

ELECTROMAGNETIC FIELD OF CHARGED PARTICLE BUNCHES MOVING IN GYROTROPIC MEDIA

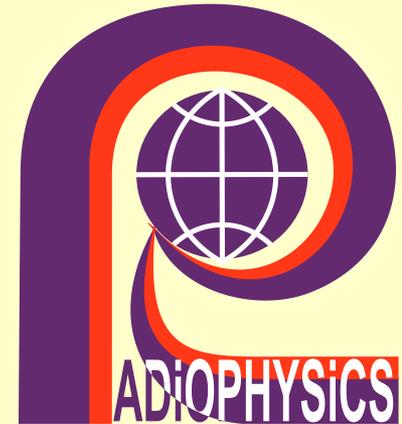
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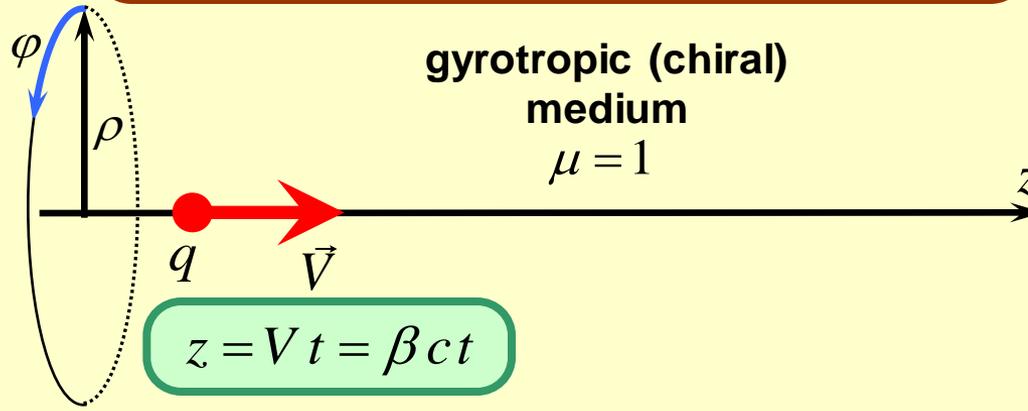


Physical
faculty



Radiophysics
Department

Formulation of the problem



Anisotropic chiral medium

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_1 & -i\varepsilon_2 & 0 \\ i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}$$

Isotropic chiral medium

$$\vec{D}_\omega = \begin{pmatrix} \varepsilon & \tilde{u} & 0 \\ \tilde{u} & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{pmatrix} \vec{E}_\omega, \quad \vec{B}_\omega = \vec{H}_\omega + \tilde{u} \vec{E}_\omega$$

$\vec{k} \uparrow \uparrow O_z$

Cold magnetized plasma

$$\varepsilon_1 = 1 - \frac{\omega_p^2 (\omega + iv)}{\omega [(\omega + iv)^2 - \omega_h^2]}$$

$$\varepsilon_2 = \frac{-\omega_p^2 \omega_h}{\omega [(\omega + iv)^2 - \omega_h^2]}$$

$$\varepsilon_3 = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\nu}$$

negligible losses

$$\nu \rightarrow 0$$

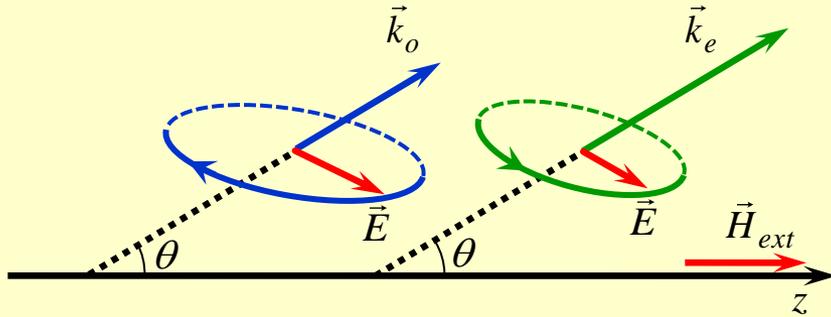
Condon medium

$$\varepsilon = 1 + \frac{\omega_p^2}{\omega_r^2 - \omega^2 - 2i\omega\nu}$$

$$\tilde{u} = \frac{\omega\omega_0}{\omega^2 - \omega_r^2}$$

General theory

Cold magnetized plasma

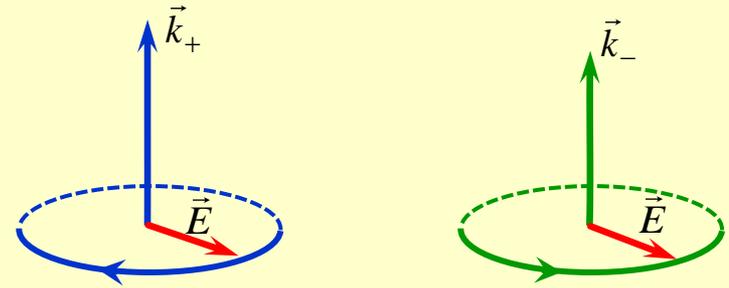


complex elliptical polarization

$$\omega_p^2 = 4\pi N e^2 / m \quad \omega_h = |e| H_{ext} / (m c)$$

$$n_{o,e}^2 = 1 - 2\omega_p^2 (\omega^2 - \omega_p^2) \left[2(\omega^2 - \omega_p^2)\omega^2 - \omega_h^2 \omega^2 \sin^2 \theta \pm \sqrt{\omega^4 \omega_h^4 \sin^4 \theta + 4\omega_h^2 \omega^2 (\omega^2 - \omega_p^2)^2 \cos^2 \theta} \right]^{-1}$$

Condon medium



right-hand circular polarization

left-hand circular polarization

$$n_{\pm} = \frac{\sqrt{\omega^2 - \omega_{\Sigma}^2} \sqrt{\omega^2 - \omega_r^2} \pm \omega_0 \omega}{\omega^2 - \omega_r^2}$$

$$\omega_{\Sigma}^2 = \omega_r^2 + \omega_p^2$$

General theory

Cold magnetized plasma

$$\begin{Bmatrix} E_{\rho,\varphi,z} \\ H_{\rho,\varphi,z} \end{Bmatrix} = 2 \operatorname{Re} \int_0^{+\infty} \begin{Bmatrix} E_{\omega\rho,\varphi,z}^{(o)} + E_{\omega\rho,\varphi,z}^{(e)} \\ H_{\omega\rho,\varphi,z}^{(o)} + H_{\omega\rho,\varphi,z}^{(e)} \end{Bmatrix} e^{-i\omega t} d\omega$$

$$E_{\omega\rho}^{(o,e)} = \frac{\mp i q s_{o,e} H_1^{(1)}(\rho s_{o,e})}{2\nu \varepsilon_1 (s_e^2 - s_o^2)} \left(\frac{\omega^2 (1 - \varepsilon_1 \beta^2)}{\nu^2} + s_{o,e}^2 \right) \exp\left(\frac{i\omega z}{\nu}\right)$$

