COHERENT BREMSSTRAHLUNG OF RELATIVISTIC ELECTRONS UNDER THE EXTERNAL ACOUSTIC FIELD

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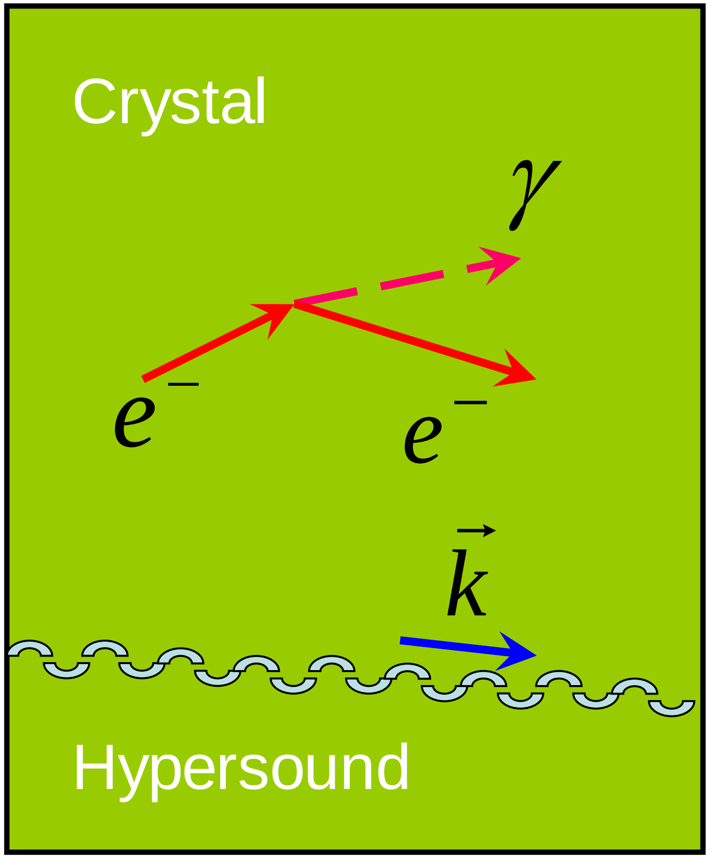
Outline

- Motivation
- Analysis of conditions for the influence of hypersound on the cross-section of bremsstrahlung
- Cross-section of the bremsstrahlung in crystals in presence of hypersonic vibrations
- Numerical results and discussion

Motivation

- In crystals the cross-sections of the high-energy electromagnetic processes can change essentially compared with the corresponding quantities for a single atom
- From the point of view of controlling the parameters of the high-energy electromagnetic processes in a medium it is of interest to investigate the influence of external fields (acoustic waves, temperature gradient) on the corresponding characteristics
- Investigation of bremsstrahlung by high-energy electrons is of interest from the viewpoint of the underlying physics and from the viewpoint of practical applications for generation of intense photon beams

Problem setting and notations



Photon energy ω

Photon momentum $\vec{k}$

Initial energy of electron E_1

Initial momentum of electron $\vec{p}_1$

Final energy of electron E_2

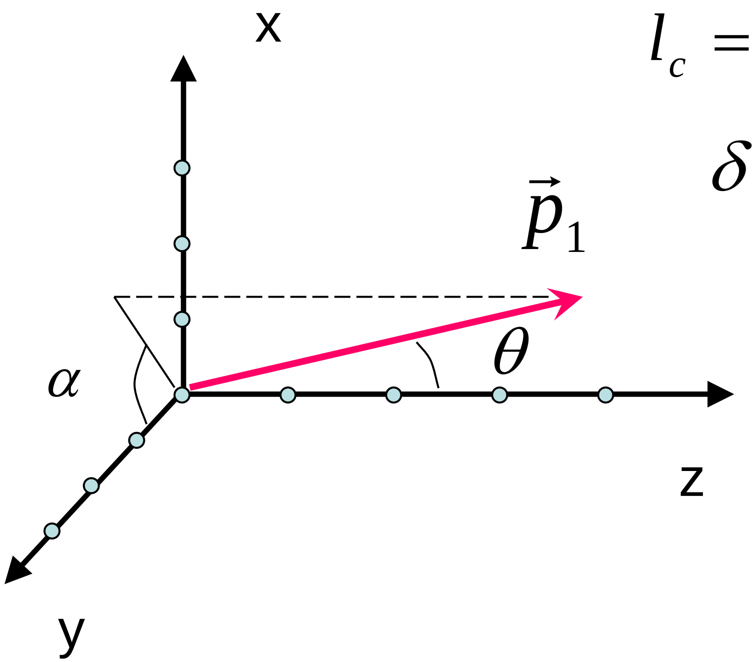
Final momentum of electron $\vec{p}_2$

Hypersound wave vector $\vec{k}_s$

Displacements of atoms due to the hypersound $\vec{u} = \vec{u}_0 f(\vec{k}_s \vec{r})$

Geometry of the problem

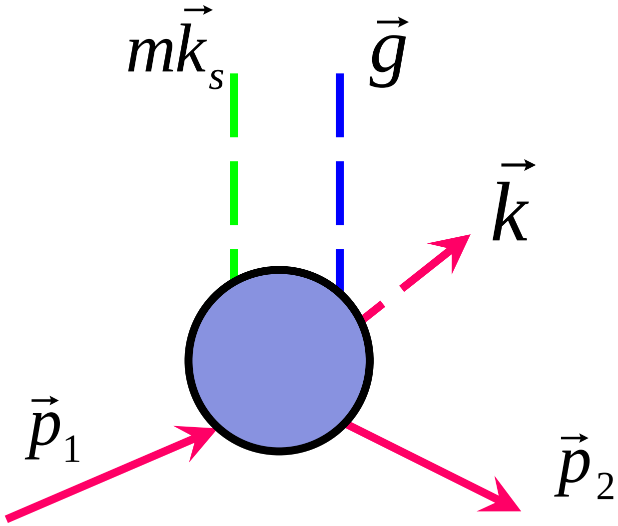
Coherence effects are essential if the electron enters into the crystal at small angle with respect to the crystallographic axis



