



Electromagnetic fields and the radiation of a charged particle rotating along a closed trajectory around a dielectric cylinder

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Content

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- ✧ **Conclusion**

Motivation

✦ Due to the unique characteristics of synchrotron radiation as:

- Broad Spectrum
- High Flux
- High Brilliance
- Polarization: both linear and circular

this radiation has wide applications

✦ Synchrotron light is an ideal tool for many types of research and also has industrial applications including:

Materials science

Biological and Life Sciences

Medicine

Chemistry

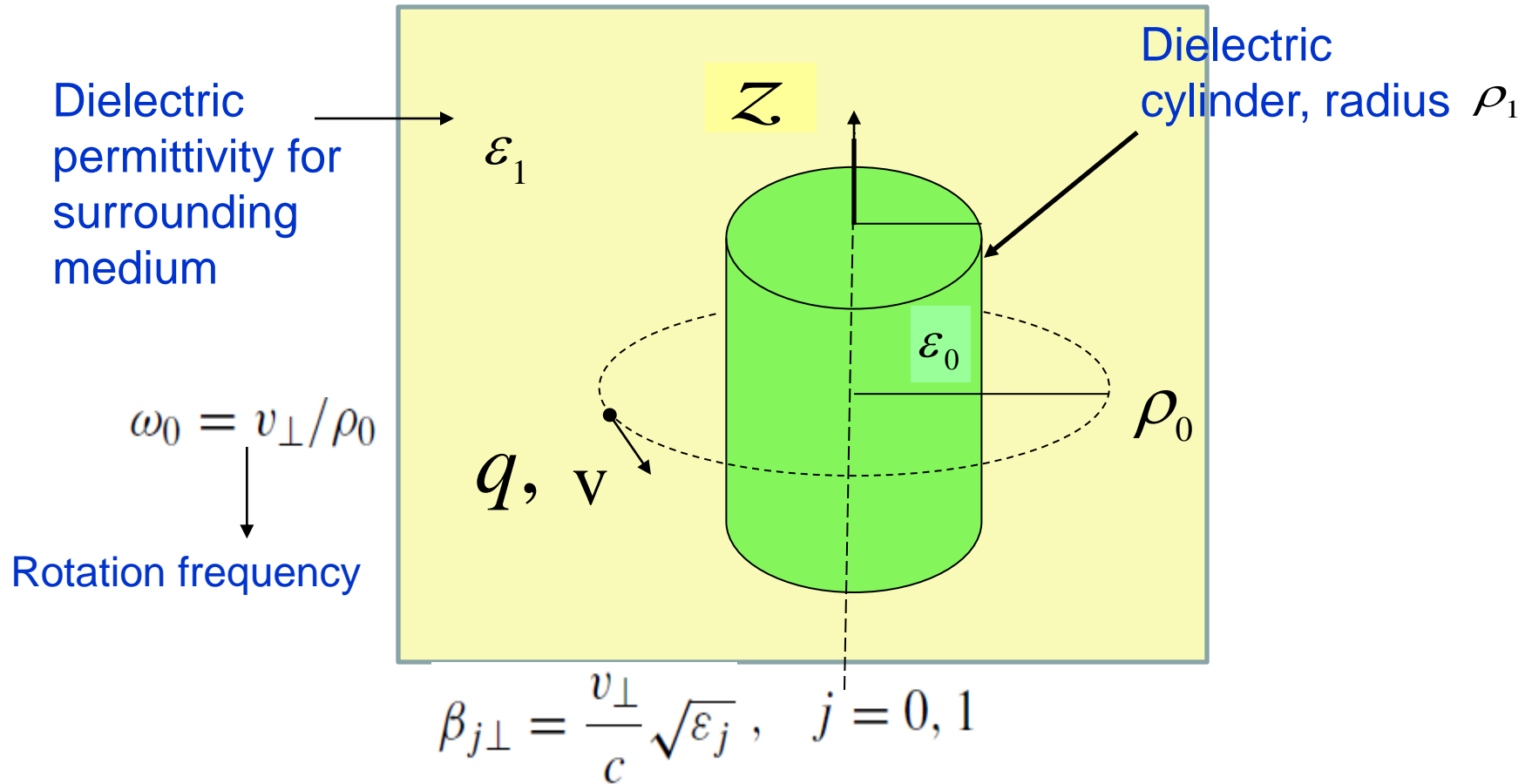
Radiation processes in medium

- ★ The wide applications of the synchrotron radiation motivate the importance of investigations for various mechanisms of control of the radiation parameters
- ★ From this point of view, it is of interest to investigate the influence of a medium on the spectral and angular distributions of the synchrotron emission
- ★ The radiation from a charged particle circulating in a homogeneous medium was considered by V.N. Tsytovich (V. N. Tsytovich. Vestnik MGU, 11, 27 (1951)). It has been shown that the interference between the synchrotron and Cerenkov radiations leads to interesting effects.
