

Wave Packets in Electromagnetic Processes at High Energy in Matter (the Problem of Half-Bare Electron)

N. F. Shul'ga

Akhiezer Institute for Theoretical Physics of NSC KIPT
shulga@kipt.kharkov.ua

- Coherent length
- Spreading of wave packets
- LPM and TSF effects
- Ultra thin crystals
- Ionization energy losses
- Other processes

Coherent length

Discussion: E.Feinberg and M.Ter- Mikaelian with L.Landau and I. Pomeranchuk (1952)

T. - M. – Interference effect in radiation by ultrarelativistic electrons in crystals.

Landau – That is impossible because the interference effect is possible only for

$$\lambda = \frac{\hbar}{p} \geq a \quad , \quad \text{but not for } \lambda \ll a$$

The discussion was stopped !

Coherent length

In the theory of high energy electrons' radiation besides the length $\lambda \sim \hbar/p$ there exists another length responsible for the radiation,

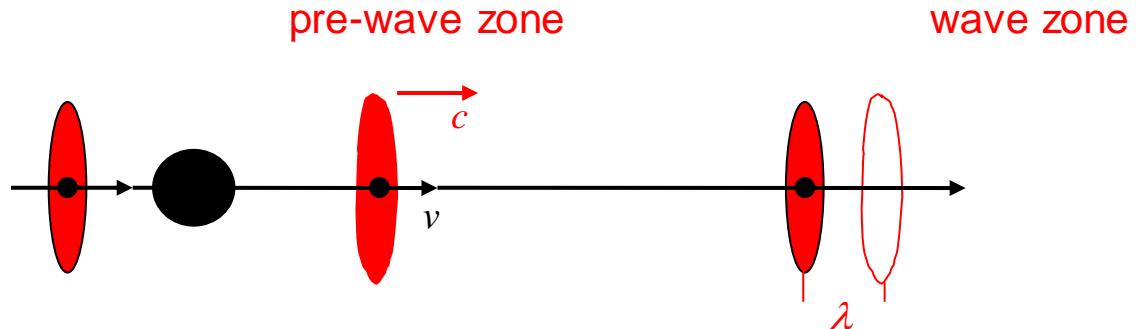
$$l_c = \frac{2\epsilon\epsilon'}{m^2\omega}.$$

Interpretations of l_c

- Bethe, Heitler (1933)
Ter-Mikaelian (1952) It is based on the first Born Approximation
- Landau, Pomeranchuk (1953) It is based on classical electrodynamics
- Frish, Olsen (1959),
Akhiezer, Shul'ga (1982) It is based on the behavior of the wave packets
- Feinberg (1966)
Akhiezer, Shul'ga, Fomin (1982) Development of the process of radiation in space and time

ULTRATRAHIGH FORMATION (COHERENT) LENGTHS

The excitation is small



$$v_{rel} = c - v$$

$$l_{coh} = 2\gamma^2 \lambda \gg \lambda$$

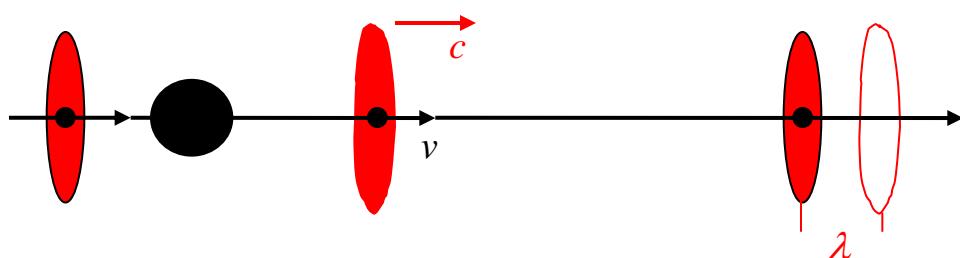
$$\lambda = (c - v)\Delta t_c \rightarrow \Delta t_c \approx 2\gamma^2 \lambda$$

$$E \sim 100 \text{ GeV} \quad \omega \sim 500 \text{ MeV} \quad l_c \sim 10^{-3} \text{ cm}$$

$$E \sim 50 \text{ MeV} \quad \lambda \sim 0.1 \text{ cm} \quad l_c \sim 20 \text{ m}$$

EFFECTIVE CONSTANT OF INTERACTION FOR LARGE COHERENCE LENGTH

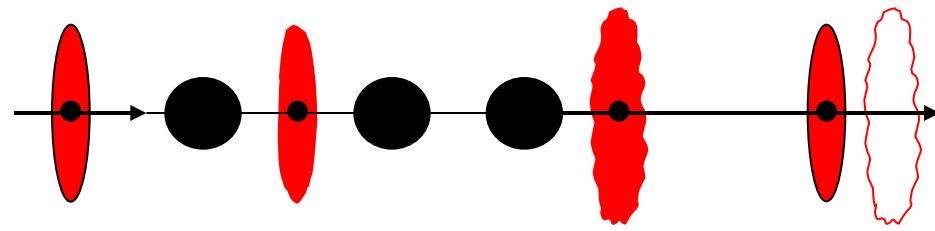
The excitation is small



$$\gamma\theta_{scat} \ll 1$$

$$\alpha_{eff} = \frac{Ze^2}{\hbar c}$$

The excitation is increased



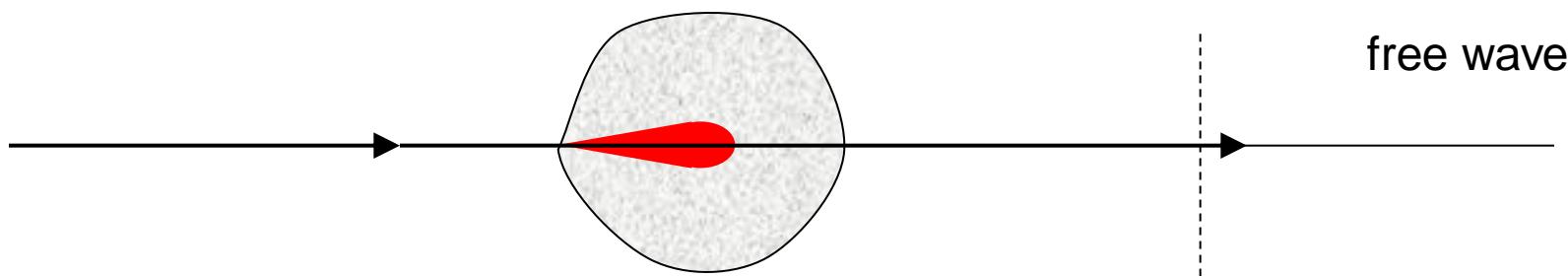
$$\gamma\theta_{scat} \approx 1$$

$$\begin{aligned}\alpha_{eff} &= N_{coh} \frac{Ze^2}{\hbar c} = \\ &= \frac{l_{coh}}{a} \frac{Ze^2}{\hbar c}\end{aligned}$$

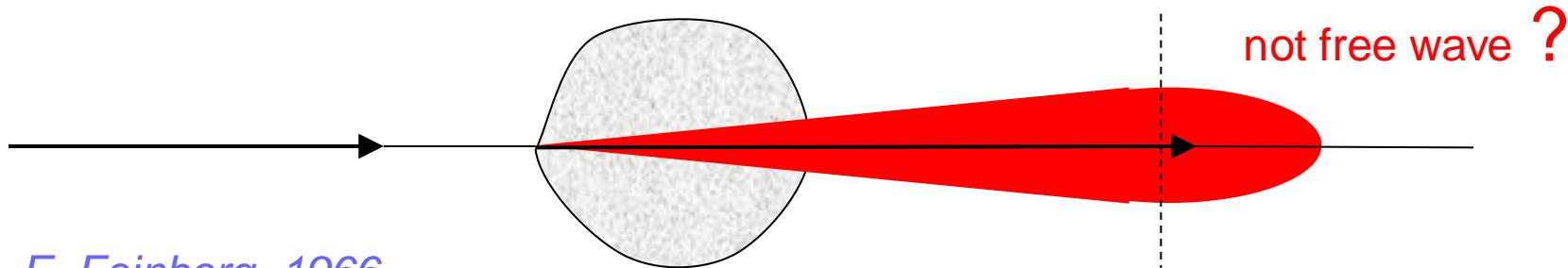
Problems

- **Methods of description of radiation at $\alpha_{eff} > 1$
(eikonal, semiclassical, operator semiclassical,
classical electrodynamics, ...)**
- **Evolution in space and time**
- **Medium influence on radiation**
- **S-matrix and boundary conditions**

S-matrix and Boundary Conditions for Ultra-High Coherent Lengths



$$S = T \exp \left\{ ie^2 \int_{-\infty}^{\infty} dt \int d^3 r J_\mu A_\mu \right\}$$



E. Feinberg, 1966

$$S = T \exp \left\{ ie^2 \int_{-\infty}^T dt \int d^3 r J_\mu A_\mu \right\}$$

???

Evolution of electromagnetic field at electron's scattering

$$\left(\Delta - \frac{\partial^2}{\partial t^2} \right) \phi = 4\pi e \delta(\vec{r} - \vec{r}(t))$$

$$\varphi_v(\vec{r}, t) = \frac{e}{\sqrt{(z-vt)^2 + \rho/\gamma^2}}, \quad t < 0$$

$$\begin{aligned} \varphi_{ret}(\vec{r}, t) \Big|_{t>0} &= \frac{e}{2\pi^2} \operatorname{Re} \int \frac{d^3 k}{k} e^{i\vec{k}\vec{r}} \left\{ \frac{1 - e^{-i(k - \vec{k}\vec{v}_1)t}}{\omega - \vec{k}\vec{v}} e^{-i\vec{k}\vec{v}_1 t} + \frac{1}{k - \vec{k}\vec{v}} e^{-ikt} \right\} = \\ &= \Theta(t-r) \varphi_{v_1}(\vec{r}, t) + \Theta(r-t) \varphi_v(\vec{r}, t) \end{aligned}$$

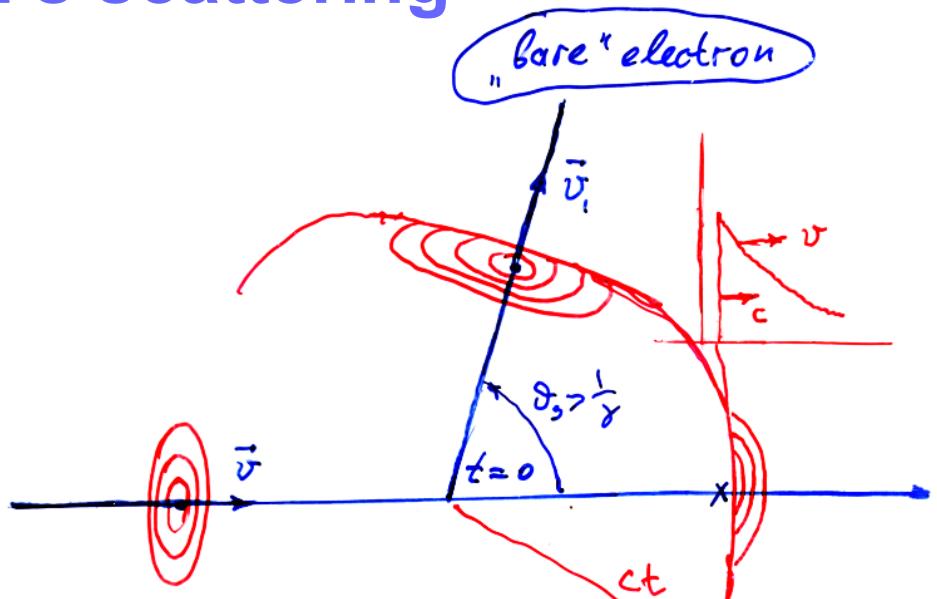
$$\Delta t \square (k - kv_1)^{-1} \approx 2\gamma^2/v = l_c$$