$l_c = 2E_1E_2/\omega m_e^2$ ⇨ formation length

$\delta = 1/l_c$ ⇨ minimum longitudinal momentum transfer

Condition for the influence



- Momentum conservation

$$\vec{p}_1 = \vec{p}_2 + \vec{k} + \vec{g} - m\vec{k}_s$$

- Dominant contribution comes from

$$|m| \lesssim \lambda_s / a, \qquad \lambda_s = 2\pi / k_s$$

a — interatomic distance

- Influence of the deformation field may be considerable if $|mk_{s\|}| \gtrsim \delta$

- Condition for the influence of the hypersound to be essential

$$u_0 / \lambda_s \gtrsim a / (4\pi^2 l_c)$$

Cross-section

- Cross-section $d\sigma = N_0(d\sigma_n + d\sigma_c)\,,\; N_0$ number of atoms
- Coherent part of the cross-section

$$\frac{d\sigma_c}{d\omega} = \frac{e^2 N}{N_0 E_1^2 \Delta} \sum_{m,\vec{g}} \frac{g_{m\perp}^2}{g_{m||}^2} \left|F_m(\vec{g}_m \vec{u}_0)\right|^2 \left|S(\vec{g}_m, \vec{g})\right|^2 \times \left[1 + \frac{\omega^2}{2E_1E_2} - 2\frac{\delta}{g_{m||}}\left(1 - \frac{\delta}{g_{m||}}\right)\right] \qquad \delta = 1/l_c$$

N number of cells

Δ unit cell volume

$$\vec{g}_m = \vec{g} - m\vec{k}_s$$

$$m = 0, \pm 1, \pm 2, \ldots$$

Reciprocal lattice vector

$g_{m||}$ and $g_{m\perp}$ are the parallel and perpendicular components of $\vec{g}$ with respect to the photon momentum

- Summation goes under the constraint $g_{m||} \geq \delta$

Cross-section

$$F_m(x) = \frac{1}{2\pi}\int_{-\pi}^{+\pi} e^{ixf(t)-imt}dt\,,\quad S(\vec{g})$$ structure factor of the crystal

- For $u_0 = 0$ one has $F_m = \delta_m^0$ and from the general formula the bremsstrahlung cross-section in an undeformed crystal is obtained

- Sinusoidal deformation field $f(z) = \sin(z+\varphi_0)$

$$\boxed{F_m(z) = e^{im\varphi_0} J_m(z)}$$

Bessel function

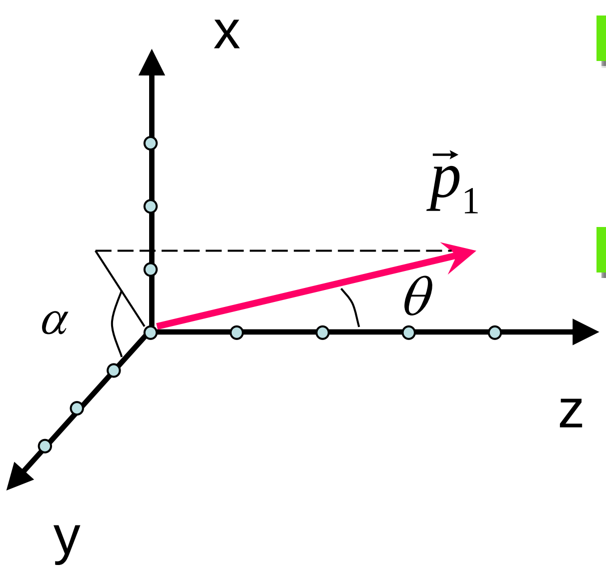
- In the case of the presence of the hypersound the formula for the cross-section differs from the corresponding formula for an undeformed crystal by the replacement $\vec{g} \to \vec{g}_m$, and additional summation over *m* with weights $\left|F_m(\vec{g}_m\vec{u}_0)\right|^2$

Qualitatively different cases

- Orthogonal crystal lattice with the reciprocal lattice vector components

$$g_i = 2\pi n_i/a_i,\; n_i = 0, \pm 1, \pm 2, \ldots,\; i = 1, 2, 3$$

- Coherent effects appear when the electron enters into the crystal at small angles θ



- Main contribution to the cross-section give the terms with $g_z = 0$

- Qualitatively different cases

  a) Angles α and $\pi/2-\alpha$ are not small

  b) Angle α is small and $\delta \sim 2\pi\theta / a_2$

  c) Angle α is small and $\delta \sim 2\pi\theta\alpha / a_1$

Qualitatively different cases

- Angles α and $\pi/2-\alpha$ are not small

$$\sum_{g_x,g_y} \to (a_1 a_2/4\pi^2)\int dg_x g_y$$

- Angle α is small and $\delta \sim 2\pi\theta/a_2$

$$g_{m\|} \approx -mk_{s\|} + \theta g_y \geq \delta\,,\ \sum_{g_x} \to (a_1/2\pi)\int dg_x,$$