- ★ The radiation from a charge rotating around a dielectric cylinder enclosed by a homogeneous medium has been investigated (L.Sh. Grigoryan, A.S. Kotanjyan, A.A. Saharian, Sov. J. Contemp. Phys. 30, 239 (1995): A.S. Kotanjyan, H.F. Khachatryan, A.V. Petrosyan, A.A. Saharian, Sov. J. Contemp. Phys. 35, 1 (2000): A.A. Saharian, A.S. Kotanjyan, J.Phys. A42,135402, (2009): A.S. Kotanjyan, A.A. Saharian, <http://www.jacow.org>, Proceedings of ICAP2012, pp. 96-98, (2012): A.S. Kotanjyan, A.A. Saharian, Nuclear Instruments and Methods in Physics Research B, 309 (2013), pp177–182.

Motivation

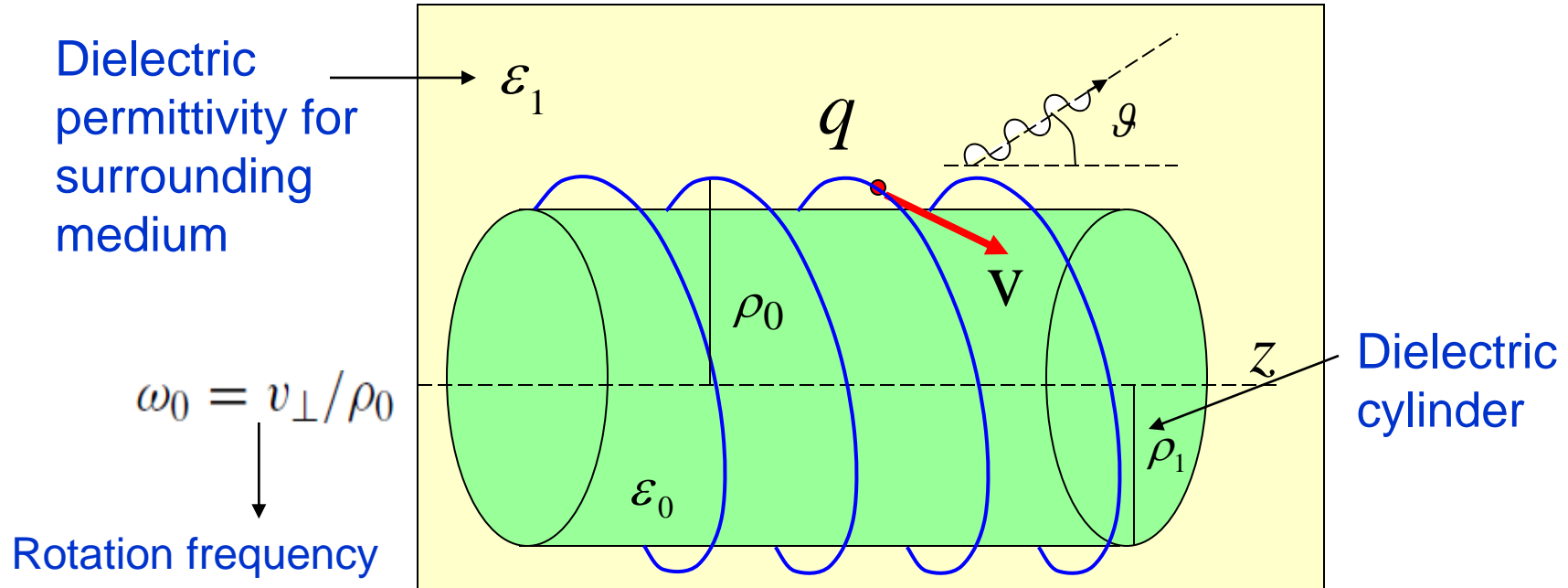
- ✦ The investigation of the radiation from a charged particle circulating around a dielectric cylinder immersed in a homogeneous medium has shown that under the Cerenkov condition for the material of the cylinder and the velocity of the charge projection on the cylinder surface there are narrow peaks in the angular distribution of the number of quanta emitted **into the exterior space**.
- ✦ For some values of parameters the density of the number of quanta at these peaks exceeds the corresponding quantity for the radiation in the vacuum by several orders of magnitude
- ✦ Present work aims to study the sensitivity of the peaks with respect to the shift of particle trajectory from coaxial circular one