$$E_{\omega\varphi}^{(o,e)} = \pm \frac{q}{2\nu} \frac{\omega^2 \varepsilon_2 s_{o,e} H_1^{(1)}(\rho s_{o,e})}{c^2 \varepsilon_1 (s_e^2 - s_o^2)} \exp(i\omega z/\nu)$$

$$E_{\omega z}^{(o,e)} = \frac{\mp q}{2c^2} \frac{\omega H_0^{(1)}(\rho s_{o,e})}{\varepsilon_1 (s_e^2 - s_o^2)} \exp(i\omega z/\nu) \times$$

$$\times \left[s_{o,e}^2 \left(\frac{1}{\beta^2} - \varepsilon_1 \right) + \frac{\omega^2}{c^2} \left(\varepsilon_1^2 - \varepsilon_2^2 - \frac{2\varepsilon_1}{\beta^2} + \frac{1}{\beta^4} \right) \right]$$

$$s_{o,e}^2 = \frac{-B \mp \sqrt{B^2 - 4\varepsilon_1 C}}{2\varepsilon_1}$$

$$B = -\frac{\omega^2}{\nu^2} \left[(\varepsilon_1 \beta^2 - 1)(\varepsilon_1 + \varepsilon_3) - \varepsilon_2^2 \beta^2 \right]$$

$$C = -\frac{\omega^4}{\nu^4} \varepsilon_3 \left[\beta^4 \varepsilon_2^2 - (\varepsilon_1 \beta^2 - 1)^2 \right]$$

$$s_{\pm} = \sqrt{s_{\pm}^2}$$

$$s_{o,e} = \sqrt{s_{o,e}^2}$$

$$\operatorname{Im} s > 0$$

Condon medium

$$\begin{Bmatrix} E_{\rho,\varphi,z} \\ H_{\rho,\varphi,z} \end{Bmatrix} = 2 \operatorname{Re} \int_0^{+\infty} \begin{Bmatrix} E_{\omega\rho,\varphi,z}^+ + E_{\omega\rho,\varphi,z}^- \\ H_{\omega\rho,\varphi,z}^+ + H_{\omega\rho,\varphi,z}^- \end{Bmatrix} e^{-i\omega t} d\omega$$

$$E_{\omega\rho}^{\pm} = \frac{i q}{4\nu} \frac{\mu}{\sqrt{\varepsilon\mu}} \frac{1}{n_{\pm}} H_1^{(1)}(s_{\pm}\rho) s_{\pm} \exp(i\omega z/\nu)$$

$$E_{\omega\varphi}^{\pm} = \pm \frac{q}{4c} \frac{\mu}{\sqrt{\varepsilon\mu}} H_1^{(1)}(s_{\pm}\rho) s_{\pm} \exp(i\omega z/\nu)$$

$$E_{\omega z}^{\pm} = \frac{-q}{4} \frac{\mu}{\sqrt{\varepsilon\mu}} H_0^{(1)}(s_{\pm}\rho) \frac{s_{\pm}^2}{\omega n_{\pm}} \exp(i\omega z/\nu)$$

$$\vec{H}_{\omega}^+ = i \frac{\sqrt{\varepsilon\mu}}{\mu} \vec{E}_{\omega}^+$$

$$\vec{H}_{\omega}^- = -i \frac{\sqrt{\varepsilon\mu}}{\mu} \vec{E}_{\omega}^-$$

$$s_{\pm}^2 = \frac{\omega^2}{\nu^2} (n_{\pm}^2 \beta^2 - 1)$$

$$n_{\pm} = \sqrt{\varepsilon\mu} \pm \dot{u}$$

General theory

Cold magnetized plasma

$$\sqrt{B^2(-\bar{\omega}) - 4\varepsilon_1(-\bar{\omega})C(-\bar{\omega})} = -\sqrt{B^2(\omega) - 4\varepsilon_1(\omega)C(\omega)}$$

$$s_o(-\bar{\omega}) = -\overline{s_e(\omega)}$$

$$\left\{ \begin{array}{l} E_{\omega\rho,\varphi,z}^{(o)}(-\bar{\omega}) \\ H_{\omega\rho,\varphi,z}^{(o)}(-\bar{\omega}) \end{array} \right\} = \left\{ \begin{array}{l} \overline{E_{\omega\rho,\varphi,z}^{(e)}(\omega)} \\ \overline{H_{\omega\rho,\varphi,z}^{(e)}(\omega)} \end{array} \right\}$$

$$\left\{ \begin{array}{l} E_{\rho,\varphi,z} \\ H_{\rho,\varphi,z} \end{array} \right\} = 2 \operatorname{Re} \int_{-\infty}^{+\infty} d\omega \left\{ \begin{array}{l} E_{\omega\rho,\varphi,z}^{(e)} \\ H_{\omega\rho,\varphi,z}^{(e)} \end{array} \right\} \exp(-i\omega t)$$

Condon medium

$$\sqrt{\varepsilon(-\bar{\omega})\mu(-\bar{\omega})} = \sqrt{\varepsilon(\omega)\mu(\omega)}$$

$$n_{\pm}(-\bar{\omega}) = \overline{n_{\mp}(\omega)}$$

$$s_+(-\bar{\omega}) = -\overline{s_-(\omega)}$$

$$\left\{ \begin{array}{l} E_{\omega\rho,\varphi,z}^{-}(-\bar{\omega}) \\ H_{\omega\rho,\varphi,z}^{-}(-\bar{\omega}) \end{array} \right\} = \left\{ \begin{array}{l} \overline{E_{\omega\rho,\varphi,z}^{+}(\omega)} \\ \overline{H_{\omega\rho,\varphi,z}^{+}(\omega)} \end{array} \right\}$$

$$\left\{ \begin{array}{l} E_{\rho,\varphi,z} \\ H_{\rho,\varphi,z} \end{array} \right\} = 2 \operatorname{Re} \int_{-\infty}^{+\infty} d\omega \left\{ \begin{array}{l} E_{\omega\rho,\varphi,z}^{+} \\ H_{\omega\rho,\varphi,z}^{+} \end{array} \right\} \exp(-i\omega t)$$

Asymptotic properties of integrands

$$s_{o,e} \xrightarrow{|\omega| \rightarrow \infty} i \frac{\sqrt{1-\beta^2}}{\nu} \omega \operatorname{sgn}(\operatorname{Re} \omega)$$

$$s_{\pm} \xrightarrow{|\omega| \rightarrow \infty} i \frac{\sqrt{1-\beta^2}}{\nu} \omega \operatorname{sgn}(\operatorname{Re} \omega)$$

SDP asymptote

$$\operatorname{Im} \omega = \frac{|\operatorname{Re} \omega|(z - vt)}{\rho\sqrt{1-\beta^2}}$$