Formula for the cross-section is further simplied in the case when the amplitude of the deformation field is perpendicular to the crystallographic *x*-axis

$$\frac{d\sigma_c}{d\omega} \approx \frac{e^2 N}{2\pi E_1^2 a_2 a_3 N_0}\sum_{m,g_y}\left[1+\frac{\omega^2}{2E_1E_2}-2\frac{\delta}{g_{m\|}}\left(1-\frac{\delta}{g_{m\|}}\right)\right]$$

$$\times\frac{|F_m(\mathbf{g}_m\mathbf{u}_0)|^2}{g_{m\|}^2}\int dg_x g_\perp^2|S(\mathbf{g}_m,\mathbf{g})|^2,$$

Effective structure factor

Qualitatively different cases

- Angle α is small and $\delta \sim 2\pi\theta\alpha/a_1$

  Dominant contribution comes from the terms $g_y = 0$

$$g_{m\|} \approx -mk_{z\|} + \psi g_x, \qquad \psi \equiv \alpha\theta,$$

Numerical calculation

- Numerical calculations have been performed for $\mathrm{SiO_2}$ single crystal at low temperatures

- For the Fourier transforms of the atomic potentials the Moliere parametrization is used

$$u_q^{(j)} = \sum_{i=1}^{3} \frac{4\pi Z_j e^2 \alpha_i}{q^2 + (\chi_i / R_j)^2}, \; \alpha_i = \{0.1, 0.55, 0.35\}, \; \chi_i = \{6.0, 1.2, 0.3\},$$

Screening radius of the *j*-th atom

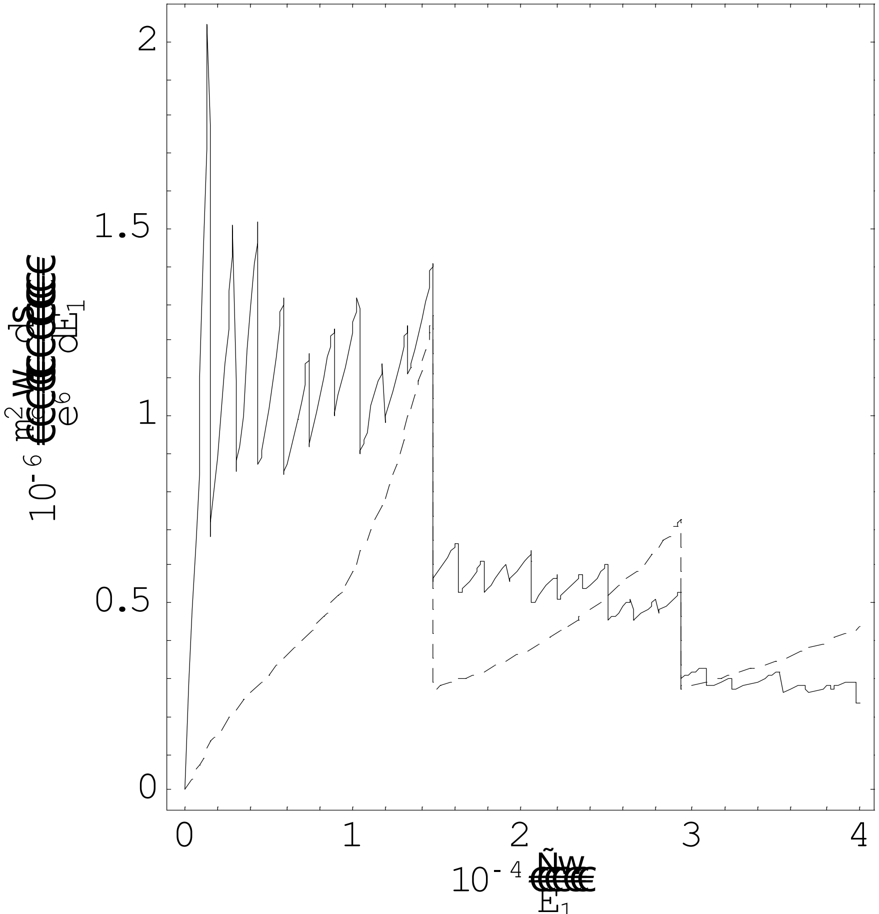
- Sinusoidal transversal acoustic wave of the S-type