Geometry of the problem: The case of circular motion



The charge velocity will be denoted by v_{\perp}

Geometry of the problem: The case of helical motion



Notations used:

$$\beta_{j\perp} = \frac{v_{\perp}}{c} \sqrt{\epsilon_j}, \quad \beta_{j\parallel} = \frac{v_{\parallel}}{c} \sqrt{\epsilon_j}, \quad j = 0, 1$$

The components of the charge velocity along the axis of the cylinder (drift velocity) and on the perpendicular plane will be denoted by v_{\parallel} and v_{\perp} respectively

Radiation intensity in the exterior medium

- ★ Under the Cherenkov condition, $\beta_{1\parallel} > 1$, the total radiation intensity at large distances from the charge trajectory is presented in the form

$$I = I_0 + I_{m \neq 0}$$

- ★ I_0 describes the radiation with a continuous spectrum propagating along the Cherenkov cone of the external medium

$$\vartheta = \vartheta_0 \equiv \arccos(\beta_{1\parallel}^{-1})$$

- ★ $I_{m \neq 0} = \sum_{m=1}^{\infty} \int d\Omega \frac{dI_m}{d\Omega}$, $d\Omega = \sin \vartheta d\vartheta d\phi$, describes the radiation, which, for a given angle ϑ , has a discrete spectrum determined by

$$\omega_m = \frac{m\omega_0}{|1 - \beta_{1\parallel} \cos \vartheta|}, \quad m = 1, 2, \dots,$$

- Normal Doppler effect $\beta_{1\parallel} < 1$ and $\beta_{1\parallel} > 1, \vartheta > \vartheta_0$,
- Anomalous Doppler effect $\vartheta < \vartheta_0$, in the case $\beta_{1\parallel} > 1$

Radiation intensity in the exterior medium: The case of circular motion

$$\frac{dI_m}{d\Omega} = \frac{q^2 m^2 \omega_0^2}{8\pi c \sqrt{\varepsilon_1}} \beta_1^2 [|B_m^{(+1)} - B_m^{(-1)}|^2 + |B_m^{(+1)} + B_m^{(-1)}|^2 \cos^2 \vartheta].$$

$$B_m^{(\pm 1)} = J_{m\pm 1}(\lambda_1 \rho_0) - \frac{H_{m\pm 1}(\lambda_1 \rho_0) W(J_{m\pm 1}, J_{m\pm 1})}{W(J_{m\pm 1}, H_{m\pm 1})} \pm \frac{i_1 \lambda_0 J_m(\lambda_0 \rho_1) J_{m\pm 1}(\lambda_0 \rho_1)}{\pi \alpha_m \rho_1 W(J_{m\pm 1}, H_{m\pm 1})} \sum_{p=\pm 1} \frac{H_{m+p}(\lambda_1 \rho_0)}{W(J_{m+p}, H_{m+p})},$$

$$\omega_m = m\omega_0, \quad m = 1, 2, \dots$$

$$\alpha_m = \frac{\varepsilon_0}{(\varepsilon_1 - \varepsilon_0)} - \frac{\lambda_0}{2} J_m(\lambda_0 \rho_1) \sum_{l=\pm 1} l \frac{H_{m+l}(\lambda_1 \rho_1)}{W(J_{m+l}, H_{m+l})}.$$

$$\lambda_0 = \frac{m\omega_0}{c} \sqrt{\varepsilon_0 - \varepsilon_1 \cos^2 \vartheta}, \quad \lambda_1 = \frac{m\omega_0}{c} \sqrt{\varepsilon_1} \sin \vartheta, \quad m = 1, 2, \dots$$

$$\alpha_m = 0 \quad \longrightarrow \quad \text{the equation determining the eigenmodes for the dielectric cylinder}$$

