

Coherent Bremsstrahlung from planar channeled positron



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Coherent bremsstrahlung and coherent electron-positron pair photoproduction

was predicted by

Ter-Mikaelian *High Energy Electromagnetic processes in Condensed Media*, Wiley Interscience, New-York, 1972.

Uberall H. *High-energy interference effect of bremsstrahlung and pair production in crystals. // Physical review volume 103, number 4. 1956. p.1055-1067.*

Now coherent effects are divided into two types: A and B

Saenz A.W. and Uberall H., *Theory of coherent bremsstrahlung. - in: Coherent Radiation Sources, eds. A.W. Saenz and H. Uberall. - Berlin: Springer-Verlag.- 1985.- p.5-32* «RREPS-13» and "Meghri-13"

Review of experimental and theoretical work on coherent type A bremsstrahlung and coherent electron-positron pair photoproduction in crystals was given by **Diambrini - Palazzi G.** . // *Rev.Mod.Phys., 1968, v.10, p.611.*

Combined effect arise when channeled particle emits coherent bremsstrahlung and manifest it self in changing of coherent peaks

For the first time combined effect in coherent type B bremsstrahlung was observed in Tomsk

Amosov K.Yu., Vnukov I.E., Naumtnko G.F., Potulisin A.P., Saruchev V.P. *Pis'ma v ZhETP 1992 v. 55 p. 587*

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«RREPS-13» and "Meghri-13"

A diagram illustrating the planar channeling of a positron through a crystal. A thick red arrow labeled "Discrete axis" points diagonally upwards from a black dot representing the positron's path. This path follows the bottom of a deep blue U-shaped potential well. A blue arrow labeled "Planar continuous potential" points along the horizontal base of the well.

Discrete axis

**Positron moves through crystal
in the regime of planar channeling.
Longitudinal positron momentum
is parallel to crystal axis.**

Planar continuous potential

Photon scattering by channeled positron

We use virtual photon methods therefore we need cross-section of Photon scattering by channeled positron. It is convenient to consider scattering of photons by channeled positron in a coordinate system moving with the longitudinal velocity of channeled positron. In the moving reference frame one can use non relativistic approach to the scattering process.

Cross - section

$$d\sigma_{fi} = \frac{2\pi}{\hbar} \left| M_{fi} \right|^2 \delta(E_i - E_f - \hbar\omega) \frac{1}{J} d^3 p_{\parallel f} d\rho, \quad d\rho = \frac{d^3 k}{(2\pi)^3};$$

J initial positron flux

matrix element (n is intermediate state)

$$M_{fi} = \sum_n \left\{ \frac{(\mathbf{e}_2 \mathbf{R}_{fn})(\mathbf{e}_1 \mathbf{A}_{ni})}{(E_i + \hbar\omega_1) - E_n} + \frac{(\mathbf{e}_1 \mathbf{A}_{fn})(\mathbf{e}_2 \mathbf{R}_{ni})}{E_i - E_n - \hbar\omega} \right\}$$

Photon scattering by channeled positron

matrix element of photon emission

$$\mathbf{R}_{kl} = -\frac{e}{mc} \int \Psi_k^*(\mathbf{r}) \hat{\mathbf{p}} \sqrt{\frac{2\pi\hbar^2 c^2}{\hbar\omega_2}} \exp[-i\mathbf{k}_2 \cdot \mathbf{r}] \Psi_l(\mathbf{r}) d\mathbf{r}$$

matrix element of the photon absorption

$$\mathbf{A}_{kl} = -\frac{e}{mc} \int \Psi_k^*(\mathbf{r}) \hat{\mathbf{p}} \sqrt{\frac{2\pi\hbar^2 c^2}{\hbar\omega_1}} \exp[-i\mathbf{k}_1 \cdot \mathbf{r}] \Psi_l(\mathbf{r}) d\mathbf{r}$$

Photon scattering by channeled positron

$$\Psi_{ph}(\mathbf{r}) = \sqrt{\frac{2\pi\hbar^2 c^2}{\hbar\omega}} \exp[-i\mathbf{k}\mathbf{r}] \quad \text{Photon wave function}$$

$$\Psi_{i(f)}(\mathbf{r}) = \varphi_{i(f)}(x) \exp(i\mathbf{k}_{||i(f)} \mathbf{r}_{||}) \quad \text{Channeled positron wave function}$$

