Spectral properties of Compton inverse radiation

Application of Compton Beams

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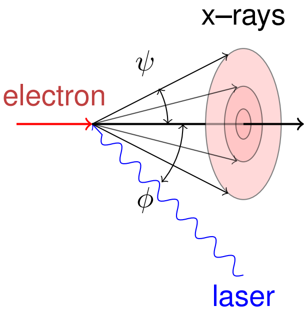
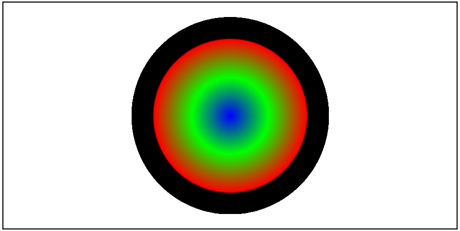
Outline

- Spectral properties of Compton (Thomson) radiation
- Monochromatisation with collimation
- X-ray imaging with Compton (subtracting scheme)
- Simulation of a ‘proof-of-principle’ experiments





X–ray photon with a definite energy scattered off at the definite angle from electron's trajectory:

$$E_x \approx \frac{2\gamma^2(1+\cos\phi)E_{\text{las}}}{1+\gamma^2\psi^2} = \frac{E_x^{\max}}{1+\gamma^2\psi^2}$$

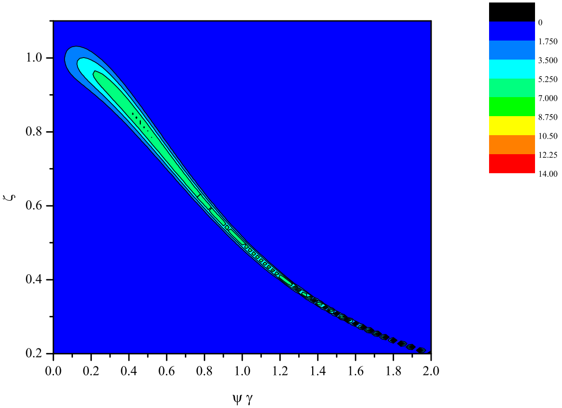
γ is the Lorentz-factor of the electron

- Since $\lambda_{\text{und}} \simeq 1$ cm , $\lambda_{\text{las}} \simeq 1\ \mu$m
  $\Rightarrow \quad \gamma_{\text{Com}} \sim \gamma_{\text{und}}/200$
- Mainly the first (fundamental) harmonic emitted, no radiation $\mathcal{E} > \mathcal{E}_{\text{max}}$
- Small cross section of the emitting area

- Compactness of Compton x–ray sources (determined by dimensions of the electron accelerator – storage ring)
- Possibility of subtracting scheme to be discussed
- Enhances spatial resolution in imaging

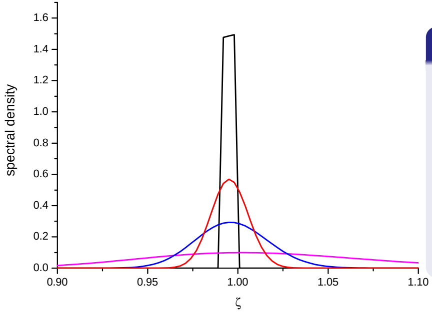
Deterioration Factor: Energy Spread

Spectral–angular density. Zero emittances, finite energy spread



Spectral angular density is diluted off vertically

Spectrum Passed Collimator



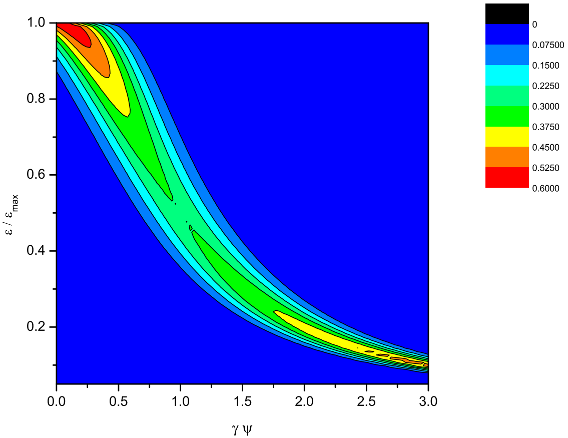
Collimated Spectrum

$$\mathcal{G}_\gamma(\zeta, x_\mathrm{i}, x_\mathrm{f}) = \frac{3}{4}\left[1 - 2\zeta\left(1-\zeta\right)\right] \times \left(\mathrm{Erf}\left(\eta_\mathrm{i}\right) - \mathrm{Erf}\left(\eta_\mathrm{f}\right)\right) \ ,$$

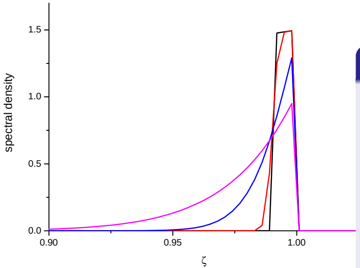
where $\eta_{\mathrm{i,f}} = -\left(1/\zeta - 1 - x_{\mathrm{i,f}}^2\right)/2\sqrt{2}s_\gamma$, $\mathrm{Erf}(z)$ the error integral function.