For $\varepsilon = 50 \text{ MeV}$, $\lambda = 1 \text{ cm}$, $l_c = 200 \text{ m}$

E.Feinberg JETP 50(1966)202,

A. Akhiezer, N.Shul'ga, S.Fomin Sov.Phys.Usp. 30(1987)197

Phys.Lett.A 114(1986)148



Spreading of high energy wave packets

SPREADING OF RELATIVISTIC WAVE PACKETS

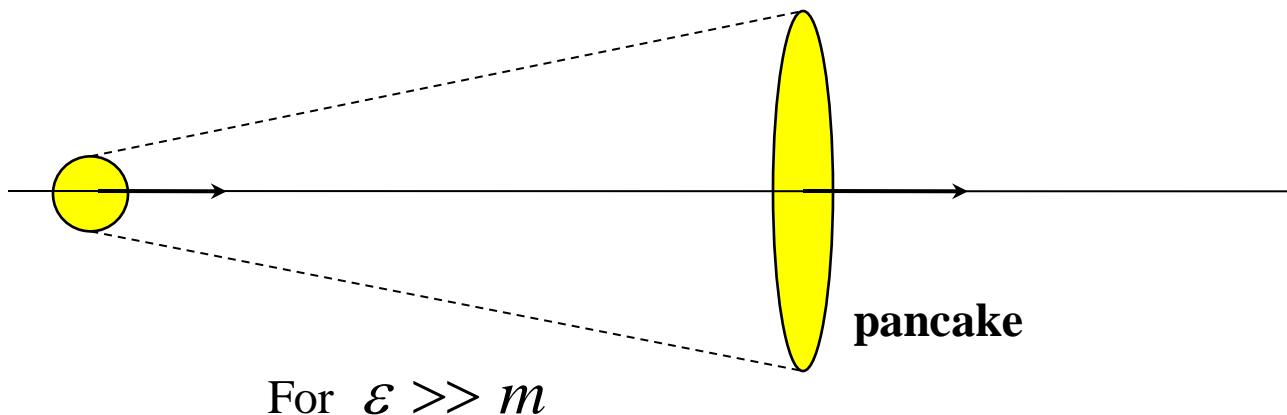
(D.I. Blokhintsev, 1967)

$$\left(\partial_t^2 - \nabla^2 - m^2\right) \varphi(\vec{r}, t) = 0$$

$$\phi(\vec{r}, t) = e^{i(\vec{p}\vec{r} - \varepsilon t)} A(t) \exp \left\{ i\alpha(\vec{r}, t) - \frac{(z-t)^2}{2\Delta_{||}^2} - \frac{\rho^2}{2\Delta_{\perp}^2} \right\}$$

$$\Delta_{||}^2(t) = a_{||}^2 + \left(t / a_{||} \varepsilon \gamma^2 \right)^2$$

$$\Delta_{\perp}^2(t) = a_{\perp}^2 + \left(t / a_{\perp} \varepsilon \right)^2$$



1. Strong stabilizing effect
2. Dispersion mostly in transverse direction

SPREADING OF HIGH-ENERGY ELECTROMAGNETIC PACKETS

N.F. Shul'ga, S.V. Trofymenko, *in the book*
“*Electromagnetic Waves*”, InTech, 2012

$$\phi(\vec{r}, t) = e^{i(\vec{k}\vec{r} - \omega t)} A(t) \exp\left\{i\alpha(\vec{r}, t) - \frac{(z-t)^2}{2\Delta_{||}^2} - \frac{\rho^2}{2\Delta_{\perp}^2}\right\}$$

$$\Delta_{||}^2(t) = a_{||}^2 \quad \Delta_{\perp}^2(t) = a_{\perp}^2 + (t / a_{\perp} \omega)^2$$

$$A(t) \exp\{i\alpha(\vec{r}, t)\} \rightarrow \frac{1}{r} e^{i\omega r} \quad \text{for } t \approx z \rightarrow \infty$$

1. The equivalent photon method
2. Bremsstrahlung, coherent and transition radiation etc.
3. Ionization energy losses

APPROXIMATION OF THE COULOMB FIELD BY THE PACKET OF PLANE WAVES (EQUIVALENT PHOTON METHOD)

$$\varphi_{free}(\vec{r}, t) = \text{Re} \int \frac{d^3 k}{(2\pi)^3} e^{i(\vec{k}\vec{r} - kt)} C_k$$

$$C_k = \frac{8\pi e \Theta(k_z)}{k_\perp^2 + k_z^2 / \gamma^2}$$

$$\varphi_{free}(\vec{r}, t) = \text{Re} \int dk \varphi_k(\vec{r}, t)$$

$$\varphi_k(\vec{r}, t) = \frac{2}{\pi} e^{ik(z-t)} \int_0^\infty \frac{\theta d\theta}{\theta^2 + \gamma^{-2}} J_0(k\rho\theta) e^{-ikz\theta^2/2}$$

WAVE AND PRE-WAVE ZONES

$$\varphi_k(\vec{r}, t) = \frac{2}{\pi} e^{ik(z-t)} \int_0^\infty \frac{\theta d\theta}{\theta^2 + \gamma^{-2}} J_0(k\rho\theta) e^{-ikz\theta^2/2}$$

pre-wave zone

$$\varphi_k(\vec{r}, t) \approx \frac{2}{\pi} K_0(k\rho/\gamma) e^{ik(z-t)} \quad kz\vartheta^2/2 \ll 1$$

$$\varphi(\vec{r}, t) = \frac{e}{\sqrt{(z-t)^2 + \rho^2/\gamma^2}} \quad z \ll l_c$$

wave zone

$$\varphi_k(\vec{r}, t) = -\frac{2i}{\pi} \frac{1}{\vartheta_0^2 + \gamma^{-2}} \frac{1}{kr} e^{ik(r-t)} \quad kz\vartheta^2/2 \gg 1$$

$$\vartheta_0 = \rho/z \quad z \gg l_c$$

N.Shul'ga, V.Syshchenko, S.Shul'ga. Phys. Lett. A 374 (2009) 331

Evolution of electromagnetic field at electron's scattering

$$\left(\Delta - \frac{\partial^2}{\partial t^2} \right) \phi = 4\pi e \delta(\vec{r} - \vec{r}(t))$$

$$\varphi_v(\vec{r}, t) = \frac{e}{\sqrt{(z-vt)^2 + \rho/\gamma^2}}, \quad t < 0$$

$$\begin{aligned} \varphi_{ret}(\vec{r}, t) \Big|_{t>0} &= \frac{e}{2\pi^2} \operatorname{Re} \int \frac{d^3 k}{k} e^{i\vec{k}\vec{r}} \left\{ \frac{1 - e^{-i(k - \vec{k}\vec{v}_1)t}}{\omega - \vec{k}\vec{v}} e^{-i\vec{k}\vec{v}_1 t} + \frac{1}{k - \vec{k}\vec{v}} e^{-ikt} \right\} = \\ &= \Theta(t-r) \varphi_{v_1}(\vec{r}, t) + \Theta(r-t) \varphi_v(\vec{r}, t) \end{aligned}$$

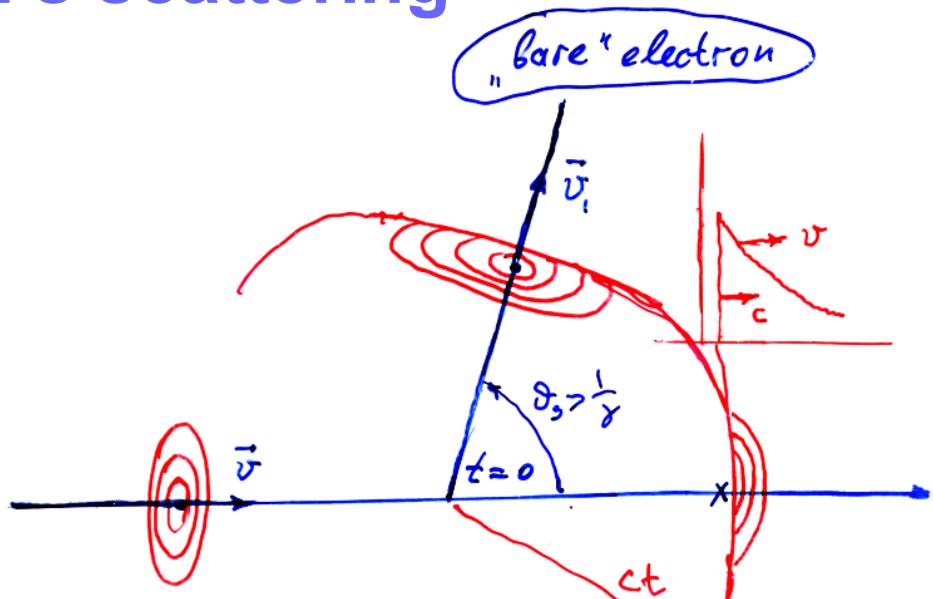
$$\Delta t \square (k - kv_1)^{-1} \approx 2\gamma^2/v = l_c$$

For $\varepsilon = 50 \text{ MeV}$, $\lambda = 1 \text{ cm}$, $l_c = 200 \text{ m}$