Channeled positron transverse wave function

$$\left[\varepsilon_{\perp i(f)} - \frac{1}{2m} p_x^2 + \gamma e V(x) \right] \varphi_{i(f)}(x) = 0$$

$$eV(x) = \frac{4V_0}{d^2} x^2$$

$$\varphi_n(x) = C_n \exp\left(-\xi^2 x^2 / 2\right) H_n(\xi x)$$

$$\xi = \sqrt{m\omega}, \quad \omega = \frac{2\sqrt{2\gamma V_0}}{d\sqrt{m}}$$

$$C_n = \frac{\xi^{1/2}}{\pi^{1/4} \sqrt{2^n n!}}$$

$$\varepsilon_{\perp n} = \hbar\omega \left(n + \frac{1}{2}\right)$$

Photon scattering by channeled positron

$$d\sigma = \frac{\alpha^2 \hbar^2}{m^2 c^2} \frac{\hbar\omega_2}{\hbar\omega_1} \left| \sum_n \left\{ \frac{(\mathbf{e}_2 \mathbf{R}_{fn})(\mathbf{e}_1 \mathbf{A}_{ni})}{(E_i + \hbar\omega_1) - E_n} + \frac{(\mathbf{e}_1 \mathbf{A}_{fn})(\mathbf{e}_2 \mathbf{R}_{ni})}{E_i - E_n - \hbar\omega} \right\} \right|^2 \times \delta[(\mathbf{k}_{\parallel i} + \mathbf{k}_{\parallel 1}) - (\mathbf{k}_{\parallel f} + \mathbf{k}_{\parallel})] \delta[(E_i + \hbar\omega_1) - (E_f + \hbar\omega)] \times d^2 \mathbf{k}_{\parallel f} d\Omega d\hbar\omega_2.$$

From the δ function it follows **conservation laws:**

$$\begin{aligned} (\mathbf{k}_{\parallel i} + \mathbf{k}_{\parallel 1}) &= (\mathbf{k}_{\parallel f} + \mathbf{k}_{\parallel}) \\ (E_i + \hbar\omega_1) &= (E_f + \hbar\omega_2) \end{aligned}$$

In the initial state *i* positron at rest

$$\mathbf{k}_{\parallel i} = 0$$

Photon scattering by channeled positron

If the incident photon moves along the OZ axis $\mathbf{k} = \{0, 0, k\}$, $k = \omega/c$ then its wave vector is defined as follows:

the wave vector of a scattered photon

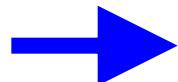
$$\mathbf{k}' = \{k' \sin \vartheta \cos \Phi, k' \sin \vartheta \sin \Phi, k' \cos \vartheta\}, \quad k' = \omega'/c$$

$$\left. \begin{aligned} \frac{\hbar\omega_1}{c} &= \hbar k_{||f,z} + \frac{\hbar\omega_2}{c} \cos \vartheta \\ 0 &= \hbar k_{||f,y} + \frac{\hbar\omega}{c} \sin \vartheta \\ \hbar\omega_1 + \varepsilon_{\perp i} &= \hbar\omega_2 + \varepsilon_{\perp f} + \frac{(\hbar k_{||f,z})^2 + (\hbar k_{||f,y})^2}{2m} \end{aligned} \right\}$$

Photon scattering by channeled positron

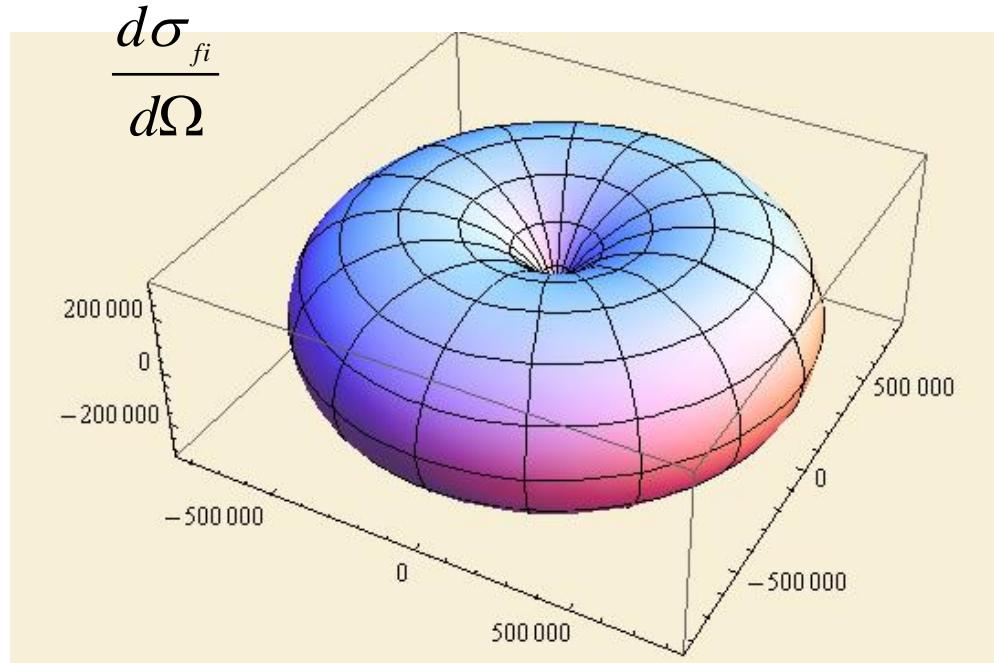
Connected between scattered and initial photon energies