For pin-hole collimation energy spread of gammas is the doubled spread of electrons
(plus spread of the laser photons energy)

Deterioration Factor: Angular Spread
Zero spread, finite emittances



Spectral angular density smeared off horizontally



X-ray energy spectra for angular spread 0.01, 0.1, 0.5, 1.0 into collimating range 0...0.1.

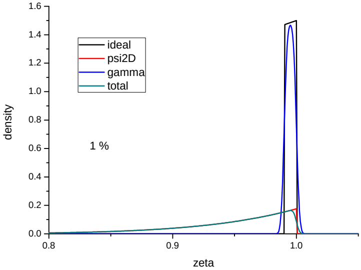
Collimated Spectrum

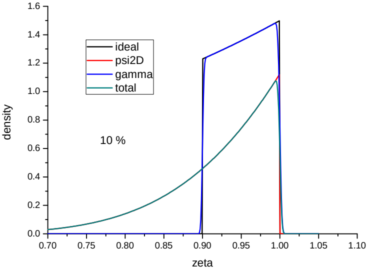
$$\mathcal{G}_\psi(\zeta, x_\mathrm{i}, x_\mathrm{f}) = \frac{3\left[\zeta^2 + \left(1-\zeta^2\right)^2\right]}{2\sqrt{2\pi}s_\psi} \times$$
$$\left(\mathrm{erf}\left(\eta_\mathrm{f}^-\right) + \mathrm{erf}\left(\eta_\mathrm{f}^+\right) - \mathrm{erf}\left(\eta_\mathrm{i}^-\right) - \mathrm{erf}\left(\eta_\mathrm{i}^+\right)\right) \quad ,$$

with $\eta_{\mathrm{i,f}}^\pm = \left(x_{\mathrm{i,f}} \pm \sqrt{1/\zeta - 1}\right)/\sqrt{2}s_\psi$.

2D beam (round) produces wider spectrum

Integral Spectra Passed Collimator
Energy spread 0.001, angular spread $s\gamma = 0.2$



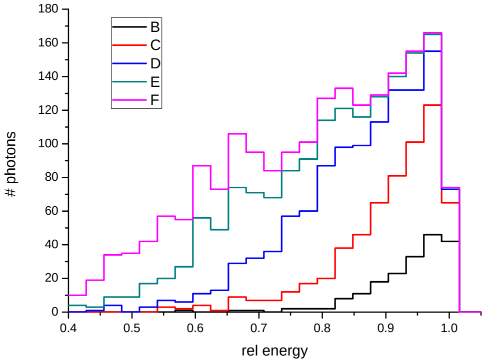


fwmh $\approx 5.6\,\%$

Angular spread dominant

fwmh $\approx 8.4\,\%$

Integral Spectra Passed Collimator – Simulation
(similar to LUCX)



$\beta_x = 0.200\,\text{m}$
$\beta_z = 0.014\,\text{m}$
$\varepsilon_x = 2.00 \times 10^{-7}\,\text{m}$
$\varepsilon_z = 2.00 \times 10^{-7}\,\text{m}$
$E_{\text{las}} = 1.164\,\text{eV}$
$\phi = 0.34907$
$E_{\text{e}} = 40.0\,\text{MeV}$
$E_{\text{x}}^{(\max)} = 27670.3\,\text{eV}$

$\psi\gamma = 0.2, 0.4, 0.6, 0.8, 1.0$ (B to F histogram)

Steep high–energy cutoff !