Sectors of integrands vanishing

$$\operatorname{Im} \omega > -|\operatorname{Re} \omega| \frac{\rho\sqrt{1-\beta^2}}{z - vt}, \quad z - vt < 0; \quad \operatorname{Im} \omega < |\operatorname{Re} \omega| \frac{\rho\sqrt{1-\beta^2}}{|z - vt|}, \quad z - vt > 0$$

Analytical results (with surprises)

Cold magnetized plasma

$$s_e^2 = \frac{1}{v^2(\omega^2 - \omega_\Sigma^2)} \left[(1 - \beta^2) \left(-\omega^4 + \omega^2 \omega_\Sigma^2 - \omega_p^2 \omega_h^2 / 2 \right) + \beta^2 \omega_p^4 - \beta^2 \omega^2 \omega_p^2 + \beta \omega_p^2 \omega_h \sqrt{\omega^2 - \omega_c^2} \right]$$

$$\omega_\Sigma = \sqrt{\omega_p^2 + \omega_h^2} \quad \omega_c^2 = \omega_p^2 - \omega_h^2 (1 - \beta^2)^2 / 4\beta^2$$

exact roots

$$s_e^2 = \frac{(\beta^2 - 1)(u - u_1)(u - u_3)(u - u_4)}{v^2(u - u_2)} \quad \sqrt{\omega^2 - \omega_c^2} = u$$

$$u_{1,2} = \frac{\omega_h(1 \mp \beta^2)}{2\beta} \quad u_{3,4} = \frac{\omega_h \beta}{2} \mp \frac{1}{2\beta} \sqrt{\frac{\omega_h^2}{4} - \frac{\omega_p^2 \beta^2}{1 - \beta^2}}$$

$$\omega_1 = \omega_p \quad \omega_2 = \omega_\Sigma \quad \omega_{3,4} = \frac{\omega_h}{2} \pm \sqrt{\frac{\omega_h^2}{4} - \frac{\omega_p^2 \beta^2}{1 - \beta^2}}$$

relativistic motion

$$\beta > \beta_1 = \frac{\omega_h}{\sqrt{\omega_h^2 + 4\omega_p^2}} \quad \omega_{3,4} - \text{complex}$$

Condon medium

$$s_+^2 = \frac{\omega^2}{v^2(\omega^2 - \omega_r^2)^{3/2}} \left[2\beta^2 \omega_0 \omega \sqrt{\omega^2 - \omega_\Sigma^2} + \left[(1 - \beta^2)(\omega_r^2 - \omega^2) - \omega_p^2 \beta^2 \right] \sqrt{\omega^2 - \omega_r^2} \right]$$

$$\omega_\Sigma = \sqrt{\omega_r^2 + \omega_p^2}$$

ultrarelativistic motion

$$\gamma = 1/\sqrt{1 - \beta^2} \gg 1$$

weak chirality

$$\omega_0 \ll \omega_r, \omega_p$$

low frequencies

$$\omega \sim \omega_r, \omega_p$$

$$\omega_{l,2} = \omega_r \left(1 - \omega_0^2 / 2\omega_p^2 \pm \omega_r \omega_0^3 / \beta \omega_p^4 \right)$$

$$\omega_c(\alpha) = \omega_r + r(\alpha) \exp(i\alpha) \quad -\pi < \alpha < \pi$$

$$r(\alpha) = \frac{\omega_r \omega_0^2 \left[\cos(3\alpha/2) - \sqrt{(1 + \cos \alpha)/2} \right]^2}{2\omega_p^2 \sin^2 \alpha}$$

high frequencies

$$|\omega| \gg \omega_r, \omega_p$$

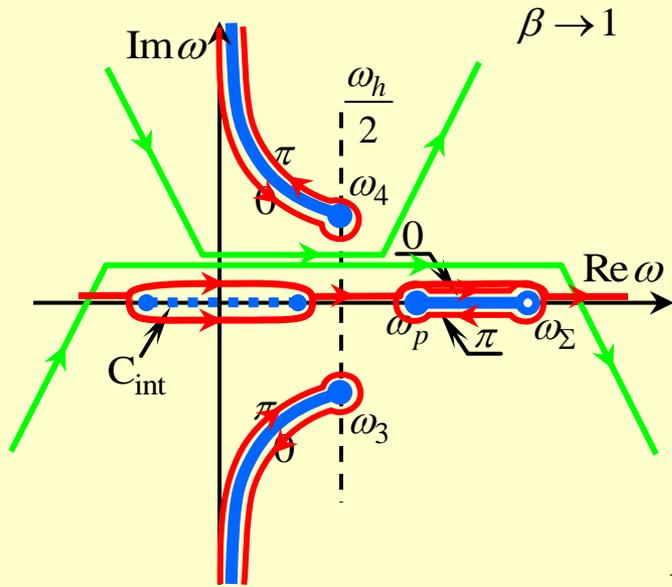
$$\omega_{h,1,2} = \omega_0 \gamma^2 \pm \sqrt{\omega_0^2 \gamma^4 - \omega_p^2 \gamma^2}$$

$$\gamma_{\max} = \omega_p / \omega_0$$

$$\gamma > \gamma_{\max} \Rightarrow \omega_{h,1,2} - \text{real}$$

Analytical results

Cold magnetized plasma



$$E_\rho = E_\rho^W + E_\rho^C$$

wave field (VCR)