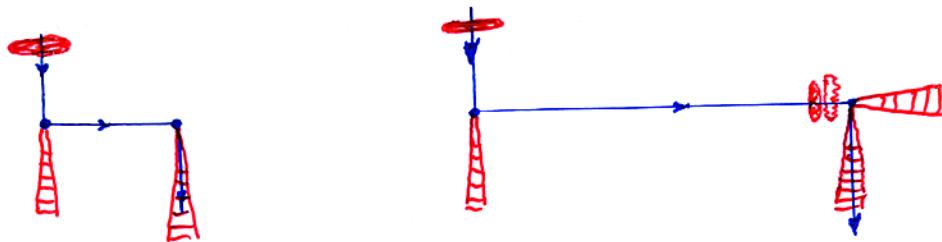
E.Feinberg JETP 50(1966)202,

A. Akhiezer, N.Shul'ga, S.Fomin Sov.Phys.Usp. 30(1987)197

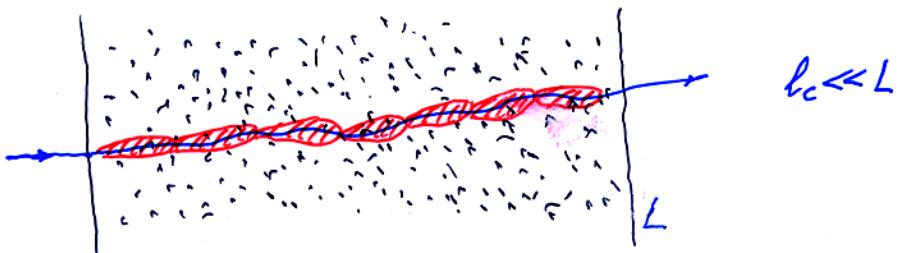
Phys.Lett.A 114(1986)148



LPM and TSF effects

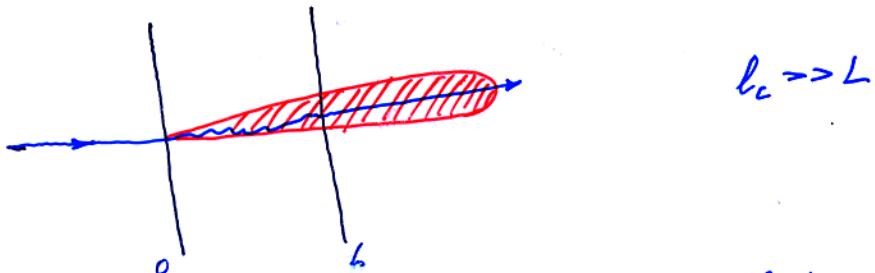


LPM case

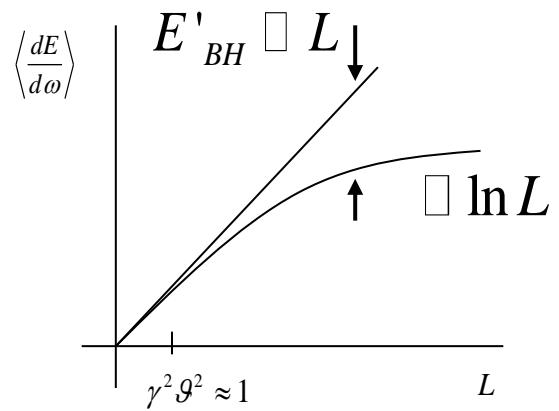


balance = e^- is undressed + e^- is dressed

N. Shul'ya, S. Fomin (1978)

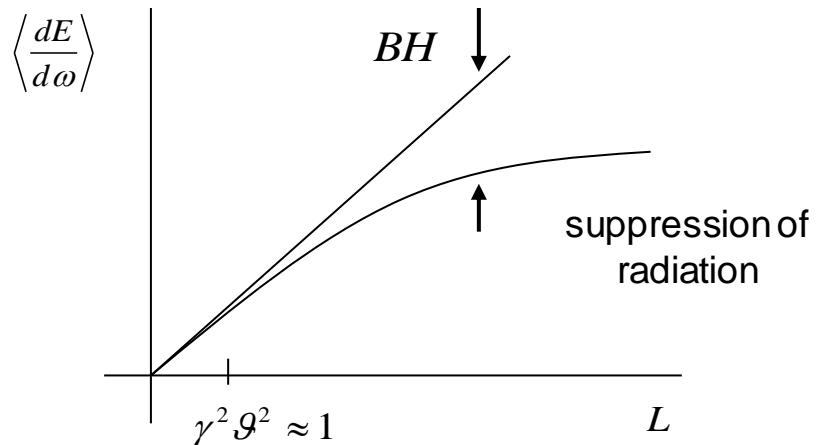
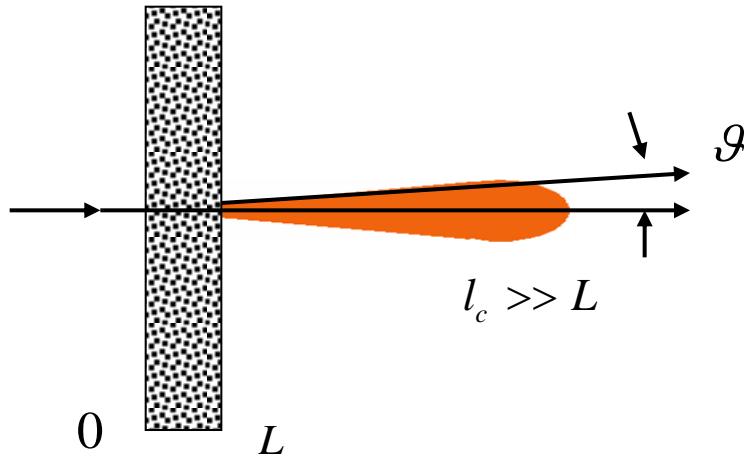


Electron is "bare" for all collisions !!!



Radiation in thin target (TSF-effect)

F. Ternovskii, JETP 1960, N. Shul'ga, S. Fomin JETP Lett. 1978, 1996



$$l_c = \frac{2\gamma^2}{\omega} \square L$$

$$\left\langle \frac{dE}{d\omega} \right\rangle = \frac{2e^2}{\pi} \left\langle \left[\frac{2\xi^2 + 1}{\xi \sqrt{\xi^2 + 1}} \ln \left(\xi + \sqrt{\xi^2 + 1} \right) - 1 \right] \right\rangle \approx$$

$$\approx \frac{2e^2}{3\pi} \left\{ \frac{\gamma^2 \vartheta^2}{3 \ln \gamma^2 \vartheta^2} \right\} \approx \begin{cases} E'_{BH} \\ < E'_{BH} \end{cases}$$

$$\xi = \frac{\gamma \vartheta}{2}$$

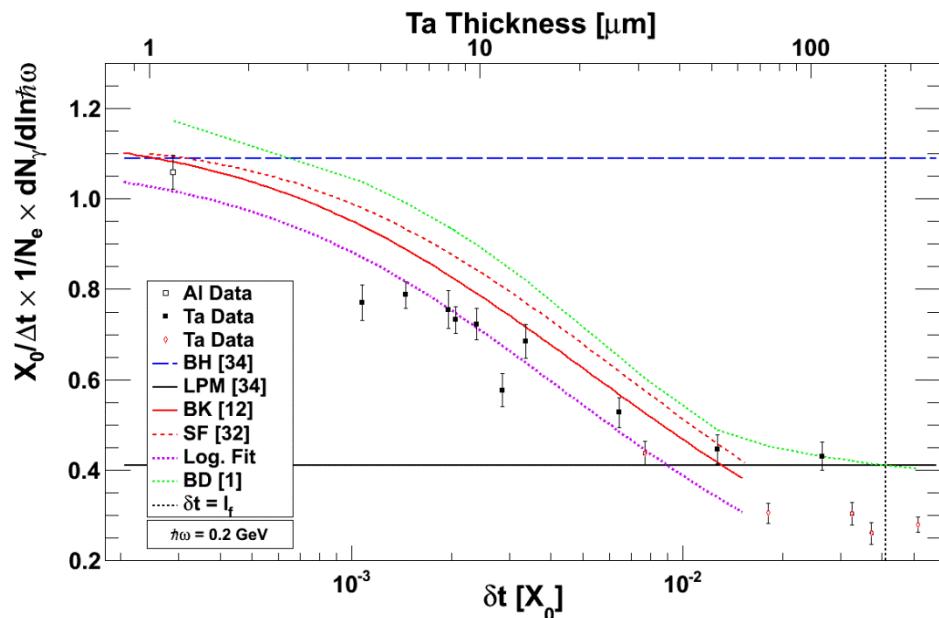
Suppression of radiation by relativistic electrons in a thin layer of matter (TSF effect)

Predicted at KIPT - 1978 - *N.F.Shul'ga, S.P.Fomin, JETP Letters, 27(1978)126.*

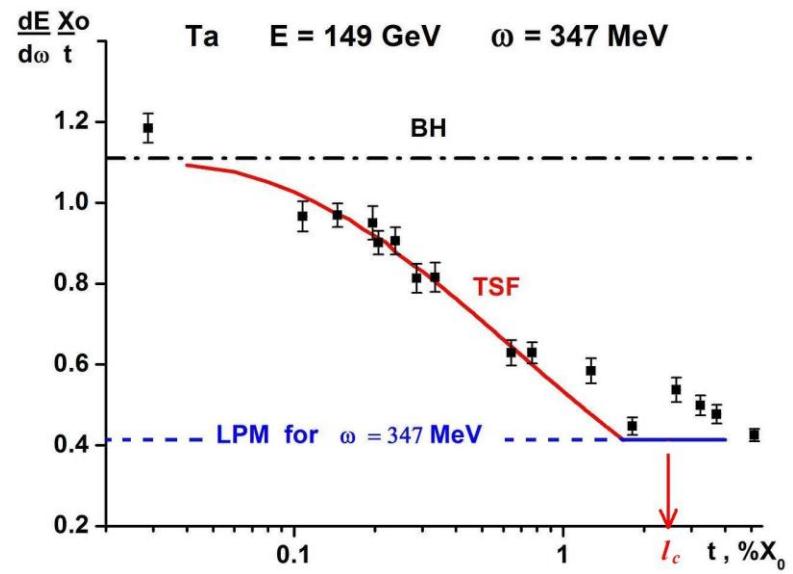
Confirmed at CERN - 2009 - *H.D.Thomsen et al., Physics Letters B 672 (2009) 323.*

H.D.Thomsen et al., Physical Review D 81 (2010) 052003.

CERN NA63 SPS E = 149 GeV



A.S.Fomin, S.P.Fomin, N.F.Shul'ga
Nuovo Cimento (2011)

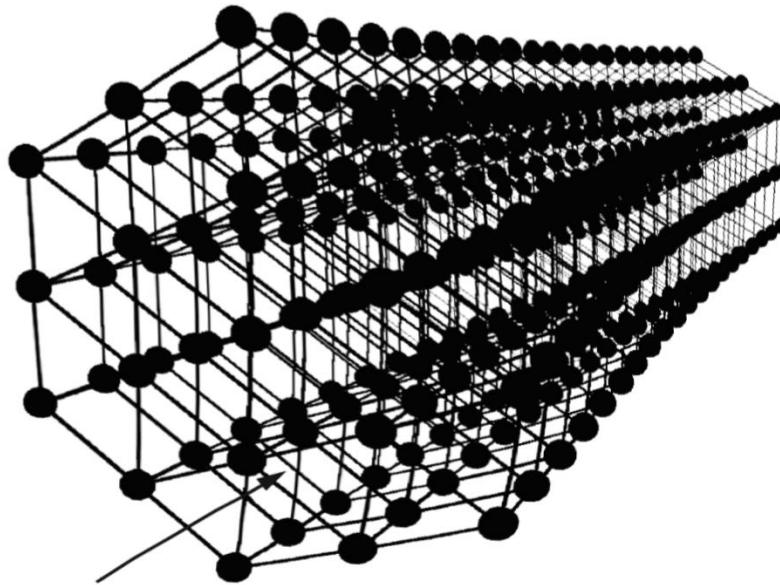


U. Uggerhoj : ... we have seen the half - bare electron !

Coherent radiation in ultra thin crystals

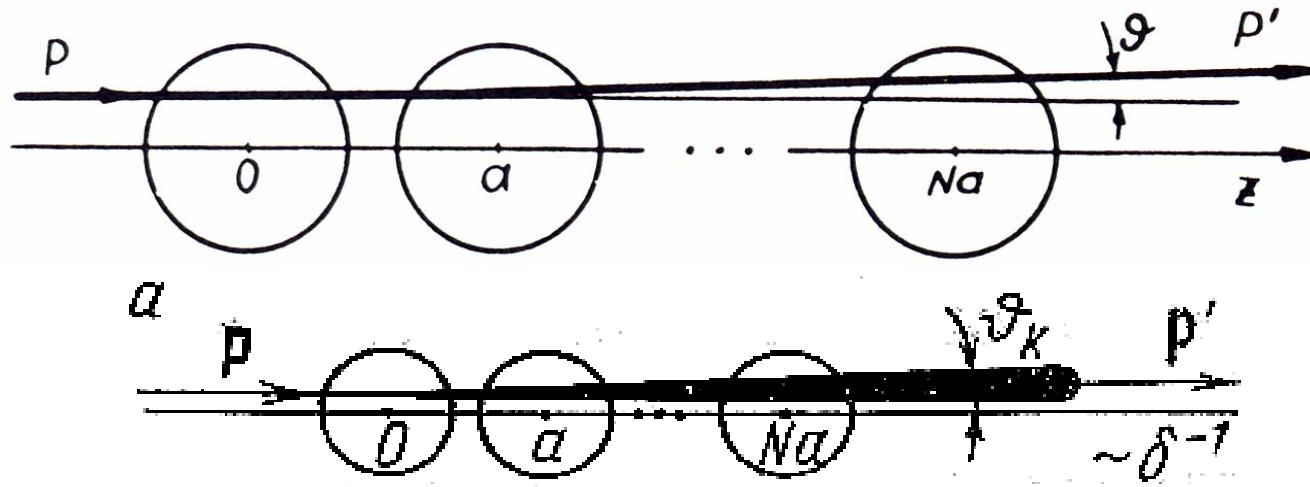
Coherent radiation in ultra thin crystals

(N.F. Shul'ga, S.N. Shul'ga, 2013)



The creation of ultra thin crystal for high-energy experiments
(V. Guidi, A. Mazzolari, et al. 2010...)