$$\hbar\omega_2 = \hbar\omega_1 \cos \vartheta - mc^2 +$$
$$+ mc^2 \sqrt{1 - \frac{\hbar\omega_1^2 + 4mc^2(\varepsilon_{\perp i} + \hbar\omega_1 - \varepsilon_{\perp f}) + \hbar\omega_1(\hbar\omega_1 \cos 2\vartheta - 4mc^2 \sin \vartheta)}{2m^2 c^4}} \left(ctg^2 \vartheta \right)$$



$$\hbar\omega_2 \approx \varepsilon_{\perp i} + \hbar\omega_1 - \varepsilon_{\perp f}$$

(“Compton formula”)



«RREPS-13» and "Meghri-13"

The cross section of the coherent bremsstrahlung from channeled positron

electrostatic potential of the crystal axis

$$V(\vec{r}) = \sum_{i=1}^N V_1(|\vec{r} - \vec{r}_i|), \quad V_1(r) = \frac{Ze}{r} \exp\left(-\frac{r}{R}\right).$$

R is the screening radius, N is the number of atoms in the axis, Z is atomic number.

$$n(\omega)d\omega = \frac{Z^2 e^2}{\pi^2} N \times \text{The spectrum of virtual photons of crystal axis}$$

$$\left\{ \left[L - B \exp\left[-\left(\frac{\hbar\omega}{\gamma\hbar c} \bar{u}\right)^2\right] \right] + \frac{2\pi}{d} \sum_{g_n} B \exp\left[-\left(\frac{\hbar\omega}{\gamma\hbar c} \bar{u}\right)^2\right] \delta(k_1 - g_n) |S|^2 \right\} \frac{d\omega}{\omega}$$

$$B(x) = \pi \left\{ -(1+x)e^x Ei(-x) - 1 \right\}, \quad x = \left[\left(\frac{\hbar\omega}{\gamma\hbar c} \right)^2 + R^{-2} \right] \bar{u}^2 \quad L = \pi \ln \left[\frac{a\lambda^{-2}}{(\hbar\omega/\gamma\hbar c)^2 + R^{-2}} \right], \quad a \approx 1$$

Here d is the lattice constant, ω is the frequency of virtual photon

$\lambda = mc/\hbar$ is Compton wave length

\bar{u}^2 is the mean-square displacement of a crystal atom from equilibrium position

The cross section of the coherent bremsstrahlung from channeled positron

$Ei(-x)$ is the exponential integral function,

$g_n = 2\pi n / d$ is the 1D reciprocal lattice vector,

$|S|$ is the structure factor of a crystal axis, γ is the relativistic factor.

If we neglect the terms $(\hbar\omega/\gamma\hbar c)$ in the functions L and B

than we obtain the virtual photons spectrum from well – known

Ter- Mikaelian book.