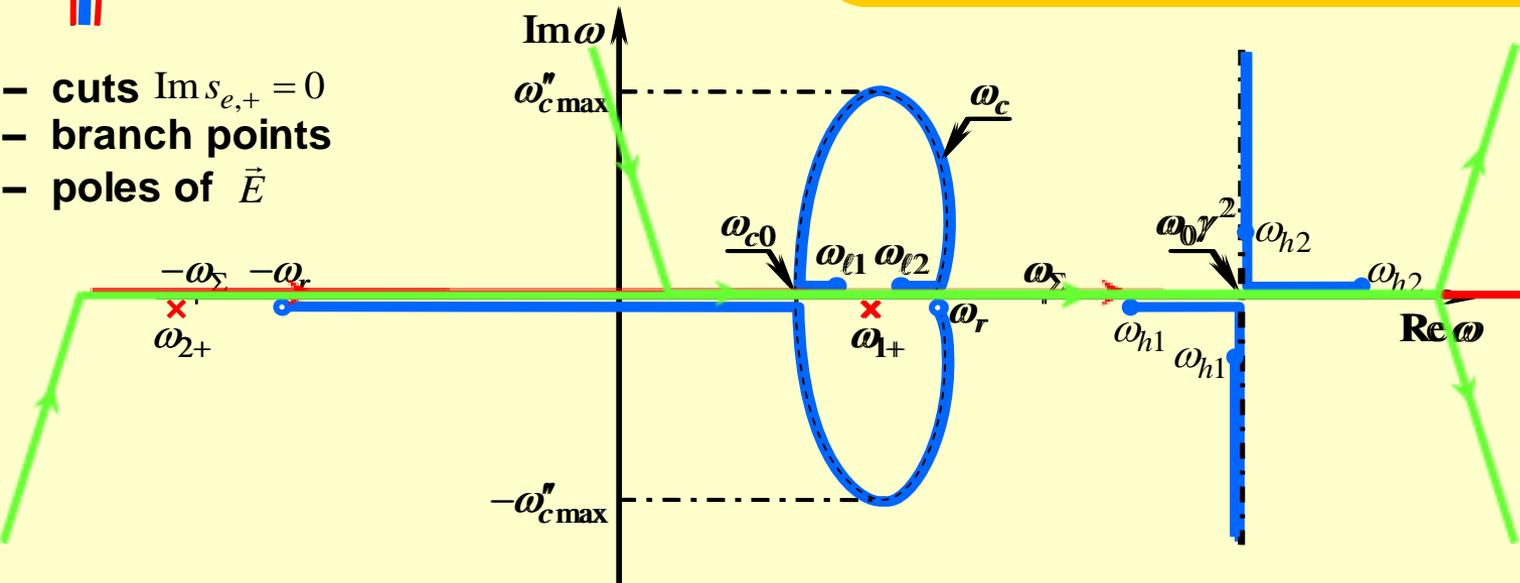
$$E_\rho^W = \Phi(-\zeta) \int_{\omega_p}^{\omega_\Sigma} \frac{qcs_e}{\omega_p^2 \omega_h} \frac{\omega_h^2 - \omega^2}{\sqrt{\omega^2 - \omega_c^2}} \left(s_e^2 + \frac{\omega^2}{v^2} - \frac{\omega^2 \epsilon_1}{c^2} \right) J_1(\rho s_e) \sin\left(\frac{\omega \zeta}{v}\right) d\omega$$

quasistatic field

$$E_\rho^C = \int_{\omega_4}^{+i\infty} \text{Im} \left[\frac{qcs_e}{\omega_p^2 \omega_h} \frac{\omega_h^2 - \omega^2}{\sqrt{\omega^2 - \omega_c^2}} \left(s_e^2 + \frac{\omega^2}{v^2} - \frac{\omega^2 \epsilon_1}{c^2} \right) J_1(\rho s_e) \exp\left(\frac{i\omega|\zeta|}{v}\right) d\omega \right]$$

Condon medium

- - cuts $\text{Im } s_{e,+} = 0$
- - branch points
- × - poles of \vec{E}



Analytical results

Cold magnetized plasma

$$\rho \rightarrow 0$$

$$\{E_\rho, H_{z,\varphi}\} \approx \{E_{\rho 0}, H_{z0,\varphi 0}\} \sin(\omega_\Sigma \zeta / v)$$

$$\{H_\rho, E_{z,\varphi}\} \approx \{H_{\rho 0}, E_{z0,\varphi 0}\} \cos(\omega_\Sigma \zeta / v)$$

singular

$$E_{\rho 0} = \frac{2q\omega_p^2}{v\omega_\Sigma \rho} \quad E_{z0} = \frac{2q\omega_p^2}{v^2} \ln\left(\frac{\rho\omega_p}{c}\right)$$

$$H_{z0} = \frac{2q\omega_p^2 \omega_h}{vc\omega_\Sigma} \ln\left(\frac{\rho\omega_p}{c}\right)$$

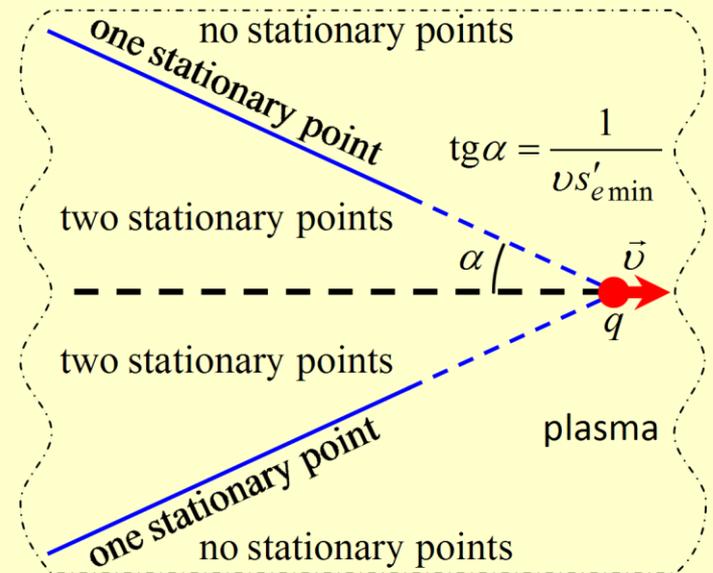
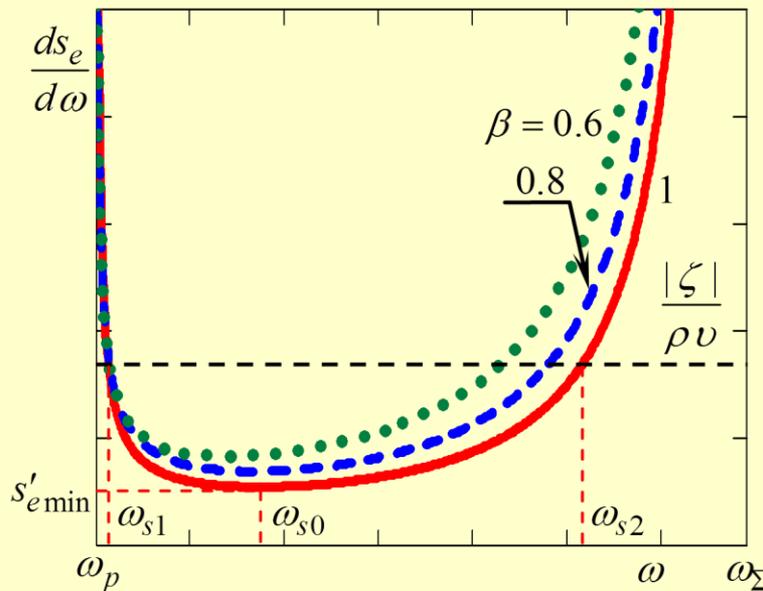
vanishing

$$H_{\rho 0} \approx \frac{-E_{\varphi 0}}{\beta} \quad E_{\varphi 0} = \frac{q\omega_p^2 \omega_h}{vc^2} \rho \ln\left(\frac{\rho\omega_p}{c}\right)$$

$$H_{\varphi 0} = \frac{q\omega_p^2 \omega_h^2}{cv^2 \omega_\Sigma} \rho \ln\left(\frac{\rho\omega_p}{c}\right)$$