Electron angular spread dominant

Pros

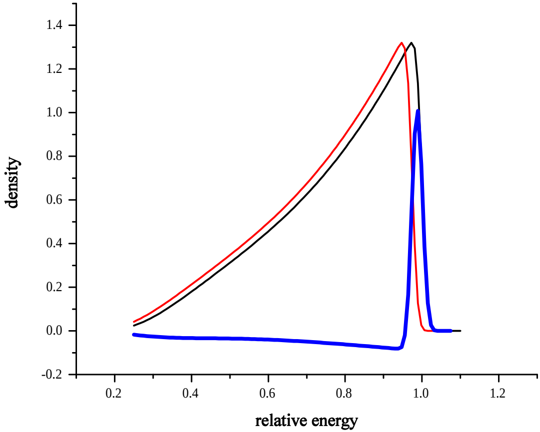
- higher total yield attainable due to lower β: $\sigma_\perp^2 \sim \epsilon\beta_\perp$
- wider area exposed to x–rays: $\sim \epsilon/\beta_\perp$

Cons

- lower brightness
- no sufficient monochromatization with collimation

IMPORTANT: Angular spread do not change the maximal x–ray energy

Subtracting scheme – ‘digital’ monochromatization
Employing steep high-energy edge



Substraction of the images produced by spectra with slightly different energies of electrons resembles spectral width $\geq 2\times$ beam energy spread

Subtracting scheme widely used in medicine

- Subtracting two images can reveal a tiny difference
- Medical x–ray imaging: images w/ and w/o radiocontrast agents
  - angiography/venography: contrast produced by iodine, maximal contrast $E_x \gtrsim 33\,\text{keV}$
  - the imaging of the digestive system: contrast produced by barium sulfate, maximal contrast $E_x \gtrsim 37\,\text{keV}$

proposal

- Both images with the contrast agent
- First image at $E_x^{\max} < E_{\mathrm{K}}$
- Second image at $E_x^{\max} > E_{\mathrm{K}}$

Difference in images will reveal the contrast agent location

Expected speed and sensitiveness (up to x–ray movie)

Proof-of-principle experiment for LUCX

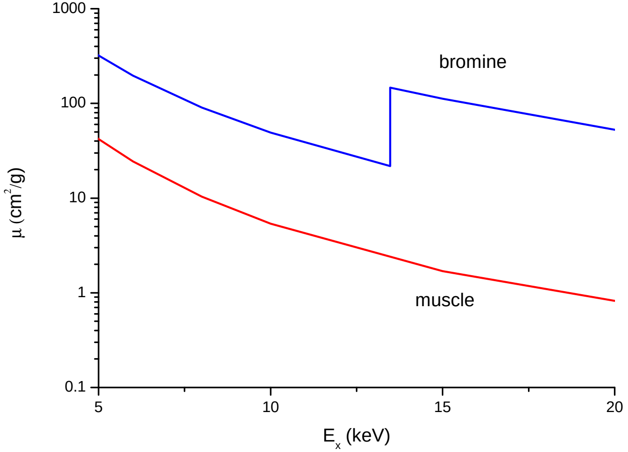
- LUCX + YAG laser $E_x^{\max} < E_{\mathrm{K}}$(iodine, boron)
- Proposal – bromine as contrast agent $E_{\mathrm{K}}(\text{bromine}) = 13\,\text{keV}$

Simulation model

- Collimated ideal Compton spectra – Monte Carlo – collimation angle preserved
- Tissue: uniform muscle slab with adding bromine and nonuniformity. X–ray mass attenuation curves from *www.nist.gov*
- Square mesh, 20 by 20 pixels
- Impinging quanta distribution – uniform, random (Monte Carlo)

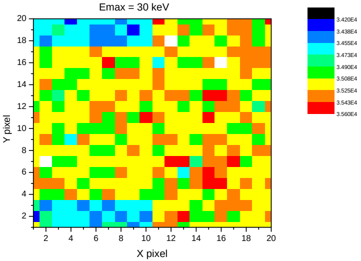
X–Ray Mass Attenuation Coefficients

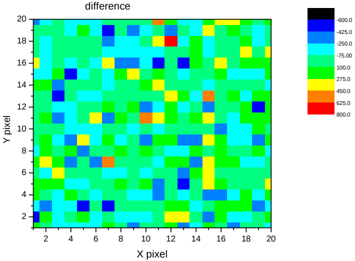
Bromine and muscle μ (data taken from NIST): $I = I_0 e^{-\mu x}$

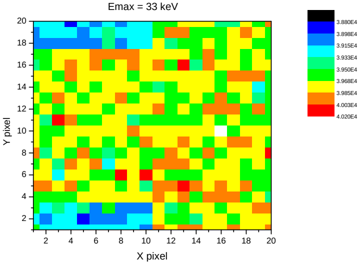


Simulation Results

$E_{1,2}^{\max} > E_{\mathrm{K}}$ (similar to bremsstrahlung/synchrotron) – no difference