Radiation intensity in the exterior medium: Helical motion

$$\frac{dI_m}{d\Omega} = \frac{q^2 \omega_0^2 m^2 \sqrt{\varepsilon_1}}{2\pi^3 c |1 - \beta_{1\parallel} \cos \vartheta|^3} \left[\left| D_m^{(1)} - D_m^{(-1)} \right|^2 + \left| D_m^{(1)} + D_m^{(-1)} \right|^2 \cos^2 \vartheta \right],$$

$$D_m^{(p)} = -\frac{v_{\parallel} \pi}{2ic} \sqrt{\beta_{1\parallel}^2 - 1} J_m(\lambda_1^{(0)} \rho_0) + v_{\parallel} \frac{H_m(\lambda_1^{(0)} \rho_0)}{c V_m^H} \\ \times \left[p k_z \frac{J_m(\lambda_0^{(0)} \rho_1) J_{m+p}(\lambda_0^{(0)} \rho_1)}{\rho_1 \alpha_m V_{m+p}^H} + \frac{\pi}{2i} V_m^J \sqrt{\beta_{1\parallel}^2 - 1} \right]$$

$$\omega_m = \frac{m\omega_0}{|1 - \beta_{1\parallel} \cos \vartheta|}, \quad m = 1, 2, \dots,$$

$$\lambda_0 = \frac{m\omega_0}{c} \frac{\sqrt{\varepsilon_0 - \varepsilon_1 \cos^2 \vartheta}}{1 - \beta_{1\parallel} \cos \vartheta}, \quad \lambda_1 = \frac{m\omega_0}{c} \frac{\sqrt{\varepsilon_1} \sin \vartheta}{1 - \beta_{1\parallel} \cos \vartheta}, \quad m = 1, 2, \dots$$

Strong peaks in the radiation intensity

- ★ Strong narrow peaks are present in the angular distribution for the radiation intensity at a given harmonic m
- ★ The condition for the appearance of the peaks is obtained from the equation determining the eigenmodes for the dielectric cylinder by the replacement

$$H_m \rightarrow Y_m$$

Hankel function
of the first kind

Neumann function

Bessel function

$$\sum_{l=\pm 1} \left[\frac{\lambda_1}{\lambda_0} \frac{J_{m+l}(\lambda_0 \rho_1) Y_m(\lambda_1 \rho_1)}{J_m(\lambda_0 \rho_1) Y_{m+l}(\lambda_1 \rho_1)} - 1 \right]^{-1} = \frac{2\varepsilon_0}{\varepsilon_1 - \varepsilon_0}$$

Imaginary part of $\alpha_m = 0$

This equation has no solutions for the case $\varepsilon_0 < \varepsilon_1$

Strong peaks in the radiation intensity

- As necessary conditions for the presence of the strong narrow peaks in the angular distribution for the radiation intensity one has

$$\frac{\omega_0 \rho_0}{c} \sqrt{\varepsilon_1} \sin \vartheta < |1 - \beta_{1\parallel} \cos \vartheta| < \frac{\omega_0 \rho_1}{c} \sqrt{\varepsilon_0 - \varepsilon_1 \cos^2 \vartheta}.$$

- These conditions can be satisfied only if we have

$$\varepsilon_0 > \varepsilon_1, \quad \tilde{v} \sqrt{\varepsilon_0}/c > 1,$$

$$\text{where } \tilde{v} = \sqrt{v_{\parallel}^2 + \omega_0^2 \rho_1^2}$$

velocity of the charge image
on the cylinder surface

Analytic estimates for the heights and widths of peaks

- Angular dependence of the radiation intensity near the peak

$$\frac{dI_m}{d\Omega} \propto \frac{1}{(\vartheta - \vartheta_p)^2/b_p^2 + 1} \left(\frac{dI_m}{d\Omega} \right)_{\vartheta=\vartheta_p},$$