$$\rho |s_e| \gg 1$$

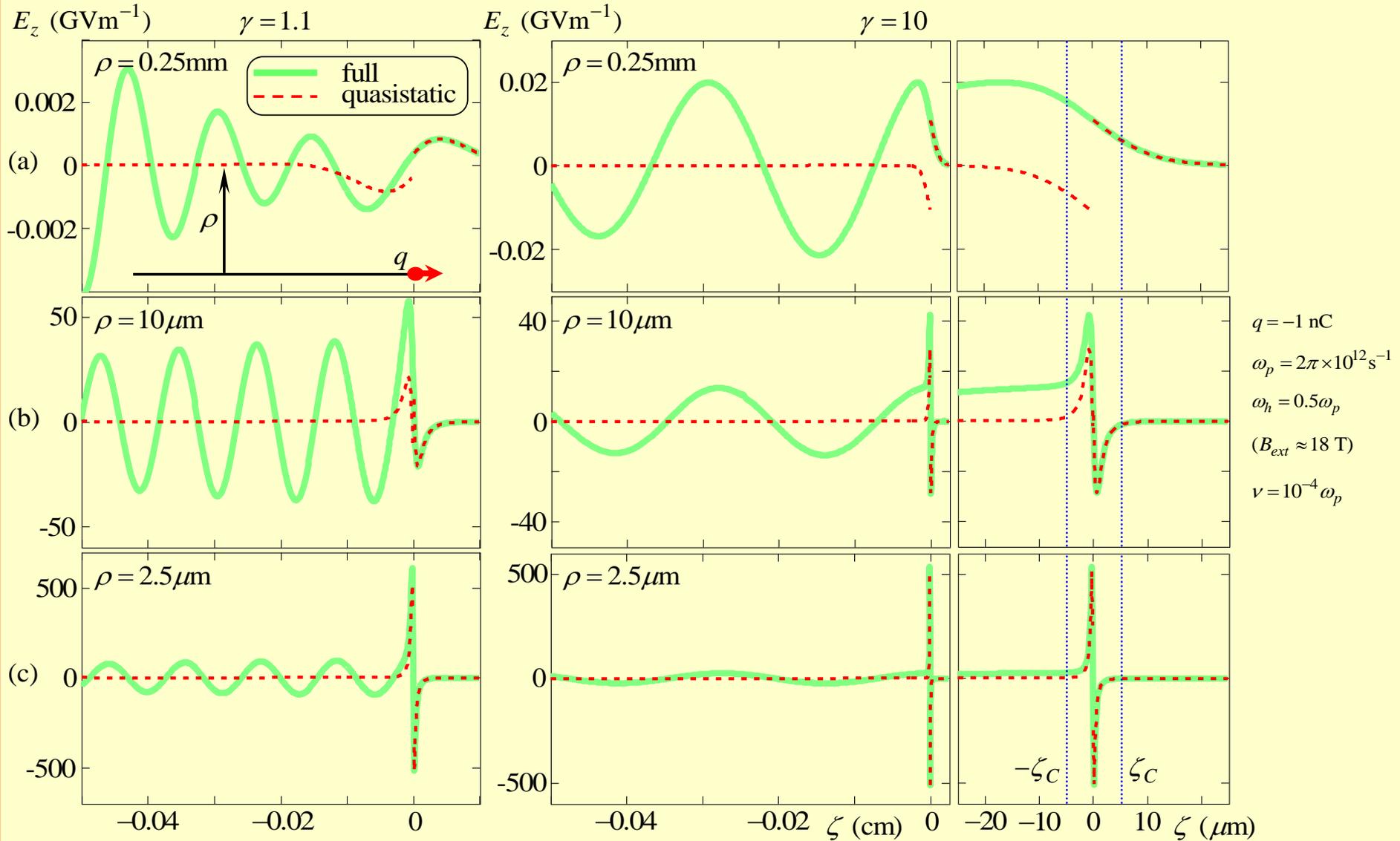
stationary point approach



$$E_\rho^W \approx \frac{1}{\rho} \left[\frac{-\tilde{e}_\rho^W(\omega_{s1})}{\sqrt{s_{e1} |s_{e1}''|}} \sin\left(\rho s_{e1} - \frac{\omega_{s1} |\zeta|}{v}\right) + \frac{\tilde{e}_\rho^W(\omega_{s2})}{\sqrt{s_{e2} |s_{e2}''|}} \cos\left(\rho s_{e2} - \frac{\omega_{s2} |\zeta|}{v}\right) \right]$$

Numerical results: point charge

Cold magnetized plasma

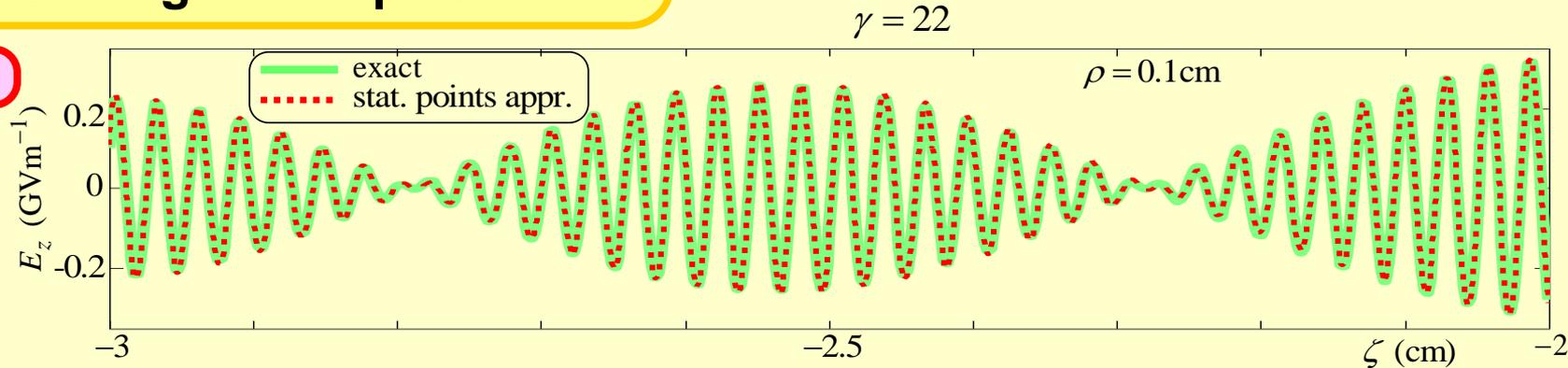


Numerical results: point charge & bunches

Cold magnetized plasma

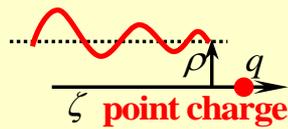
$$\rho |s_e| \gg 1$$

$q = -1 \text{ nC}$
 $\omega_p = 2\pi \times 10^{12} \text{ s}^{-1}$
 $\omega_h = 0.5\omega_p$
 $(B_{ext} \approx 18 \text{ T})$
 $\nu = 10^{-4} \omega_p$

