



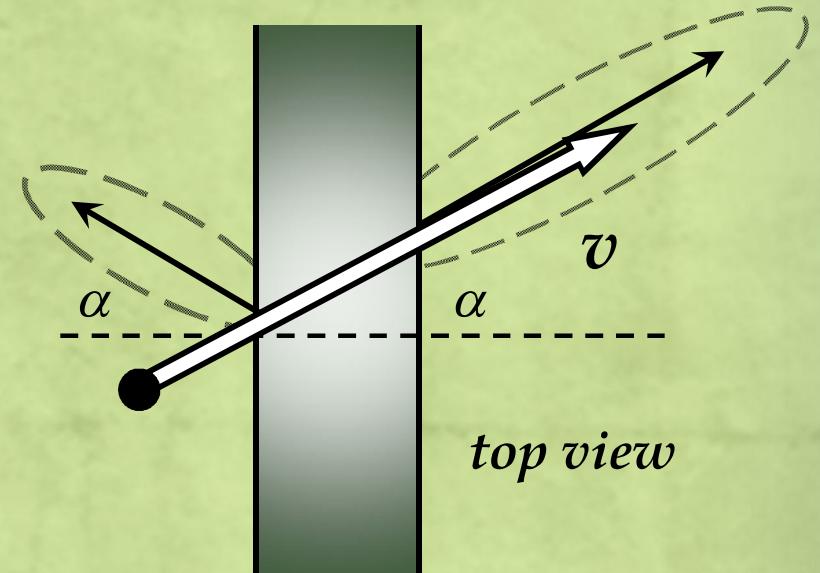
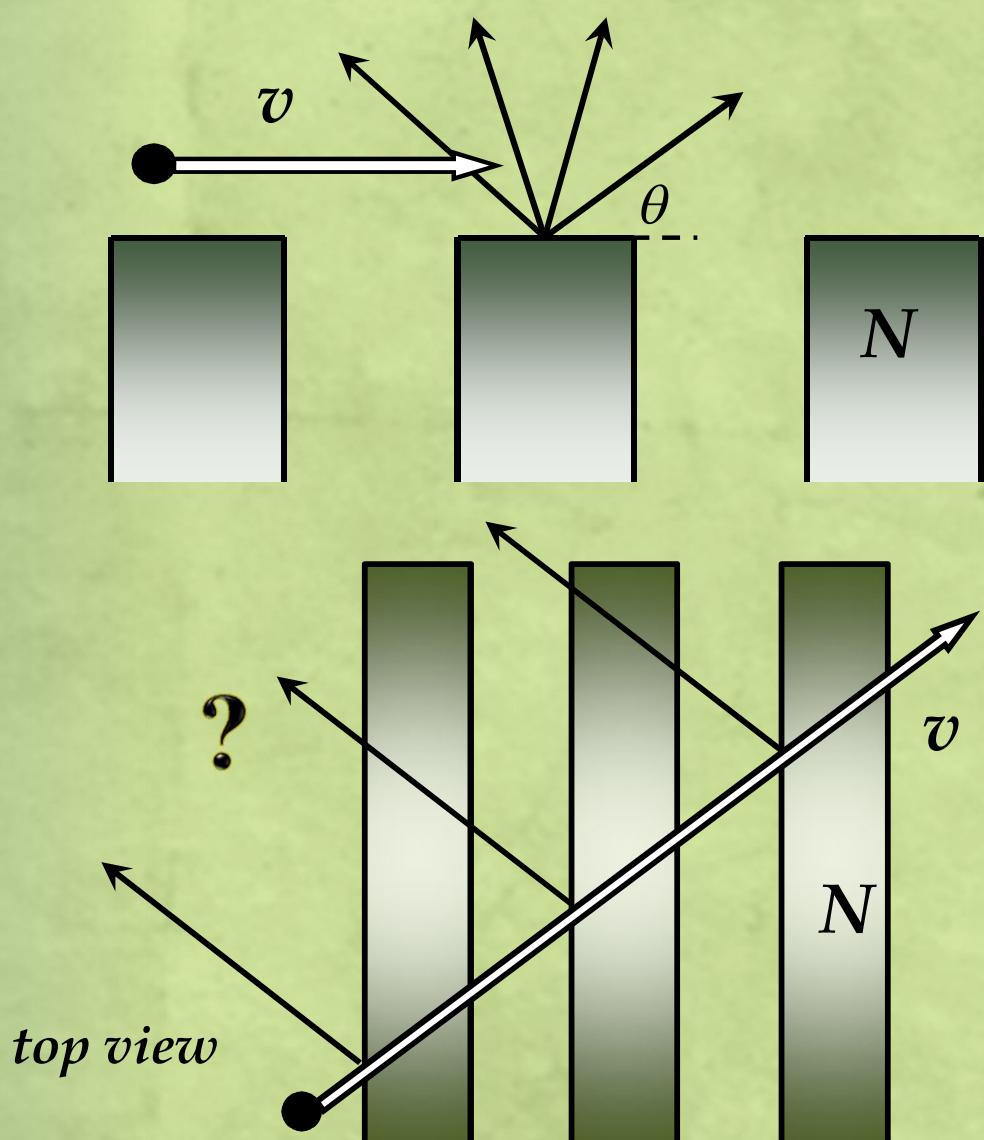
Backward Smith-Purcell Radiation