Coherent radiation in ultra thin crystals



$$\frac{NZe^2}{\hbar c} \gg 1$$

- eikonal approximation

$$\frac{NZe^2}{\varepsilon R} \ll \frac{R}{Na}$$

- factorization

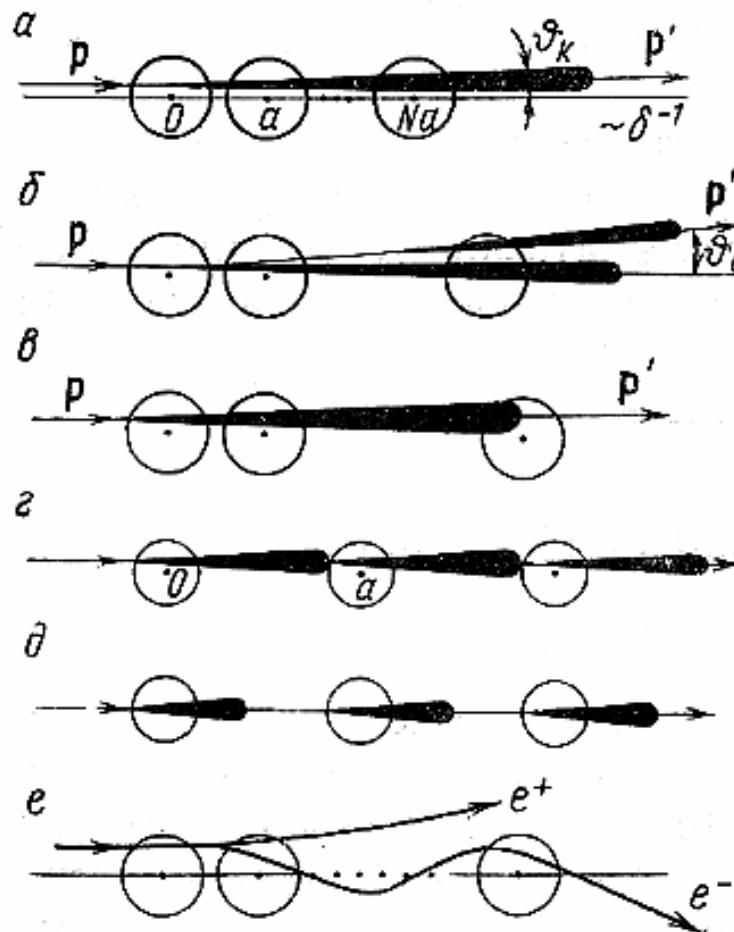
$$Na \ll l_c = \frac{2\varepsilon\varepsilon'}{m^2\omega}$$

At $\varepsilon \approx 100 \text{ GeV}$ and $\omega \sim 50 \text{ GeV}$ these conditions are fulfilled for

$$L = Na = 10^{-4} \text{ cm}$$

Interaction with atomic string

(A.Akhiezer, N.Shul'ga, 1975)



Semi-classical description of CB

(A. Akhiezer and N. Shul'ga, 1975)



$$\frac{dE^{(N)}}{d\omega} = \hbar\omega \int_0^\infty \frac{dW(q_\perp)}{d\omega} d\sigma_e^{(N)}(q_\perp) \left(1 + O(\frac{\chi_1}{\chi_0}, \frac{L}{l_c}) \right)$$

$$\hbar\omega \frac{dW}{d\omega} = \frac{2e^2}{\pi} \frac{\epsilon'}{\epsilon} \left[\frac{2\xi^2 \left(1 + \frac{(\hbar\omega)^2}{2\epsilon\epsilon'} \right) + 1}{\xi \sqrt{\xi^2 + 1}} \ln(\xi + \sqrt{\xi^2 + 1}) - 1 \right], \quad \xi = \frac{q_\perp}{2m}$$

$$\alpha_{eff}^{(N)} = \frac{\chi_0^{(N)}}{\hbar} = N \frac{Ze^2}{\hbar c} \square 1$$

Coherent Bremsstrahlung on N atoms of string

$$U(r) = \frac{Ze^2}{r} e^{-r/R}$$

$$\frac{NZe^2}{\hbar c} \gg 1 \quad \frac{NZe^2}{mR} \square \frac{q}{m} \ll 1$$

$$\hbar\omega \frac{d\sigma^{(N)}}{d\omega} \approx \frac{16e^2(NZe^2)^2}{3m^2} \frac{\varepsilon'}{\varepsilon} \left[\left(1 + \frac{3}{4} \frac{\hbar^2\omega^2}{\varepsilon\varepsilon'} \right) \ln \frac{mR}{NZe^2} \right]$$