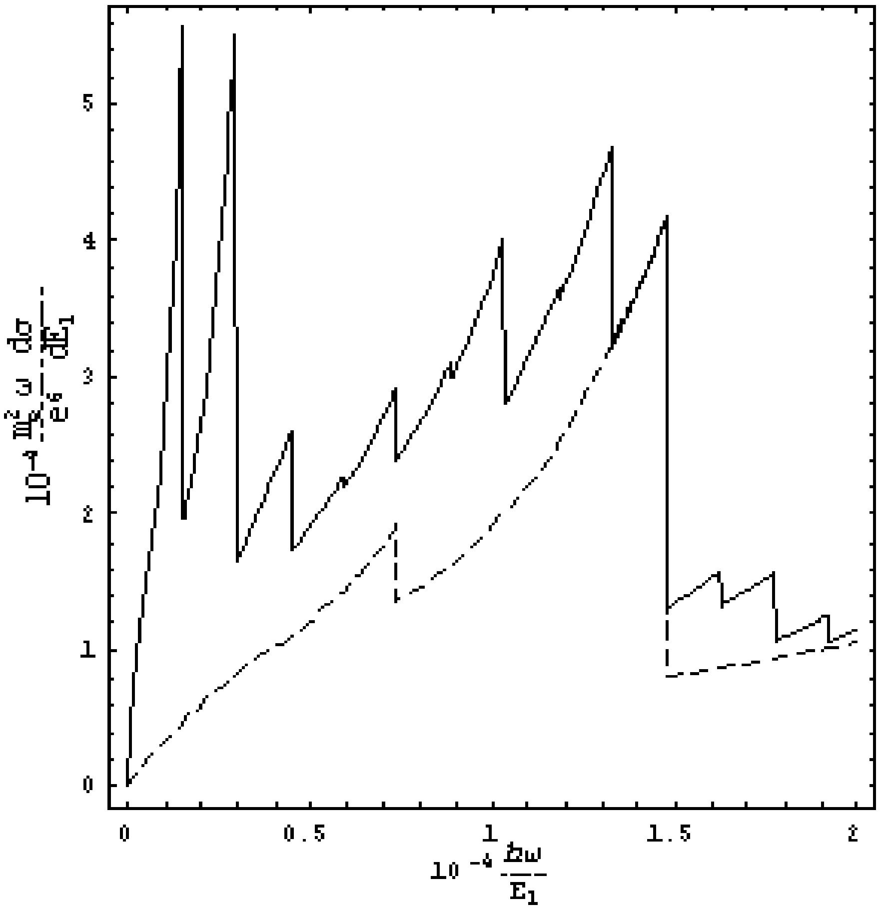
  The vector determining the direction of the hypersound propagation lies in the *yz*-plane and forms the angle 0.295 rad with the *z*-axis

- Numerical calculations show that, in dependence of the values for the parameters the external excitation can either enhance or reduce the bremsstrahlung cross-section