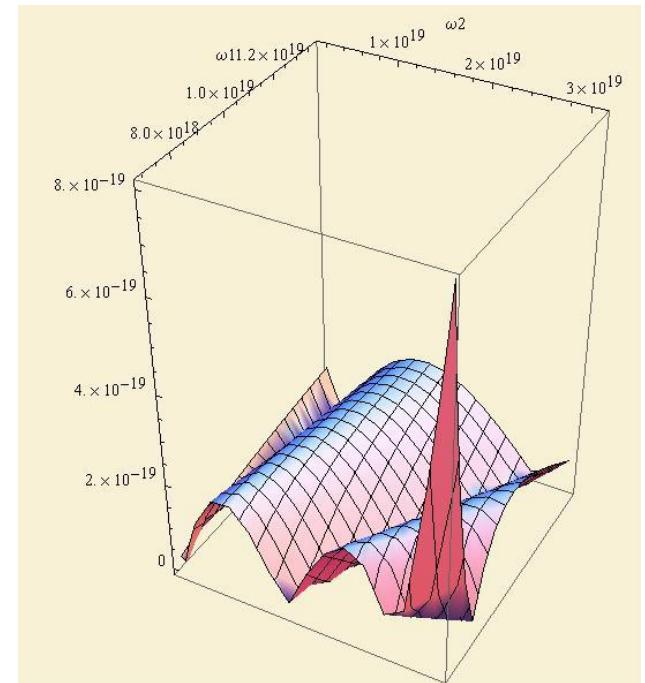
The cross section of the coherent bremsstrahlung from channeled positron

For transform the photons scattering cross-section into the laboratory coordinate system we use the "**Compton formula**" and the Lorentz transformation for the photon energy (the Doppler shift): $\hbar\omega_r = \gamma \hbar\omega(1 - \beta \cos \vartheta)$

$$\frac{\sigma_{fi}(\omega_r, \omega)}{d\omega_r d\omega}$$

$\hbar\omega$ *is the energy of virtual photon*

$\hbar\omega_r$ *is the energy of emitted photon*



The cross section of the coherent bremsstrahlung from channeled positron

$$\frac{\sigma_{fi}^{CR}(\omega_r)}{d\omega_r} = \int_{\omega_{MIN}}^{\omega_{MAX}} \frac{\sigma_{fi}(\omega_r, \omega)}{d\omega_r d\omega} n(\omega) d\omega$$

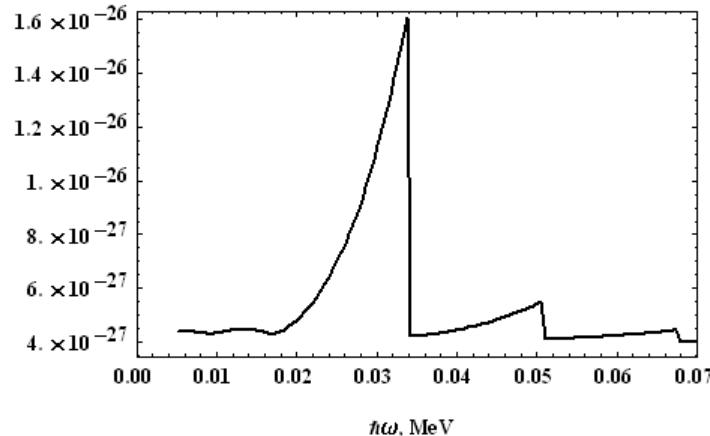
The limits of integration are determined by the condition $-1 \leq \cos\vartheta \leq 1$

$$\hbar\omega_{MIN} = \frac{\hbar\omega_r + \gamma(\hbar\omega + \varepsilon_{\perp i} - \varepsilon_{\perp f})}{(\beta + 1)\gamma}$$

$$\omega_{MAX} = \frac{\hbar\omega_r + \gamma(1 - \beta)(\hbar\omega + \varepsilon_{\perp i} - \varepsilon_{\perp f})}{\gamma(1 - \beta)}$$

