2)$

$$\Phi_s(\eta^2) = -\frac{(1+\beta)}{2} \left[1 - \frac{\beta\alpha - \chi_0'(1+\beta) - 2\beta(\alpha - \Delta_s''/\chi_0'')/(1+\beta)}{\sqrt{[\beta\alpha - \chi_0'(1+\beta)]^2 + 4\beta(\chi_0'\alpha - \Delta_s')}} \right]$$

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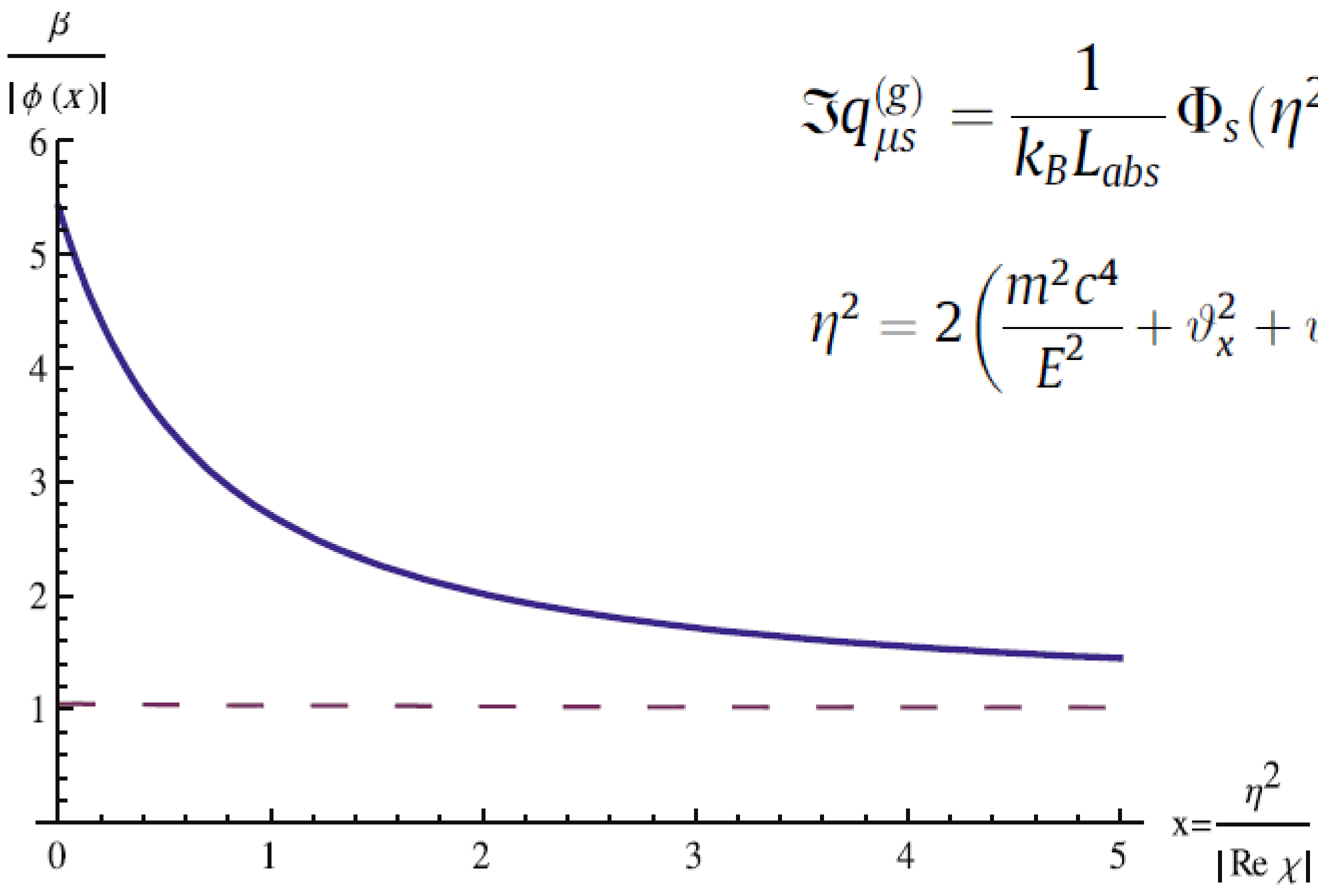


$$\Im q_{\mu s}^{(g)} = \frac{1}{k_B L_{abs}} \Phi_s(\eta^2)$$

$$\eta^2 = 2\left(\frac{m^2c^4}{E^2} + \vartheta_x^2 + \vartheta_y^2 + \theta_s^2\right)$$

Characteristic behavior of $\beta/\Phi_\sigma(x)$ (solid line) and $\beta/\Phi_\pi(x)$ (dashed line) for the reflection (220) in crystal Si. The radiation parameters are following: $\hbar\omega_B = 5.166$ keV, $\theta_B = 38.7^\circ$, $\beta = 0.219$.