Coherent part of the cross-section: Numerical examples

$$E_1 = 70\ \mathrm{MeV},\ \nu_s = 2.5\ \mathrm{GHz}$$



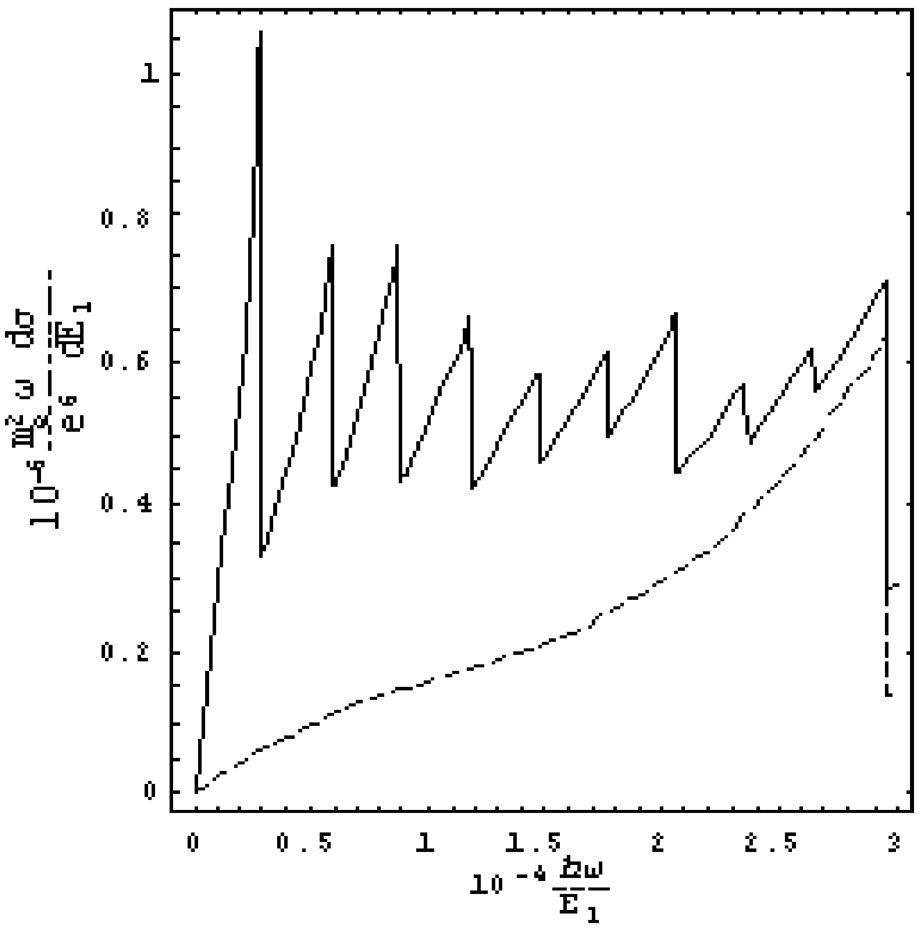


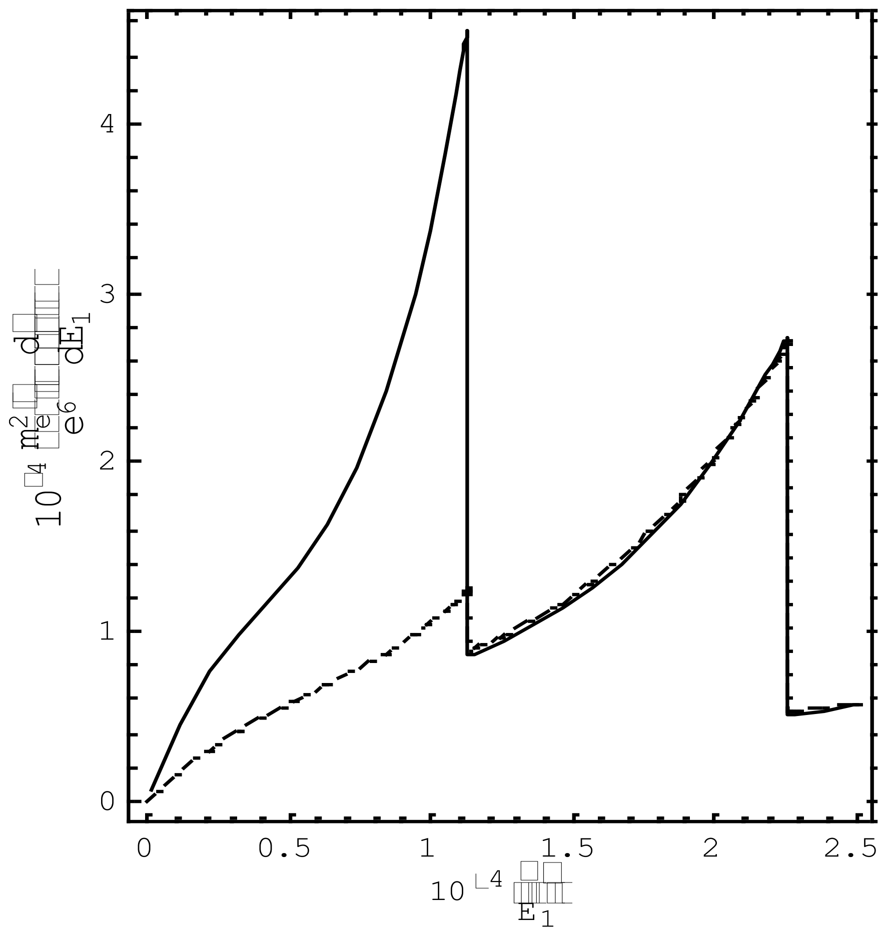
$$2\pi u_0 / a_2 = 0.55,\ \ \theta = 0.00012$$