Results of calculation

σ_{comb} , cm^2



$$\sum_{fi} \frac{\sigma_{fi}^{CR}(\omega_r)}{d\omega_r}$$

W (100) plane

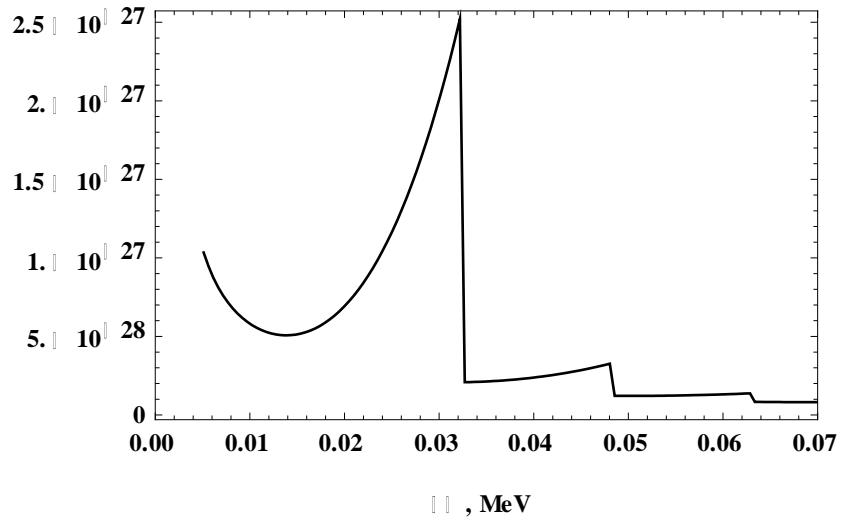
W (220) axis

$N_s = 10^4$

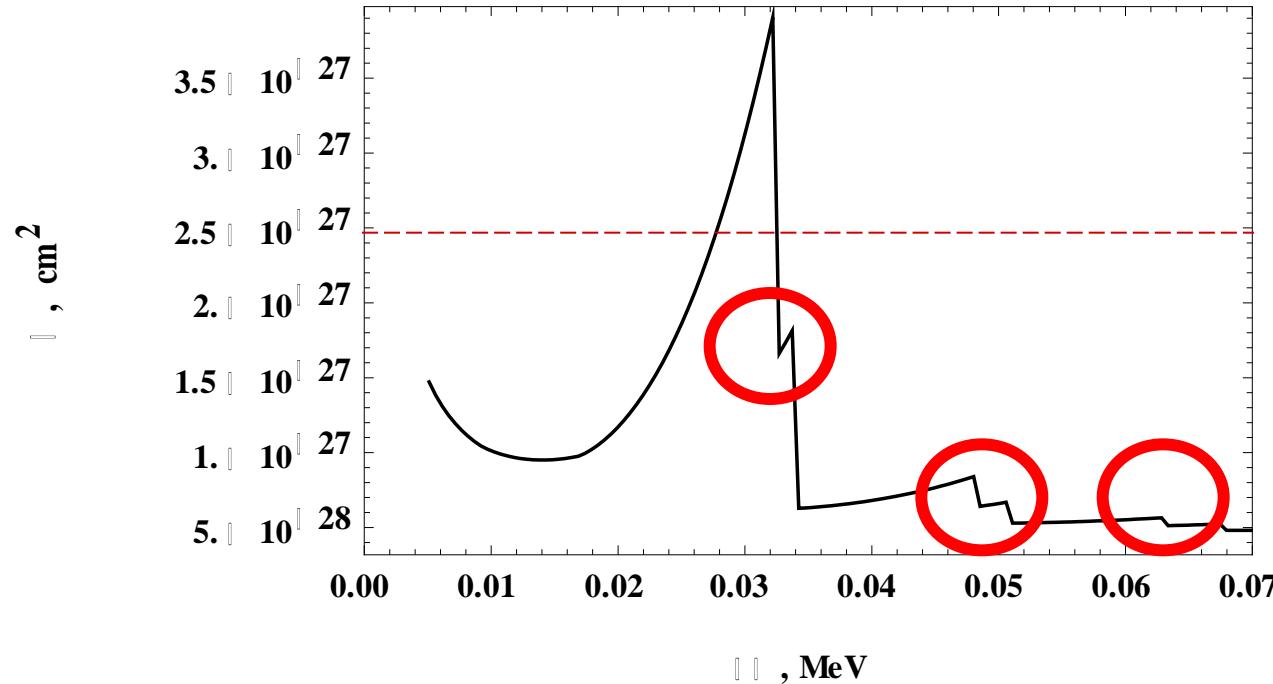
$$\gamma = 2$$

$$\frac{\sigma_{TM}(\omega_r)}{d\omega_r}$$

σ_{st} , cm^2



Results of calculation



$$\frac{\sigma_{TM}(\omega_r)}{d\omega_r} + \frac{1}{N} \sum_{fi} \frac{\sigma_{fi}^{CR}(\omega_r)}{d\omega_r}$$

The cross section of the coherent bremsstrahlung from channeled positron

Conclusion

**It is possible the coherent bremsstrahlung from planar
channeled positron**

**The contribution to cross-section of the coherent
bremsstrahlung from channeled positron result in
splitting of coherent peek and appearance of fine
structure of photon spectrum**

Thanks for attention