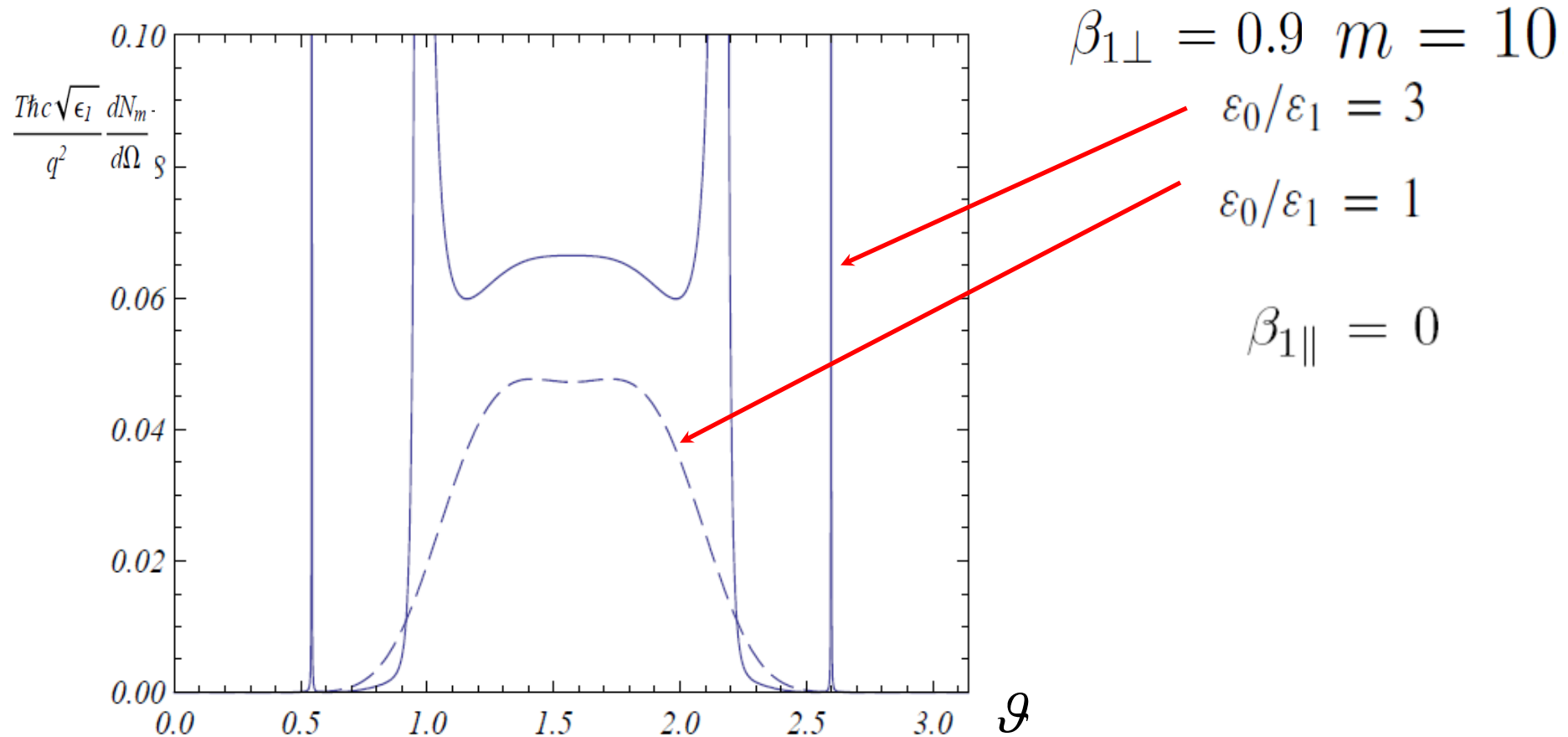
$$b_p \propto \exp[-2m\zeta(\lambda_1 \rho_1/m)]$$

- Angular widths of the peaks:

$$\Delta\vartheta \propto \exp[-2m\zeta(\lambda_1 \rho_1/m)]$$

$$\zeta(z) = \ln \frac{1 + \sqrt{1 - z^2}}{z} - \sqrt{1 - z^2},$$

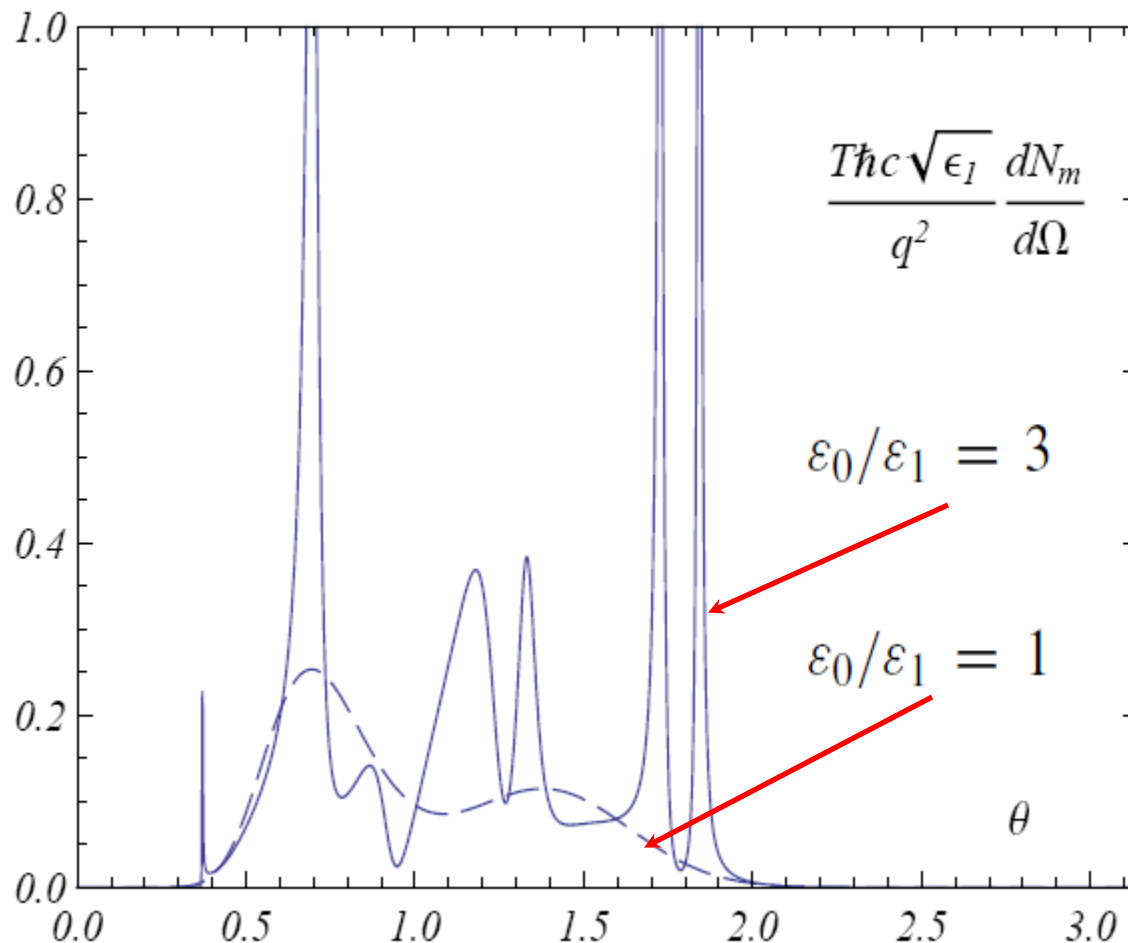
Numerical example



At the peaks of the right panel one has $\vartheta \approx 0.5435$, $(T\sqrt{\varepsilon_1}\hbar c/q^2)dN_m/d\Omega \approx 138$, with the width of the peak $\Delta\vartheta \approx 10^{-4}$, and $\vartheta \approx 0.973$, $(T\sqrt{\varepsilon_1}\hbar c/q^2)dN_m/d\Omega \approx 12.9$, $\Delta\vartheta \approx 6 \cdot 10^{-3}$.

The dependence of the quantity $(T\sqrt{\varepsilon_1}\hbar c/q^2)dN_m/d\Omega$ on the angle ϑ for $\rho_1/\rho_0 = 0.95$

Numerical example



$$\beta_{1\perp} = 0.9 \quad m = 10$$

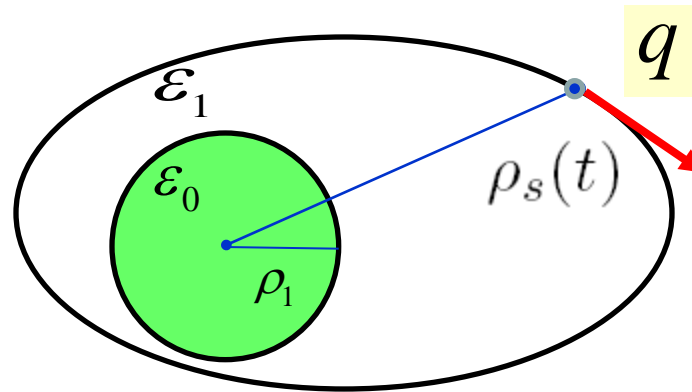
$$\beta_{1\parallel} = 0.5$$

The dependence of the quantity
 $(T\sqrt{\epsilon_1}\hbar c/q^2)dN_m/d\Omega$
 on the angle ϑ for $\rho_1/\rho_0 = 0.95$