$$2\pi u_0 / a_1 = 0.28,\ \ \psi = \alpha\theta = 0.00006$$

Numerical examples

$$E_1 = 70\ \mathrm{MeV},\ \nu_s = 5\,\mathrm{GHz}$$



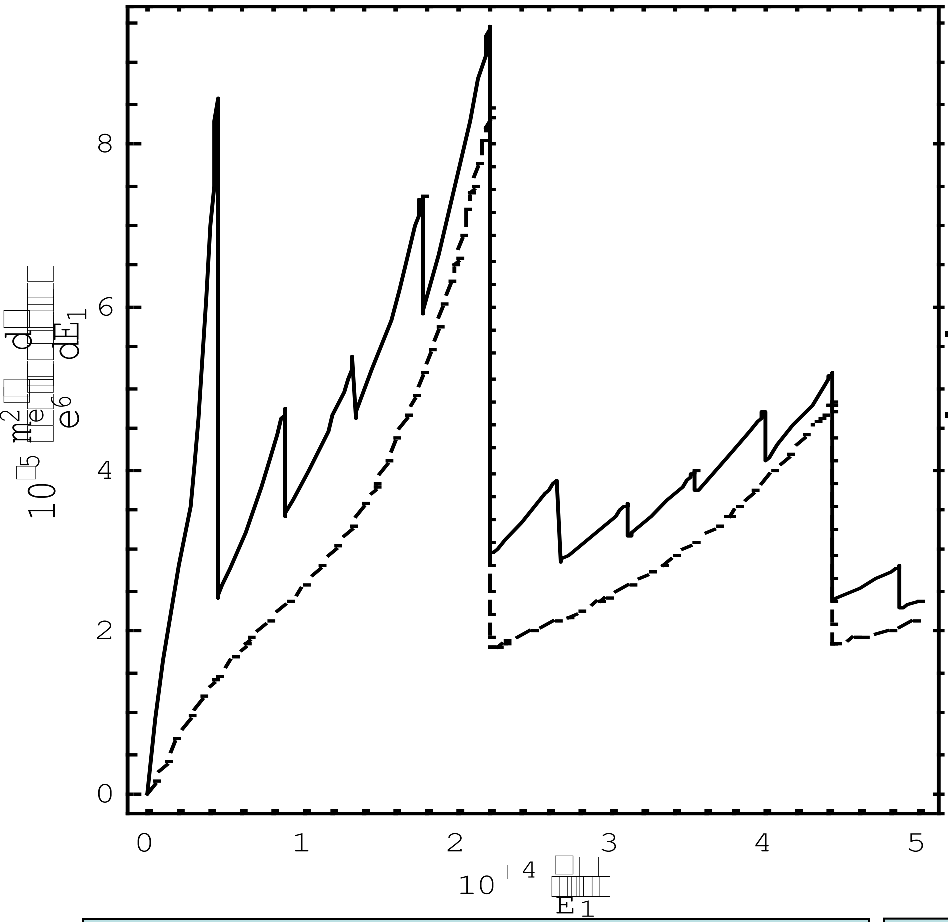


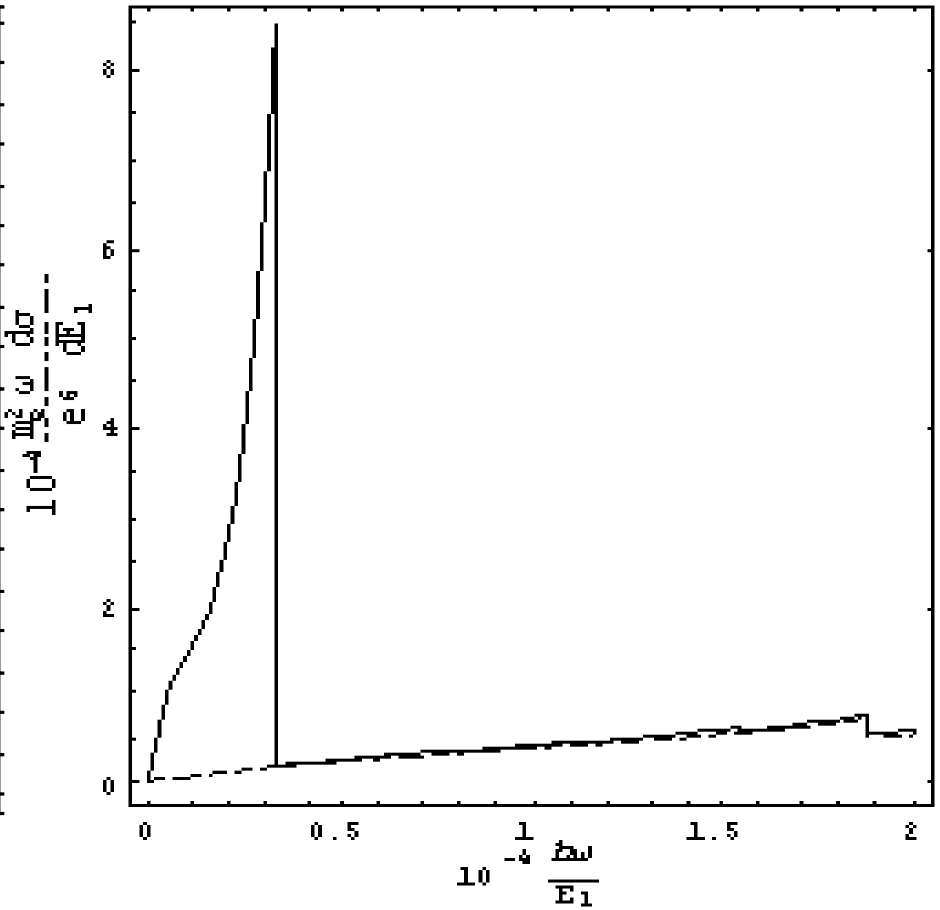
$$2\pi u_0 / a_2 = 0.55,\ \theta = 0.00024$$