Spectral-angular distribution: dynamical *vs.* kinematical (σ polarization)

Dynamical theory result

$$\boxed{\frac{\partial^2 N_\sigma^g}{\partial\vartheta_x\partial\vartheta_y} = N_g \frac{\vartheta_y^2 + 1/2\theta_s^2}{\left[\vartheta_x^2 + \vartheta_y^2 + \theta_0^2\right]^2 - \Delta'_\sigma - \left[\vartheta_x^2 + \vartheta_y^2 + \theta_{ph}^2\right]\Delta''_\sigma/\chi''_0}}$$

Kinematical

$$\boxed{\frac{\partial^2 \widetilde{N}_\sigma^g}{\partial\vartheta_x\partial\vartheta_y} = N_g \frac{\vartheta_y^2 + 1/2\theta_s^2}{\left[\vartheta_x^2 + \vartheta_y^2 + \theta_{ph}^2\right]^2}}$$

$$\theta_0^2 = \frac{m^2c^4}{E^2} + \theta_s^2$$

$$\theta_{ph}^2 = \frac{m^2c^4}{E^2} + \theta_s^2 + |\chi'_0|$$

Borrmann parameter $\Delta_s = \chi_0^2 - c_s^2\chi_g\chi_{-g} = \Delta'_s + i\Delta''_s$

Outcome: reduced density effect

Lengths that bound PXR outcome in **kinematical** approach:

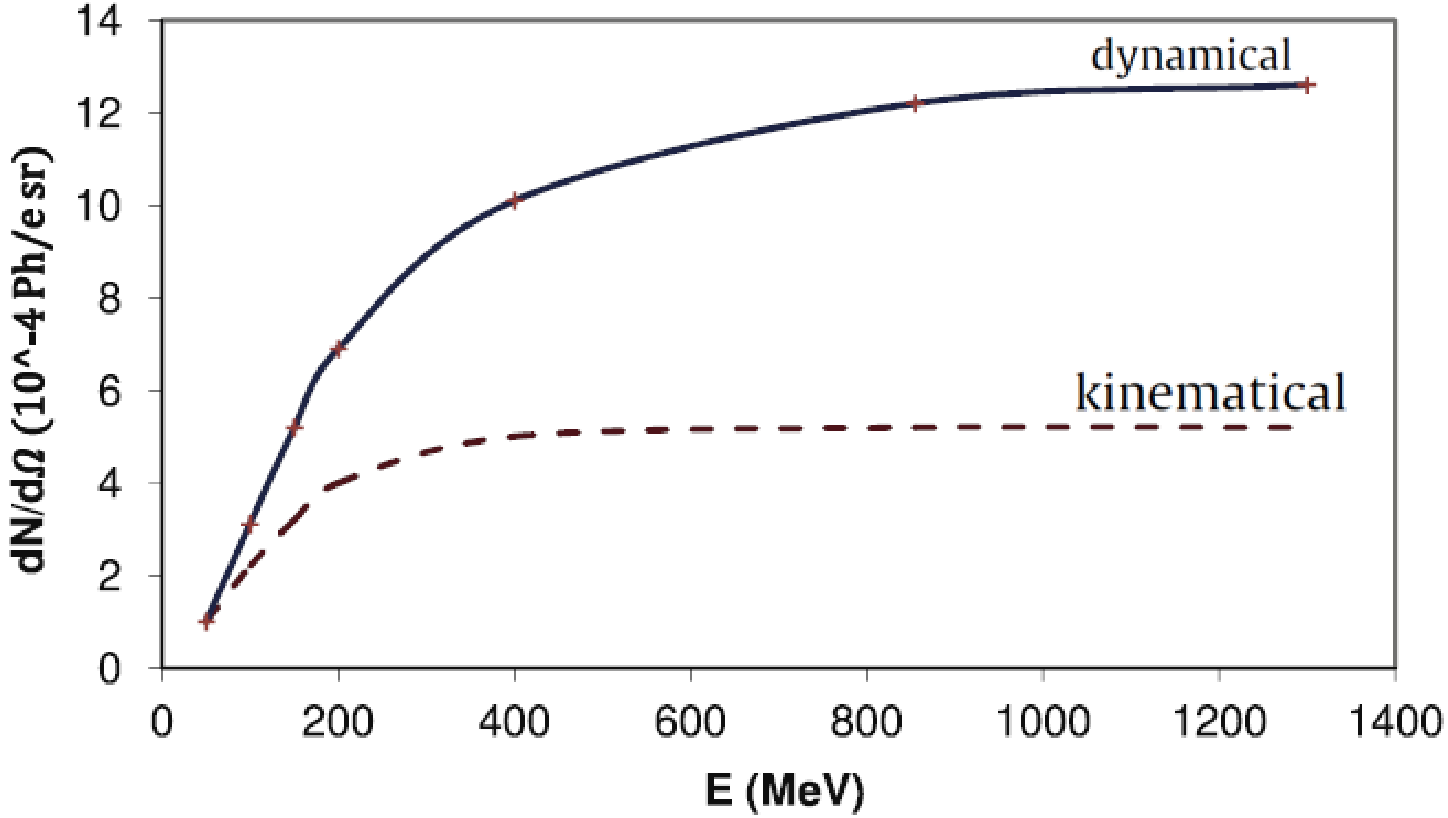
Crystal thickness L , absorption length $L_{abs} = \dfrac{1}{k_B \chi_0''(\omega_B)}$,