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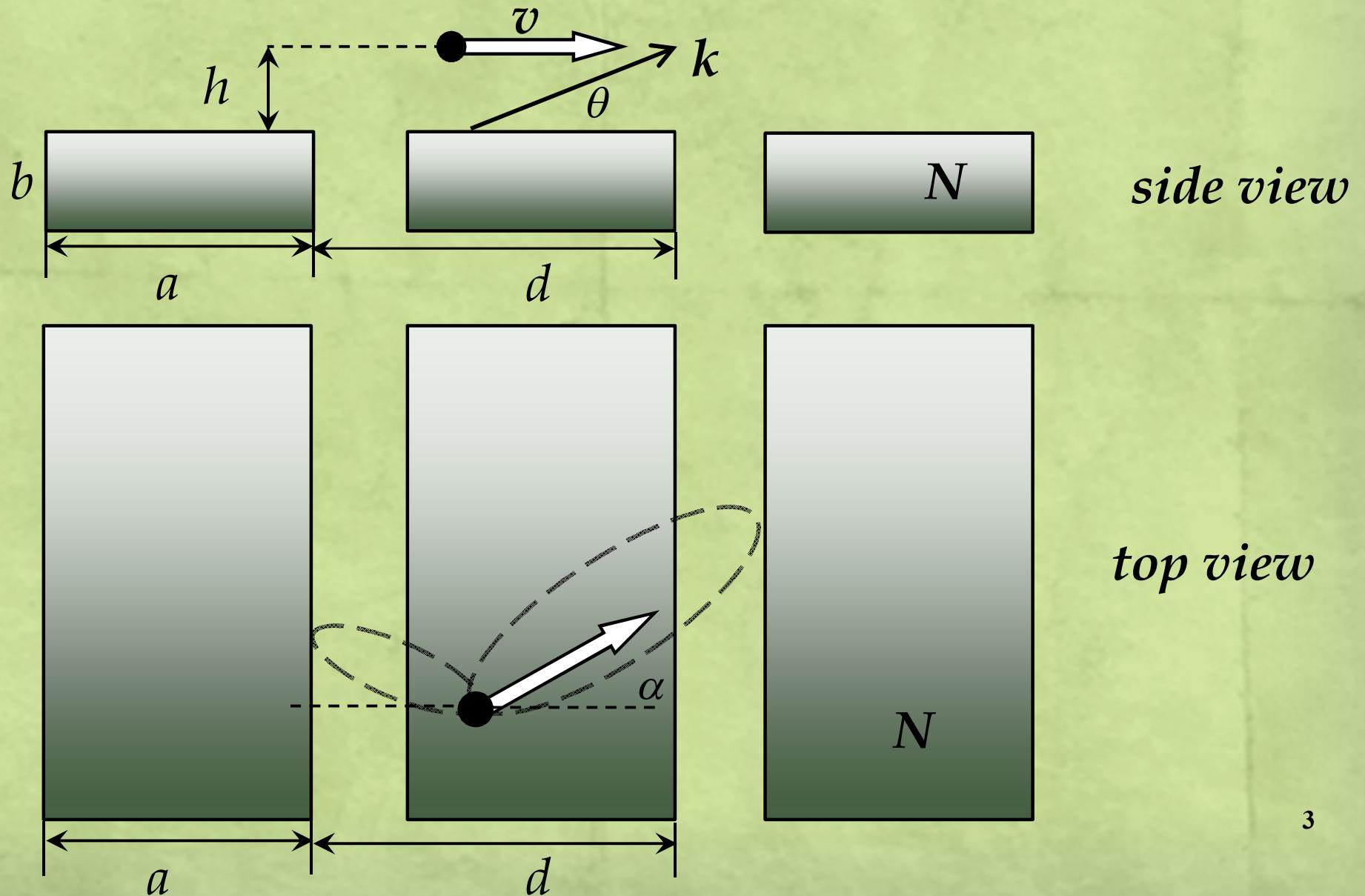
INTRODUCTION



Backward Smith-Purcell radiation:
...exists or not?..
...properties?..



GEOMETRY




THEORY, X-Ray

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2} \quad \omega \gg \omega_p$$

The polarization current density: $\mathbf{j}(\mathbf{r}, \omega) = \frac{\omega}{4\pi i} (\varepsilon(\omega) - 1) \mathbf{E}_0(\mathbf{r}, \omega)$

The radiation field: $\mathbf{E}(\mathbf{r}, \omega) = \frac{i\omega}{c^2} \frac{e^{ikr}}{r} \left[\mathbf{n} \left[\mathbf{n} \int d^3 r \mathbf{j}(\mathbf{r}, \omega) e^{-i\mathbf{k}\mathbf{r}} \right] \right]$



$$\frac{d^2 W_N(\mathbf{n}, \omega)}{d\Omega d\omega} = \frac{d^2 W_1(\mathbf{n}, \omega)}{d\Omega d\omega} F_N$$



X-Rays

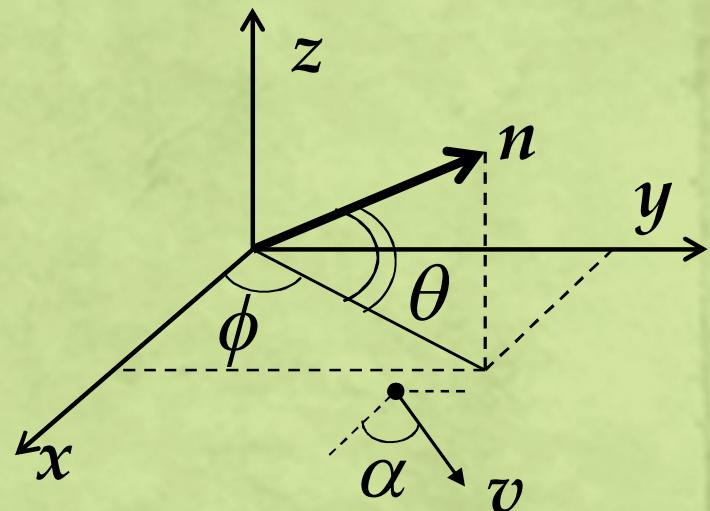
$$\frac{d^2W(\mathbf{n}, \omega)}{d\Omega d\hbar\omega} = \frac{1}{137} \frac{(\varepsilon(\omega) - 1)^2}{4\pi^2 \beta_x^2} e^{-2\rho h} \frac{\omega^4}{c^4} \frac{\sin^2\left(\frac{a\varphi}{2}\right)}{\varphi^2} \frac{\sin^2\left(N\frac{d\varphi}{2}\right)}{\sin^2\left(\frac{d\varphi}{2}\right)} F_b \frac{\left\{ \frac{1-n_z^2}{\varepsilon(\omega)} + \frac{\mathbf{A}^2}{\rho^2} - \frac{(\mathbf{A}, \mathbf{n})^2}{\varepsilon(\omega)\rho^2} \right\}}{\left| \rho - i\frac{\omega}{c} \sqrt{\varepsilon(\omega) - 1 + n_z^2} \right|^2}$$

$$\varphi = \frac{\omega}{c} \frac{1}{\beta_x} (1 - n_x \beta_x - n_y \beta_y)$$

$$\rho = \frac{\omega}{c} \frac{1}{\beta_x} \sqrt{1 - 2n_y \beta_y + n_y^2 \beta^2 - \beta_x^2}$$

$$\mathbf{A} = \frac{\omega}{c} \left\{ \frac{1 - n_y \beta_y}{\beta_x} - \beta_x, n_y - \beta_y, 0 \right\}$$

$$F_b = \left| 1 - \exp\left(-b\rho + ib \frac{\omega}{c} \sqrt{\varepsilon(\omega) - 1 + n_z^2} \right) \right|^2$$





MAXIMAL RADIATION

$$\frac{d^2W(\mathbf{n}, \omega)}{d\Omega d\hbar\omega} \propto e^{-2\rho h} \frac{\sin^2\left(N\frac{d\varphi}{2}\right)}{\sin^2\left(\frac{d\varphi}{2}\right)} \rightarrow \text{MAX}$$



$$\rho = \rho_{\min}$$



$$\frac{\sin^2\left(N\frac{d\varphi}{2}\right)}{\sin^2\left(\frac{d\varphi}{2}\right)} \xrightarrow{N \gg 1} 2\pi N \sum_m \delta(d\varphi - 2\pi m)$$

$$\varphi = \frac{\omega}{c} \frac{1}{\beta \cos \alpha} (1 - n_x \beta \cos \alpha - n_y \beta \sin \alpha), \quad \rho = \frac{\omega}{c} \frac{1}{\beta \cos \alpha} \sqrt{1 - 2n_y \beta \sin \alpha + n_y^2 \beta^2 - \beta^2 \cos^2 \alpha}$$