For the left peak one has $\vartheta \approx 0.695$, $(T\sqrt{\epsilon_1}\hbar c/q^2)dN_m/d\Omega \approx 1.53$, $\Delta\vartheta \approx 0.05$;
 $\vartheta \approx 1.726$, $(T\sqrt{\epsilon_1}\hbar c/q^2)dN_m/d\Omega \approx 1.67$, $\Delta\vartheta \approx 0.02$;
 $\vartheta \approx 1.842$, $(T\sqrt{\epsilon_1}\hbar c/q^2)dN_m/d\Omega \approx 2.37$, $\Delta\vartheta \approx 0.01$.

Geometry of the problem: The case of arbitrary motion

Point charge q moving along the arbitrary trajectory $\rho_s(t)$ in medium with dielectric permittivity ε_1 outside a dielectric cylinder ε_0 with radius ρ_1 .



The components of the current density created by charge are given by the formula

$$j_l = \frac{v_l q}{\rho} \delta(\rho - \rho_s(t)) \delta(\phi - \phi_s(t)) \delta(z - v_{\parallel} t), \quad |v| = \sqrt{v_{\rho}^2 + v_{\phi}^2 + v_{\parallel}^2}, \quad l = \rho, \phi, z.$$

Our aim is to study the sensitivity of peaks with respect to distortions of the particle's trajectory

Electromagnetic potentials and fields in the exterior region

Scheme for evaluation

I step As all properties of the electromagnetic field are encoded in corresponding Green Function (a tensor of the second rank), we have developed a recurrent scheme for constructing the Green Function of the electromagnetic field for a medium consisting of an arbitrary number of coaxial cylindrical layers

II step The 4-potential $A_i(\mathbf{r}, t)$ of the electromagnetic field can be determined via the GF and the current density by the following formula

$$A_i(x) = -\frac{1}{2\pi^2 c} \int G_{il}(x, x') j_l(x') d^4 x', \quad x = (t, \mathbf{r}), \quad i, l = 0, 1, 2, 3,$$

III step Having 4-potential, we can derive the strengths of electromagnetic field, which are obtained by means of standard formulae of electrodynamics

IV step Finally we proceed to the consideration of the intensity for the radiation to the exterior medium. The average energy flux per unit time through the cylindrical surface coaxial with the dielectric cylinder is given by the Poynting vector \mathbf{S} :

$$I = \frac{c}{4\pi T} \int_0^T dt \int_0^{2\pi} d\phi \int_{-\infty}^{+\infty} dz \rho \mathbf{n}_\rho \cdot [\mathbf{E} \times \mathbf{H}]$$

The radiation intensity in the exterior region

For the angular density of the radiation intensity at a given harmonic n one finds

$$\frac{dI_n}{d\Omega} = \frac{q^2}{4\pi^3 c} \frac{n^2 \omega_0^2 \sqrt{\varepsilon_1}}{|1 - \beta_{1\parallel} \cos \theta|^3} \sum_{m=-\infty}^{+\infty} \left[\left| \sum_{p=\pm 1} p D_m^{(p)} \right|^2 + \left| \sum_{p=\pm 1} D_m^{(p)} \right|^2 \cos^2 \theta \right],$$