maximal vacuum coherence length $L_0^{(max)} = \dfrac{E^2}{k_B}\left[m^2 c^4 + E_s^2 \dfrac{L_{abs}}{L_R}\right]$

For maximal outcome it should be $L > L_{abs}; \quad L_0^{(max)} > L_{abs}$

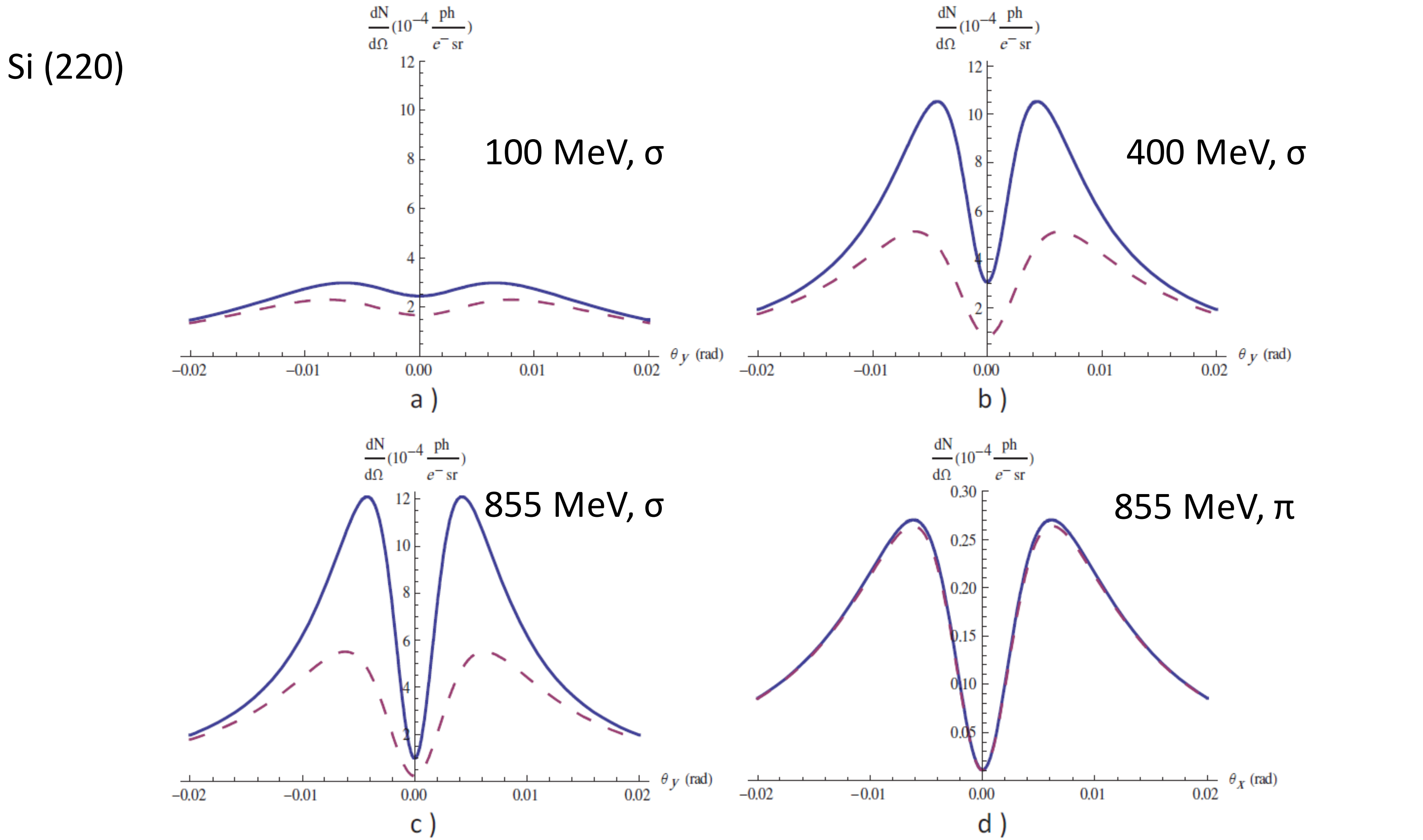
hence $$E > E_{th} = \sqrt{\frac{m^2 c^4 + E_s^2 \frac{L_{abs}}{L_R}}{|\chi_0''|}}$$