MAXIMAL RADIATION

$$\frac{d^2W(\mathbf{n}, \omega)}{d\Omega d\hbar\omega} \propto e^{-2\rho h} \frac{\sin^2\left(N\frac{d\phi}{2}\right)}{\sin^2\left(\frac{d\phi}{2}\right)} \rightarrow \text{MAX}$$

$$\begin{cases} \theta^{\max} = \arccos \sqrt{\left(\frac{\cos \alpha}{\beta} - \frac{\lambda m}{d}\right)^2 + \left(\frac{\sin \alpha}{\beta}\right)^2} \\ \phi^{\max} = \arcsin \frac{\sin \alpha}{\beta \cos \theta^{\max}} \end{cases}$$

Forward Smith-Purcell radiation

$$\begin{cases} \theta^{\max} = \arccos \sqrt{\left(\frac{\cos \alpha}{\beta} - \frac{\lambda m}{d}\right)^2 + \left(\frac{\sin \alpha}{\beta}\right)^2} \\ \phi^{\max} = \pi - \arcsin \frac{\sin \alpha}{\beta \cos \theta^{\max}} \end{cases}$$

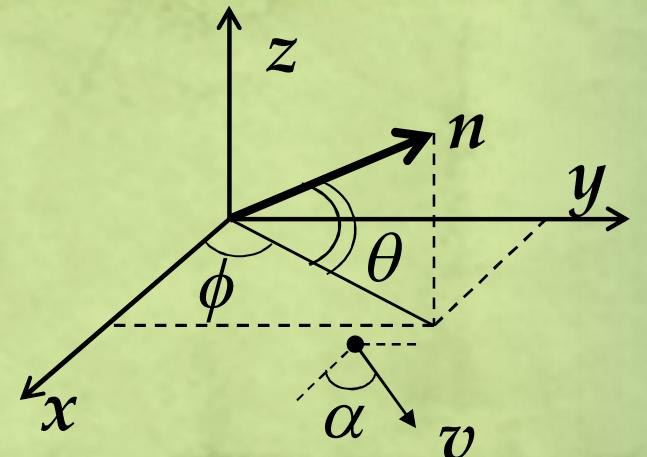
Backward Smith-Purcell radiation

$$\varphi = \frac{\omega}{c} \frac{1}{\beta \cos \alpha} (1 - n_x \beta \cos \alpha - n_y \beta \sin \alpha), \quad \rho = \frac{\omega}{c} \frac{1}{\beta \cos \alpha} \sqrt{1 - 2n_y \beta \sin \alpha + n_y^2 \beta^2 - \beta^2 \cos^2 \alpha}$$



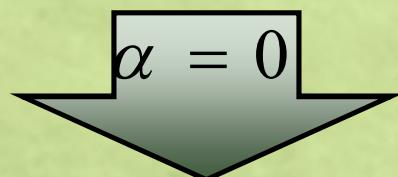
DISPERSION RELATION

$$\frac{\sin^2\left(N \frac{d\phi}{2}\right)}{\sin^2\left(\frac{d\phi}{2}\right)} \rightarrow 2\pi N \sum_m \delta(d\phi - 2\pi m)$$



Smith-Purcell radiation, both forward and backward:

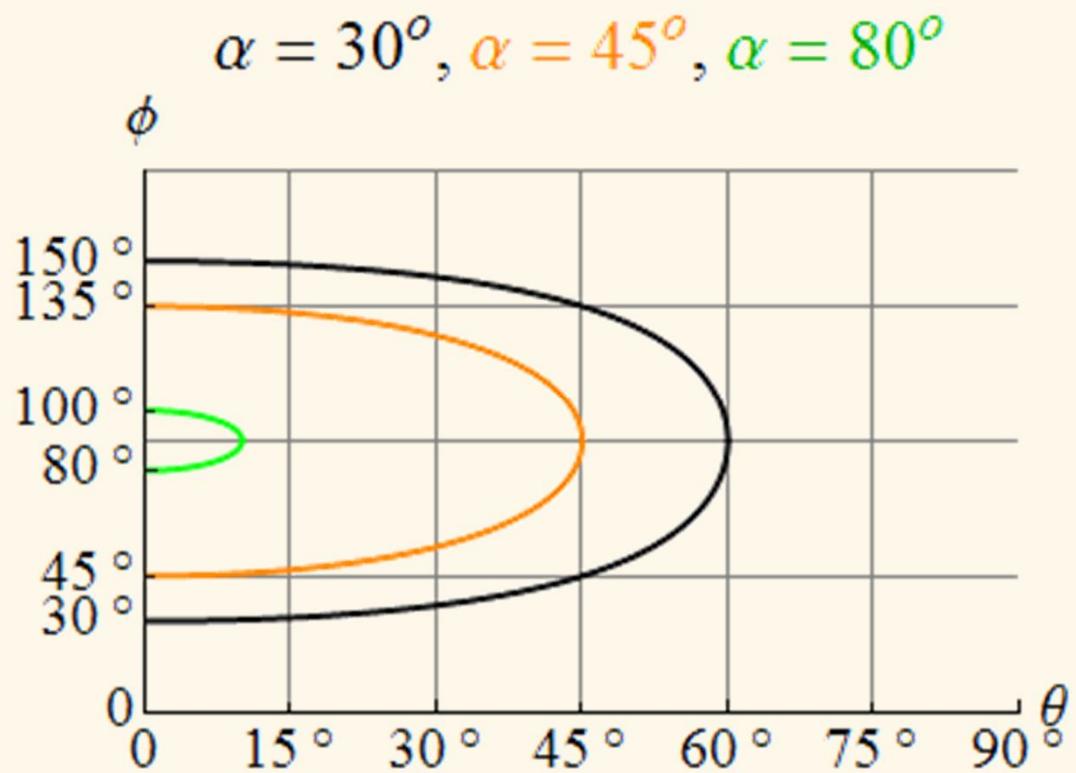
$$\frac{1}{\beta \cos \alpha} (1 - \beta \cos \theta \cos \phi \cos \alpha - \beta \cos \theta \sin \phi \sin \alpha) = \frac{\lambda m}{d}, \quad m = 1, 2..$$



$$\frac{1}{\beta} (1 - \beta \cos \theta \cos \phi) = \frac{\lambda m}{d}, \quad m = 1, 2..$$

RADIATION ANGLES

$$\gamma = 4 \cdot 10^4$$



Transition and Diffraction radiation:

$$\phi \approx \alpha$$

$$\phi \approx \pi - \alpha$$

BUT

Smith-Purcell radiation:

$$\phi \approx \alpha$$

$$\phi \approx \pi - \alpha$$

only if θ is rather small !!