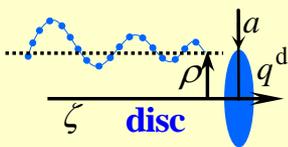
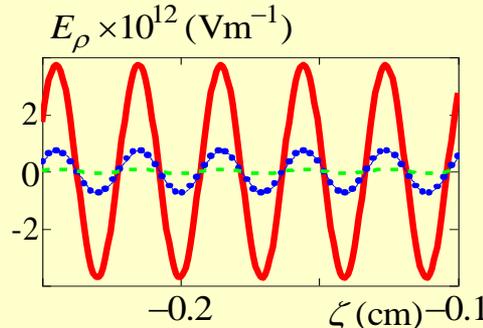


$$\rho \rightarrow 0$$

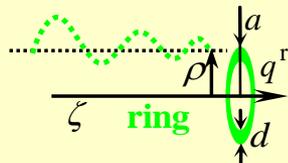
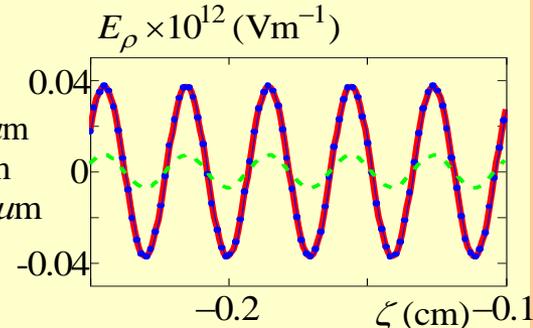
$q = q^d = -1 \text{ nC}$
 $q^r = \frac{q(2ad - d^2)}{a^2}$
 $\omega_p = 2\pi \times 10^{12} \text{ s}^{-1}$
 $\omega_h = 0.14\omega_p$
 $(B_{ext} \approx 5 \text{ T})$



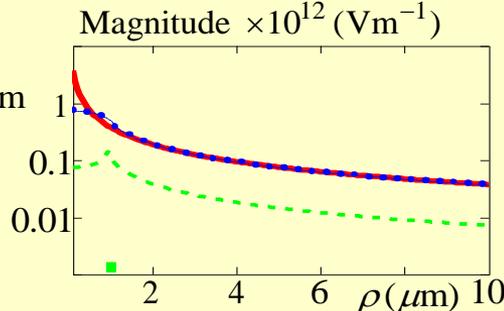
$\rho = 0.1 \mu\text{m}$
 $a = 1 \mu\text{m}$
 $d = 0.1 \mu\text{m}$



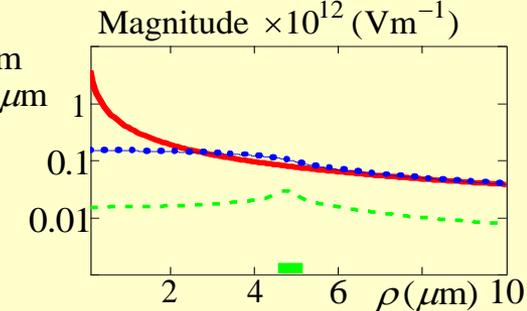
$\rho = 10 \mu\text{m}$
 $a = 1 \mu\text{m}$
 $d = 0.1 \mu\text{m}$



$a = 1 \mu\text{m}$
 $d = 0.1 \mu\text{m}$



$a = 5 \mu\text{m}$
 $d = 0.5 \mu\text{m}$

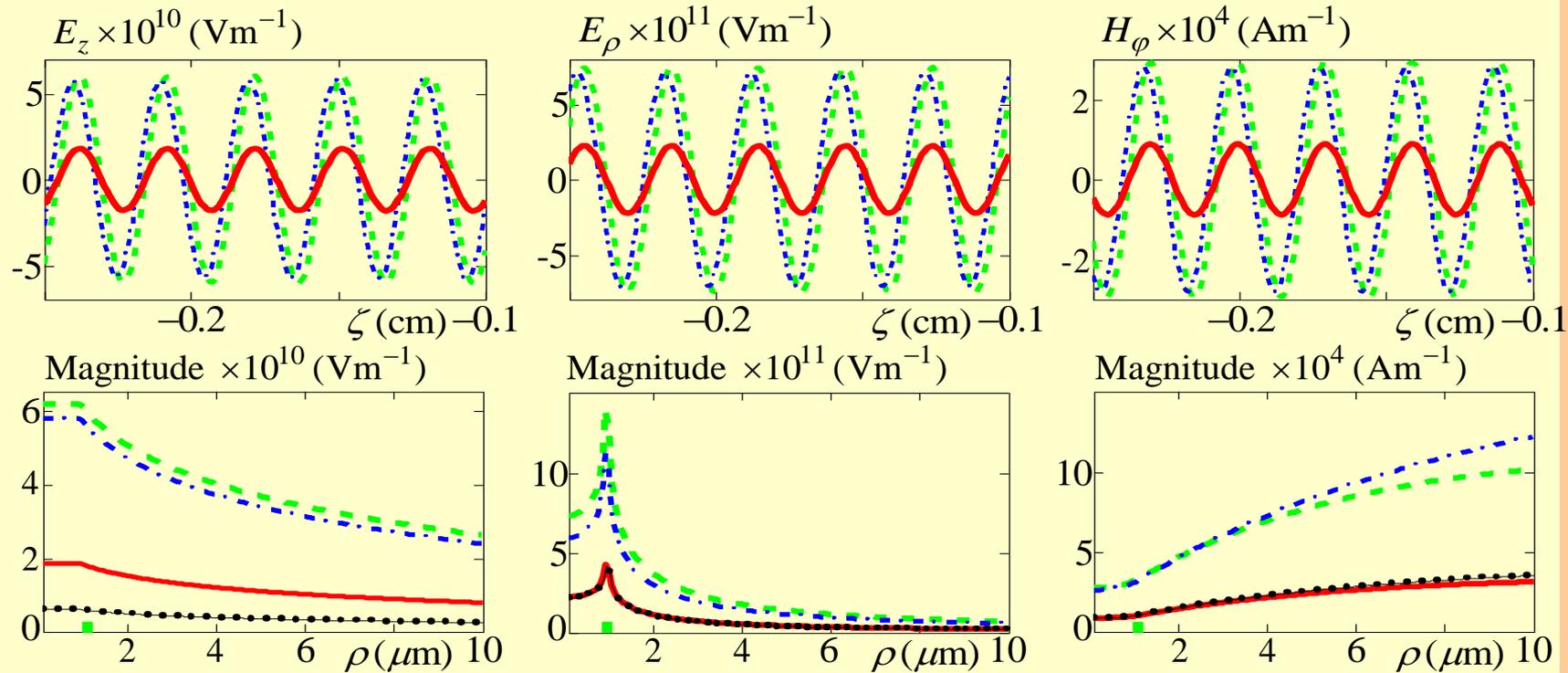
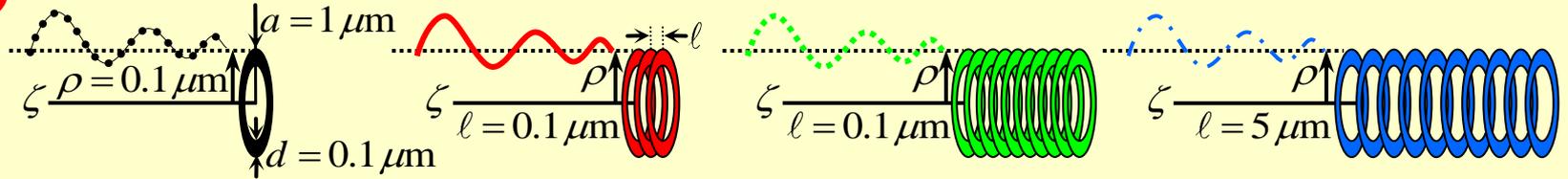