$$2\pi u_0 / a_1 = 0.25,\ \psi = \alpha\theta = 0.00092$$

Numerical examples

$E_1 = 70$ MeV, $\nu_s = 15$ GHz



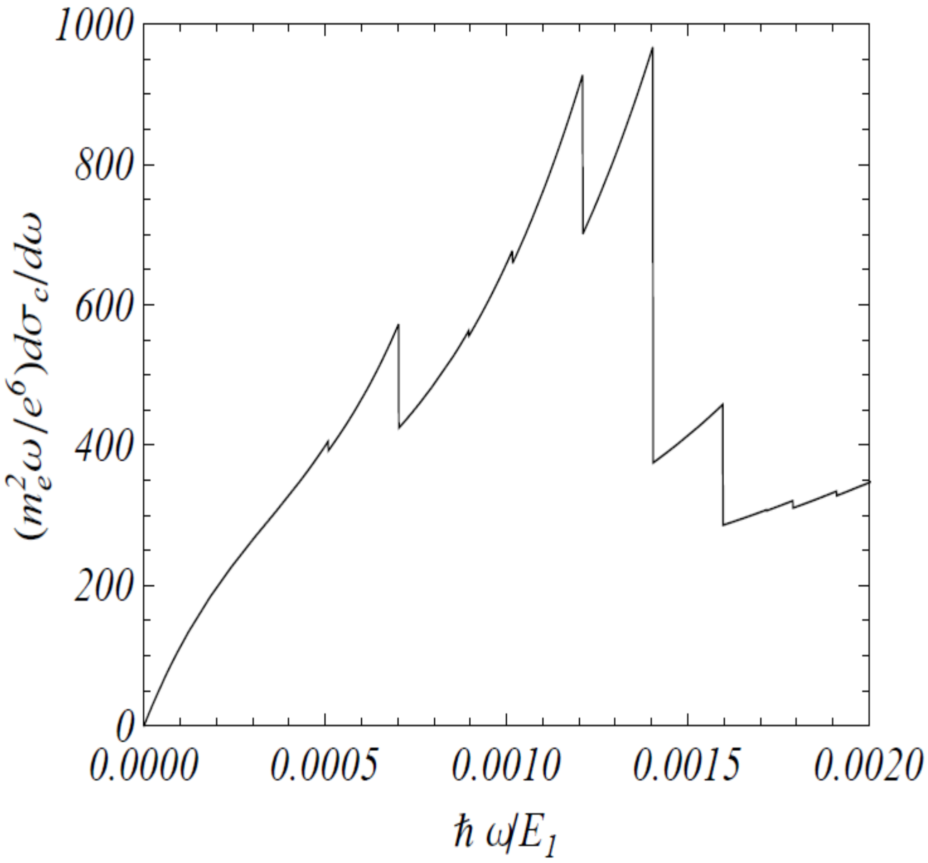


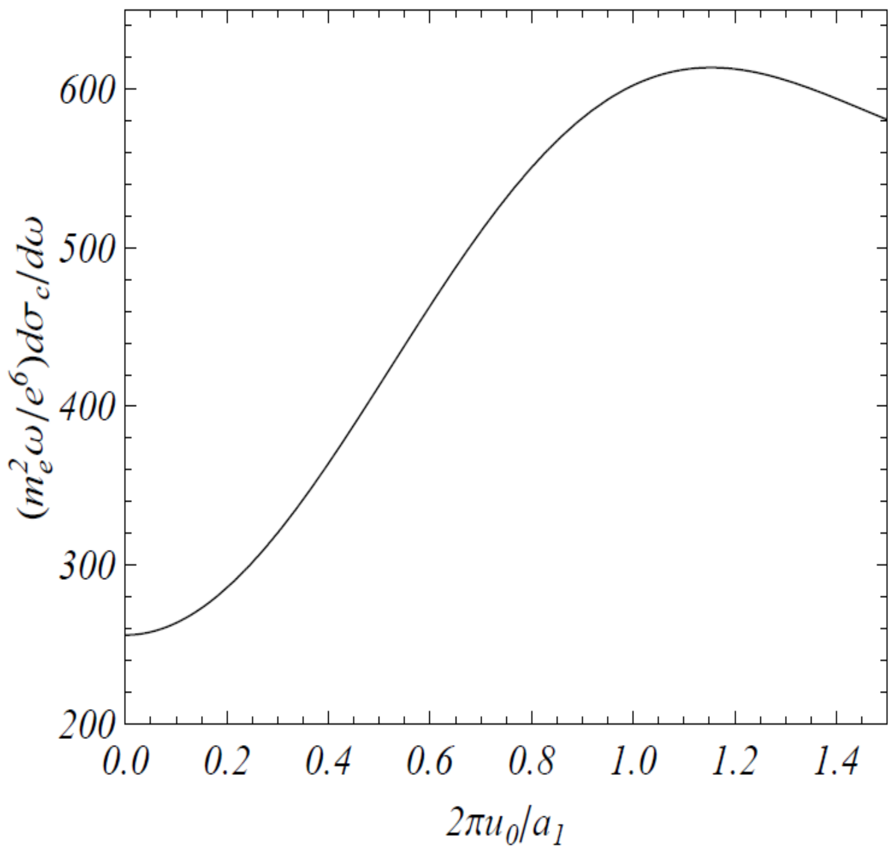
$2\pi u_0 / a_2 = 0.18,\ \theta = 0.00018$