A.A. Tishchenko, M.N. Strikhanov, A.P. Potylitsyn, X-ray transition radiation from an ultrarelativistic charge passing near the edge of a target or through a thin wire, Nucl. Instr. and Meth. B **227**, 63 (2005).

A.P. Potylitsyn, M.I. Ryazanov, M.N. Strikhanov, A.A. Tishchenko,
Diffraction Radiation from Relativistic Particles, Springer, 2010.



BACKWARD Smith-Purcell radiation

$$\mathbf{E}^{F \max}(\mathbf{r}, \omega) \propto \frac{1}{\varphi(\theta^{\max}, \phi^{\max})}$$

$$\mathbf{E}^{B \max}(\mathbf{r}, \omega) \propto \frac{1}{\varphi(\theta^{\max}, \phi^{\max}) + 2\Delta}$$

$$\Delta = \beta_x \cos \theta^{\max} \cos \phi^{\max}$$

$$\frac{d^2W_F - d^2W_B}{d\hbar\omega d\Omega} \rightarrow \min \quad \longleftrightarrow$$

$$\boxed{\frac{\cos \alpha}{\beta} - \frac{\lambda m}{d} \rightarrow 0}$$

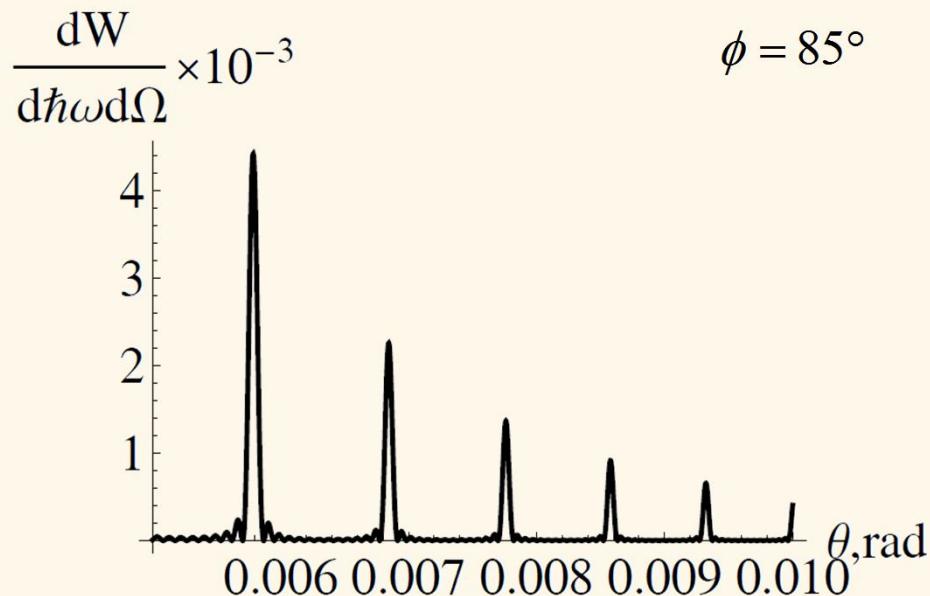
$$\varphi = \frac{\omega}{c} \frac{1}{\beta \cos \alpha} (1 - n_x \beta \cos \alpha - n_y \beta \sin \alpha)$$