$$\{\bar{E}^b, \bar{H}^b\} = \iiint_{V^b} \{\bar{E}, \bar{H}\} \left(\sqrt{(x-x')^2 + (y-y')^2}, \zeta - \zeta' \right) \rho^b(x', y', \zeta') dx' dy' d\zeta'$$

Numerical results: bunches

Cold magnetized plasma

$\rho \rightarrow 0$



$$q = q^d = -\ln C$$

$$\omega_p = 2\pi \times 10^{12} \text{ s}^{-1}$$

$$\omega_h = 0.14 \omega_p$$

$$(B_{ext} \approx 5 \text{ T})$$

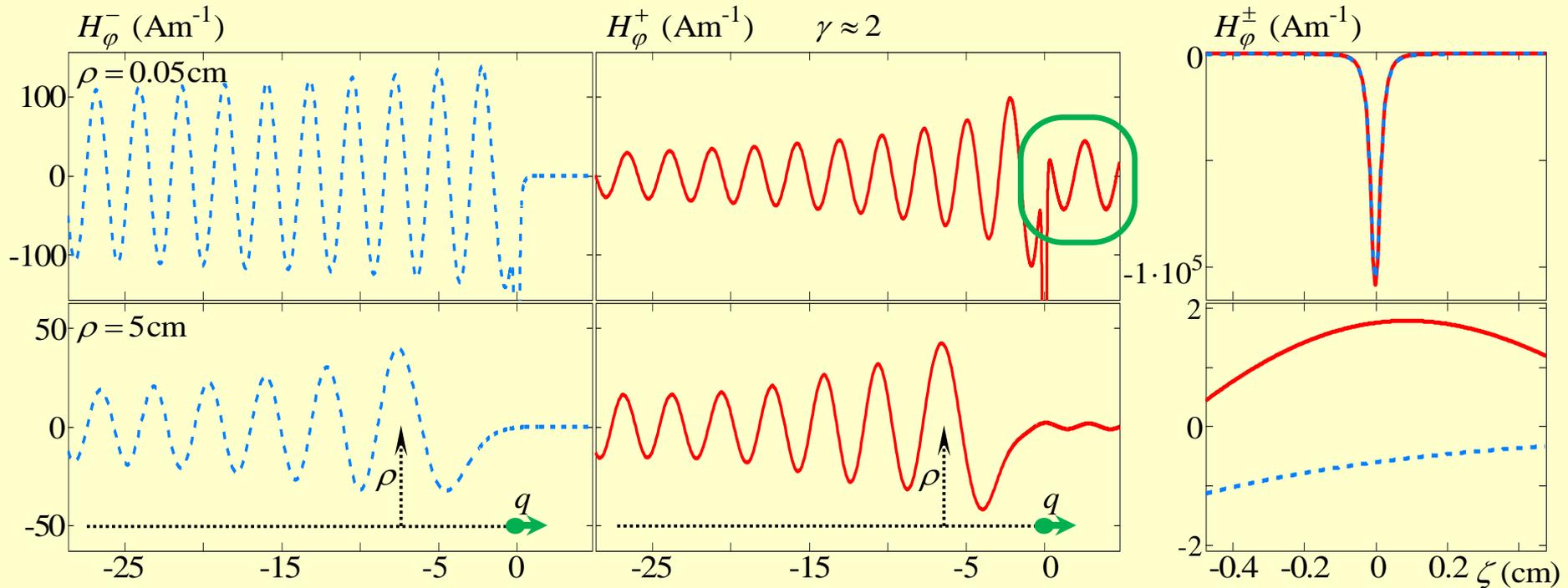
$$q^r = \frac{q(2ad - d^2)}{a^2}$$

$$\{\vec{E}^b, \vec{H}^b\} = \iiint_{V^b} \{\vec{E}, \vec{H}\} \left(\sqrt{(x-x')^2 + (y-y')^2}, \zeta - \zeta' \right) \rho^b(x', y', \zeta') dx' dy' d\zeta'$$