$2\pi u_0 / a_1 = 0.165,\ \psi = \alpha\theta = 0.000146$

Numerical examples

$$E_1 = 20\ \mathrm{MeV},\ \nu_s = 5\,\mathrm{GHz}$$



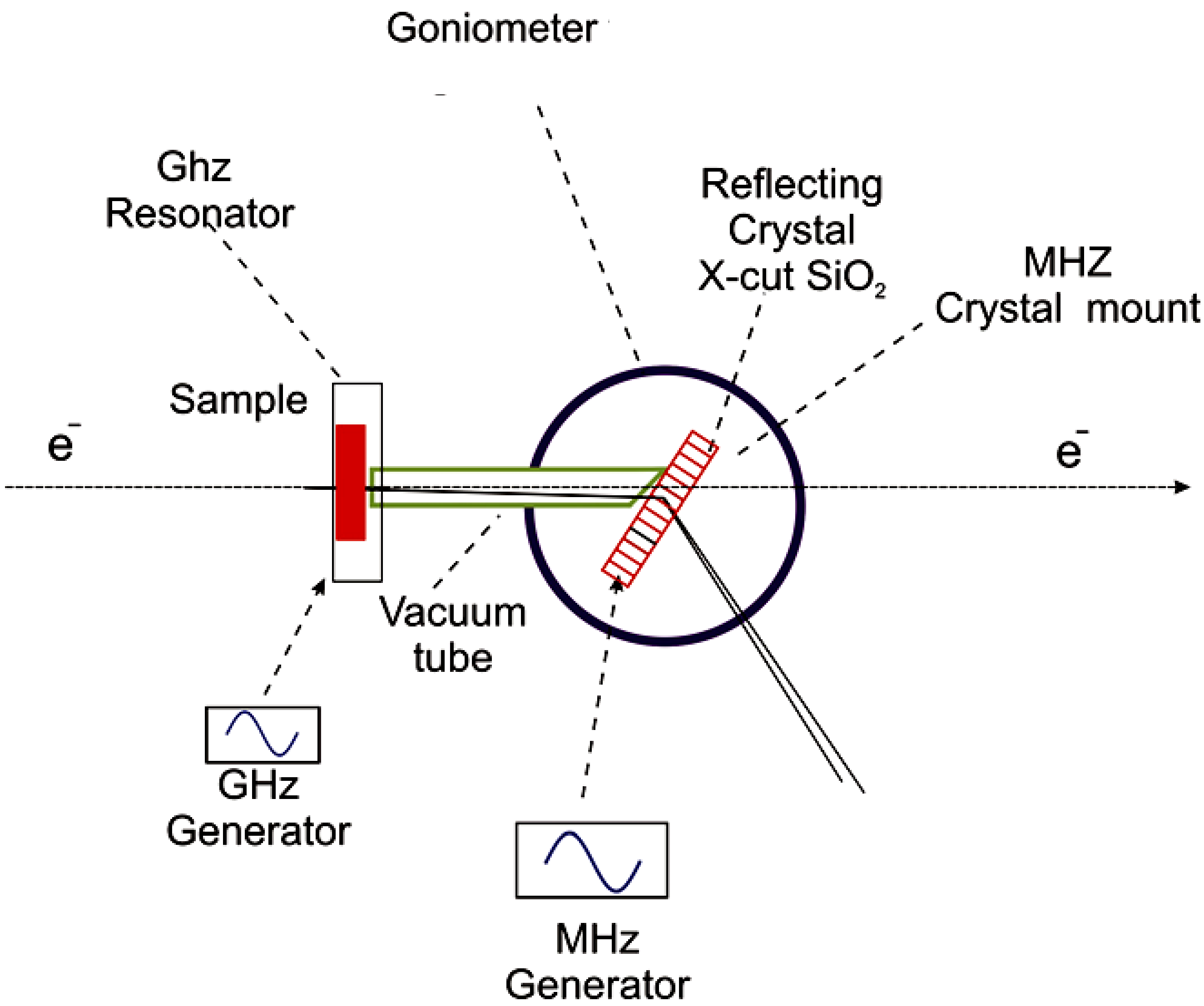


$$\psi = 0.002$$

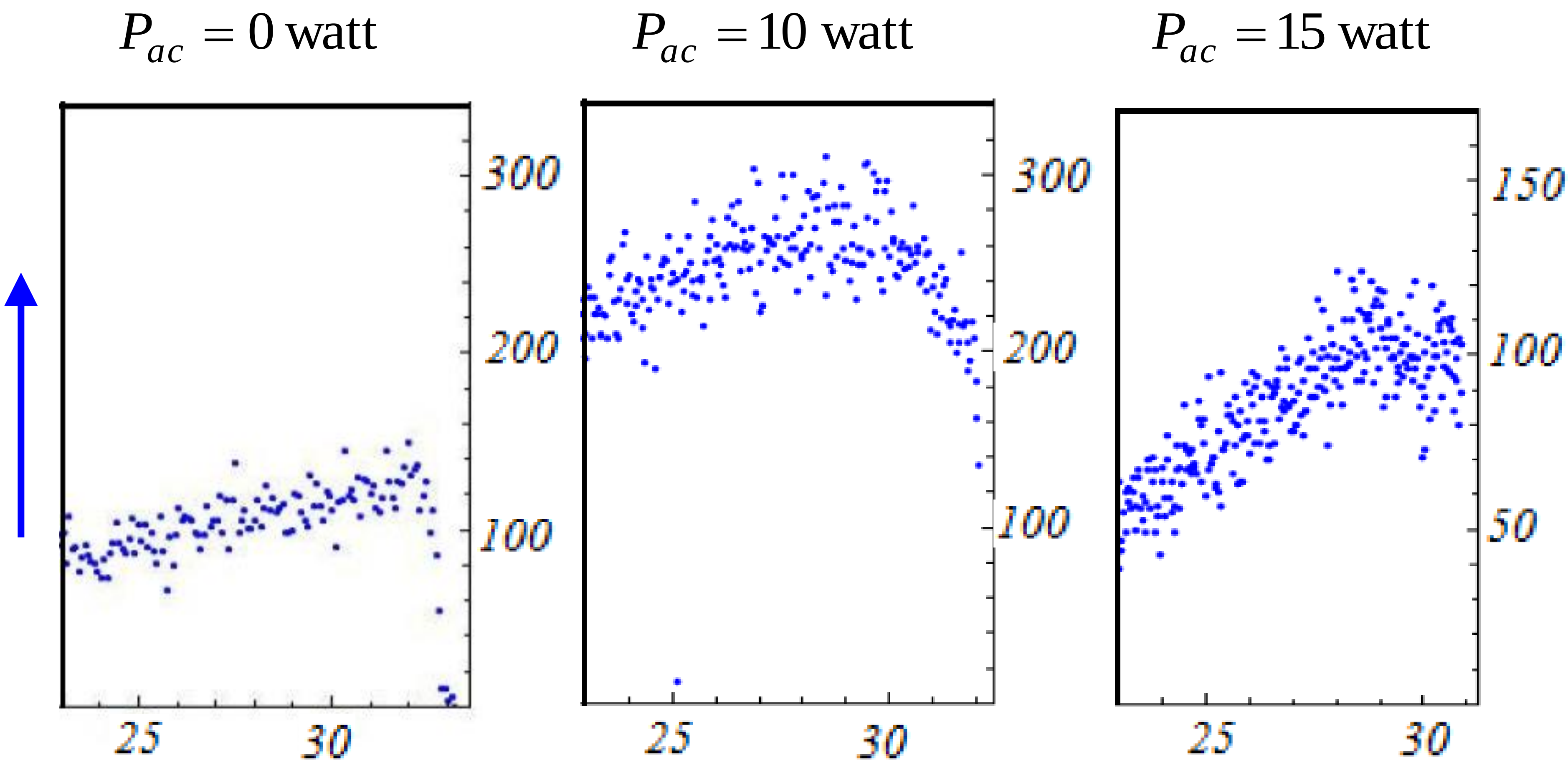
$$2\pi u_0/a_1 = 0.5$$

$$\omega/E_1 = 0.0015$$

Experimental scheme for 20 MeV electrons



Number of photons



Photon energy (keV)

Conclusions

- Formula is derived for the coherent part of the differential cross-section for the bremsstrahlung in crystals in the presence of hyperacoustic vibrations
- Conditions are specified under which the influence of the hypersound is essential
- In dependence of the parameters the hypersonic waves can either enhance or reduce the cross-section
- Presence of an ultrasonic wave leads to the appearance of new peaks. This is related to the point that in the presence of the ultrasonic waves the number of possibilities which satisfy the condition $g_{m\|} \geq \delta$ increases
- New peaks are relatively strong in the range of the ratio ω/E_1 from zero up to the first peak of the cross-section in the case when the ultrasonic wave is absent