X-Rays

$$\frac{\lambda}{d} \ll 1 \Rightarrow \alpha \rightarrow \frac{\pi}{2}$$

Forward SPR

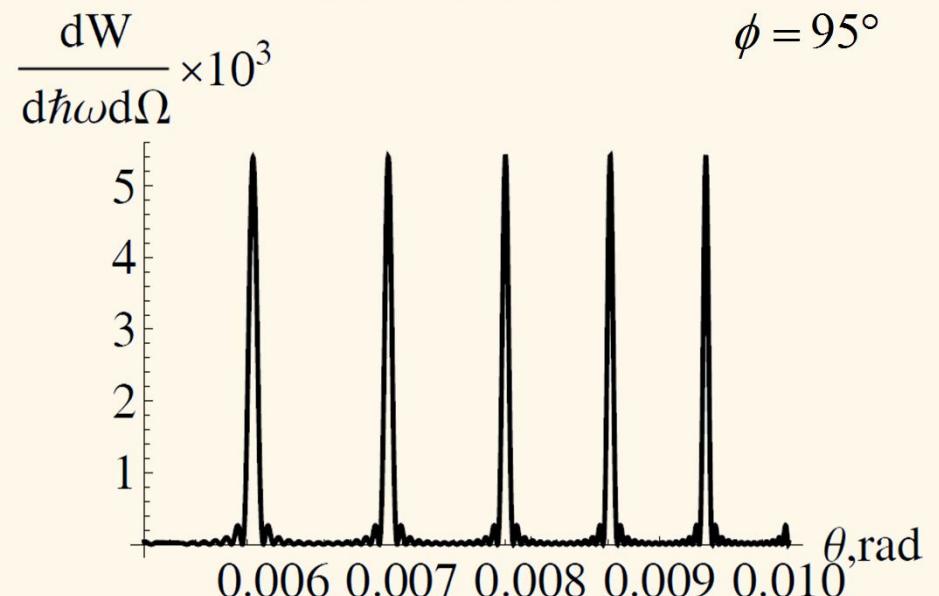


$h = 1 \mu m, d = 130 \mu m, a = 65 \mu m, \omega_p = 14.8 eV$

$\phi = \phi^{\max}(\theta), N = 7, b = 2 \mu m$

$$\alpha = 85^\circ \quad \gamma = 4 \cdot 10^4 \quad \lambda = 5 nm$$

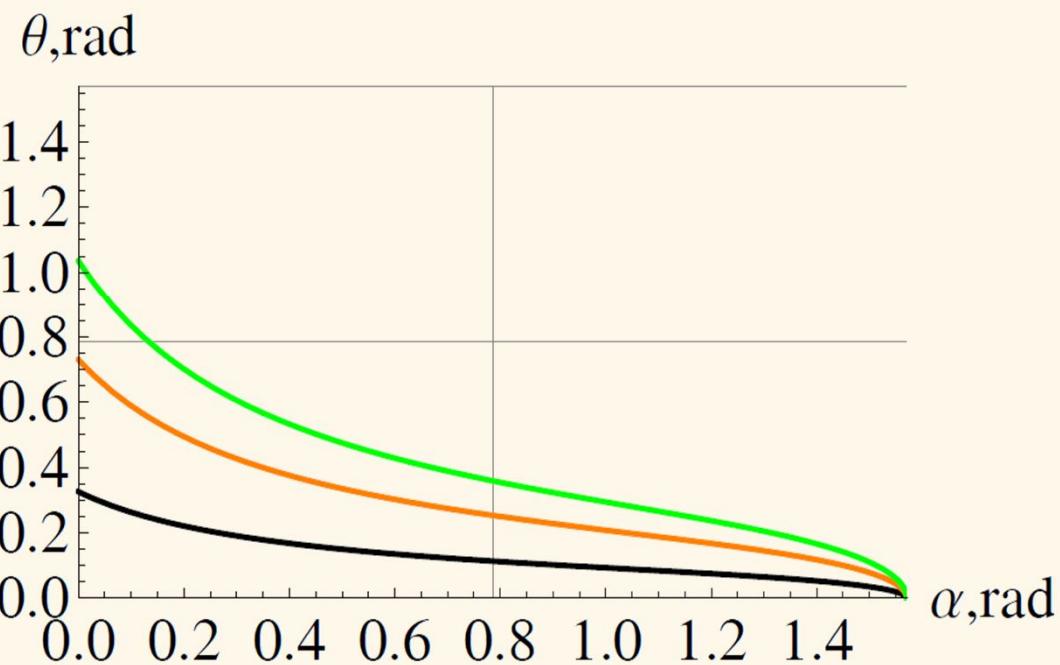
Backward SPR



Optical

$$\frac{d^2W(\mathbf{n}, \omega)}{d\Omega d\hbar\omega} \propto \frac{\sin^2\left(N\frac{d\phi}{2}\right)}{\sin^2\left(\frac{d\phi}{2}\right)} \Rightarrow \theta(\phi)$$

$m = 2$, $m = 10$, $m = 20$



$\alpha \searrow \Rightarrow \theta \nearrow$

$$\gamma = 4 \cdot 10^2 \quad \lambda = 0.5 \mu m \quad \phi = 1.2 rad \quad d = 130 \mu m$$



Optical

$$\frac{d^2W_N(\mathbf{n},\omega)}{d\Omega d\omega} = \frac{d^2W_\infty(\mathbf{n},\omega)}{d\Omega d\omega} F_{cell} F_N$$

$$\begin{aligned} \frac{dW_\infty}{d\hbar\omega d\Omega} &= \frac{1}{137} \frac{1}{4\pi^2} \frac{\beta_x(1-\beta_y n_y)}{\sqrt{1-n_y^2}} \exp\left(-\frac{4\pi h}{\lambda\beta_x} \sqrt{(1-\beta_y n_y)^2 - \beta_x^2(1-n_y^2)}\right) \times \\ &\quad \left((1-\beta_y n_y)^2 - \beta_x^2(1-n_y^2)\right) \left(1 + \frac{\beta_x \sqrt{1-n_y^2}}{1-\beta_y n_y}\right) \left(1 - \frac{n_x}{\sqrt{1-n_y^2}}\right) + (\beta_y - n_y)^2 \left(1 - \frac{\beta_x \sqrt{1-n_y^2}}{1-\beta_y n_y}\right) \left(1 + \frac{n_x}{\sqrt{1-n_y^2}}\right) \\ &\times \frac{\left((1-\beta_y n_y)^2 - \beta_x^2(1-n_y^2)\right)(1-\beta_y n_y - \beta_x n_x)^2}{\left((1-\beta_y n_y)^2 - \beta_x^2(1-n_y^2)\right)(1-\beta_y n_y - \beta_x n_x)^2} \end{aligned}$$

$$F_{cell} = 4 \sin^2 \left[\frac{a}{2} \frac{\omega}{c} \frac{1}{\beta_x} (1 - n_x \beta_x - n_y \beta_y) \right]$$

$$F_N = \frac{\sin^2 \left(N \frac{d\varphi}{2} \right)}{\sin^2 \left(\frac{d\varphi}{2} \right)}$$

N.A. Potylitsyna-Kube, X. Artru, *Diffraction radiation from ultrarelativistic particles passing through a slit. Determination of the electron beam divergence.* Nucl. Instr. Meth Phys. Res. B **201** (2003) 13



Optical

$$\frac{d^2W_N(\mathbf{n}, \omega)}{d\Omega d\omega} = \frac{d^2W_\infty(\mathbf{n}, \omega)}{d\Omega d\omega} F_{cell} F_N$$

$$\frac{dW_\infty}{d\hbar\omega d\Omega} = \frac{1}{137} \frac{1}{4\pi^2} \frac{\beta_x(1 - \beta_y n_y)}{\sqrt{1 - n_y^2}} \exp\left(-\frac{4\pi h}{\lambda\beta_x} \sqrt{(1 - \beta_y n_y)^2 - \beta_x^2(1 - n_y^2)}\right) \times \\ \times \frac{\left((1 - \beta_y n_y)^2 - \beta_x^2(1 - n_y^2)\right) \left(1 + \frac{\beta_x \sqrt{1 - n_y^2}}{1 - \beta_y n_y}\right) \left(1 - \frac{n_x}{\sqrt{1 - n_y^2}}\right) + (\beta_y - n_y)^2 \left(1 - \frac{\beta_x \sqrt{1 - n_y^2}}{1 - \beta_y n_y}\right) \left(1 + \frac{n_x}{\sqrt{1 - n_y^2}}\right)}{\left((1 - \beta_y n_y)^2 - \beta_x^2(1 - n_y^2)\right) \left(1 - \beta_y n_y - \beta_x n_x\right)^2}$$

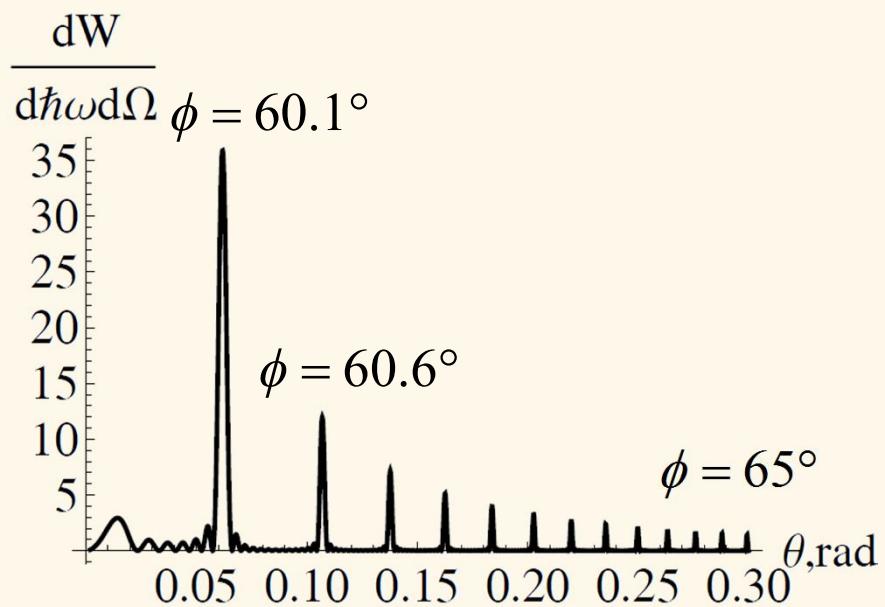
$$F_{cell} = 4 \sin^2 \left[\frac{a}{2} \frac{\omega}{c} \frac{1}{\beta_x} (1 - n_x \beta_x - n_y \beta_y) \right]$$