Where $d\Omega = \sin \theta d\theta d\varphi$ is the solid angle element and

$$I = \sum_{n=-\infty}^{+\infty} \int d\Omega \frac{dI_n}{d\Omega}.$$

We have introduced the notation

$$\begin{aligned} D_m^{(p)} = & \frac{\pi}{2ic} \left[J_{m+p,m}(n, \lambda_1) - H_{m+p,m}(n, \lambda_1) \frac{V_{m+p}^J}{V_{m+p}^H} \right] + \frac{p\lambda_0 J_m(\lambda_0 \rho_1) J_{m+p}(\lambda_0 \rho_1)}{2c\rho_1 \alpha_m V_{m+p}^H} \sum_{l=\pm 1} \frac{H_{m+l,m}(n, \lambda_1)}{V_{m+l}^H} \\ & + \frac{pk_z}{c\rho_1} H_{m,m,z}(n, \lambda_1) \frac{J_m(\lambda_0 \rho_1) J_{m+p}(\lambda_0 \rho_1)}{\alpha_m V_m^H V_{m+p}^H} + \frac{i\pi\lambda_1}{2ck_z} \left[J_{m,m,z}(n, \lambda_1) - H_{m,m,z}(n, \lambda_1) \frac{V_m^J}{V_m^H} \right] \end{aligned}$$

Notations

$$F_{m',m}(n, \lambda_1) = F_{m',m,\phi}(n, \lambda_1) - ipF_{m',m,\rho}(n, \lambda_1), \quad F = J, H.$$

$$F_{m',m,l}(n, \lambda_1) = \frac{1}{T} \int_0^T dt v_l(t) F_{m'}(\lambda_1 \rho_s(t)) e^{-im\phi_s(t) + in\omega_0 t}$$

$$H_{m',m,l}(n, \lambda_1) = \frac{1}{T} \int_0^T dt v_l(t) H_{m'}(\lambda_1 \rho_s(t)) e^{-im\phi_s(t) + in\omega_0 t}$$

$$J_{m',m,l}(n, \lambda_1) = \frac{1}{T} \int_0^T dt v_l(t) J_{m'}(\lambda_1 \rho_s(t)) e^{-im\phi_s(t) + in\omega_0 t}$$

Radiation frequency

$$\omega_n = \frac{n\omega_0}{1 - \beta_{1\parallel} \cos \theta}$$

Angle between the wave vector of the radiated photon and the axis z

$$\lambda_1 = \frac{n\omega_0}{c} \frac{\sqrt{\varepsilon_1} \sin \theta}{1 - \beta_{1\parallel} \cos \theta}, \quad \lambda_0 = \frac{n\omega_0}{c} \frac{\sqrt{\varepsilon_0 - \varepsilon_1 \cos^2 \theta}}{1 - \beta_{1\parallel} \cos \theta}.$$

Theoretical analyses

The radiation intensity in the case of general trajectory is obtained from the corresponding formula for circular motion by the replacement

$$F_{m'}(\lambda_1 \rho_0) \Rightarrow F_{m',m,l}(n, \lambda_1)$$

For circular/spiral case ***m*** = ***n*** remains only

For general motion we also have an additional summation over ***m*** for a given harmonic ***n***

Peaks in the radiation intensity: Arbitrary motion

✦ For a general motion peaks may appear in the angular distribution for the radiation intensity at a given harmonic n

✦ The condition for the appearance of the peaks again is obtained from the equation determining the eigenmodes for the dielectric cylinder by the replacement

$$H_m \rightarrow Y_m$$

Hankel function
of the first kind
Neumann function

$$\sum_{l=\pm 1} \left[\frac{\lambda_1}{\lambda_0} \frac{J_{m+l}(\lambda_0 \rho_1) Y_m(\lambda_1 \rho_1)}{J_m(\lambda_0 \rho_1) Y_{m+l}(\lambda_1 \rho_1)} - 1 \right]^{-1} = \frac{2\varepsilon_0}{\varepsilon_1 - \varepsilon_0}$$

✦ For a single peak at a given harmonic n in the case of arbitrary motion one has set of peaks corresponding to different values of m

✦ Heights and widths of the peaks depend on the special form of the trajectory

Conclusions

- ✦ We have investigated electromagnetic fields and the radiation intensity of a charged particle rotating along an arbitrary trajectory around a dielectric cylinder
- ✦ Similar to the case of coaxial circular motion under certain conditions for the parameters of the trajectory and dielectric cylinder strong narrow peaks appear in the angular distribution of the radiation intensity in the exterior medium
- ✦ Opposite to a single peak in the case of a coaxial circular motion in case of arbitrary motion set of peaks appear
- ✦ Heights and widths of the peaks depend on the form of trajectory
- ✦ From the general formula given above, as a special case, we obtain the formula for the radiation of a charge moving along helix with an arbitrary cross section in the vacuum or in a homogeneous medium