Outcome: reduced density effect



Dependence of the maximal PXR intensity on the electron energy for (220) reflection in crystal Si with thickness $L = 124\ \mu m$, $L_{abs} = 19.56\ \mu m$, $L_R = 9.4$ cm $\hbar\omega_B = 5.166$ keV, $\theta_B = 38.7^\circ$, $\beta = 0.219$

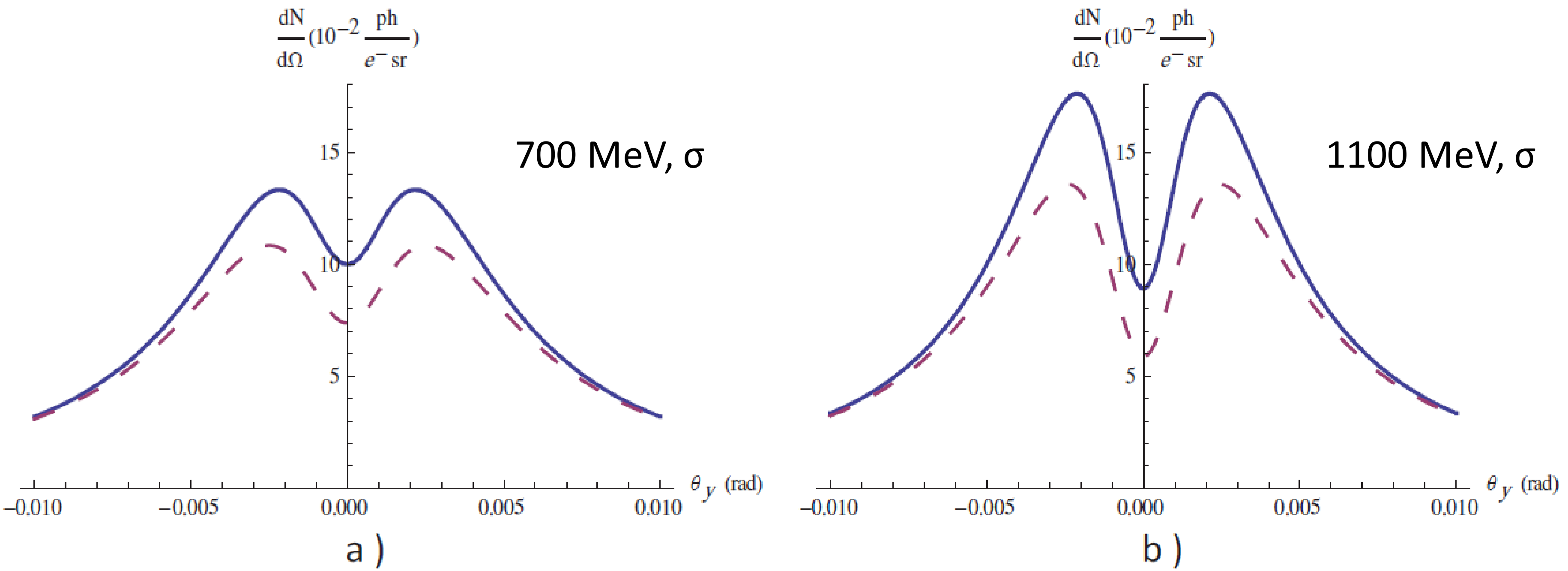
Outcome: increased intensity



Increase of the PXR intensity due to the Borrmann effect for (220) reflection in crystal Si with thickness $L = 124\ \mu m$, $L_{abs} = 19.56\ \mu m$, $L_R = 9.4$ cm, $\hbar\omega_B = 5.166$ keV, $\theta_B = 38.7^\circ$, $\beta = 0.219$ (solid curves correspond to the dynamical theory, dashed lines describe results of the kinematical theory): (a) $E = 100$ MeV, σ-polarization; (b) $E = 400$ MeV, σ-polarization; (c) $E = 855$ MeV, σ-polarization; (d) $E = 855$ MeV, π-polarization.

Outcome: increased intensity

Si (111)

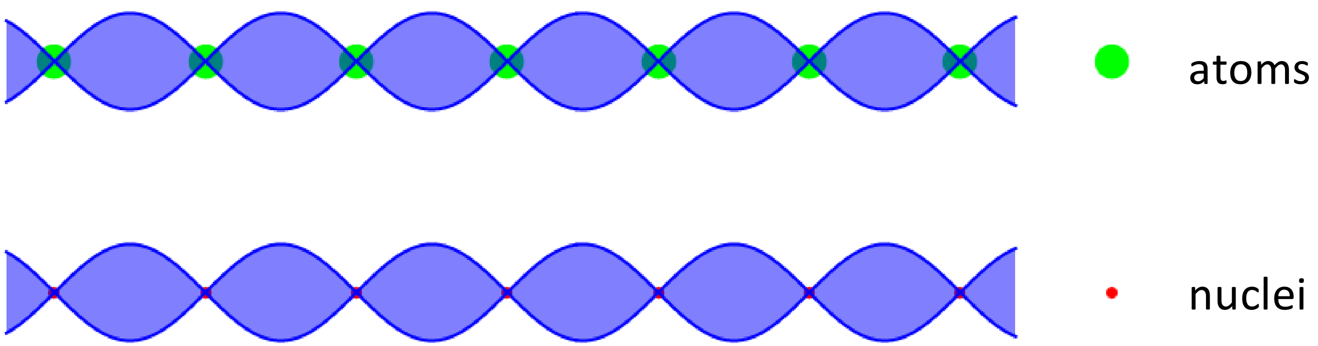


Increase of the PXR intensity for (111) reflection in crystal Si with thickness $L = 500$ µm, $L_{abs} = 392.6$ µm; $\hbar\omega_B = 14.4$ keV, $\theta_B = 7.89°$, $\beta = 0.962$, σ-polarization (solid curves correspond to the dynamical theory, dashed lines correspond to the kinematical one): (a) $E = 700$ MeV, (b) $E = 1100$ MeV.

Outlook

- Correct description of high-resolution experiments
- Lower threshold for Parametric Beam Instability
- More pronounced effect can be expected for

Parametric *Gamma* Radiation (PGR) *)



Borrmann parameter $\Delta_s = \chi_0^2 - \chi_g \chi_{-g}$ is smaller,

but assumption $|\chi_0''| \ll |\chi_0'|$ is wrong.

*) A. Ahmadi, I. Feranchuk, EPJAP 62 (2013) 10702

Conclusions

- Expressions for PXR distributions taking into account the dynamical diffraction treatment are formulated in compact way highlighting Borrmann parameter
- Dynamical treatment results in higher energies for density effect and higher PXR intensity
- These effects are expected to be more pronounced in the case of PGR

Thank you for your attention!