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N.A. Potylitsyna-Kube, X. Artru, *Diffraction radiation from ultrarelativistic particles passing through a slit. Determination of the electron beam divergence.* Nucl. Instr. Meth Phys. Res. B **201** (2003) 14

Optical

Forward SPR

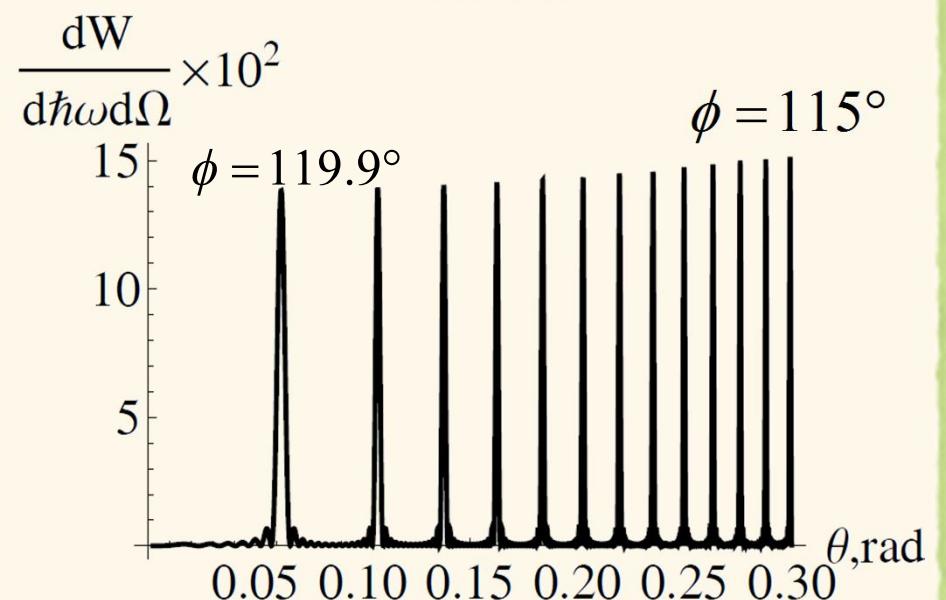


$$h = 5 \mu m \quad d = 130 \mu m \quad a = 65 \mu m$$

$$\phi = \phi^{\max}(\theta) \quad N = 7$$

$$\alpha = 60^\circ \quad \gamma = 2500 \quad \lambda = 0.5 \mu m$$

Backward SPR



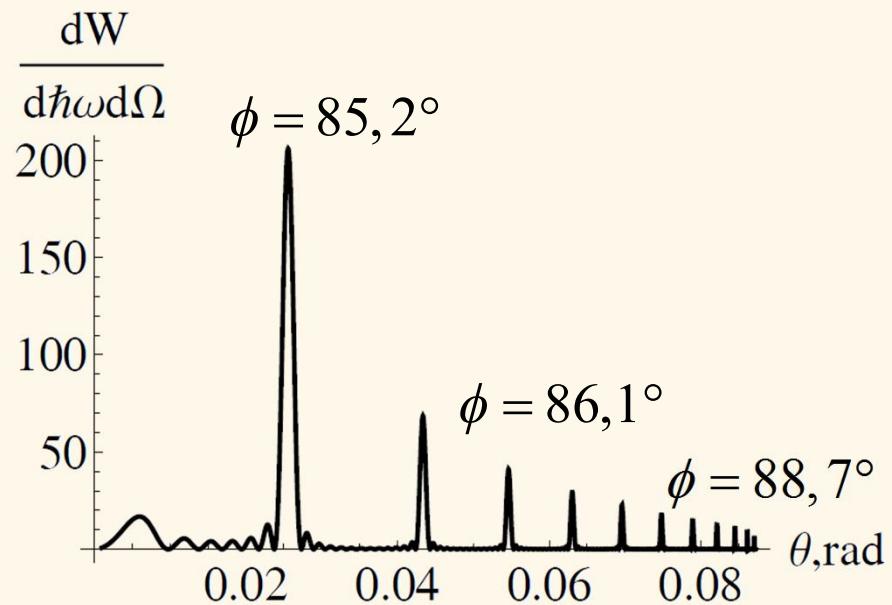
Optical

$$\alpha = 85^\circ \quad \gamma = 2500 \quad \lambda = 0.5 \mu m$$

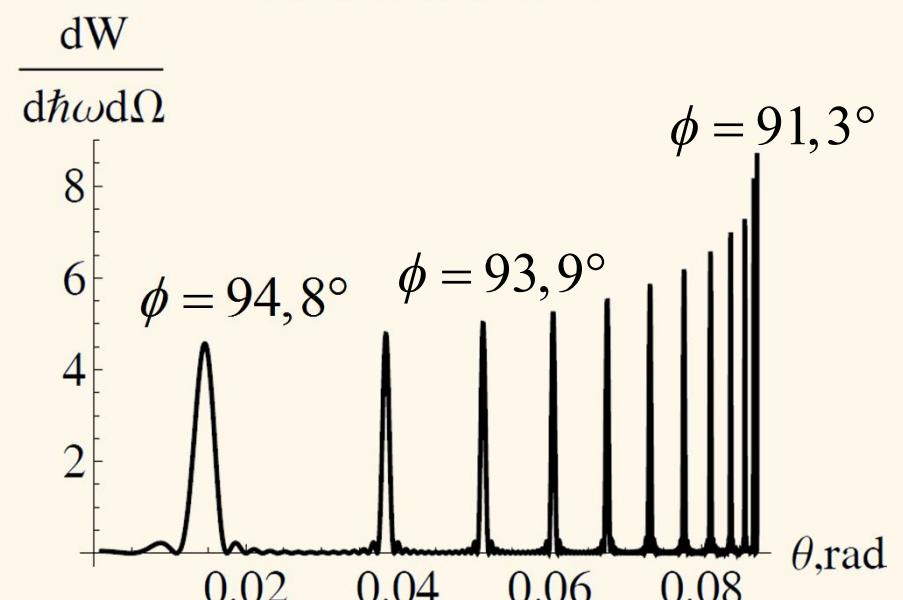
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Forward SPR