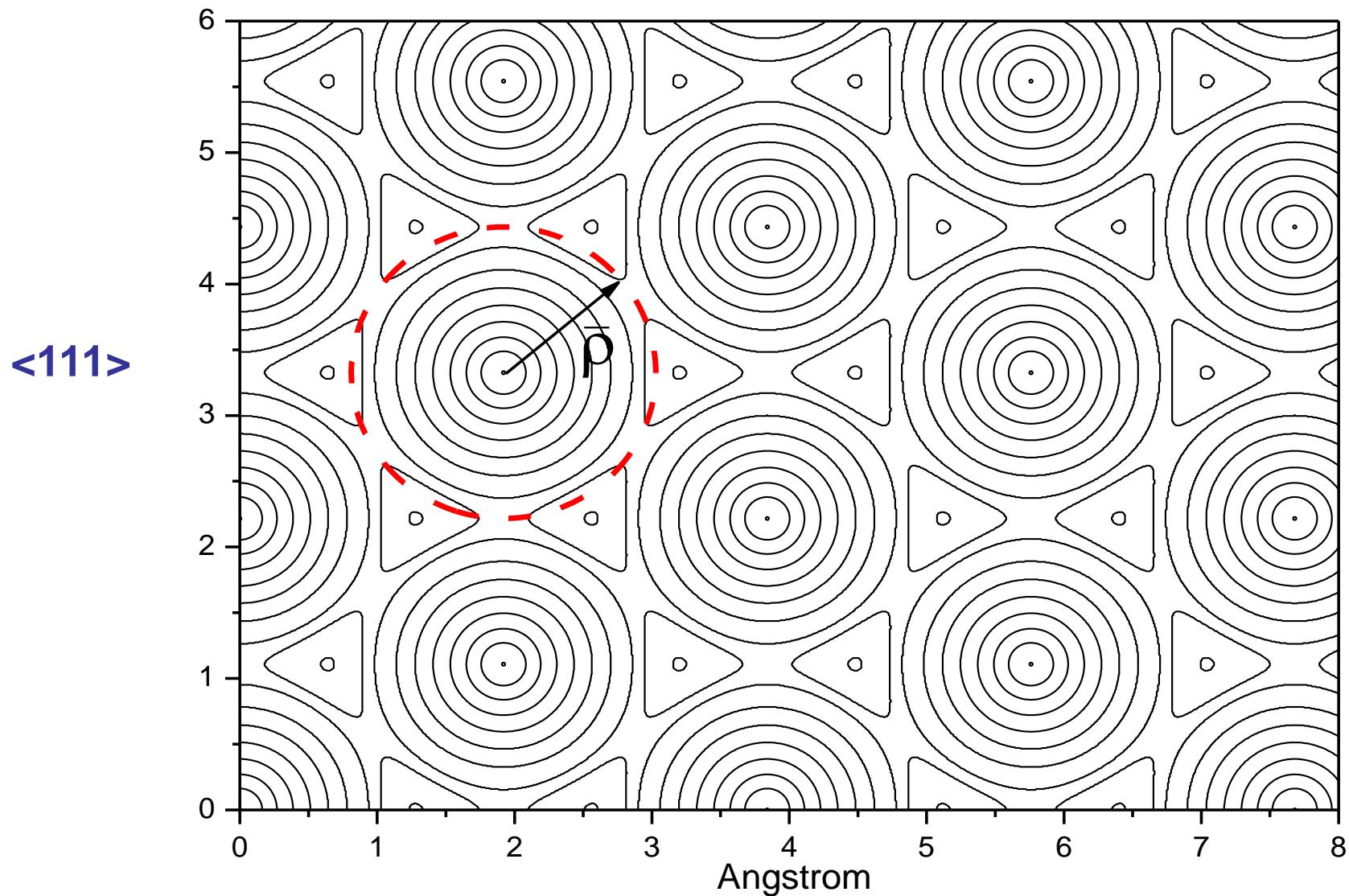
CB suppression

$$\frac{NZe^2}{\hbar c} \gg 1 \quad \frac{NZe^2}{mR} \square \frac{q}{m} \gg 1$$

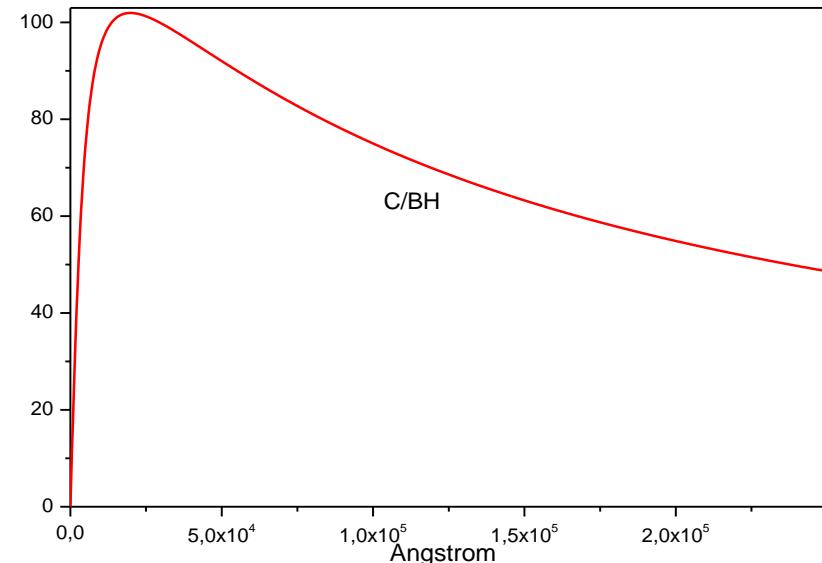
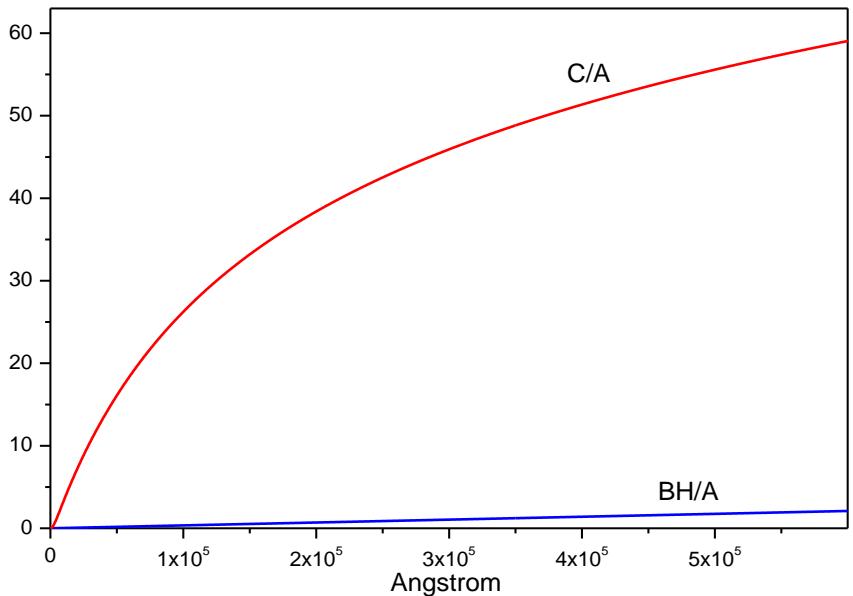
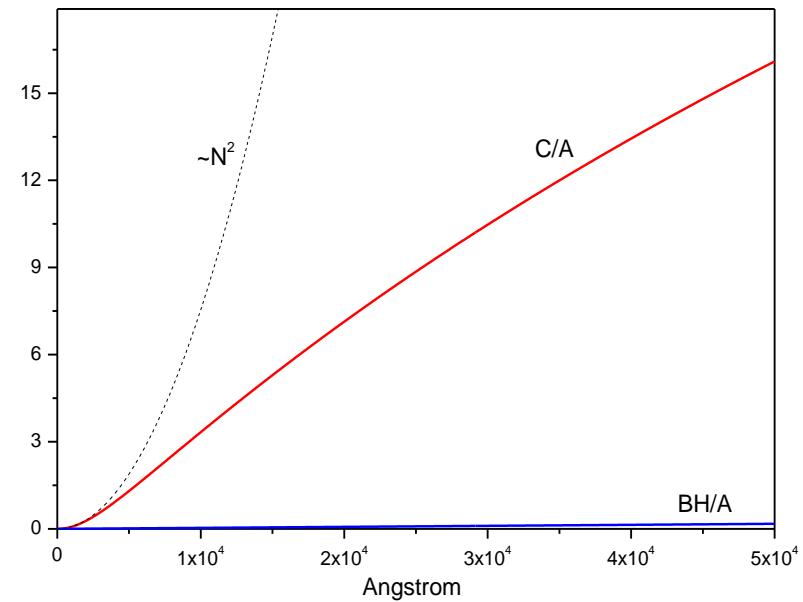
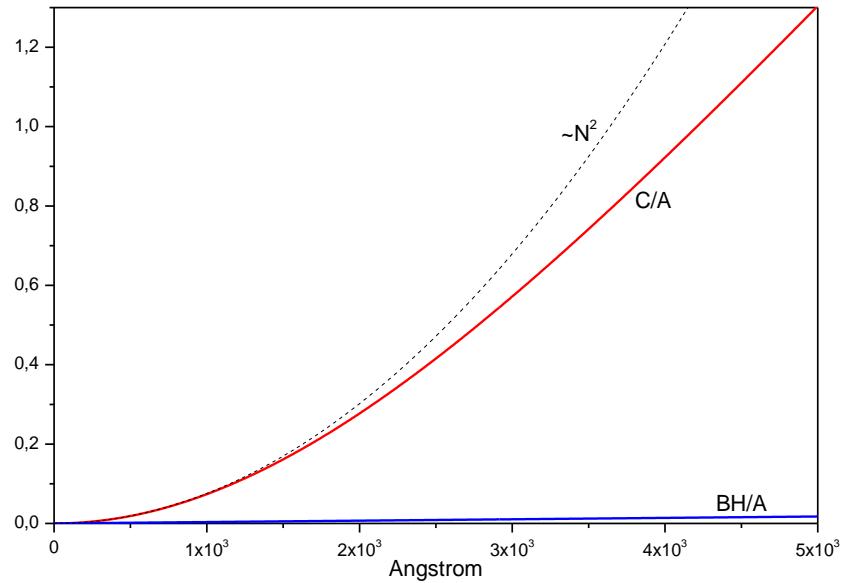
$$\hbar\omega \frac{d\sigma^{(N)}}{d\omega} \approx \frac{4e^2 R^2}{3} \frac{\varepsilon'}{\varepsilon} \left(1 + \frac{\hbar^2\omega^2}{2\varepsilon\varepsilon'} \right) \left(\ln \frac{NZe^2}{mR} \right)^3$$

Potential of atomic strings in a real Si crystal

$$U_c(\vec{\rho}) = \sum_n U_R(\vec{\rho} - \vec{\rho}_n)$$



CB for real potential of atomic string



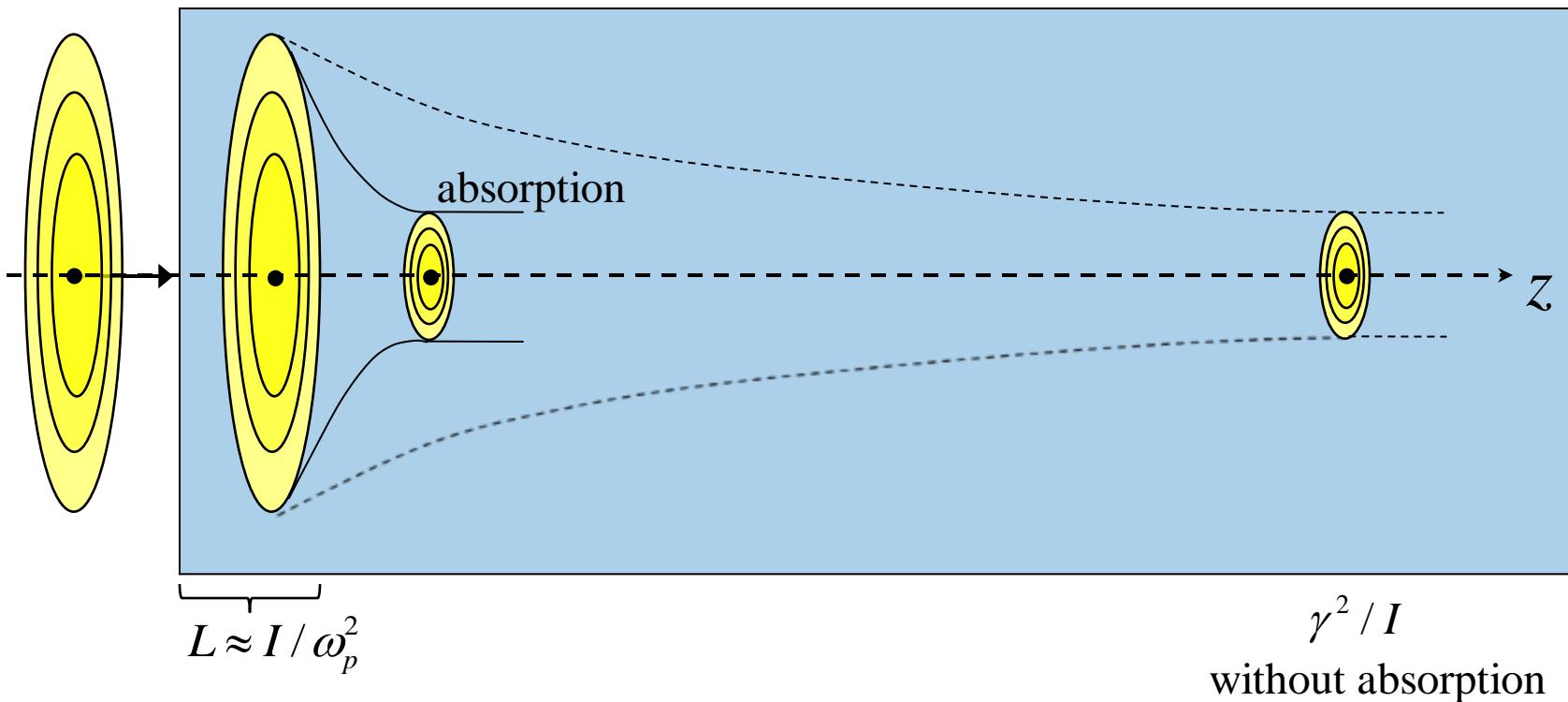
Ionization energy losses

(N. Shul'ga, S. Trofymenko Phys. Lett. A 2012)

IONIZATION ENERGY LOSSES

N. Shul'ga, S. Trofymenko, Phys. Lett. A, 2012

Bethe-Bloch



G. Garibian, 1959
A. Sørensen, 1987

29

29

Thin Target

Bethe-Bloch and Fermi formulas are related to the in infinite medium

Garibian G.M. // JETP, 1959

Sørensen A. // Phys.Rev.A, 1987

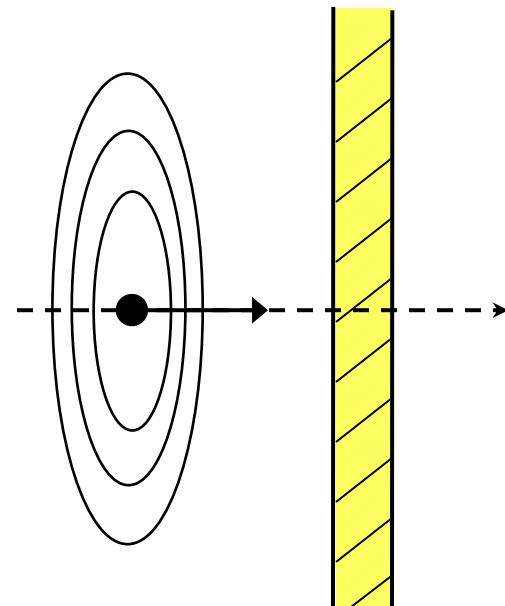
The density effect is absent for the targets with thickness

$$L \leq I / \omega_p^2$$

Energy losses by a particle:

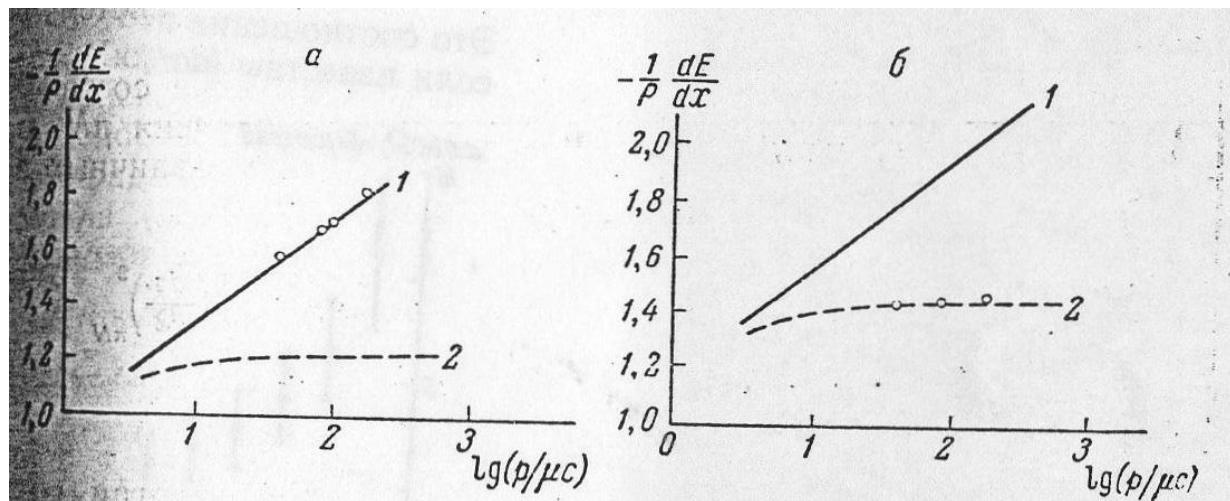
$$\frac{dE}{dz} = \frac{\omega_p^2 e^2}{v^2} \ln \frac{\gamma/I}{\rho_{\min}}$$

$$\begin{cases} 1 \leq \gamma \leq I / \omega_p \\ \gamma > I / \omega_p \end{cases}$$