Numerical results

Condon medium

$$\gamma < \gamma_{\max} = 10$$



$$q = -1 \text{ nC}$$

$$\omega_r = \omega_p = 2\pi \cdot 10 \text{ GHz}$$

$$\lambda_r = 3 \text{ cm}$$

$$\omega_0 = 0.1 \omega_p$$

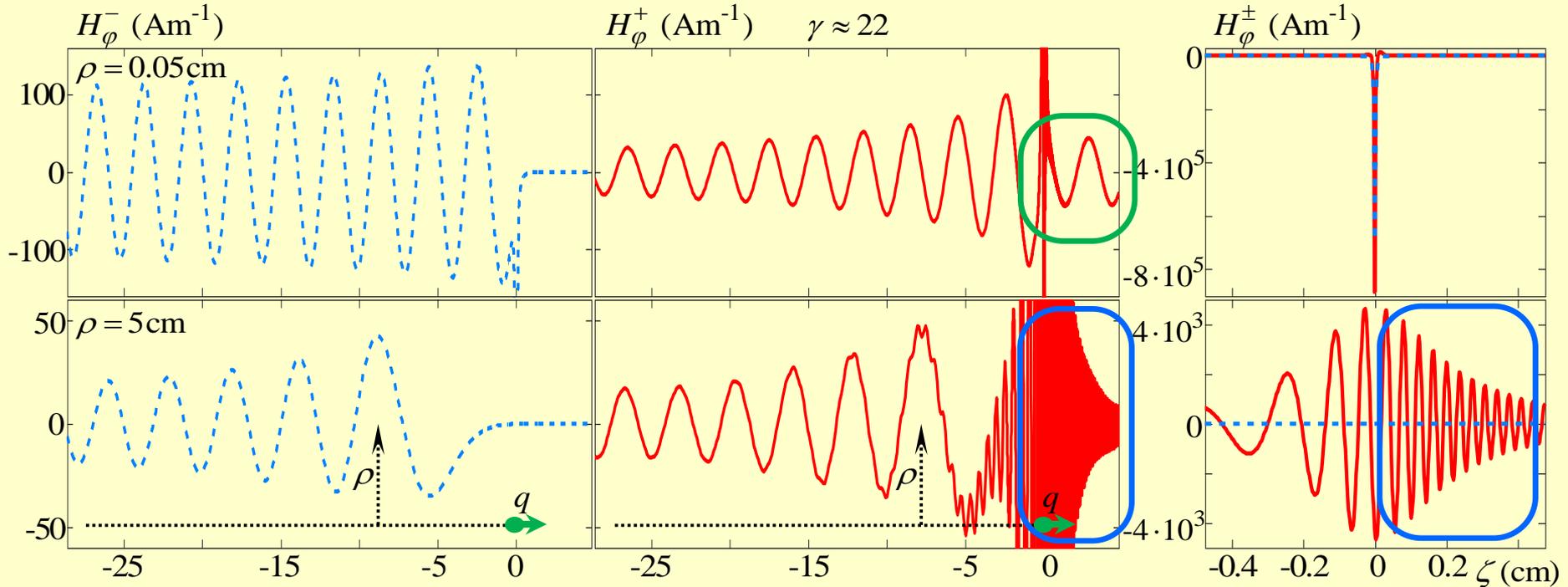
$$\omega_d = 0.001 \omega_p$$

Low frequency
wave forerunner

Numerical results

Condon medium

$$\gamma > \gamma_{\max} = 10$$



$$q = -1 \text{ nC}$$

$$\omega_r = \omega_p = 2\pi \cdot 10 \text{ GHz}$$

$$\lambda_r = 3 \text{ cm}$$

$$\omega_0 = 0.1 \omega_p$$

$$\omega_d = 0.001 \omega_p$$

Low frequency
wave forerunner

High frequency
wave forerunner

$$\omega_{h1} = 5.3 \omega_r$$

$$\omega_{h2} = 91.5 \omega_r$$

$$\lambda_{h1} = 0.6 \text{ cm}$$

$$\lambda_{h2} = 0.3 \text{ mm}$$

Numerical results

Condon medium

$$\vec{W} = \int_{-\infty}^{+\infty} \vec{S} dt = W_{\rho} \vec{e}_{\rho} + W_z \vec{e}_z + W_{\varphi} \vec{e}_{\varphi}$$

$$\vec{W} = \int_0^{+\infty} (W_{\rho\omega} \vec{e}_{\rho} + W_{z\omega} \vec{e}_z + W_{\varphi\omega} \vec{e}_{\varphi}) d\omega$$

$$\rho |s_{\pm}| \gg 1$$

$$W_{\varphi\omega} \approx 0$$

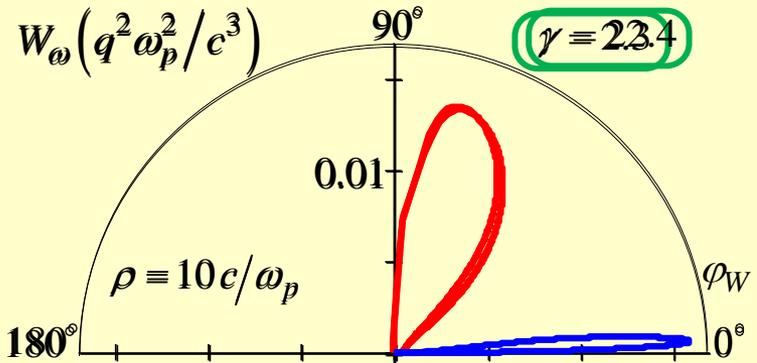
$$W_{\omega} = \sqrt{W_{\rho\omega}^2 + W_{z\omega}^2} \quad \theta_W = \arctg(W_{\rho\omega}/W_{z\omega})$$

$$W_{z\omega} \approx \frac{q^2}{4\pi^2 \beta c \rho} \frac{\mu}{\sqrt{\epsilon\mu}} \left(\frac{|s_+|}{n_+} F(\text{Im } s_+) + \frac{|s_-|}{n_-} F(\text{Im } s_-) \right)$$

$$W_{\rho\omega} \approx \frac{q^2}{4\pi\rho} \frac{\mu}{\omega\sqrt{\epsilon\mu}} \left(\frac{s_+ |s_+|}{n_+} F(\text{Im } s_+) + \frac{s_- |s_-|}{n_-} F(\text{Im } s_-) \right)$$

$$F(\xi) = 1 \text{ if } \xi = 0, \quad F(\xi) = 0 \text{ otherwise}$$

$$\gamma > \gamma_{\max} = 10$$



Gaussian bunch

$$\gamma \approx 22$$

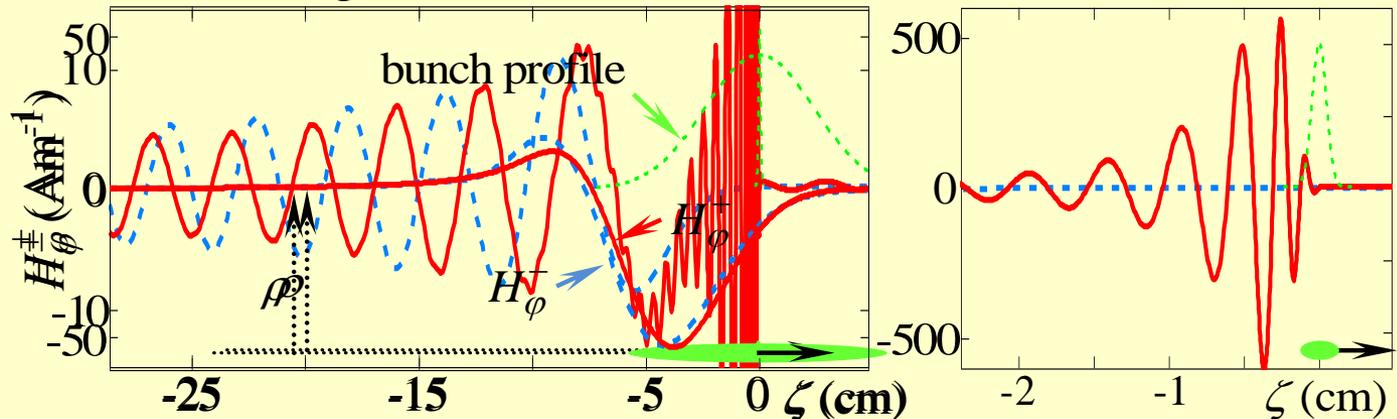
$$q = -1 \text{ nC}$$

$$\omega_r = \omega_p = 2\pi \cdot 10 \text{ GHz}$$

$$\omega_0 = 0.1\omega_p$$

$$\omega_d = 0.001\omega_p$$

$\rho = 5 \text{ cm}$ long bunch, $\sigma = 2.5 \text{ cm}$ short bunch, $\sigma = 0.5 \text{ mm}$



Thank you for your attention!