Backward SPR





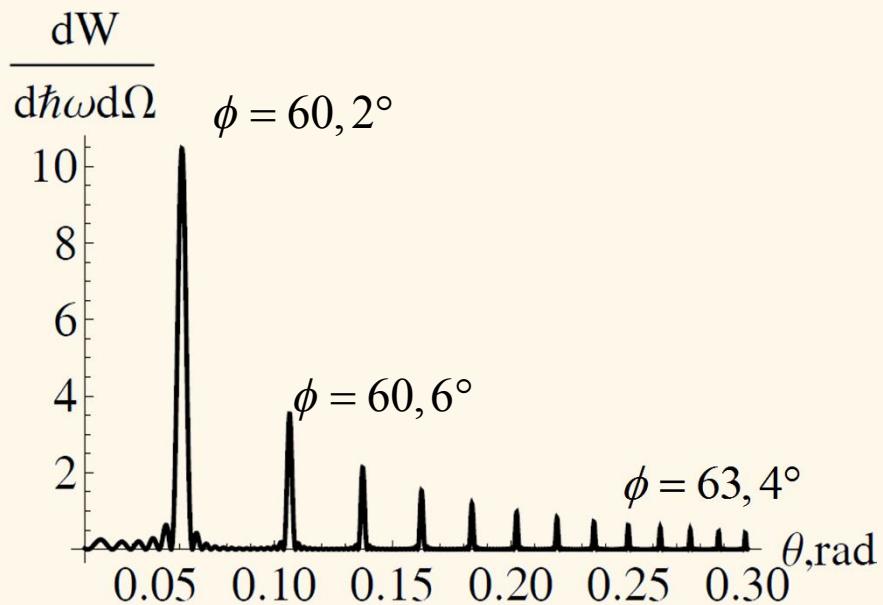
Optical

$$\alpha = 60^\circ \quad \gamma = 100 \quad \lambda = 0.5 \mu m$$

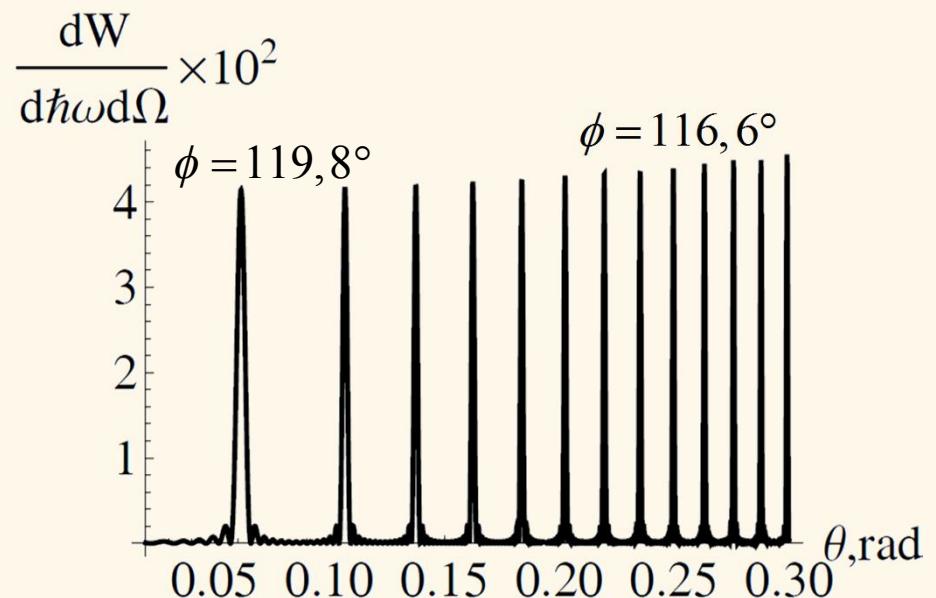
$$h = 5 \mu m \quad d = 130 \mu m \quad a = 65 \mu m$$

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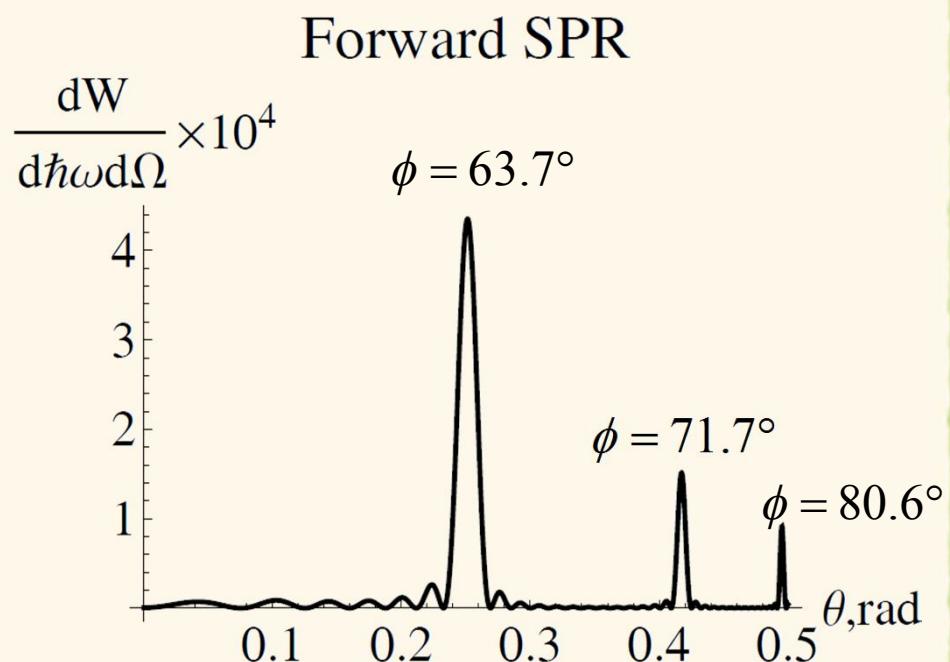
Forward SPR