FIRST EXPERIMENT (Kharkov, 1963)

A.I. Alikhanian, G.M. Garibian, M.P. Lorikian, A.K. Walter,
I.A. Grishaiev, V.A. Petrenko, G.L. Fursov

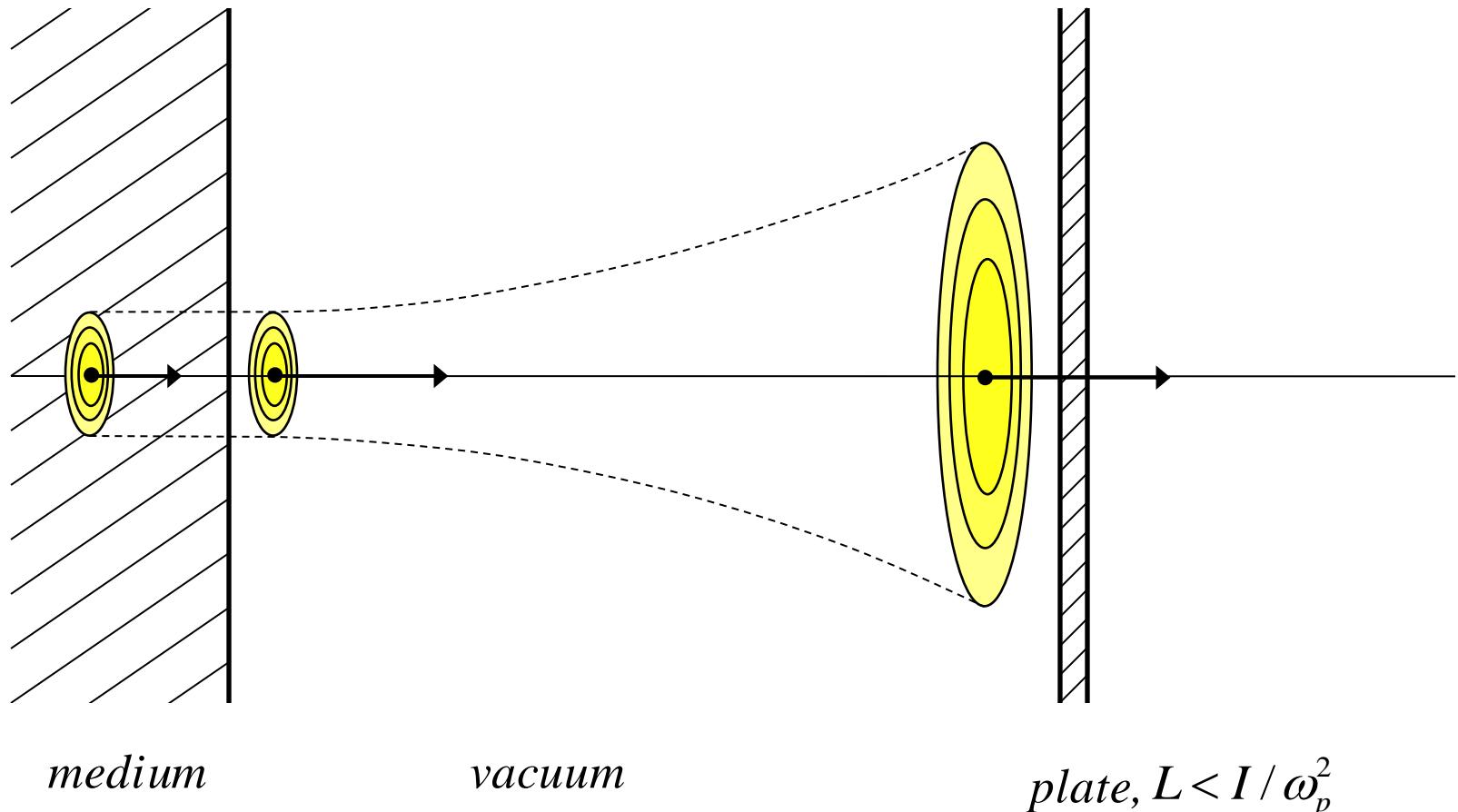


Electron energy losses in thin films of polystyrene of
thicknesses $10^{-6} cm$ (a) and $2 \times 10^{-3} cm$ (b)
1 – theoretical curve without density effect
2 – theoretical curve with density effect
circles show the measurement results

20 MeV < ϵ < 100 MeV

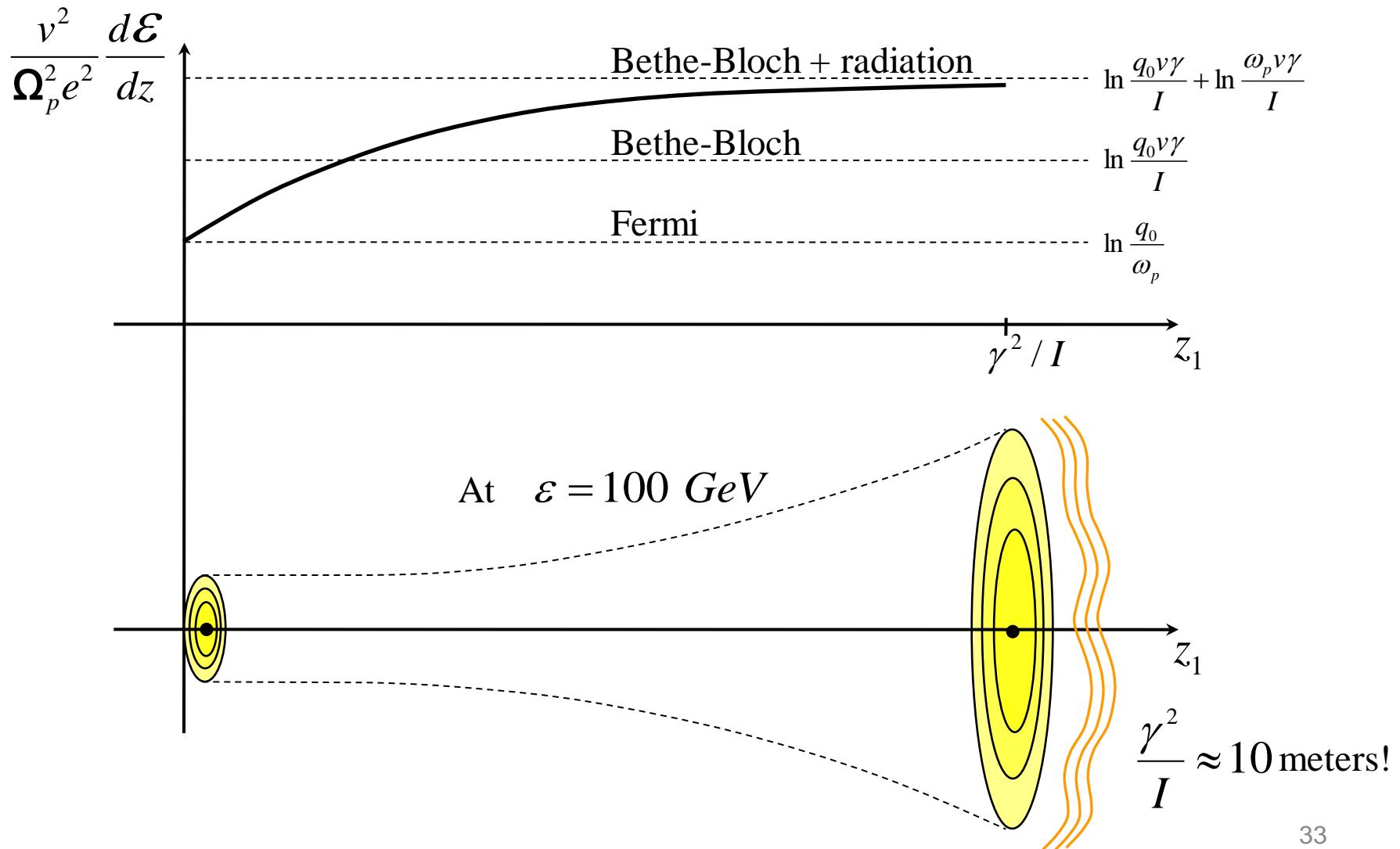
IONIZATION ENERGY LOSSES BY HALF-BARE ELECTRON

N. Shul'ga, S. Trofymenko, Phys. Lett. A, 2012



IONIZATION ENERGY LOSSES BY HALF-BARE ELECTRON

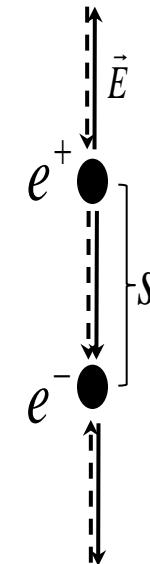
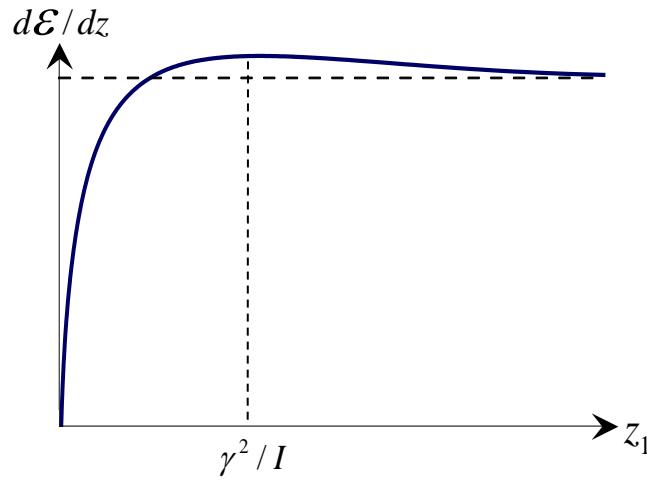
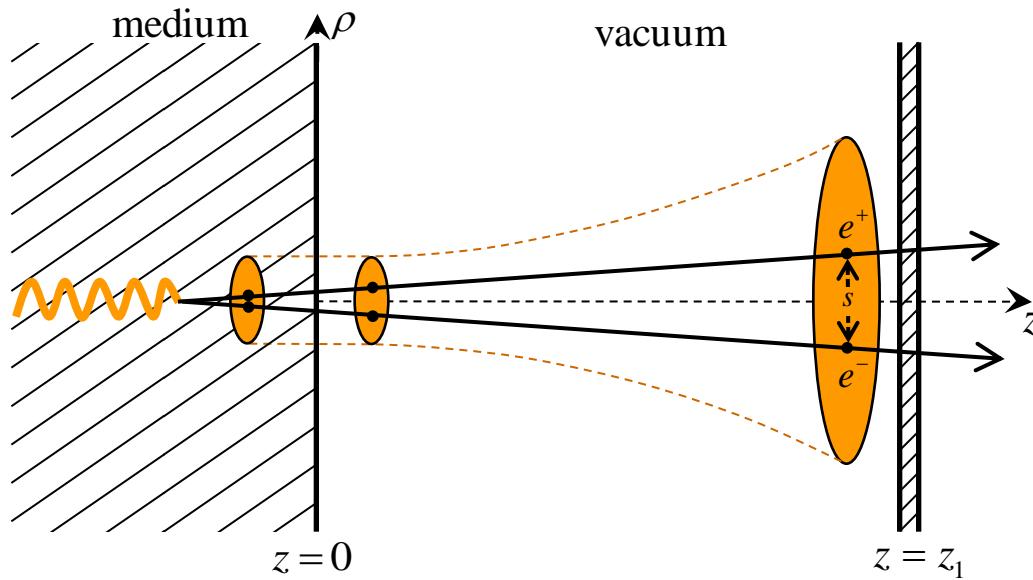
(from Fermi to Bethe-Bloch formula)