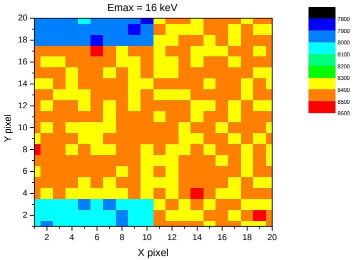


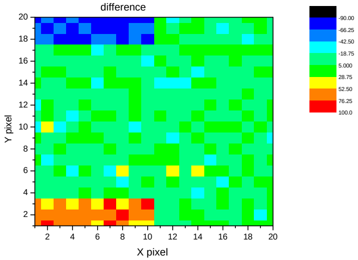
X-ray pictures of a tissue:

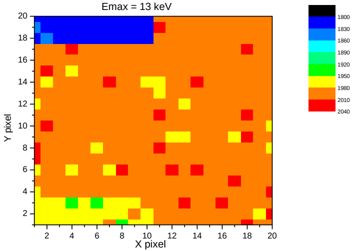
- muscle thickness 1 g/cm^2
- additional thickness 30 mg/cm^2
- bromine 0.5 mg/cm^2
- collimation $E_{\min} = E_{\max}/10$ at 30 keV
- # input x-ray photons 2×40 M

Simulation Results

$E_1^{\max} < E_K < E_2^{\max}$ clear difference: opposite signs





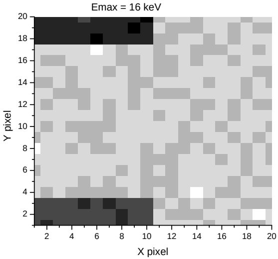


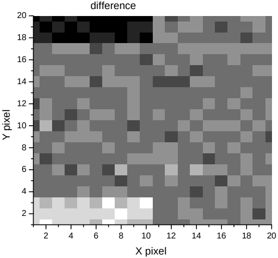
X-ray pictures of a tissue:

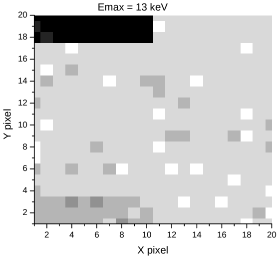
- muscle thickness 1 g/cm^2
- additional thickness 30 mg/cm^2
- bromine 0.5 mg/cm^2
- collimation $E_{\min} = E_{\max}/2$ at 13 keV
- # input x-ray photons 2×40 M

Simulation Results

$E_1^{\max} < E_K < E_2^{\max}$ clear difference: opposite signs BW







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Summary and Outlook

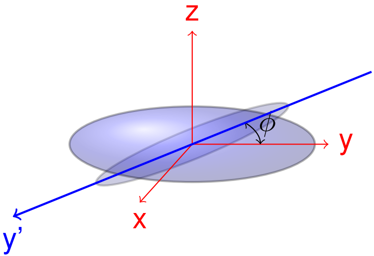
Summary

- Compton spectrum similar to undulator one
- Compton sources are much more compact (and cheaper)
- Focus upon the laser system
- Angular spread of electrons limits monochromatization by collimation

Outlook

- Advantage of subtracting scheme in x-ray imaging with Compton
- Possible proof–of–principle experiment on LUCX

Backup: Yield – major figure of merit
E.Bulyak, V.Skomorokhov. PRST-AB, 8, 030703 (2005)



Yield per crossing

$$Y = \frac{N_{\text{las}} N_e \sigma_{\text{C}}}{2\pi\sqrt{\sigma'^2_z + \sigma^2_z}} \times \frac{1}{\sqrt{\sigma^2_x + \sigma'^2_x + \left(\sigma^2_y + \sigma'^2_y\right)\tan^2\phi/2}}$$

$\sigma'_{x,y,z}$, $\sigma_{x,y,z}$ are rms dimensions of the laser pulse and the electron bunch

Backup: LUCX Experiment

- Electron energy 24.7/30.4 MeV for 13/16 keV x-rays (1.164 eV laser)
- $(0.5 \times 10^{-9}\text{Coul}) \times (2 \times 10^{3}\,\text{bunch/train}) \times 3.13\,\text{train/s}$
  $\approx 2 \times 10^{13}\text{electron/s}$
- conversion at 5 mJ, 20° $\sim 4 \times 10^{-5}$
- X–ray photons/second 8×10^{8}

Specifity

- short period, wavelength ranges from tens of micrometers down to submicrometer
- travelling wave, scattered off radiation – frequency doubled $\lambda_{\mathrm{rad}} \sim \lambda_{\mathrm{las}}/4\gamma^2$
- macro structures (undulators) $\lambda_{\mathrm{rad}} \sim \lambda_{\mathrm{struct}}(1 + a_0^2)/2\gamma^2$
- transverse localization $\sim 10\lambda_{\mathrm{las}}$
- ...