Backward SPR



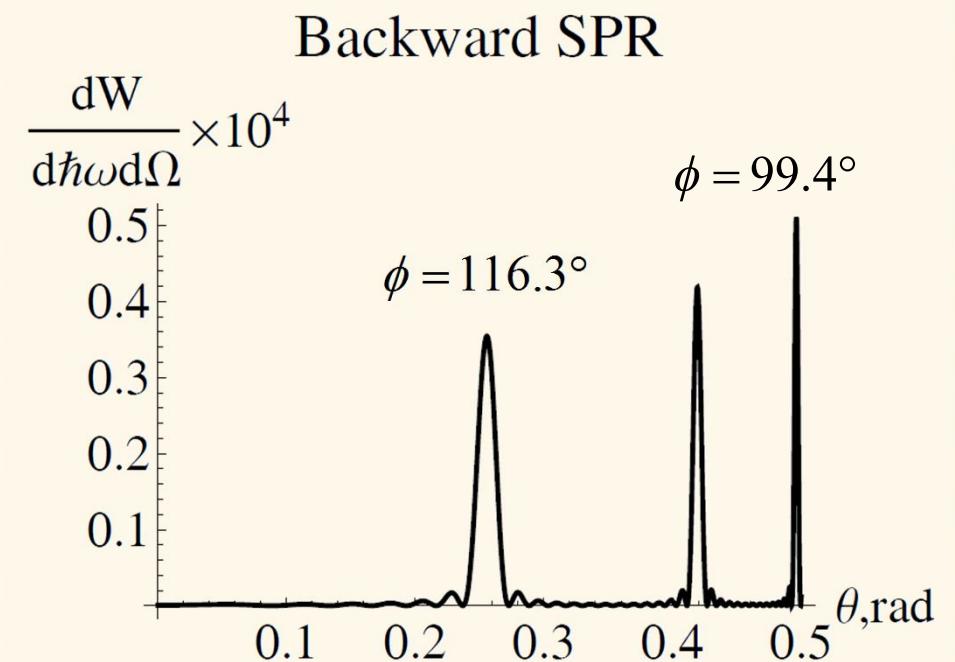
 THz



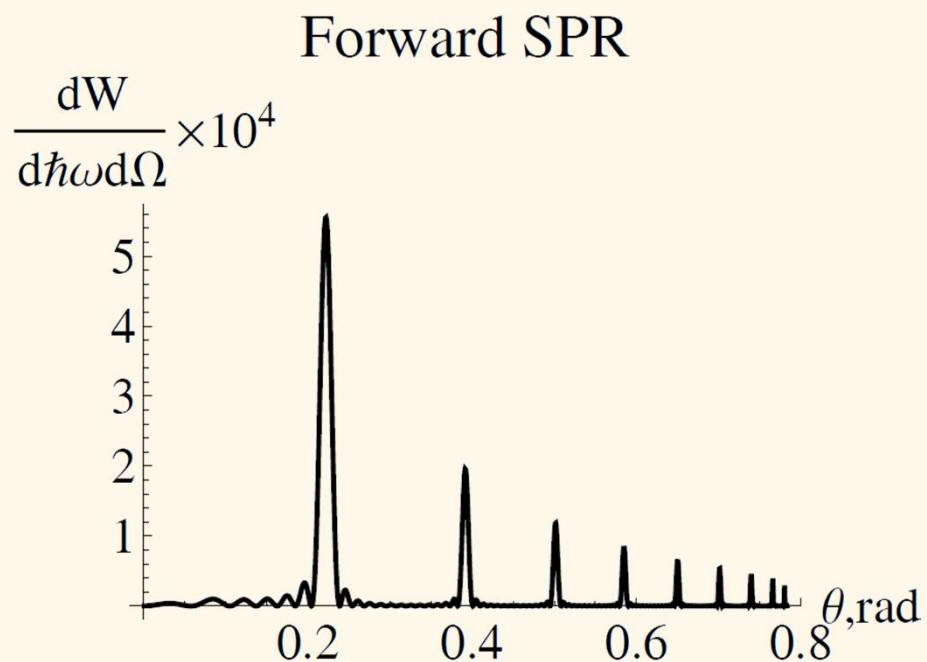
$$h = 5 \text{ mm} \quad d = 13 \text{ mm} \quad a = 6.5 \text{ mm}$$

$$\phi = \phi^{\max}(\theta) \quad N = 7$$

$$\alpha = 60^\circ \quad \gamma = 15 \quad \lambda = 0.5 \text{ mm}$$



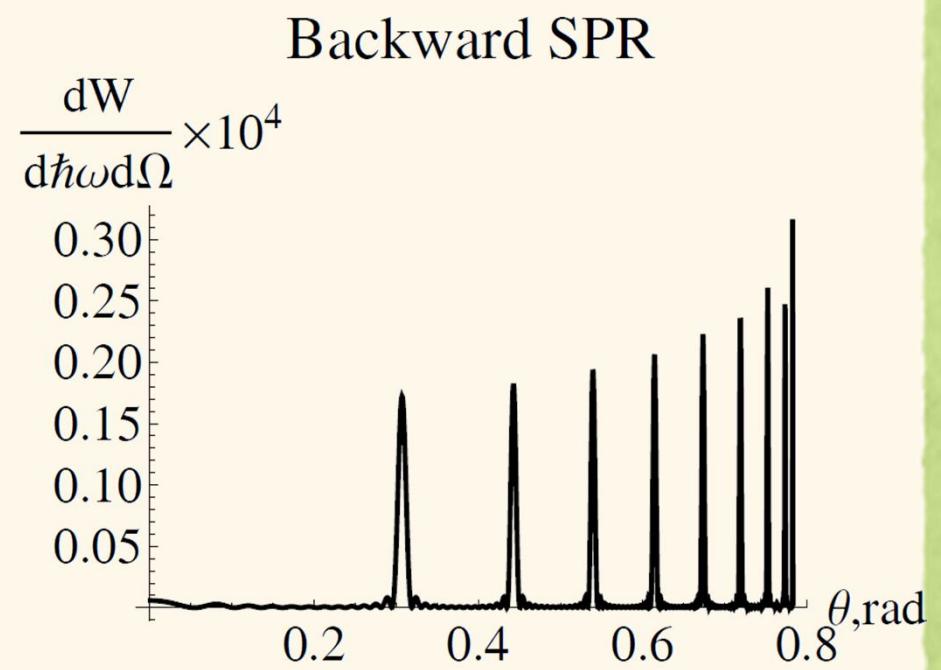
 THz



$$h = 5 \text{ mm} \quad d = 13 \text{ mm} \quad a = 6.5 \text{ mm}$$

$$\phi = \phi^{\max}(\theta) \quad N = 7$$

$$\alpha = 45^\circ \quad \gamma = 15 \quad \lambda = 0.5 \text{ mm}$$





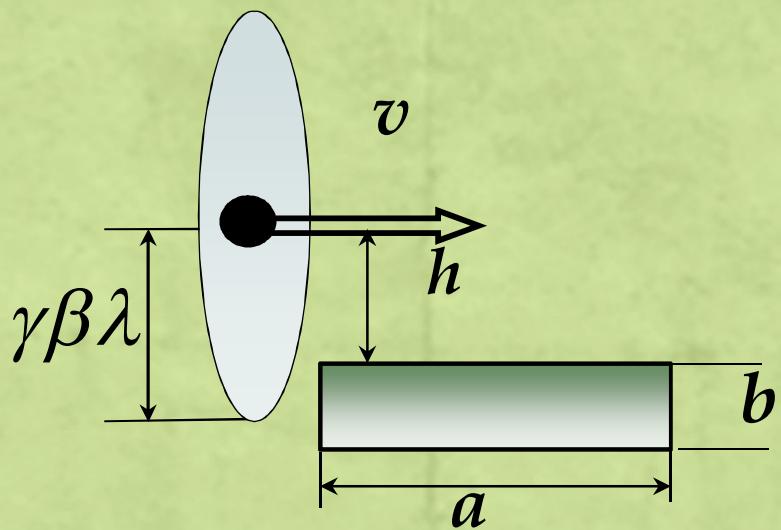
SUMMARY

- ✓ Smith-Purcell radiation for oblique incidence has been investigated in different frequency regions: X-Ray, optical, THz and for different energies.
- ✓ In case of oblique incidence the surface of maximal Smith-Purcell radiation is not a plane. It is a plane only for normal incidence.
- ✓ The backward Smith-Purcell radiation intensity in X-Ray region is much less than forward one; in optical and THz ranges they are comparable.

**THANK YOU
FOR ATTENTION!**



X-Rays



$$b^* \ll a \tan \theta \frac{1}{\cos \phi}$$

$$b^* = \min \{ \gamma\beta\lambda, b \}$$