Ionization energy losses of high-energy electron-positron pair in thin targets

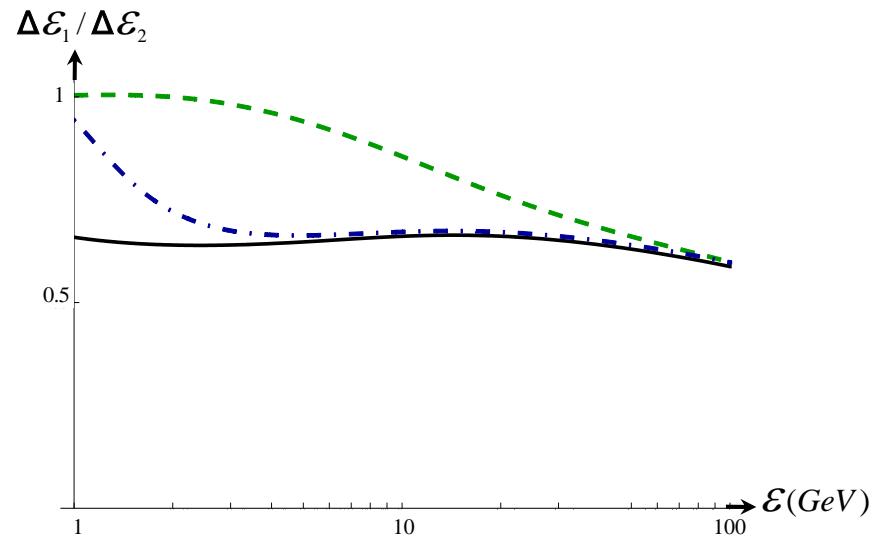
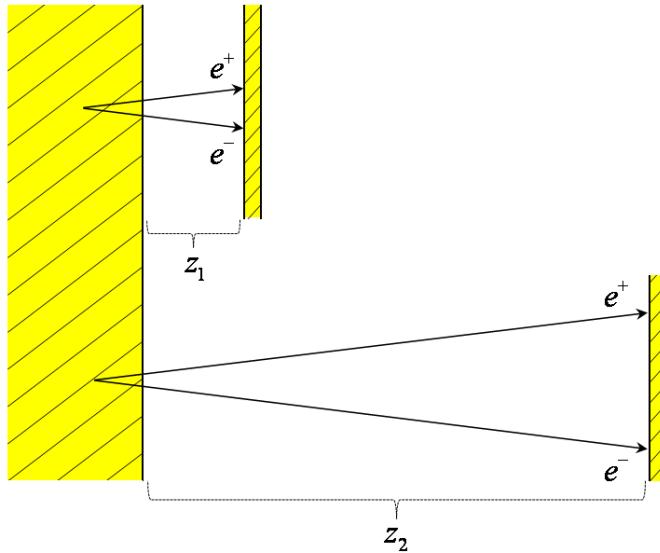
(S. Trofymenko, N. Shul'ga, 2013)

Anti-Chudakov effect



Ionization energy losses of e^+e^- pair under conditions of CERN experiment

Phys.Rev.Lett., 2008, v.100, 164802



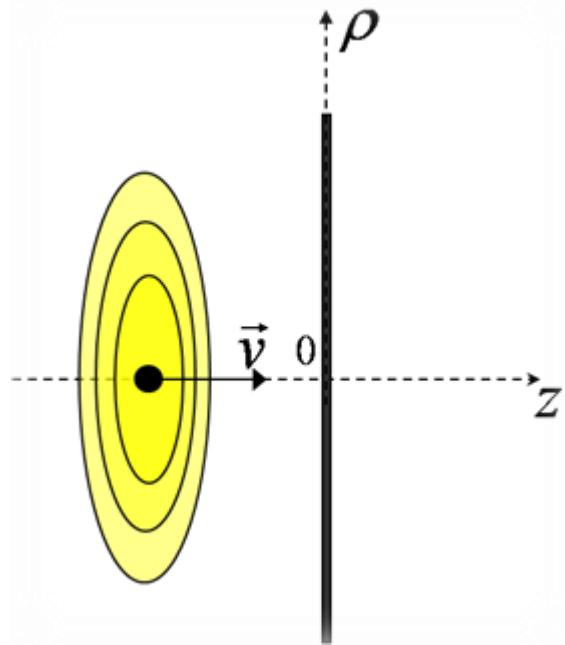
SOME OTHER EFFECTS WITH HALF-BARE PARTICLES

- transition radiation
- coherent radiation at beam-beam collisions
- LPM effect in thick crystals
- e^+e^- pair production in thin targets
- electromagnetic showers
- quark-gluon plasma
- moving polaron in solid
- ...
- ...
- ...

**THANK YOU FOR YOUR
ATTENTION!**

Transition radiation

TRANSITION RADIATION BY ELECTRON WITH EQUILIBRIUM FIELD



Total field:

$$\varphi = \varphi^C + \varphi^f$$

Boundary condition:

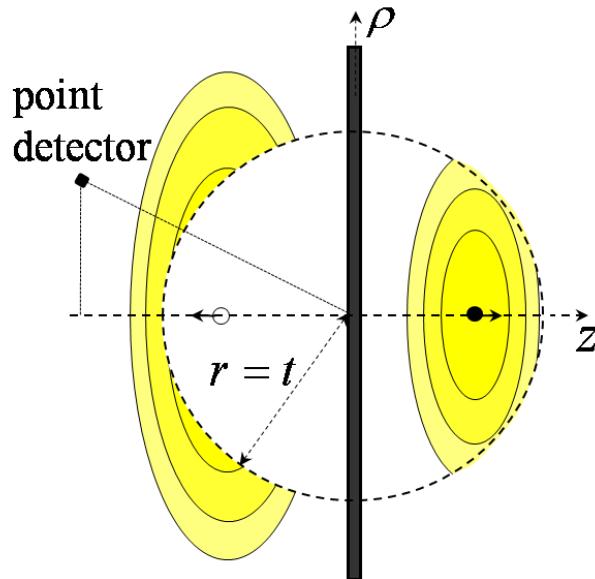
$$\vec{E}_\perp^C(\vec{\rho}, z = 0, t) + \vec{E}_\perp^f(\vec{\rho}, z = 0, t) = 0$$

Fourier integral for radiation field:

$$\varphi^f(\vec{r}, t) = -\frac{e}{2\pi^2 v} \int d^2 k_\perp \int_{-\infty}^{\infty} d\omega \frac{1}{k_\perp^2 + \omega^2 / p^2} e^{i(z\omega\sqrt{1-k_\perp^2/\omega^2} - \omega t + \vec{k}_\perp \cdot \vec{\rho})}$$

STRUCTURE OF TR ELECTROMAGNETIC FIELD

N.Shul'ga, S. Trofymenko, V. Syshchenko, Nuovo Cimento (2011)



$$E = 50 \text{ Mev} \quad \lambda \approx 0.1 \text{ cm}$$

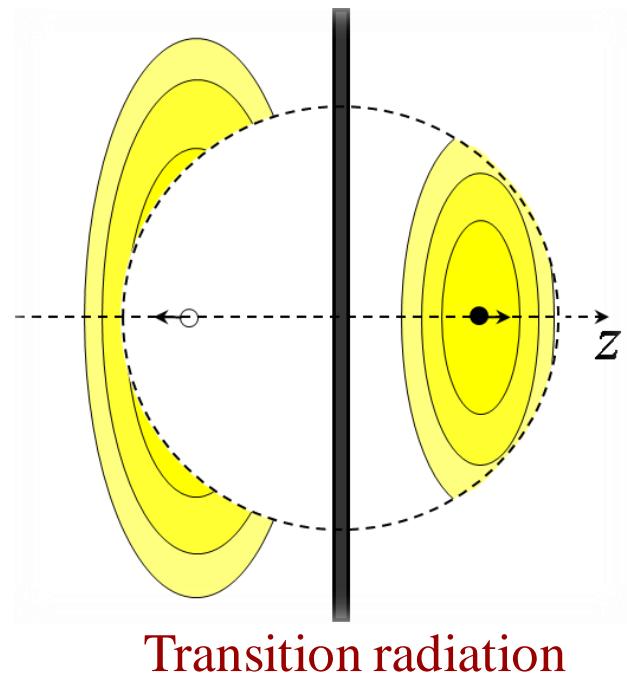
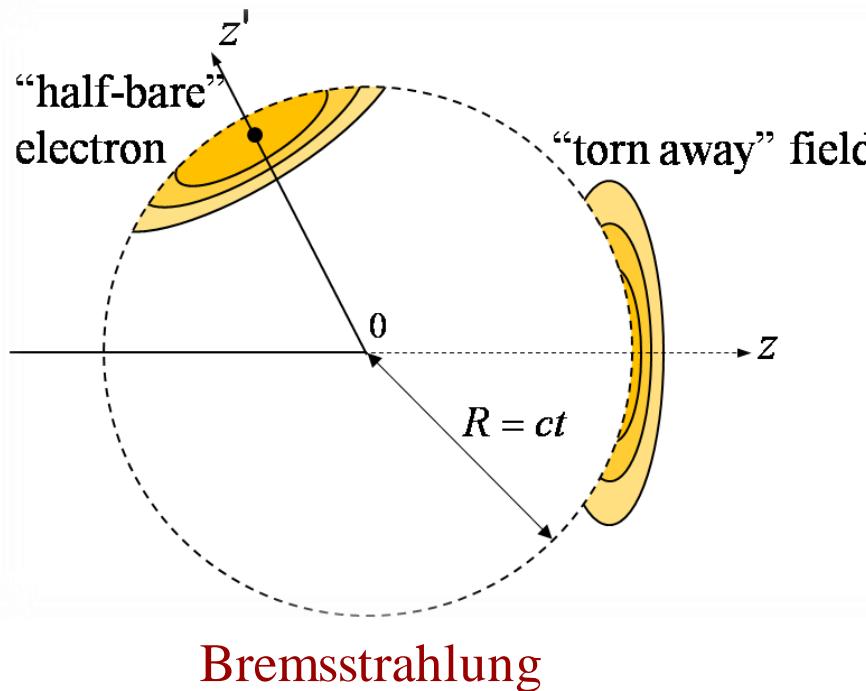
$$l_c \approx 2\gamma^2\lambda \approx 20 \text{ m} \quad l_T \approx \gamma\lambda \approx 10 \text{ cm}$$

For $t > 0$:

$$\mathbf{z} > \mathbf{0}: \quad \varphi(\vec{r}, t) = \left[\frac{e}{\sqrt{\rho^2\gamma^{-2} + (z - vt)^2}} - \frac{e}{\sqrt{\rho^2\gamma^{-2} + (z + vt)^2}} \right] \theta(t - r)$$

$$\mathbf{z} < \mathbf{0}: \quad \varphi(\vec{r}, t) = \left[-\frac{e}{\sqrt{\rho^2\gamma^{-2} + (|z| - vt)^2}} + \frac{e}{\sqrt{\rho^2\gamma^{-2} + (z - vt)^2}} \right] \theta(r - t)$$

THE ANALOGY IN BREMSSTRAHLUNG



The total field for $t > 0$:

$$\varphi(\vec{r}, t) = \theta(r - t)\varphi_{\vec{v}}(\vec{r}, t) + \theta(t - r)\varphi_{\vec{v}'}(\vec{r}, t)$$

A. Akhiezer, N. Shul'ga *High Energy Electrodynamics in Matter*, 1996
N. Shul'ga, V. Syshchenko, S. Shul'ga // Phys. Lett. A, 2009

Ionization energy losses

(N. Shul'ga, S. Trofymenko Phys. Lett. A 2012)

Bethe-Bloch Formula

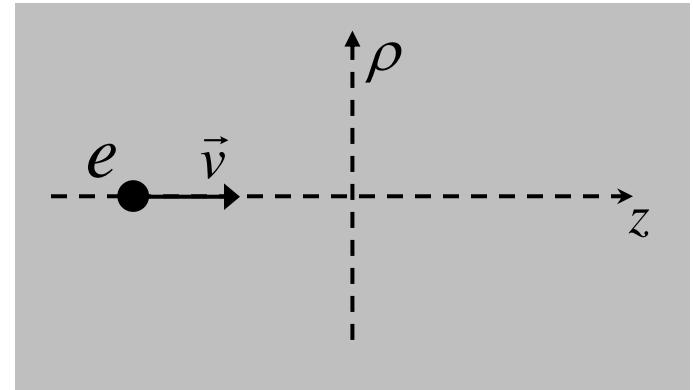
Infinite medium

Ionization energy losses with $q < q_0$

are considered

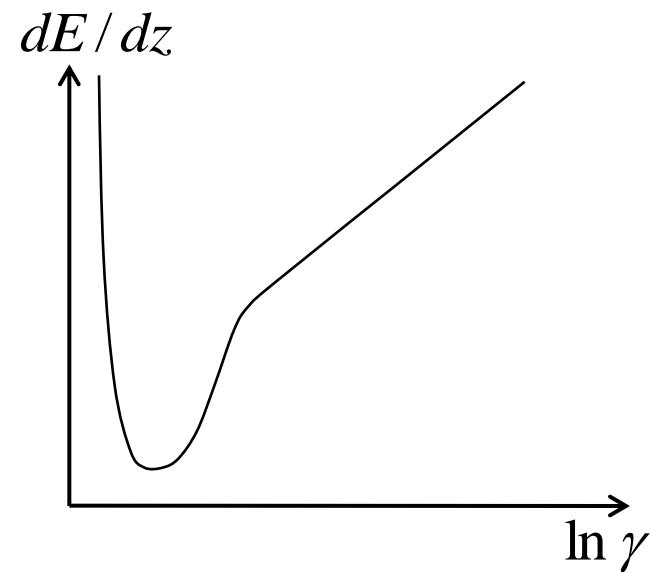
$$q_0$$

$$\rho > \rho_{\min} = 1/q_0$$



At $1 \leq \gamma \leq I / \omega_p$

$$\frac{dE}{dz} = \frac{\omega_p^2 e^2}{v^2} \ln \frac{\rho_{\max}}{\rho_{\min}} = \gamma / I$$



Fermi Formula

Infinite medium

At $\gamma > I / \omega_p$

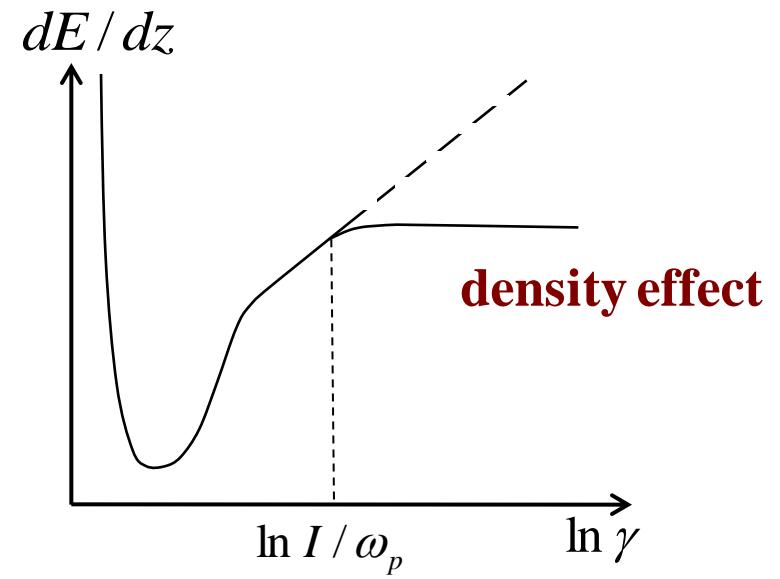
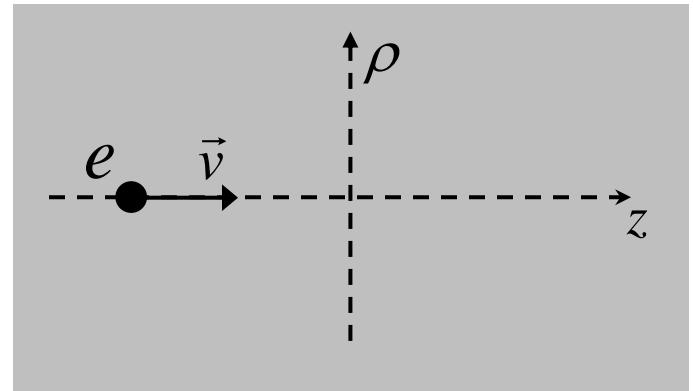
$$\phi(\vec{r}, t) = \phi_{Coul} e^{-\omega_p \sqrt{\rho^2 + \gamma^2(z - vt)^2}}$$

Screening at

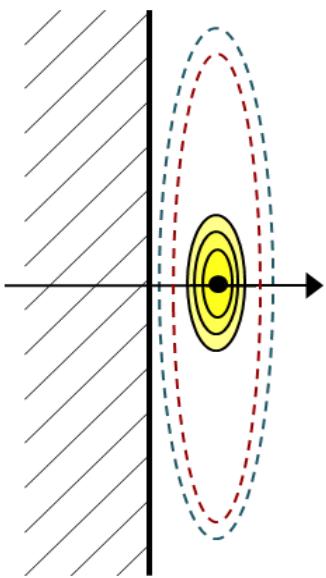
$$\rho > v / \omega_p$$

Ionization energy losses:

$$\frac{dE}{dz} = \frac{\omega_p^2 e^2}{v^2} \ln \frac{\rho_{\max}}{\rho_{\min}} = v / \omega_p$$



EVOLUTION OF THE FIELD AROUND THE ELECTRON IN VACUUM AT ULTRA HIGH ENERGIES



Full field:

$$\vec{E} = \vec{E}^C + \vec{E}^F$$

Boundary conditions:

$$\left. \begin{array}{l} (E_\rho^C + E_\rho^F)^{dielectric} \\ (E_z^C + E_z^F)^{dielectric} \end{array} \right|_{z=0} = \left. \begin{array}{l} (E_\rho^C + E_\rho^F)^{vacuum} \\ (E_z^C + E_z^F)^{vacuum} \end{array} \right|_{z=0}$$

$$+ \operatorname{div} \vec{E}^F = 0$$

Fourier decomposition of a free field

$$E_\rho^F(\vec{r}, t) = \frac{e}{\pi v} \int_0^\infty dq q^2 J_1(q\rho) \times$$

$$\times \int_{-\infty}^{+\infty} d\omega \frac{\sqrt{\omega^2 - q^2}}{\sqrt{\omega^2 \epsilon - q^2} + \epsilon \sqrt{\omega^2 - q^2}} \left[\frac{1 + \frac{v}{\omega} \sqrt{\omega^2 \epsilon - q^2}}{q^2 + \frac{\omega^2}{v^2} - \epsilon \omega^2} - \frac{\epsilon + \frac{v}{\omega} \sqrt{\omega^2 \epsilon - q^2}}{q^2 + \frac{\omega^2}{v^2} - \omega^2} \right] e^{i\sqrt{\omega^2 - q^2} z - i\omega t}$$

FIELD IN VACUUM

Fourier component of full field ($\gamma \gg 1$, $\omega \gg \omega_p$):

$$E_\omega^\rho(\vec{r}) = 2 \frac{e}{v} \int_0^\infty dq q^2 J_1(q\rho) \left\{ \underbrace{\left[\frac{1}{q^2 + \omega_p^2 + \frac{\omega^2}{\gamma^2}} - \frac{1}{q^2 + \frac{\omega^2}{\gamma^2}} \right]}_{\text{Free wave packet}} e^{i\omega z - \frac{q^2 z}{2\omega}} + \underbrace{\frac{e^{i\frac{\omega}{v}z}}{q^2 + \frac{\omega^2}{\gamma^2}}}_{\text{Coulomb field}} \right\}$$

For $z \rightarrow 0$:

$$E_\omega^\rho(\rho) = \frac{2e}{v} \sqrt{\frac{\omega^2}{\gamma^2} + \omega_p^2} K_1 \left(\rho \sqrt{\frac{\omega^2}{\gamma^2} + \omega_p^2} \right) e^{i\frac{\omega}{v}z} \quad \text{The electron is "half-bare" for } \omega \leq \gamma\omega_p$$

For $z > 2\gamma^2/\omega$:

$$E_\omega^\rho(\rho, z) = \frac{2e}{v} \left\{ \frac{\omega}{\gamma} K_1 \left(\frac{\omega}{\gamma} \rho \right) e^{i\frac{\omega}{v}z} + \frac{e^{i\omega r}}{r} F(\rho/z) \right\} \quad r = \sqrt{\rho^2 + z^2}$$

IONIZATION OF MEDIUM BY EXTERNAL FIELD

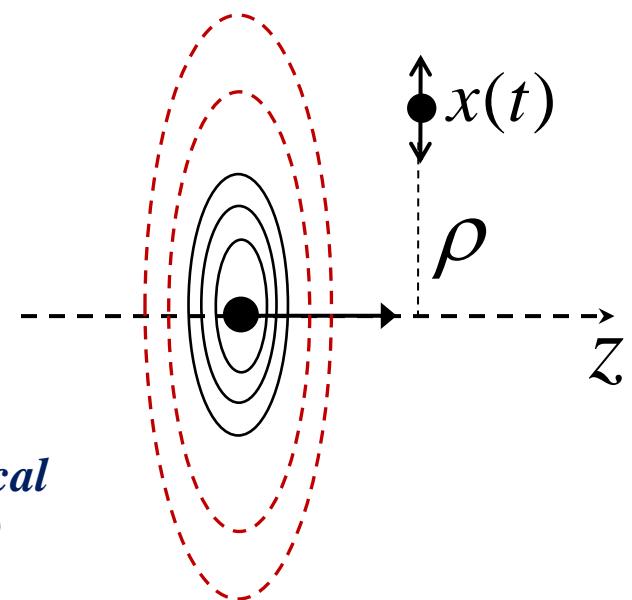
$$\omega_0 = I$$

$$\ddot{x} + \beta \dot{x} + \omega_0^2 x = \frac{e}{m} (E_\rho^C + E_\rho^F)$$

Transmission of energy to a harmonic oscillator by the external field:

$$\Delta \mathcal{E} = \frac{e^2}{2m} |E_{\omega_0}(\vec{r})|^2$$

J.D. Jackson // Classical electrodynamics, 1999



Summary ionization on the unity of length

$$\frac{d\mathcal{E}}{dz} = n \frac{e^2}{2m} \int_0^\infty d\rho 2\pi\rho |E_{\omega_0}^\rho(\vec{r})|^2$$